Moduli Masses and Inflationary Observables

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June, 2017

References

Moduli Vacuum Misalignment and Precise Predictions in String Inflation

arXiv: 1604.08512[hep-th]

Michele Cicoli, Koushik Dutta, Anshuman Maharana, Fernando Quevedo

Inflationary Predictions and Moduli Masses

arXiv:1506.05745[hep-ph]

Kumar Das, Koushik Dutta, Anshuman Maharana

Inflationary constraints on modulus dominated cosmology

arXiv:1409.7037[hep-ph]

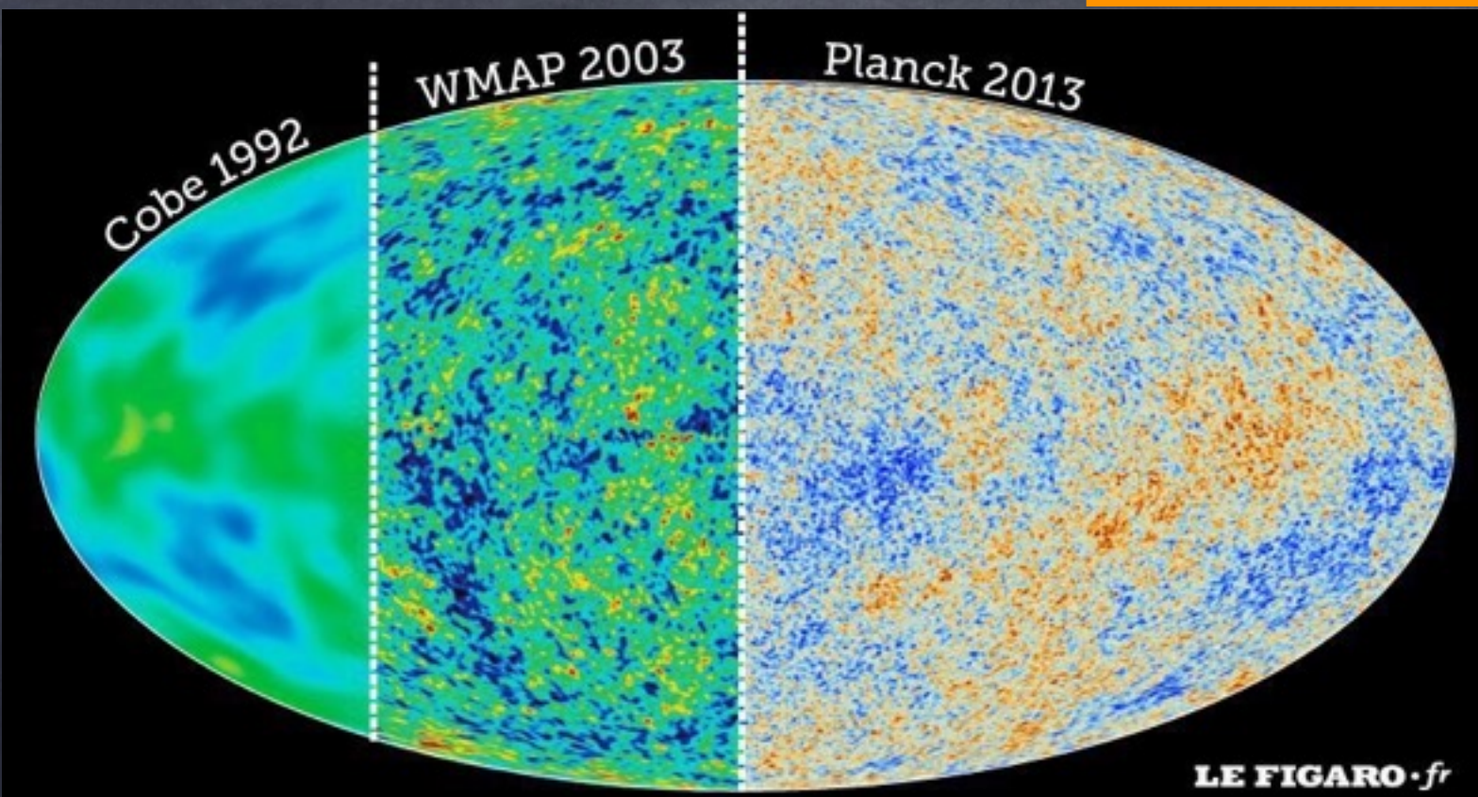
Koushik Dutta, Anshuman Maharana

Take Home

The number of e-foldings is sensitive to the post-inflationary history of the universe.

The generic presence of light scalar fields (in SUSY/String Theory) leads to a late-time period of matter domination which lowers the required number of e-foldings and, in turn, modifies the exact predictions of any inflationary model.

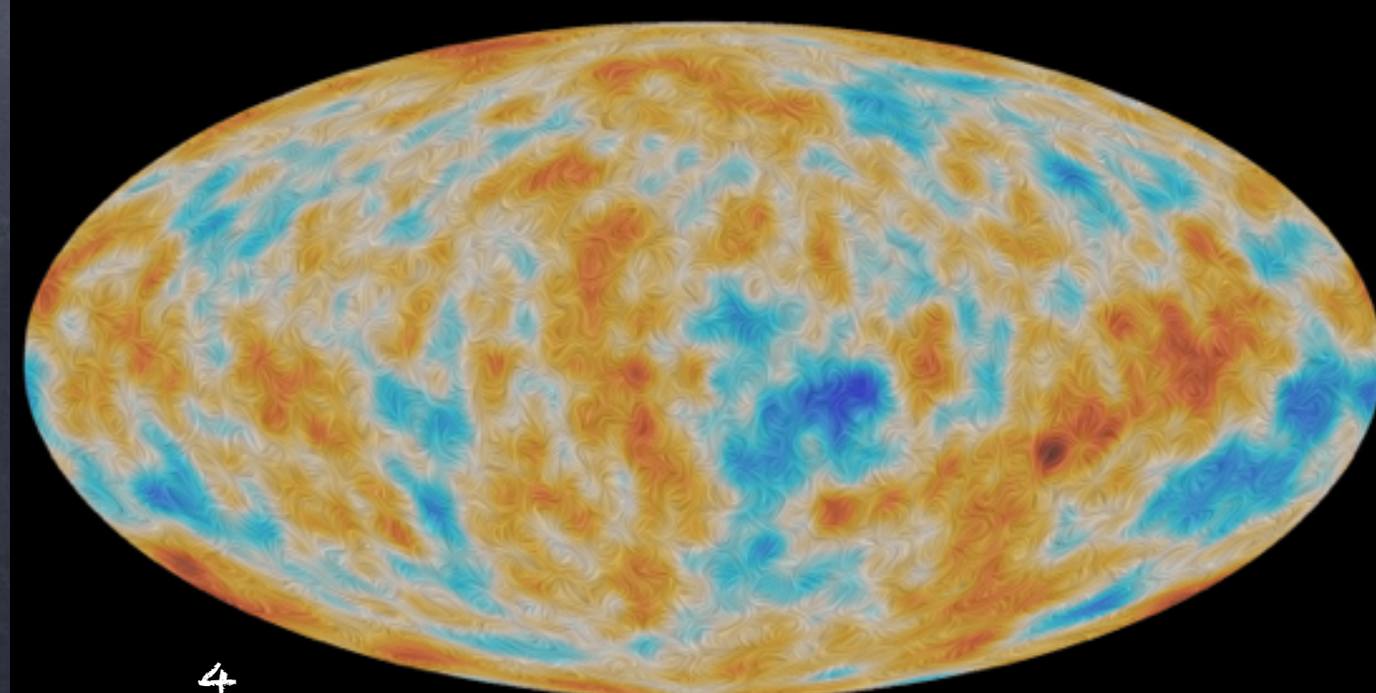
CMB



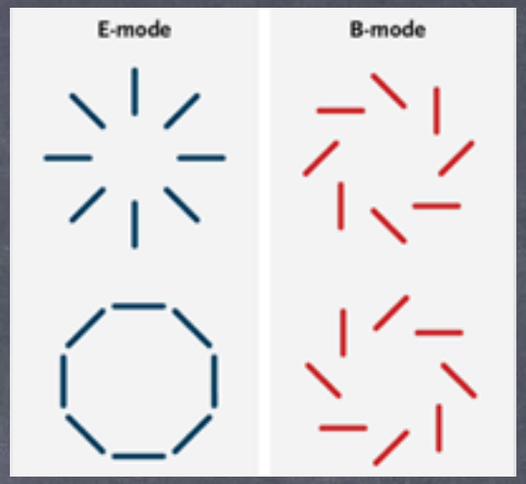
$$\frac{\Delta T(\hat{n})}{\bar{T}} = \sum_{l=0}^{\infty} \sum_{m=-l}^{m=+l} a_{lm} Y_{lm}(\hat{n})$$

$$C(\theta) \equiv \left\langle \frac{\Delta T}{\bar{T}}(\hat{n}) \frac{\Delta T}{\bar{T}}(\hat{n}') \right\rangle$$

$$C_l^{obs} = \frac{1}{2l+1} \sum_m |a_{lm}|^2$$

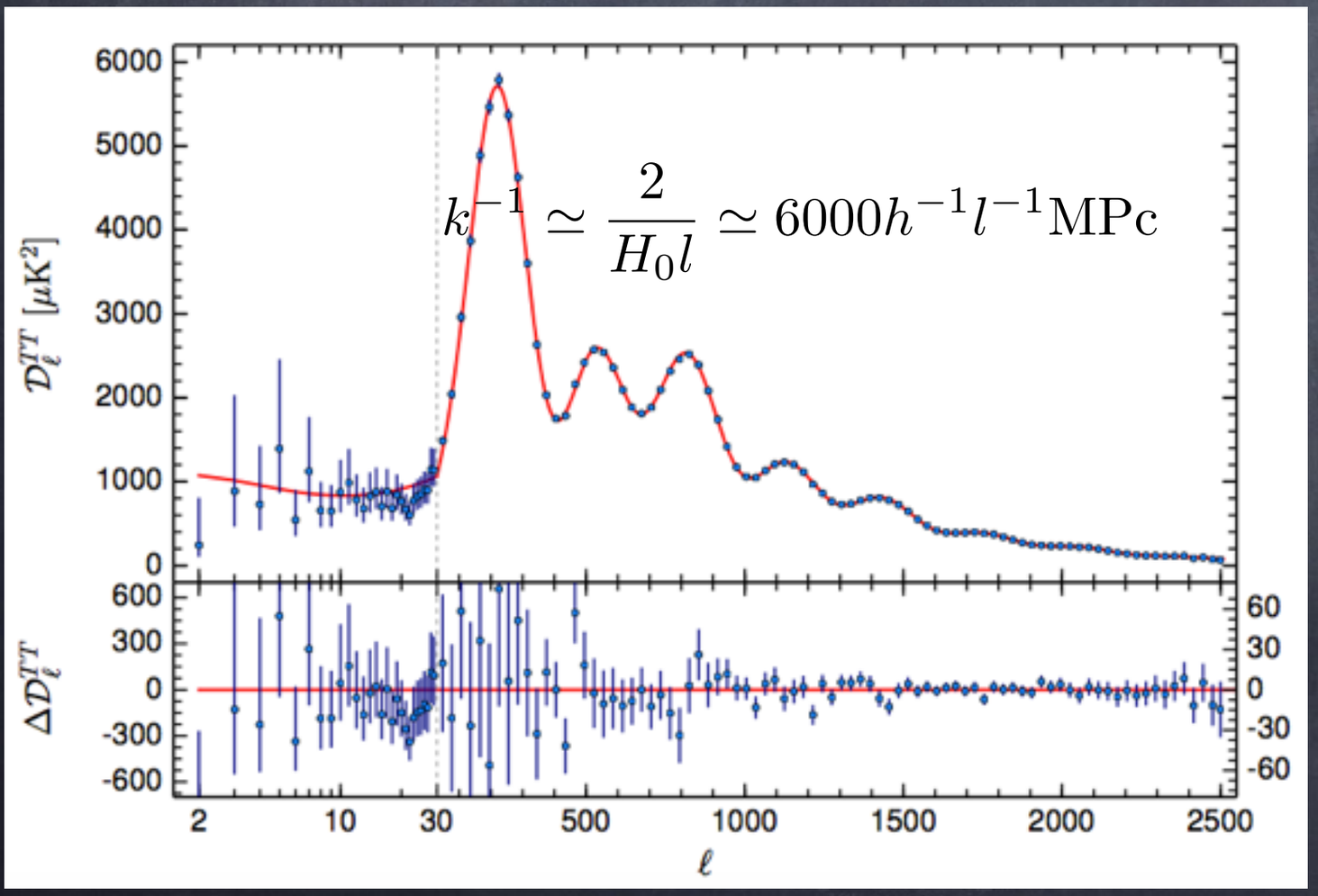


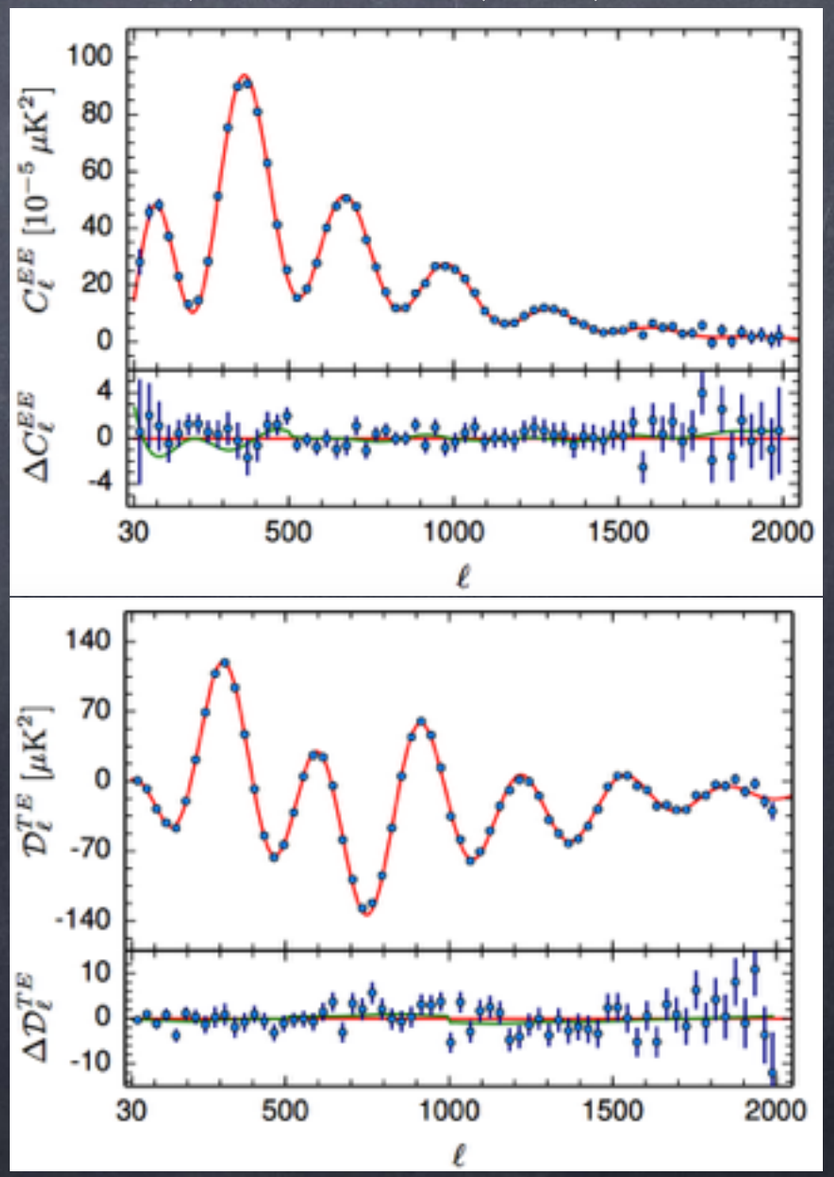
CMB Data



Photons are linearly polarised

$$\langle a_{X,lm} a^*_{Y,l'm'} \rangle = C_l^{XY} \delta_{ll'} \delta_{mm'}, \quad X, Y = T, E, B$$





$$\mathcal{D}_l = l(l+1)C_l/2\pi$$

CMB Theory

Strongly coupled plasma - sound waves

(red line in previous plot) $C_l = \int d\, lnk\, \Delta^2_{\mathcal{R}}(k) T_l^2(k)$

primordial spectrum

(given in radiation dominated era)

A_s, n_s

transfer function

$\Omega_b h^2, \Omega_c h^2, \Omega_\Lambda, \tau$

$\Delta^2_{\mathcal{R}}(k) = A_s (k/k_*)^{n_s - 1}$

ΛCDM cosmology

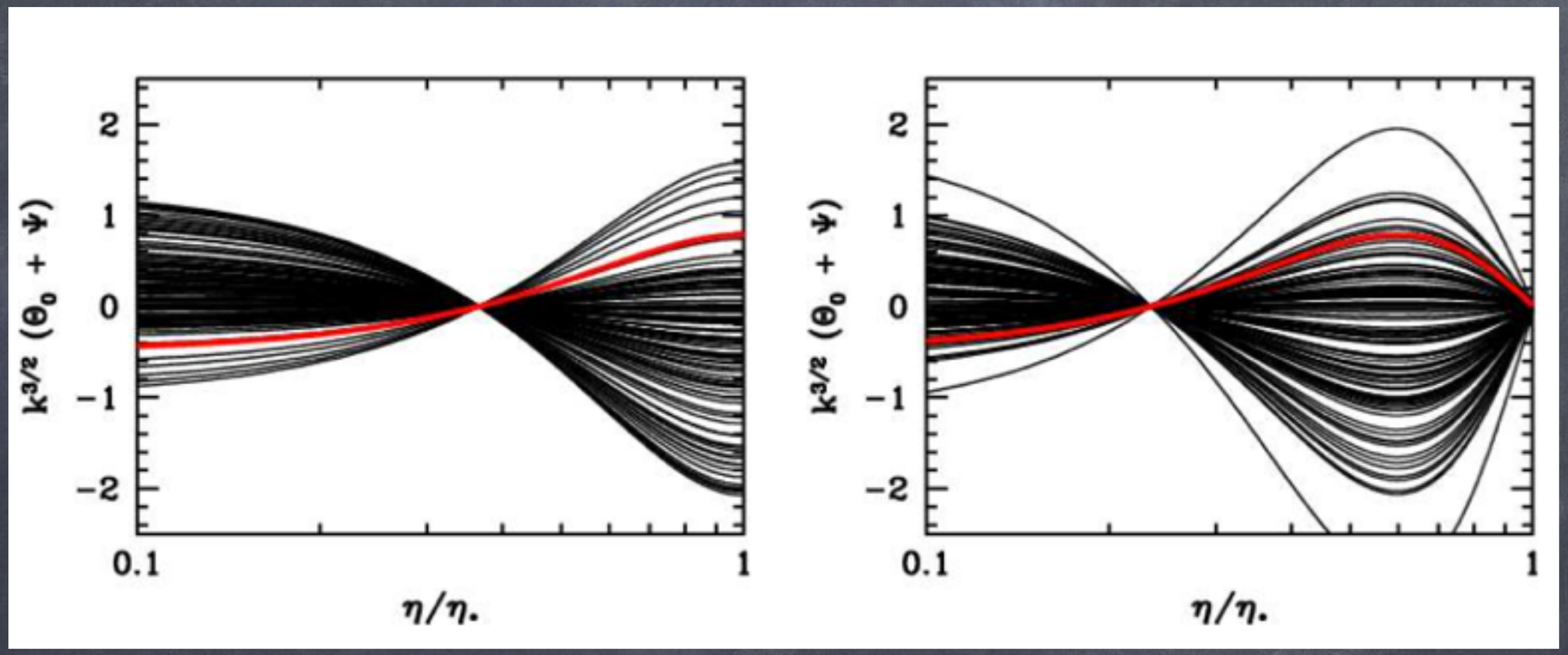
Parameter	Planck	$\cdots$ + WMAP + ACT	CMB + BAO
$\Omega_b h^2$	0.02207 ± 0.00067	0.02207 ± 0.00054	0.02214 ± 0.00048
$\Omega_c h^2$	0.1196 ± 0.0061	0.1198 ± 0.0052	0.1187 ± 0.0034
Ω_Λ	0.683 ± 0.040	0.685 ± 0.033	0.692 ± 0.021
τ	0.097 ± 0.080	0.091 ± 0.027	0.092 ± 0.026
$10^9 A_s$	2.23 ± 0.32	2.20 ± 0.11	2.20 ± 0.11
n_s	0.962 ± 0.019	0.959 ± 0.014	0.961 ± 0.011

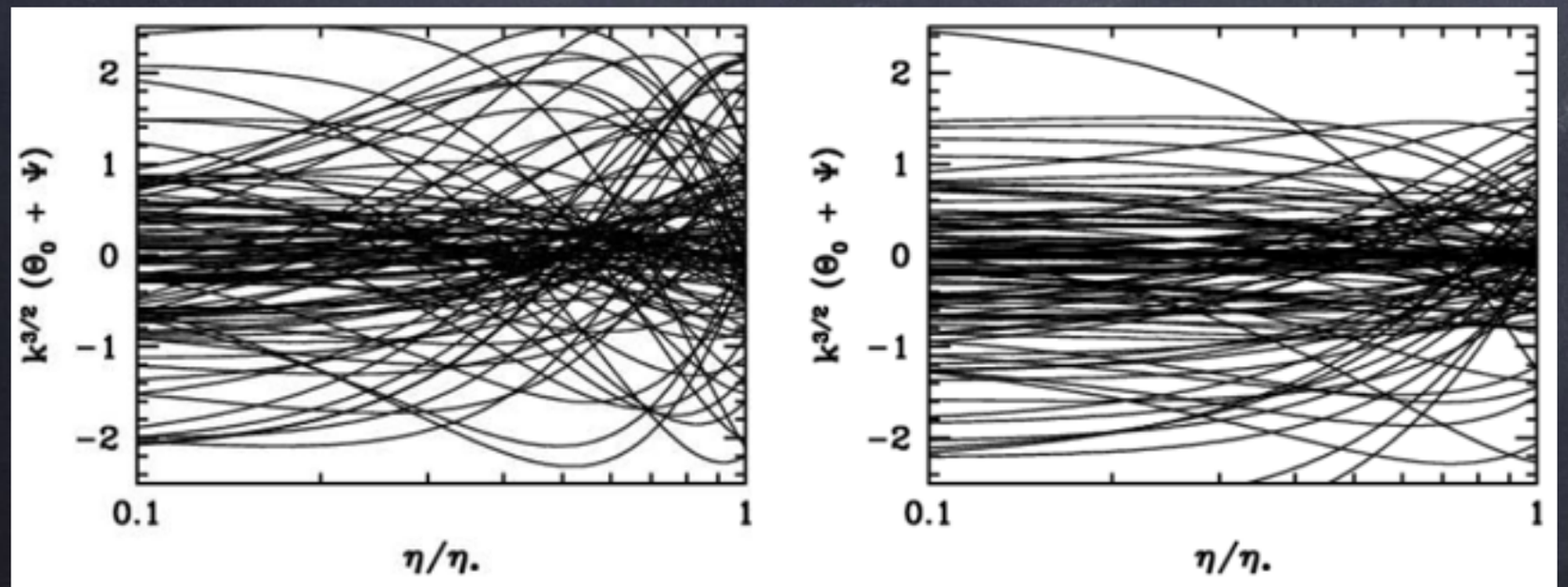
Complete story

Coherent Phase

(of sub-Hubble modes)

given an wavenumber, infinite number of modes



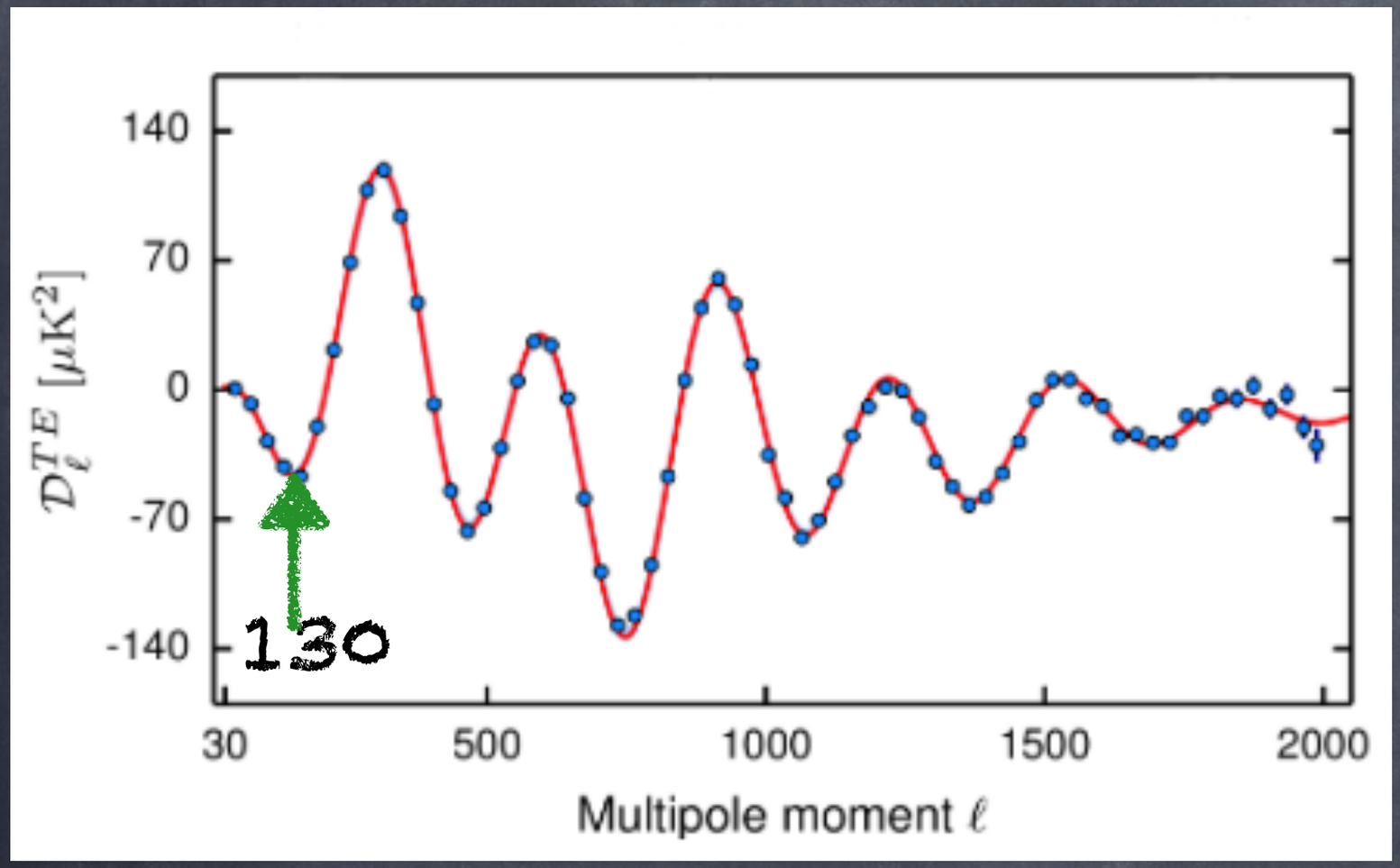


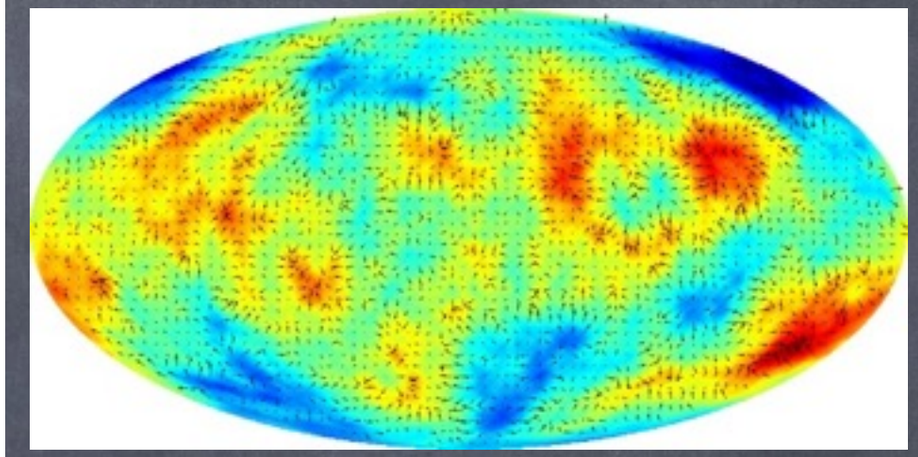
with random phase

This is still sub-horizon physics

Coherent Phase

(of super-Hubble modes)





$$D_l^{TE} \sim \langle Q_{lm}^T Q_{lm}^E \rangle$$

$$\Delta T \sim \delta_\gamma, \quad E \sim \partial v_\gamma$$

$$\delta_\gamma = A\ Cos(wt + \phi) e^{i\vec{k}\cdot\vec{x}}$$

$$\partial v_\gamma = -\dot{\delta}_\gamma$$

If phases were uniformly distributed

$$D_l^{TE} = 0$$

phase is not random

From data ONLY

- At the time of BBN, the Universe was radiation dominated
- The existence of primordial spectrum

$$\Delta^2_{\mathcal{R}}(k) = A_s (k/k_*)^{n_s - 1}$$

- Coherent super-Hubble perturbations
- (possibly) dark matter .. gravitational collapse

From data ONLY

- At the time of BBN, the Universe was radiation dominated
- The existence of primordial spectrum

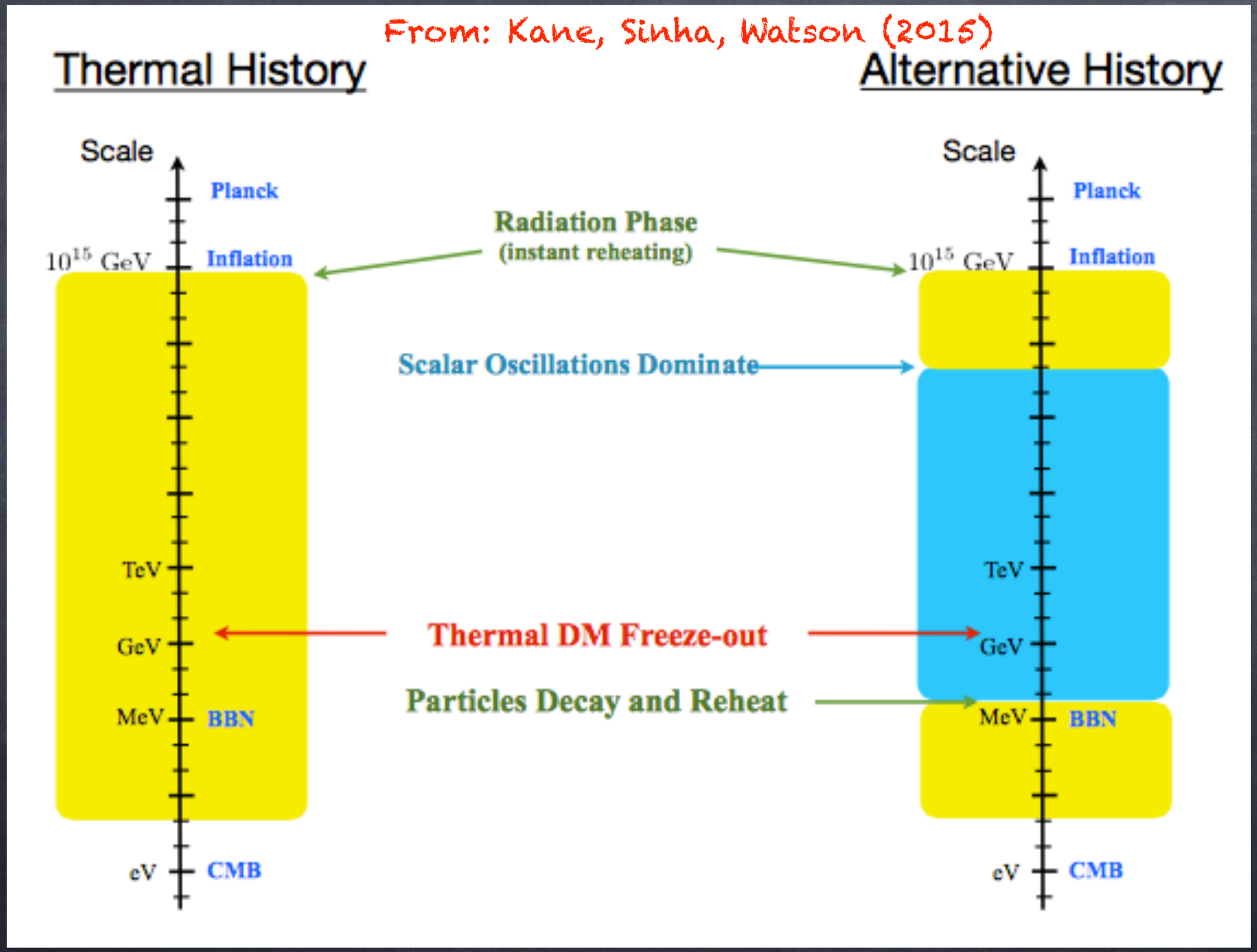
$$\Delta^2_{\mathcal{R}}(k) = A_s (k/k_*)^{n_s - 1}$$

- Coherent super-Hubble perturbations

INFLATION

- (possibly) dark matter .. gravitational collapse

INFLATION also solves horizon and flatness problem


From: Kane, Sinha, Watson (2015)
Thermal History
Alternative History
Scale
Planck
Inflation
10^{15} GeV
TeV
GeV
MeV
BBN
eV
CMB
Radiation Phase
(instant reheating)
Scalar Oscillations Dominate
Thermal DM Freeze-out
Particles Decay and Reheat

Outline

- Cosmology with moduli
- Constraint on moduli mass/inflation

  Dutta, Maharana (2014)
- Implications ..

  Das, Dutta, Maharana (2015)
- An example: Kahler moduli inflation

  Cicoli, Dutta, Maharana, Quevedo (2016)

Framework

'Modular Cosmology'

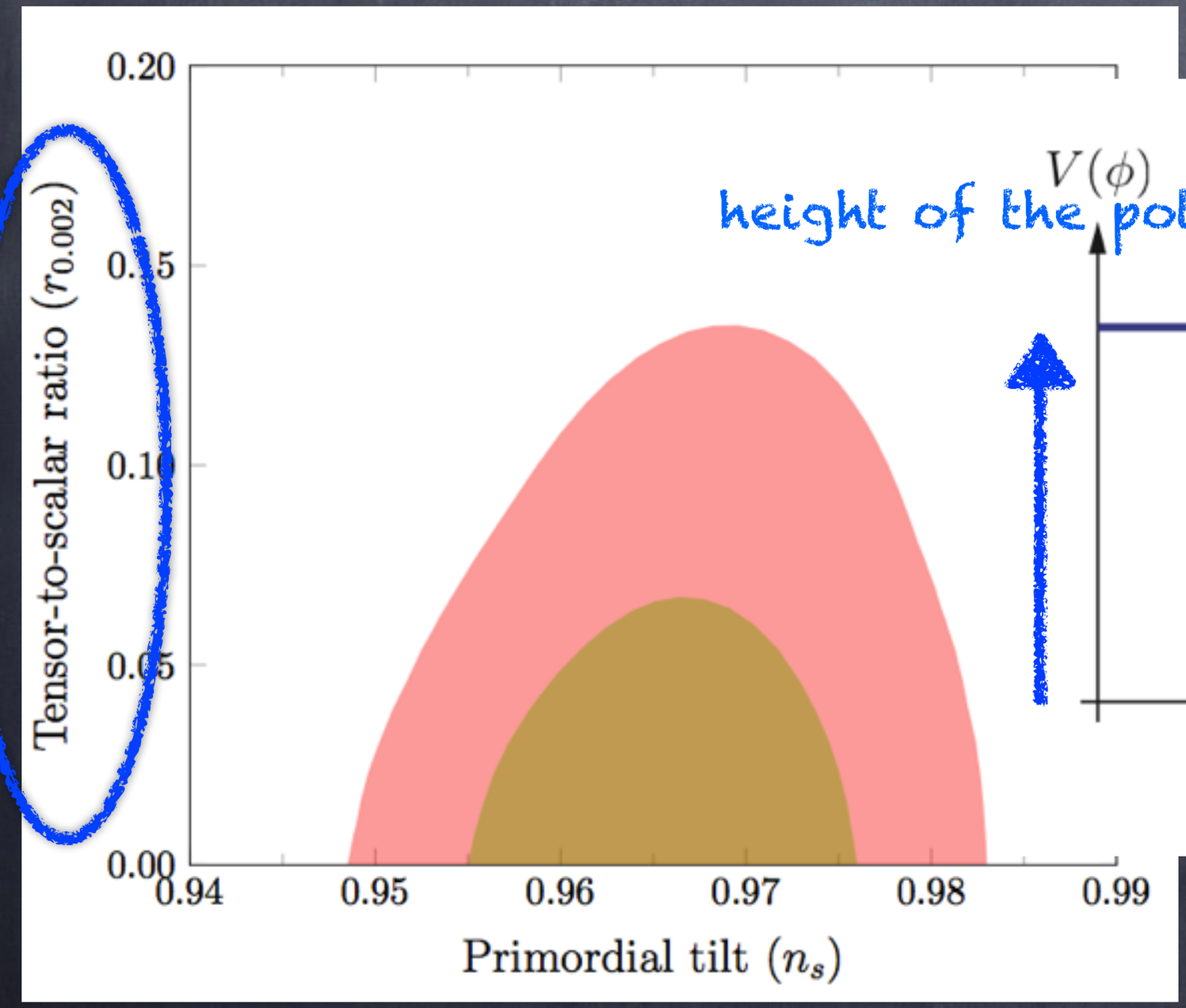
Moduli φ

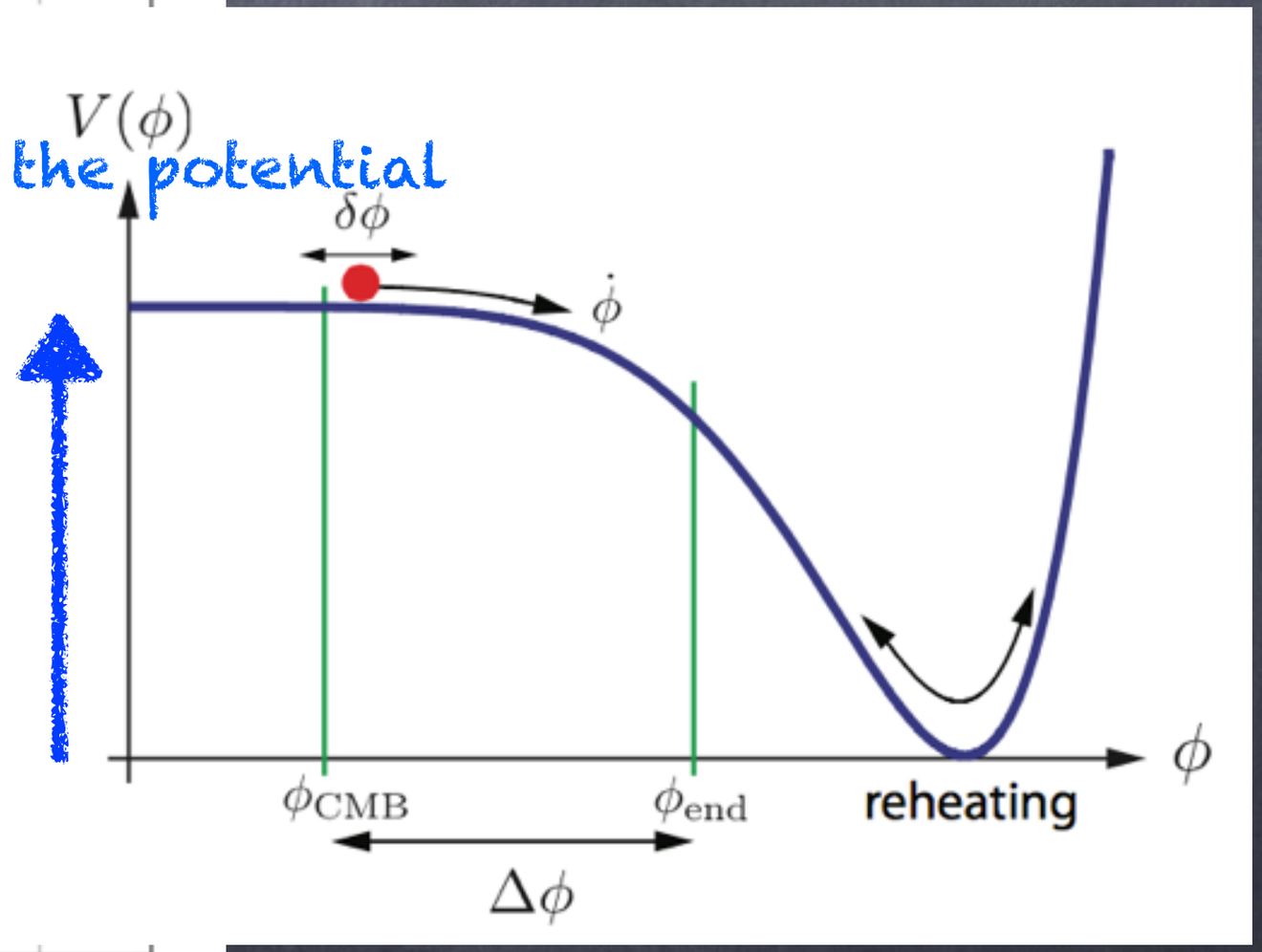
potential at the end of inflation

post-inflationary moduli mass $m < H_{inf}$

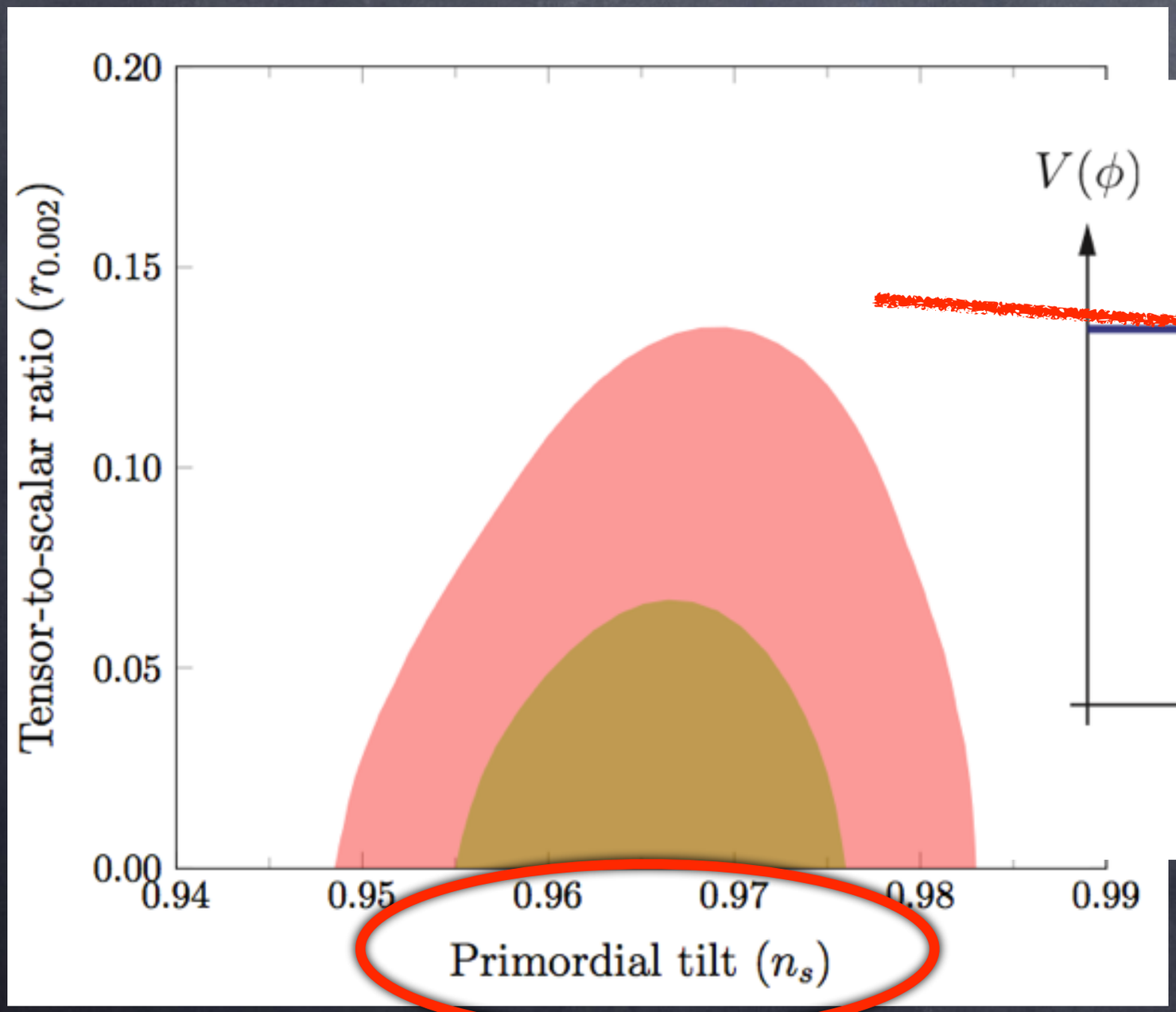
$$Y = \hat{\varphi}/M_{Pl} \sim 1$$

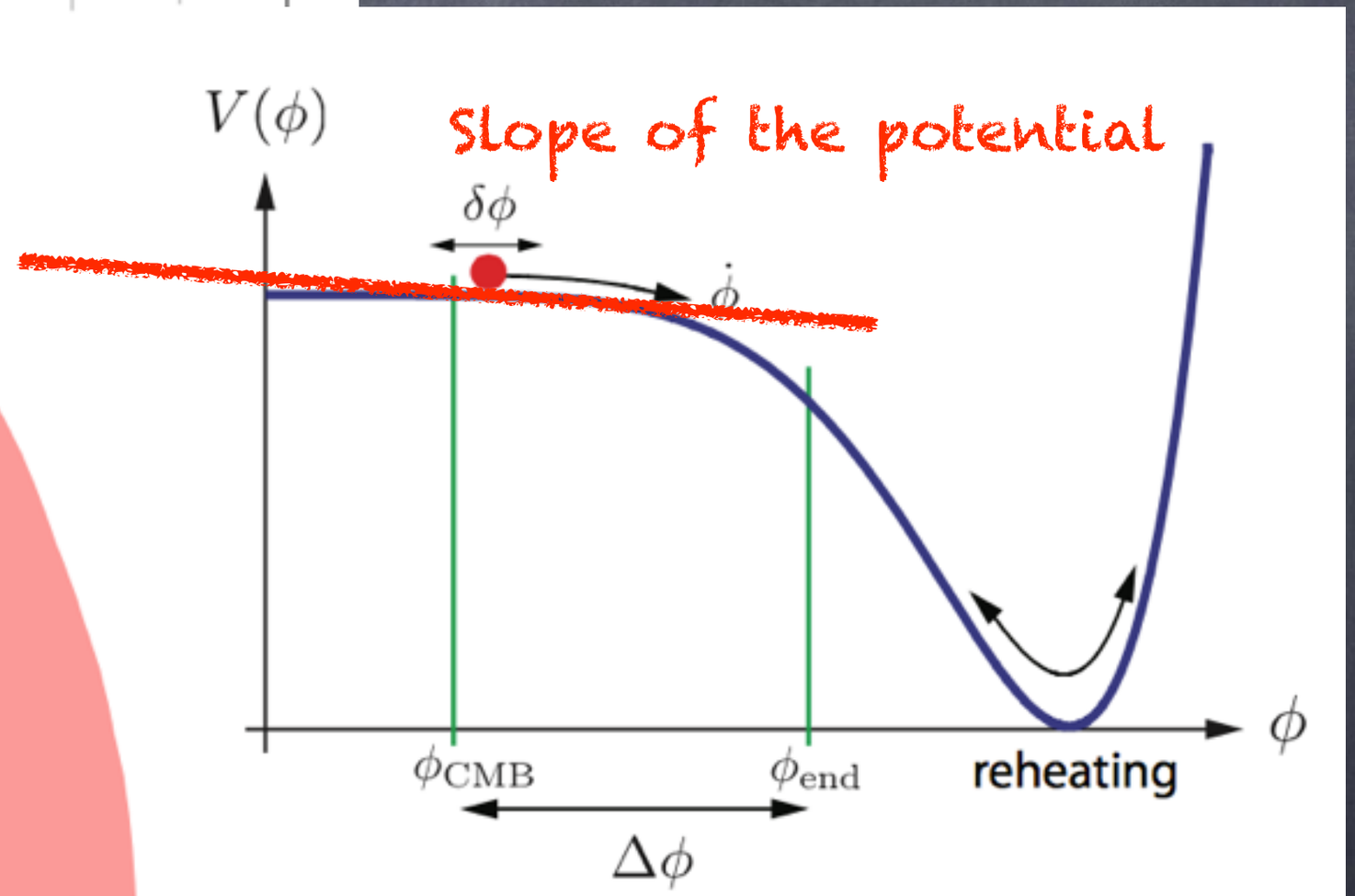
Inflation Observables





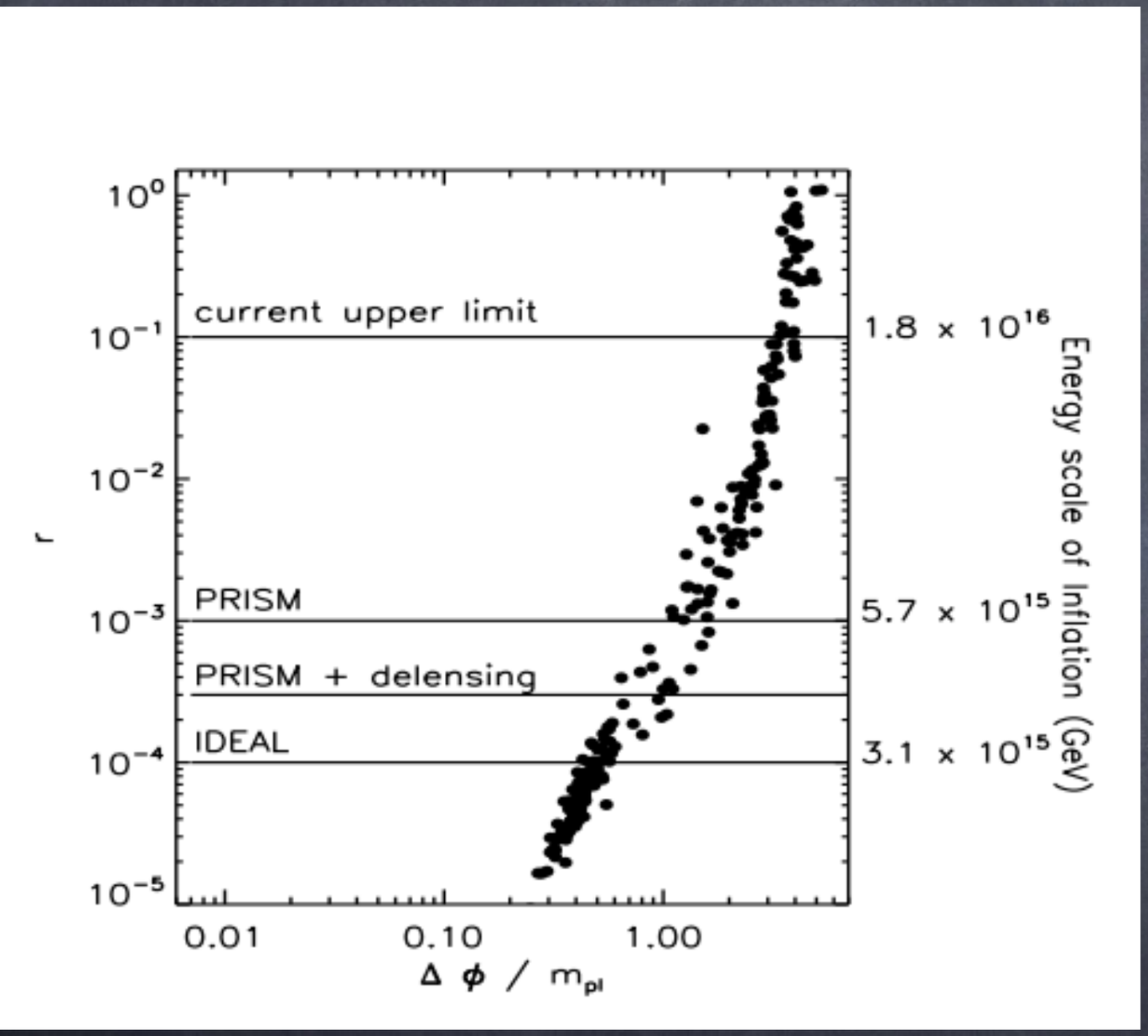
Inflation Observables





Few observables and other observables seem to be not required

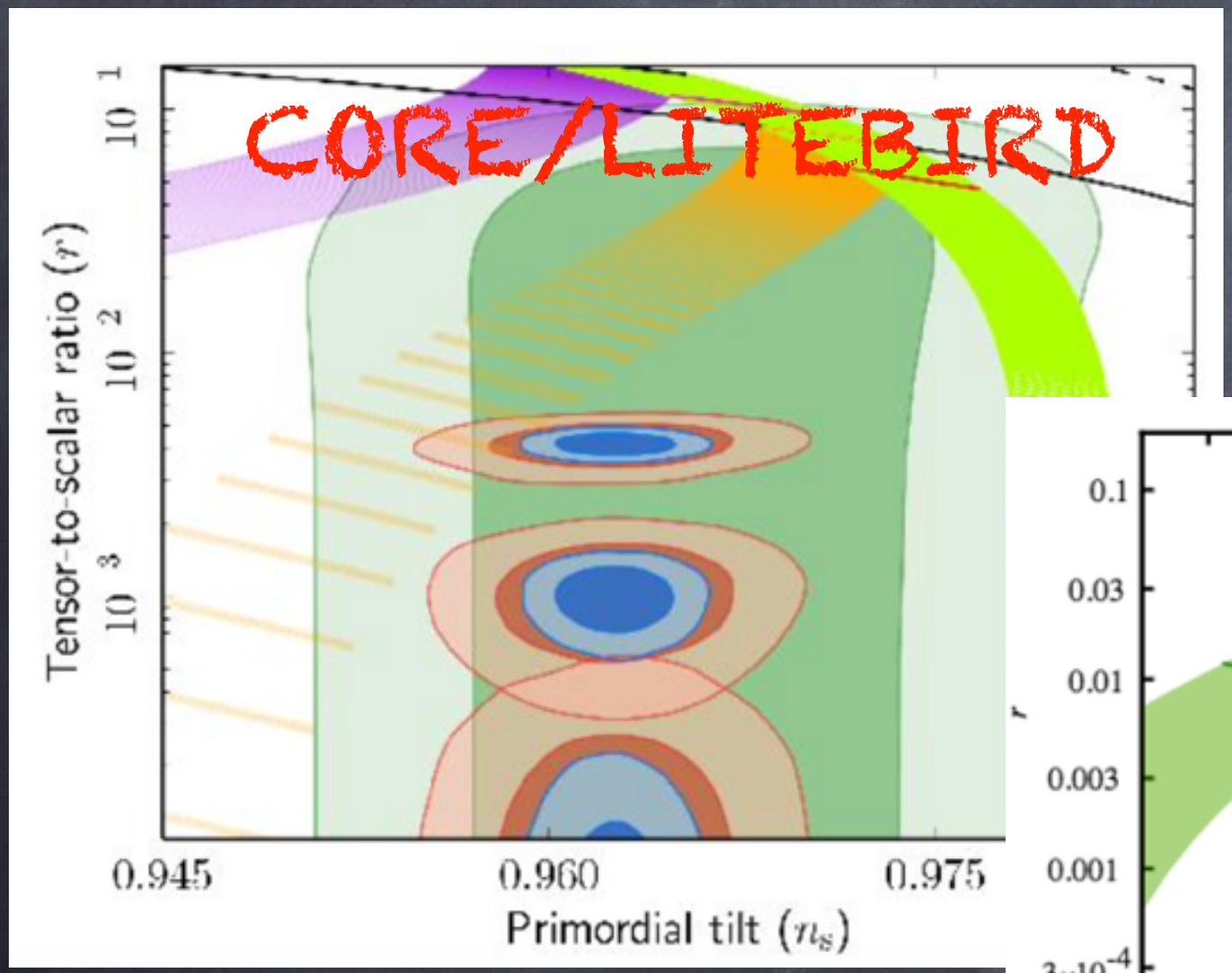
Where we stand

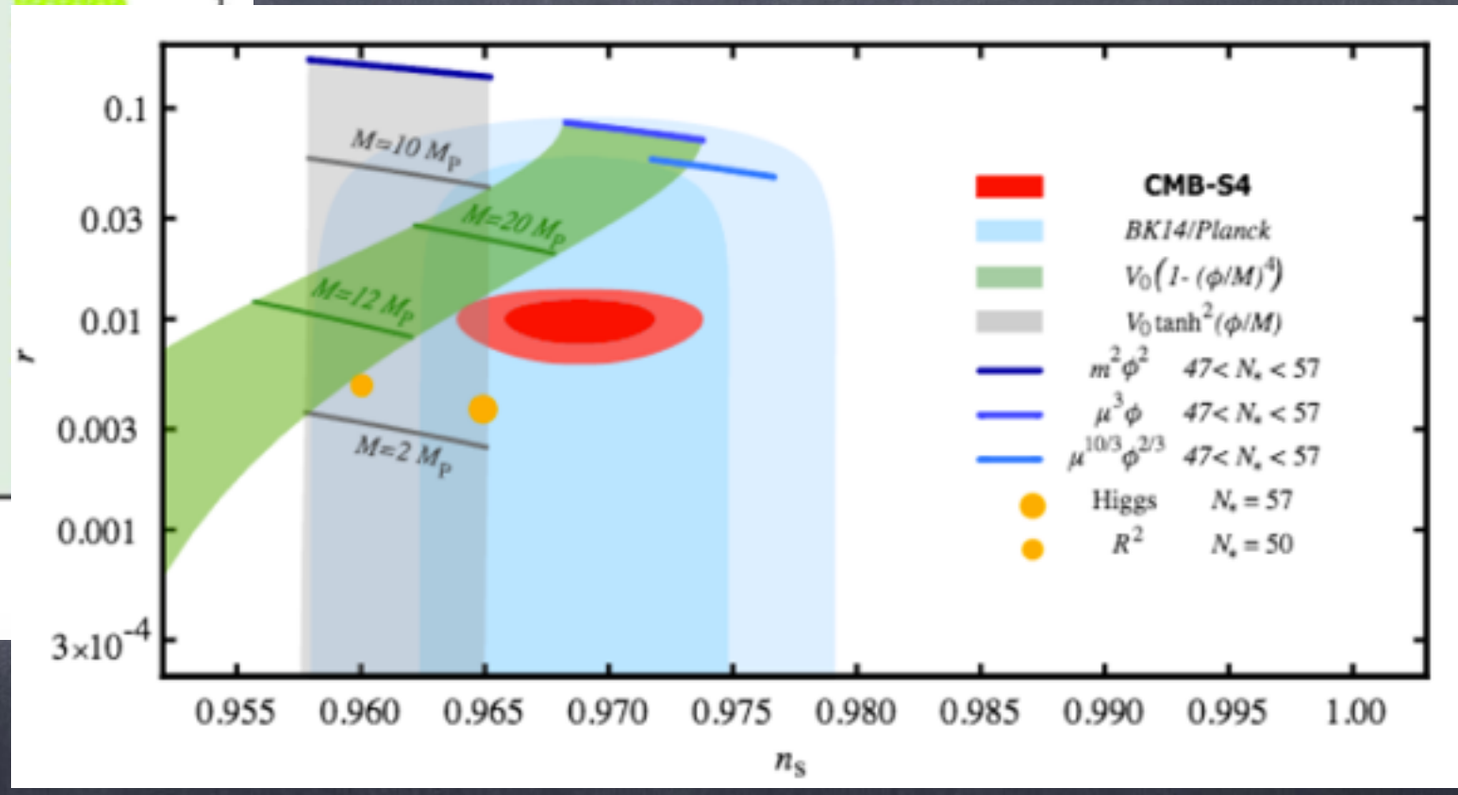


$$V^{1/4} \sim \left(\frac{r}{0.01}\right)^{1/4} 10^{16}\ GeV$$

We need some luck here!

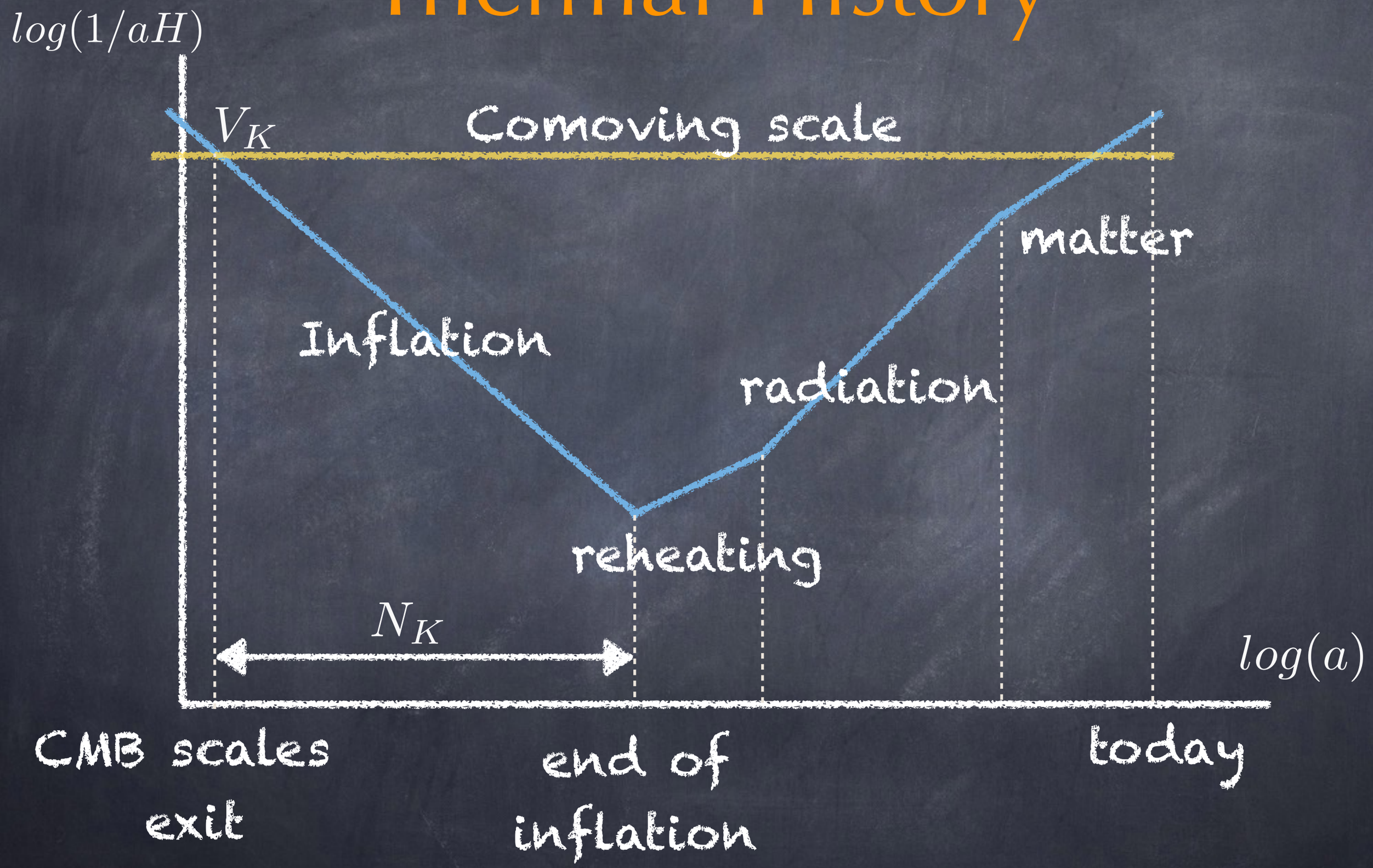
Future





precise measurements of spectral index is crucial! sensitivity $n_s \sim 10^{-3}$

Thermal History



Consistency

V_k must be evolved to H_0

Any post inflationary phase must be evolved to the present energy density

Making predictions ..

$$N_{inf} + \frac{1}{4}(1 - 3w_{rh})N_{rh} = 55 + \frac{1}{4}lnr + \frac{1}{4}ln(\rho_k/\rho_{end})$$

$$N_{inf} = 55 \pm 5$$

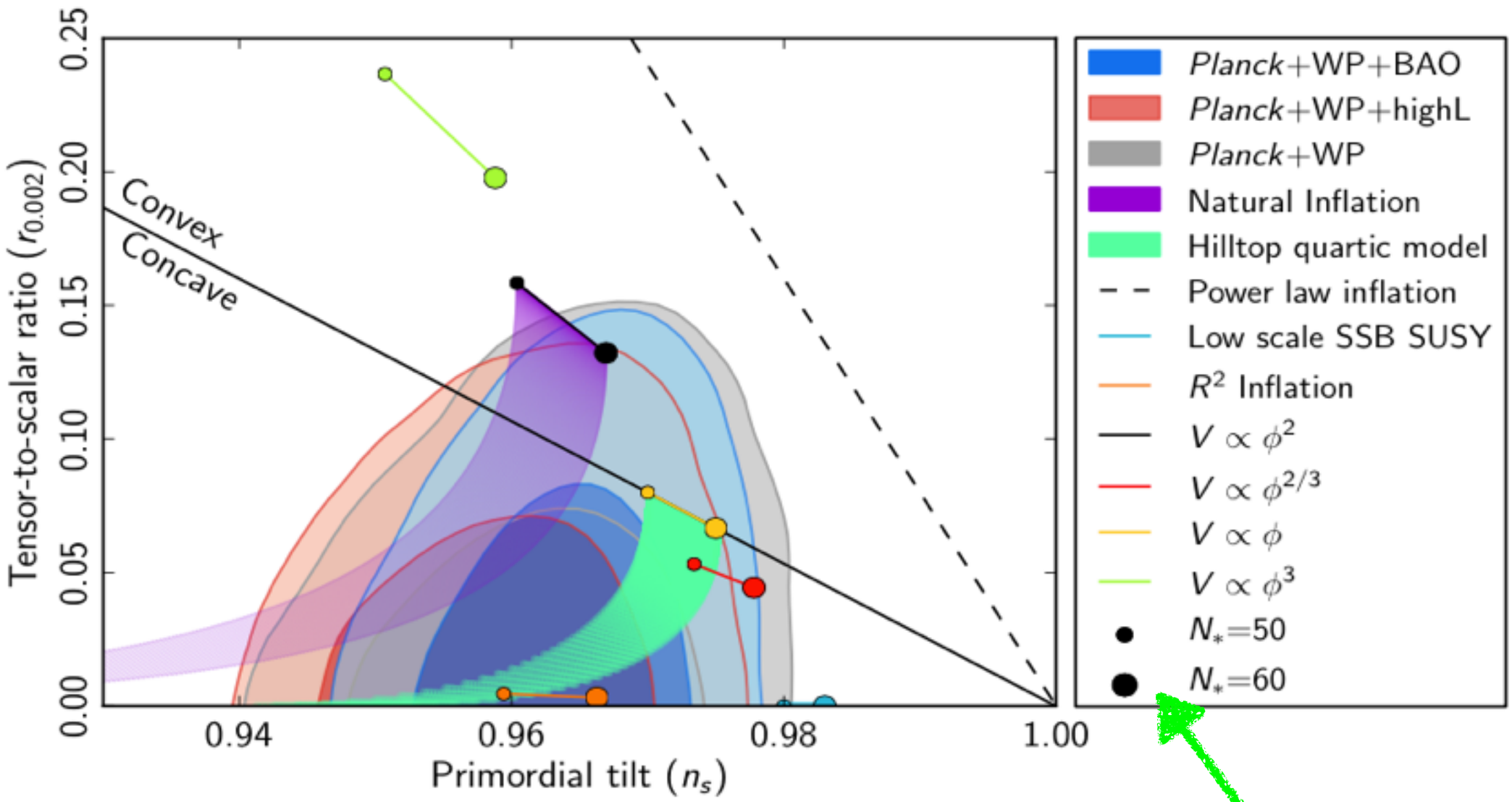
Liddle, Leach (2003)

PLANCK (2016)

'Theoretical prior'

compute observables in terms of N_{inf} and see whether it fits data for N = 50-60!

$$V(\chi) = \frac{1}{2}m^2\chi^2 \qquad n_s - 1 = -\frac{2}{N_k} \qquad r = 8/N_K$$



'Theoretical prior'

How does making predictions change for modular cosmology?

Moduli ..

- moduli: light scalar fields with Planck suppressed interactions

- at tree level effective Lagrangian of string theory/SUGRA, moduli are massless

- moduli must acquire masses (thus fixed vev) to become phenomenologically viable

- moduli stabilisation: KKLT etc....

Moduli ..

- Conservative approach: Make ALL modulus much heavier than the Hubble scale .. decouple from inflation

- Wishful ....

- In practice, few fields remain parametrically light in the post-inflationary vacua .. (e.g LVS constructions .. / talk by John Ellis yesterday) see later ..

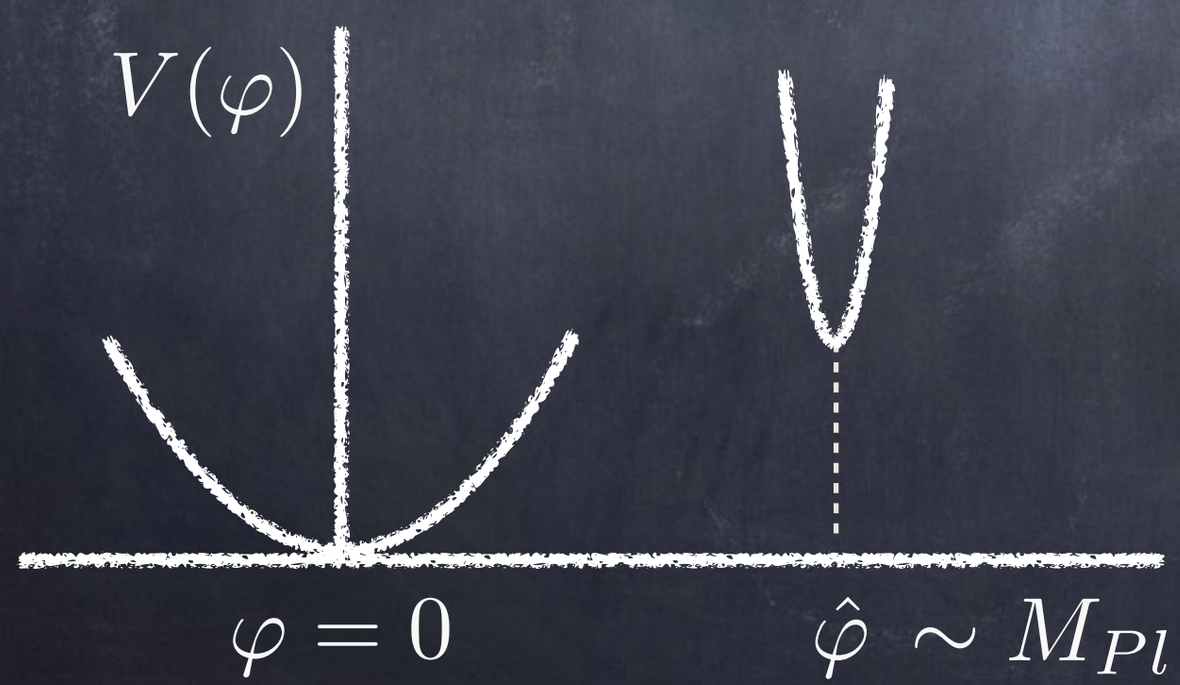
A typical case

$$\mathcal{L} \supset -\frac{1}{2}m^2\varphi^2 - \frac{1}{2}H^2(\varphi - \hat{\varphi})^2 - V_{inf}(\chi)$$

$m < H_{inf}$

post-inflationary moduli mass

minima during inflation



$$Y = \hat{\varphi}/M_{Pl} \sim 1$$

Dine, Randall, Thomas

Supergravity

$$V = e^{K[\varphi,\bar{\varphi}]} V_0[\varphi,\chi] \sim H^2 M_{Pl}^2 f\left(\frac{\varphi}{M_{Pl}}\right)$$

$V'' \sim H^2$

$\eta -$ problem

Scale of variations M_{Pl}

$$Y = \hat{\varphi}/M_{Pl} \sim 1$$

Dine, Randall, Thomas

Dvali

Supergravity

$$V = e^{K[\varphi,\bar{\varphi}]} V_0[\varphi,\chi] \sim H^2 M_{Pl}^2 f\left(\frac{\varphi}{M_{Pl}}\right)$$

$V'' \sim H^2$ $\qquad$ $\eta -$ problem

Scale of variations M_{Pl} $\qquad$ $Y = \hat{\varphi}/M_{Pl} \sim 1$

Dine, Randall, Thomas

Dvali

Toy example ..

$$V = (m_{3/2}^2 - a^2 H^2)|\varphi|^2 + \frac{1}{2M_{Pl}^2}(m_{3/2}^2 + b^2 H^2)|\varphi|^4$$

$$\hat{\varphi} \sim (a/b) M_{Pl}$$

Different source of moduli stabilisation during and after inflation

Antusch, K.D, Halter

Decay of Modulus

moduli must decay so that it does not overclose the Universe

$$\Gamma_{mod} \sim \frac{m_\varphi^3}{16\pi M_{Pl}^2}$$

G. D. Coughlan, W. Fischler, E. W. Kolb, S. Raby and G. G. Ross 1984

$$\rho_{mod}(t_{decay}) = g\frac{\pi^2}{30}T_{reheat}^4 \quad = 3H^2 M_{Pl}^2$$

decay happens

Γ_{mod}

$$T_{reheat} \sim \sqrt{\Gamma M_{Pl}}$$

$T_{reheat} > MeV$ successful BBN

$m_\varphi > 30\ TeV$ BBN bound

phenomenological implications .. SUSY breaking ..

Tom Banks, David B. Kaplan, and Ann E. Nelson

B. de Carlos, J. Casas, F.Quevedo, E.Roulet

Inflation $\longrightarrow$ reheating $\longrightarrow$ radiation

N_{inf} N_{rh1} N_{rad}

reheating $\longleftarrow$ moduli (matter)

N_{rh2} 'non-standard' N_{mod}

BBN

today

Inflation $\longrightarrow$ reheating $\longrightarrow$ radiation

N_{inf} N_{rh1} N_{rad}

reheating $\longleftarrow$ moduli (matter)

N_{rh2} 'non-standard' N_{mod}

BBN

today

K.D, Maharana

arXiv:1409.7037[hep-ph]

Liddle, Leach (2003)

$$N_{inf} + \frac{1}{4}(1-3w_{rh1})N_{rh1} + \frac{1}{4}N_{mod} + \frac{1}{4}(1-3w_{rh2})N_{rh2}$$

$$= 55.43 + \frac{1}{4}ln\ r + \frac{1}{4}ln(\frac{\rho_k}{\rho_{end}})$$

non-thermal history

Constraint ..

$$\Gamma_{mod} \sim \frac{m_\varphi^3}{16\pi M_{Pl}^2} \qquad N_{mod} \sim \frac{2}{3} ln(\frac{16\pi M_{Pl}^2 Y^4}{m_\phi^2})$$

initial displacement $Y = \hat{\varphi}/M_{Pl}$

post-inflationary details

$$\frac{1}{6} ln(\frac{16\pi M_{Pl}^2 Y^4}{m_\varphi^2}) + \frac{1}{4}(1 - 3w_{rh1})N_{re1} + \frac{1}{4}(1 - 3w_{rh2})N_{re2}$$

$$= 55.43 - N_{inf} + \frac{1}{4} ln\ r + \frac{1}{4} ln(\frac{\rho_k}{\rho_{end}})$$

inflationary details

inflationary potentials

Implications I

Central value of e-folding shifts

$$N_{inf} = 55 \pm 5$$

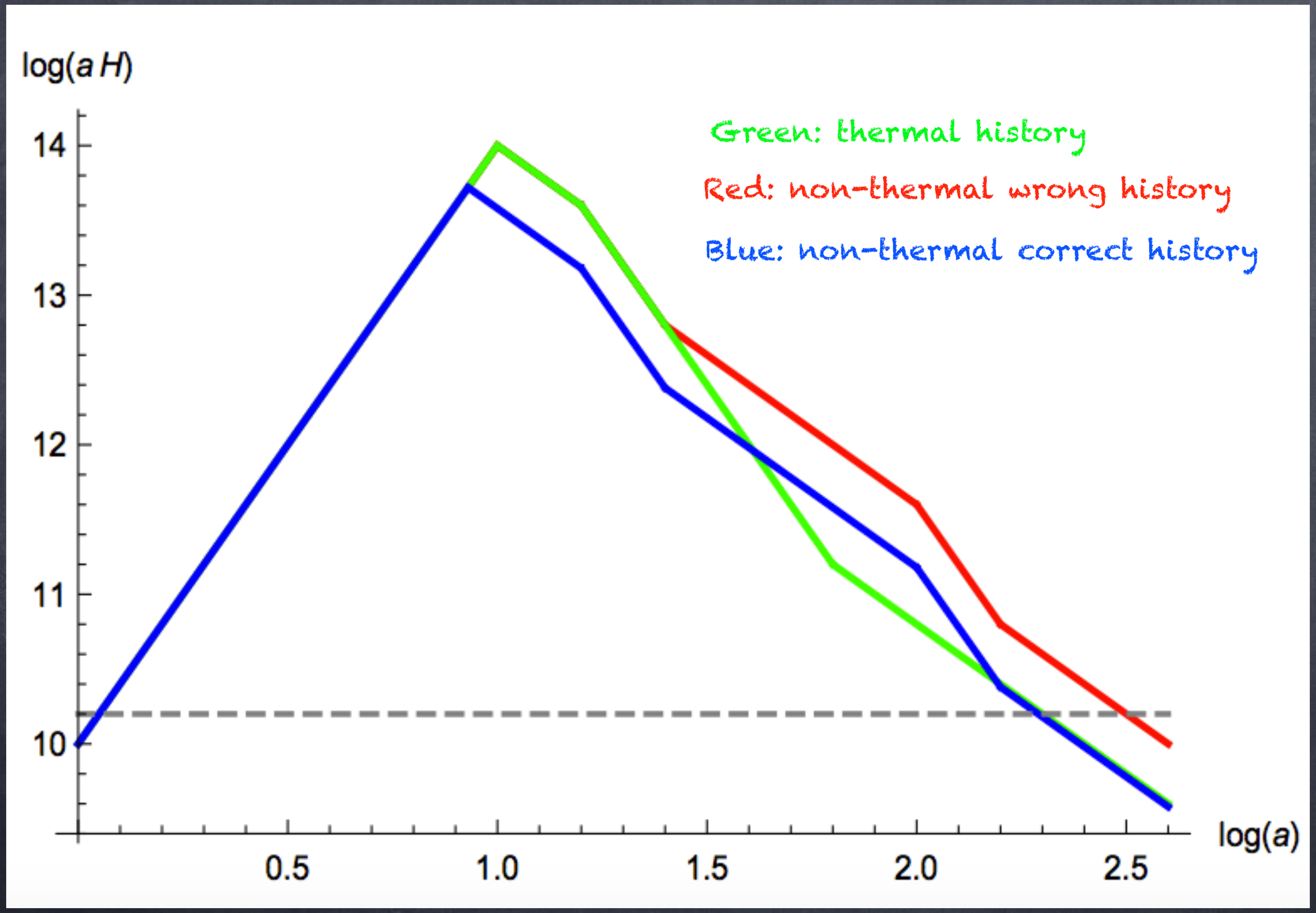
$$N_{inf} = \left(55 - \frac{1}{3} ln \left(\frac{\sqrt{16\pi} M_{pl} Y^2}{m_\varphi}\right)\right) \pm 5$$

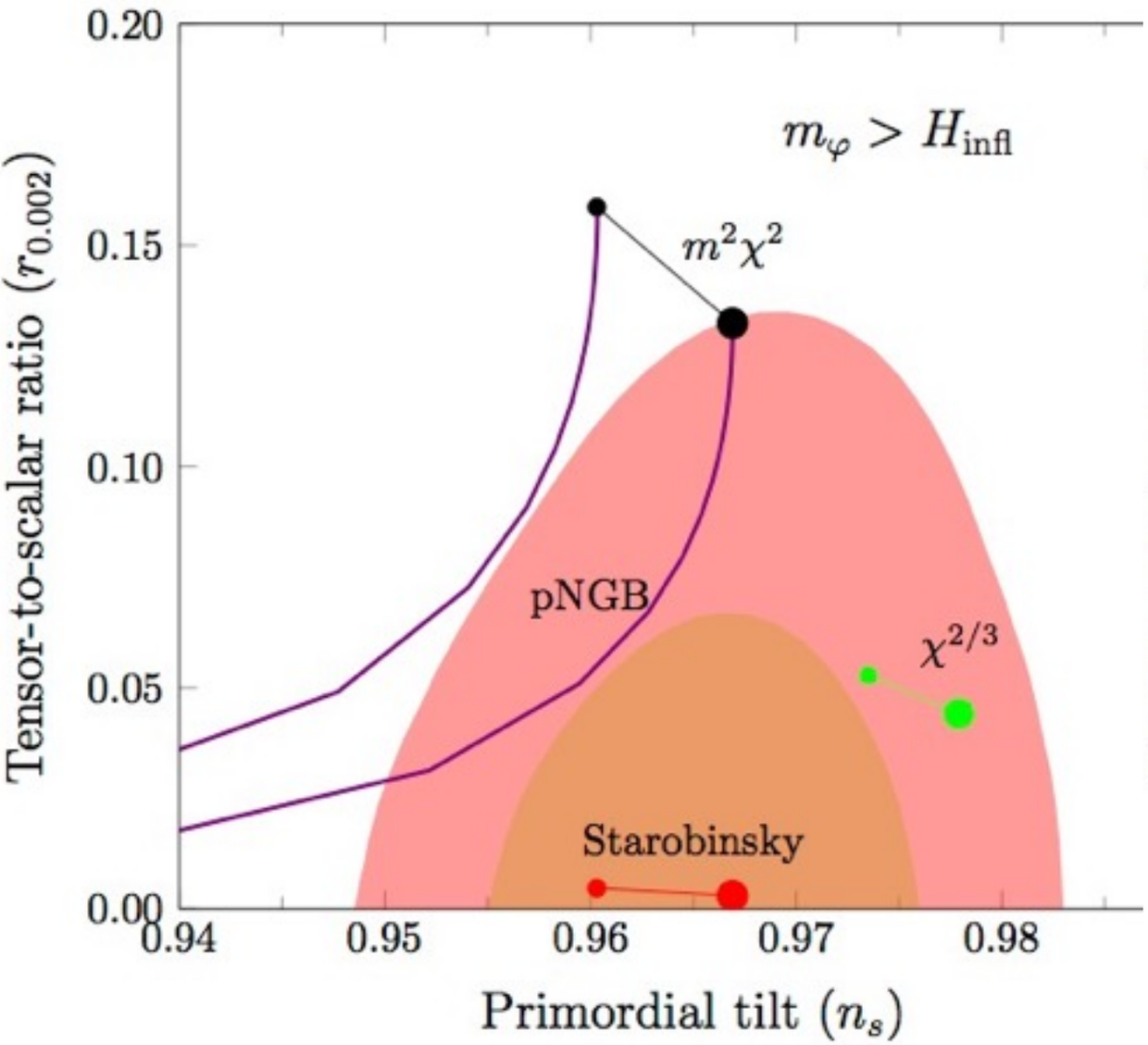
For $m_\varphi \sim 10^3$ TeV : $N_{inf} = 41 \quad - \quad 51$

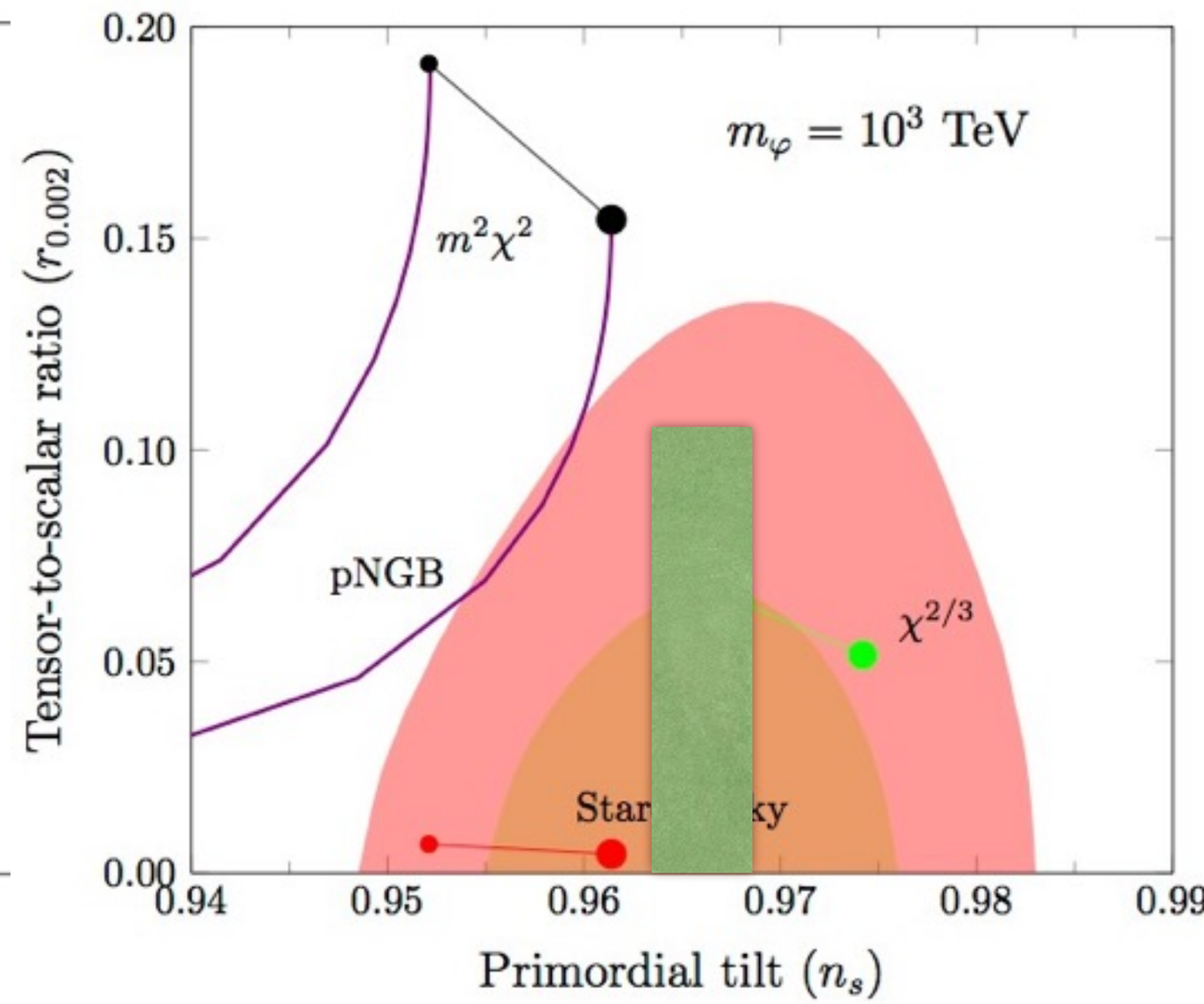
For $m_\varphi \sim 10^6$ TeV : $N_{inf} = 43 \quad - \quad 53$

(used to be 50 - 60)

(Y ~ 0.1 assumed)


log(a H)
14
13
12
11
10
0.5
1.0
1.5
2.0
2.5
log(a)
Green: thermal history
Red: non-thermal wrong history
Blue: non-thermal correct history





Das, K.D, Maharana

sensitivity $n_s \sim 10^{-3}$ CMB IV, CORE

Implications

- moduli masses tied to soft masses in SUSY
- typical moduli mass 100/1000 TeV

Preferred value of inflation e-folds

$$\hat{N}_{inf} = 55 - \frac{1}{3}\left(\frac{\sqrt{16\pi}M_{Pl}Y^2}{m_\varphi}\right) \simeq 45$$

The effects of modulus mass must be taken for inflation models for $m_\varphi \lesssim 10^{10} GeV$

Implications II

$$\frac{1}{6}ln(\frac{16\pi M_{Pl}^2 Y^4}{m_\varphi^2}) + \frac{1}{4}(1 - 3w_{rh1})N_{re1} + \frac{1}{4}(1 - 3w_{rh2})N_{re2}$$

usually positive definite

$$= 55.43 - N_{inf} + \frac{1}{4}ln\ r + \frac{1}{4}ln(\frac{\rho_k}{\rho_{end}})$$

analytical/numerical understanding of reheating: $w_{re} < 1/3$

Ellis, Garcia, Nanopoulos, Olive (2015)

Constraint ..

$$m_\varphi \gtrsim \sqrt{16\pi} M_{\rm pl} Y^2 \; e^{-3\left(55.43 - N_k + \frac{1}{4}\ln(\frac{\rho_{\rm k}}{\rho_{\rm end}}) + \frac{1}{4}\ln r\right)}$$

- Dependence correlated

Das, K.D, Maharana

- larger the value of N_{k}, stronger the bound

- smaller the value of 'r' stronger the bound

- bound depends on the nature of inflationary potentials via the ratio of energy densities

Small field models

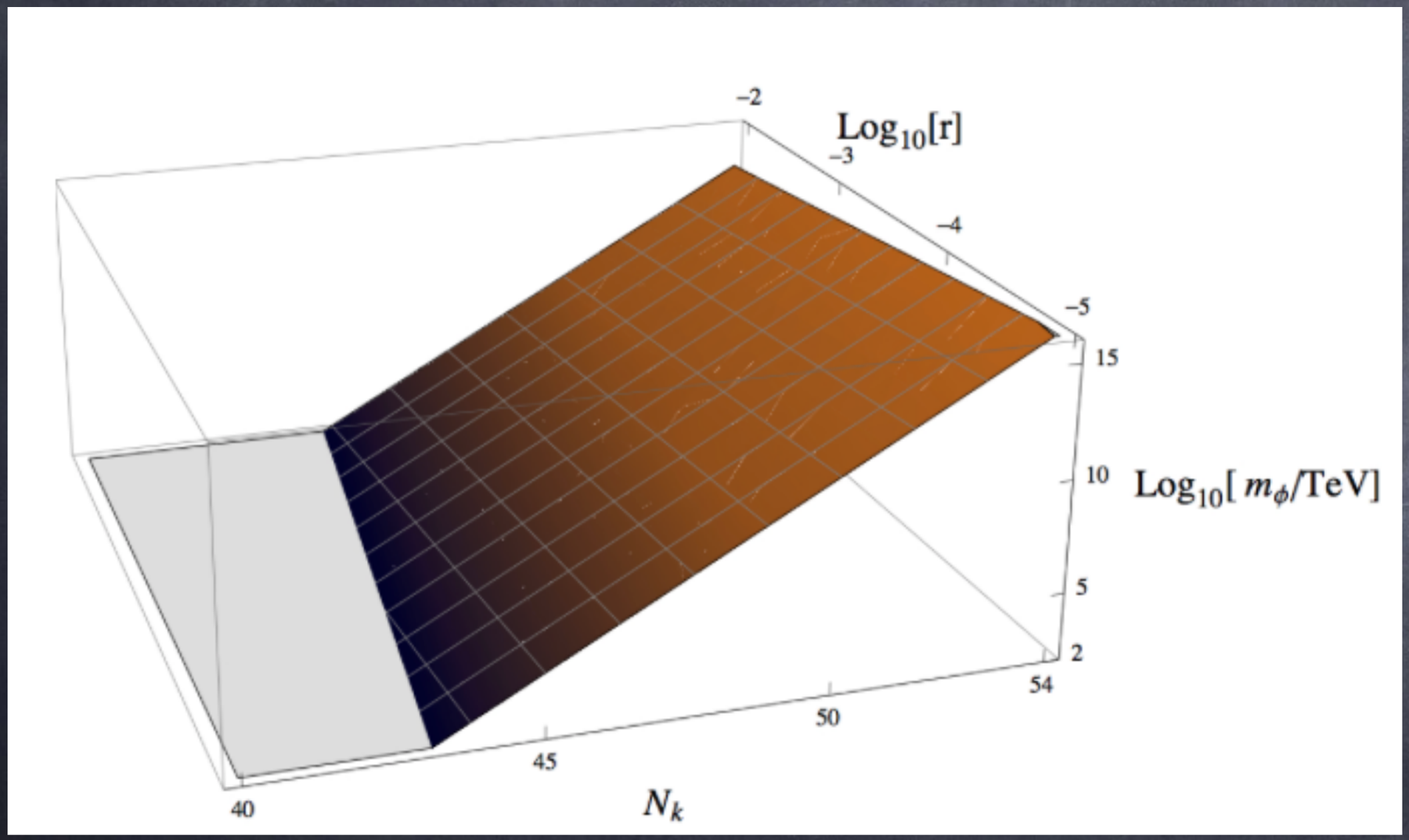
$$m_\varphi \gtrsim \sqrt{16\pi} M_{\rm pl} Y^2 \; e^{-3\left(55.43 - N_k + \frac{1}{4}\ln(\frac{\rho_{\rm k}}{\rho_{\rm end}}) + \frac{1}{4}\ln r\right)}$$

- conservative estimate r ~ 0.01 stronger the bound
- potential plateau like .. ratio of energy densities negligible
- take Y = 0.1, then for N = 50

$$m_\varphi \gtrsim 4.5 \times 10^6 TeV$$

much stronger than BBN bound

Small field models



for N > 44, the bound much stronger than BBN bound

Das, Maharana, K.D

large field models

$$m_\varphi \gtrsim \sqrt{16\pi} M_{\rm pl} Y^2 \; e^{-3\left(55.43 - N_k + \frac{1}{4}\ln(\frac{\rho_{\rm k}}{\rho_{\rm end}}) + \frac{1}{4}\ln r\right)}$$

$$V_\chi = m^{4-\alpha}\chi^\alpha$$

chaotic inflation
axion monodormy

$$m_\phi \gtrsim$$

$$\sqrt{16\pi} M_{\rm pl} Y^2 e^{-3\left(55.85 - \frac{(2+\alpha)}{2(1-n_s)} + \frac{\alpha}{8}\ln 2 + \frac{1}{8}(\alpha-2)\ln\left(\frac{2+\alpha}{\alpha(1-n_s)}\right)\right)}$$

$\alpha = 2 \quad m_\varphi >> 10^{10}$ TeV

PLANCK: Central value

$\alpha = 2/3$

Bound insignificant

What to calculate now?

Cicoli, K.D, Maharana, Quevedo

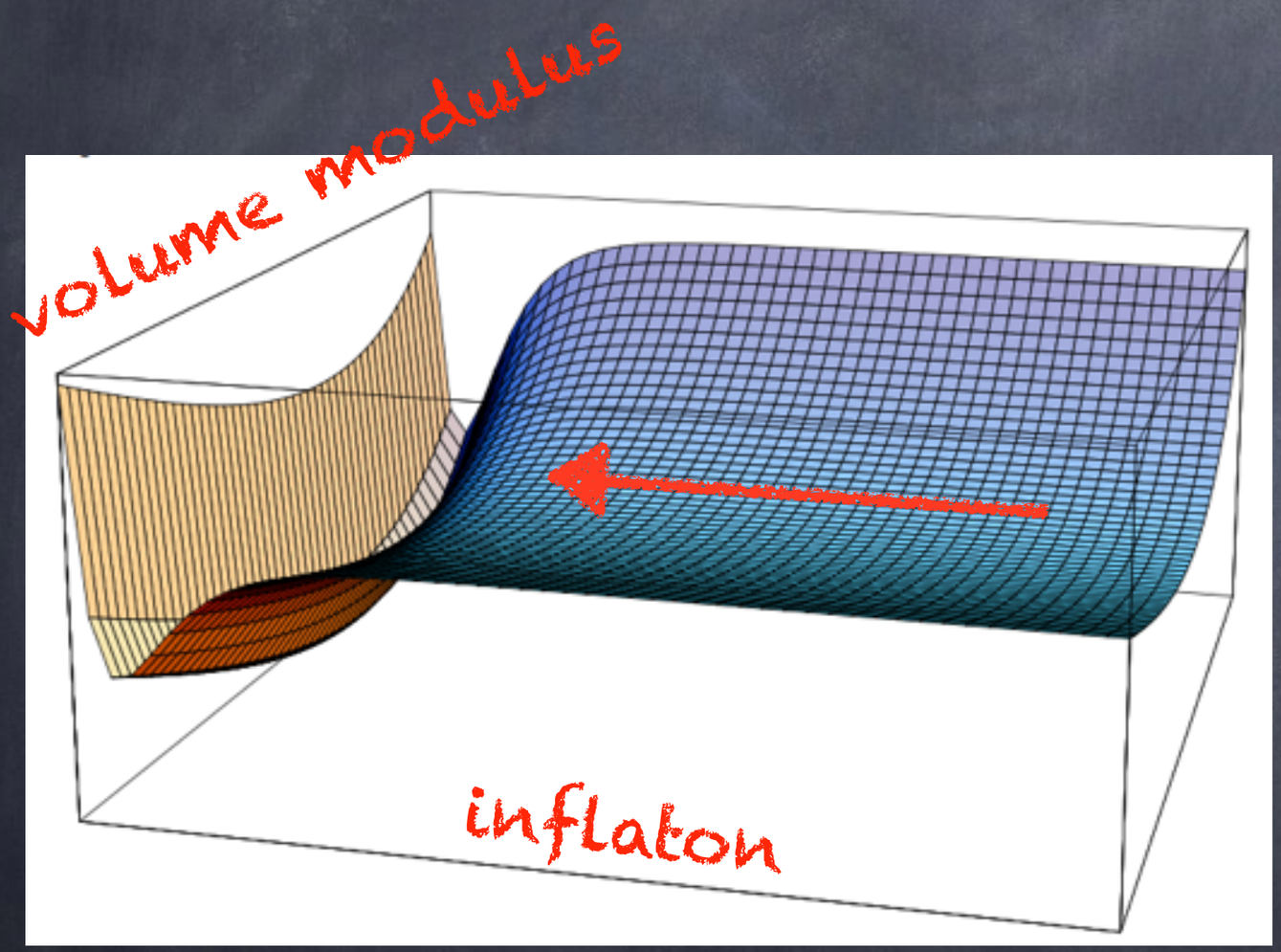
$$N_{mod} \sim \frac{2}{3} ln(\frac{16\pi M_{Pl}^2 Y^4}{m_\phi^2})$$

$$\frac{1}{6} ln(\frac{16\pi M_{Pl}^2 Y^4}{m_\varphi^2}) + \frac{1}{4}(1-3w_{rh1})N_{re1} + \frac{1}{4}(1-3w_{rh2})N_{re2}$$

$$= 55.43 - N_{inf} + \frac{1}{4} ln\ r + \frac{1}{4} ln(\frac{\rho_k}{\rho_{end}})$$

Kahler moduli inflation

Conlon, Quevedo (2005)



- In LVS scenario of string theory.
- A concrete set-up where inflationary potential is known
- Post-inflationary modulus domination happens

$$\Gamma_{\tau_n} \simeq 0.1 \frac{m_{\tau_n}^3}{M_s^2} \gg \quad \Gamma_\nu \simeq \frac{m_\nu^3}{16\pi M_{\rm pl}^2}$$

Cicoli, Mazumdar (2010)

Details ..

Volume $\mathcal{V} = \alpha\left(\tau_1^{3/2} - \sum_{i=2}^{n} \lambda_i \tau_i^{3/2}\right)$

$$V = \sum_{i=2}^{n} \frac{8(a_i A_i)^2 \sqrt{\tau_i}}{3\mathcal{V}\lambda_i} e^{-2a_i\tau_i} - \sum_{i=2}^{n} \frac{4a_i A_i W_0}{\mathcal{V}^2} \tau_i e^{-a_i\tau_i} + \frac{3\hat{\xi} W_0^2}{4\mathcal{V}^3} + \frac{D}{\mathcal{V}^\gamma}$$

$$m_{\tau_i}^2 \simeq \frac{W_0^2 (\ln \mathcal{V})^2 M_{\rm pl}^2}{\mathcal{V}^2} \gg m_{\mathcal{V}}^2 \simeq \frac{W_0^2 M_{\rm pl}^2}{\mathcal{V}^3 \ln \mathcal{V}}$$

taking all small moduli at minima: two field potential

$$V_{\rm inf} = -\frac{3W_0^2}{2\mathcal{V}^3}\left(\sum_{i=2}^{n-1}\left[\frac{\lambda_i \alpha}{a_i^{3/2}}\right](\ln \mathcal{V})^{3/2} - \frac{\xi}{2}\right) + \frac{D}{\mathcal{V}^\gamma} - \frac{4a_n A_n W_0}{\mathcal{V}^2}\tau_n e^{-a_n\tau_n}$$

modulus

inflaton

Shift in Volume Modulus

Potential experienced by the volume modulus depends on the inflaton: Vacuum misalignment

Other modulus are not shifted and having masses much larger than the Hubble scale.

$$V = -\frac{3W_0^2}{2\mathcal{V}^3}\left(\sum_{i=2}^{n}\left[\frac{\lambda_i\alpha}{a_i^{3/2}}\right](\ln\mathcal{V})^{3/2} - \frac{\hat{\xi}}{2}\right) + \frac{D}{\mathcal{V}^\gamma}$$

$$Y = \frac{\delta\varphi}{M_{pl}} = \sqrt{\frac{2}{3}}\delta\phi \simeq 2\sqrt{\frac{2}{3}}R\phi_* \simeq 0.1 - 1$$

We validate the generic arguments with explicit calculations

e-foldings

t1: end of inflation

Barnaby, Bond, Huang, Kofman (2009)

$$\rho_{\tau_n}(t_1) \approx \frac{M_{\rm pl}^4 W_0^2 \beta}{\mathcal{V}^3}$$

$$H(t_1) \approx \frac{M_{\rm pl} W_0 \beta^{1/2}}{\mathcal{V}^{3/2}} \simeq m_{\mathcal{V}}$$

Volume modulus starts to oscillate immediately at the end of inflation

$$\rho_{\mathcal{V}}(t_1) \approx m_{\mathcal{V}}^2 \varphi_{\rm in}^2 \approx \frac{M_{\rm pl}^4 W_0^2 Y^2}{\mathcal{V}^3 \ln \mathcal{V}} \qquad \frac{\rho_{\mathcal{V}}(t_1)}{\rho_{\tau_n}(t_1)} \approx \frac{Y^2}{\beta \ln \mathcal{V}} \equiv \theta^2 \ll 1$$

short matter dominated epoch until inflaton decays

$$N_{\rm mod1} = \ln\left(\frac{a(t_2)}{a(t_1)}\right) = \frac{1}{3}\ln\left(\frac{\rho_{\tau_n}(t_1)}{\rho_{\tau_n}(t_2)}\right) \simeq \frac{2}{3}\ln\left(\frac{H(t_1)}{\Gamma_{\tau_n}}\right) \simeq \frac{2}{3}\ln\left(\frac{10\beta^{1/2}\mathcal{V}^{1/2}}{W_0^2(\ln\mathcal{V})^3}\right)$$

t2: inflation decay time

e-foldings

Inflaton may decay to hidden sectors Cicoli, Mazumdar (2010)

When inflaton decays, radiation domination

$$H(t_2) = H(t_1)\left(\frac{a(t_1)}{a(t_2)}\right)^{3/2} = H(t_1)\, e^{-\frac{3}{2}N_{\rm mod1}} \simeq \frac{H(t_1)W_0^2(\ln\mathcal{V})^3}{10\beta^{1/2}\mathcal{V}^{1/2}}$$

At t_eq, radiation and volume modulus oscillations density become equal

$$\rho_{\rm rad}(t_2)\left(\frac{a(t_2)}{a(t_{\rm eq})}\right)^4 = \rho_{\mathcal{V}}(t_2)\left(\frac{a(t_2)}{a(t_{\rm eq})}\right)^3 \qquad H(t_{\rm eq}) = \frac{H(t_1)W_0^2(\ln\mathcal{V})^3\theta^4}{10\beta^{1/2}\,\mathcal{V}^{1/2}}$$

$$N_{\rm mod2} \simeq \frac{2}{3}\ln\left(\frac{H(t_{\rm eq})}{\Gamma_{\mathcal{V}}}\right)$$

Similar to the previous expressions

$$N_{\rm mod2} \approx \frac{2}{3}\ln\left(\frac{16\pi\mathcal{V}^{5/2}(\ln\mathcal{V})^{5/2}Y^4}{10\beta^2}\right) \approx \frac{2}{3}\ln\left(\frac{16\pi\mathcal{V}^{5/2}Y^4}{10P^2R^2(\ln\mathcal{V})^{1/2}}\right)$$

A benchmark example

$$W_0 = \alpha = \lambda_i = 1, a_i = 2\pi, g_s = 0.06$$

$$\mathcal{V}_{\rm in} \simeq 1.38 \cdot 10^5, \beta \simeq 3.88$$

$$\tau_n \simeq 1.12 \cdot 10^8 \, N_e^{-4} \qquad \text{and} \qquad r = 16\epsilon \simeq 1.04 \cdot 10^{-10}$$

plays important role

$$m_{\mathcal{V}} \sim 10^8 - 10^9 \;\; GeV$$

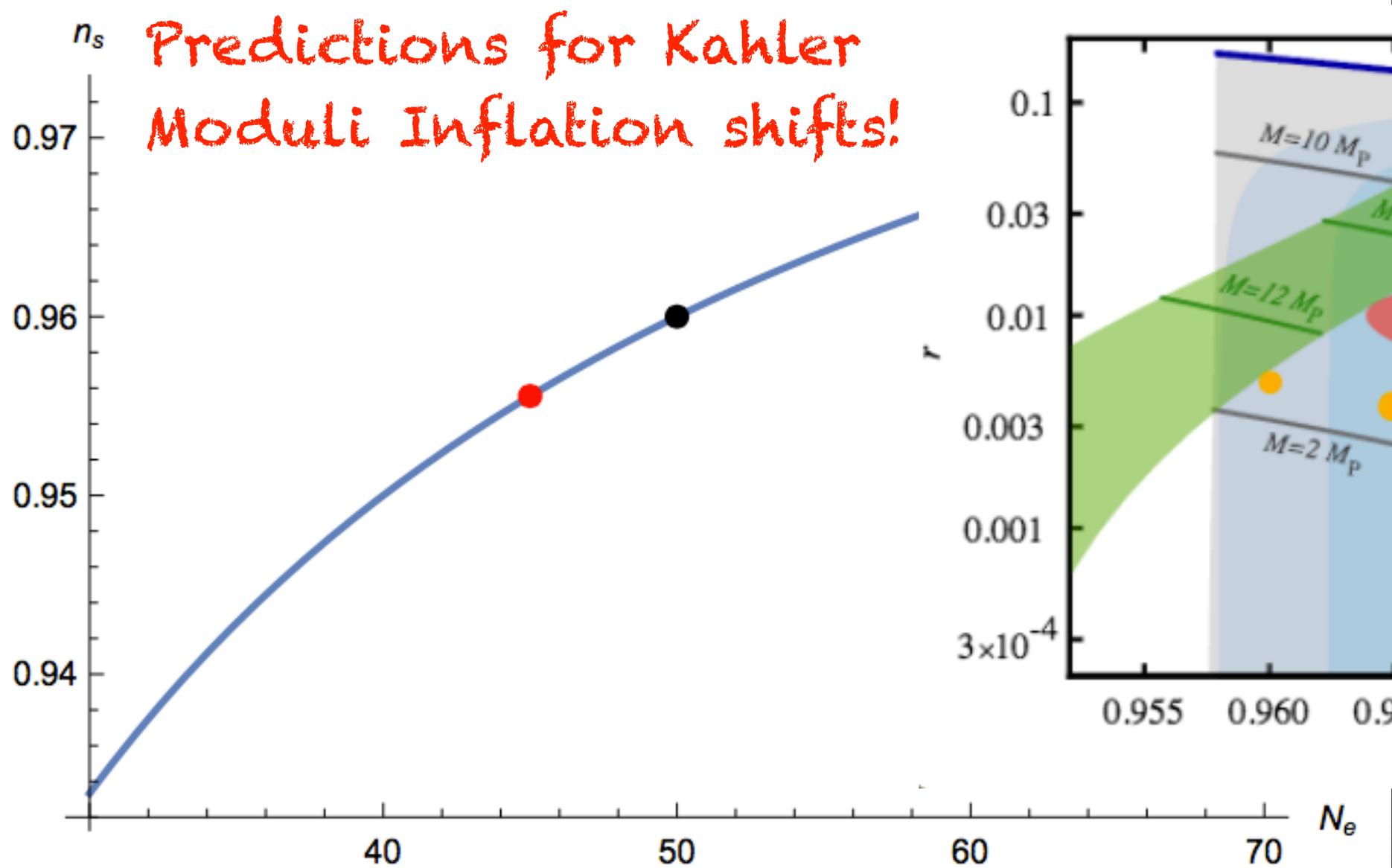
$$N_{\rm mod1} \simeq 0.99 \qquad \text{and} \qquad N_{\rm mod2} \simeq 25.4$$

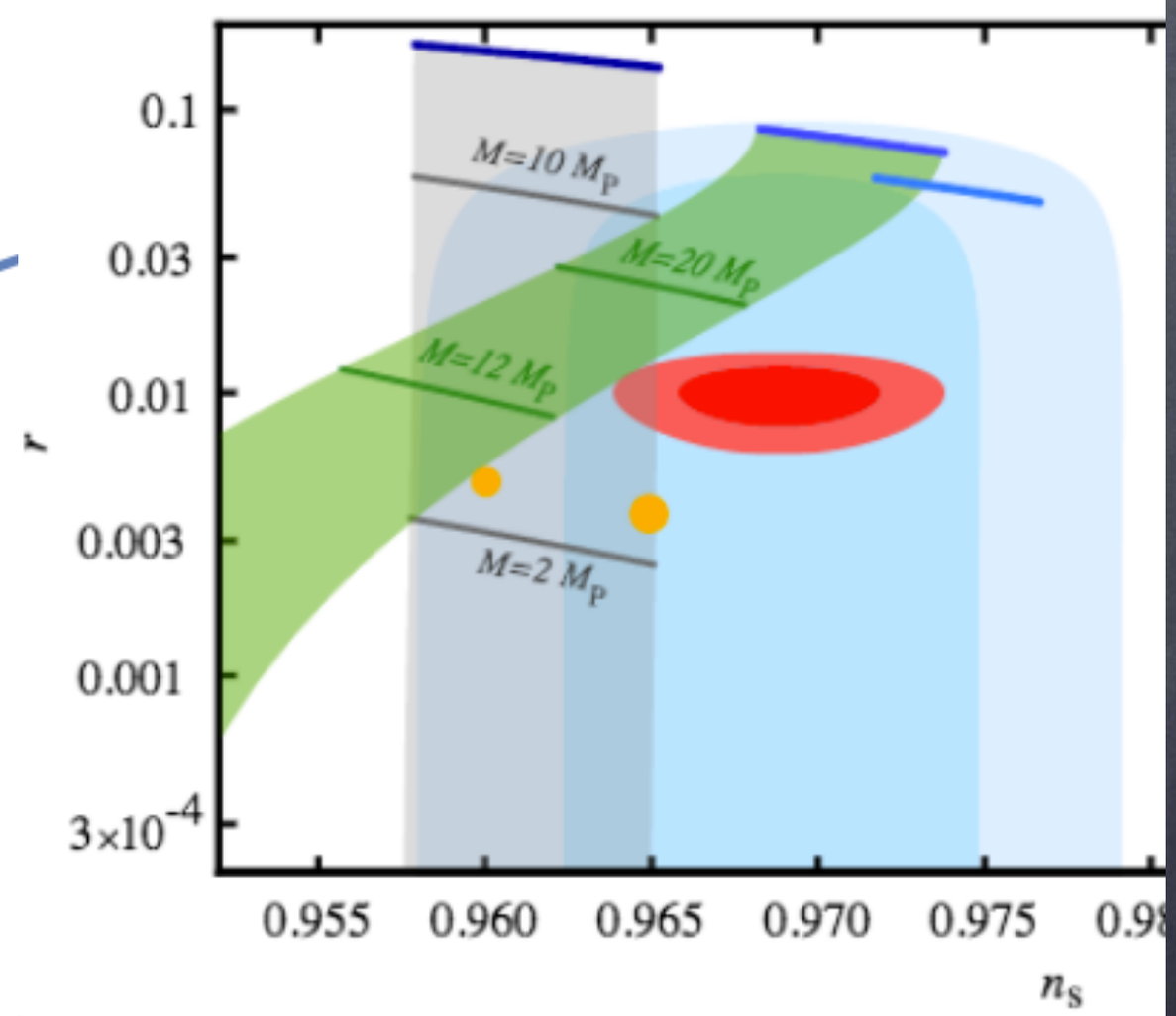
$$N_e \simeq 44.65 + \frac{1}{4} \ln \left(\frac{\rho_*}{\rho_{\rm end}} \right) \simeq 45 \qquad \Rightarrow \qquad \tau_n \simeq 27.3 \qquad \text{and} \qquad n_s \simeq 0.955$$

$$T_{\rm rh} \gtrsim 10^3 \;\; GeV$$

OK with BBN bound

Predictions for Kahler Moduli Inflation shifts!





Yesterday's talk

Starobinsky-like Inflation, Supercosmology and Neutrino Masses in No-Scale Flipped SU(5)

1704.07331

John Ellis[a], Marcos A. G. Garcia[b], Natsumi Nagata[c], Dimitri V. Nanopoulos[d] and Keith A. Olive[e]

Entropy production from a non-inflaton field

$$N_* = N_*^{\mathrm{STH}} - \frac{1}{3}\ln\Delta$$

For 'weak reheating', the spectral-index prediction is changed substantially!

Post-inflationary history is crucial to understand!

Conclusions

- modulus dominated cosmology is a generic feature of string/sugra motivated scenario

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- modulus dominated cosmology is a generic feature of string/sugra motivated scenario

$$\hat{N}_{inf} = 55 - \frac{1}{3}\left(\frac{\sqrt{16\pi}M_{Pl}Y^2}{m_\varphi}\right)$$

sensitivity $n_s \sim 10^{-3}$ CORE/CMB IV

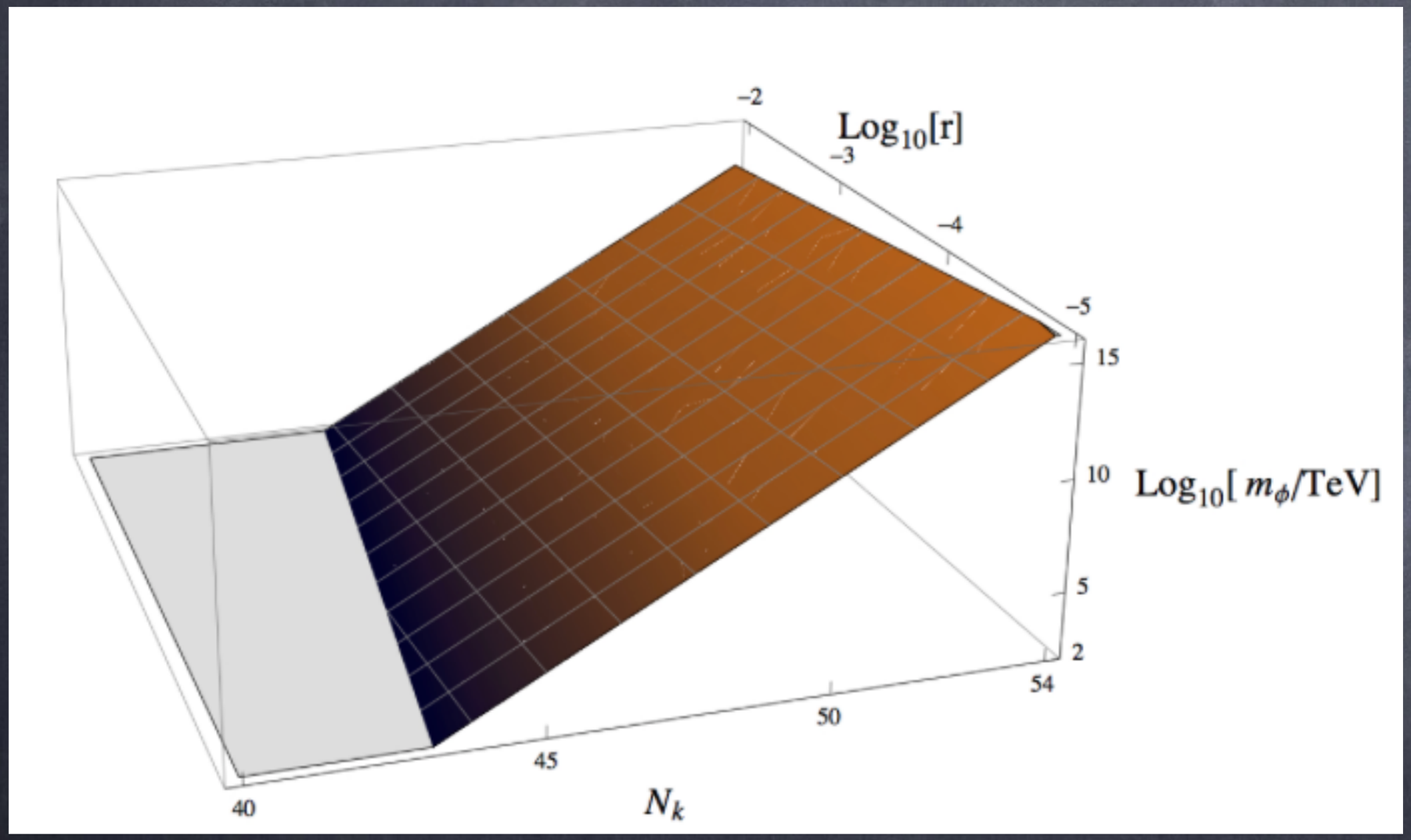
Conclusions

- modulus dominated cosmology is a generic feature of string/sugra motivated scenario

$$\hat{N}_{inf} = 55 - \frac{1}{3}\left(\frac{\sqrt{16\pi}M_{Pl}Y^2}{m_\varphi}\right) \simeq 45$$

- Independent constraint on modulus mass derived using precision CMB data

Bound..



for N > 44, the bound much stronger than BBN bound

Conclusions

- modulus dominated cosmology is a generic feature of string/sugra motivated scenario

$$\hat{N}_{inf} = 55 - \frac{1}{3}\left(\frac{\sqrt{16\pi}M_{Pl}Y^2}{m_\varphi}\right) \simeq 45$$

- Independent constraint on modulus mass derived using precision CMB data

- Explicit calculations for Kahler Moduli inflation

Thank You

Shift in Volume Modulus

Potential experienced by the volume modulus depends on the inflaton: Vacuum misalignment

Other modulus are not shifted and having masses much larger than the Hubble scale.

$$V = -\frac{3W_0^2}{2\mathcal{V}^3}\left(\sum_{i=2}^{n}\left[\frac{\lambda_i\alpha}{a_i^{3/2}}\right](\ln\mathcal{V})^{3/2} - \frac{\hat{\xi}}{2}\right) + \frac{D}{\mathcal{V}^\gamma}$$

$$V(\mathcal{V}_*) = \frac{\partial V(\mathcal{V}_*)}{\partial\mathcal{V}} = 0 \qquad -\frac{3W_0^2}{2}\,e^{-3\phi_*}\left(P\phi_*^{3/2} - \frac{\hat{\xi}}{2}\right) + D\,e^{-2\phi_*} = 0$$

$$\frac{3W_0^2}{2}\,e^{-3\phi_*}\left(3P\phi_*^{3/2} - \frac{3}{2}P\phi_*^{1/2} - \frac{3\hat{\xi}}{2}\right) - 2D\,e^{-2\phi_*} = 0$$

$$\phi \equiv \ln\mathcal{V} \quad \text{and} \quad P \equiv \alpha\sum_{i=2}^{n}\lambda_i\,a_i^{-3/2} = \frac{\alpha}{R}\,\lambda_n\,a_n^{-3/2} \qquad R \equiv \frac{\lambda_n\,a_n^{-3/2}}{\sum_{i=2}^{n}\lambda_i\,a_i^{-3/2}} \ll 1$$

Shift in Volume Modulus

$$\phi_*^{3/2} - \frac{3}{2}\phi_*^{1/2} - \frac{\hat{\xi}}{2P} = 0$$

$$D = \frac{9W_0^2}{4} P e^{-\phi_*} \phi_*^{1/2}$$

$$V_{\rm in}(\phi) = -\frac{3W_0^2}{4} e^{-3\phi} \left[2P\left(1-R\right)\phi^{3/2} - \hat{\xi} - 3P\phi_*^{1/2} e^{(\phi-\phi_*)} \right]$$

$$\left(1-R\right)\phi_{\rm in}^{3/2} - \frac{1}{2}\left(1-R\right)\phi_{\rm in}^{1/2} - e^{(\phi_{\rm in}-\phi_*)}\phi_*^{1/2} - \frac{\hat{\xi}}{2P} = 0$$

$$V_{\rm in}(\phi) = V(\phi) + \delta V(\phi)$$

$$\delta V(\phi) = \frac{3W_0^2}{2} e^{-3\phi} P R \phi^{3/2}$$

$$\delta\phi = -\frac{\delta V'(\phi_*)}{V''(\phi_*)} = 4R\,\frac{\phi_* + \frac{\hat{\xi}}{2P}\,\phi_*^{1/2}}{2\phi_* - 1} \simeq 2R\phi_*$$

$$\delta\phi = \phi_{\rm in} - \phi_*$$

$$Y = \frac{\delta\varphi}{M_{pl}} = \sqrt{\frac{2}{3}}\delta\phi \simeq 2\sqrt{\frac{2}{3}}R\phi_* \simeq 0.1 - 1$$

We validate the generic arguments with explicit calculations

Inflationary Phenomenology

$$V = V_0 - \frac{4W_0 a_n A_n}{\mathcal{V}_{\rm in}^2}\left(\frac{3\mathcal{V}_{\rm in}}{4\lambda}\right)^{2/3}\sigma^{4/3}\exp\left[-a_n\left(\frac{3\mathcal{V}_{\rm in}}{4\lambda}\right)^{2/3}\sigma^{4/3}\right]$$

Canonical inflaton field

$$V(\sigma) = C_0(1 - e^{-b\sigma})$$

Effective single field dynamics

Volume modulus is stabilised during inflation and heavy

$$\epsilon = \frac{M_{\rm pl}^2}{2}\left(\frac{V'}{V}\right)^2 = \frac{32\mathcal{V}_{\rm in}^3}{3\beta^2 W_0^2\lambda_n}a_n^2 A_n^2\sqrt{\tau_n}\,(1 - a_n\tau_n)^2\,e^{-2a_n\tau_n}\,,$$

$$\eta = M_{\rm pl}^2\frac{V''}{V} = -\frac{4\mathcal{V}_{\rm in}^2}{3\beta W_0\lambda_n\sqrt{\tau_n}}a_n A_n\left[\left(1 - 9a_n\tau_n + 4a_n^2\tau_n^2\right)e^{-a_n\tau_n}\right]\,.$$

Inflationary Phenomenology

$$N_e(\sigma) = \int_{\sigma_{\rm end}}^{\sigma} \frac{1}{\sqrt{2\epsilon(\sigma)}}\, d\sigma \;\simeq\; \frac{3\beta W_0 \lambda_n}{16 \mathcal{V}_{\rm in}^2 a_n^{3/2} A_n} \frac{e^{a_n \tau_n}}{(a_n \tau_n)^{3/2}}$$

$$\epsilon \simeq \left(\frac{3\lambda_n}{8 a_n^{3/2} \mathcal{V}_{\rm in}} \right) \frac{1}{N_e^2 \sqrt{a_n \tau_n}} \quad \ll \quad \eta \simeq -\frac{1}{N_e}$$

COBE normalisation: $$\frac{V^{3/2}}{M_{\rm pl}^3 V'} = 5.2 \times 10^{-4}$$

$$\tau_n \simeq 7.31 \cdot 10^{-14} \left(\frac{6\pi \lambda_n}{g_s\, \beta\, e^{K_{\rm cs}}} \right)^2 \left(\frac{\mathcal{V}_{\rm in}^4}{W_0^4\, a_n^4} \right) \frac{1}{N_e^4}$$

$$r = 16\epsilon \simeq 16 \times 3.7 \cdot 10^6 \left(\frac{g_s\, \beta\, e^{K_{\rm cs}}}{16\pi} \right) \left(\frac{W_0^2}{\mathcal{V}_{\rm in}^3} \right) \ll 10^{-4} N_e^{-3}$$

$$n_s = 1 + 2\eta - 6\epsilon \simeq 1 - \frac{2}{N_e}$$

spectral index depends only on N_e