Effective theories of dense matter and applications in neutron stars

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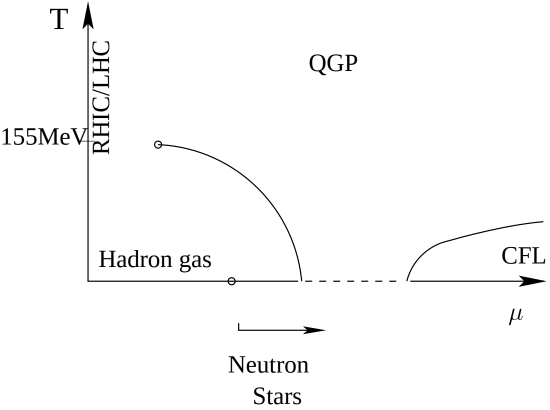
Summary

- Overview of the QCD phase diagram
- Quark matter at high density: High Density Effective Theory (HDET)
- Illustrative example: Transport properties
- Pairing: An effective theory of Goldstone modes
- Other phases: Some results on viscosity in FF phases

Phase diagram of QCD



High temperature and low baryon density

- At low temperatures (T), pions
- The quark gluon plasma (QGP) at higher T
- At high T entropy dominates: Chiral condensate tends to 0
- At high enough T, weak coupling expansions
- Crossover between the hadronic and the QGP phase understood from lattice QCD *(TIFR, BNL, HOT-QCD, Bielefeld)*
- Explored in heavy ion collisions at RHIC and LHC

Low temperature and high baryon density (high chemical potential)

- A rough estimate of the typical momentum exchange is $1/n^{1/3}$ where n is the baryon density
- If the density is high enough then the momentum exchange is large and coupling is weak
- Chiral condensate tends to 0, quarks are deconfined
- If μ large enough then weak coupling $g(\mu)$ expansions can be made
- At lower density hadronic matter of protons and neutrons
- We don't know at what density or μ the transition occurs

Quark matter at high density

- Physically interesting regime between dense hadronic matter and dense quark matter
- With this philosophy we study the properties of quark matter at high density
- Starting point, weakly interacting, nearly massless quarks (assuming the strange quark mass can be ignored), interacting weakly via gluons

Quark matter at high density: illustrative example

- Know from basic statistical physics that quarks will fill up energy levels up to a Fermi surface
- If the only other scale in the problem is T (unpaired quark matter), and we are interested in $\mu \gg T$, only excitations near the Fermi surface participate in dynamics
- This calls out for an effective theory with an expansion in T/μ
- Quarks well below the Fermi surface, and anti-quarks can be integrated out
- Systematic method: High Density Effective theory

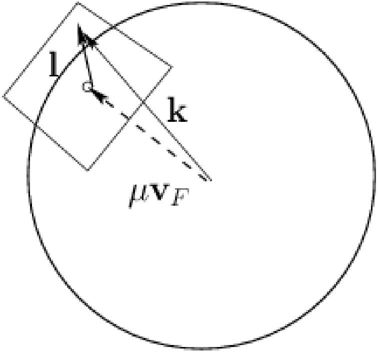
HDET lagrangian

- Instead of the full lagrangian

$$\mathcal{L} = \bar{\psi} i \not{D} \psi + \mu \bar{\psi} \gamma^0 \psi$$

- The magnitude of the momentum is close to μ

Patches



Hong (1998, 1999); Casalbuoni, Gatto, Nardulli, (2001); Schaefer (2003)

HDET lagrangian

- Instead of the full lagrangian

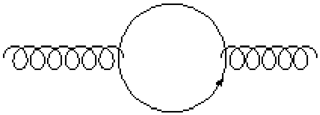
$$\mathcal{L} = \bar{\psi} i \not{D} \psi + \mu \bar{\psi} \gamma^0 \psi$$

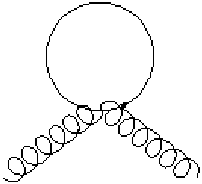
- An effective lagrangian

$$\mathcal{L} = \sum_{v_F} [\psi_+^\dagger i V \cdot D \psi_+ - \psi_+^\dagger D_\perp \frac{1}{2\mu} D_\perp \psi_+]$$

- Additional contact terms suppressed by higher powers of μ
- Formal similarities to HQET
- Similar to Fermi liquid theory in condensed matter physics

Polarization diagrams





Gluon screening

- Longitudinal gluons are Debye screened

$$\Delta_L(q) = i\frac{\hat{q}^i\hat{q}^j}{(q^0)^2 - \mathbf{q}^2 - \Pi_L(q)} \tag{1}$$

- $\Pi_L(0) = m_D^2 = g^2 N_f g_S \frac{\mu^2}{2\pi^2}$
- Transverse gluons are Landau damped

$$\Delta_t(q) = i\frac{\delta_{ij} - \hat{q}^i\hat{q}^j}{(q^0)^2 - \mathbf{q}^2 - \Pi_t(q)} \tag{2}$$

- $\Pi_t(q^\mu \to 0) = ig^2 N_f g_S \frac{\pi}{4}\frac{q^0}{q}\frac{\mu^2}{2\pi^2}$

Physical implications

- Weakly interacting quark matter has been used to calculate the equation of state for quark matter in perturbative QCD (not HDET) *Kurkela, Romatschke, Vuorinen (2010); Fraga, Kurkela, Vuorinen, (2015)*
- Dynamical, or transport properties

Physical implications

- Specific heat standard for fermions at small T,

$$c_V = \frac{1}{3v_F} p_F^2 T \tag{3}$$

  where $v_F \sim 1$ and $p_F \sim \mu$
- Introducing effective operators for weak interactions one can show

$$\epsilon_\nu \sim G_F^2 p_F^3 T^6 \tag{4}$$

  Iwamoto (1980)
- Has implications for the neutrino cooling of neutron stars
- Will focus on another transport property, which is the viscosity

Shear viscosity in the unpaired phase

- Shear viscosity measures the ability to transport momentum between two layers of a fluid
- $\eta \sim n \langle p \rangle \langle \tau \rangle$
- $n = \frac{p_F^3}{3\pi^2}$
- $p \sim p_F$
- τ is inversely proportional to the scattering cross-section

$$\tau \propto \frac{1}{|\mathcal{M}|^2} \tag{5}$$

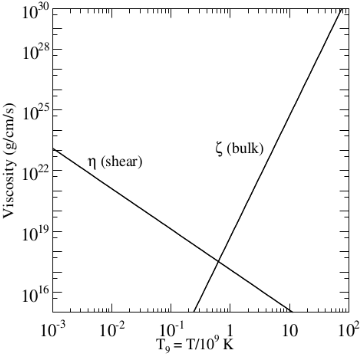
- $\mathcal{M} \sim \frac{g^2}{((q^0)^2 - \mathbf{q}^2 - \Pi)}$
- A simplification that the Landau damped transverse gluons dominate at small T *Heiselberg, Pethick (1993)*
- $\tau \sim \frac{\mu}{g^3 T^2} \left(\frac{T}{g\mu}\right)^{1/3}$
- $\eta \sim \frac{\mu^5}{g^3 T^2} \left(\frac{T}{g\mu}\right)^{1/3}$

Bulk viscosities

- Similarly one can calculate the bulk viscosity
- Bulk viscosity is related to particle production during compression and expansion
- For example expansion will break the weak equilibrium between u and d. Electro-weak processes changing u to d re-establish the equilibrium
- $\zeta = A\frac{\Gamma}{\Omega^2+\Gamma^2}$. Has a Lorentzian shape with the peak at $\Gamma = \Omega$
- $\Gamma \sim T^2\mu$ *Madsen (1998)*

Hadronic matter

- For contrast, assuming only hadronic matter in neutron stars
- $\eta \sim T^{-2}$
- Turns out that $\Gamma(\sim T^6) \ll \Omega$. Therefore $\zeta \sim T^6/\Omega^2$. *Flowers and Itoh (1979)*



- Plot at $2n_{\rm sat}$ *Jaikumar, Rupak, Steiner (2008)*

Implications: Neutron stars

- Neutron stars are compact objects that are remnants of supernova explosions
- $M \sim 1 - 2M_{\odot}$
- $R \sim 10 - 15\text{km}$
- The density at the core, over 5 times the nuclear density

Implications: *r*-modes

- Rotating neutron stars ($\Omega = 2\pi f$) feature an unstable fluid dynamics mode *Andersson (1998), Friedman and Morsink (1998)*
- First treating the fluid as an ideal fluid one obtains in a rotating frame

$$\mathbf{v}(\mathbf{r}) \approx a\Omega f(r)\mathbf{Y}_{lm}e^{i(m\phi - \sigma_r t)}$$

- $\sigma_r \approx -\frac{2m\Omega}{l(l+1)} < 0$ for $m > 0$
- $\sigma_I = \sigma_r + m\Omega > 0$ for $m > 2$

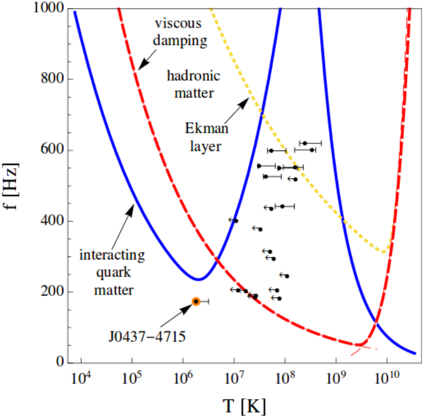
r-modes

- Including "damping" from gravitational waves: couple gravitons to the fluid motion
- $E \approx E_0 e^{-2t/\tau_{\mathrm{GR}}}$
- $\tau_{\mathrm{GR}} < 0$, implying instability
- The mode grows with time
- Note that an inertial observer far away sees the angular momentum as well as the energy of the star decrease
- $1/\tau_{\mathrm{GR}} \sim -(G_N)\Omega^{2l+2}$: instability increases with Ω
- ($l = m = 2$ is the dominant mode and is most studied)

r-modes

- Viscosities in the fluid indeed damp the fluid flow
- Including damping from gravitational waves, shear viscosity η, and bulk viscosity ζ
- $E \approx E_0 e^{-2(t/\tau_{\mathrm{GR}}+1/\tau_\eta+1/\tau_\zeta)}$
- $\frac{1}{\tau_\eta} \propto \int d^3x \eta \delta\sigma^{ab} \delta\sigma_{ab}$
- In the absence of microscopic damping mechanisms, the loss in angular momentum is very rapid (the rotational speed of about 500Hz drops by a substantial fraction in 1 year)
- The non-observation of such spin down constrains the microscopic properties of neutron stars

Quark matter



Jaikumar, Rupak, Steiner (2008); Alford, Schwenzer (2014)

Additional damping effects

- Caveat is that there could be additional damping effects
- Friction between the crust-core interface *Bildsten, Ushomirsky (1999); Lindblom, Ushomirsky (2000); Jaikumar, Rupak (2010)*
- Non-linear saturation of the $r-$modes to a small magnitude *Alford, Mahmoodifar, Schwenzer (2012); Alford, Han, Schwenzer (2012)*

Color superconductivity

- But quark matter is expected to be in a paired phase because the interaction between quarks is attractive in the color antisymmetric channel *Alford, Rajagopal, Wilczek and Shuryak, Schaefer, Rapp (1998)*
- At asymptotically high densities where the strange quark mass can be ignored, quark matter is in the CFL phase
- The di-quark condensate is antisymmetric in color and in spin, and therefore also in flavor

$$\langle \psi_{\alpha i}(p)(C\gamma^5)\psi_{\beta j}(-p)\rangle \propto \Delta \sum_I \epsilon_{I\alpha\beta}\epsilon_{Iij} \tag{6}$$

- $U(1) \times SU_c(3) \times SU_L(3) \times SU_R(3) \to Z_2 \times SU_{c+L+R}(3)$
- The ϵ tensors "lock" color and flavor, and hence CFL

Complete change in low energy excitations

- In the CFL phase all fermionic quasi-particle excitations are gapped due to pairing
- Energy scales $\mu > 500$MeV, $\Delta \sim 10$MeV, $T \sim 0.001 - 1$MeV where Δ is proportional to the condensate and is the gap in the fermionic spectrum
- $E = \sqrt{(p - \mu)^2 + \Delta^2}$
- This is the analog of electronic superconductivity where the electrons form Cooper pairs, and to break a Cooper pair one needs to supply an energy Δ
- Therefore a hierarchy of scales $\mu \gg \Delta \gg T$

EFT for CFL

- Therefore the fermionic contribution to transport properties is exponentially suppressed $e^{-\Delta/T}$
- They can be integrated out and an effective theory based only on the Goldstone modes is sufficient to describe phenomena for $T \ll \Delta$
- $U(1) \times SU_c(3) \times SU_L(3) \times SU_R(3) \to Z_2 \times SU_{c+L+R}(3)$
- Ignoring the gauged part of the symmetry breaking, the breaking pattern of the continuous symmetry is $U(1) \times SU_L(3) \times SU_R(3) \to SU_{L+R}(3)$ *Alford, Rajagopal, Wilczek, (1998)*
- This pattern is familiar from chiral symmetry breaking in vacuum, except for the additional $U(1)_B$

Mesonic EFT

- $$\begin{aligned}\mathcal{L} = &\frac{1}{4f_\pi^2}\mathrm{tr}[\partial_0\Sigma\partial_0\Sigma] - v_\pi^2\frac{1}{4f_\pi^2}\mathrm{tr}[\partial_i\Sigma\partial_i\Sigma] \\ &+ \frac{1}{2f_\phi^2}[\partial_0\phi\partial_0\phi] - v_\phi^2\frac{1}{2f_\phi^2}[\partial_i\phi\partial_i\phi] \\ &+ c_4[(\partial_0\phi)^4 + (\partial_i\phi)^4 - 2(\partial_i\phi)^2(\partial_0\phi)^2] \\ &+ c_3(\partial_i\phi)^2(\partial_0\phi) + \ldots\end{aligned} \tag{7}$$

- ϕ associated with $U_B(1)$ breaking
- $\Sigma = \exp(\frac{it^a\pi^a}{f_\pi})$ associated with $L - R$
- *Son, Stephanov (1999), Casalbuoni, Gatto (1999, 2000), Schaefer (2000)*

Mesonic EFT coefficients

- In perturbation theory to lowest order in g
- $f_\pi^2 = \frac{21-8\log(2)}{18}\frac{\mu^2}{2\pi^2}$, $v_\pi = 1/3$
- $f_\phi^2 = 9\frac{\mu^2}{2\pi^2}$, $v_\phi = 1/3$
- $c_4 = \frac{3}{4\pi^2}$
- $c_3 = \frac{3\mu}{\pi^2}$
- Can include small quark mass corrections in the standard manner *Son, Stephanov (1999), Casalbuoni, Gatto (1999, 2000), Schaefer (2000)*

Degrees of freedom: gluons

- Since color symmetry is broken, gluons t^1 to t^7 are gapped because of the Meissner effect
- The t^8 gluon mixes with the photon A_μ^Q to give one linear combination $A_\mu^{\tilde{Q}}$ that does not have a Meissner effect and one X_μ that does
- This can be understood by noting that if we define $t^8 = \frac{1}{\sqrt{3}}\mathrm{diag}(-2, 1, 1)$ in color space and $Q = \mathrm{diag}(2/3, -1/3, -1/3)$ in flavor space, $\tilde{Q} = Q + \frac{1}{\sqrt{3}}t^8$
- The condensate is neutral under $\tilde{Q}$
- Therefore, $A_\mu^{\tilde{Q}}$ is the only long distance carrier of forces and we can ignore the other gluons while calculating scattering

Scattering of mesons

- An important feature is that mesons only interact via derivative interactions
- Consequently at least $|\mathcal{M}| \propto T^4$ for ϕ
- *Manuel, Dobado, Estrada (2005); Mannarelli, Manuel, 'Saad (2008); Mannarelli, Manuel (2010)*
- A detailed calculation gives $\tau \propto \mu^4/T^5$
- Finally $\eta \sim \mu^8/T^5$
- Naiively this corresponds to a large viscosity because $\mu \gg T$
- In reality, this corresponds to mean free path larger than the size of the neutron star: no damping

Bulk viscosity

- Similarly unpaired quarks can be ignored for the evaluation of the bulk viscosity
- A similar analysis to the shear viscosity shows that Goldstone contribution much less than unpaired quark matter *Manuel, Estrada (2007); Alford, Braby, Schaefer, Reddy (2007); Mannarelli, Manuel (2010)*

Constraints on CFL

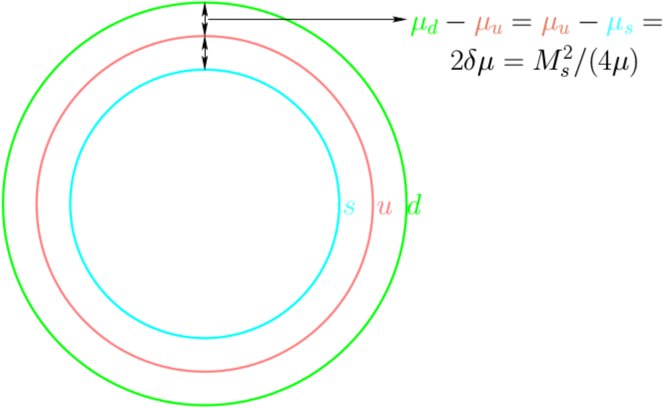
- CFL phase is inconsistent with r-mode stability constraints
  Manuel, Mannarelli, S'ad (2008), Jaikumar, Rupak (2010)

Strange quark mass and neutrality

- Ignoring M_s is not a good approximation if μ is not very large
- $\sqrt{M_s^2 + (p_s^F)^2} = \mu \implies p_s^F \approx \mu - M_s^2/(2\mu)$, but this leaves an unbalanced positive charge.
- Need to introduce a chemical potential, μ_e, to restore neutrality.
- Weak equilibrium implies $\mu_d - \mu_s = 0$, $\mu_d - \mu_u = \mu_e$
- Electrical neutrality is imposed by demanding $\frac{\partial \Omega}{\partial \mu_e} = 0$.
- Similarly, color neutrality by desiring $\frac{\partial \Omega}{\partial \mu_{3,8}} = 0$

Neutral unpaired quark matter

- For unpaired quark matter we obtain $\mu_e = M_s^2/(4\mu)$, $\mu_3 = \mu_8 = 0$.



Alford, Burgess, Rajagopal (1999)

Introduction to LOFF phases

- CFL involves pairing between different flavors

$$\langle u(\mathbf{p}) d(-\mathbf{p}) \rangle \propto \Delta \tag{8}$$

or in position space

$$\langle u(x) d(x) \rangle \propto \Delta \tag{9}$$

- This is preferred if the Fermi surfaces are equal in size
- An inhomogeneous pairing pattern may be preferred if $\delta\mu$ is large enough

$$\langle u(\mathbf{p}+\mathbf{q}) d(-\mathbf{p}+\mathbf{q}) \rangle \propto \Delta \tag{10}$$

or in position space

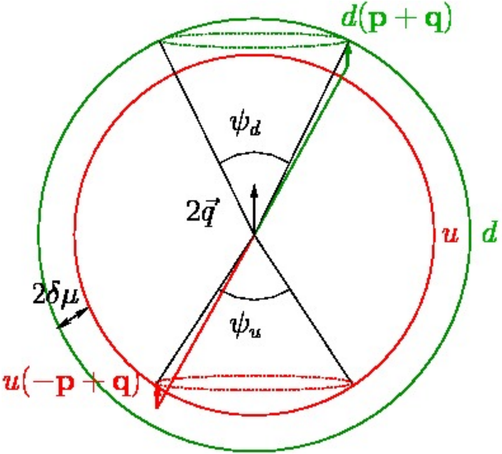
$$\langle u(x) d(x) \rangle \propto \Delta e^{i2\mathbf{q}\cdot\mathbf{r}} \tag{11}$$

Alford, Bowers, Rajagopal (2001)

Introduction to LOFF phases

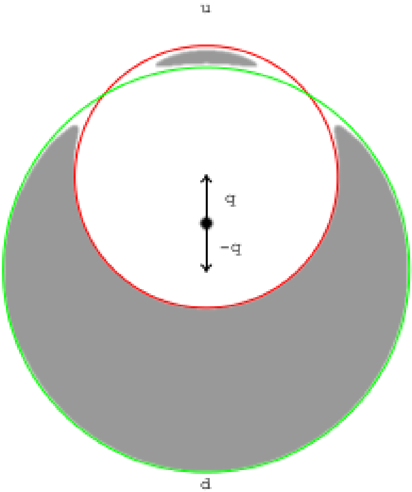
- The inhomogeneous (FF) phase thermodynamically preferred state compared to isotropic states for $\delta\mu \sim [0.707, 0.754]\Delta$, where Δ is the gap for $\delta\mu = 0$
- A detailed analysis (*Mannarelli, Rajagopal, RS (2005), Ippolito, Nardulli, Ruggieri (2007)*) suggests that for three flavors $440 \lesssim \mu \lesssim 520$MeV an inhomogenous state might be the ground state. This is the relevant region for neutron star cores
- We take the simplest phase with only one momentum direction $\mathbf{q}$
- We only consider two flavors of quarks *u* and *d* in this first analysis

Intuition for favoured inhomogeneous pairing



Gapless fermionic modes

- $E = -\delta\mu - q\cos\theta + \sqrt{(p-\mu)^2 + \Delta^2}$
- This dispersion relation has gapless surfaces (if $|\delta\mu + q| < \Delta$)



Low energy degrees of freedom

- Gapless modes of the u and d quarks
- In general, lattice phonons associated with translational symmetry breaking
- Gauge bosons of which only transverse gluons, t^1, t^2, and t^3 are relevant because they are long ranged
- The polariziation tensor for these was calculated in *RS EPJA (2017)*

Low energy lagrangian

$$\mathcal{L} = \frac{1}{2}\sum_{v_F}\Psi^{\dagger}_{Lv_F}\begin{pmatrix} V\cdot\partial - q\cos\theta - \delta\mu & \Delta \\ \Delta & \tilde{V}\cdot\partial - q\cos\theta - \delta\mu \end{pmatrix}\Psi_{Lv_F}$$
$$+ \frac{1}{2}\sum_{v_F} g A^a_\mu \Psi^{\dagger}_{Lv_F}\begin{pmatrix} V^\mu t^a & 0 \\ 0 & -\tilde{V}^\mu t^{a*} \end{pmatrix}\Psi_{Lv_F}$$
$$+ \frac{c_\mu}{f_\varphi}\partial_\mu\varphi^a\bar{\psi}_{Lv_F}\gamma^\mu\psi_{Lv_F} + (L \to R) \tag{12}$$

Gluonic and Goldstone contribution

- Gluons have a short mean free path and their contribution to viscosity is subdominant
- Because of scattering off gapless quarks, the contribution of the Goldstone mode is also sub-dominant

$$\eta_\phi \sim \frac{1}{v_\phi^3}\frac{f_\phi^2}{\mu^2}T^3 \tag{13}$$

- Therefore the dominant contribution comes from quarks
- The dominant scattering mechanism is the exchange of transverse t^1, t^2, t^3 gluons

Shear viscosity in the FF phase

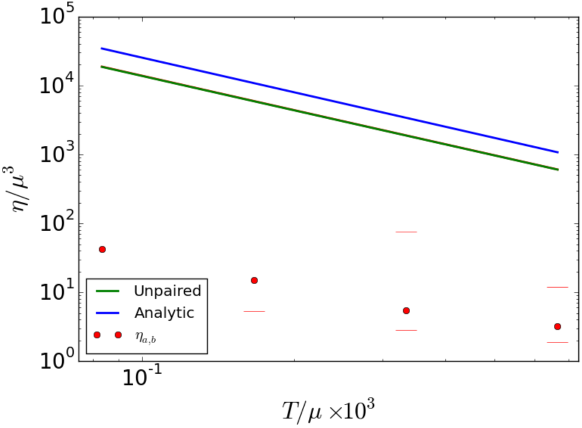
- The modification of the density of states is simple — geometric
- $\eta^{(0)} \approx \frac{\mu^4}{5\pi^2}(1 - \frac{\Delta}{q})\tau^{(0)}$
- $\tau^{(0)}$ is related to the collision integral

$$\frac{1}{\tau^{(0)}} \propto \frac{1}{T}\int \frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3}\frac{d^3p_3}{(2\pi)^3}\frac{d^3p_4}{(2\pi)^3}$$
$$|\mathcal{M}(12 \to 34)|^2$$
$$(2\pi)^4\delta(\sum p^\mu)[f_1 f_2(1-f_3)(1-f_4)]$$
$$\phi_i^{ab}.\Pi^{(0)}_{abcd}.\phi_i^{cd}$$

with $\phi_i^{ab} = v^a p^b$, $\Pi_{ijkl} = \frac{3}{2}(\hat{e}_i\hat{e}_j - \delta_{ij})(\hat{e}_k\hat{e}_l - \delta_{kl})$

- Complicated because the distribution functions f depend on the angles in addition to the magnitude of the momentum. Needs to be done numerically

Results for the FF phase

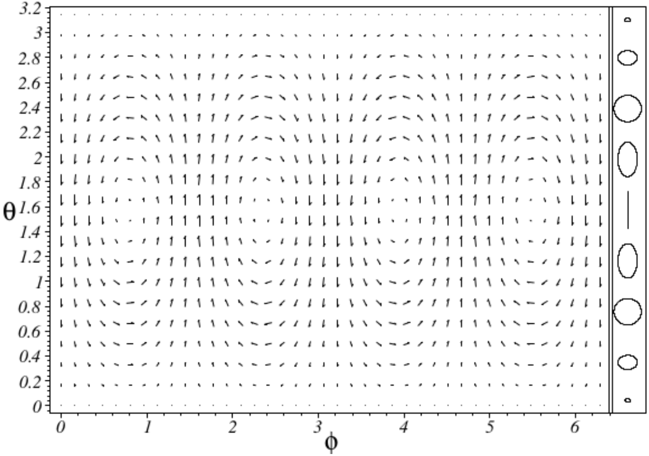


Sarkar, RS (2017)

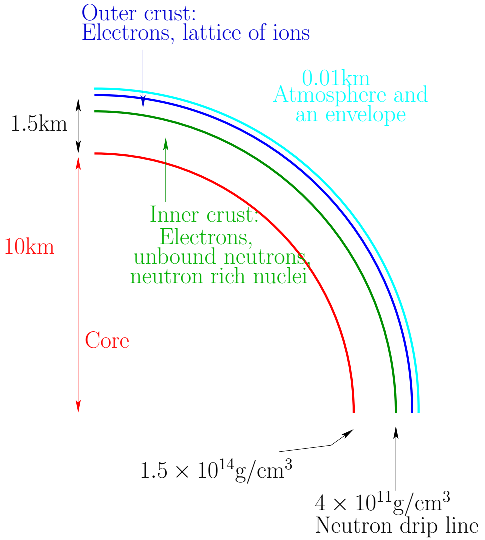
Conclusions and future work

- Data on the angular velocity of neutron stars puts constraints on the viscosity of the matter the cores of neutron stars: possibly suggesting the presence of a (1) deconfined phase with (2) gapless fermionic excitations
- Crystalline color superconducting phases are natural candidates for a paired quark matter phase with gapless excitations. Preliminary calculations suggest that the shear viscosity is not too suppressed compared to unpaired quark matter in the two flavor case
- Will be interesting to see if results of the full three flavor problem consistent with the data

Profile of a r-mode



Profile of a neutron star



Deconfined quarks?

- Qualitatively different possibilities
  - Proton-neutron (hadronic) matter persists in the core
  - Quarks are deconfined
- One can imagine using the equation of state to distinguish the two possibilities
- For example, the pressure in the equation of state should be able to sustain the most massive neutron stars known ($\sim 2M_\odot$ PSR J1614-2230, J0348+0432)
- The equation of state for (interacting) quark matter and hadronic matter are very similar. (For experts, interactions are important. Non-interacting quark matter has a much “softer” equation of state and is ruled out. *Alford, Reddy*)
- Therefore consider dynamical properties which are more sensitive to the low energy degrees of freedom