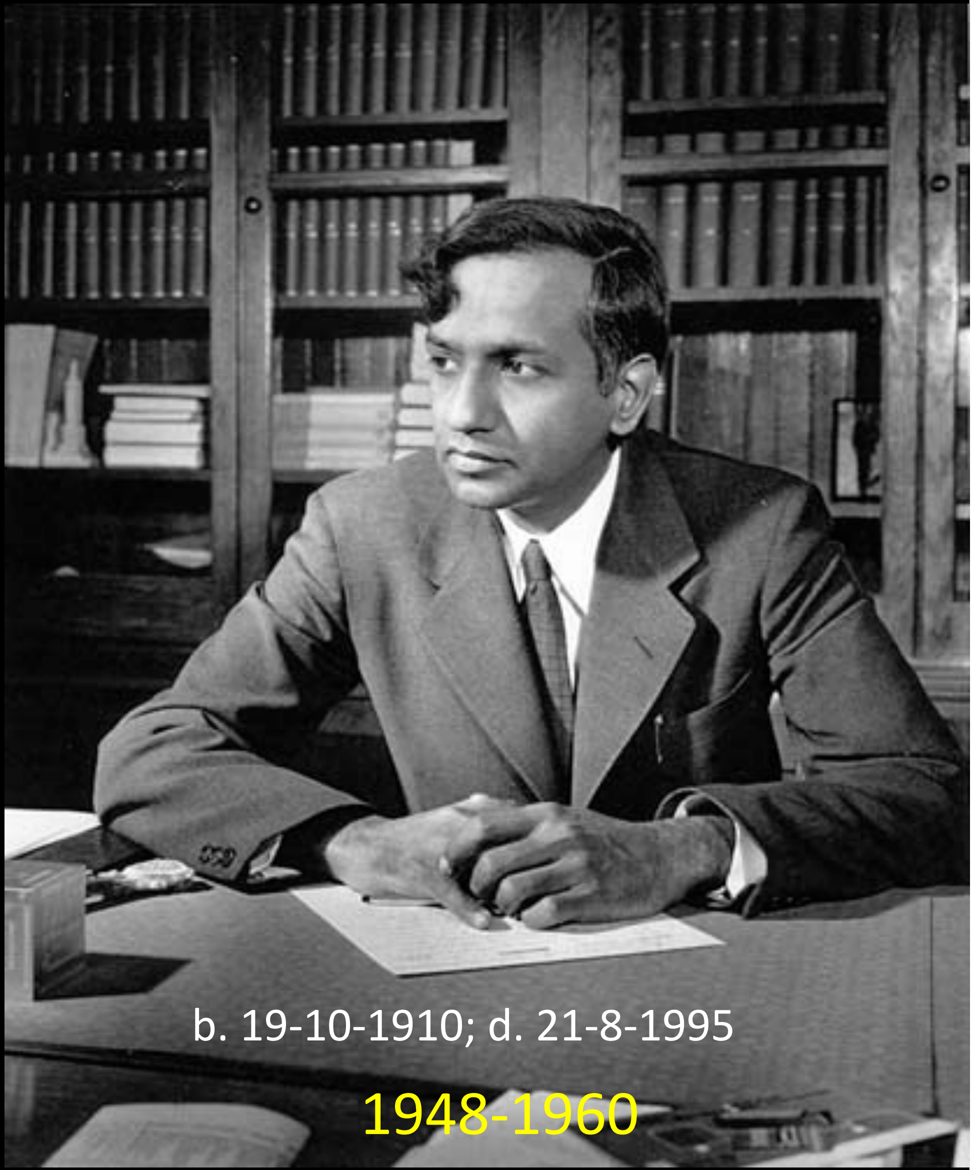
Chandrasekhar's fluid dynamics

K.R. Sreenivasan

ICTS Program on

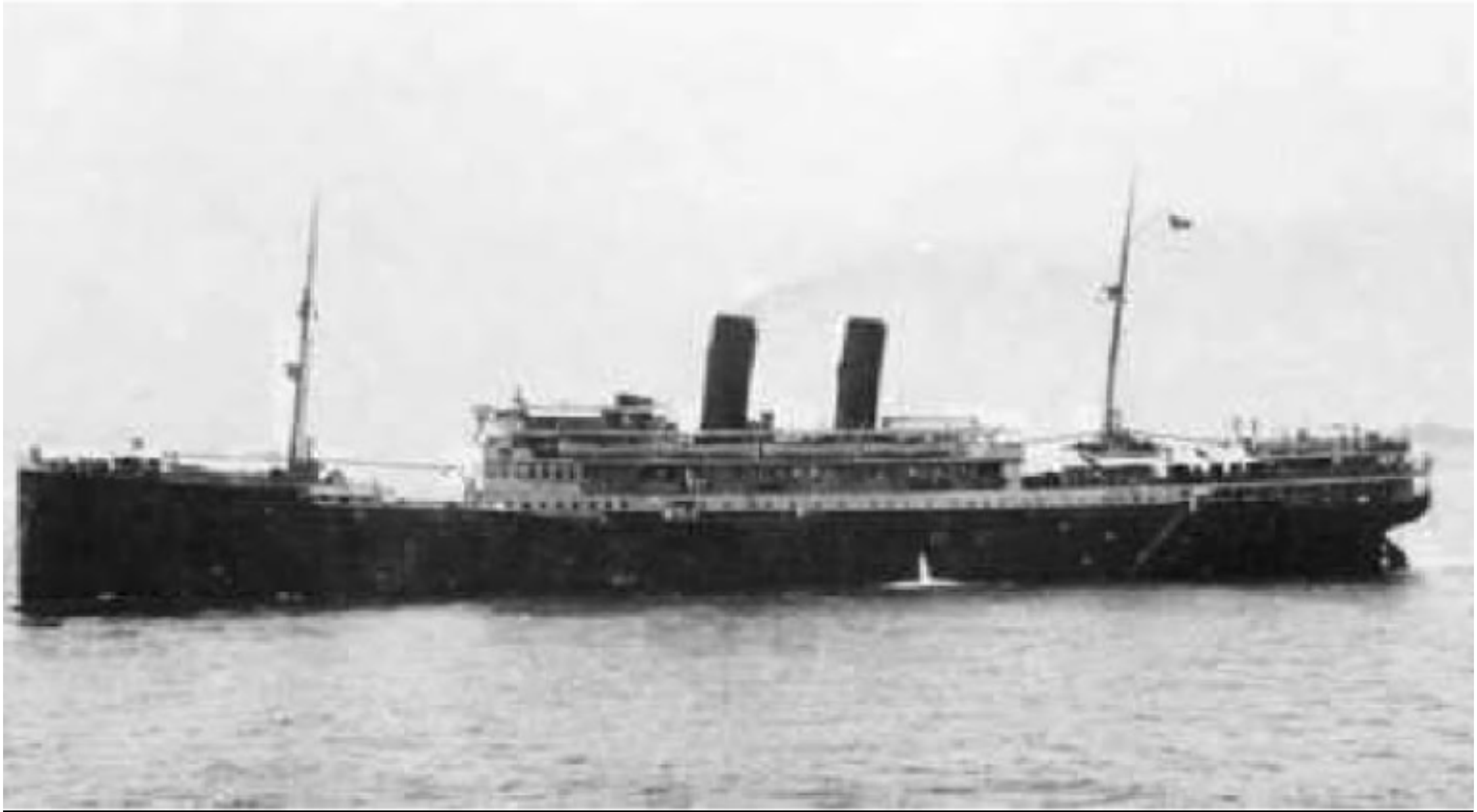
Turbulence from Angstroms to Light Years

Chandrasekhar Lecture – I



b. 19-10-1910; d. 21-8-1995

1948-1960



On August 1, 1930, the *SS Pilsna*, a member of the Lloyd Triestino fleet, carried the 19-year old Chandrasekhar from Bombay to Venice, en route to Cambridge, who reached England on August 19.

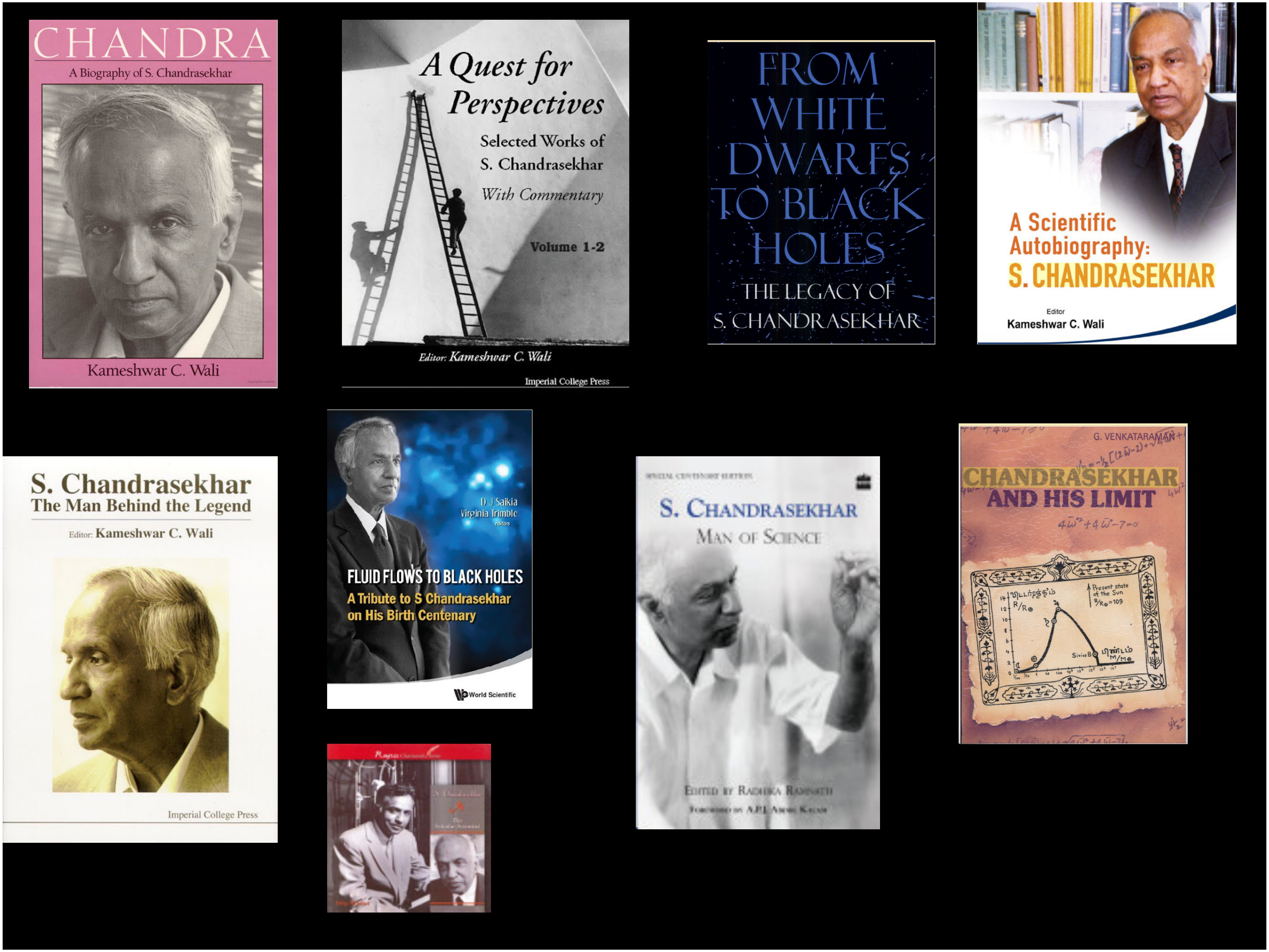
Chandrasekhar receiving the 1983 Physics Nobel Prize from King Gustav of Sweden for his work of *ca.* 1930 (inset)

Nobel Prize nomination (suggested nomination)

The nomination concerns the discovery (invention) of the direct connection between the principles of relativistic quantum mechanical degeneracy and the masses of the white dwarf stars.

And the supporting paragraphs included

From that brilliant beginning Chandrasekhar went on to fundamental work on the dynamics of stars, radiative transfer, hydromagnetic theory, the stability and the evolution of rotating self-gravitating fluid bodies, and the development of the post-Newtonian approximation to the equations of general relativity. A more recent feat has been his separation of the variables of the Dirac equation and the Newman-Penrose equations in the Kerr metric, providing the tool to deal effectively with quantum electrodynamics in strong gravitational field. Chandrasekhar has been a central figure in the development of all the major branches of modern astrophysics through his fundamental contributions to the physics of radiation, particles, and gravitational fields.



CHANDRA
A Biography of S. Chandrasekhar
Kameshwar C. Wali
A Quest for Perspectives
Selected Works of S. Chandrasekhar
With Commentary
Volume 1-2
Editor: Kameshwar C. Wali
Imperial College Press
FROM WHITE DWARFS TO BLACK HOLES
THE LEGACY OF S. CHANDRASEKHAR
A Scientific Autobiography: S. CHANDRASEKHAR
Editor
Kameshwar C. Wali
S. Chandrasekhar
The Man Behind the Legend
Editor: Kameshwar C. Wali
Imperial College Press
D J Saikia
Virginia Trimble
editors
FLUID FLOWS TO BLACK HOLES
A Tribute to S Chandrasekhar on His Birth Centenary
World Scientific
S. CHANDRASEKHAR
MAN OF SCIENCE
G. VENKATARAMAN
CHANDRASEKHAR AND HIS LIMIT
Present state of the Sun


great
INDIANS

Chandra had more than 65 years of outstanding scientific productivity:

(1) Stellar structure, including the theory of white dwarfs (1929-1939)

(2) Stellar dynamics, including the theory of Brownian motion (1938-43)

(3) The theory of radiative transfer (1943-50)

(4) Hydrodynamic and hydromagnetic stability and turbulence (1952-61)

(5) The equilibrium and stability of ellipsoidal figures of equilibrium (1961-68)

(6) The general theory of relativity and relativistic astrophysics (1962-71)

(7) The mathematical theory of black holes (1974-83)

For a dozen more years (1983-95), he worked on (a) the theory of colliding gravitational waves and non-radial perturbations of relativistic stars and, finally, on (b) *Newton's* Principia *for the Common Reader*, published just a few weeks before his death.

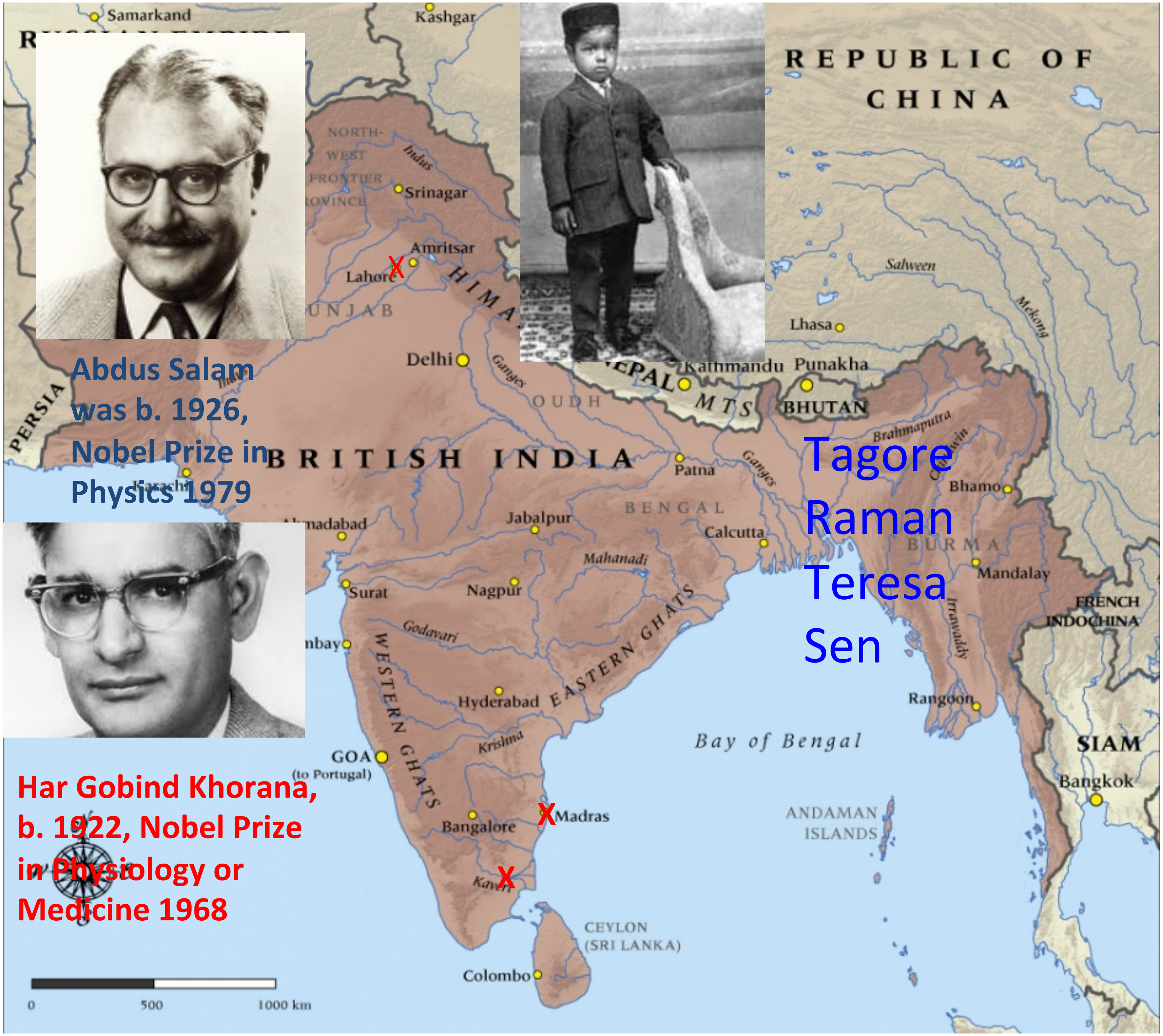
- Chandrasekhar limit
- Chandra X-ray Observatory
- Chandrasekhar friction
- Chandrasekhar-Kendall function
- Chandrasekhar number
- Chandrasekhar virial equations
- Chandrasekhar's Variational Principle, etc

Some scientific terms bearing Chandra's name


AN INTRODUCTION TO THE STUDY OF STELLAR STRUCTURE
S. CHANDRASEKHAR
DOVER PHOENIX EDITIONS
Principles of Stellar Dynamics
S. CHANDRASEKHAR
S. Chandrasekhar
Radiative Transfer
Hydrodynamic and Hydromagnetic Stability
S. Chandrasekhar
EDDINGTON
The most distinguished astrophysicist of his time
S. Chandrasekhar
Plasma Physics
Chandrasekhar, S.
Oxford Classic Texts
IN THE PHYSICAL SCIENCES
The Mathematical Theory of Black Holes
TRUTH AND BEAUTY
Aesthetics and Motivations in Science
S. Chandrasekhar
Ellipsoidal Figures of Equilibrium
S. Chandrasekhar
Indian Edition
S. Chandrasekhar
Newton's Principia
for the Common Reader
Lecture Notes in Physics 810
Edward A. Spiegel
Editor
The Theory of Turbulence
Subrahmanyan Chandrasekhar's 1954 Lectures
Springer

S. Chandrasekhar receiving the National Medal of Science from President Lyndon Johnson in 1967.

Influences



Abdus Salam was b. 1926, Nobel Prize in Physics 1979

Har Gobind Khorana, b. 1922, Nobel Prize in Physiology or Medicine 1968

Tagore
Raman
Teresa
Sen

“The single most important event in my life”

Sommerfeld’s visit to the Presidency College in 1928 and Chandra’s meeting with him

Chandra had mastered Sommerfeld’s “Atomic Structure and Spectral Lines”, but was told by Sommerfeld that his book was superseded by the new quantum mechanics, and was given the galley proofs of his article on the application of Fermi-Dirac statistics to electron theory of metals.

Arnold Sommerfeld

- Size of stars is determined by a balance between gravity and pressure due to fusion
- In course of time, fusion diminishes and the star contracts---but not to a point
- Indeed, there are objects, white dwarfs, which are “small” (size of the earth), but their density is very high (million times the Sun’s)
- What prevents the collapse from continuing on? No mechanism exists in classical physics
- Enter Ralph Fowler. The enormous internal pressure of stars strips atoms of electrons and we have a sea of protons and electrons. Quantum mechanically, Pauli’s exclusion principle (no two particles can occupy the same quantum state) produces a repulsive pressure to balance gravity.

R.W. Fowler
(Chandra’s thesis advisor)

Chandra recognized that the degenerate gas is relativistic and redid the calculations. Lo and behold, for stars exceeding a limit value of about 1.5 solar masses, the protection that Fowler had concocted does not exist. So massive stars are free to contract or suffer other fates (supernovae and neutron stars, black holes, etc). Our galaxies are no longer calm and quiet, but prone to violence and grandeur.

The Astrophysical Journal 74, 81-82 (1931)

THE MAXIMUM MASS OF IDEAL WHITE DWARFS

By S. CHANDRASEKHAR

ABSTRACT

The theory of the *polytropic gas spheres* in conjunction with the equation of state of a *relativistically degenerate electron-gas* leads to a *unique value for the mass of a star* built on this model. This mass ($=0.91\odot$) is interpreted as representing the upper limit to the mass of an ideal white dwarf.

In a paper appearing in the *Philosophical Magazine*,[1] the author has considered the density of white dwarfs from the point of view of the theory of the polytropic gas spheres, in conjunction with the degenerate non-relativistic form of the Fermi-Dirac statistics. The expression obtained for the density was

$$\rho=2.162\times10^{6}\times\left(\frac{M}{\odot}\right)^{2}, \tag{1}$$

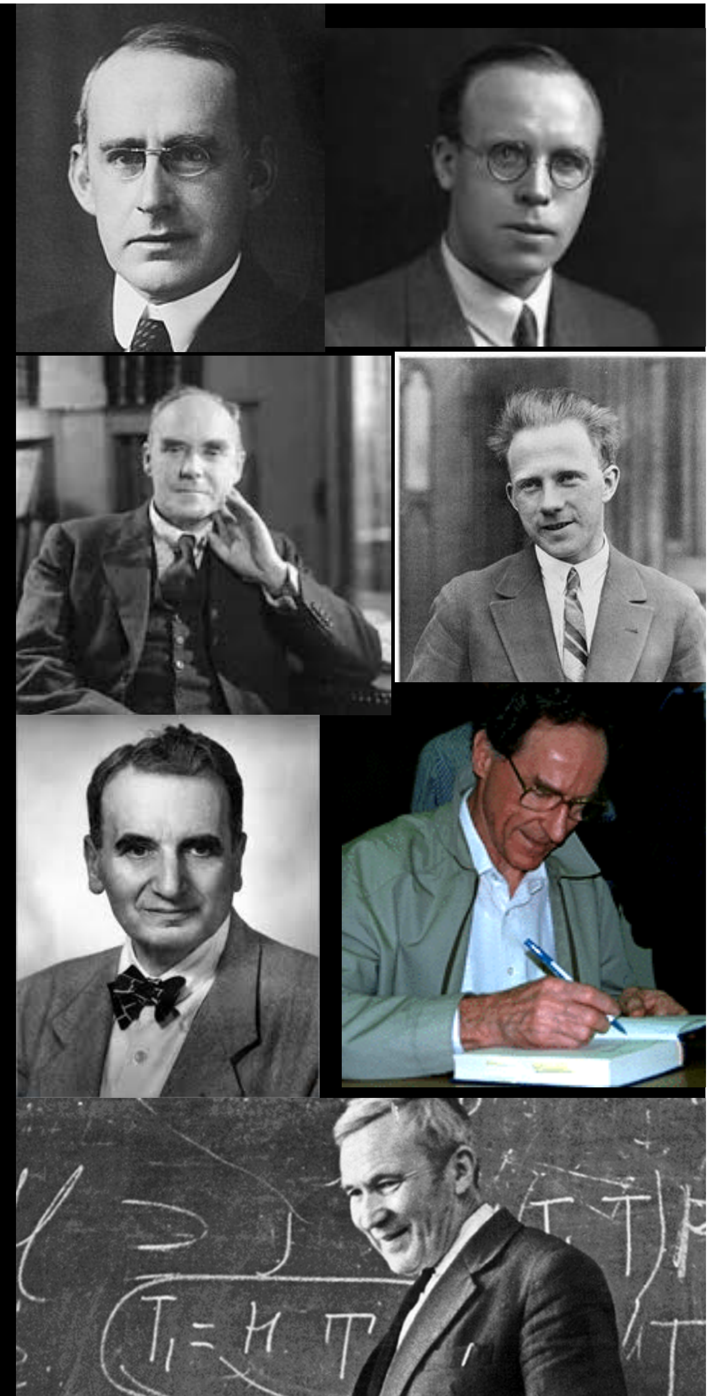
where $M/\odot$ equals the mass of the star in units of the sun. This formula was found to give a much better agreement with facts than the theory of E. C. Stoner,[2] based also on Fermi-Dirac statistics but on uniform distribution of density in the star which is not quite justifiable.

In this note it is proposed to inquire as to what we are able to get when we use the relativistic form of the Fermi-Dirac statistics for the degenerate case (an approximation applicable if the number of electrons per cubic centimeter is $> 6\times10^{29}$). The pressure of such a

The second paper in Zietschrift für Astrophysik 5, 321-327 (1932)

Entry into fluid mechanics

- When Chandra started work on radiation, he had left behind the influence of the stars of his student days and felt, for the first time, that he was on his own, no longer “intimidated by bigger people in front of me.” His two books, on stellar structure and dynamics, were becoming standard references, and he had just then been elected FRS in 1944 at the age of 34. His felt that his standing was secure.
- Brimming with confidence, Chandra began to explore a new area to work on, one in which problems “will find their solutions only a decade or two later," and settled on turbulence. It was clear to him that many interesting problems in astrophysics could not be solved unless turbulence was understood better: ``We cannot expect to incorporate the concept of turbulence in any essential manner without a basic physical theory of the phenomenon of turbulence itself.”
- In his so-called Monday evening seminars starting late 1940’s, he began to discuss the works of Taylor, Karman & Howarth, Kolmogorov, Batchelor, and Heisenberg.
- During the dozen or so years he spent on fluid dynamics (until 1961), he enjoyed his association with many young stars: Bill Reid, Norman Lebovitz, Russ Donnelly, Dave Fultz, Peter Vandervoort, Eugene Parker, Y. Nakagawa, and others.



“Analytical” theories (1948-1951)

- Heisenberg’s theory
- Axisymmetric turbulence
- Density fluctuations in compressible turbulence
- MHD turbulence
- Convective turbulence
- Effect of turbulence on gravitational instability

THE STORY OF THE 3-YEAR GAP

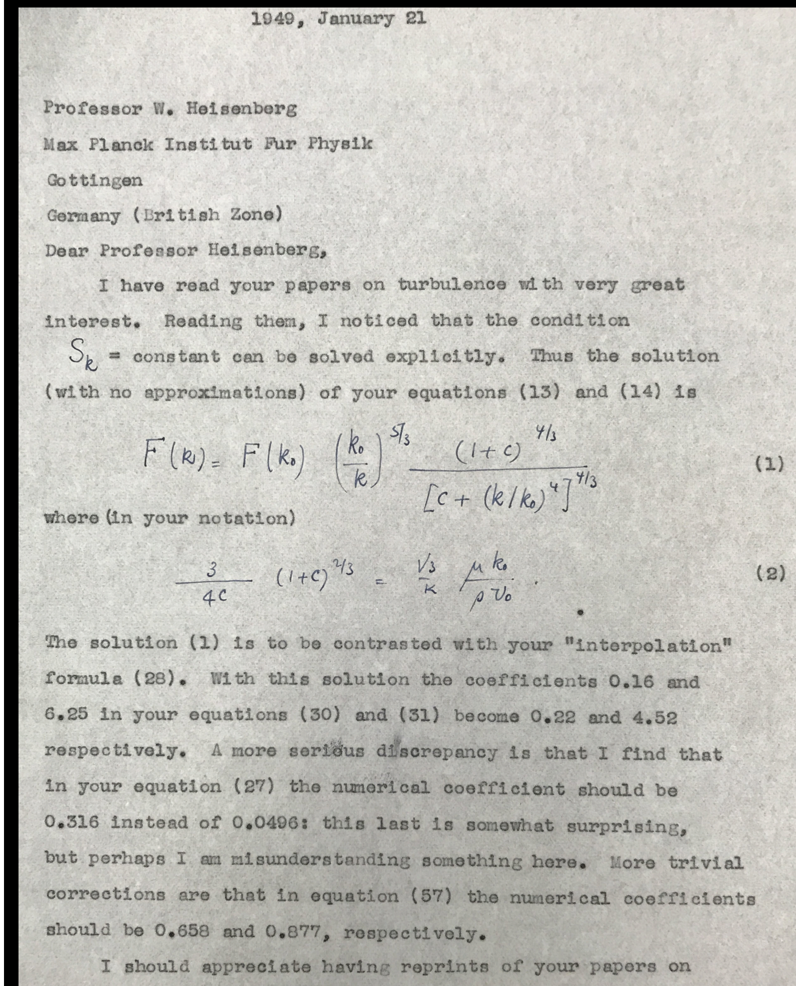
Dynamical theory (1954-56)

A theory of turbulence. *Proc. Roy. Soc. Lond.* **A229**, 1-19, 1955

Hydromagnetic turbulence. *Proc. Roy. Soc. Lond.* **A233**, 322-330, 1955

Theory of turbulence. *Phys. Rev* **102**, 941-51, 1956

Heisenberg, W. (1948). "On the theory of statistical and isotropic turbulence". *Proc. Roy. Soc.* **A195**: 402–6.



1949, January 21

Professor W. Heisenberg
Max Planck Institut Fur Physik
Gottingen
Germany (British Zone)

Dear Professor Heisenberg,

I have read your papers on turbulence with very great interest. Reading them, I noticed that the condition S_k = constant can be solved explicitly. Thus the solution (with no approximations) of your equations (13) and (14) is

$$F(k) = F(k_0)\left(\frac{k_0}{k}\right)^{5/3}\frac{(1+c)^{4/3}}{[c+(k/k_0)^4]^{4/3}} \qquad (1)$$

where (in your notation)

$$\frac{3}{4c}(1+c)^{2/3} = \frac{V_3}{\kappa}\,\frac{\mu k_0}{\rho v_0}. \qquad (2)$$

The solution (1) is to be contrasted with your "interpolation" formula (28). With this solution the coefficients 0.16 and 6.25 in your equations (30) and (31) become 0.22 and 4.52 respectively. A more serious discrepancy is that I find that in your equation (27) the numerical coefficient should be 0.316 instead of 0.0496: this last is somewhat surprising, but perhaps I am misunderstanding something here. More trivial corrections are that in equation (57) the numerical coefficients should be 0.658 and 0.877, respectively.

I should appreciate having reprints of your papers on

Eddy viscosity

$$\alpha \int_{\kappa}^{\infty} \{E(\kappa)/\kappa^3\}^{1/2}\, d\kappa \,,$$

α is a constant and $E(\kappa)$ is the energy spectral density in wavenumber κ. Heisenberg's integro-differential equation for $E(\kappa)$ is

$$\varepsilon = 2\left[\nu + \int_{\kappa}^{\infty} d\kappa'' \{E(\kappa'')/\kappa''^3\}^{1/2}\right]$$

$$\times \int_0^{\kappa} E(\kappa')\kappa'^2\, d\kappa'.$$

where ν is the coefficient of kinematic viscosity and

$$S_{ij} = \frac{\partial}{\partial \xi_k} (\overline{u_i u_k u'_j} - \overline{u_i u'_k u'_j}) + \frac{1}{\rho} \left(\overline{\frac{\partial p u'_j}{\partial \xi_i}} - \overline{\frac{\partial p' u_i}{\partial \xi_j}} \right). \tag{117}$$

In (117), the primes and the lack of primes distinguish the values of the quantities taken at x'_i and x_i, respectively; also ρ denotes the density and p the pressure.

We shall return to a detailed consideration of S_{ij} in § 10, but it is evident, meantime, that since both Q_{ij} and $\nabla^2 Q_{ij}$ are symmetrical in their indices and solenoidal, the tensor S_{ij} must also be symmetrical in its indices and solenoidal. Accordingly, S_{ij} must be derivable from a skew tensor of the form (49). Let S_1 and S_2 denote the defining scalars of S_{ij}.

Since the representation of the tensors and vectors we have adopted in terms of certain defining scalars is gauge-invariant and unique, we can directly pass from the tensor equation (116) to one between its defining scalars. Thus, remembering that the defining scalars of $\nabla^2 Q_{ij}$ are given by (61), we have

$$\frac{\partial Q_1}{\partial t} = 2\nu \Delta Q_1 + S_1 \tag{118}$$

and

$$\frac{\partial Q_2}{\partial t} = 2\nu (\Delta Q_2 + 2 D_{\mu\mu} Q_1) + S_2, \tag{119}$$

where it may be recalled that

$$\Delta = \frac{\partial^2}{\partial r^2} + \frac{4}{r}\frac{\partial}{\partial r} + \frac{1-\mu^2}{r^2}\frac{\partial^2}{\partial \mu^2} - \frac{4\mu}{r^2}\frac{\partial}{\partial \mu} \quad \text{and} \quad D_{\mu\mu} = \frac{1}{r^2}\frac{\partial^2}{\partial \mu^2}. \tag{120}$$

Equations (118) and (119) are the fundamental equations in the theory of axisymmetric turbulence and replace the equation of von Kármán & Howarth in the theory of isotropic turbulence.

9. The rate of change of the mean squares of the velocity and the vorticity components: the viscous dissipation of energy

The equations governing the rate of change of the mean squares of the velocity and the vorticity components can be derived from equations (118) and (119) in the following manner:

First, we note that with the series expansions (104) for Q_1 and Q_2 we obtain for the defining scalars of $\nabla^2 Q_{ij}$ the expansions

$$\left.\begin{aligned} \Delta Q_1 &= (10\alpha_{02} + 2\alpha_{22}) + r^2[(28\alpha_{04} + 2\alpha_{24}) + \mu^2(12\alpha_{44} + 18\alpha_{24})] + \dots, \\ \Delta Q_2 + 2D_{\mu\mu} Q_1 &= (10\beta_{02} + 2\beta_{22} + 4\alpha_{22}) + r^2[(28\beta_{04} + 2\beta_{24} + 4\alpha_{24}) \\ &\qquad + \mu^2(12\beta_{44} + 18\beta_{24} + 24\alpha_{44})] + \dots. \end{aligned}\right\} \tag{121}$$

Also, let

$$\left.\begin{aligned} S_1 &= \eta_{00} + r^2(\eta_{02} + \eta_{22}\mu^2) + \dots \\ S_2 &= \zeta_{00} + r^2(\zeta_{02} + \zeta_{22}\mu^2) + \dots. \end{aligned}\right\} \tag{122}$$

and

Substituting these expansions for Q_1, Q_2, etc., in equations (118) and (119), we obtain the equations:

$$\left.\begin{aligned} \frac{d\alpha_{00}}{dt} &= 2\nu(10\alpha_{02} + 2\alpha_{22}) + \eta_{00}, \\ \frac{d\beta_{00}}{dt} &= 2\nu(10\beta_{02} + 2\beta_{22} + 4\alpha_{22}) + \zeta_{00} \end{aligned}\right\} \tag{123}$$

A beginning in the theory of axisymmetric turbulence was made by Batchelor (1946). However, Batchelor did not develop the theory of *axisymmetric tensors* and *forms* far enough to derive the basic equations of the problem.

In this paper the theory of axisymmetric tensors will be developed to the same degree of completeness that Robertson developed the theory of isotropic tensors. It turns out that the essential part of the theory is that which pertains to solenoidal tensors and their representation as the curl of certain basic *skew* tensors—an aspect of the theory which Batchelor did not consider. With the theory of axisymmetric tensors and forms completed, the reduction of the equations of motion is straightforward. In this manner a pair of equations will be derived which, under the conditions of axisymmetric turbulence, replace the well-known equation of von Kármán & Howarth in the theory of isotropic turbulence.

A. Chandrasekhar, S. 1950a.
The theory of axisymmetric turbulence.
Phil. Trans. Roy. Soc. A **242**:557—77

B. Chandrasekhar, S. 1950b.
The decay of axisymmetric turbulence.
Proc. Roy. Soc. A **203**:358--64

The main idea was that solenoidal axisymmetric tensors can be represented by two scalar functions for which Chandra derived in A equations (# 118 and 119) to the level of the Karman-Howarth equation (1938) for isotropic turbulence. In B he worked out the final period of axisymmetric turbulence to the same level of completion as had Batchelor & Townsend (1948) for isotropic turbulence.

The story of the 3-year gap

``The description in terms of E(k) only (or f(r) only) would be complete only if there were no phase relationships between the different Fourier components of the velocity field. But this is not the case. Phase relationships must exist: without them there would be no exchange of energy between the different Fourier components which is, after all, the essence of the phenomenon of turbulence. A theory, albeit an approximate theory, must incorporate in itself some element which describes these phase relationships… It would appear that by introducing the correlations in the velocity components at two different points and at two different times, we can incorporate features which are the result of these phase relationships."

PHYSICAL REVIEW VOLUME 102, NUMBER 4 MAY 15, 1956

Theory of Turbulence*

S. CHANDRASEKHAR
Enrico Fermi Institute for Nuclear Studies, University of Chicago, Chicago, Illinois
(Received February 10, 1956)

It is pointed out if there are aspects of the turbulence phenomenon which are truly universal, then they should be capable of being characterized in terms of the two parameters ϵ and ν which denote the constant rate of dissipation of energy per unit mass and the kinemetic viscosity respectively and these two parameters only without reference either to the mean square velocity $\langle u_1^2\rangle_{Av}$ or to the size of the largest energy containing eddies. This is a slight modification of Kolmogoroff's similarity principles as currently formulated. It appears that

$$\chi=\partial\psi(r,t)/\partial r, \quad \text{where} \quad \psi(r,t)=\tfrac{1}{2}\langle(u_1'-u_1'')^2\rangle_{Av},$$

and u_1' and u_1'' are the velocities in the x-direction (say) at two points on the x-axis separated by a distance r and at times an interval t apart, can be so specified. The similarity principles require that if this is the case, χ should be of the form

$$\chi\equiv(\epsilon^3/\nu)^{\frac{1}{4}}X(r(\epsilon/\nu^3)^{\frac{1}{4}},\, t(\epsilon/\nu)^{\frac{1}{2}}),$$

where X is a universal function of the arguments specified. In the limit of zero viscosity, χ must have the more special form

$$\chi\rightarrow r^{-\frac{1}{3}}\sigma(t/r^{\frac{2}{3}}) \quad (\nu\rightarrow 0).$$

The boundary conditions on $\sigma(x)$ are that $\sigma=\sigma_0(>0)$ and $d\sigma/dx=0$ at $x=0$ and $\sigma\rightarrow 0$ as $x\rightarrow\infty$.

It is shown that with a slight modification of the premises of the theory described in an earlier paper, an equation for χ can be derived which is compatible with the requirements of the similarity principles as formulated. In particular the ordinary differential equation for σ to which the theory leads can be solved. The solution for σ which is found satisfies all the boundary conditions of the problem and is unique, apart from adjustable scale factors. The predicted evolutions of χ and the vorticity correlations are illustrated.

Letter of November 15, 1955 to Heisenberg (who had already made encouraging remarks about the work) complaining about the referee reports: ``Meanwhile, the Royal Society has rejected the paper on the basis of referees' report which among other things calls the paper `fallacious' and 'of no value'. I have tried to be as critical as I can, but I cannot see that there is anything unsound in what I have said in sections 2 and 3 of the paper. These are the sections to which the referees have objected. If you have had a chance to examine those sections, I shall be most grateful for any criticisms you may have. The referees of my paper have used rude language; but they have not stated arguments with substance. If I have gone astray I should like to know where; and I should appreciate any comments you have on these general ideas."

PHYSICAL REVIEW VOLUME 107, NUMBER 6 SEPTEMBER 15, 1957

Relation of Fourth-Order to Second-Order Moments in Stationary Isotropic Turbulence*

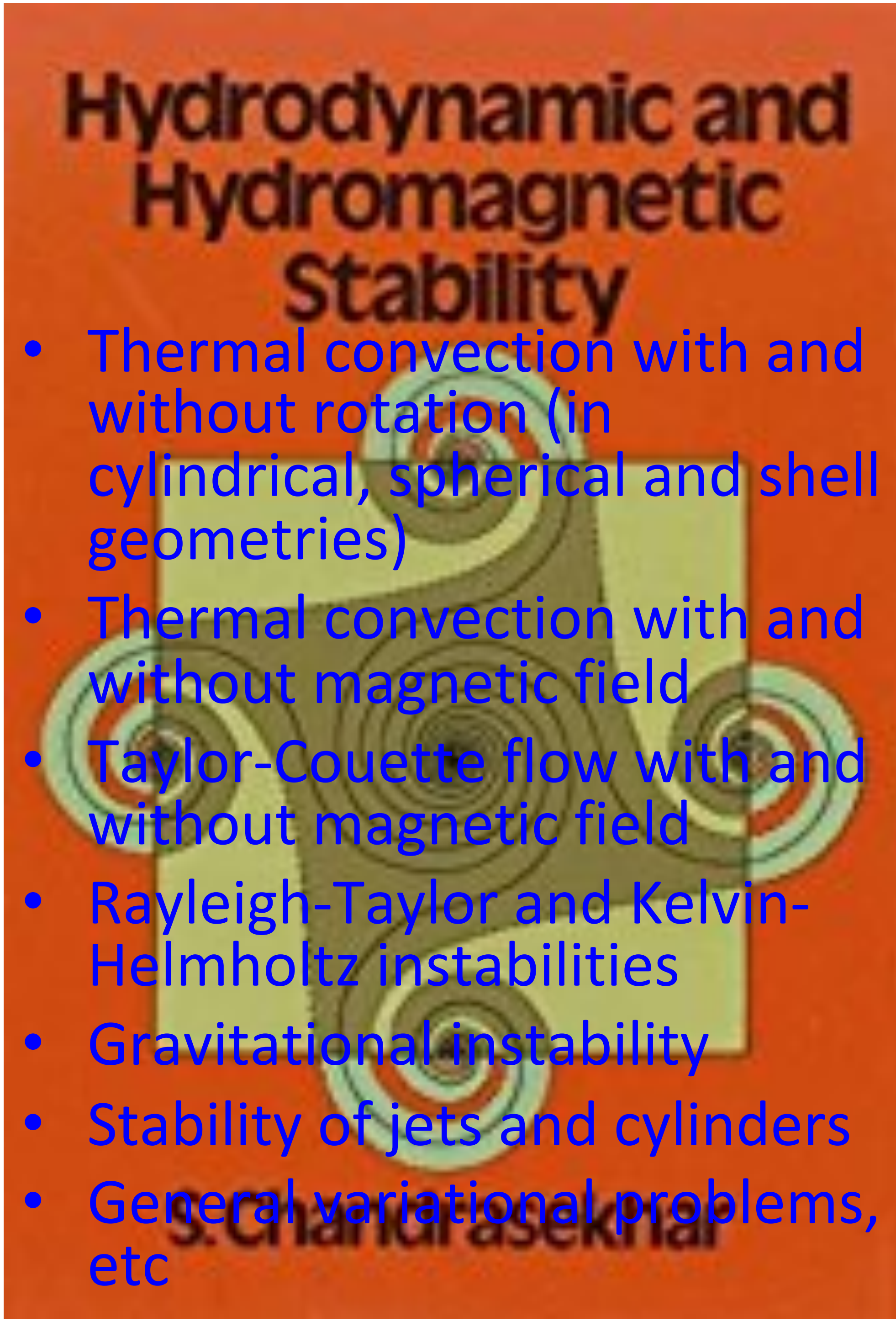
ROBERT H. KRAICHNAN
Institute of Mathematical Sciences, New York University, New York, New York
(Received December 14, 1956; revised manuscript received June 24, 1957)

An investigation is made of the hypothesis that the fourth-order moments of the two-time velocity amplitude distribution in stationary, isotropic, incompressible turbulence are related to second-order moments as in a normal distribution. It is concluded that this hypothesis is inconsistent with the equations of motion, and the inconsistency is exhibited as a gross violation of energy conservation in the inertial range. The origin of the inconsistency is discussed. The arguments developed are used to demonstrate inconsistencies in Chandrasekhar's recent theory of turbulence. Qualitative considerations are presented with regard to the consistency of the hypothesis that fourth-order moments of the simultaneous amplitude distribution are related to second-order moments as in a normal distribution. The general validity of this restricted hypothesis is questioned also.

Chandra published the rejected paper in Phys. Rev and moved on--- never again to return to turbulence.

Impact of the abrupt departure

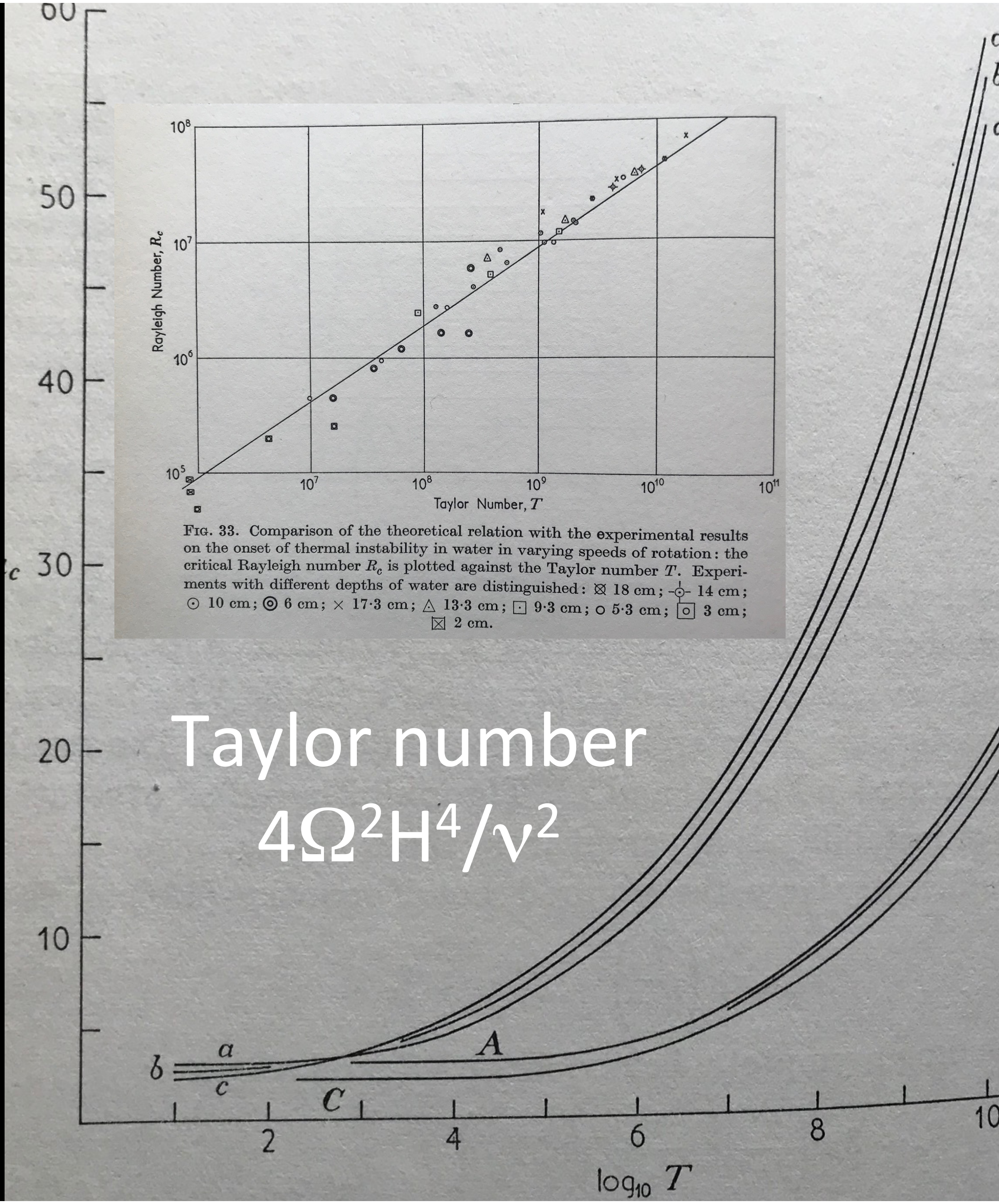
- The characterization that Chandra was wrong in some part of his work was generalized, and many people kept away from it.
- He himself did not produce any students in the field, stick around to defend the work, or clarify the subtleties.
- He was aware of it himself and expressed it sometimes (e.g., to Mahinder Uberoi).
- Others (e.g., Annick Pouquet) rederived some of his MHD results and the results are less often attributed to Chandra.



- Thermal convection with and without rotation (in cylindrical, spherical and shell geometries)
- Thermal convection with and without magnetic field
- Taylor-Couette flow with and without magnetic field
- Rayleigh-Taylor and Kelvin-Helmholtz instabilities
- Gravitational instability
- Stability of jets and cylinders
- General variational problems, etc

Gillis 1962, Physics Today: ``It is now at least half a century since it became clear to applied mathematicians that it would henceforth be prudent, before ever publishing any of their research, to check whether it had not already been done by Rayleigh. The time has come to amend this rule to read ``Rayleigh or Chandrasekhar". The latter's newest book, representing only one facet of his many-sided work, will stand for a long time as a text on problems and methods, a reference work of results, and a monument to the scientific power and erudition of its author."

Howard 1962, JFM: "The systematic theoretical treatment, the compact presentation of the results of many difficult numerical calculations, the discussion of experimental results and the extensive bibliography make this an extremely useful book for reference purposes---one which will be wanted in the library by all, and on the desk by many, of those whose work is connected with hydrodynamic or hydromagnetic stability."



Inhibition effects of rotation in thermal convection: Critical Rayleigh number and the critical wavenumber, as functions of the Taylor number.

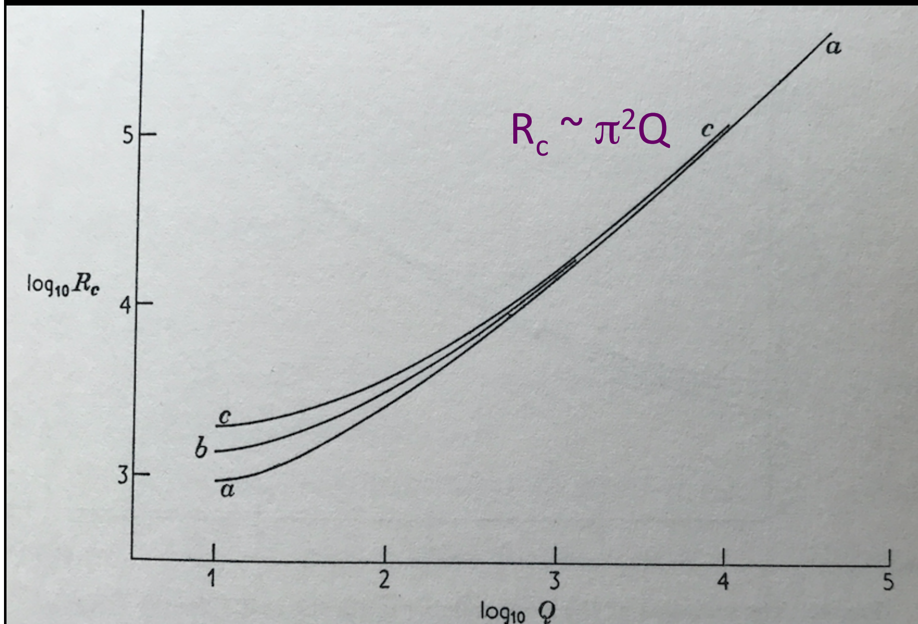


FIG. 39. The variation of the critical Rayleigh number R_c for the onset of instability as a function of Q for the three cases (i) both bounding surfaces free (curve labelled *aa*), (ii) one bounding surface free and the other rigid (curve labelled *b*), and (iii) both bounding surfaces rigid (curve labelled *cc*).

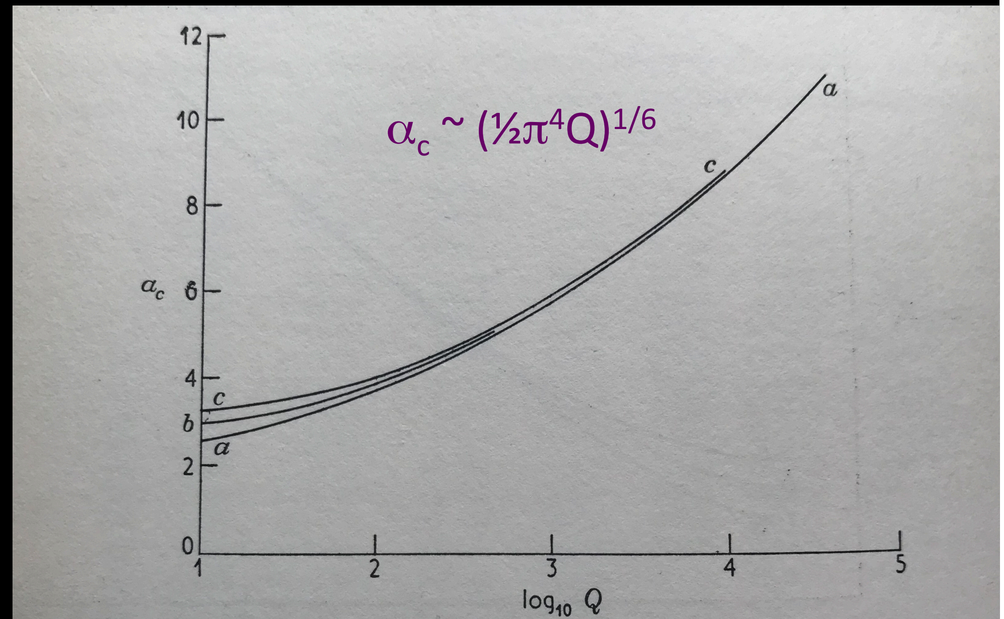


FIG. 40. The variation of the wave number a_c (in the unit $1/d$) at the onset of instability as a function of Q for the three cases (i) both bounding surfaces free (curve labelled *aa*), (ii) one bounding surface free and the other rigid (curve labelled *b*), and (iii) both bounding surfaces rigid (curve labelled *cc*).

Chandrasekhar number $Q = \dfrac{B^2 H^2}{\mu_o \rho \nu \lambda}$ is the ratio of Lorentz force to viscous force

B = magnetic field strength, H = height between walls, μ_0 = magnetic permeability,
ρ = density of fluid, ν = fluid viscosity, λ = magnetic diffusivity

Inhibitory effects of the magnetic field

Two virtues of the book

1. With a few exceptions, everything of interest in linear stability of classical hydrodynamic and hydrodynamic stability can be found in the book. Many problems of stability are discussed in the book using the same style and the same techniques, so if a new student of stability masters the techniques once, gaining entry into all other problems is easy (despite some quaint terminology).
2. It still acts as an inspiration for the application of modern computing in the context of magnetohydrodynamic problems of astrophysical context.

But it is heavily driven by astrophysical contexts

1. By design, the paradigm problem of stability that was not considered is the viscous shear flow, because Lin's (1955) book had just covered it.
2. Internal gravity waves, baroclinic instability, Rossby waves

- The book may appear as a masterly account of many stability problems in classical astrophysical fluid dynamics, written in leisure and quiet.
- But a look at Chandra's autobiographical remarks (see Wali 2011) shows the enormous pressure under which he operated, or chose to operate.
- Chandra had committed to a deadline of spring of 1960 to deliver the manuscript to the publisher, and was racing against time to meet it. A few selected quotes may describe the frenzy of the three weeks that preceded the submission.
- ``Only three weeks were now left... Starting Chapter XIII under extreme pressure, I realized that the virial theorem should have to be formulated in tensor form. The existing treatments had many loopholes and were quite unsatisfactory. I developed a whole new approach...
- I had to organize all the figures... When all this was finished, I was so tired that I decided to go to New York to give my invited talk to the American Mathematical Society...
- On returning from New York, the weekend and Monday were spent on various sections of the book which had been incomplete...
- It was finally on Tuesday morning that I started on Chapter XIV... I actually thought I would abandon the idea... I knew this would disappoint Donna (Chandra's secretary) and so I decided that I would start on the chapter anyway...
- The theory was worked out by late Wednesday evening; and I wrote up a first draft before going to bed. Early on Thursday morning, I started my second draft. By noon I was ready for the nth draft. (By this time, I was in a constant state of nausea.)…it was finally completed by 9:30 p.m. I called Donna at that time and she came over to start typing the last chapter…
- Most of the Friday morning was occupied by filling in the formulae...
- Early on Saturday, Norman Lebovitz drove us to O'Hare...
- In London the following day, April 24, the manuscript was handed over to Mr. Wood of the Clarendon Press."

Chandra did take a four-month `break' in the fall of 1961 in India, but delivered some seventy lectures at various academic and research institutions.

War-time work on shock waves

The normal reflection of a blast wave. Ballistics Research Laboratories Report, #439, 1943

Stability of Taylor-Couette flow of He-II

Chandrasekhar, S. & Donnelly, R.J. *Proc. Roy. Soc.* **A241**:9-28, 1957

Chandrasekhar, S. The hydrodynamic stability of helium II between rotating cylinders. II, *Proc. Roy. Soc.* **A241**:29-36, 1957

Some personal comments

From Wali (1984), pp. 305-6

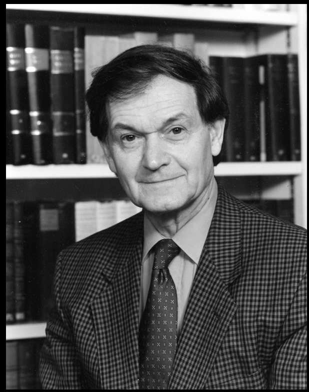
"The hope for contentment and a peaceful outlook on life as a result of pursuing a goal has remained unfulfilled"… "I don't really have a sense of fulfillment"…"I find it very difficult to reconcile with: namely, to pursue certain goals all your life only to become doubtful of those goals at the end"…

"My unhappiness or discontent is…because of the distortion, in some sense, of my life, of its one-sidedness, of the consequent loneliness, and my inability to escape from it all."

- A highly accomplished and devoted scientist with a huge capacity for disciplined work
- Wrote extremely well
- Was a great teacher
- Sure of his science, not particularly arrogant
- More of the "last word" type
- Didn't invoke intuition often
- Held himself to very high standards
- Hard task master
- Expected high standards also of others (and was often disappointed)
- Had strong likes and dislikes
- Didn't handle criticism well
- Felt lonely and unhappy despite his many attainments
- Felt underappreciated sometimes

A slightly dubious anecdote about Chandra

Chandra to Roger Penrose: Roger, do you use artistic considerations in your scientific work?



R (diplomatically): Do you, Chandra?

C: What do *you* think?

R: I think your work is like Claude Monet's.

Note: Monet lived 86 years, painted nearly to the end, and in the impressionism style which he more or less created, prodigiously painted between 1200 and 2000 canvasses, small and large.

Apparently for some time after this conversation, Chandra used to repeat that Monet was underappreciated most of his life. When others pointed out that it was really not so, Chandra would say, "No, it is true: he was underappreciated." Acclaim was slow in coming to Monet, but he had the support of many even during the early years.









Sample paintings of Claude Monet 1840-1926



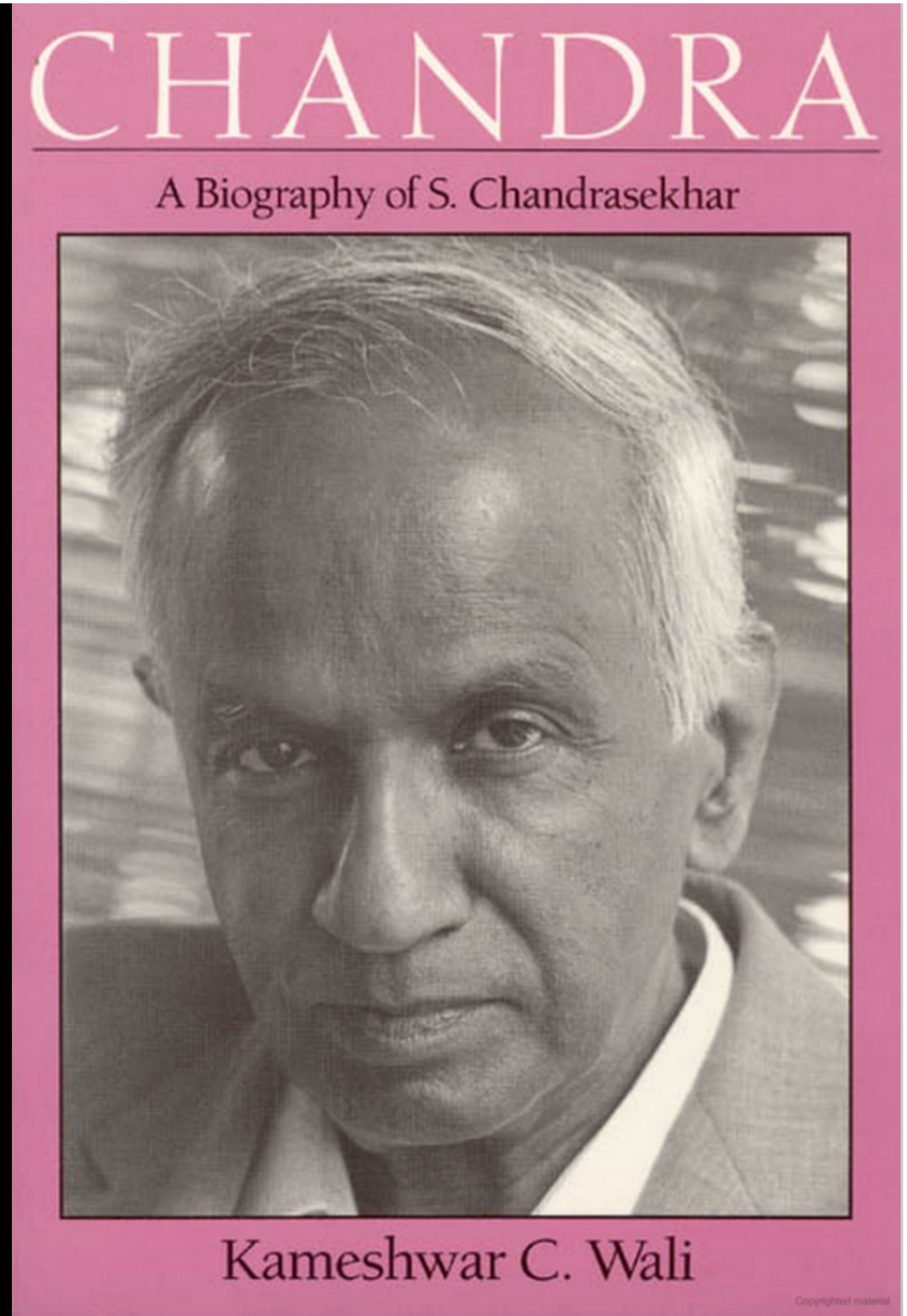
“On one occasion, now more than 50 years ago, Milne reminded me that posterity, in time, will give us all our true measure and assign to each of us our due and humble place; and in the end it is the judgment of posterity that really matters. And Milne further added: He really succeeds who perseveres according to his lights, unaffected by fortune, good or bad. And it is well to remember that there is in general no correlation between the judgment of posterity and the judgment of contemporaries.”

Chandrasekhara

From Wali (1984), pp. 305-6

"The hope for contentment and a peaceful outlook on life as a result of pursuing a goal has remained unfulfilled"… "I don't really have a sense of fulfillment"…"I find it very difficult to reconcile with: namely, to pursue certain goals all your life only to become doubtful of those goals at the end"…

"My unhappiness or discontent is…because of the distortion, in some sense, of my life, of its one-sidedness, of the consequent loneliness, and my inability to escape from it all."



Thank you for your attention

[redacted]

(1) Stellar structure, including the theory of white dwarfs (1929-1939)

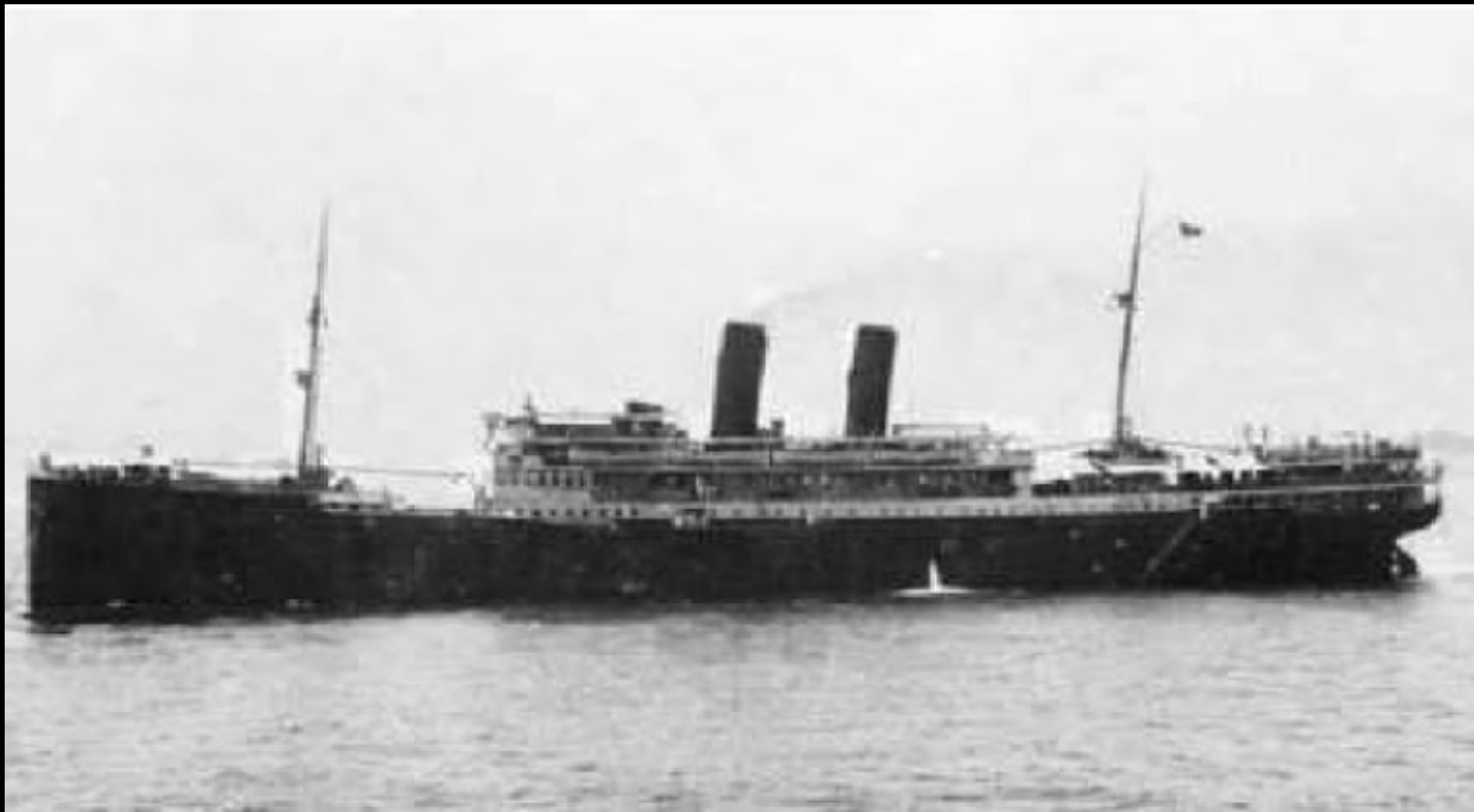
Nobel Prize nomination (suggested citation)

The nomination concerns the discovery (invention) of the direct connection between the principles of relativistic quantum mechanical degeneracy and the masses of the white dwarf stars.

And the supporting paragraphs included

From that brilliant beginning Chandrasekhar went on to fundamental work on the dynamics of stars, radiative transfer, hydromagnetic theory, the stability and the evolution of rotating self-gravitating fluid bodies, and the development of the post-Newtonian approximation to the equations of general relativity. A more recent feat has been his separation of the variables of the Dirac equation and the Newman-Penrose equations in the Kerr metric, providing the tool to deal effectively with quantum electrodynamics in strong gravitational field. Chandrasekhar has been a central figure in the development of all the major branches of modern astrophysics through his fundamental contributions to the physics of radiation, particles, and gravitational fields.

Chandra receiving the 1983 Physics Nobel Prize from King Gustav of Sweden for his work of *ca.* 1930 (inset)



The SS Pilsna, a member of the fleet of Lloyd Triestino fleet, set sail to Venice on August 1, 1930, and carried the 19-year old Chandra, en route to Cambridge.

Chandra had more than 65 years of outstanding scientific productivity:

(1) Stellar structure, including the theory of white dwarfs (1929-1939)

(2) Stellar dynamics, including the theory of Brownian motion (1938-43)

(3) The theory of radiative transfer (1943-50)

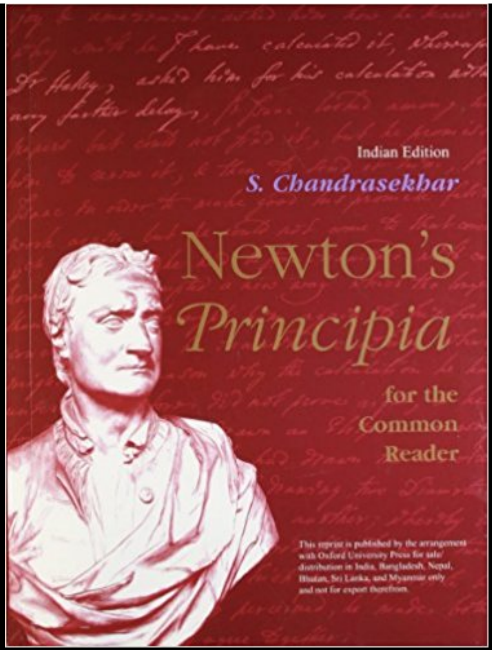
(4) Hydrodynamic and hydromagnetic stability and turbulence (1952-61)

(5) The equilibrium and stability of ellipsoidal figures of equilibrium (1961-68)

(6) The general theory of relativity and relativistic astrophysics (1962-71)

(7) The mathematical theory of black holes (1974-83)

For a dozen more years (1983-95), he worked on (a) the theory of colliding gravitational waves and non-radial perturbations of relativistic stars and, finally, on (b) *Newton's* Principia *for the Common Reader*, published just a few weeks before his death.



"Chandrasekhar is well known and justly praised for the high standards he enforced during his nineteen years as editor of the *Astrophysical Journal.* The history of science has its standards, too… If *Newton's* Principia *for the common reader* had met these standards, it would have been of singular value. As it is, it is a very useful, though flawed, addition to the Newton literature. Anyone seriously interested in the Principia ought to read it. It will serve the readers best, however, if they do not restrict themselves to it alone."

George E. Smith, Tufts U, J. History of Astronomy, 96: 353-361, 1996

Nobel Prize nomination (suggested nomination)

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[redacted]

physical circumstance that, fundamentally, a description of turbulence in terms only of its spectrum $F(k)$ (or, equivalently, in terms only of the scalar $Q(r)$ defining the tensor $\overline{u_i u'_j}$) cannot be a complete one. A description in terms of $F(k)$ only (or $Q(r)$ only) would be complete only if there were no phase relationships between the different Fourier components of the velocity field. But this is not the case. Phase relationships must exist: without them there would be no exchange of energy between the different Fourier components which is, after all, the essence of the phenomenon of turbulence. A theory, *albeit* an approximate theory, must incorporate in itself some element which describes these phase relationships; without such an element the theory would lack the means of accounting for the essence of the phenomenon. It would appear that by introducing the correlations in the velocity components at two different points and at two different times, we can incorporate features which are the result of these phase relationships. As we shall see, by

A theory of turbulence, Proc. Roy. Soc. Lond. Axxx xx-xx, 1955
Theory of turbulence, Phys. Rev 102, 941-51, 1956

Dynamical theory

Define

$$\chi = -\delta\psi(r,t)/dr, \text{ where } \psi(r,t) = \langle(u' - u'')^2\rangle,$$

with u' and u" denoting the velocities in the x-direction (say) at two points on the x-axis separated by a distance r and a time interval t.

The Kolmogorov similarity principles when applied to χ show that it should be of the form

$$\chi = (\varepsilon^3/\nu)^{1/4} X[r(\varepsilon/\nu^3)^{1/4}, t(\varepsilon/\nu)^{1/2}],$$

where X is a universal function of its arguments, and ε and ν are the energy dissipation rate and kinematic viscosity, respectively. In the limit of zero viscosity (or infinite Reynolds number), X must have the special form

$$X \sim r^{-1/3} \sigma(t/r^{2/3}).$$

The boundary conditions on $\sigma(x)$ are that $\sigma = \sigma^*$ (>0) and $d\sigma/dx = 0$ at x=0 and $\sigma \longrightarrow 0$ as $x \longrightarrow \infty$.

Chandra solved the equation numerically for σ under the quasi-Gaussian assumption.

S. Chandrasekhar's fluid dynamics

K.R. Sreenivasan
New York University

S. Chandrasekhar Lecture - I
International Center for Theoretical Sciences, TIFR
Bengaluru

Chandra’s 66 years of scientific productivity

(1) Stellar structure, including the theory of white dwarfs (1929-1939)

(2) Stellar dynamics, including the theory of Brownian motion (1938-43)

(3) The theory of radiative transfer, including the theory of stellar atmospheres and the quantum theory of the negative ion of hydrogen and the theory of planetary and stellar atmospheres, including the illumination and the polarization of sunlit sky (1943-50)

(4) Hydrodynamic and hydromagnetic stability, including the theory of Rayleigh-Bènard convection (1952-61)

(5) The equilibrium and stability of ellipsoidal figures of equilibrium, partly in collaboration with Norman R. Lebovitz (1961-68)

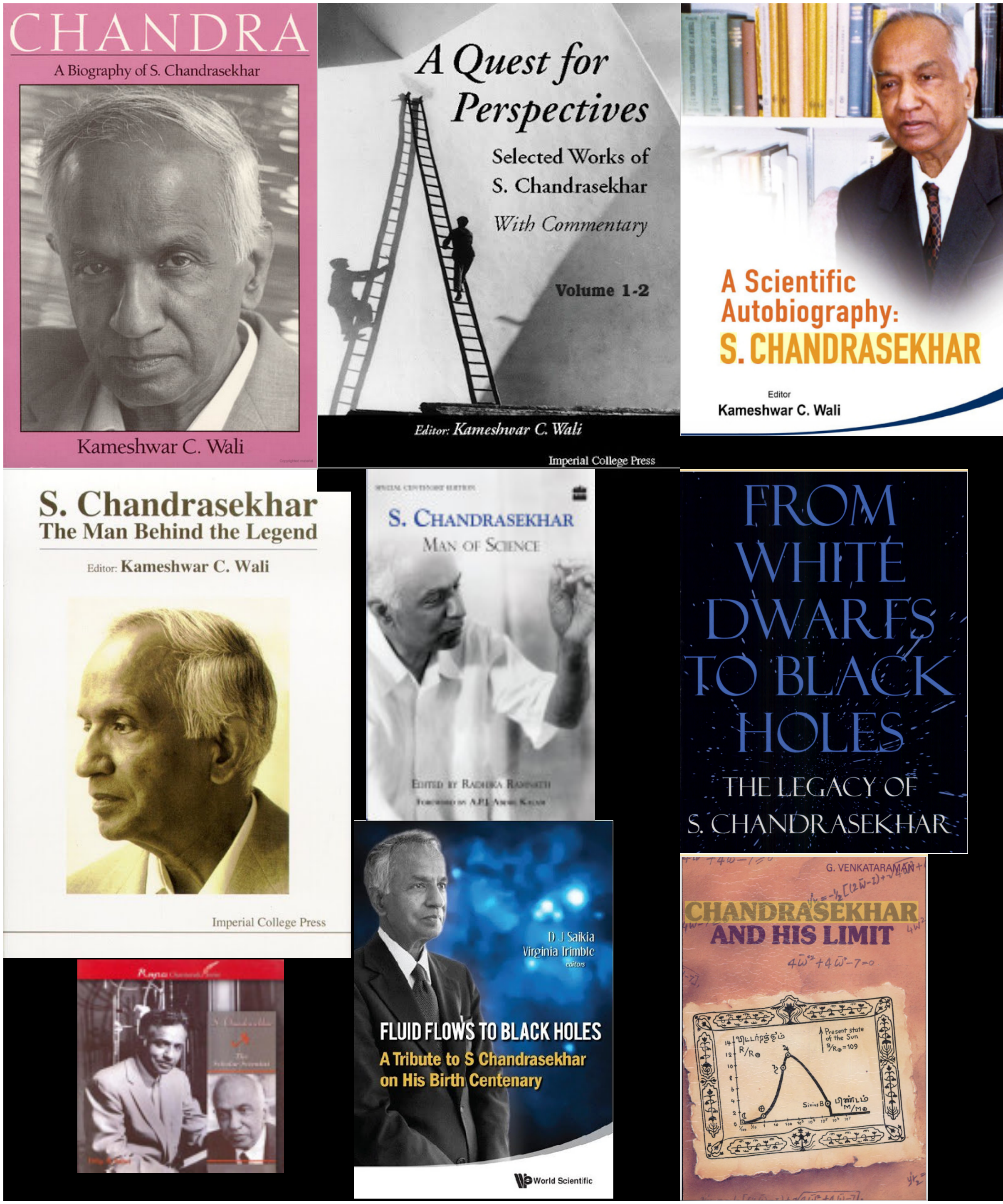
(6) The general theory of relativity and relativistic astrophysics

(7) The mathematical theory of black holes (1974-83)

For a dozen more years, he worked on (a) the theory of colliding gravitational waves and non-radial perturbations of relativistic stars (1983-95) and, finally, on (b) *Newton's Principia for the Common Reader*, published just a few weeks before his death.

[redacted]

- https://www.youtube.com/watch?v=n-lJjR7pM7k



- Many books and essays have been written about Chandra by family, friends, colleagues and admirers, so what's new in this talk at all?
- Much of that writing concerns Chandra's work in proper astrophysics and relativity---not much about his fluids work to which he devoted about a dozen years.
- This talk fills that gap in some way
- What motivated Chandra to work in fluid dynamics, a subject in which he had no formal training?
- What did he accomplish and how much of it is lasting (with the hindsight of some 50-60 years)?
- How did he interact with his contemporaries, and what were his moments of glory and agony?
- What was the larger perspective that drove his science?
- Will attempt to address these questions but will be unable to do full justice to Chandra's prolific work and to his extraordinary personality.

Warning: Tip of the Iceberg

On August 1, 1930, the *SS Pilsna*, a member of the Lloyd Triestino fleet, carried the 19-year old Chandrasekhar from Bombay to Venice, en route to Cambridge, and reached England on August