

Quantum matter without quasiparticles: graphene, random fermion models, and black holes

Discussion Meeting on Current Frontiers in Condensed Matter Research
International Center for Theoretical Sciences, Bengaluru
June 27, 2016

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Quantum matter without quasiparticles:

1. *Ground states disconnected from independent electron states: many-particle entanglement*
2. *No quasiparticles*

- Quantum criticality near the superfluid-insulator transition of ultracold bosonic atoms in an optical lattice
- Graphene
- Solvable random fermion Sachdev-Ye-Kitaev (SYK) model
- Charged black hole horizons in anti-de Sitter space

Note: Most states with long-range entanglement, like the fractional quantum Hall states, do have quasiparticles

Local thermal equilibration or phase coherence time, τ_φ :

- As $T \rightarrow 0$, there is an *lower bound* on τ_φ in all many-body quantum systems of order $\hbar/(k_B T)$,

$$\tau_\varphi > C \frac{\hbar}{k_B T},$$

and the lower bound is realized by systems *without* quasiparticles.

- In systems *with* quasiparticles, τ_φ is parametrically larger at low T ;
e.g. in Fermi liquids $\tau_\varphi \sim 1/T^2$,
and in gapped insulators $\tau_\varphi \sim e^{\Delta/(k_B T)}$ where Δ is the energy gap.

A bound on quantum chaos:

- The time over which a many-body quantum system becomes chaotic is given by $\tau_L = 1/\lambda_L$, where λ_L is the “Lyapunov exponent” determining memory of initial conditions (the “butterfly effect”):

$$D(t) = \langle W(t)V(0)W(t)V(0) \rangle \sim c_0 - \epsilon c_1 e^{\lambda_L t},$$

where we make a (system-dependent) choice to arrange $\epsilon \ll 1$. As $T \rightarrow 0$, this Lyapunov time is argued to obey the lower bound

$$\tau_L \geq \frac{1}{2\pi} \frac{\hbar}{k_B T}$$

- Theories holographically dual to Einstein gravity have the shortest possible $\tau_L = \hbar/(2\pi k_B T)$

A.I. Larkin and Y. N. Ovchinnikov, JETP **28**, 6 (1969)

S. H. Shenker and D. Stanford, arXiv:1306.0622

J. Maldacena, S. H. Shenker and D. Stanford, arXiv:1503.01409

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Quantum matter without quasiparticles
 \approx fastest possible many-body quantum chaos

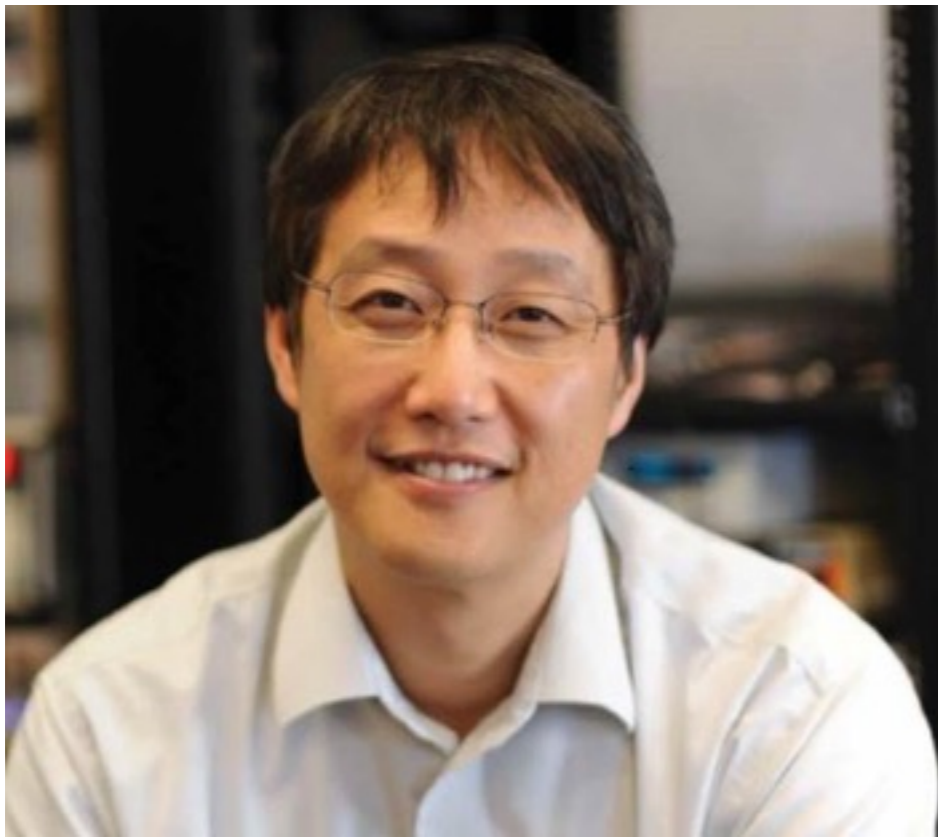
Quantum matter without quasiparticles:

- Graphene

- Solvable random fermion

Sachdev-Ye-Kitaev (SYK) model

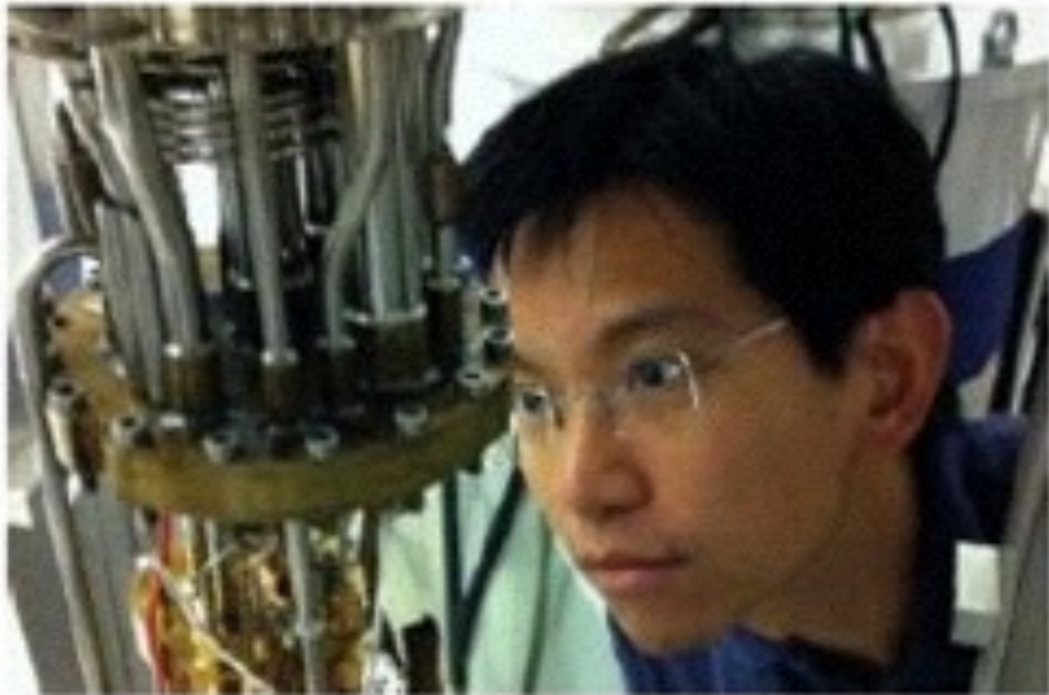
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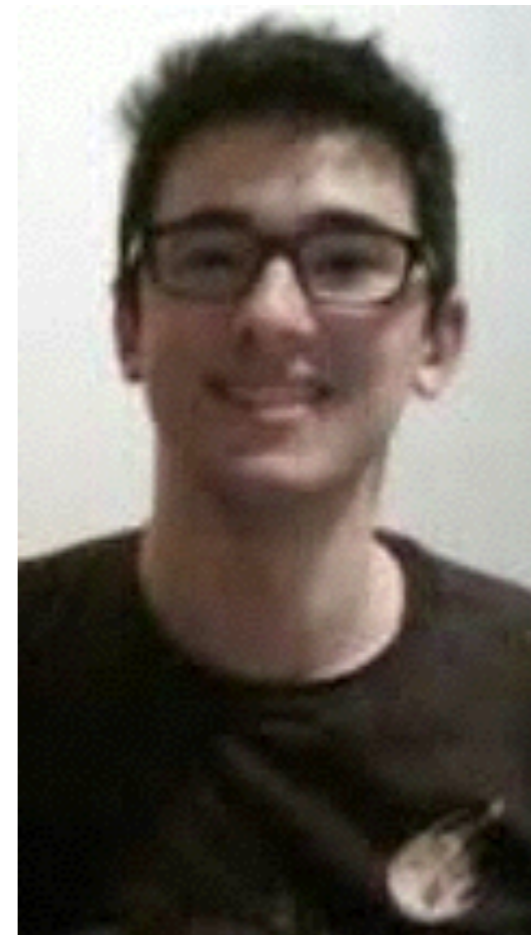
Philip Kim



Jesse Crossno

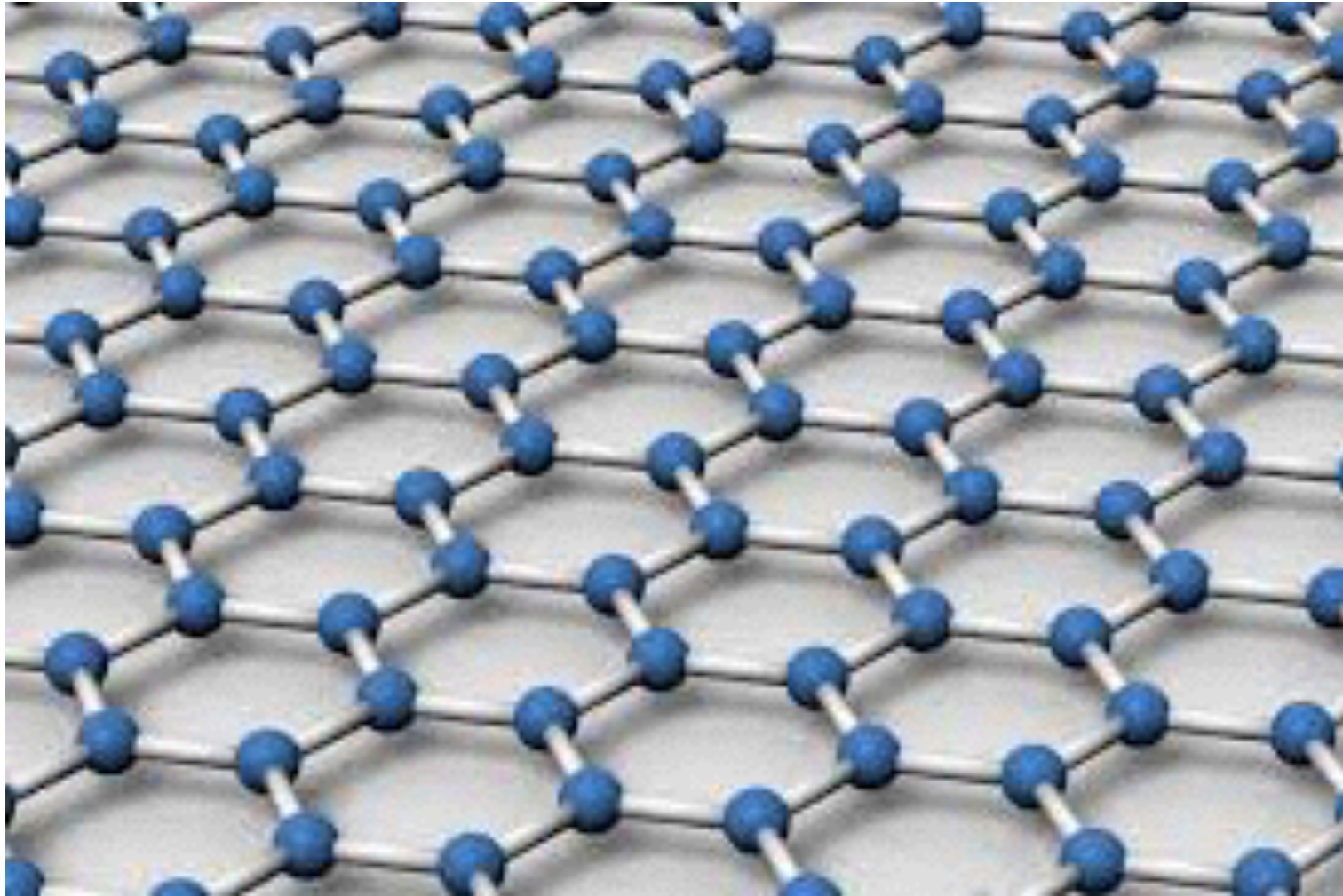


Kin Chung Fong

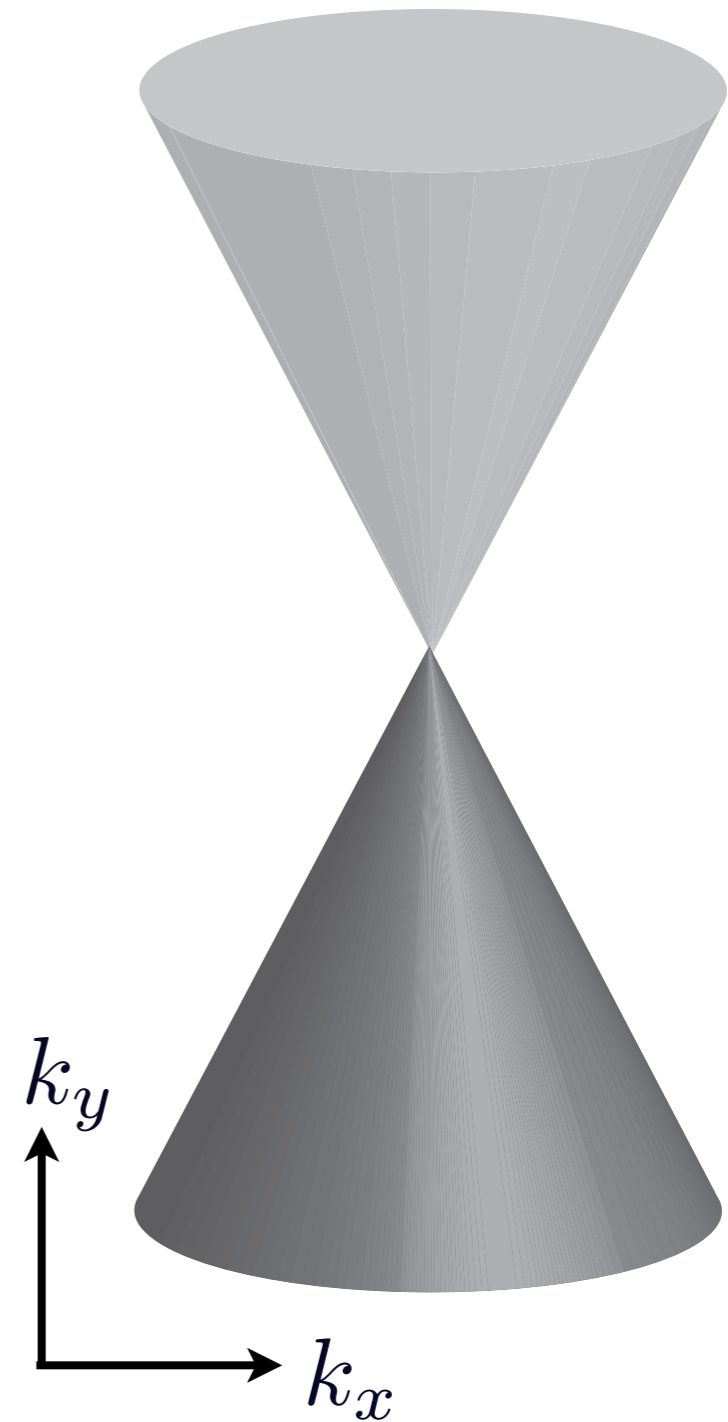


Andrew Lucas

Graphene

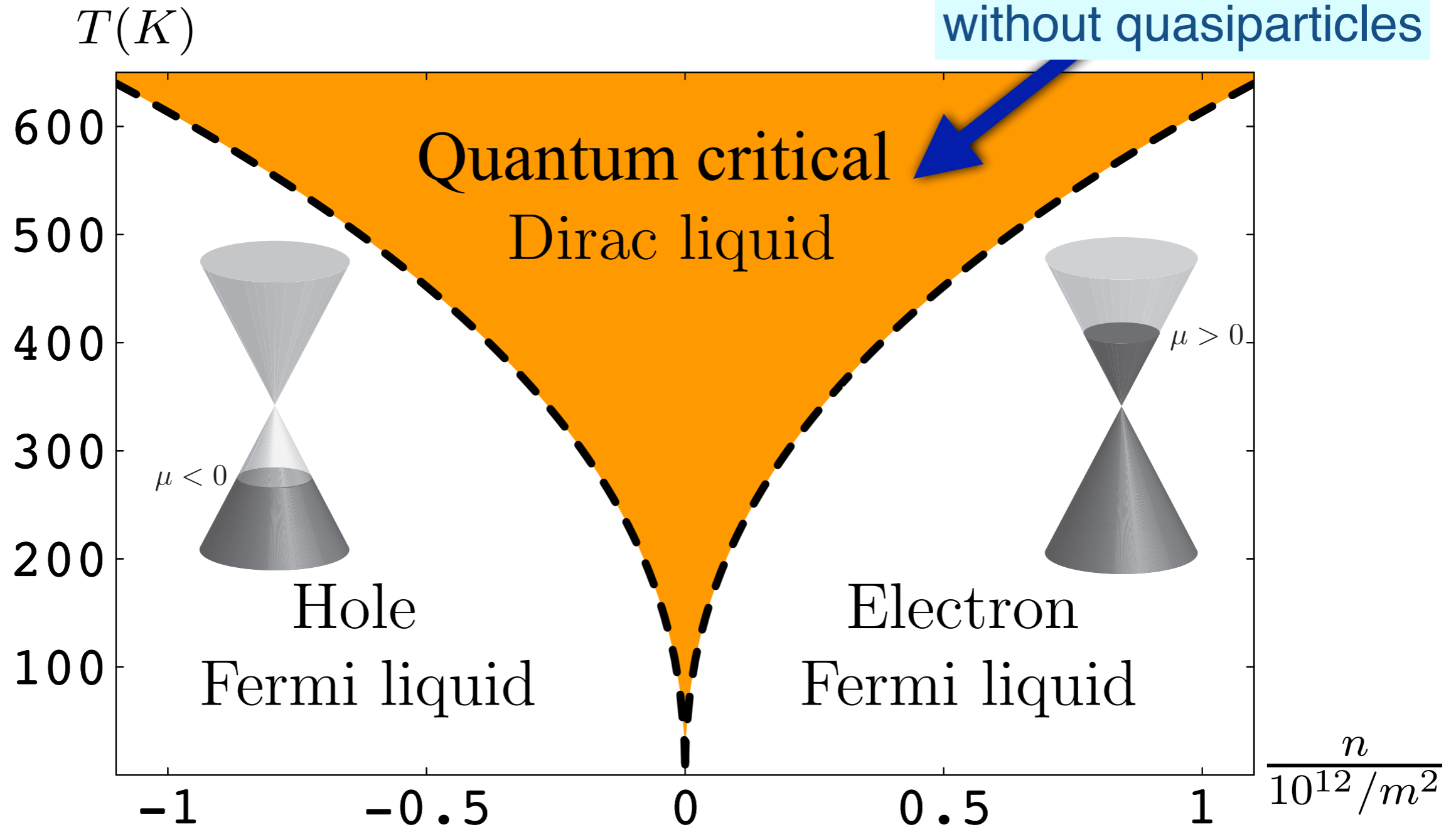


Same “Hubbard” model as for ultracold atoms, but for electrons on the honeycomb lattice



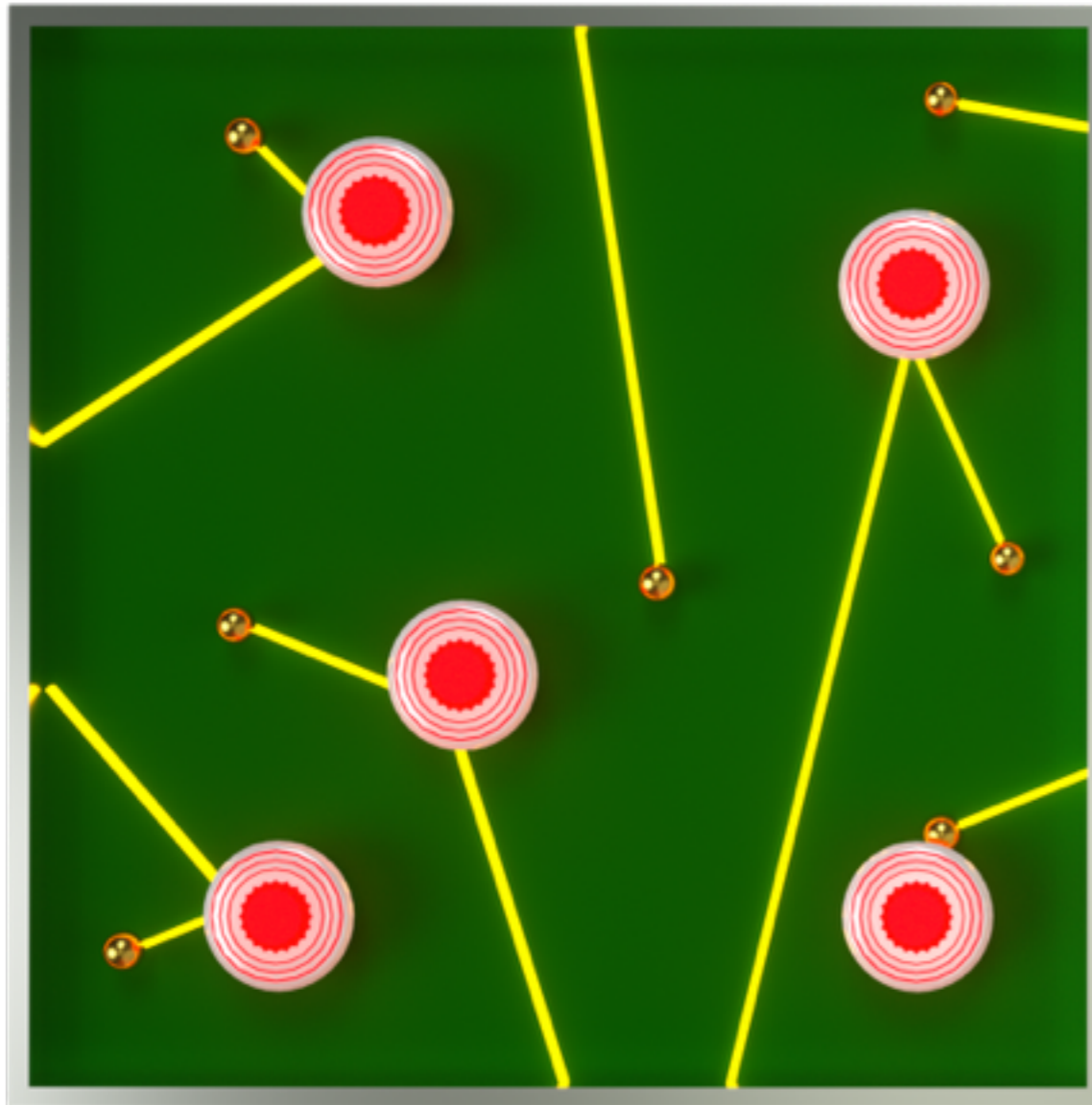
Graphene

Predicted
“strange metal”
without quasiparticles

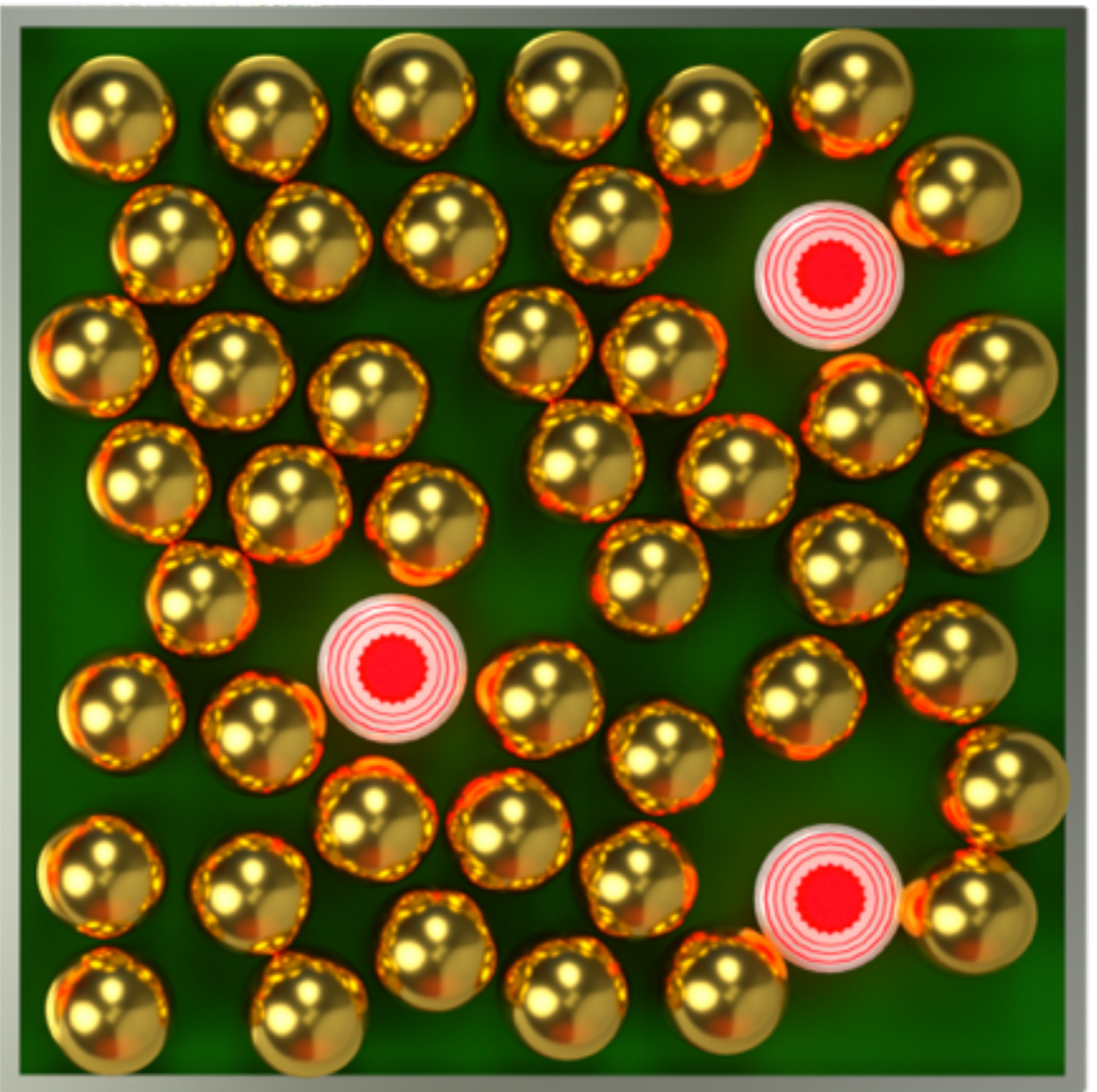


M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)

M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

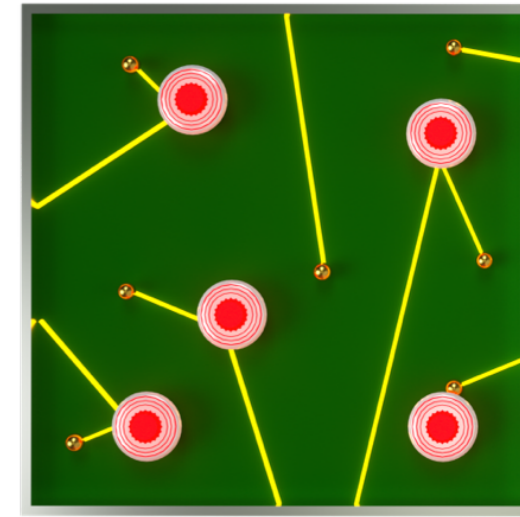


Fermi liquids: quasiparticles moving ballistically between impurity (red circles) scattering events



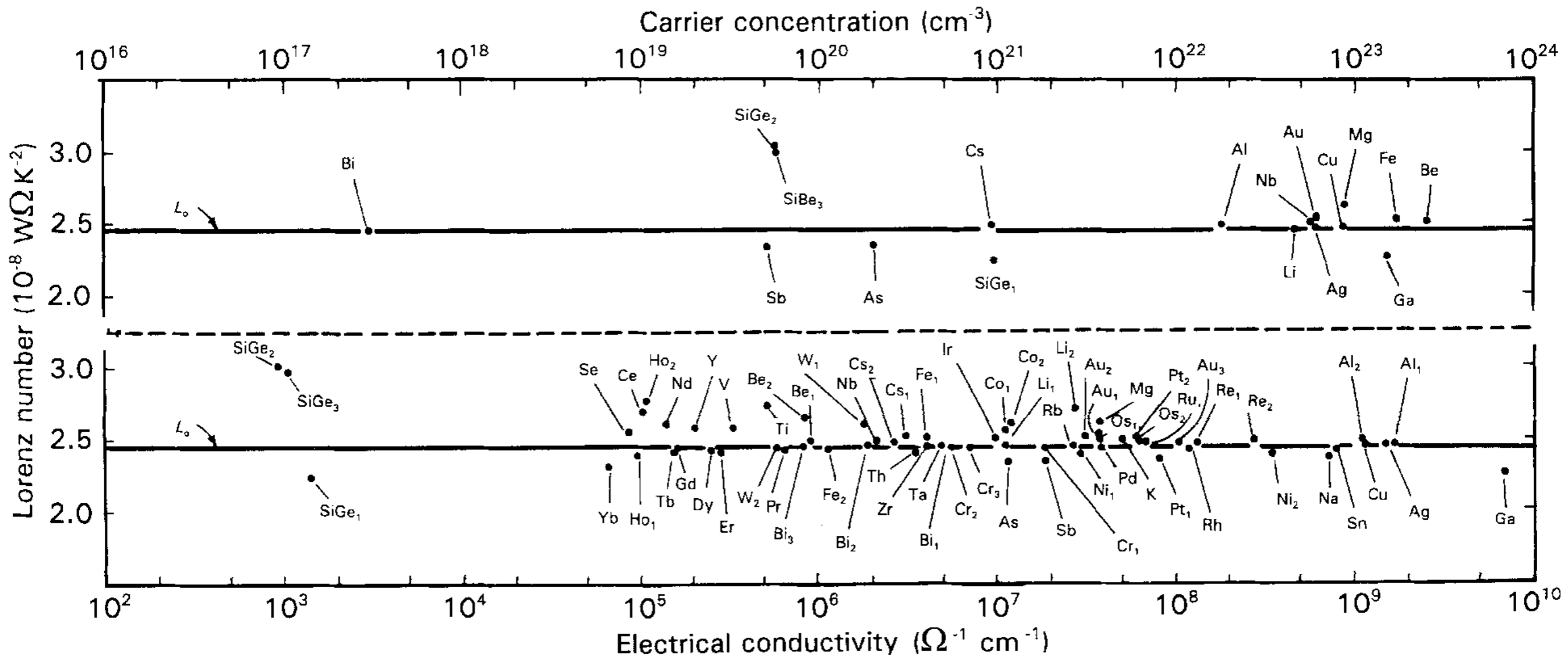
Strange metals: electrons scatter frequently off each other, so there is no regime of ballistic quasiparticle motion. The electron “liquid” then “flows” around impurities

Thermal and electrical conductivity with quasiparticles

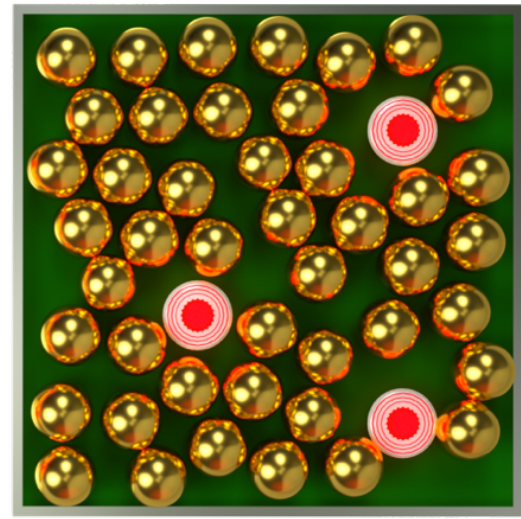


- Wiedemann-Franz law in a Fermi liquid:

$$L_0 = \frac{\kappa}{\sigma T} \approx \frac{\pi^2 k_B^2}{3e^2} \approx 2.45 \times 10^{-8} \frac{\text{W} \cdot \Omega}{\text{K}^2}.$$



Transport in Strange Metals



For a strange metal
with a “relativistic” Hamiltonian,
hydrodynamic, holographic,
and memory function methods yield

$$\text{Lorentz ratio } L = \kappa / (T\sigma) \\ = \frac{v_F^2 \mathcal{H} \tau_{\text{imp}}}{T^2 \sigma_Q} \frac{1}{\left(1 + e^2 v_F^2 Q^2 \tau_{\text{imp}} / (\mathcal{H} \sigma_Q)\right)^2}$$

$Q \rightarrow$ electron density; $\mathcal{H} \rightarrow$ enthalpy density

$\sigma_Q \rightarrow$ quantum critical conductivity

$\tau_{\text{imp}} \rightarrow$ momentum relaxation time from impurities.

Note that for a clean system ($\tau_{\text{imp}} \rightarrow \infty$ first),

the Lorentz ratio diverges $L \sim 1/Q^4$,

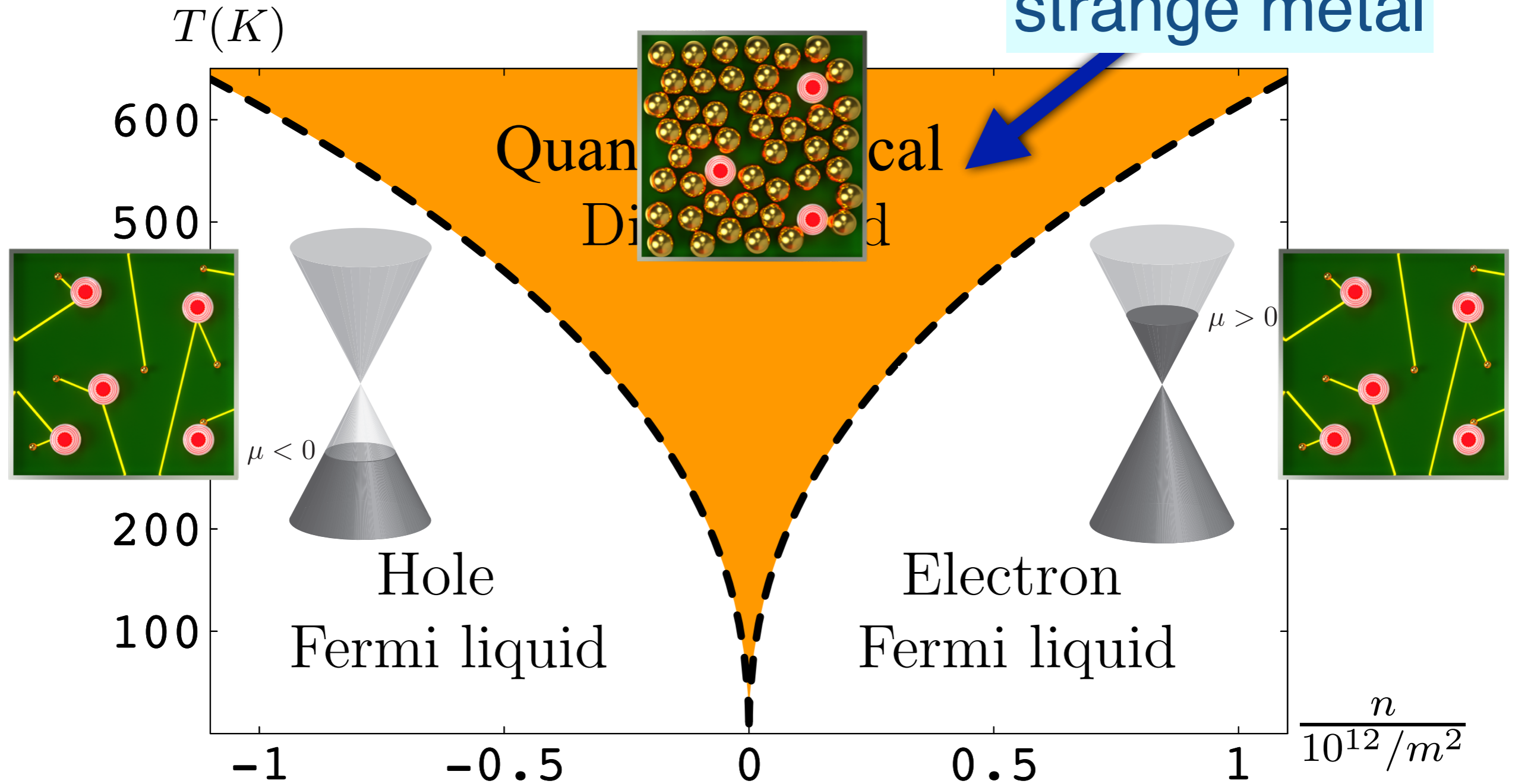
as we approach “zero” electron density at the Dirac point.

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)

M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

Graphene

Predicted
strange metal

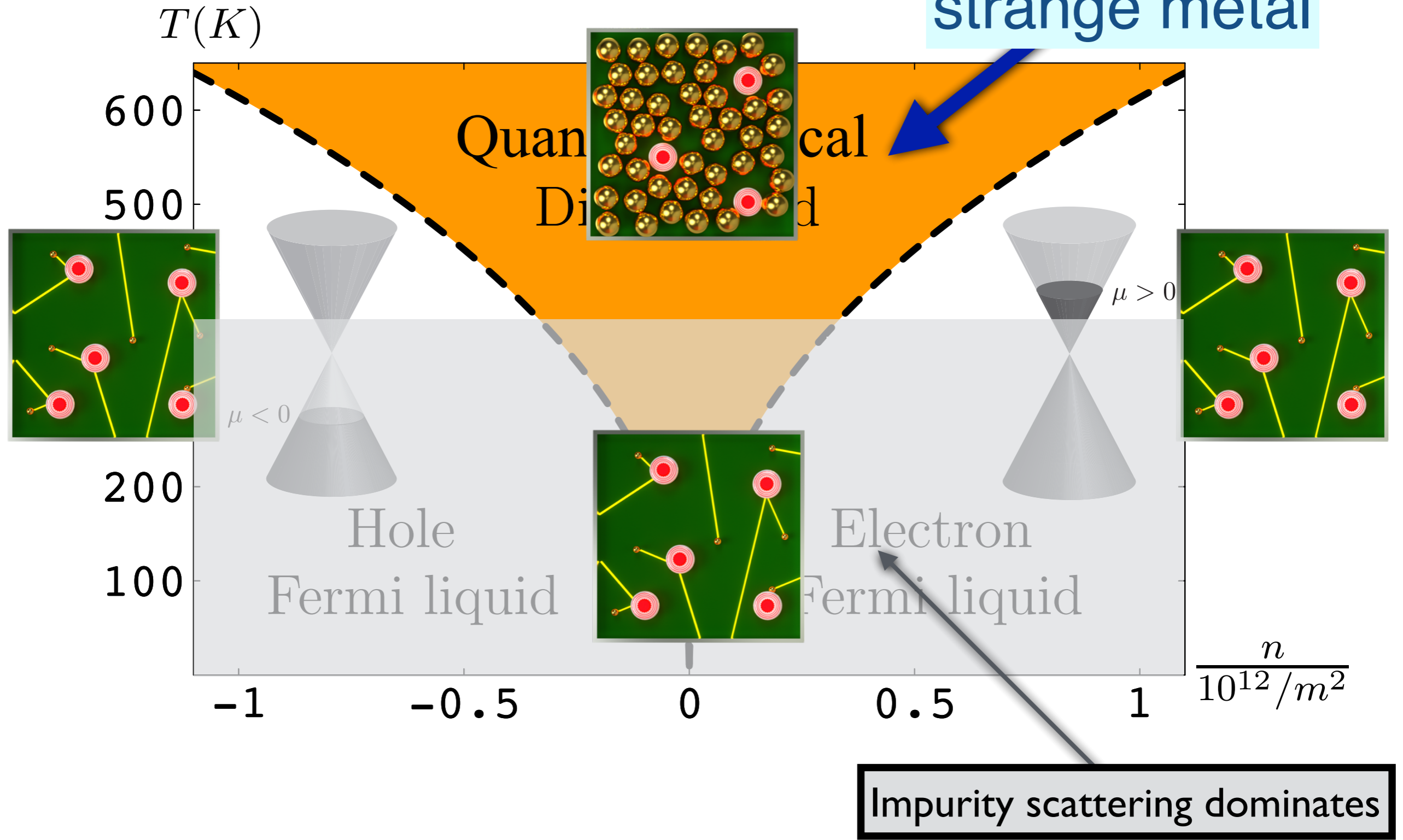


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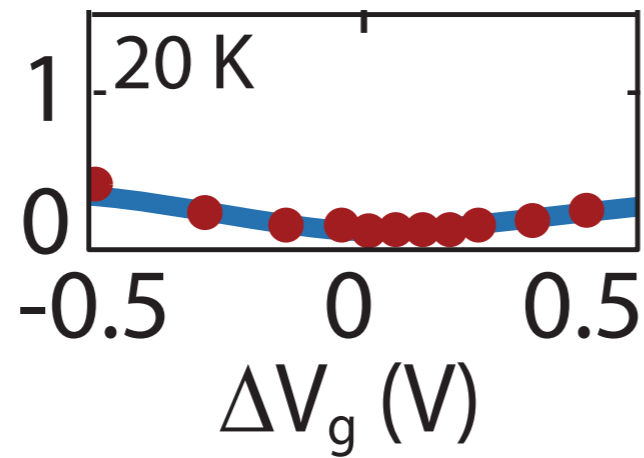
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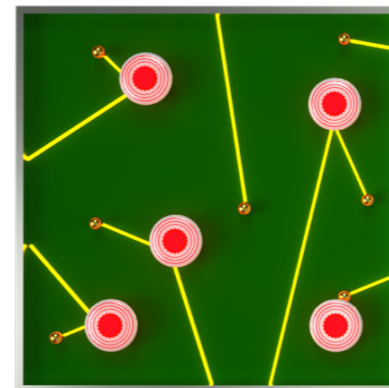
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

Thermal Conductivity (nW/K)

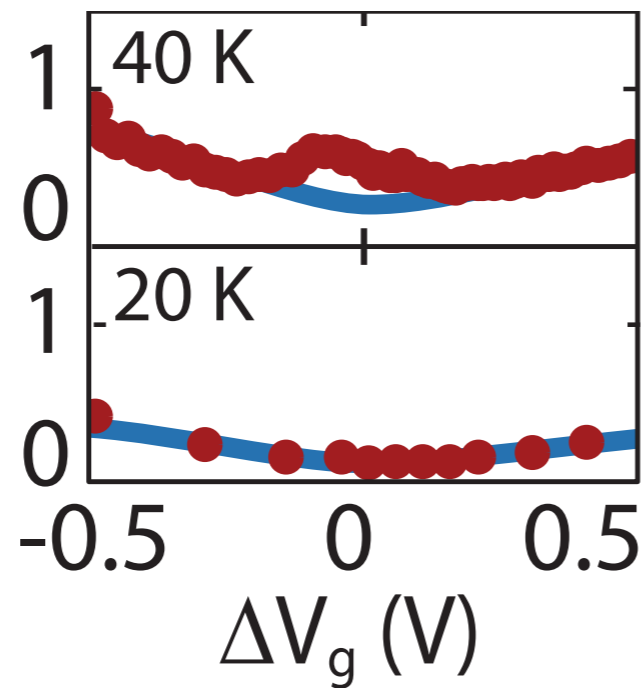


Red dots: data

Blue line: value for $L = L_0$

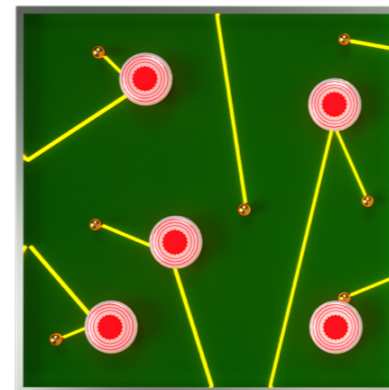


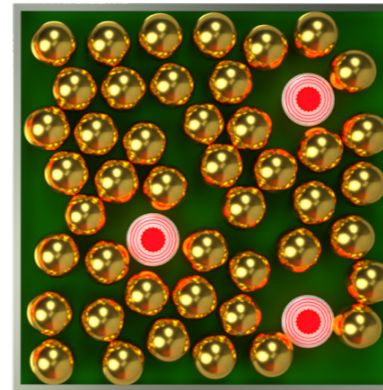
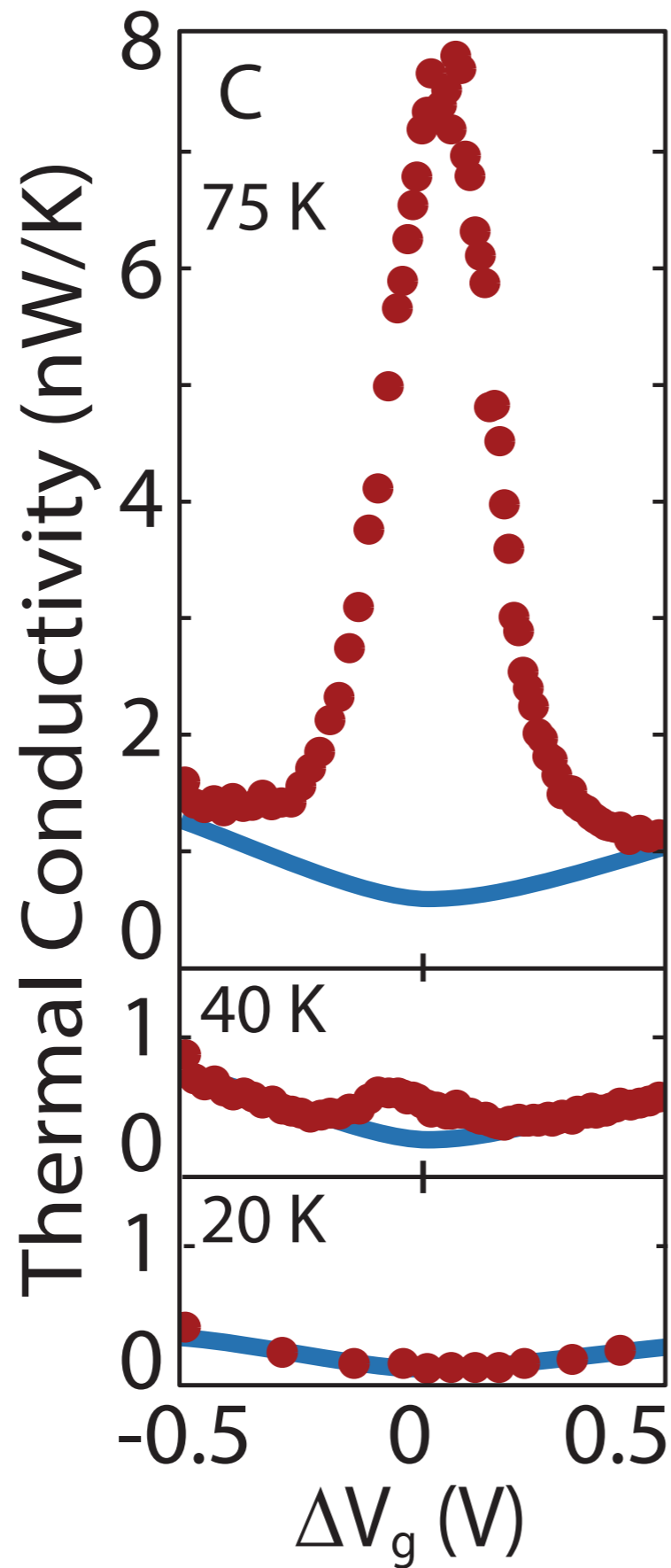
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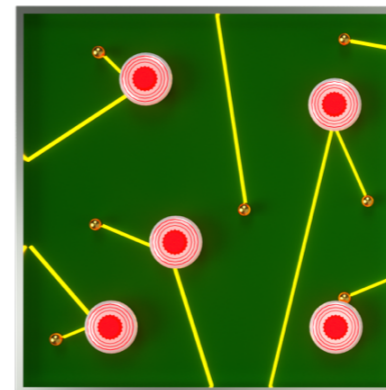
Red dots: data

Blue line: value for $L = L_0$



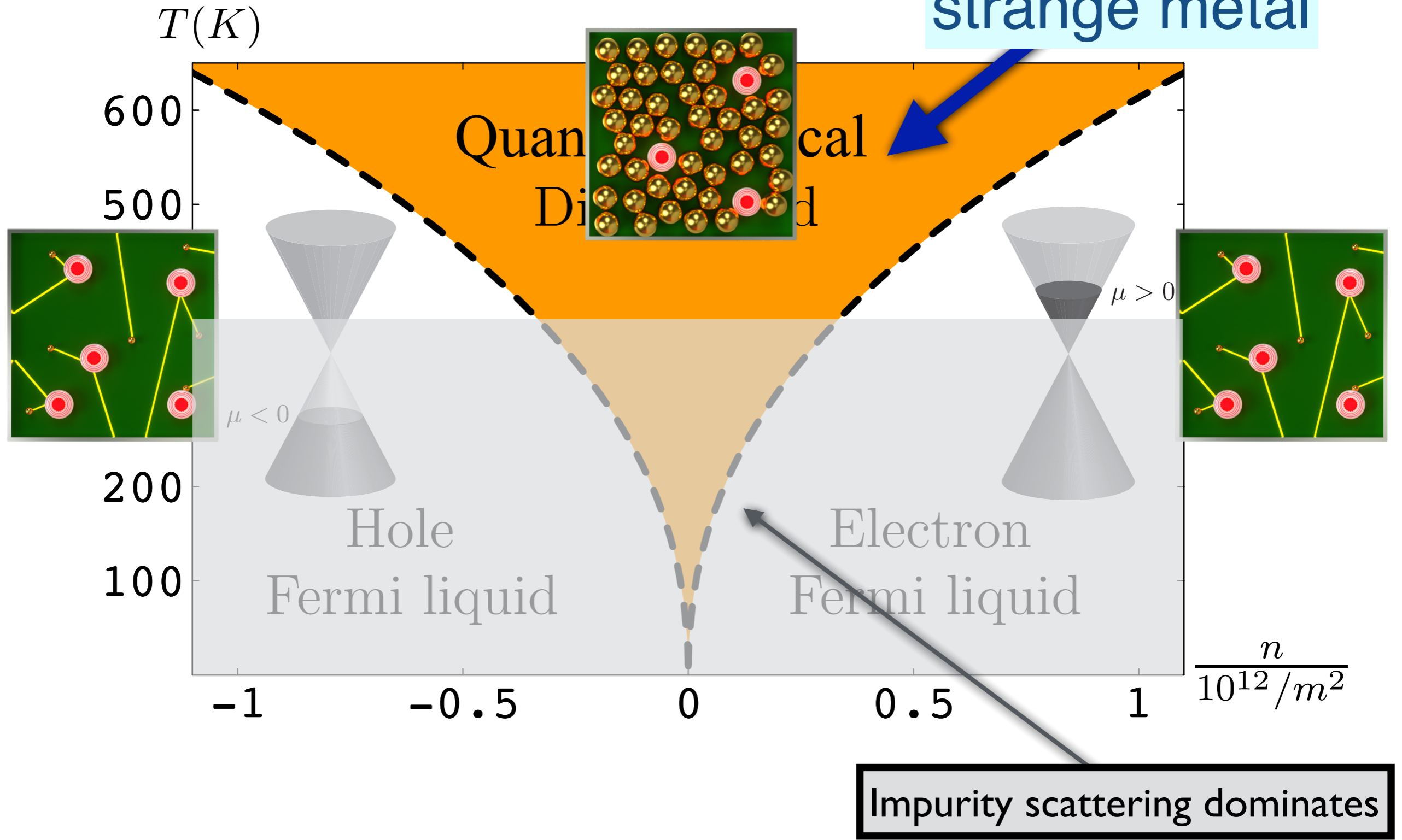


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Graphene

Predicted
strange metal

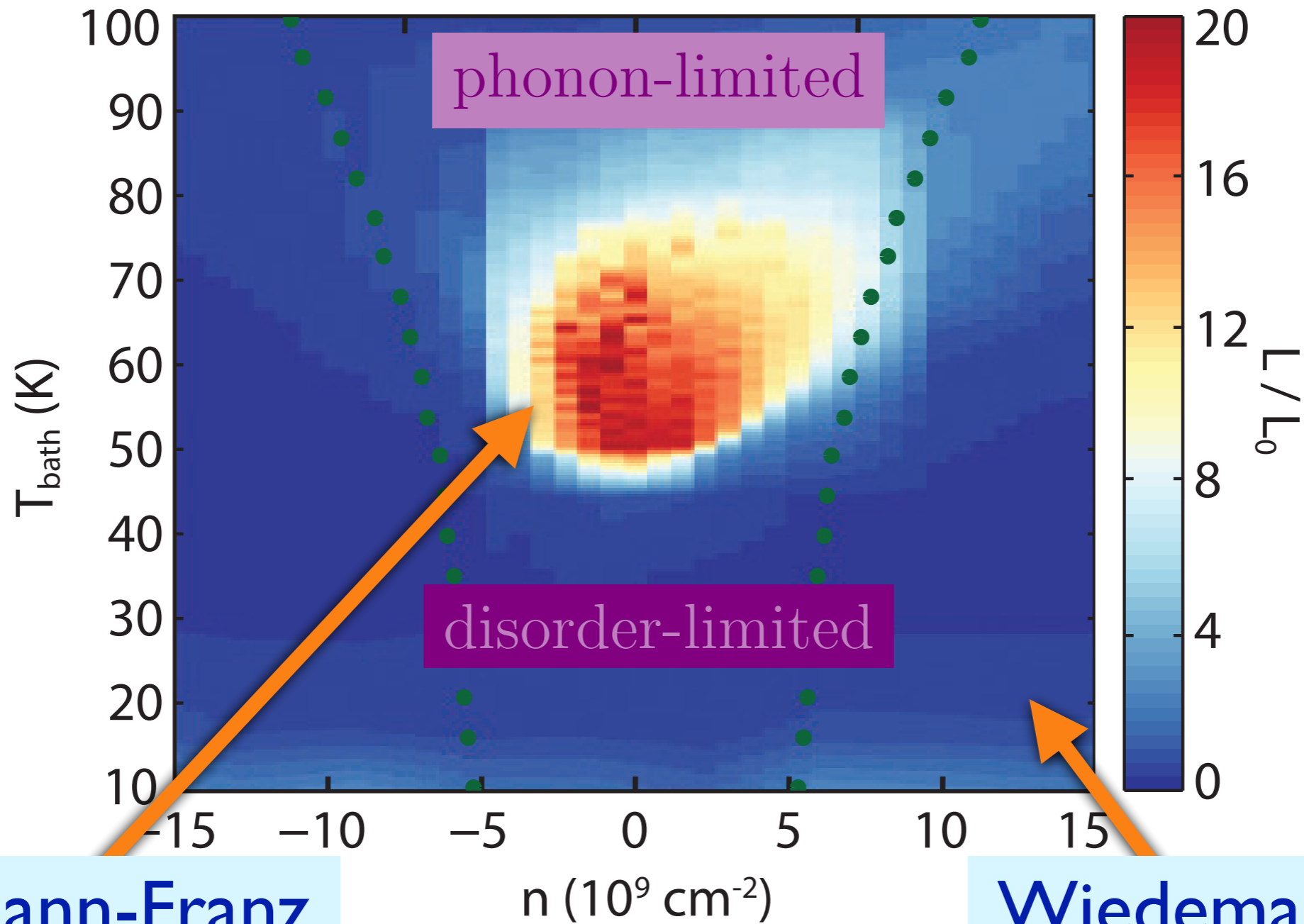
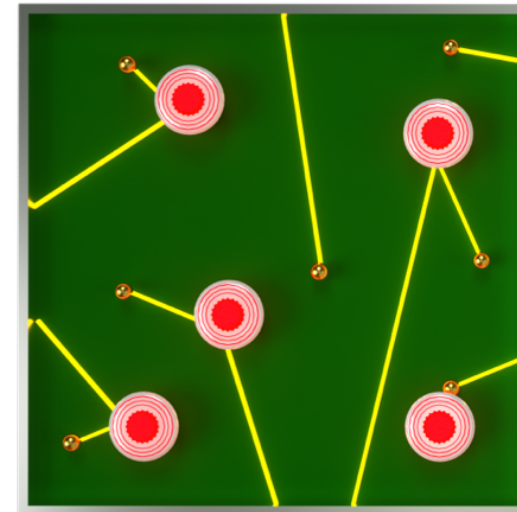


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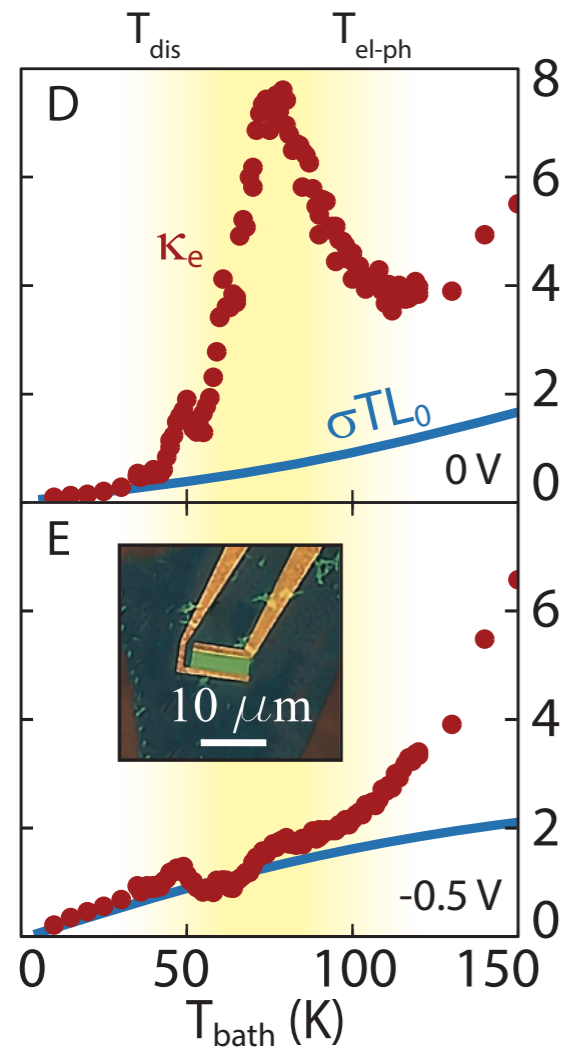
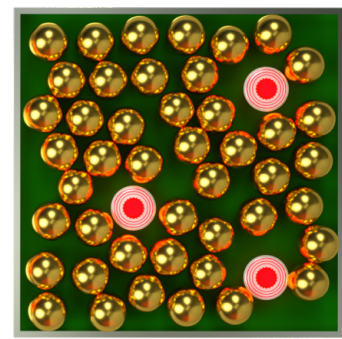
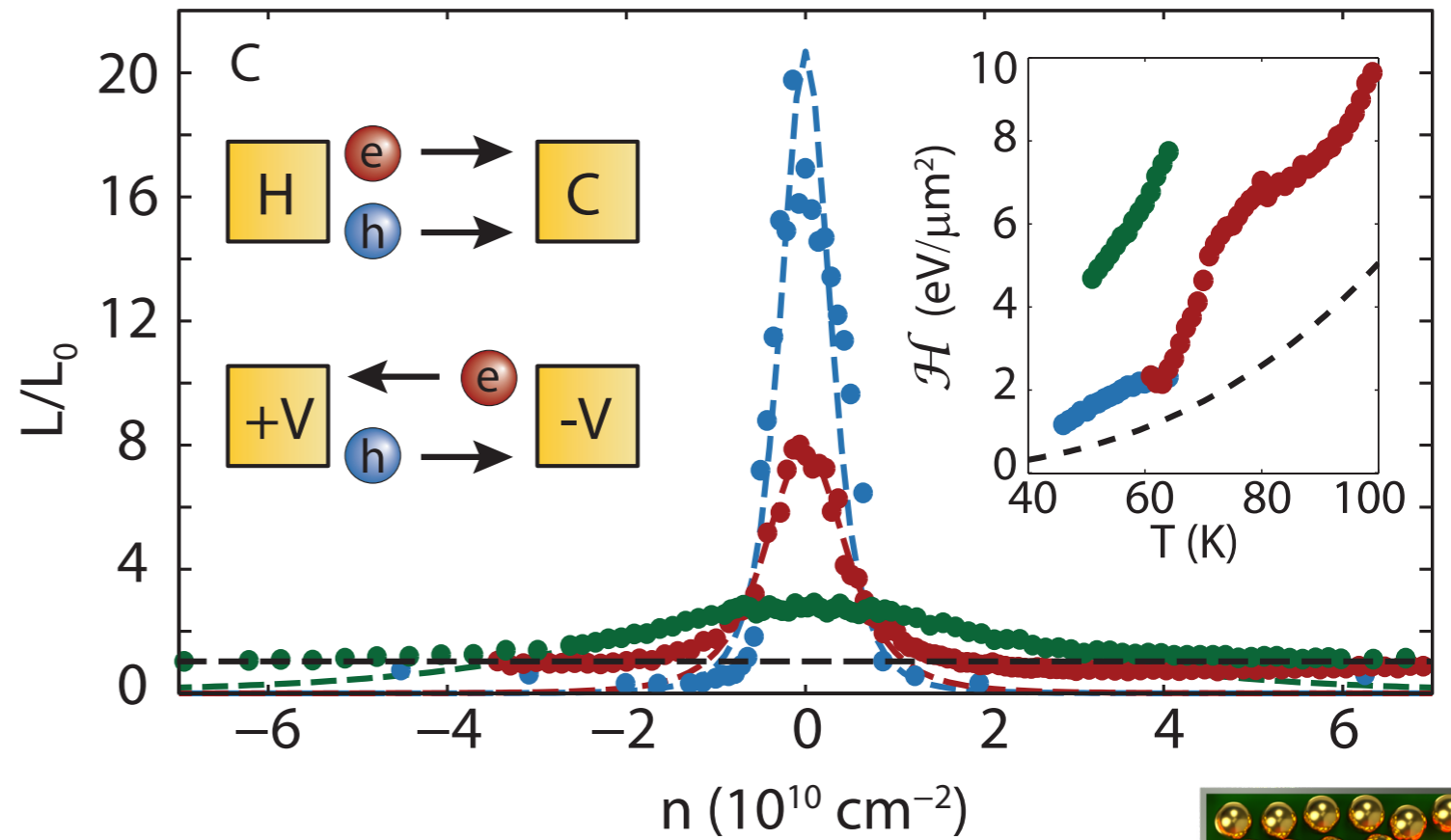
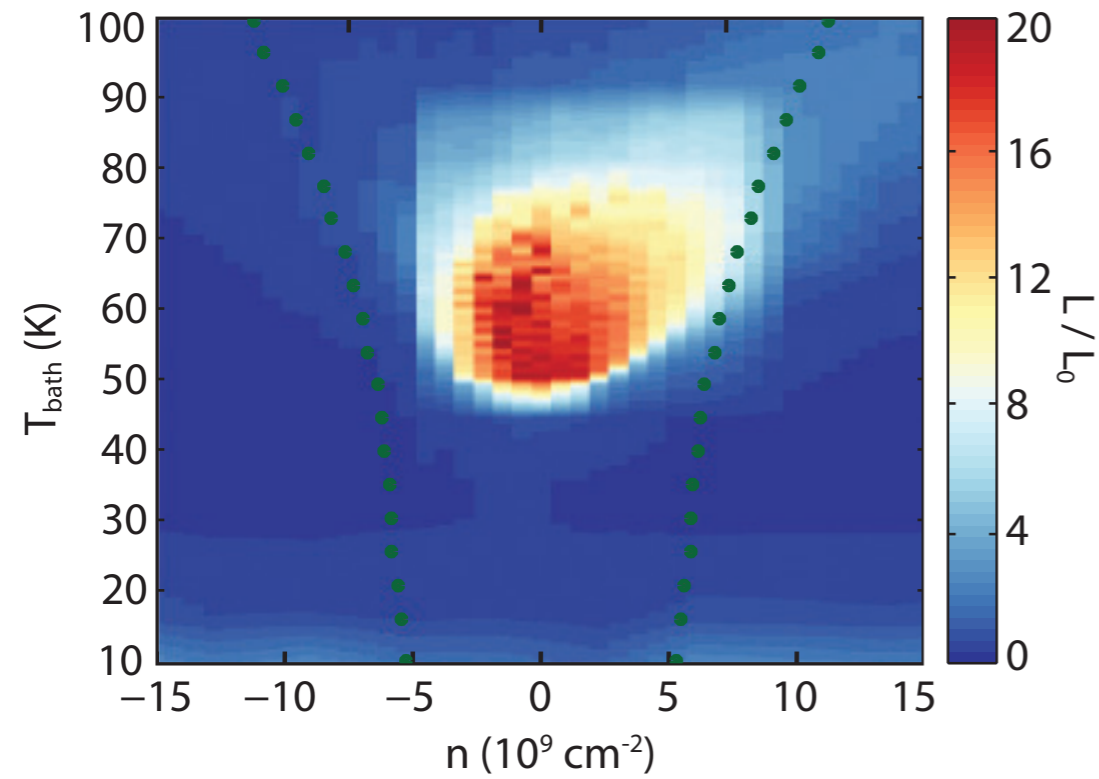
J. Crossno et al., Science **351**, 1058 (2016)

Strange metal in graphene



Wiedemann-Franz
violated !

Wiedemann-Franz
obeyed



Lorentz ratio $L = \kappa / (T\sigma)$

$$= \frac{v_F^2 \mathcal{H} \tau_{\text{imp}}}{T^2 \sigma_Q} \frac{1}{(1 + e^2 v_F^2 Q^2 \tau_{\text{imp}} / (\mathcal{H} \sigma_Q))^2}$$

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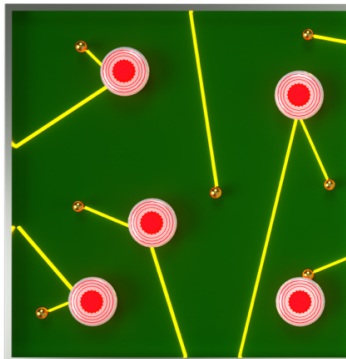
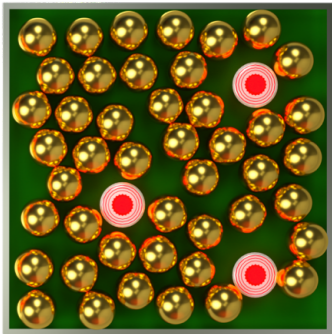
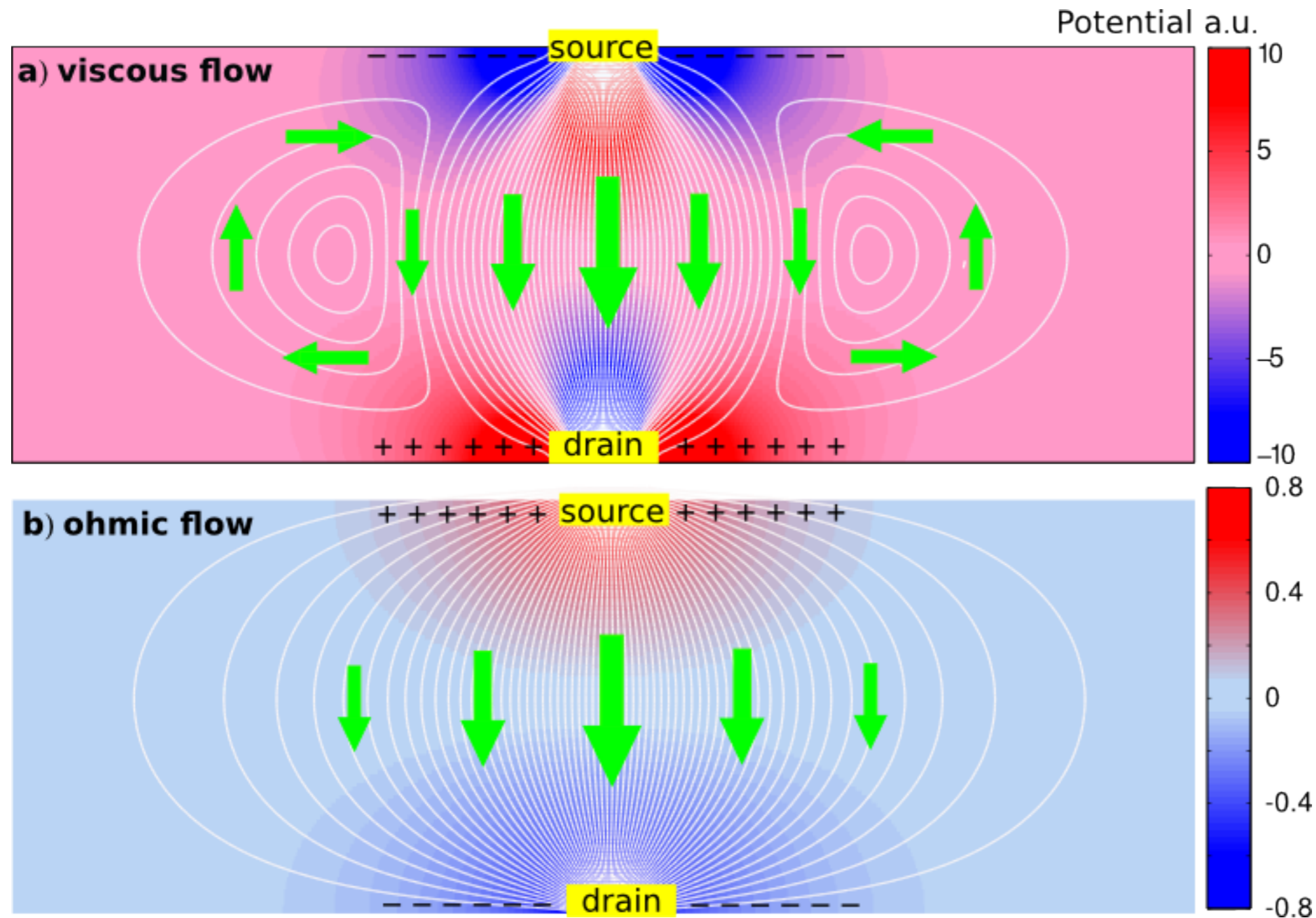
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J. Crossno et al., Science **351**, 1058 (2016)

Strange metal in graphene

Negative local resistance due to viscous electron backflow in graphene



L. Levitov and G. Falkovich, arXiv:1508.00836, *Nature Physics online*

Strange metal in graphene

Science 351, 1055 (2016)

Negative local resistance due to viscous electron backflow in graphene

D. A. Bandurin¹, I. Torre^{2,3}, R. Krishna Kumar^{1,4}, M. Ben Shalom^{1,5}, A. Tomadin⁶, A. Principi⁷, G. H. Auton⁵, E. Khestanova^{1,5}, K. S. Novoselov⁵, I. V. Grigorieva¹, L. A. Ponomarenko^{1,4}, A. K. Geim¹, M. Polini^{3,6}

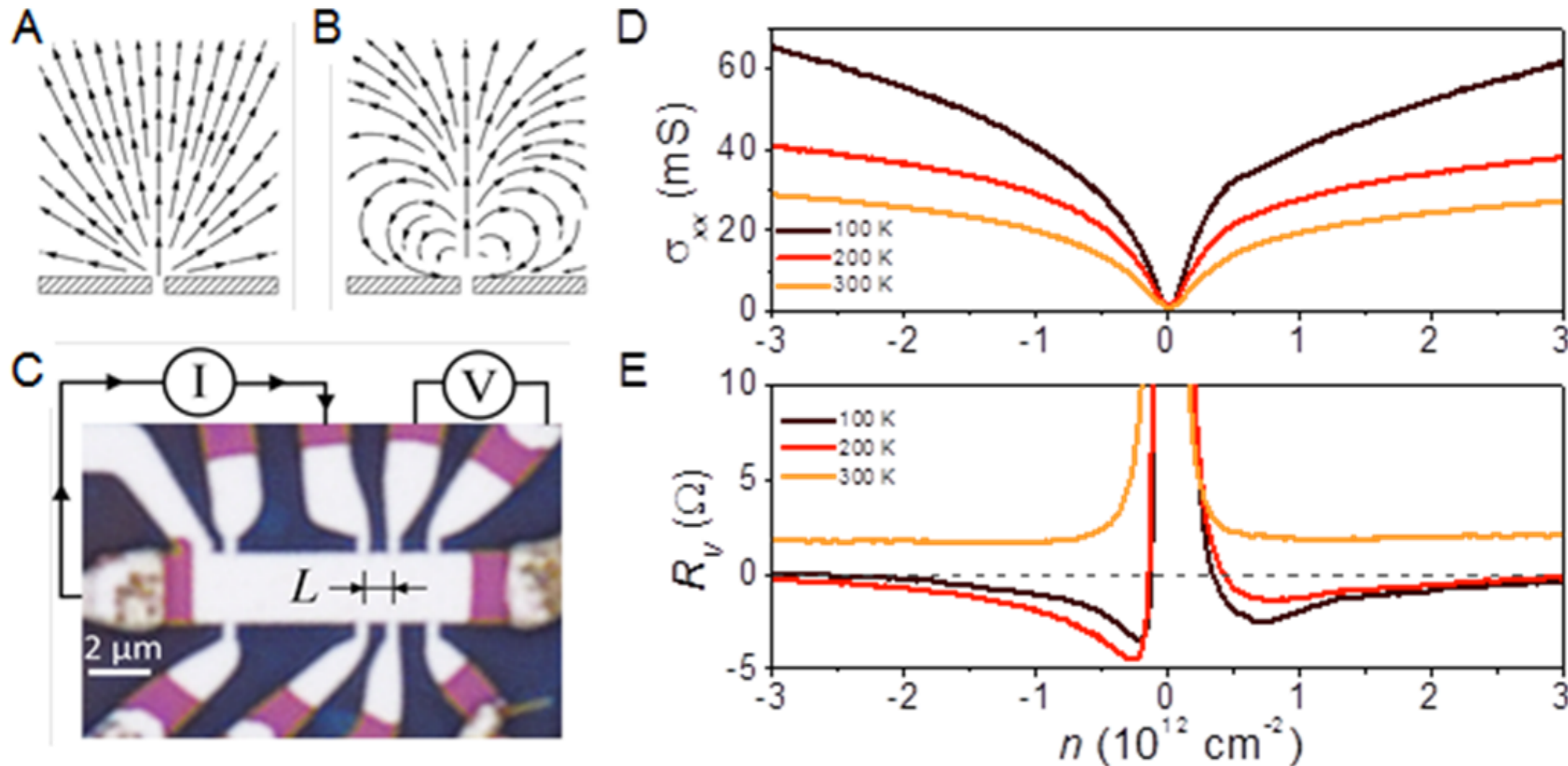


Figure 1. Viscous backflow in doped graphene. (a,b) Steady-state distribution of current injected through a narrow slit for a classical conducting medium with zero ν (a) and a viscous Fermi liquid (b). (c) Optical micrograph of one of our SLG devices. The schematic explains the measurement geometry for vicinity resistance. (d,e) Longitudinal conductivity σ_{xx} and R_V for this device as a function of n induced by applying gate voltage. $I = 0.3 \mu\text{A}$; $L = 1 \mu\text{m}$. For more detail, see Supplementary Information.

Quantum matter without quasiparticles:

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Sachdev-Ye-Kitaev (SYK) model

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Infinite-range model with quasiparticles

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^N t_{ij} c_i^\dagger c_j + \dots$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\frac{1}{N} \sum_i c_i^\dagger c_i = Q$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

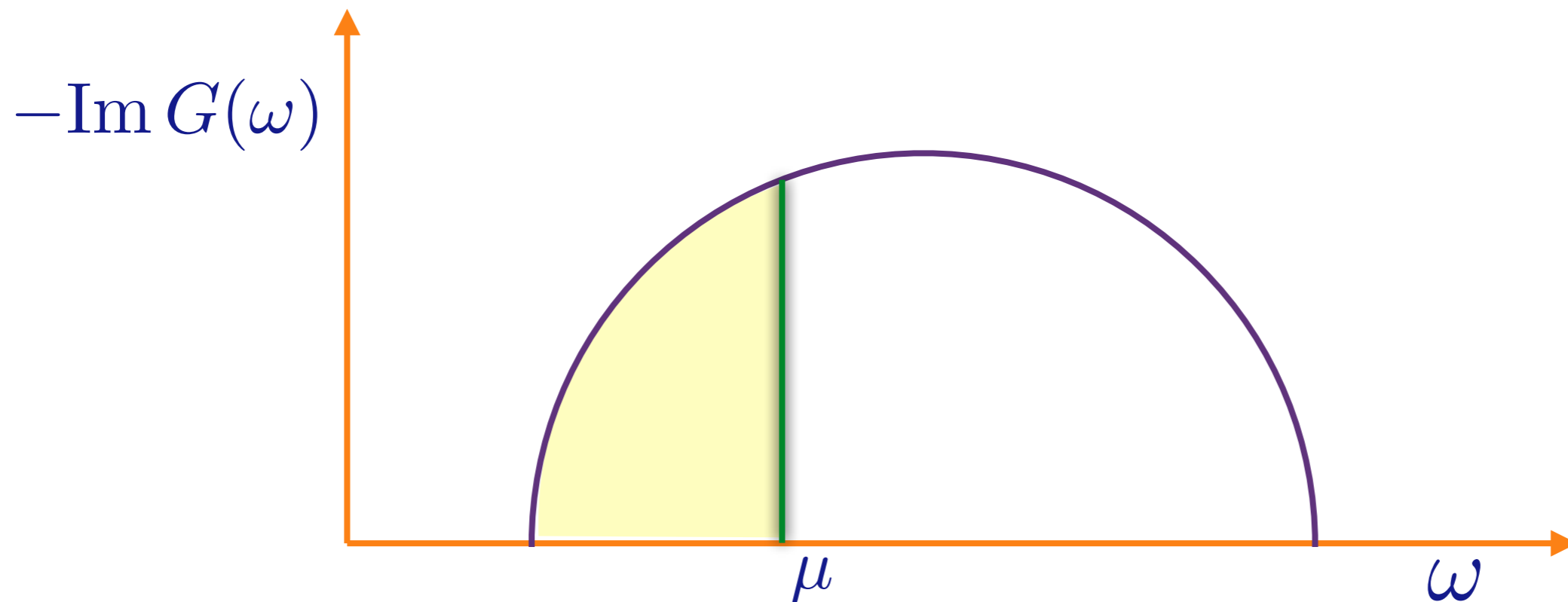
Fermions occupying the eigenstates of a
 $N \times N$ random matrix

Infinite-range model with quasiparticles

Feynman graph expansion in $t_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = t^2 G(\tau)$$
$$G(\tau = 0^-) = Q.$$

$G(\omega)$ can be determined by solving a quadratic equation.



Infinite-range model with quasiparticles

Now add weak interactions

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^N t_{ij} c_i^\dagger c_j + \frac{1}{(2N)^{3/2}} \sum_{i,j,k,l=1}^N J_{ij;kl} c_i^\dagger c_j^\dagger c_k c_l$$

$J_{ij;kl}$ are independent random variables with $\overline{J_{ij;kl}} = 0$ and $\overline{|J_{ij;kl}|^2} = J^2$. We compute the lifetime of a quasiparticle, τ_α , in an exact eigenstate $\psi_\alpha(i)$ of the free particle Hamiltonian with energy E_α . By Fermi's Golden rule, for E_α at the Fermi energy

$$\begin{aligned} \frac{1}{\tau_\alpha} &= \pi J^2 \rho_0^2 \int dE_\beta dE_\gamma dE_\delta f(E_\beta)(1 - f(E_\gamma))(1 - f(E_\delta))\delta(E_\alpha + E_\beta - E_\gamma - E_\delta) \\ &= \frac{\pi^3 J^2 \rho_0^2}{4} T^2 \end{aligned}$$

where ρ_0 is the density of states at the Fermi energy.

Fermi liquid state: Two-body interactions lead to a scattering time of quasiparticle excitations from in (random) single-particle eigenstates which diverges as $\sim T^{-2}$ at the Fermi level.

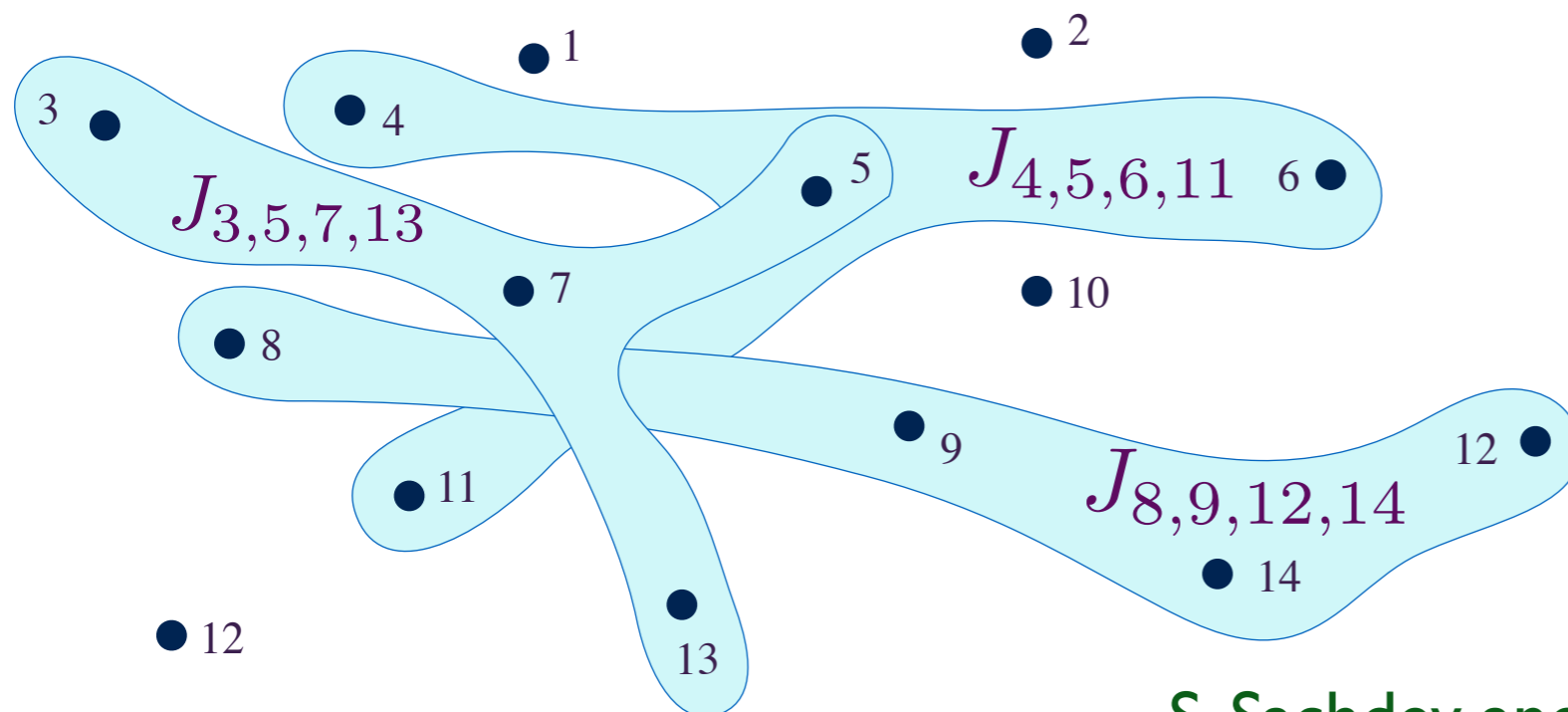
SYK model without quasiparticles

To obtain a non-Fermi liquid, we set $t_{ij} = 0$:

$$H_{\text{SYK}} = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^N J_{ij;kl} c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$Q = \frac{1}{N} \sum_i c_i^\dagger c_i$$

H_{SYK} is similar, and has identical properties, to a related model proposed by SY in 1993.



S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

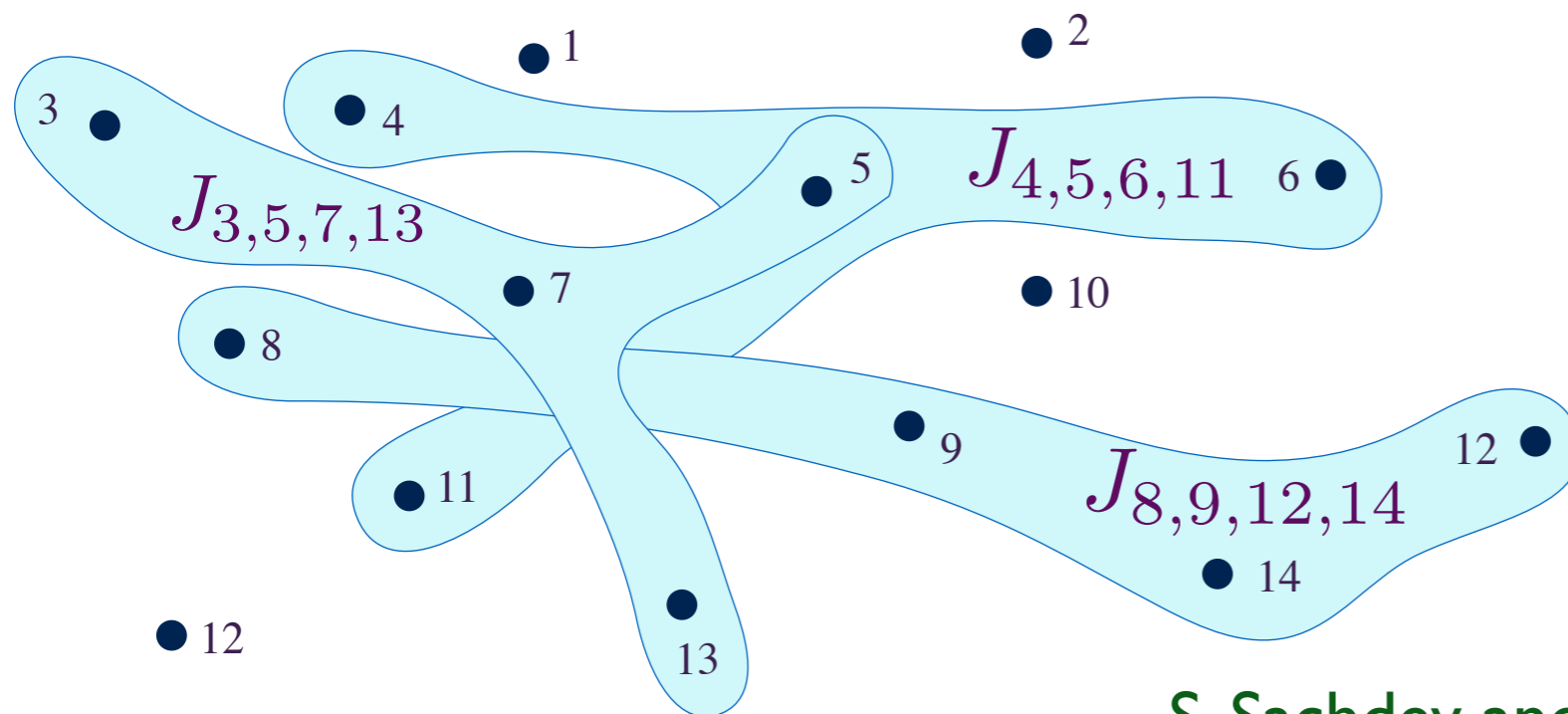
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A fermion can move only by entangling with another fermion: the Hamiltonian has “nothing but entanglement”.

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

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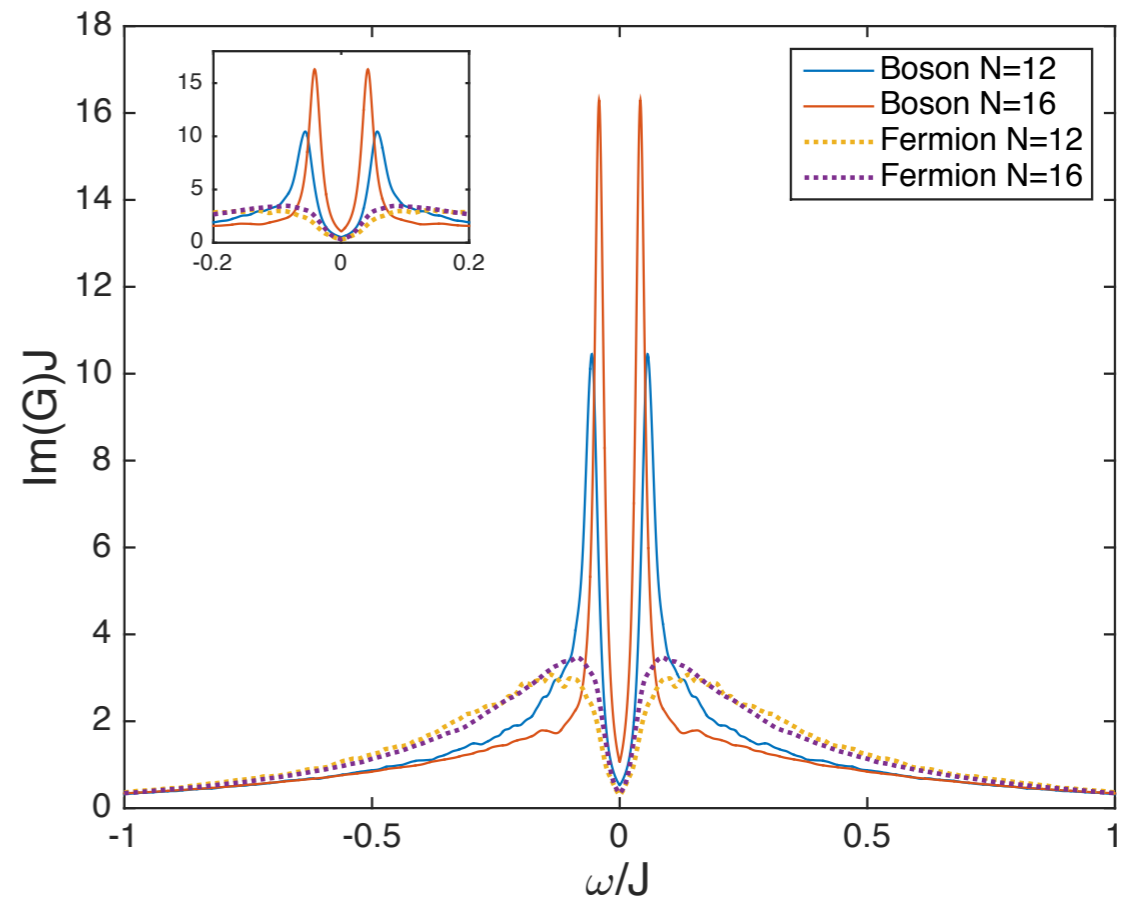
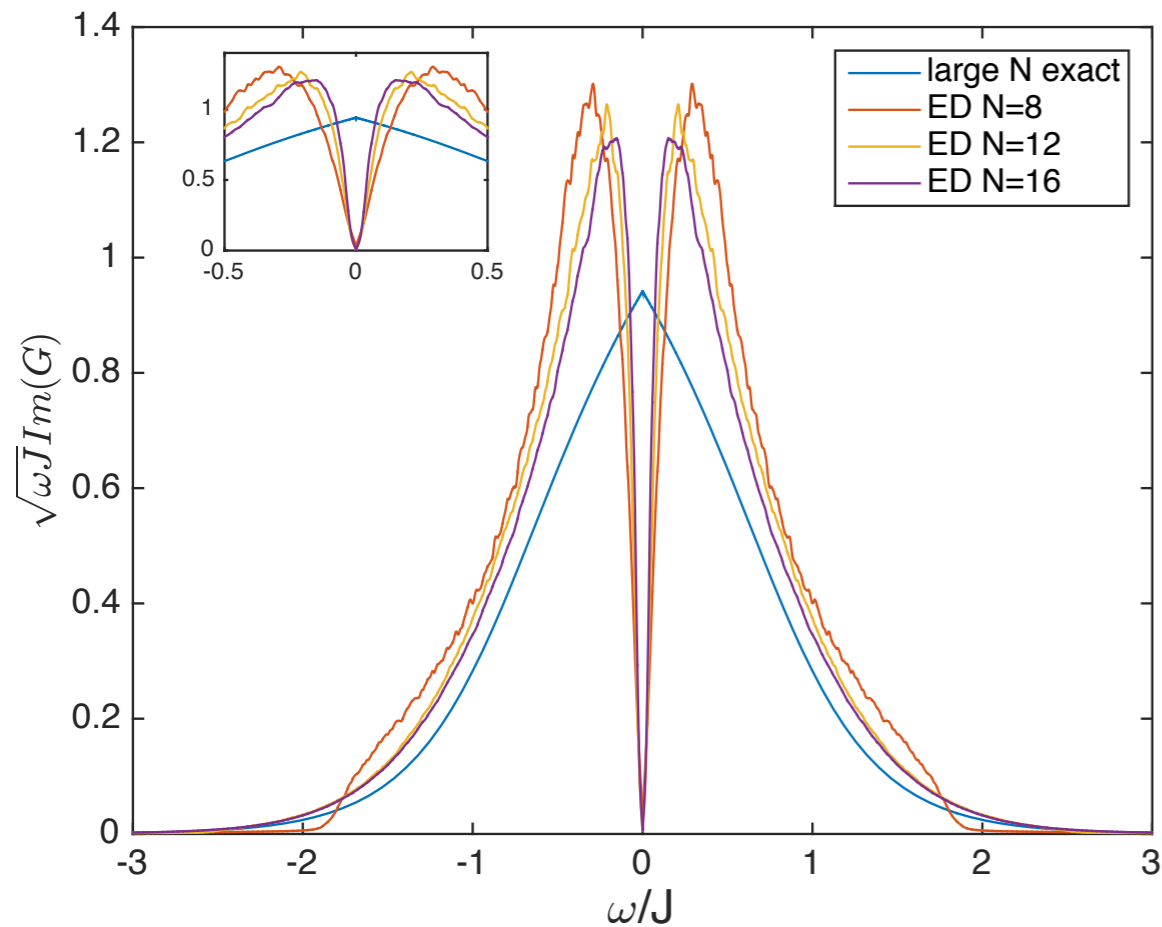
$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = Q.$$

Low frequency analysis shows that the solutions must be gapless and obey

$$\Sigma(z) = \mu - \frac{1}{A} \sqrt{z} + \dots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

for some complex A . The ground state is a non-Fermi liquid, with a continuously variable density Q .

SYK model without quasiparticles



Large N solution of equations for G and Σ agree well with exact diagonalization of the finite N Hamiltonian.

However, exact diagonalization of the same model with hard-core bosons indicates the presence of spin-glass order in the ground state.

SYK model without quasiparticles

The entropy per site, \mathcal{S} , has a non-zero limit as $T \rightarrow 0$. This is *not* due to an extensive degeneracy, but due to an energy level spacing $\sim e^{-aN}$ in the entire many-body spectrum all the way down to the ground state. At low T we write

$$\mathcal{S}(T \rightarrow 0) = \mathcal{S}_0 + \gamma T + \dots$$

where the specific heat is $\mathcal{C} = \gamma T$, and \mathcal{S}_0 obeys

$$\frac{d\mathcal{S}_0}{dQ} = 2\pi\mathcal{E},$$

with \mathcal{E} a spectral asymmetry parameter, which is a known function of Q . \mathcal{E} fully determines the Green's function at low T and ω as a ratio of Gamma functions.

Note that \mathcal{S}_0 and \mathcal{E} involve low-lying states, while Q depends upon *all* states, and details of the UV structure.

Infinite-range (SYK) model without quasiparticles

After integrating the fermions, the partition function can be written as a path integral with an action S analogous to a Luttinger-Ward functional

$$Z = \int \mathcal{D}G(\tau_1, \tau_2) \mathcal{D}\Sigma(\tau_1, \tau_2) \exp(-NS)$$

$$S = \ln \det [\delta(\tau_1 - \tau_2)(\partial_{\tau_1} + \mu) - \Sigma(\tau_1, \tau_2)] \\ + \int d\tau_1 d\tau_2 \Sigma(\tau_1, \tau_2) [G(\tau_2, \tau_1) + (J^2/2)G^2(\tau_2, \tau_1)G^2(\tau_1, \tau_2)]$$

At frequencies $\ll J$, the time derivative in the determinant is less important, and without it the path integral is invariant under the reparametrization and gauge transformations

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = [f'(\sigma_1)f'(\sigma_2)]^{-1/4} \frac{g(\sigma_1)}{g(\sigma_2)} G(\sigma_1, \sigma_2)$$

$$\Sigma(\tau_1, \tau_2) = [f'(\sigma_1)f'(\sigma_2)]^{-3/4} \frac{g(\sigma_1)}{g(\sigma_2)} \Sigma(\sigma_1, \sigma_2)$$

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.

A. Georges and O. Parcollet
PRB 59, 5341 (1999)
A. Kitaev, unpublished
S. Sachdev, PRX 5, 041025 (2015)

Infinite-range (SYK) model without quasiparticles

Let us write the large N saddle point solutions of S as

$$G_s(\tau_1 - \tau_2) \sim (\tau_1 - \tau_2)^{-1/2} \quad , \quad \Sigma_s(\tau_1 - \tau_2) \sim (\tau_1 - \tau_2)^{-3/2}.$$

These are not invariant under the reparametrization symmetry but are invariant only under a $SL(2, \mathbb{R})$ subgroup under which

$$f(\tau) = \frac{a\tau + b}{c\tau + d} \quad , \quad ad - bc = 1.$$

So the (approximate) reparametrization symmetry is spontaneously broken.

Reparametrization zero mode

Expand about the saddle point by writing

$$G(\tau_1, \tau_2) = [f'(\tau_1)f'(\tau_2)]^{1/4} G_s(f(\tau_1) - f(\tau_2))$$

(and similarly for Σ) and obtain an effective action for $f(\tau)$. This action does not vanish because of the time derivative in the determinant which is not reparameterization invariant.

J. Maldacena and D. Stanford, arXiv:1604.07818

See also A. Kitaev, unpublished, and J. Polchinski and V. Rosenhaus, arXiv:1601.06768

Infinite-range (SYK) model without quasiparticles

However the effective action must vanish for $SL(2, \mathbb{R})$ transformations because G_s, Σ_s are invariant under it. In this manner we obtain the effective action as a Schwarzian

$$N S_{\text{eff}} = -\frac{N\gamma}{4\pi^2} \int d\tau \{f, \tau\} \quad , \quad \{f, \tau\} \equiv \frac{f'''}{f'} - \frac{3}{2} \left(\frac{f''}{f'} \right)^2 ,$$

where the specific heat, $\mathcal{C} = \gamma T$.

The Schwarzian effective action implies that the SYK model *saturates* the lower bound on the Lyapunov time

$$\tau_L = \frac{1}{2\pi} \frac{\hbar}{k_B T}$$

Infinite-range (SYK) model without quasiparticles

The Schwarzian describes fluctuations of the energy operator with scaling dimension $h = 2$.

Apart from the energy operator associated with the Schwarzian, there are an infinite number of other scalar operators with irrational scaling dimensions given by the roots of

$$\tan\left(\frac{\pi(2h-1)}{4}\right) = \frac{1-2h}{3}$$

$$\Rightarrow h = 3.77354\dots, 5.67946\dots, 7.63197\dots, 9.60396\dots, \dots$$

Quantum matter without quasiparticles:

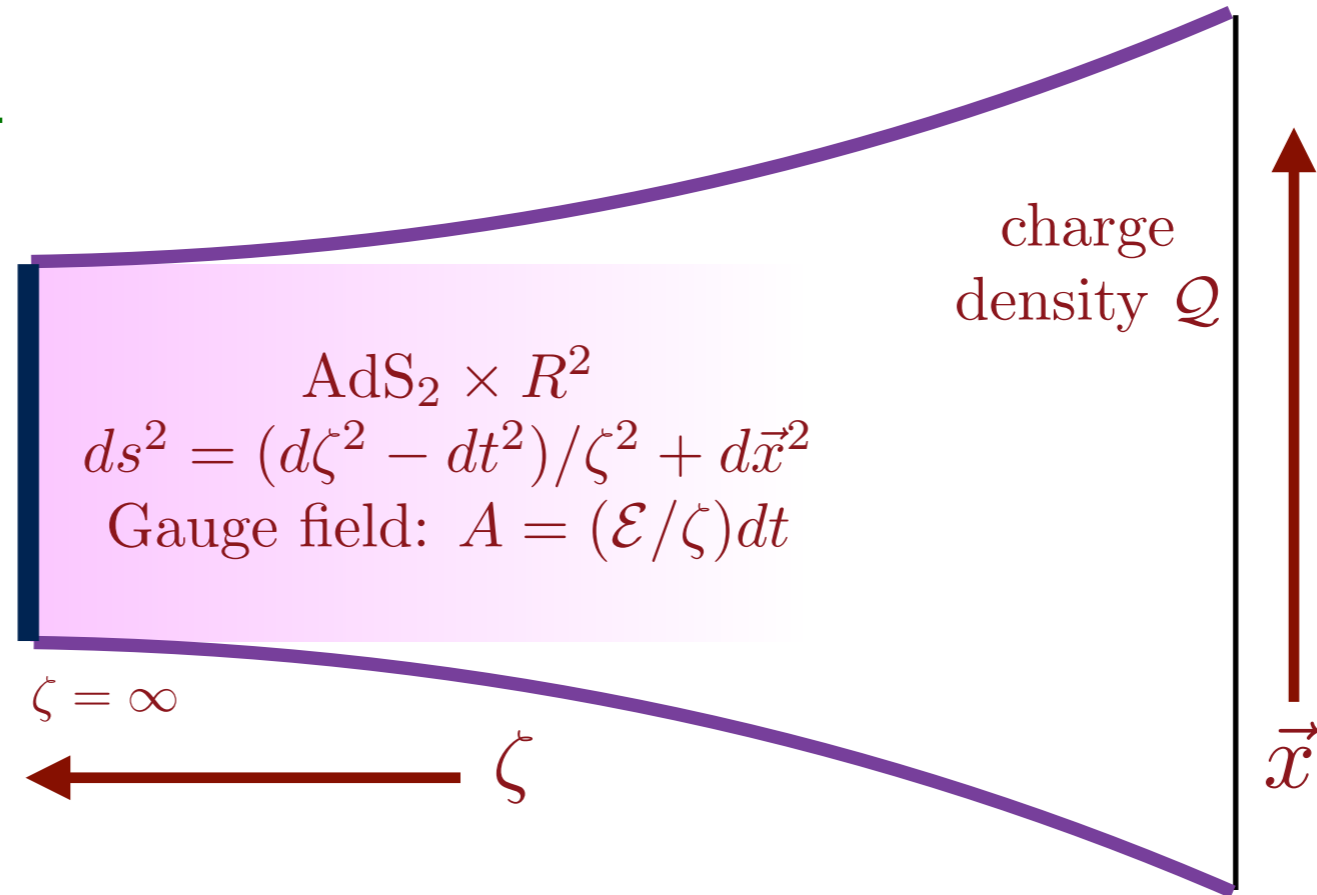
- Graphene

- Solvable random fermion

Sachdev-Ye-Kitaev (SYK) model

- Charged black hole horizons in anti-de Sitter space

SYK and AdS₂



PHYSICAL REVIEW LETTERS **105, 151602 (2010)**



Holographic Metals and the Fractionalized Fermi Liquid

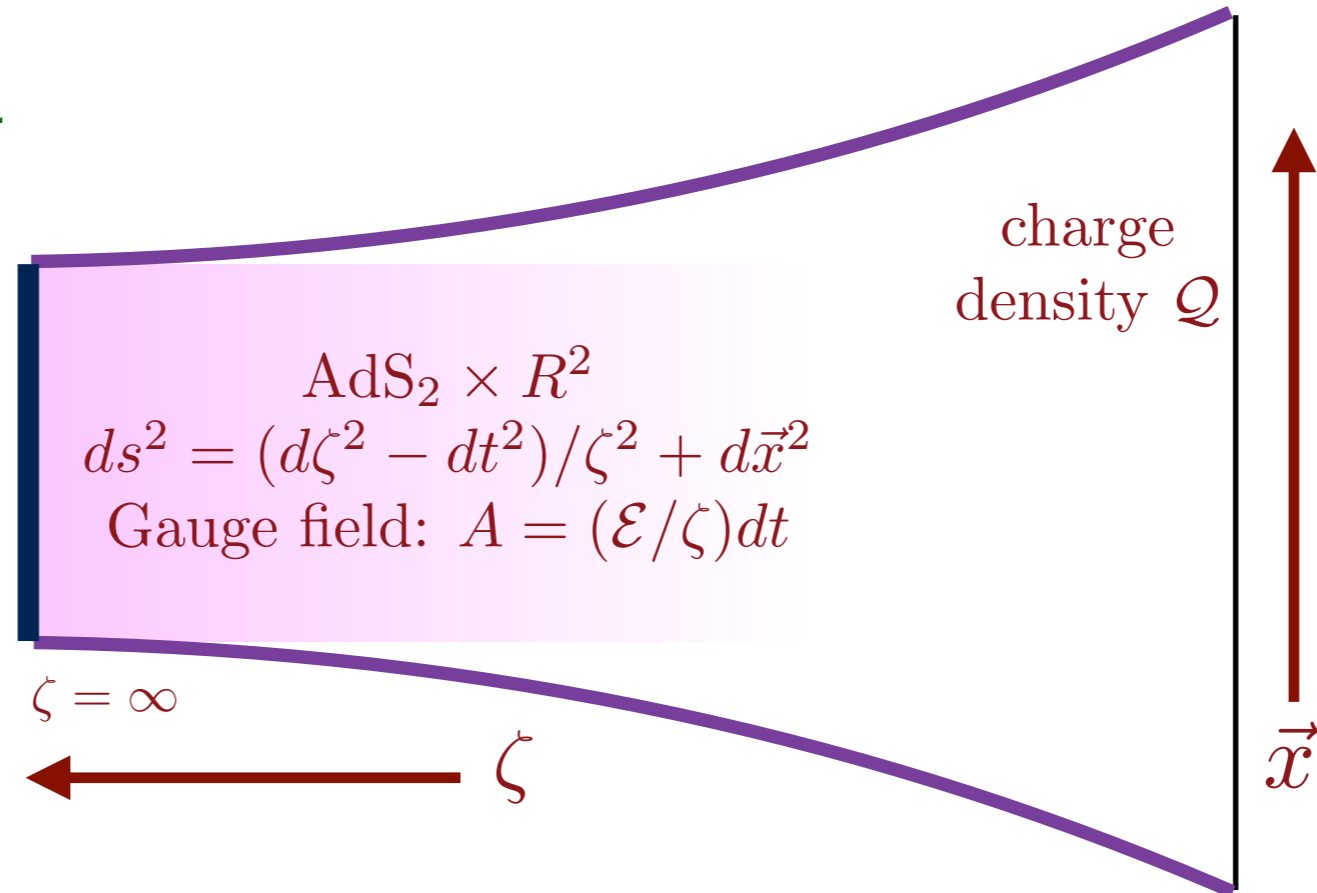
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(Received 23 June 2010; published 4 October 2010)

We show that there is a close correspondence between the physical properties of holographic metals near charged black holes in anti-de Sitter (AdS) space, and the fractionalized Fermi liquid phase of the lattice Anderson model. The latter phase has a “small” Fermi surface of conduction electrons, along with a spin liquid of local moments. This correspondence implies that certain mean-field gapless spin liquids are states of matter at nonzero density realizing the near-horizon, $AdS_2 \times R^2$ physics of Reissner-Nordström black holes.

SYK and AdS₂



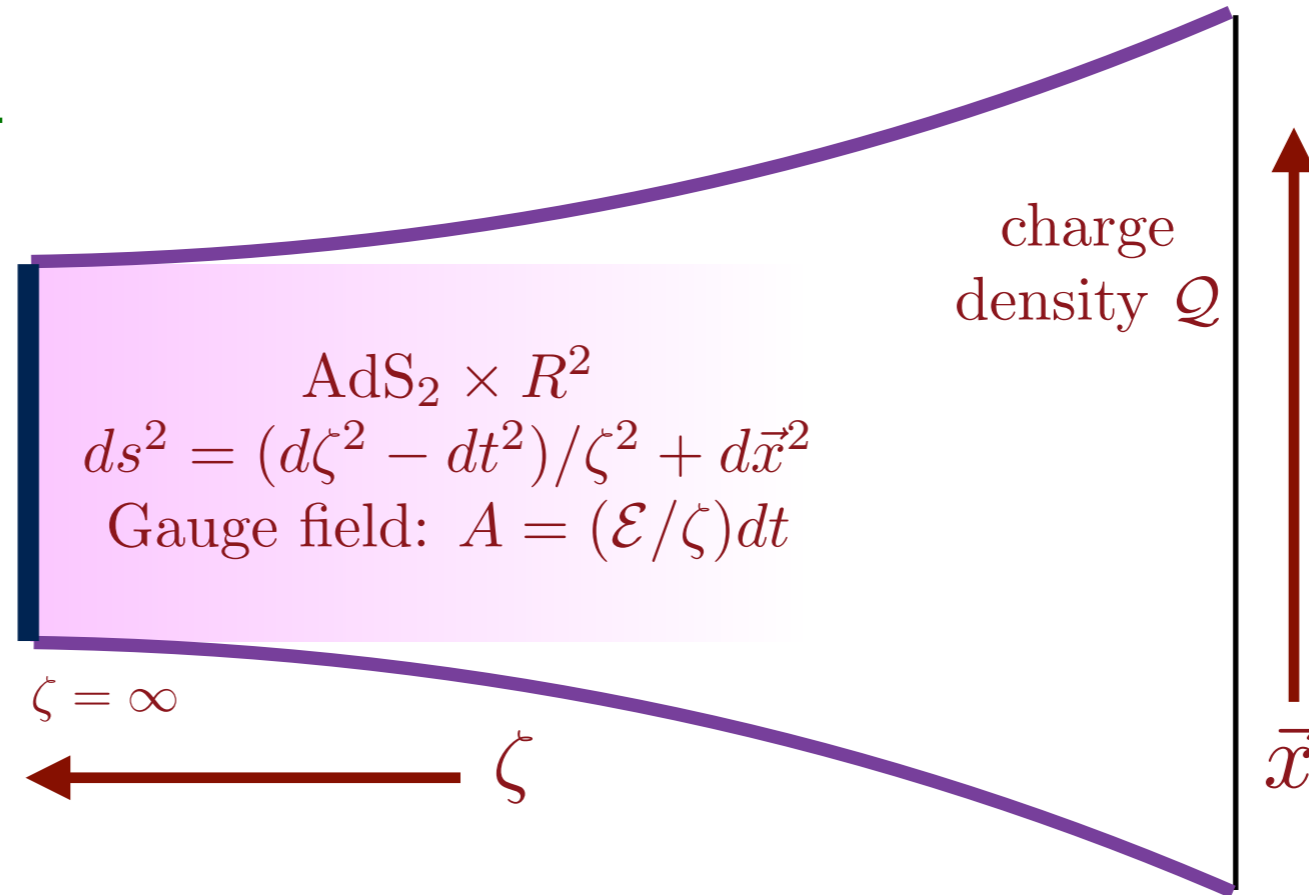
- The non-zero $T \rightarrow 0$ entropy density, \mathcal{S}_0 , matches the Bekenstein-Hawking-Wald entropy density of extremal AdS_2 horizons, and the dependence of the fermion Green's function on ω , T , and \mathcal{E} , matches that of a Dirac fermion in AdS_2 (as computed by T. Faulkner, Hong Liu, J. McGreevy, and D. Vegh, PRD **83**, 125002 (2011)).

S. Sachdev, PRL **105**, 151602 (2010)

- More recently, it was noted that the relation $d\mathcal{S}_0/d\mathcal{Q} = 2\pi\mathcal{E}$ also matches between SYK and gravity, where \mathcal{E} , the electric field on the horizon, also determines the spectral asymmetry of the Dirac fermion.

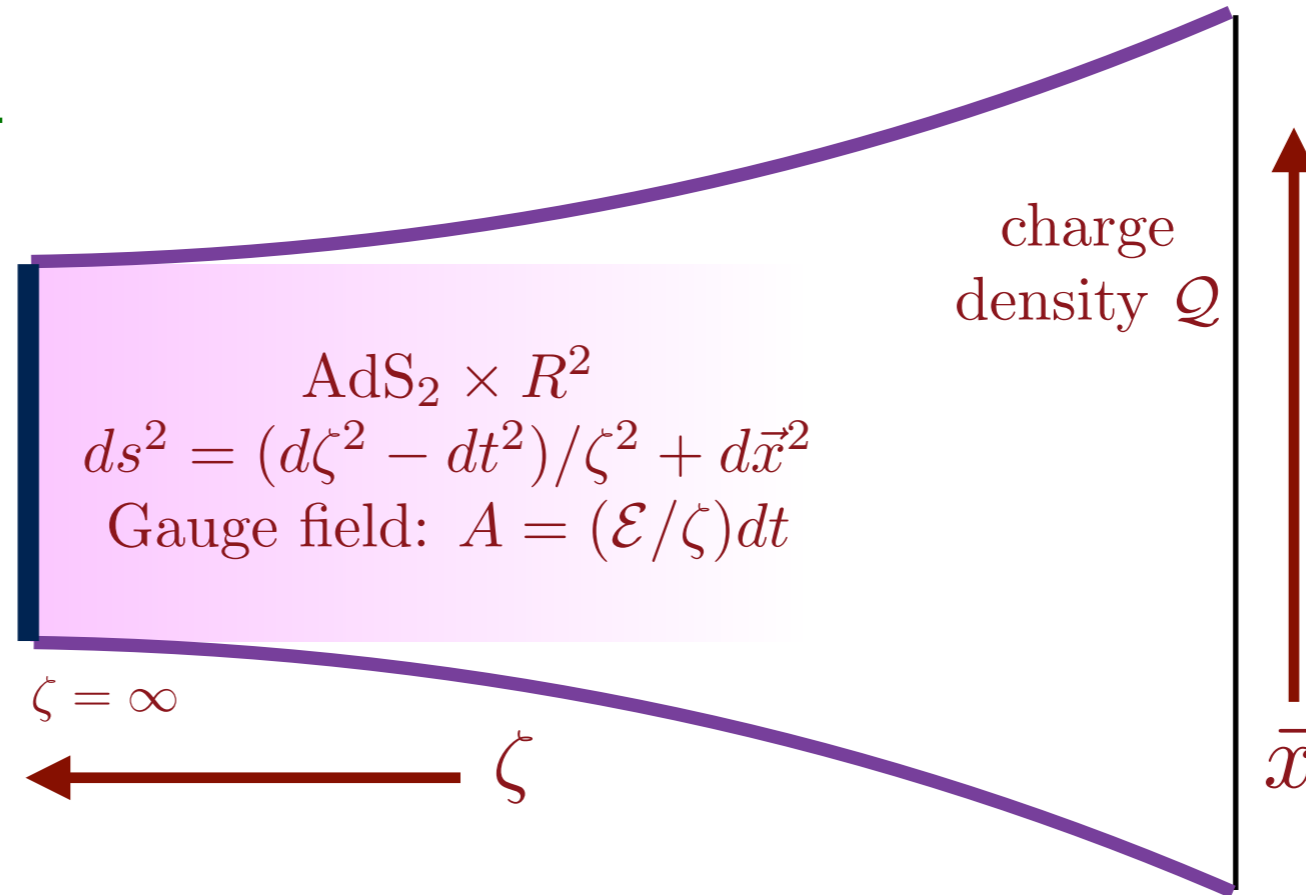
S. Sachdev, PRX **5**, 041025 (2015)

SYK and AdS₂



The same Schwarzian effective action describes low energy fluctuations on the boundary theory of gravity theories with AdS_2 near-horizon geometries (including the AdS-Reissner-Nordstrom solution of Einstein-Maxwell theory in 4 space-time dimensions). And the co-efficient of the Schwarzian, $N\gamma/4\pi^2$, determines the specific heat $\mathcal{C} = \gamma T$.

SYK and AdS₂



The Schwarzian effective action implies that both the SYK model and the AdS₂ theories *saturate* the lower bound on the Lyapunov time

$$\tau_L = \frac{1}{2\pi} \frac{\hbar}{k_B T}.$$

Quantum matter without quasiparticles:

- Solvable random fermion
Sachdev-Ye-Kitaev (SYK) model
 - Charged black hole horizons in
anti-de Sitter space
- Graphene

Entangled quantum matter without quasiparticles

- No quasiparticle excitations
- Shortest possible “phase coherence” time, fastest possible local equilibration time, or fastest possible Lyapunov time towards quantum chaos, all of order $\frac{\hbar}{k_B T}$
- Experiments on graphene agree well with predictions of a theory of a nearly relativistic quantum liquid without quasiparticles.
- Realization in solvable SYK model, which saturates the lower bound on the Lyapunov time. Its properties have some similarities to non-rational, large central charge CFT2s.
- Remarkable match between SYK and quantum gravity of black holes with AdS_2 horizons, including a $\text{SL}(2, \mathbb{R})$ -invariant Schwarzian effective action for thermal energy fluctuations.