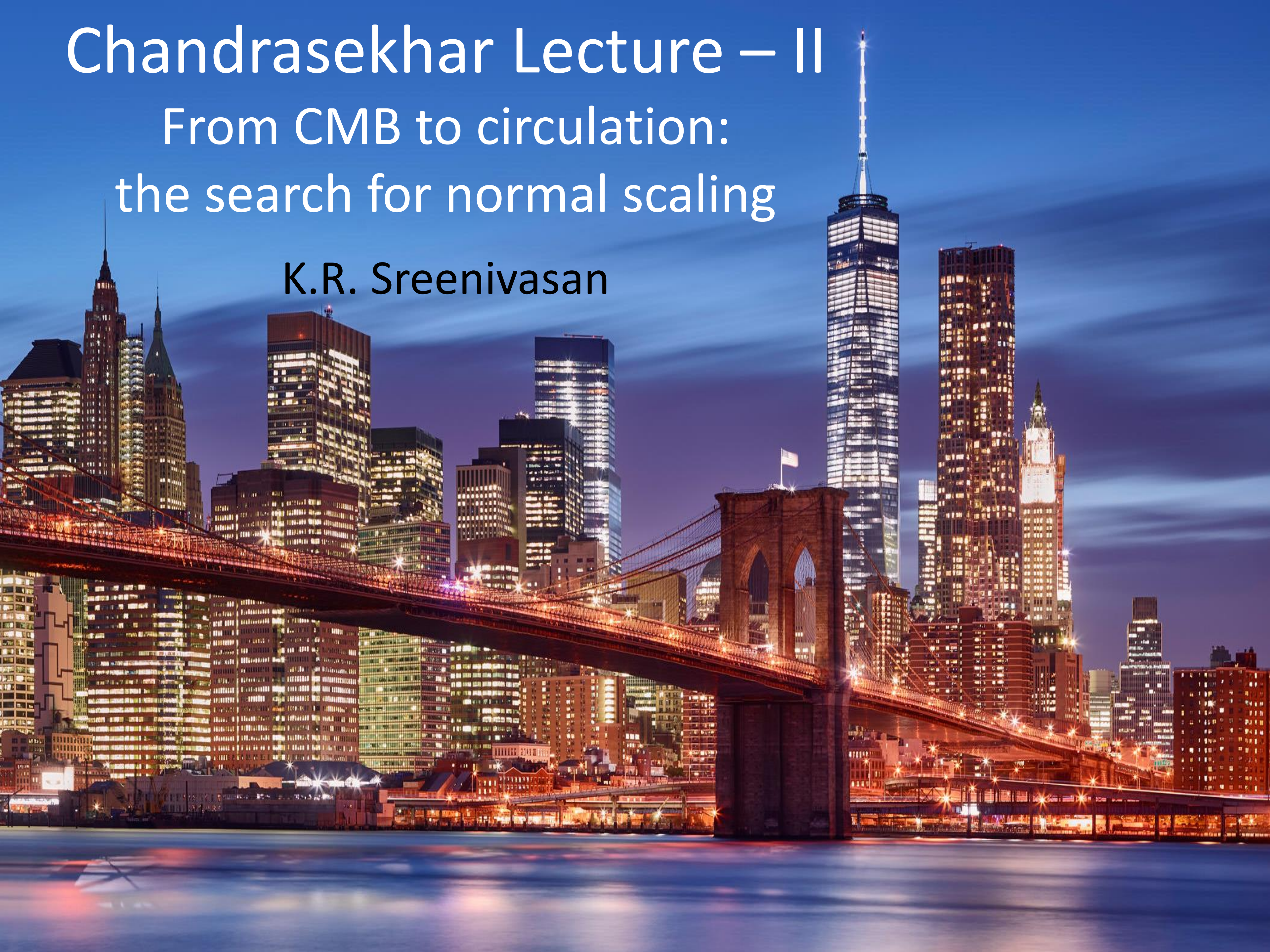
Chandrasekhar Lecture – II

From CMB to circulation: the search for normal scaling

K.R. Sreenivasan



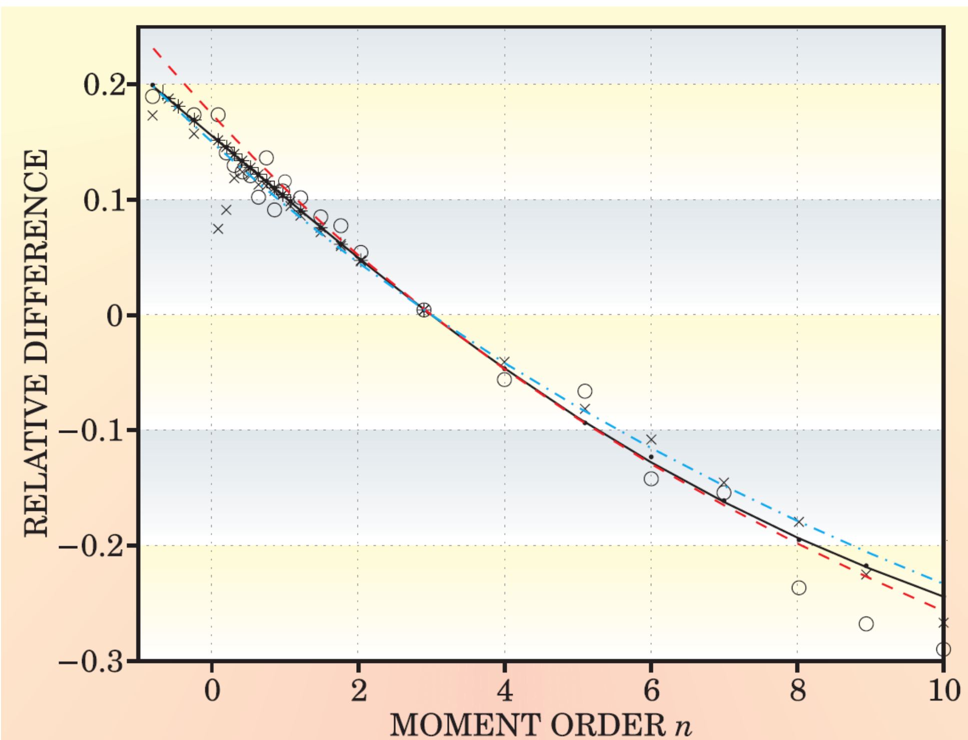
Critical scaling (W,K,F,W)

$f(t,h) \sim t^{2-\alpha} g_f(h/|t|^{\Delta})$

$\xi \quad \sim t^{-\nu} g_\xi(h/|t|^{\Delta})$

$t = (T-T_c)/T_c$

Universality



Kolmogorov

$\phi(k,Re) = C\varepsilon^{2/3}k^{-5/3}\ f(k\eta)$

$\langle \Delta u_r^2 \rangle = (\varepsilon r)^{2/3}\, g(r/\eta)$

Wall-layer scaling

$dU/dy = (1/k)(u_\tau/y)\ f(yu_\tau/\nu)$

Normal scaling

$\langle \Delta u_r^p \rangle = C_n(r\varepsilon)^{p/3}$ in IR

Anomalous scaling

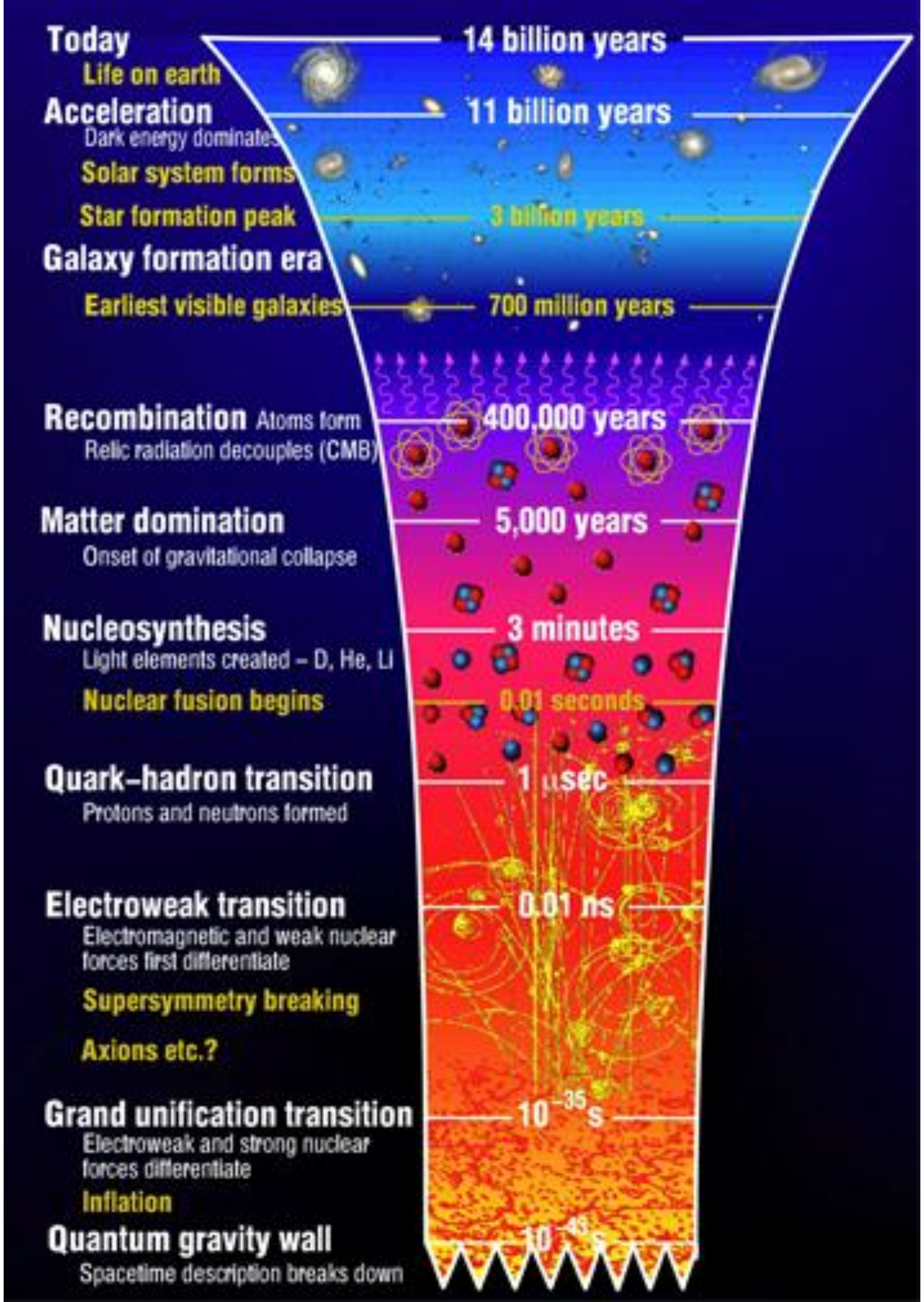
$\langle \Delta u_r^p \rangle \sim r^{\zeta_p}$

with $\langle \Delta u_r^3 \rangle = -(4/5)\varepsilon r$ (in IR)

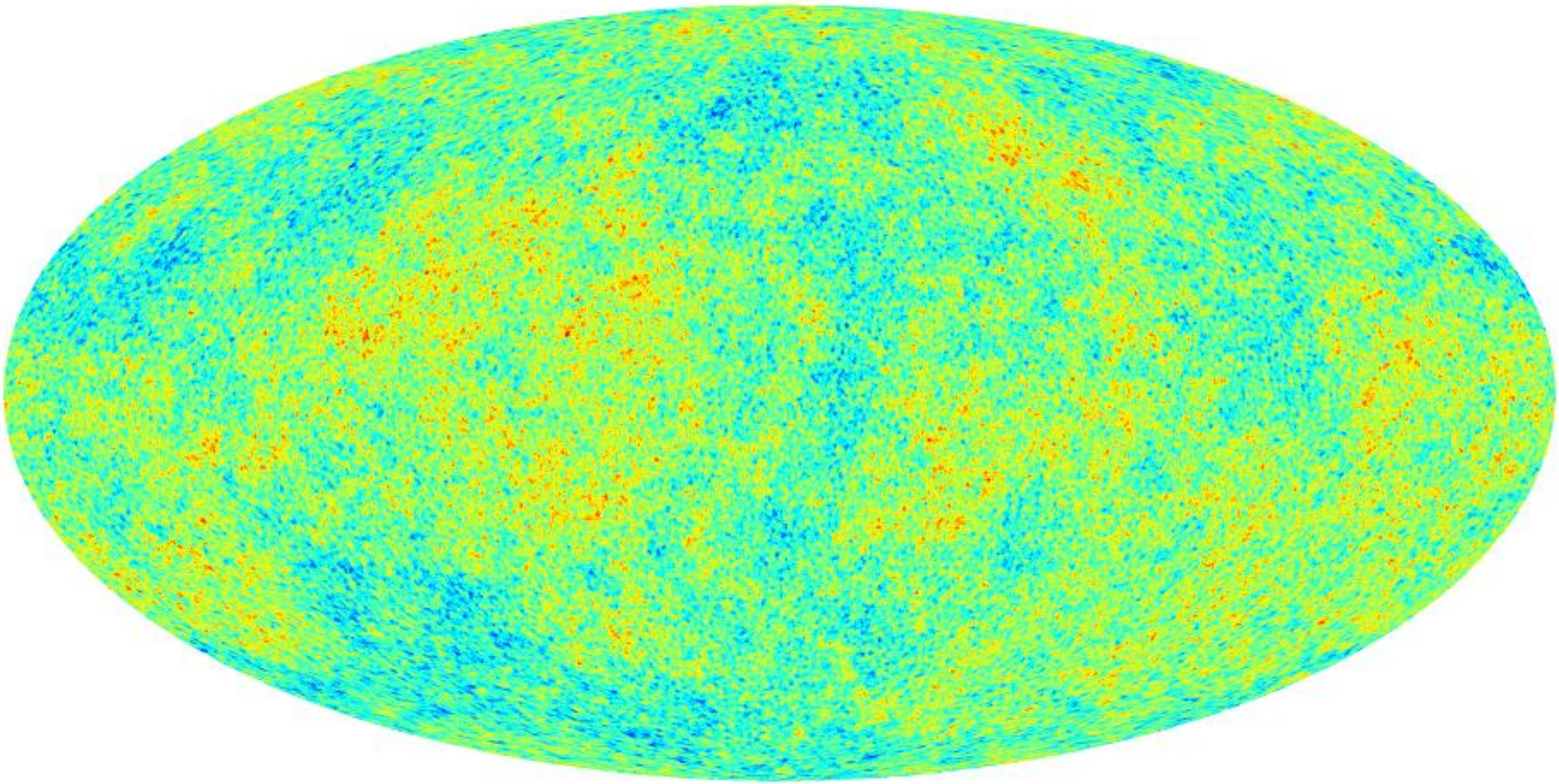
ESS

$\langle \Delta u_r^3 \rangle = -(4/5)\varepsilon r$

$\langle \Delta u_r^p \rangle \sim \langle \Delta u_r^3 \rangle^{\zeta_p}$



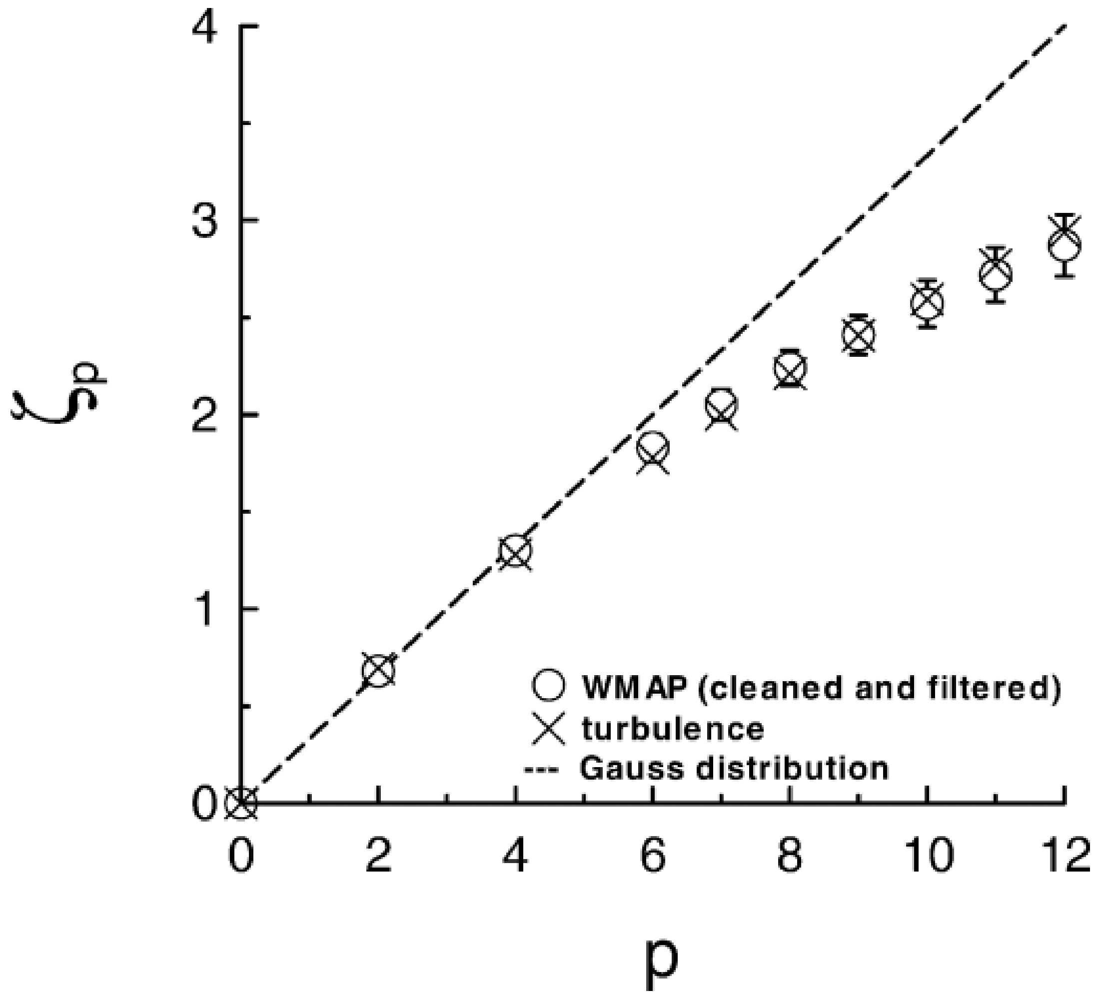
Today
Life on earth
Acceleration
Dark energy dominates
Solar system forms
Star formation peak
Galaxy formation era
Earliest visible galaxies
Recombination Atoms form
Relic radiation decouples (CMB)
Matter domination
Onset of gravitational collapse
Nucleosynthesis
Light elements created – D, He, Li
Nuclear fusion begins
Quark–hadron transition
Protons and neutrons formed
Electroweak transition
Electromagnetic and weak nuclear forces first differentiate
Supersymmetry breaking
Axions etc.?
Grand unification transition
Electroweak and strong nuclear forces differentiate
Inflation
Quantum gravity wall
Spacetime description breaks down
14 billion years
11 billion years
3 billion years
700 million years
400,000 years
5,000 years
3 minutes
0.01 seconds
1 μsec
0.01 ns
10^{-35} s
10^{-43} s



The CMB is a snapshot of the oldest light in our Universe, imprinted on the sky when the Universe was just 380 000 years old (it is now about 13.7 billion years old). It obeys blackbody radiation accurately (with a mean value of about 2.7K).

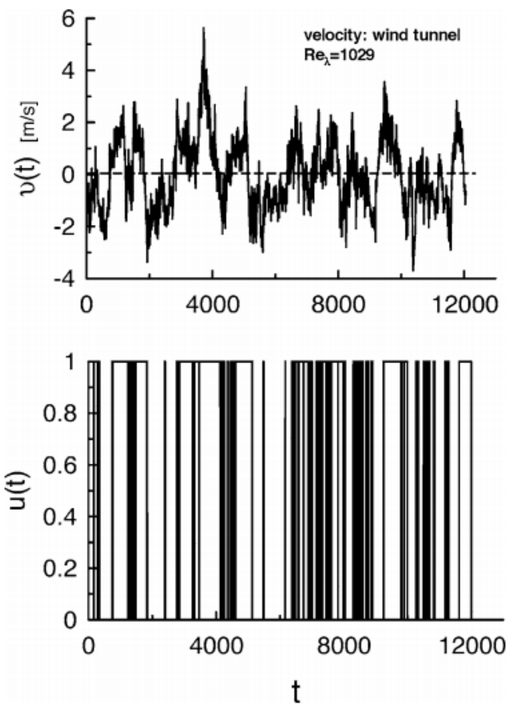
with A. Bershadskii

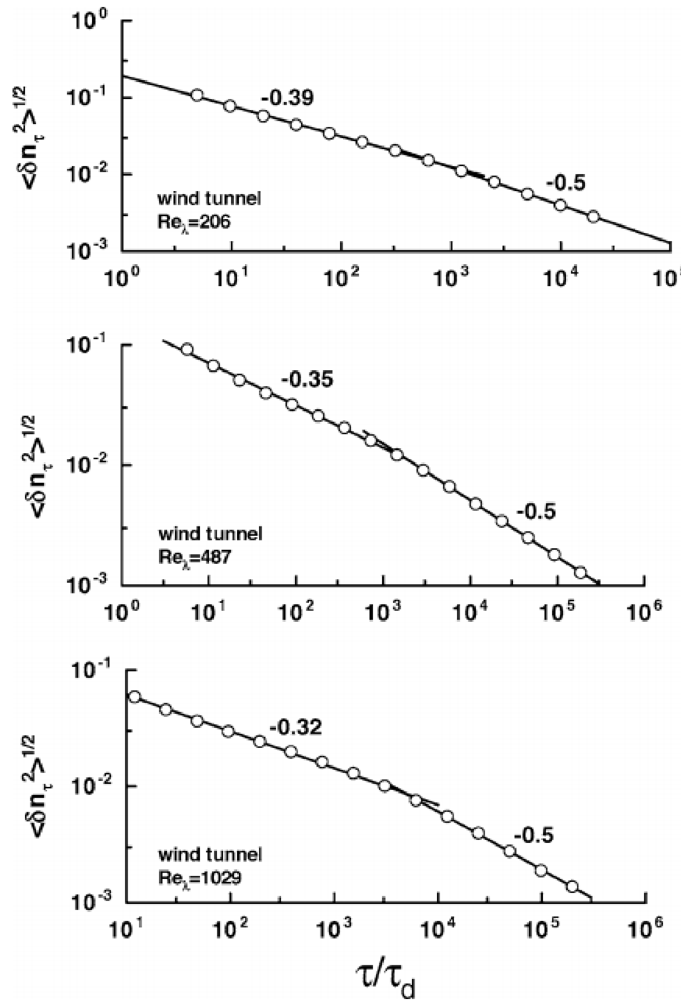
/ Physics Letters A 319 (2003) 21–23



Clustering exponent

(with Bershadskii)

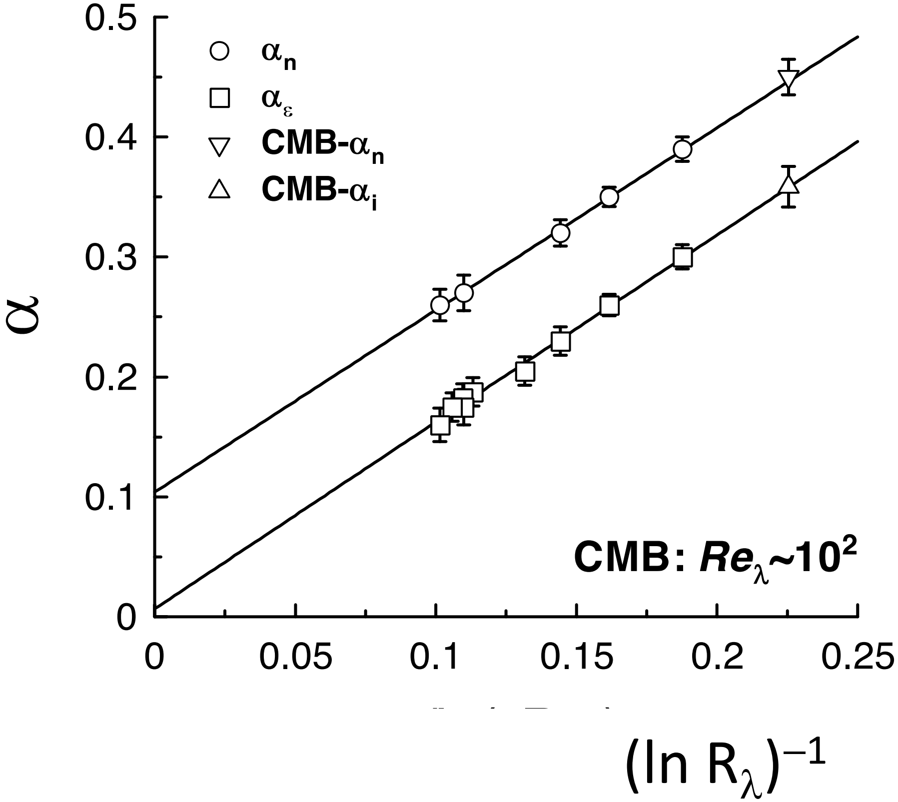




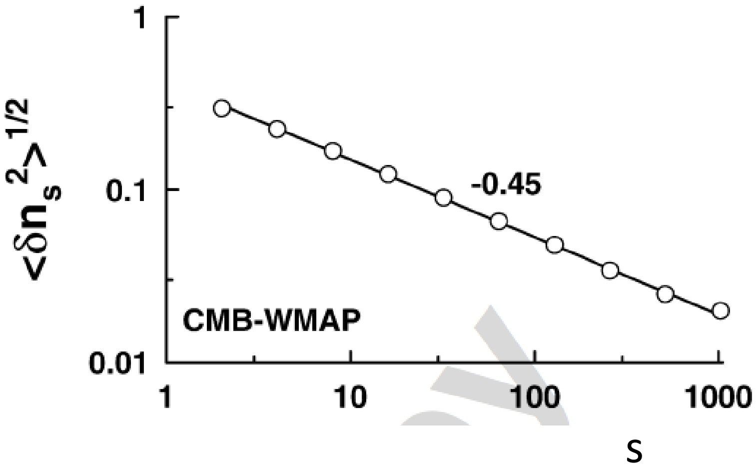
$\Delta n_\tau = n_\tau - <n_\tau>$

$<\Delta n_\tau^2>^{1/2} \sim \tau^{\alpha}$

$$\alpha(\ln Re_\lambda) = \alpha_\infty + a_1/(\ln Re_\lambda) + a_2/(\ln Re\lambda)^2 + ...$$



α
0.5
0.4
0.3
0.2
0.1
0
0.05
0.1
0.15
0.2
0.25
α_n
α_ε
CMB-α_n
CMB-α_i
CMB: Re_λ~10^2
$(\ln R_\lambda)^{-1}$



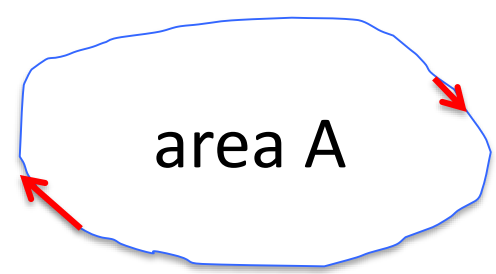
$<\delta n_s^2>^{1/2}$
1
0.1
0.01
10
100
1000
-0.45
CMB-WMAP
s

Stipulation: Circulation around a contour plays an important role in aerodynamics, and can be an interesting property to examine in turbulent flows (with K.P. Iyer & P.K. Yeung)

Circulation around a contour of area A

$$\Gamma_A = \oint_c \boldsymbol{u}.\, d\mathrm{l} = \iint \boldsymbol{\omega}.\mathbf{n}\, dS$$

$$r = sqrt\ A$$



Previous work on the subject of scaling of circulation in 3D turbulence

Theory

- Migdal AA (1994) *Int. J. Mod. Phys. A* **9**:1197–1238
- Migdal AA (1995) in a Summer School Proceedings published by Gordon and Breach (ed. V.P. Mineev), pp. 177–204

Experiment (using PIV)

- Sreenivasan KR, Juneja A, Suri AK (1995) *Phys. Rev. Lett.* **75**:433–436.
  Wake behind a cylinder: Re = VD/ν = 190 - 4540
- Zhou et al. (2008) *J. Fluid Mech.* **598**: 361-372. Rayleigh-Bènard convection

Direct numerical simulations

- Umeki M (1993) *J. Phys. Soc. Jpn*. **62**:3788–3791. DNS, 128^3
- Cao N, Chen S, Sreenivasan KR (1996) *Phys. Rev. Lett.* **76**:616–619. DNS, 256^3 and 512^3
- Benzi R, Biferale L, Struglia MV, Tripiccione R (1997) *Phys. Rev. E* **55**:3739–3742. Kolmogorov flow, DNS 160^3

Present

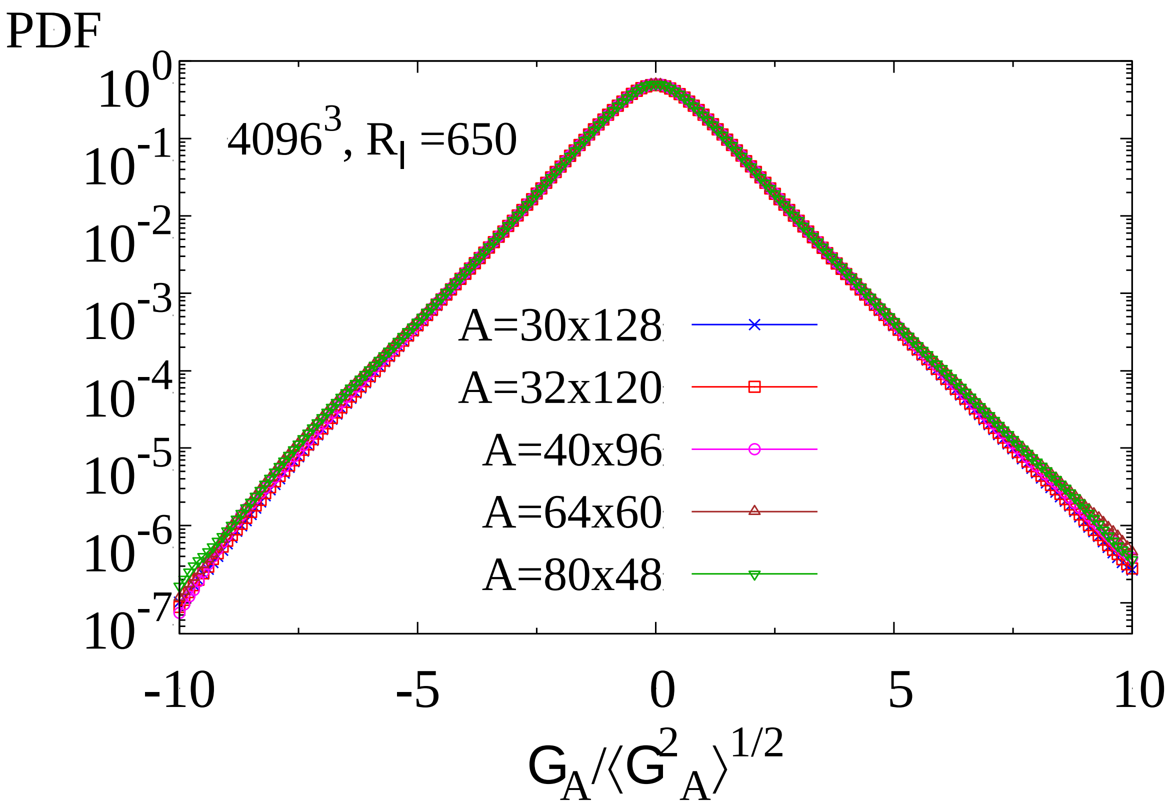
Box size 512^3 – 8192^3, R_λ = 200 – 1300

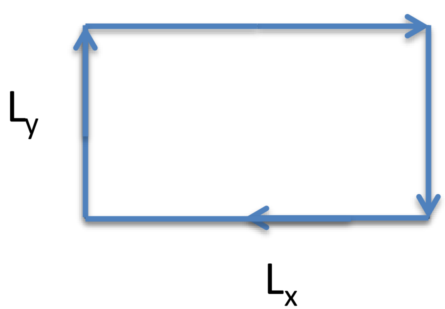
The 8192^3 simulations are discussed in Yeung et al. (2015) *PNAS* **112**: 12633; resolution about η. (Preliminary data from 16384^3 cube will not be discussed.)

Table 1. Isotropic DNS database: N^3 is the number of points on a L_0^3 grid with $L_0 = 2\pi$ units, $R_\lambda \equiv u'\lambda/\nu$ is the Taylor scale Reynolds number where u' is the root-mean-square velocity fluctuation, $\lambda \equiv u'/\sqrt{\langle(\partial u/\partial x)^2\rangle}$ is the Taylor microscale, ν is the kinematic viscosity, $L \approx L_0/5$ is the integral scale, $\eta \equiv (\nu^3/\langle\epsilon\rangle)^{1/4}$ is the Kolmogorov scale, $\Delta x = L_0/N$ is the grid spacing and $\langle\epsilon\rangle$ is the mean dissipation rate. The results have been averaged over a time span of about 5 large-eddy time scales (L/u').

N^3	R_λ	L/η
512^3	200	188
2048^3	400	402
4096^3	650	774
8192^3	1300	2274

¼ million processors, some preliminary calculations using a grid of size 16384^3

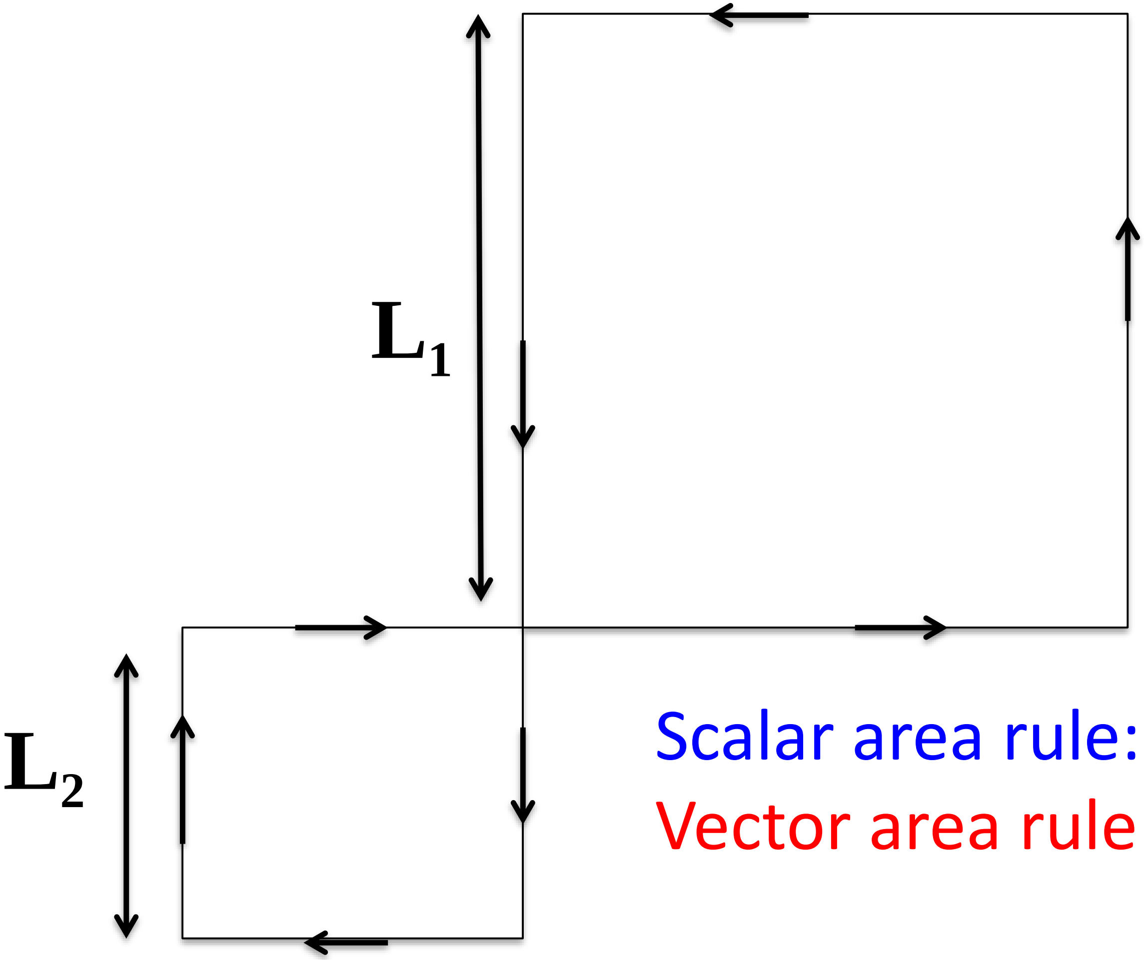




Circulation statistics depend *only* on the area of the loop, not on its shape

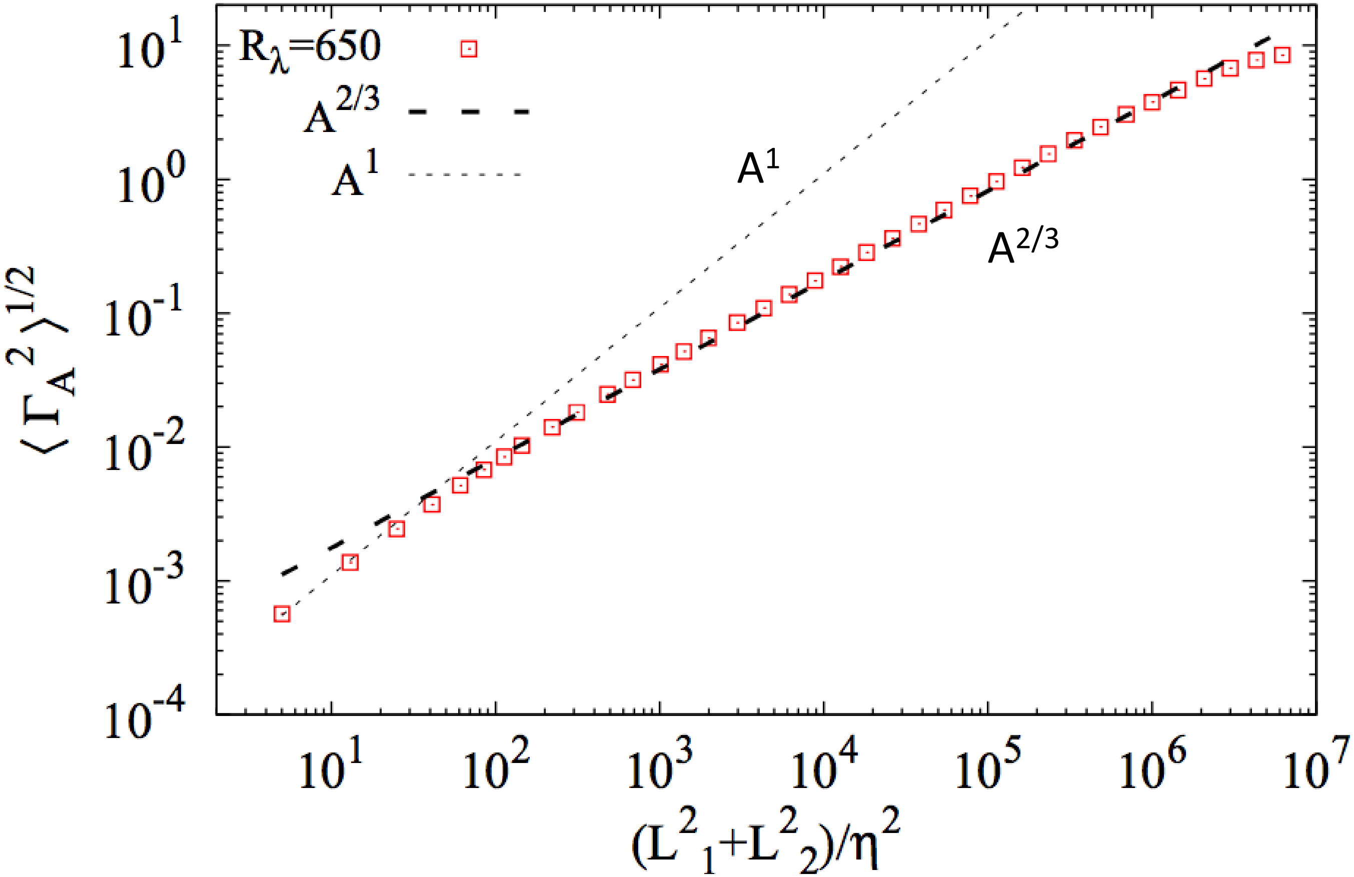
(as long as the largest dimension is contained within the inertial range)

L_x and L_y are within the inertial range



Scalar area rule: $<\Gamma_A{}^2> \sim A^{2/3}$

Vector area rule: $<\Gamma_A{}^2> \sim A^{1/3}$



Scalar area rule holds, not vector area rule

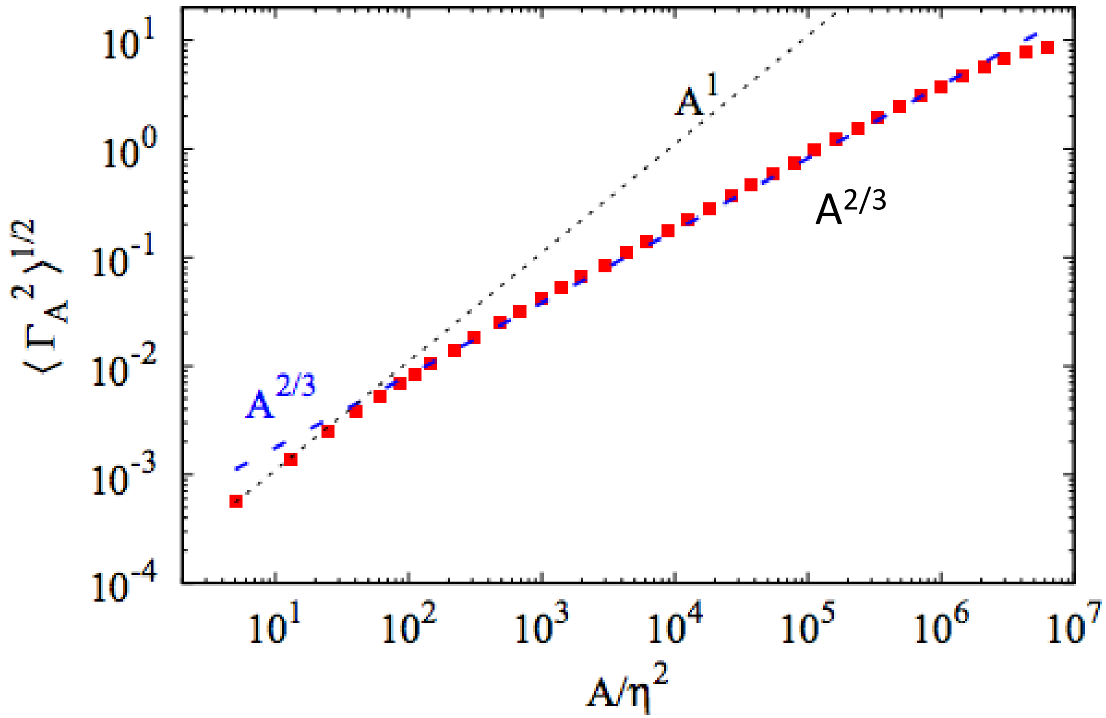
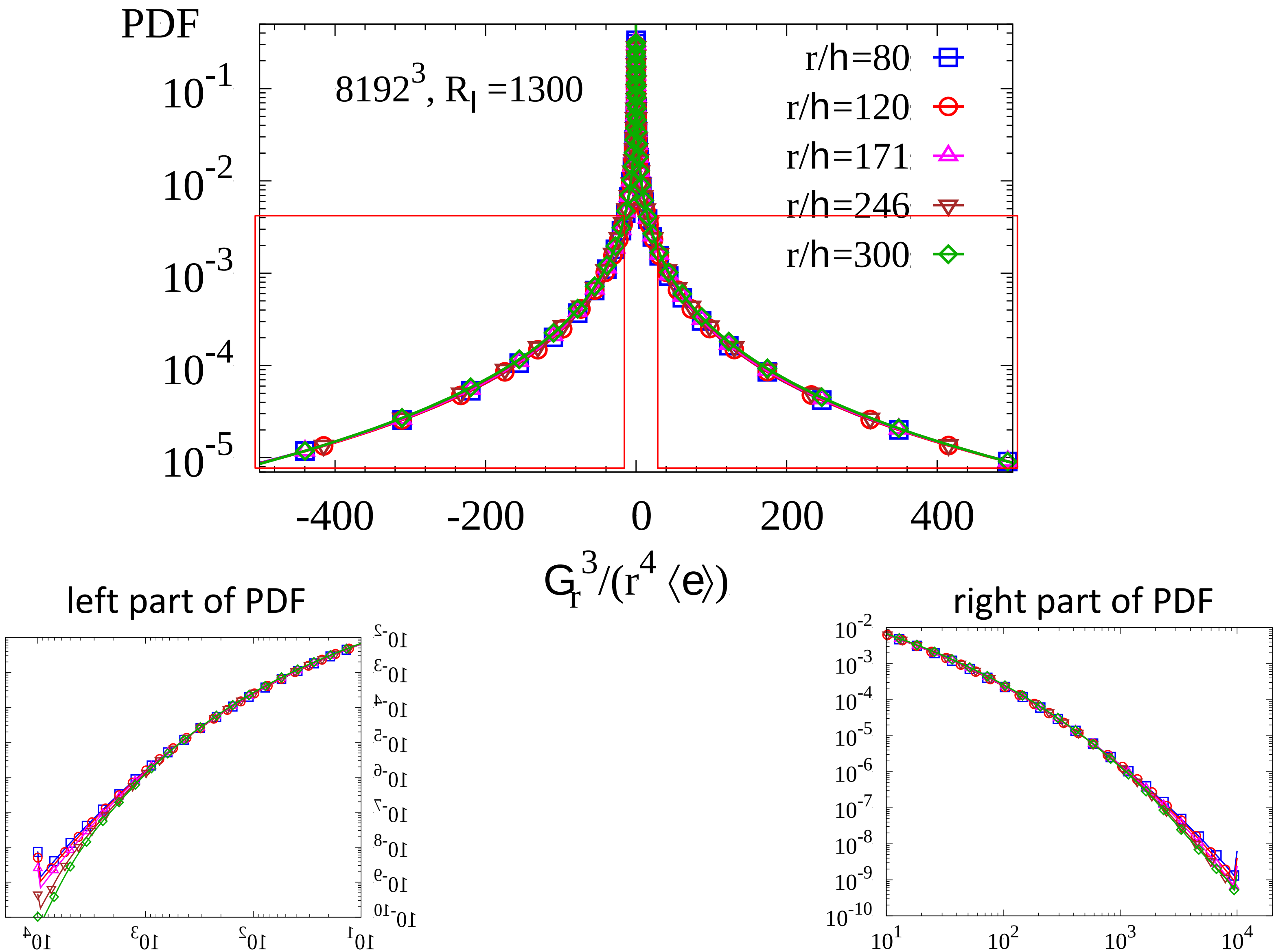
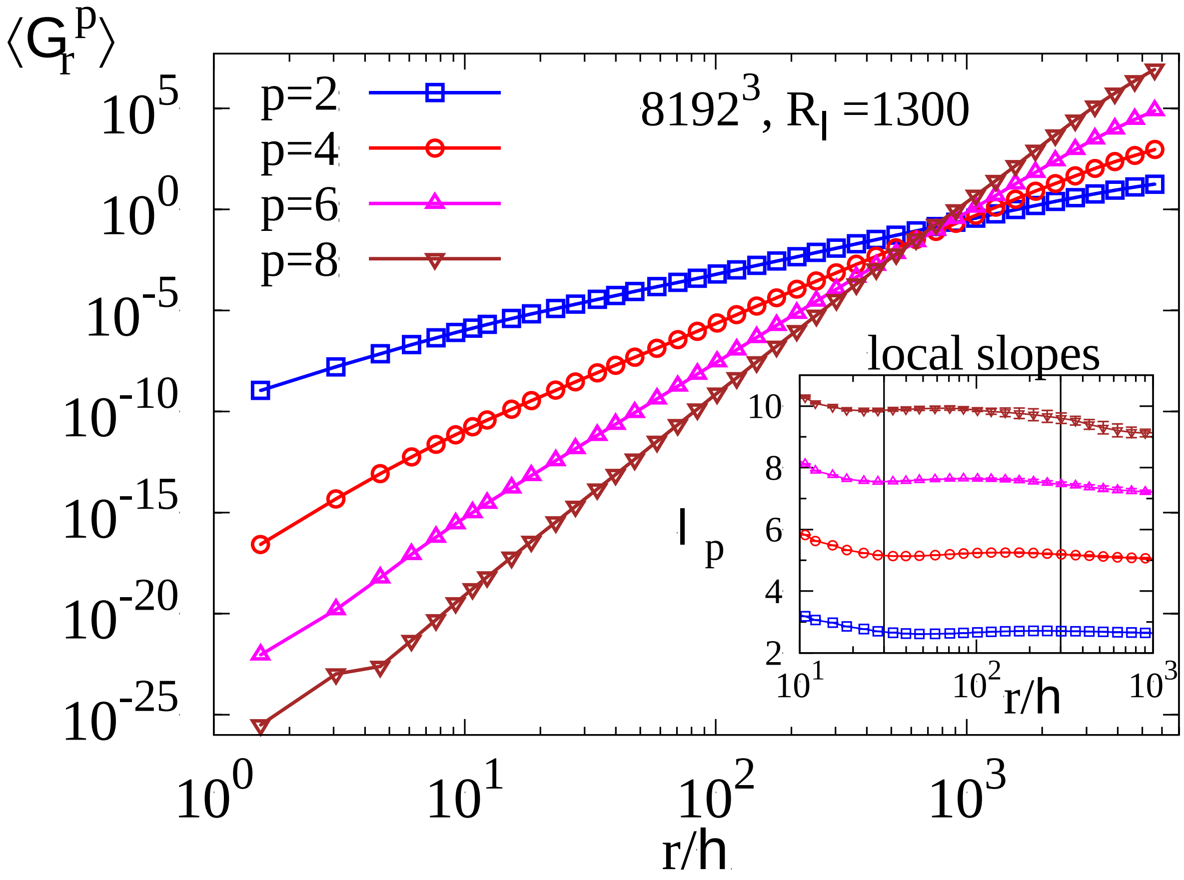


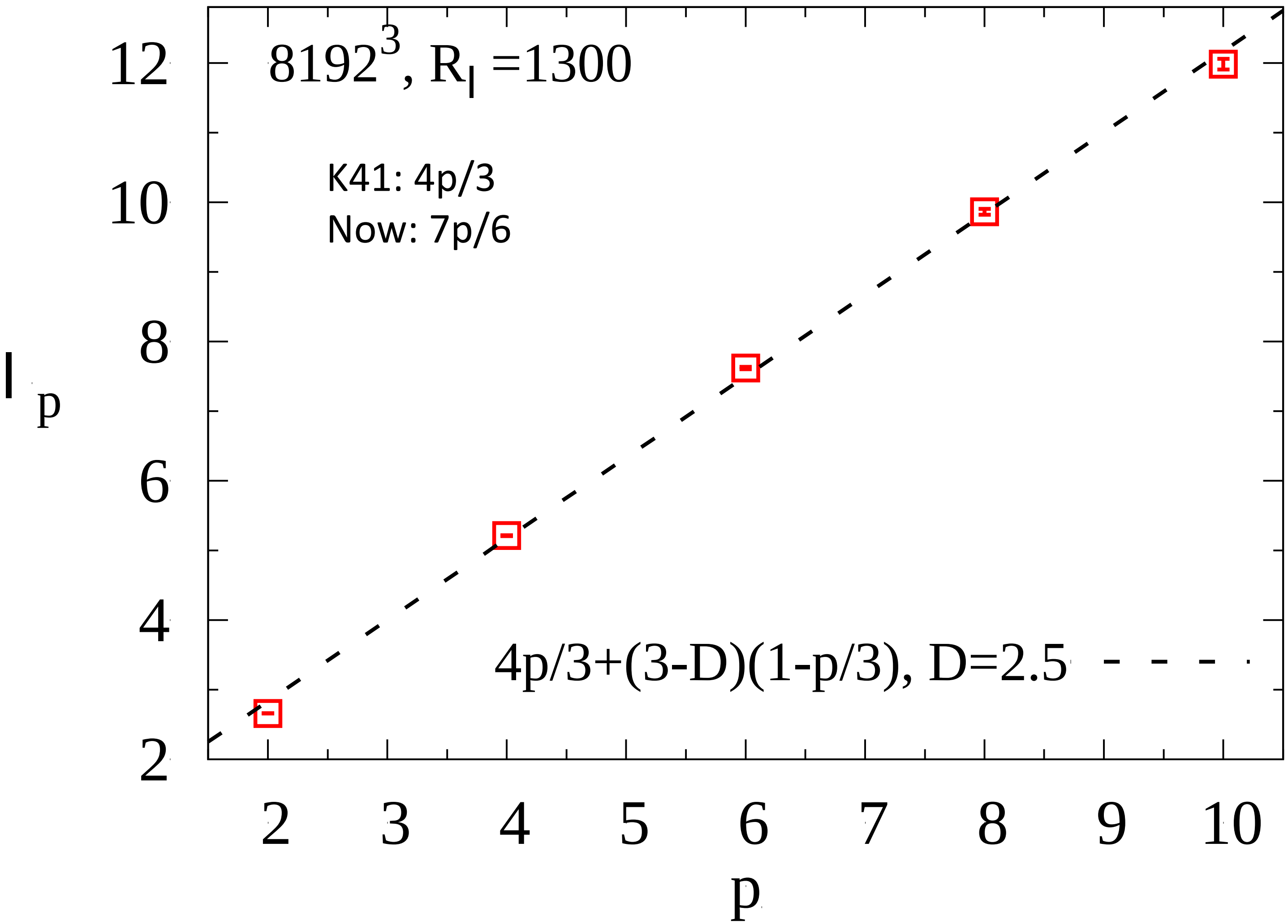
Fig. 9. Circulation standard deviation $\langle \Gamma_A^2 \rangle^{1/2}$, (■) as a function of the area A in the 8-loop contour, calculated on a 4096^3 grid at $R_\lambda = 650$. The area law follows the scalar area law, as evidenced by the $A^{2/3}$ scaling in the inertial range. The linear area dependence in the small-r regime, is also shown.



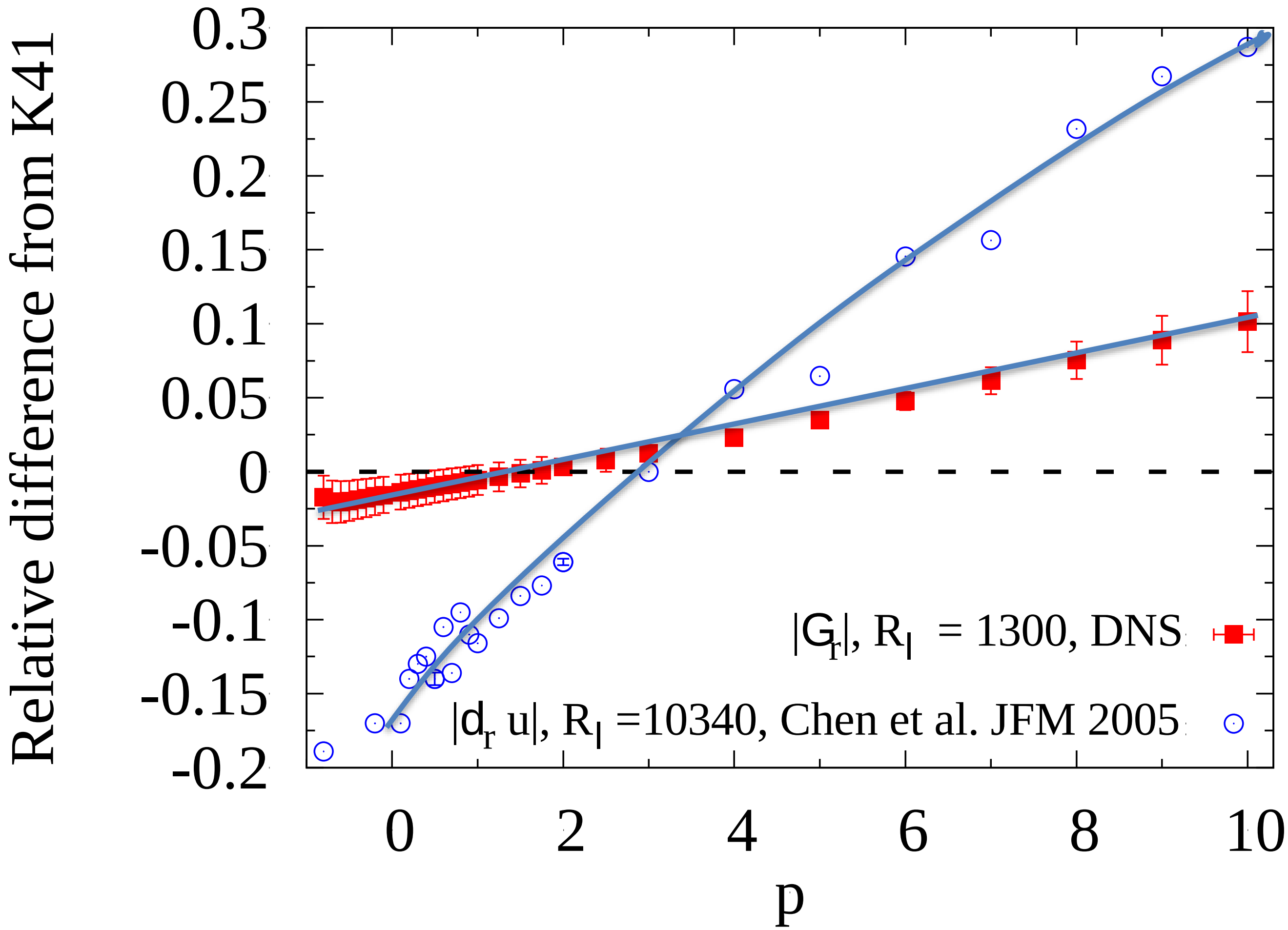
PDFs for r in the inertial range collapse well on Kolmogorov scaling (but are far from Gaussian)



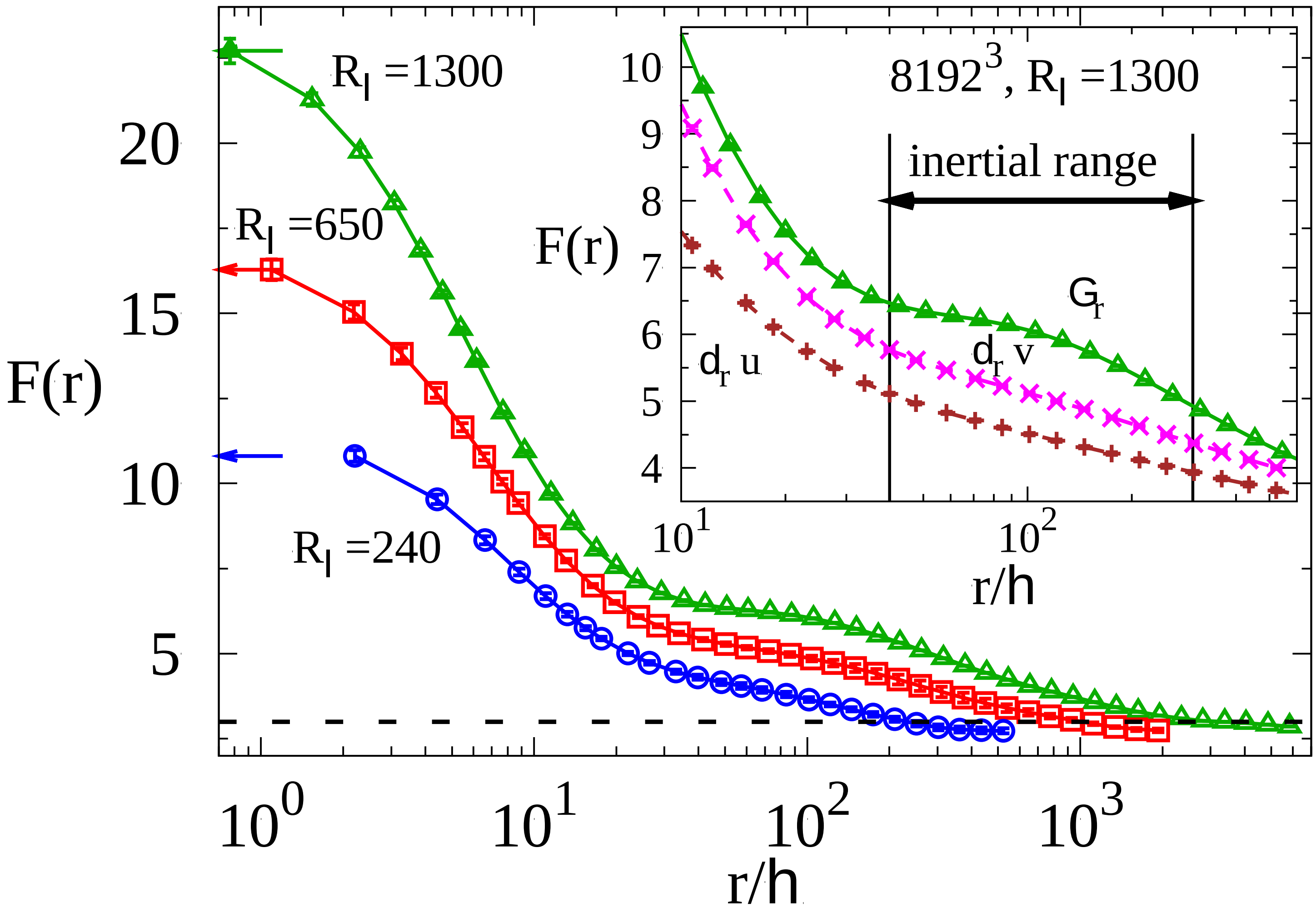
Moments of circulation scale quite well.



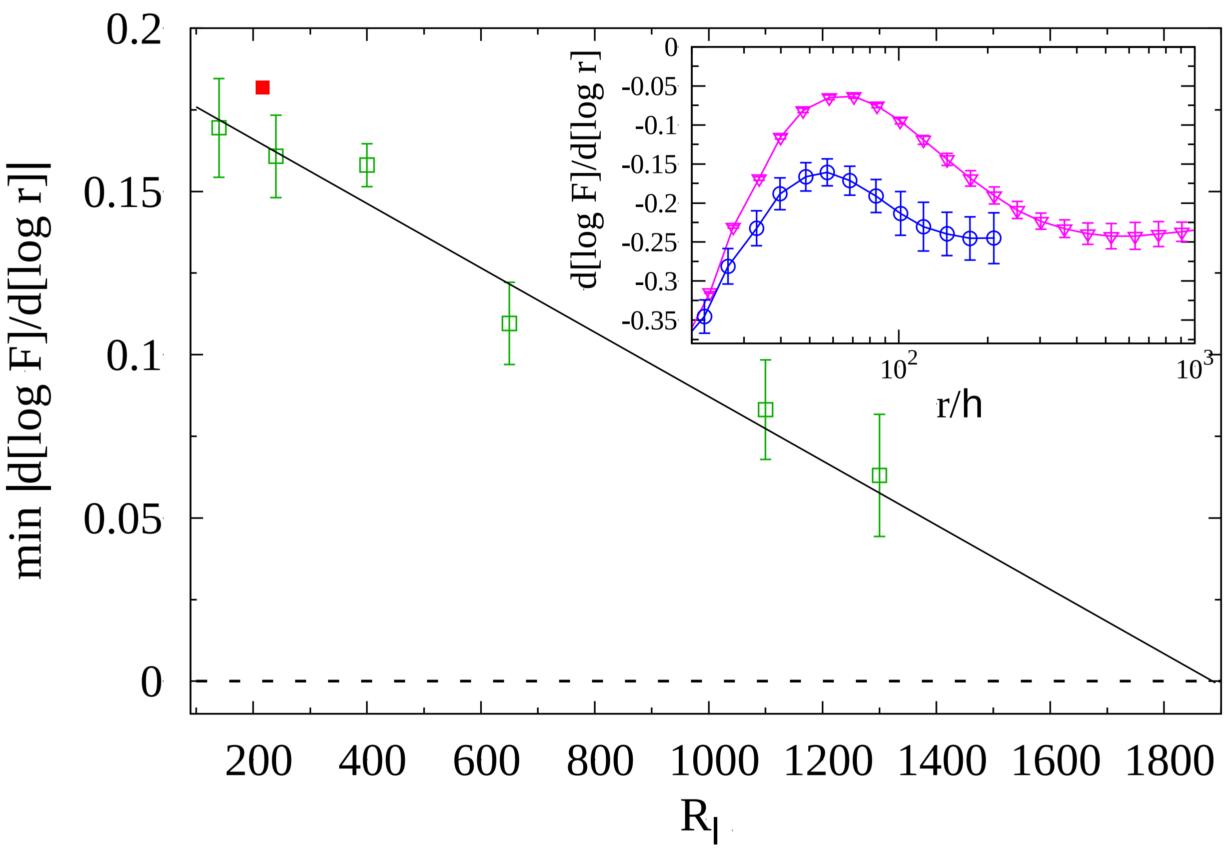
The scaling exponents are very closely linear with the order of the moment.



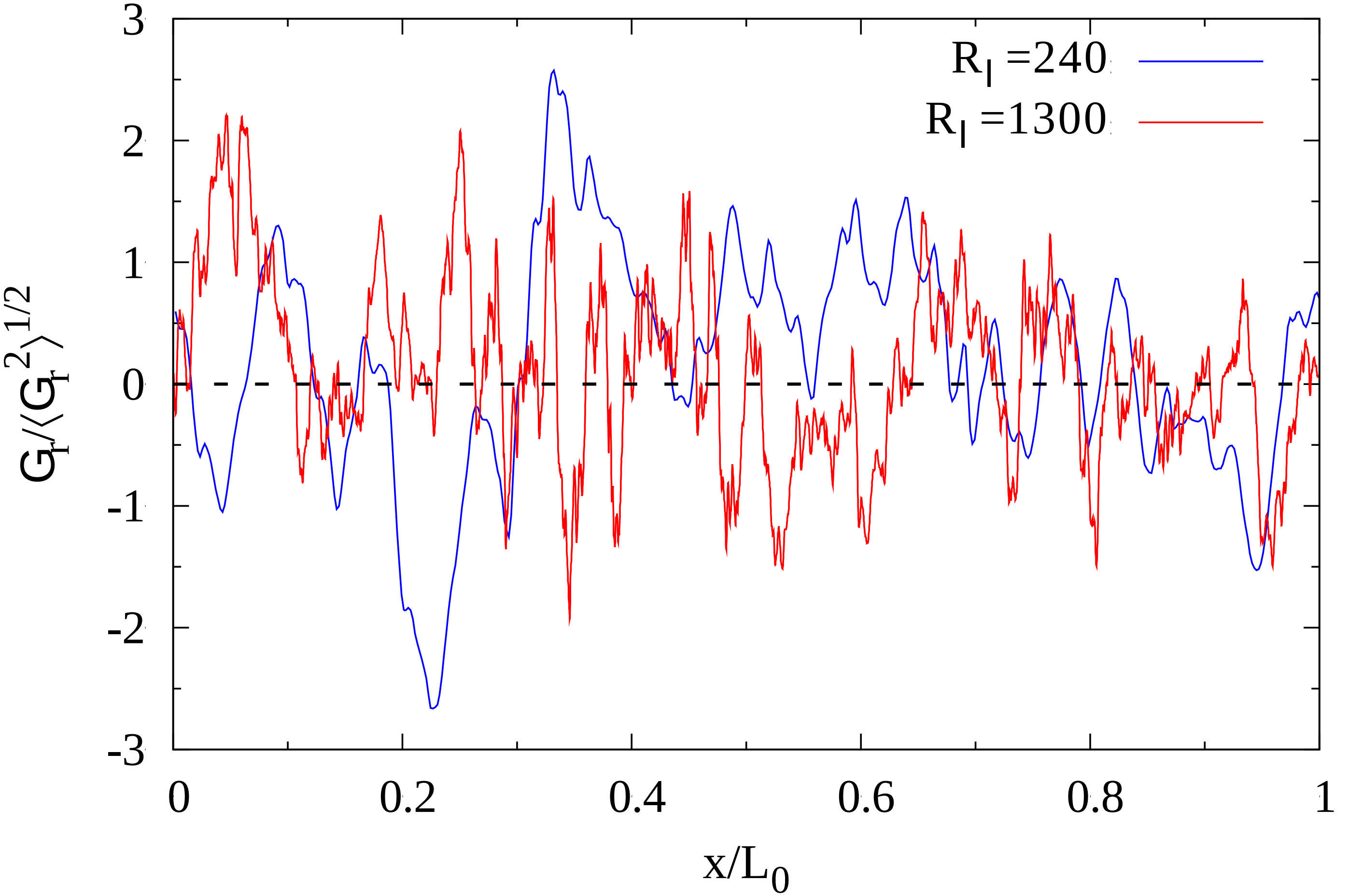
Circulation:
about a third
as much away
from K41,
essentially
linear.



Flatness approaches constancy in the inertial range as the Reynolds number goes up (unlike velocity increments).



0.2
0.15
0.1
0.05
0
200
400
600
800
1000
1200
1400
1600
1800
R_l
min |d[log F]/d[log r]|
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-0.35
d[log F]/d[log r]
10^2
10^3
r/h



$R_l = 240$
$R_l = 1300$
$G_r / \langle G_r^2 \rangle^{1/2}$
x/L_0
3
2
1
0
-1
-2
-3
0
0.2
0.4
0.6
0.8
1

Conclusions

1. I have been looking at scaling in turbulence for about 4 decades(while also doing other things) and everything has seemed anomalous---perhaps not circulation?
2. Circulation statistics depend *only* on the area of the loop, not on its shape.
3. Scalar area rule holds, not vector area rule
4. PDFs of Γ_r for r in the inertial range collapse well on Kolmogorov scaling (but are far from Gaussian)
5. Moments of circulation scale quite well, and the scaling exponents are very closely linear with the order of the moment. Circulation seems to reside on a fractal set of dimension 2.5.
6. One needs to double the Reynolds number before being conclusive. This should take another 5 years.

Some comments for the young audience

- Associate with people who are generally smarter than you, but don't lose your core to them.
- Develop a small support group of peers within which you can talk freely without worrying that you might be labeled as stupid.
- No one will give you credit for free but you will learn how to get the just credit for your work if you follow some rules.
  - Know what you want to do (this requires some contemplation and self-knowledge) and be prepared to sacrifice something for achieving it.
  - Learn to make a case for yourself without pretense or insincerity.
  - Learn the art of speaking and writing formally and effectively especially as a way to persuade or influence people.
  - Learn to defend yourself without being argumentative
  - Learn to handle criticism well without allowing it to warp your essence. This requires a degree of confidence; for some it is ingrained; for some it is acquired.