A non-Hermitian Hamiltonian description of the dynamic Mott transition

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Dynamic Mott transitions: Motivation

General interest in understanding nonequilibrium transitions in quantum many-body systems:

(a) Is the transition continuous? What symmetries are broken? What are the critical exponents? Do they differ from corresponding equilibrium transitions?

(b) How different are dynamic Mott transitions from noninteracting counterparts (e.g. dielectric breakdown in semiconductors)?

(c) What is the role played by dissipative processes? Is a "Hamiltonian" formulation possible?

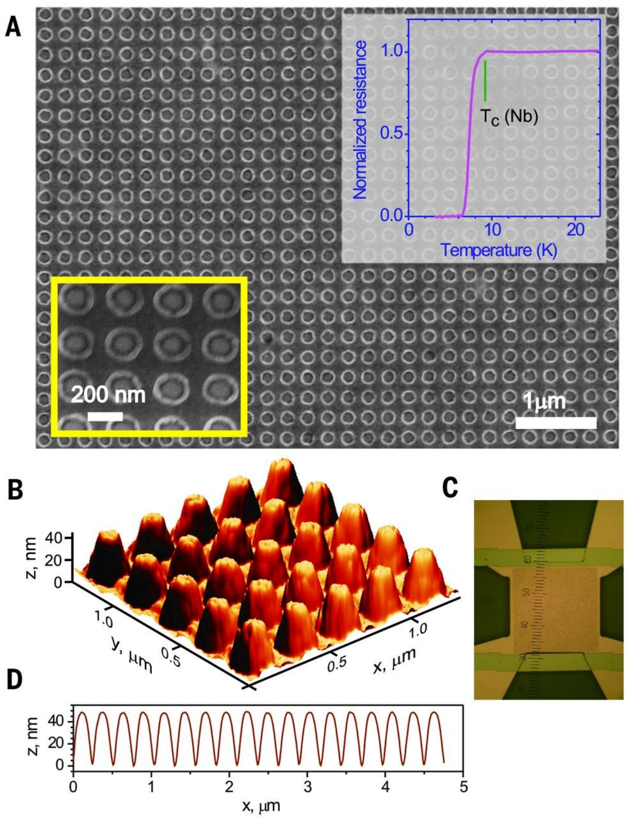
(d) How does disorder affect the nonequilibrium transition?

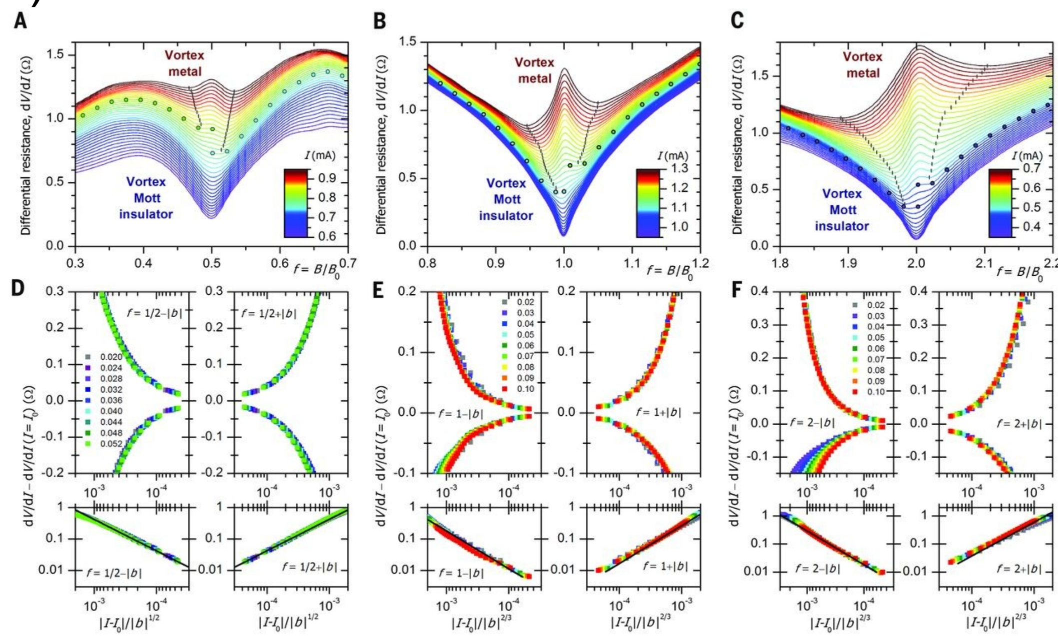
Practical: Possibility of use in switching devices

C. H. Ahn, J-M. Triscone, and J. Mannhart, Nature (2003)

A current driven vortex Mott transition

N. Poccia et al., Science (2015)





$$\frac{dV}{dI} = F\left[\frac{|I - I_0|}{|f - f_c|^{\epsilon}}\right]$$

Critical current

Frustration factor (vortex filling)

$$f = 1, 2 \Rightarrow \epsilon \approx 2/3,$$
$$f = 1/2 \Rightarrow \epsilon \approx 1/2$$

Physical mechanism

As in semiconductors, electrical conduction in gapped phase proceeds through creation of free particle-hole excitations by Landau-Zener tunnelling.
T. Oka and H. Aoki, PRB (2010); M. Eckstein, T. Oka, and P. Werner, PRL (2010); A. G. Green and S. L. Sondhi, PRL (2005)

Landau-Zener tunneling known to get enhanced in semiconductors in the presence of dissipation.
E. Shimshoni and A. Stern, PRB (1993)

Q: What happens close to the Mott transition? What is the role of dissipation?
A: Dissipation enhances Landau-Zener tunneling in Mott insulators and ultimately leads to collapse of Mott gap. In the absence of dissipation, Mott gap not renormalized.

Method

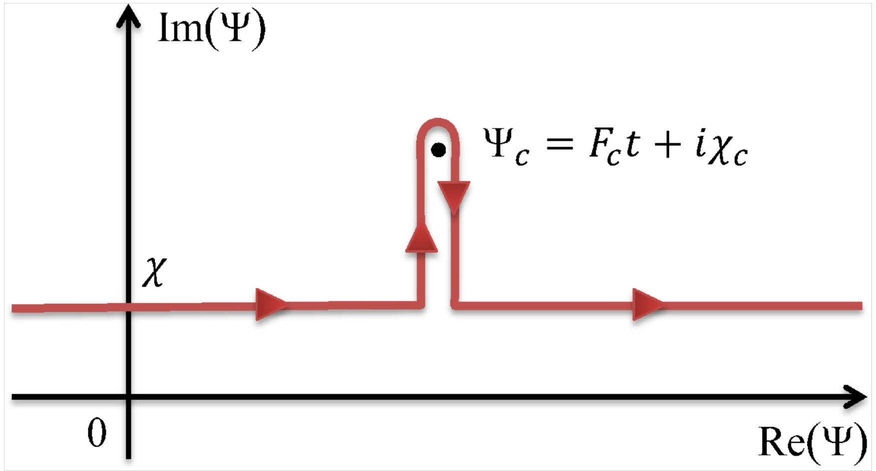
Use Landau-Dykhne formula:
A. M. Dykhne, JETP (1962)

$$P_{01}=|\langle 1|0\rangle|^2=\mathrm{e}^{-2\gamma},$$
$$\gamma=\frac{1}{\hbar}\Im\int_{-\infty}^{\infty}dt\,(E_1-E_0)[\Psi(t)],$$
$$\Psi=Ft+i\chi$$

Time-dependent phase factor related to driving field

Extension to complex plane

Finite imaginary part comes from branch points in the complex Ψ-plane where the spectral gap closes.



$$\gamma=\frac{1}{\hbar F}\Re\int_{\chi}^{\chi_c}d\chi'[E_1(\chi')-E_0(\chi')]$$

Completely determined by the imaginary part of Ψ!

Hubbard chain, no dissipation:
Oka & Aoki, PRB (2010)

$$\gamma\approx(E_1-E_0)^2/vF\equiv\Delta^2/vF,$$
$$v=|(d\Delta/dt)/F|$$

Dissipation and non-Hermiticity

Is a "Hamiltonian" description possible for nonequilibrium steady states? **Yes!**

Consider Legendre transformed Hamiltonian $H' = H - i\lambda J$

T. Antal et al., Phys. Rev. Lett. 78 (1997);

J. Cardy and P. Suranyi, Nucl. Phys. B (2000)

Model invariant under simultaneous Parity (P) & Time Reversal (T).

PT-symmetry generically arises in situation of balanced gain and loss.

C. Bender and S. Boettcher, PRL (1998).

Real λ eqv. to imaginary vector potential.

For small λ, spectral gap in $H \Rightarrow \langle J \rangle = 0$ (Real eigenvalues for H')

For large λ, eigenstates those of J $\Rightarrow \langle J \rangle \neq 0$ (Complex eigenvalues)

Dissipation and non-Hermiticity

Intuitively construct a density matrix for the non-Hermitian model:

$$\rho(t) = \frac{e^{-iH't}\rho(0)e^{iH't}}{tr[e^{-iH't}\rho(0)e^{iH't}]}, \quad \langle A \rangle \equiv tr(\rho A)$$

Finite real part of λ necessary for relaxation to a nonequilibrium steady state:

$$\frac{d\langle J \rangle}{dt} = -\lambda(\langle J^2 \rangle - \langle J \rangle^2)$$

Model then does describe a nonequilibrium (dissipative) metal-insulator transition. λ characterizes both dissipation & drive.

Below some critical field F_c (corresponding to a critical λ_c) the spectral gap is finite and <J> =0, eigenvalues of *H'* are real.
At larger fields, a finite current flows, eigenvalues are complex.
Eigenstates break PT symmetry.

Nonequilibrium phase transition brought about by tuning λ mirrors the electric field driven dynamic Mott transition!

Fermionic dynamic Mott transition

Fermionic Hubbard chain at half filling subjected to vector potential $\Psi = F\,t + i\,\chi$:

$$H = -t \sum_{\langle ij \rangle, \sigma} \left[e^{i\Psi(t)} c_{i\sigma}^{+} c_{j\sigma} + e^{-i\Psi(t)} c_{j\sigma}^{+} c_{i\sigma} \right] + \sum_{i} n_{i\uparrow} n_{i\downarrow}$$

Imaginary component of vector potential arises in an effective description of hopping in the presence of energy relaxation processes (bath).

Physical meaning: Hopping amplitudes along and opposite the driving field have different magnitudes.

Examples:
(1) Transport in disordered semiconductors in strong electric field
(B. Shklovskii, Sol. St. Comm. (1981))
(2) Mott transition in Vortex systems in tilted magnetic fields
(Lehrer & Nelson, PRB (1998))

Fermionic dynamic Mott transition

Fermionic Hubbard chain at half filling subjected to vector potential Ψ = *F t* + *i* χ:

$$H = -t \sum_{\langle ij \rangle, \sigma} [e^{i\Psi(t)} c^+_{i\sigma} c_{j\sigma} + e^{-i\Psi(t)} c^+_{j\sigma} c_{i\sigma}] + \sum_i n_{i\uparrow} n_{i\downarrow}$$

Solvable by Bethe ansatz even for complex Ψ!

Oka and Aoki, PRB (2010)

$$L k_j = 2\pi I_j + \Psi(t) - \sum_{\alpha=1}^{N_\downarrow} \theta(\sin(k_j) - \lambda_\alpha),$$

$$\sum_{j=1}^{N} \theta(\sin(k_j) - \lambda_\alpha) = 2\pi J_\alpha - \sum_{\beta=1}^{N_\downarrow} \theta\left(\frac{\lambda_\alpha - \lambda_\beta}{2}\right)$$

$$I_j = N_\downarrow/2 \,(mod\, 1),$$
$$J_\alpha = (N - N_\downarrow + 1)/2 \,(mod\, 1),$$
$$\theta(x) = -2\arctan(x/u),$$
$$u = U/4t$$

Solutions are in general complex.

Fermionic dynamic Mott transition

Real part of Ψ linearly increases the crystal momenta with time, and is responsible for Bloch oscillations.

Landau-Zener tunneling takes place at some $k = \pm\pi + ib$ since the bandgap is smallest at the Brillouin zone boundaries.

$$\Delta(b) = 4t\left[u - \cosh(b) + \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{J_1(\omega)\, e^{\omega \sinh(b)}}{1 + e^{2u|\omega|}}\right],$$

$$\Delta(b_c) \equiv 0,$$

$$b_c = \sinh^{-1}(u)$$

Lieb & Wu, PRL (1968),
Fukui & Kawakami, PRB (1998)

By increasing the imaginary part of the vector potential, the Mott gap gets ultimately closed & Landau-Zener factor collapses:

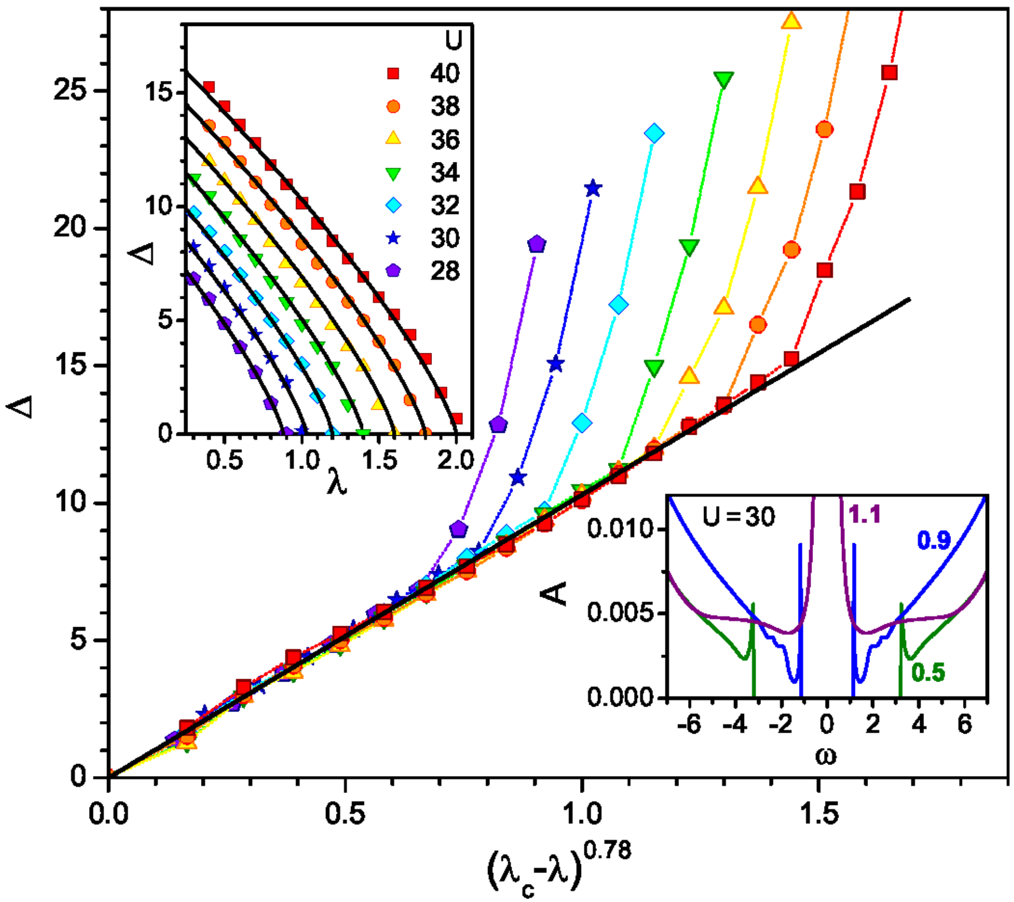
$$\Delta(\chi) \approx C_1 \sqrt{\chi_c - \chi}$$

$$\gamma = C(\chi_c - \chi)^{3/2}$$

Same dependence on electric field follows.

Fermions: D > 1

Bethe ansatz not possible in higher dimensions. We use DMFT with Iterative Perturbation Theory (IPT) based impurity solver:



See larger critical exponent for gap scaling in 2D and 3D.

$$\Delta(\lambda) \sim (\lambda_c - \lambda)^{0.78}, \quad D=2,$$
$$\Delta(\lambda) \sim (\lambda_c - \lambda)^{0.86}, \quad D=3$$

3D result lies close to exponent =1 predicted in mean-field treatments of equilibrium Hubbard models.

Florens and Georges, PRB (2004)
Zhang and Rice, PRB (1970)

Dynamic Mott transition for dissipative Bosons

Consider problem of Mott transition of field-induced vortices in superconducting 2D proximity arrays (square lattice).

External current acts like an electric field on vortex “charges”. For dissipative (overdamped case), we get PT-symmetric Schrödinger equation [also see J. Rubinstein et al. PRL (2007); N. M. Chtchelkachev, PRL (2012)]

$$\frac{\partial \psi}{\partial t} = -D\nabla^2 \psi + i(I/\rho) x \psi + m^2 \psi + 2u|\psi|^2 \psi$$

BCs: Vortices confined in normal regions of size *L*. In the surrounding superconducting phase, ψ=0. Confinement leads to spectral gap. Examine evolution of gap with current:

$$E_1 - E_0 \approx 2 E_T \sqrt{\eta \left(1 - I^2/I_c^2\right)},$$
$$E_T = D/L^2, \quad \eta = \left(\pi^2/\sqrt{2}\left(I_c L/E_T \rho\right)\right)$$

$$\gamma \sim \left(I_c - I\right)^{3/2}$$

Agrees with N. Poccia et al., Science (2015)

Summary and Outlook

- We studied dynamic Mott transition in a dissipative fermionic half-filled Hubbard model and a bosonic vortex system.
- Nonequilibrium steady states were identified as stationary states of certain non-Hermitian Hamiltonians endowed with PT-symmetry.
- Field-driven Mott transition in these systems is a PT-symmetry breaking transition. Key difference from non-dissipative case is renormalization of Landau-Zener factor which can be large even if the driving field is applied deep within the Mott phase.
- Critical exponent for Landau-Zener factor agrees with experiments on vortex Mott transition in superconducting proximity arrays.

- We need a microscopic derivation of fermionic non-Hermitian models beginning with a Hermitian system coupled to a bath.
- We would like to understand the dynamic Mott transition in the presence of disorder.