Istituto Nazionale di Fisica Nucleare
Sezione di Napoli

CANDLES OF DARKNESS
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Overview of Flavour Physics

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Collaboration with Andreas Crivellin and Julian Heeck
Phys.Rev.Lett. 114 (2015) 151801
Phys.Rev. D91 (2015) 7, 075006

Collaboration with Crivellin,A., Kitahara, T and Nierste, U.
e-Print: arXiv:1703.05786

Collaboration with Crivellin, A Hoferichter, M and Tunstall, L
Phys.Rev. D 2016

Collaboration with Cappiello,L. E. Greynat,D. EPJC

Collaboration with Coluccio-Leskow, E. Greynat,D. and Nath,A.
Phys.Rev. D 2016

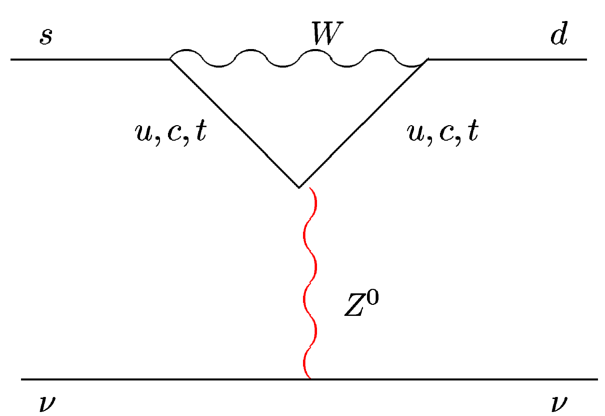
Outline

- Rare Kaon decays (golden channels)
- Direct CP violation ϵ'
- Effective field theories and QCD
- Minimal flavour violation
- LD vs SD $K^+ \to \pi^+ e^+ e^-$ $K_S \to \pi^0 e^+ e^-$

$K \to \pi\nu\overline{\nu}$

Why we need to the experiments KOTO and NA62

$$A(s \to d\nu\overline{\nu})_{\rm SM} \sim \overline{s}_L\gamma_\mu d_L \quad \overline{\nu}_L\gamma^\mu\nu_L \times \left[\sum_{q=c,t} V_{qs}^* V_{qd}\, m_q^2\right]$$



$$\sim \qquad \left[A^2\lambda^5\ (1-\rho-i\eta)m_t^2 + \lambda m_c^2\right]$$

SM $\quad \underbrace{V-A \otimes V-A}$ Littenberg

$\Downarrow$

$$\Gamma(K_L \to \pi^0\nu\overline{\nu}) \quad \begin{cases} \text{CP violating} \\ \Rightarrow\ J = A^2\lambda^6\eta \\ \text{Only } top \end{cases}$$

$K^+ \to \pi^+ \nu\bar{\nu}$

Misiak, Urban; Buras, Buchalla; Brod, Gorbhan, Stamou'11, Straub

$$B(K^+) \sim \kappa_+ \left[\left(\frac{\mathrm{Im}\lambda_\mathrm{t}}{\lambda^5} X_t \right)^2 + \left(\frac{\mathrm{Re}\lambda_\mathrm{c}}{\lambda} (P_c + \delta P_{c,u}) + \frac{\mathrm{Re}\lambda_\mathrm{t}}{\lambda^5} X_t \right)^2 \right]$$

$\lambda_q = V_{qd}{}^* V_{qs}$

- κ_+ from K_{l3}
- P_c: SD charm quark contribution (30%± 2.5% to BR)
  LD $\delta P_{c,u} \sim 4 \pm 2\%$
- $B(K^\pm) = (7.8 \pm 0.8 \pm 0.3) \times 10^{-11}$ first error parametric (V_{cb}), second non-pert. QCD
- E949 $B(K^\pm) = (1.73^{+1.15}_{-1.05}) \times 10^{-10}$

K_L

$B(K_L) = (2.43 \pm 0.39 \pm 0.06) \times 10^{-11}$ vs

E391a $B(K_L) < 2.6 \times 10^{-8}$ at 90% C.L.

K_L Model-independent bound, based on $SU(2)$ properties dim-6 operators for $\bar{s}d\bar{\nu}\nu$ Grossman-Nir

$$B(K_L) \le \frac{\tau_L}{\tau_+} \times B(K^{\pm})_{\mathrm{E949}} \le 1.4 \times 10^{-9} \quad \text{at } 90\% C.L.$$

KOTO Run62: Preliminary results

After imposing all selection cuts

New Result!

Shiomi

-Summary of

#BG inside the signal box

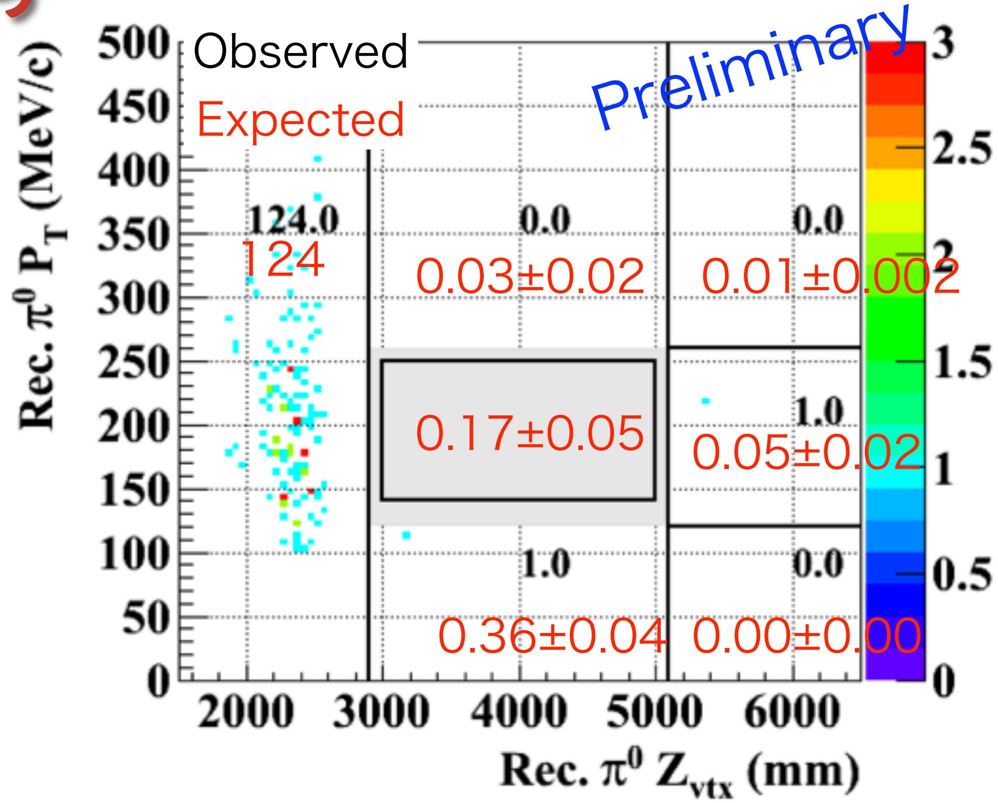
Preliminary

BG source	#BG
$K_L \to 2\pi^0$	0.04±0.03
$K_L \to \pi^+\pi^-\pi^0$	0.04±0.01
Upstream events	0.04±0.04
Hadron cluster events	0.05±0.02
Other BG sources	Under estimation

-S.E.S

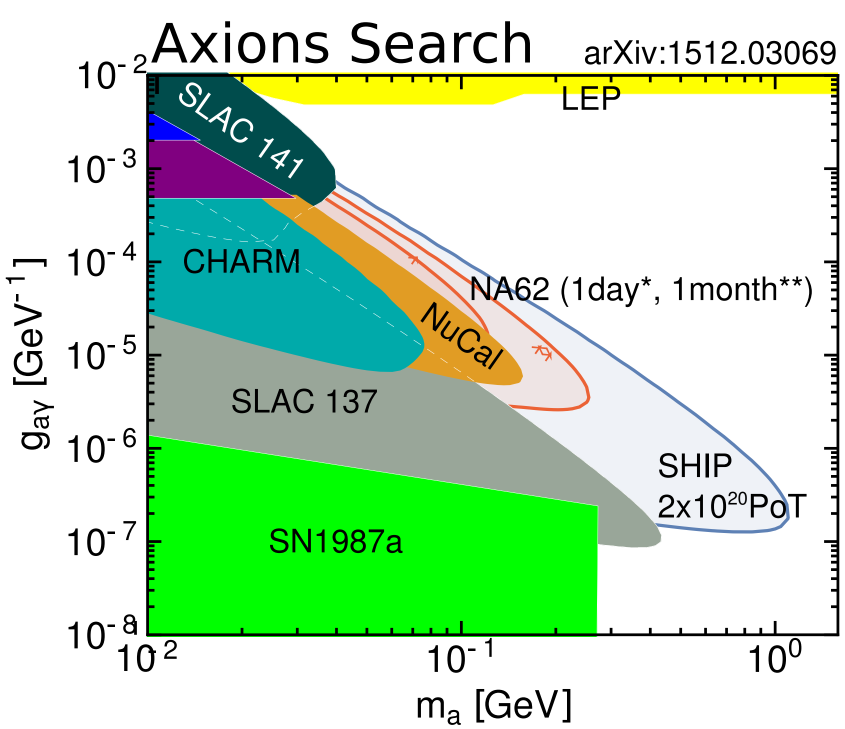
5.9×10^{-9}

Apply all selection cuts



Other Channels NA62$^{\maltese}$, E36†, TREK$^{\circledast}$, KLOE2*

- **Unitarity**: $|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2=1$ $^{\maltese\star}$[Moulson CKM2014][kloe2 1003.3868]
  V_{us}=0.2243(5) i.e. 0.21% precision, expected to reach 0.17% at KLOE2
- **Lepton Number/Flavour Violation**$^{\maltese}$
- Search for **New Particles** (axion, dark photon, inflaton...)$^{\maltese\dagger\star}$
- **Non perturbative QCD**$^{\maltese\star}$
- Test of **CPT symmetry**$^{\circledast\star}$
  [kloe2 1003.3868]



Decay	Physics	Present limit (90% C.L.) / Result	NA62
$\pi^+\mu^+e^-$	LFV	1.3×10^{-11}	0.7×10^{-12}
$\pi^+\mu^-e^+$	LFV	5.2×10^{-10}	0.7×10^{-12}
$\pi^-\mu^+e^+$	LNV	5.0×10^{-10}	0.7×10^{-12}
$\pi^-e^+e^+$	LNV	6.4×10^{-10}	2×10^{-12}
$\pi^-\mu^+\mu^+$	LNV	1.1×10^{-9}	0.4×10^{-12}
$\mu^-\nu e^+e^+$	LNV/LFV	2.0×10^{-8}	4×10^{-12}
$e^-\nu\mu^+\mu^+$	LNV	No data	10^{-12}
π^+X^0	New Particle	5.9×10^{-11} $m_{X^0}=0$	10^{-12}
$\pi^+\chi\chi$	New Particle	–	10^{-12}
$\pi^+\pi^+e^-\nu$	$\Delta S\neq\Delta Q$	1.2×10^{-8}	10^{-11}
$\pi^+\pi^+\mu^-\nu$	$\Delta S\neq\Delta Q$	3.0×10^{-6}	10^{-11}
$\pi^+\gamma$	Angular Mom.	2.3×10^{-9}	10^{-12}
$\mu^+\nu_h, \nu_h\to\nu\gamma$	Heavy neutrino	Limits up to $m_{\nu_h}=350\ MeV$	
R_K	LU	$(2.488\pm0.010)\times10^{-5}$	>×2 better
$\pi^+\gamma\gamma$	χPT	< 500 events	10^5 events
$\pi^0\pi^0e^+\nu$	χPT	66000 events	O(10^6)
$\pi^0\pi^0\mu^+\nu$	χPT	-	O(10^5)

Direct CP violation in $K \to 2\pi$

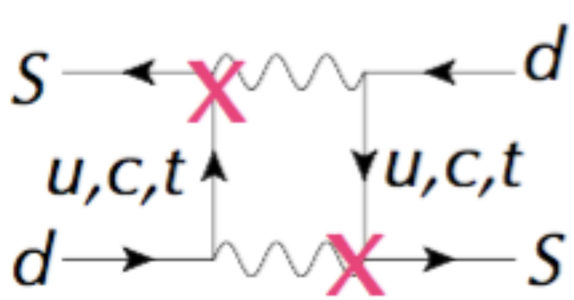
$A(K_L \to \pi^+\pi^-) \propto \epsilon + \epsilon'$ $\quad\quad$ $\epsilon \sim \mathcal{O}(10^{-3})$ $\quad\quad$ Christenson et al 64

$A(K_L \to \pi^0\pi^0) \propto \epsilon - 2\epsilon'$ $\quad\quad$ $\epsilon' \sim \mathcal{O}(10^{-6})$ $\quad\quad$ CERN, Fermilab KTeV

$$H_{\Delta S=2}$$

Indirect CP violation

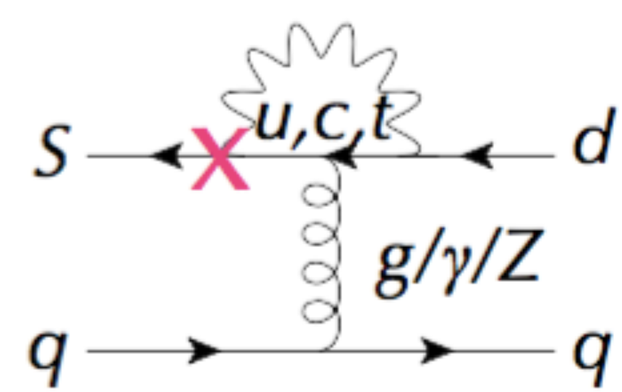
Kaon oscillation



$$H_{\Delta S=1}$$

Direct CP Violation

Penguin



$$\epsilon' = i\frac{e^{i(\delta_2-\delta_0)}}{\sqrt{2}}\frac{\Re(A_2)}{\Re(A_0)}\left[\frac{\Im(A_2)}{\Re(A_2)} - \frac{\Im(A_0)}{\Re(A_0)}\right]$$

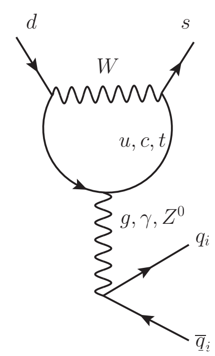
Considerations

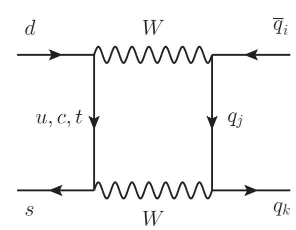
$$\epsilon' = i\frac{e^{i(\delta_2-\delta_0)}}{\sqrt{2}}\frac{\Re(A_2)}{\Re(A_0)}\left[\frac{\Im(A_2)}{\Re(A_2)} - \frac{\Im(A_0)}{\Re(A_0)}\right]$$

- large final state: $\pi/4$
- suppression $\Delta I=1/2$ rule
- two independent CP violating amplitudes
- cancellations

Theoretical Framework

- **Sensitivity to Short-Distance Scales:**





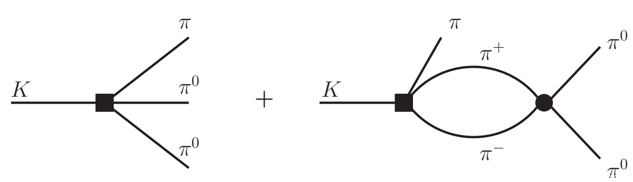
Charm mass prediction

Top quark

GIM cancellation

New Physics ?

- **Long-Distance Physics:**



Chiral Dynamics

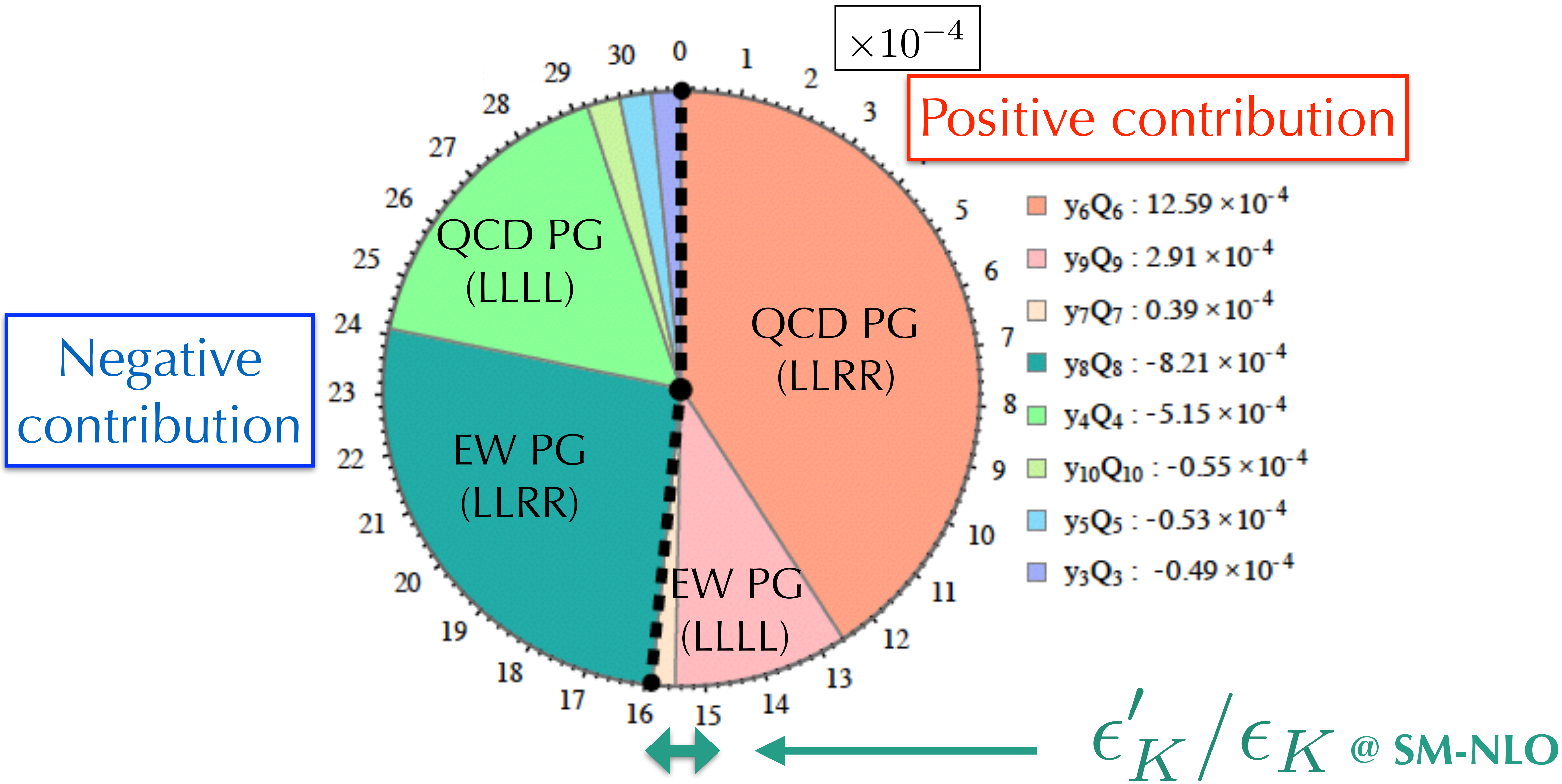
- **Multi-Scale Problem:**

$\log\left(M/\mu\right)$ **(OPE)** , $\log\left(\mu/m_\pi\right)$ **(χPT)**

Accidental cancellation

[TK, Nierste, Tremper, JHEP '16]

Composition of ϵ'_K/ϵ_K with respect to the operator basis



RBC-UK QCD

$$\varepsilon'/\varepsilon = (1.4 \pm 7.0) \cdot 10^{-4}$$

$$\left(\frac{\mathrm{Re}\,A_0}{\mathrm{Re}\,A_2}\right) = 31.0 \pm 6.6$$

$$(\varepsilon'/\varepsilon)_{\mathrm{exp}} = (16.6 \pm 2.3) \cdot 10^{-4}$$

$$\left(\frac{\mathrm{Re}\,A_0}{\mathrm{Re}\,A_2}\right)_{\mathrm{exp}} = 22.4$$

Four dominant contributions to ε'/ε in the SM

AJB, Jamin, Lautenbacher (1993); AJB, Gorbahn, Jäger, Jamin (2015)

From ReA_0

From ReA_2

$$\mathrm{Re}(\varepsilon'/\varepsilon) = \left[\frac{\mathrm{Im}(V_{td}V_{ts}^{*})}{1.4\cdot 10^{-4}}\right] 10^{-4}\left[-3.6 + 21.4\cdot B_6^{(1/2)} + 1.2 - 10.4\cdot B_8^{(3/2)}\right]$$

(Q_4)

(V-A) ⊗ (V-A) QCD Penguins	(V-A) ⊗ (V+A) QCD Penguins	(V-A) ⊗ (V-A) EW Penguins	(V-A) ⊗ (V+A) EW Penguins

Assumes that ReA_0 and ReA_2 ($\Delta I=1/2$ Rule) fully described by SM (includes isospin breaking corrections)

ε'/ε from RBC-UKQCD

Calculate all contributions directly (no isospin breaking corrections)

$$\left[-(6.5\pm 3.2) + 25.3\cdot B_6^{(1/2)} + (1.2\pm 0.8) - 10.2\cdot B_8^{(3/2)}\right]$$

ε'/ε from RBC-UKQCD

Anatomy: AJB, Gorbahn, Jäger, Jamin (2015)

Calculate all contributions directly

$$\mathrm{Re}(\varepsilon'/\varepsilon) = \left[\frac{\mathrm{Im}(V_{td}V_{ts}^*)}{1.4\cdot 10^{-4}}\right] 10^{-4}\left[-6.5 + 25.3\cdot B_6^{(1/2)} + 1.2 - 10.2\cdot B_8^{(3/2)}\right]$$

(Q_4)

(V-A) ⊗ (V-A) QCD Penguins	(V-A) ⊗ (V+A) QCD Penguins	(V-A) ⊗ (V-A) EW Penguins	(V-A) ⊗ (V+A) EW Penguins

$B_6^{(1/2)} = B_8^{(3/2)} = 1$ in the large N limit

Extracted from

RBC-UKQCD : $B_6^{(1/2)} = 0.57 \pm 0.19$ $B_8^{(3/2)} = 0.76 \pm 0.05$

EW penguins in full agreement with BGJJ but

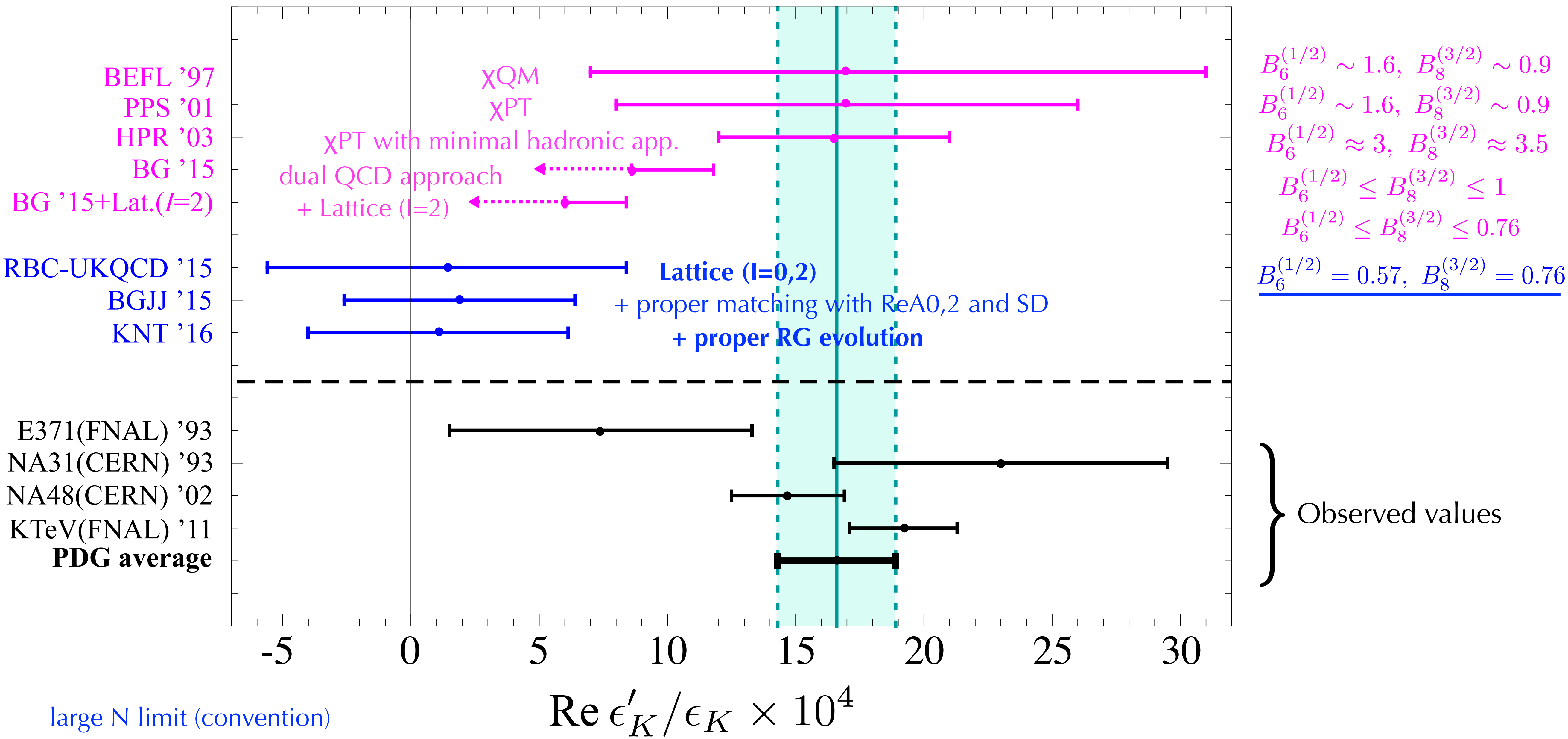
+ third term very similar to BGJJ $(\mathrm{Re}A_2)_{\mathrm{Lattice}} \approx (\mathrm{Re}A_2)_{\mathrm{exp}}$

$$\left[\frac{(\mathrm{Re}\,A_0)}{(\mathrm{Re}\,A_0)_{\mathrm{exp}}} \approx 1.4\right]$$

⇒ The negative contribution of Q_4 overestimated ⇒

$$\left(\frac{\varepsilon'}{\varepsilon}\right)_{\mathrm{Lattice}} = (1.4 \pm 7.0)\cdot 10^{-4}$$

Current situation of $\epsilon'_K/\epsilon_K \propto \mathrm{Im}A_0 - \left(\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2}\right)\mathrm{Im}A_2$



large N limit (convention)

$$B_6^{(1/2)} = B_8^{(3/2)} = 1$$

dual QCD prediction

$$B_6^{(1/2)} \le B_8^{(3/2)} < 1,\ B_8^{(3/2)} = 0.8$$

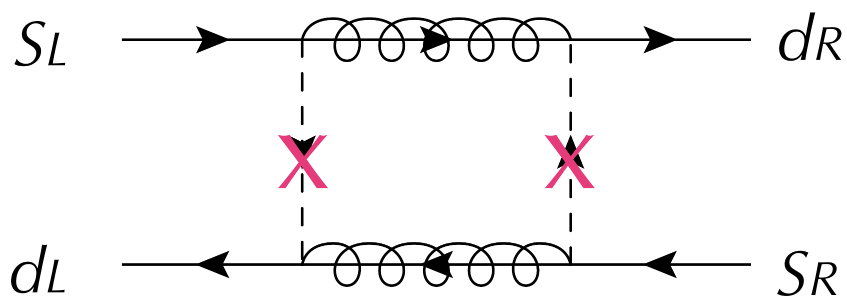
	Exp.	χPT	dual QCD	**Lattice (I=0,2)**
$\left(\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2}\right)$	22.45 ± 0.05	~ 14	16.0 ± 1.5	31.0 ± 11.1

New Physics solution

- difficult not to enhance simultaneously $\Delta S=2$ interaction, hence ϵ

Main Constraint: ϵ_K (ΔS=2, ID-CPV) cont.

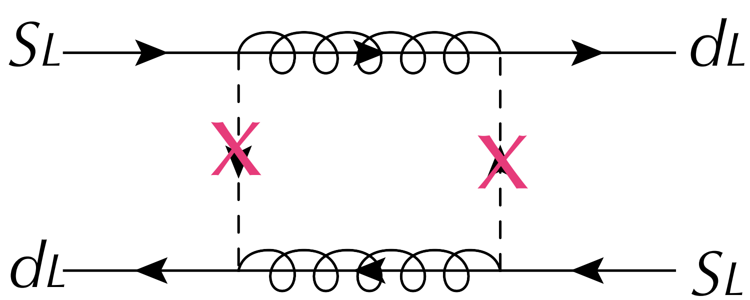
- The leading contribution is given by $\overline{d_L}s_L\overline{d_R}s_R$

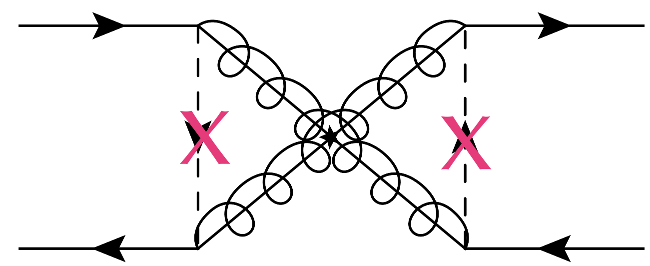


$$\propto \left(\frac{m_K}{m_s + m_d}\right)^2$$

this contribution is suppressed when $\Delta_{\bar{D},12} \simeq 0$

- The next contribution is given by $\overline{d_L}s_L\overline{d_L}s_L$





Crossed diagram gives **relatively negative** contributions

$m_{\tilde{g}} \simeq 1.5 m_{\tilde{q}}$: these contributions almost cancel out [Crivellin, Davidkov '10]

$m_{\tilde{g}} \gtrsim 1.5\ m_{\tilde{q}}$: suppressed by heavy gluing mass

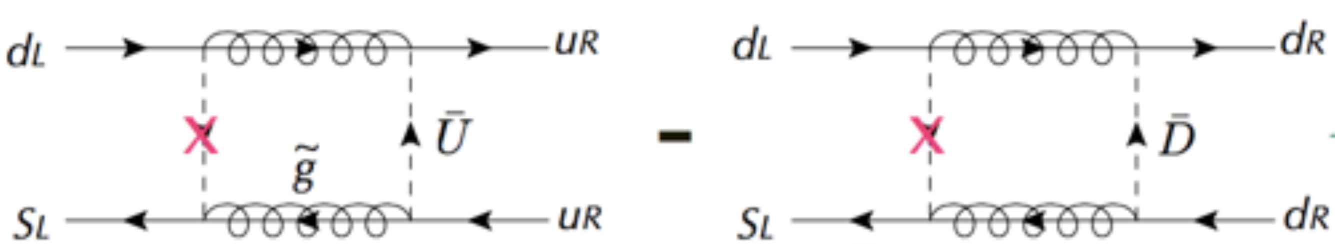
Kitahara,Nierste, Tremper PRL 16

ϵ' from isospin breaking

Kagan Neubert,99, Grossman, Kagan Neubert,99

$$\frac{\epsilon'_K}{\epsilon_K} = \frac{1}{\sqrt{2}|\epsilon_K|_{\rm exp}} \frac{\omega_{\rm exp}}{({\rm Re}A_0)_{\rm exp}} \left(-{\rm Im}A_0 + \frac{1}{\omega_{\rm exp}} {\rm Im}A_2 \right) \quad \text{where} \quad \frac{1}{\omega} \equiv \frac{{\rm Re}A_0}{{\rm Re}A_2} = 22.46 \ ({\rm exp.})$$

Assuming a discrepancy 2.9 sigmas from SM



$\rightarrow$ EW penguin(Q_8)

$m_{\bar{U}} \neq m_{\bar{D}} \rightarrow$ contribute to ${\rm Im}A_2$

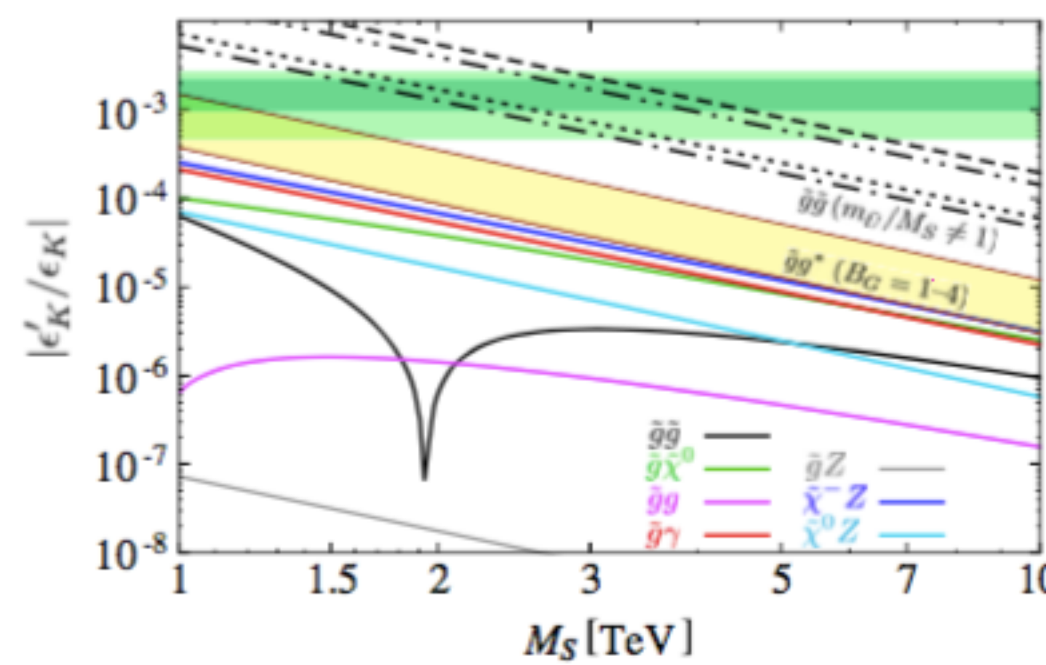
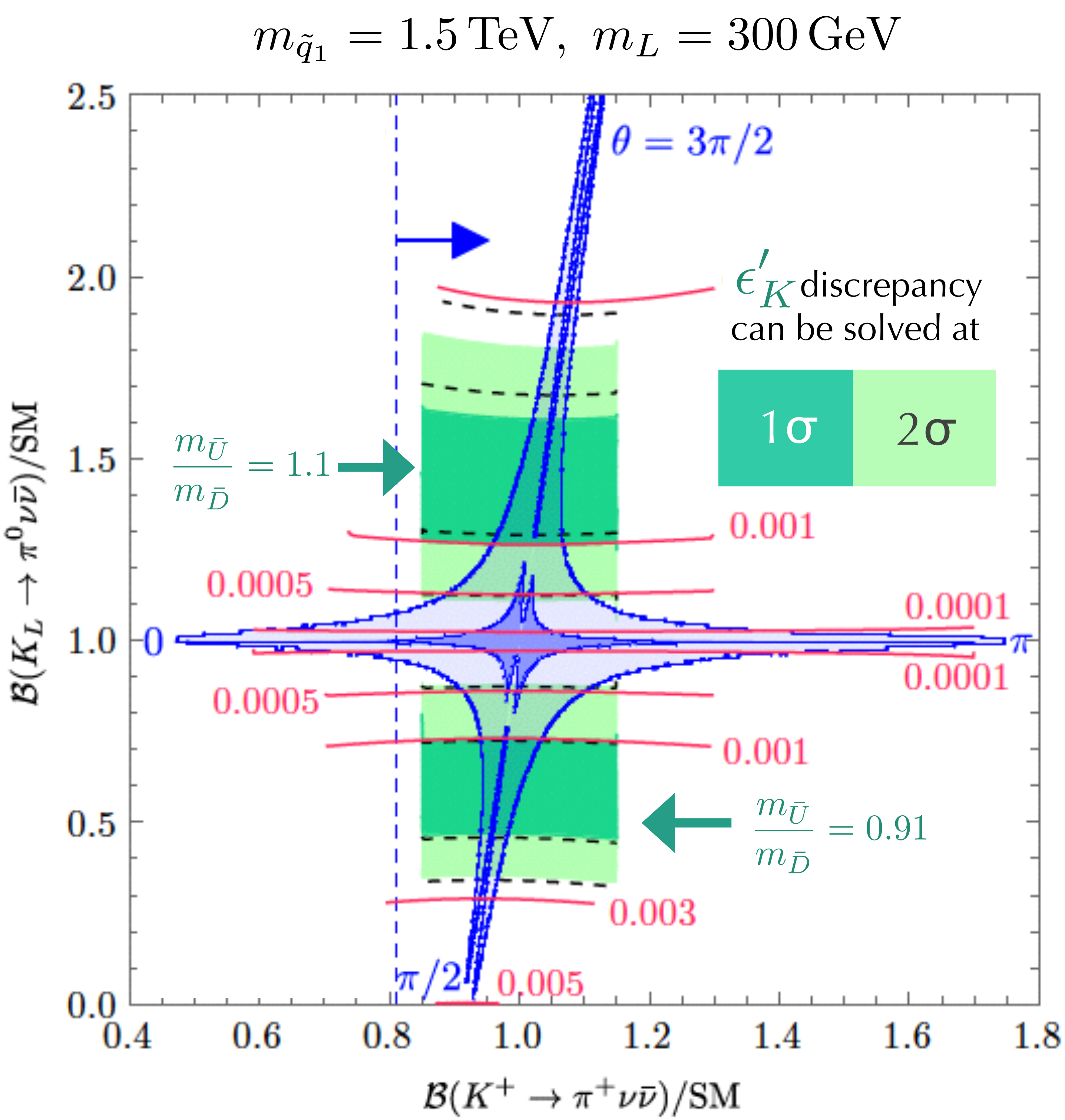


FIG. 3. Individual supersymmetric contributions to $|\epsilon'_K/\epsilon_K|$

B(K→πνν)

[Crivellin, D'Ambrosio, **TK**, Nierste, '17]



more than 10% mass shift of the gluino mass from $M_3 \simeq 1.45 M_S$ is possible in light of the constraint from ϵ_K

1-10 % mass shift of the gluino mass is possible

$\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu})/\mathrm{SM} \lesssim 2\,(1.2)$

$\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu})/\mathrm{SM} \lesssim 1.4\,(1.1)$

for a fine-tuning at the 1(10)% level

- $m_{\bar{U}}/m_{\bar{D}}$ determines a position of the green band
- Positive ϵ'_K predicts a strict correlation

$$\mathrm{sgn}\left[\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu}) - \mathcal{B}^{\mathrm{SM}}(K_L \to \pi^0 \nu\bar{\nu})\right] = \mathrm{sgn}\left[m_{\bar{U}} - m_{\bar{D}}\right]$$

$$\mathrm{sgn}\left[m_{\bar{U}} - m_{\bar{D}}\right] \xrightarrow[\epsilon'_K]{} \arg\left[m^2_{Q12}\right] \longrightarrow \mathrm{sgn}\left[\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu}) - \mathcal{B}^{\mathrm{SM}}(K_L \to \pi^0 \nu\bar{\nu})\right]$$

Modified Z-coupling scenario

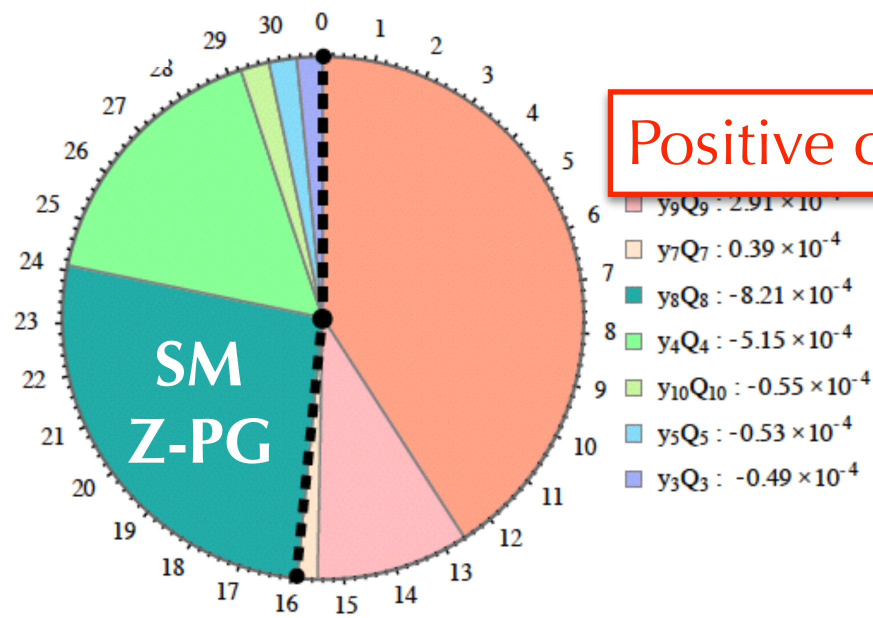
[Buras, De Fazio, Girrbach, '13, '14]
[Buras, Buttazzo, Knegjens, '15]
[Buras, '16]
[Endo, **TK**, Mishima, Yamamoto, '16]
[Bobeth, Buras, Celis, Jung,'17]

- NP contributions to *sdZ* coupling which has the same magnitude as the SM Z-penguin can explain ϵ'_K discrepancy

ϵ'_K/ϵ_K $\boxed{\times 10^{-4}}$

Negative contribution

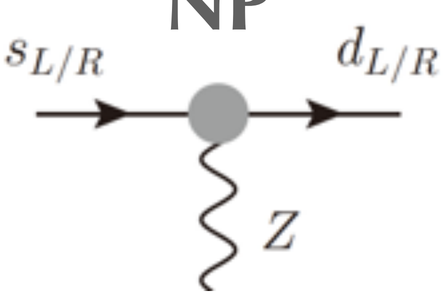
SM *Z*-penguin gives the biggest negative contribution

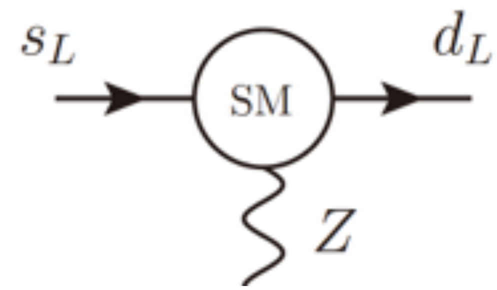


Positive contribution

$\leftrightarrow \epsilon'_K/\epsilon_K$

NP

Im 

$\simeq -$ Im 

$\longrightarrow$ [**ϵ'_K can be solved**

O(1) contribution to $\mathcal{B}(K \to \pi\nu\bar{\nu})$

Note: 

is suppressed by the GIM mechanism, so that it is a numerically small coupling

QCD and EFT

Short Distance expansion for weak interaction

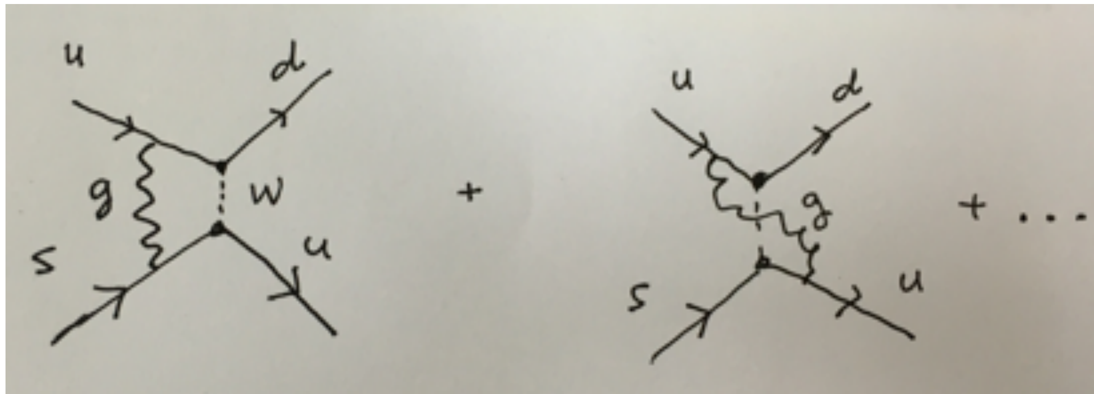
- Fermi lagrangian: description of the ΔS=1 weak lagrangian, in particular the explanation of ΔI =1/2 rule

$$\frac{A(K^+ \to \pi^+\pi^0)}{A(K_S \to \pi^+\pi^-)} \sim \frac{1}{22}$$

- Wilson suggestion (Feynman) , short distance expansion

$$-\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* C_- (\overline{s}_L \gamma^\mu u_L)(\overline{u}_L \gamma_\mu d_L)$$

- Gaillard Lee, Altarelli Maiani: right direction but not fully understood (Long distance?)



QCD theoretical tools

- analytic calculation 't Hooft, large Nc (it explains basic phenomenological facts of QCD, i.e. Zweig's rule) many implications: Skyrme model, VMD, Maldacena

- G. Parisi, `80s lattice: can we predict from QCD the proton mass at 10% level?

- Precise calculation of low energy QCD?

Chiral Perturbation theory

χPT effective field theory approach based on two assumptions

- π's Golstone bosons of $SU(3)_L \times SU(3)_R \rightarrow SU(3)_V$ $\quad$ $SU(3)_L \times SU(3)_R$ symm. $\mathcal{L}_{QCD}$ $\quad m_q = 0$
- (chiral) power counting There is a small expansion parameter $p^2/\Lambda^2_{\chi SB}$

$\Lambda_{\chi SB} \simeq 4\pi F_\pi \sim 1.2$ GeV

$$U = e^{\frac{i\sqrt{2}\ \pi}{F_\pi}} \qquad U \xrightarrow{SU(3)_L \times SU(3)_R} g_L\, U\, g_R^\dagger$$

π non-lin. real $\qquad$ U lin. transf.

Chiral sym. breaking through dim. parameter

F_π, χ related to $\langle 0|J_{5\mu}|\pi\rangle$, $\langle 0|\bar{q}_L q_R|0\rangle$

$F_\pi \approx 93$ MeV

$m_\pi^2 \sim \langle 0|\bar{q}_L q_R|0\rangle m_q$
GOR rel.

$$\mathcal{L}_{\Delta S=0} = \mathcal{L}^2_{\Delta S=0} + \mathcal{L}^4_{\Delta S=0} + \cdots = \frac{F_\pi^{\ 2}}{4} \overbrace{\langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + U \chi^\dagger \rangle}^{\pi\to l\nu,\ \pi\pi\to\pi\pi,\ K\to\pi..} + \overbrace{\sum_i L_i O_i}^{K\to\pi..} + \cdots$$

Fantastic chiral prediction

$A_{\pi\pi} \sim (s - m_\pi^2)/F_\pi^2$

L_i Gasser Leutwyler coeff determined from expts.
O_i p^4 operator

Vector Meson Dominance in the strong sector

Ecker, Gasser, de Rafael, Pich

L_i	L_i expts	V	A	Total (Scalar incl.)	Total QCD rel. incl.
L_1	0.4 ± 0.3	0,6	0	0,6	0,9
L_2	1.4 ± 0.3	1,2	0	1,2	1,8
L_3	-3.5 ± 1.1	-3,6	0	-3,0	-4,9
L_4	-0.3 ± 0.5	0	0	0	0
L_5	1.4 ± 0.5	0	0	1,4	1,4
L_6	-0.2 ± 0.3	0	0	0	0
L_7	-0.4 ± 0.2	0	0	-0,3	-0,3
L_8	0.9 ± 0.3	0	0	0,9	0,9
L_9	6.9 ± 0.7	6,9	0	6,9	7,3
L_{10}	-5.5 ± 0.7	-10	4	-6,0	-5,5

QCD inspired relations relations

$$F_V = 2G_V = \sqrt{2} f_\pi$$

$$F_A = f_\pi$$

$$M_A = \sqrt{2} M_V$$

KSFR: $G_V = \sqrt{2}\ F_\pi$
determined by dominance of pion, V,A to recover QCD short distance constraints

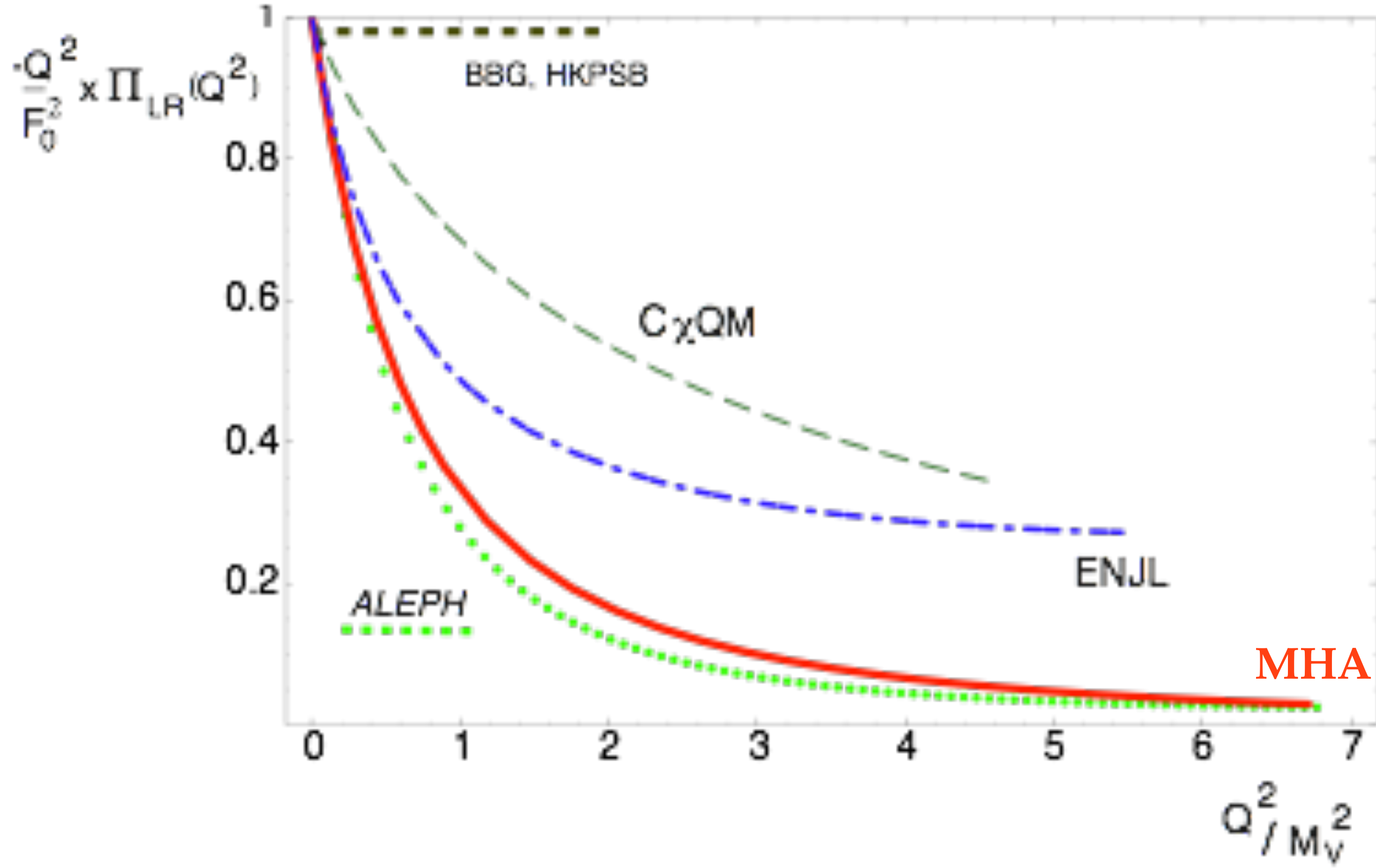
$$L_1^V = \frac{L_2^V}{2} = -\frac{L_3^V}{6} = \frac{G_V^2}{8M_V^2}, \qquad L_9^V = \frac{F_V G_V}{2M_V^2}, \qquad L_{10}^{V+A} = -\frac{F_V^2}{4M_V^2} + \frac{F_A^2}{4M_A^2}$$

QCD inspired relations relations

$$\boxed{L_1^V = L_2^V/2 = -L_3^V/6 = L_9^V/8 = -L_{10}^{V+A}/6 = f_\pi^2/(16M_V^2)}$$

Minimal Hadronic Ansatz (MHA)

- Traditional wisdom: low energy VERY WELL approximated by π's ,V,A
- Short distance: QCD
- A good interpolation among the two regimes is sufficient for a good description of the correlators



De Rafael

OPE

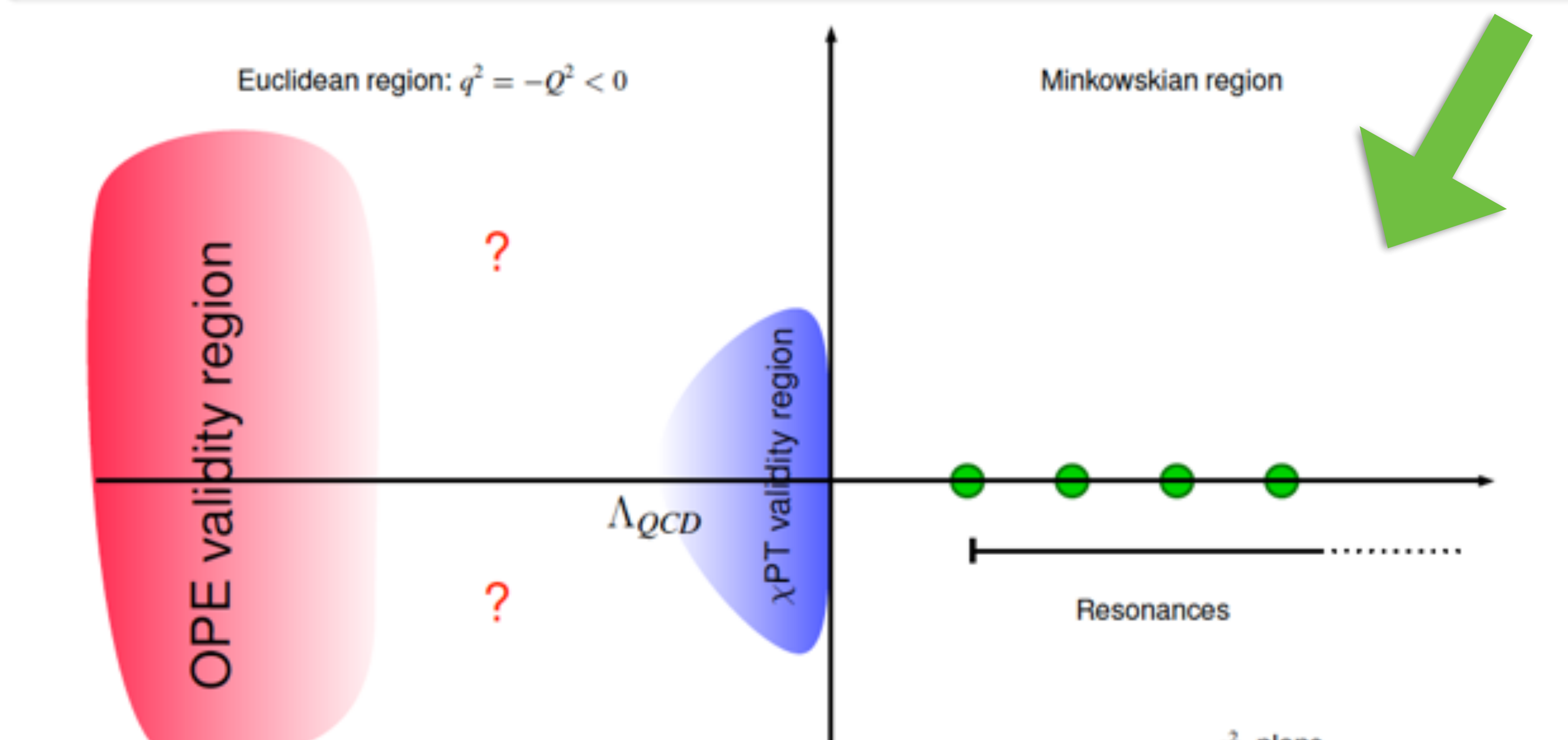
$$\Pi_V(Q^2) \underset{Q^2\to\infty}{\sim} \frac{N_c}{24\pi^2} \ln\left(\frac{\Lambda_V^2}{Q^2}\right) + \langle \mathcal{O}_2 \rangle \frac{1}{Q^2} + \langle \mathcal{O}_4 \rangle \frac{1}{Q^4} + \langle \mathcal{O}_6 \rangle_V \frac{1}{Q^6}$$

$$\begin{cases} \langle \mathcal{O}_2 \rangle = 0 \\ \langle \mathcal{O}_4 \rangle = \frac{1}{12\pi}\, \alpha_s \, \langle G^2 \rangle \\ \langle \mathcal{O}_6 \rangle_V = -\frac{28\pi}{9}\, \alpha_s \, \langle \bar{\psi}\psi \rangle^2 \end{cases}$$

– Second Weinberg's sum rule –

Large Nc QCD

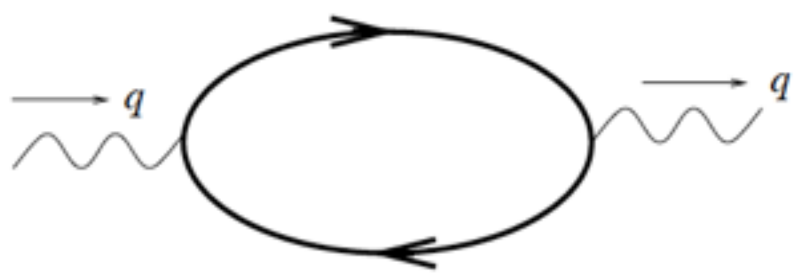
$$\boxed{\Pi_V(Q^2) = \sum_{n=0}^{\infty} \frac{F_V(n)^2}{Q^2 + M_V(n)^2}}$$



Holographic QCD at low energies

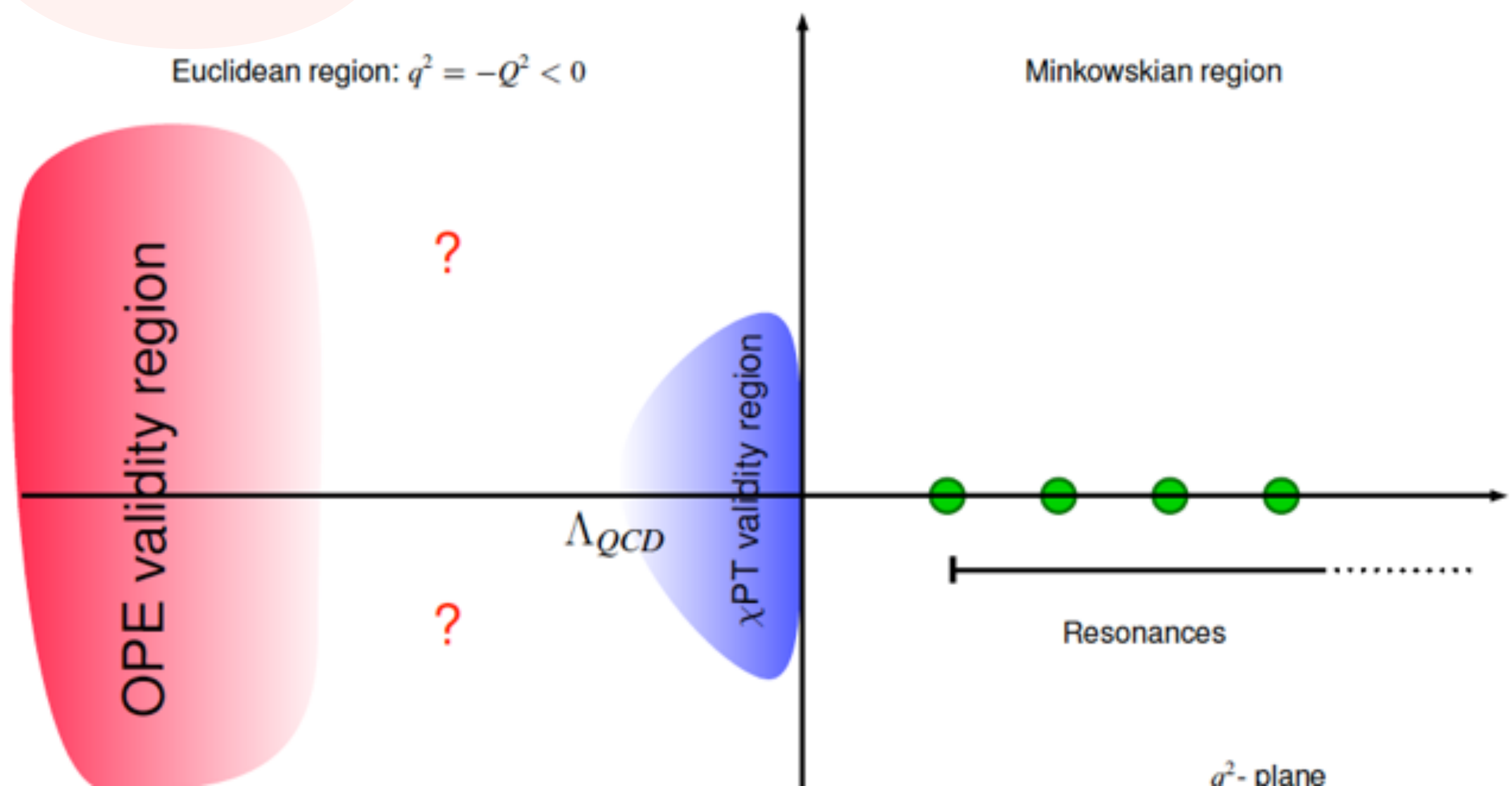
Large Nc QCD properties => conformal symmetry in 5d

$$\int d^4x\, e^{ip\cdot x}\langle J_\mu(x)J_\nu(0)\rangle = \left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\Pi(p^2)$$

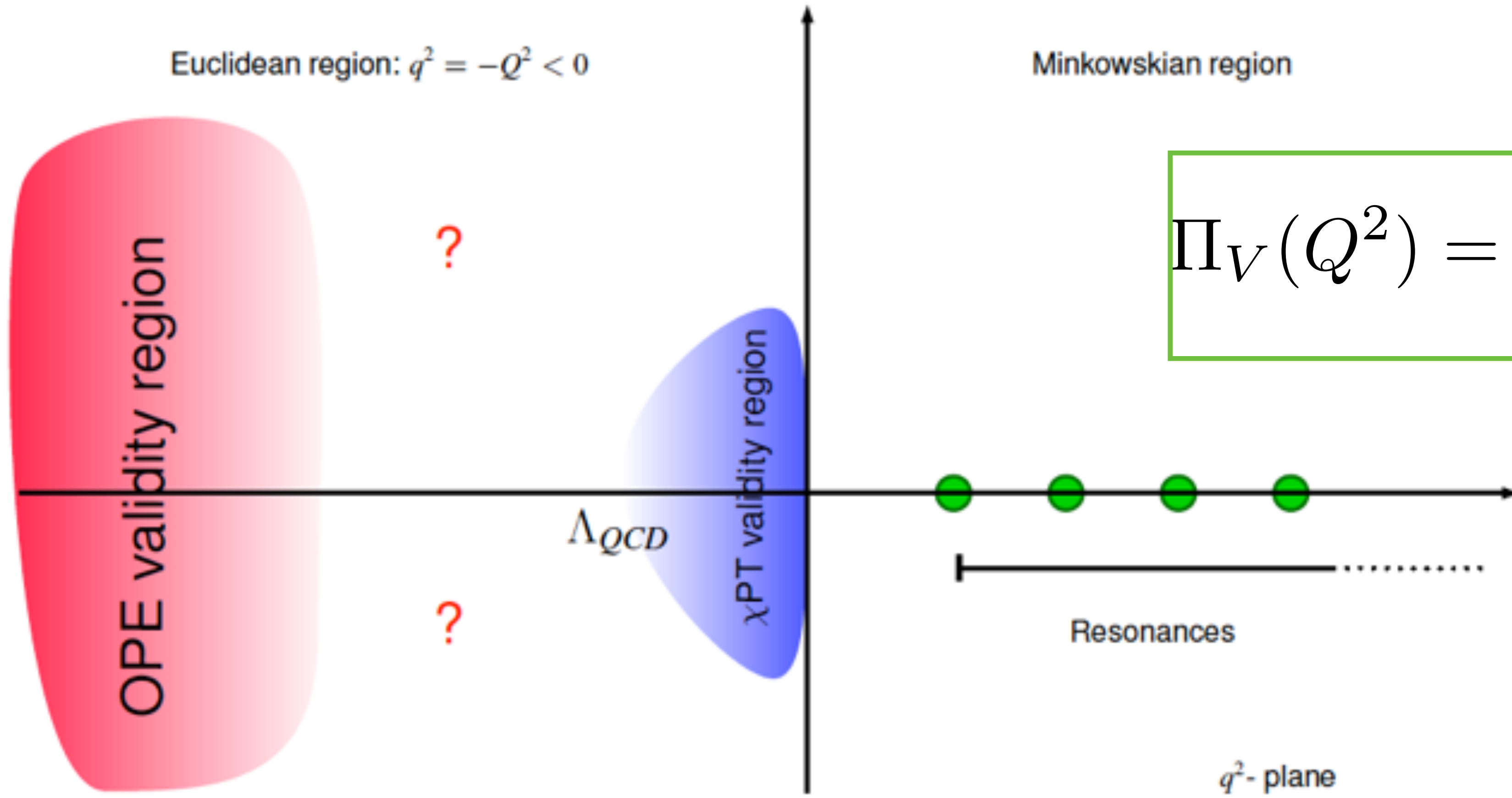


$$\Pi_V(Q^2) = -\frac{N_c}{12\pi^2}\ln Q^2 \qquad Q^2 \equiv -q^2$$

confinement: either IR cut-off or field modifying metric



Holographic QCD: Hard Wall



$$\boxed{\Pi_V(Q^2) = \sum_{n=0}^{\infty} \frac{F_V(n)^2}{Q^2 + M_V(n)^2}}$$

$$\Pi_V(Q^2) = -\frac{N_c}{12\pi^2} \ln Q^2 \quad Q^2 \equiv -q^2$$

IR very good, but

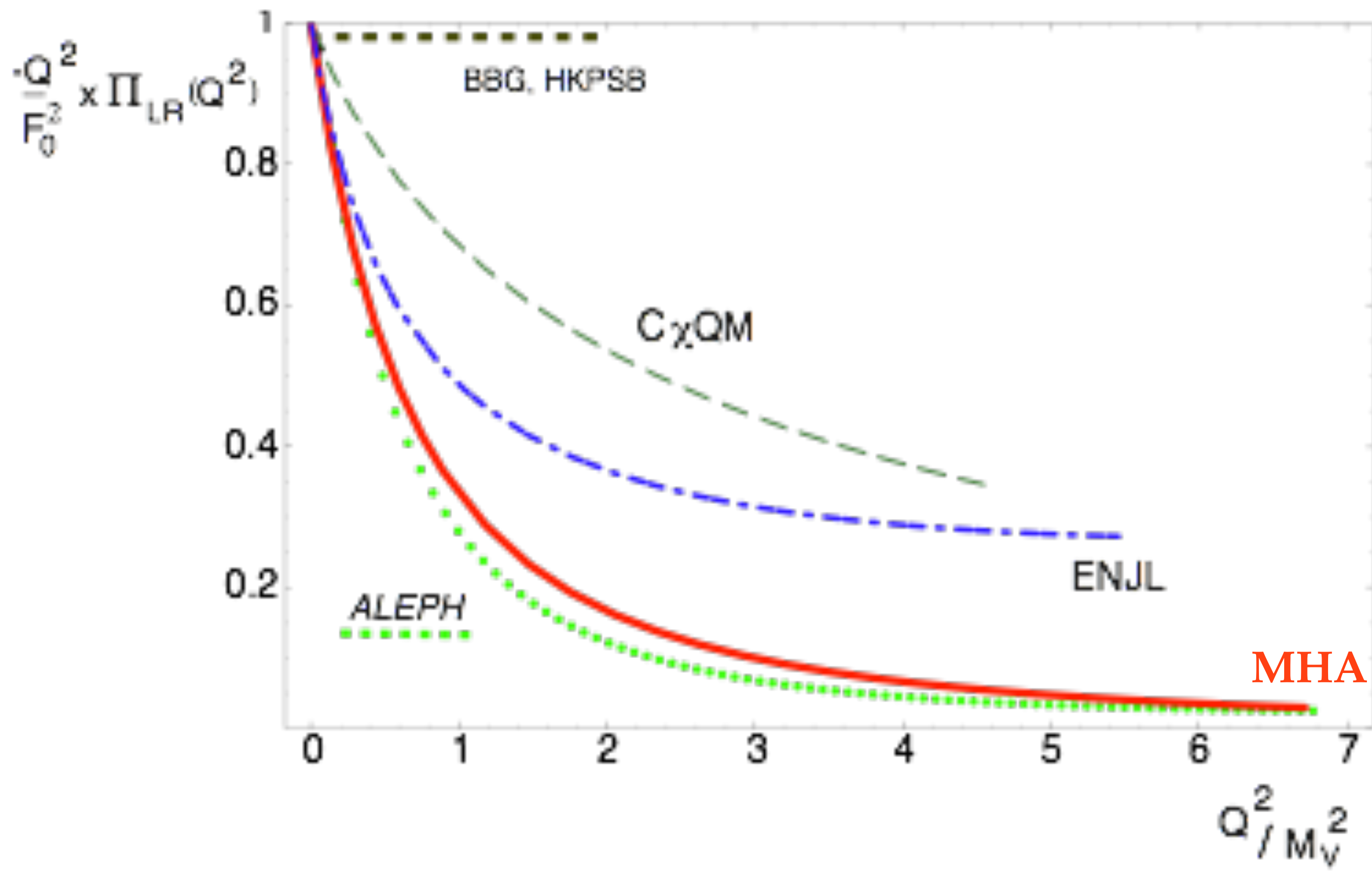
$$M_V^2(n) \sim n^2$$

HW

$$\Pi_V(q^2) = -\frac{4}{3}\frac{N_c}{(4\pi)^2}\left[\log\frac{q^2}{\mu^2} - \pi\frac{Y_0(qz_0)}{J_0(qz_0)}\right]$$

Minimal Hadronic Ansatz (MHA)

- Traditional wisdom: low energy VERY WELL approximated by π's ,V,A
- Short distance: QCD
- A good interpolation among the two regimes is sufficient for a good description of the correlators



De Rafael

Regge/Veneziano model

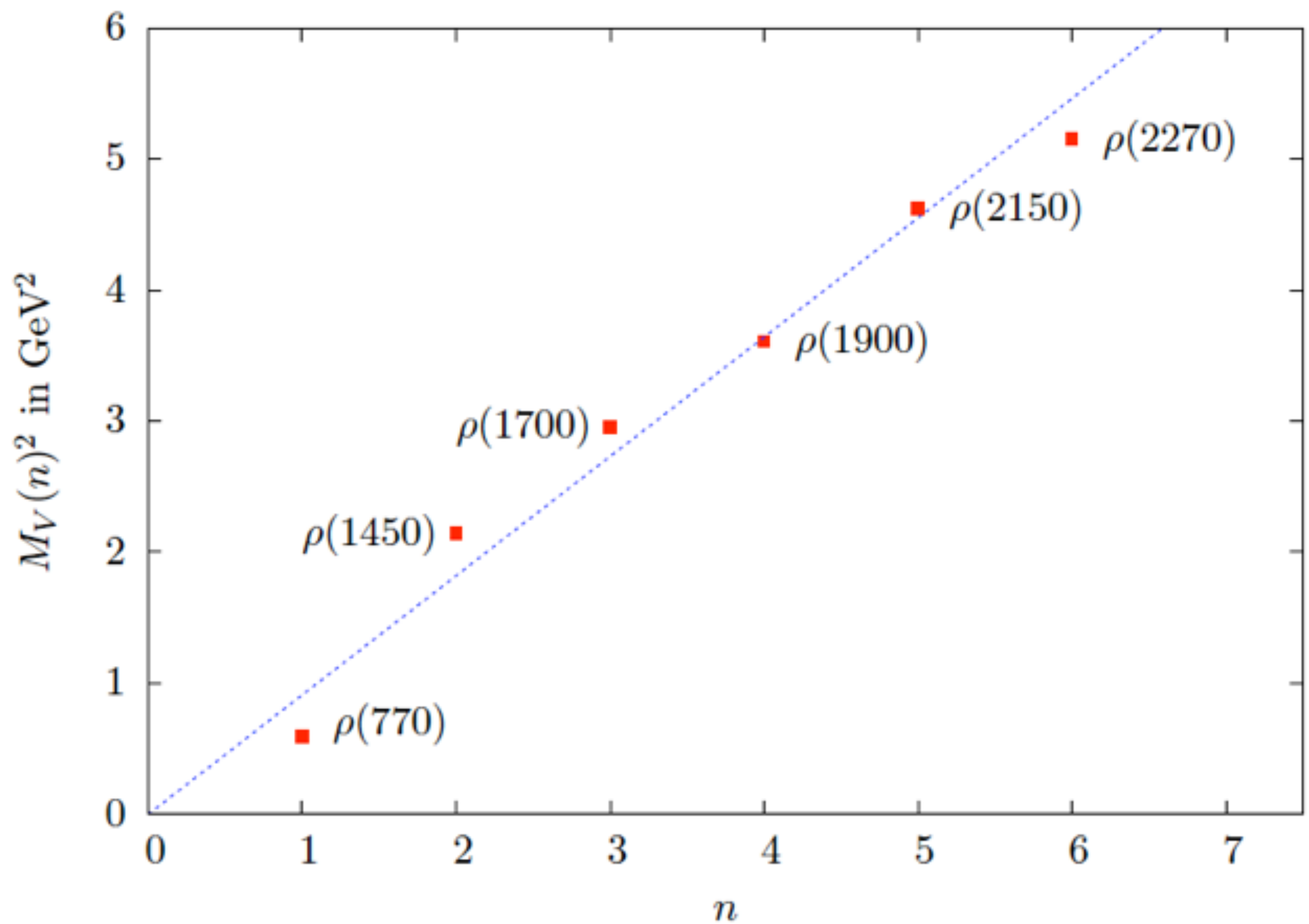
$$\boxed{\Pi_V(Q^2) = \sum_{n=0}^{\infty} \frac{F_V(n)^2}{Q^2 + M_V(n)^2}}$$

$$M_V(n)^2 \underset{n\to\infty}{\sim} \sigma\ n$$

σ 0.90 GeV^2

String tension/confining

$F_V(n)$ constant



Strong sector, KSFR: 5D vs 4d

$M_{Vn} \sim n$ ➪ first resonances OK
but not Regge behaviour.
UV OK

4d

$$\frac{1}{2} \equiv \frac{g_{V_1}^2 M_{V_1}^2}{f^2}$$

5d

$$\sum_n g_{V_n}^2 M_{V_n}^2 = f^2/3$$

Weak interaction

The symmetry of the short distance hamiltonian $-\frac{G_F}{\sqrt{2}} V_{ud} V^*_{us} C_- (\overline{s}_L \gamma^\mu u_L)(\overline{u}_L \gamma_\mu d_L)$

described in CHPT

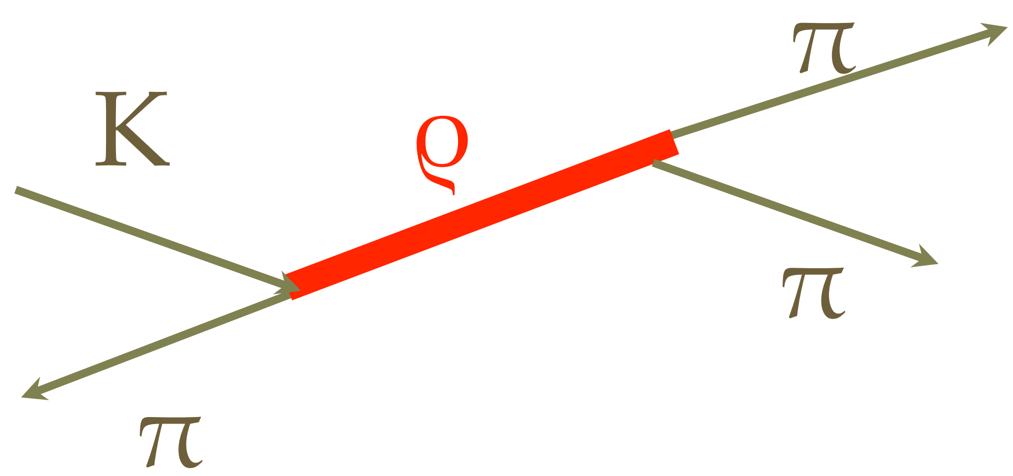
$$\mathcal{L}_{\Delta S=1} = \mathcal{L}^2_{\Delta S=1} + \mathcal{L}^4_{\Delta S=1} + \cdots = G_8 F^4 \underbrace{\langle \lambda_6 D_\mu U^\dagger D^\mu U \rangle}_{K\to 2\pi/3\pi} + \underbrace{G_8 F^2 \sum_i N_i W_i}_{K^+\to\pi^+\gamma\gamma, K\to\pi l^+ l^-} + \cdots$$

VMD not as successful, in particular for K->3pi, where in principle large VMD important

Hard Wall weak interactions: K->3pi

Luigi Cappiello, Oscar Cata and G.D.

In this channel there is a large VMD in the phenomenological slope



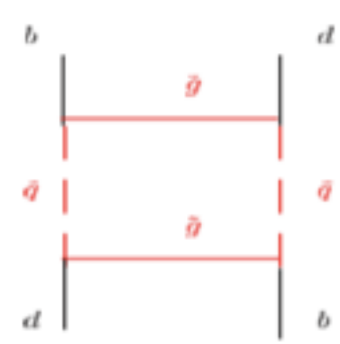
However this is proportional to $L_3 + 3/4\, L_9$

4D $L_3 + 3/4\, L_9 = 0$

5D $L_3 + 3/4\, L_9 \neq 0$ and in agreement with phenomenology

Back to Flavor vs naturalness

- Technicolor , susy , extra dimensions



$$\mathcal{H}^{\tilde{g}}_{\Delta F=2} \sim \frac{\alpha_s^2}{9M^2_{\tilde{Q}}}\left[(\delta^{LL}_{12})^2\ (\bar{s}_L\gamma_\mu d_L)^2 + \cdots\right]$$

δ^{LL}_{12} departure from identity matrix m^2_Q

Gabrielli,Masiero,Silvestrini

- $K \underset{\Longrightarrow}{-} \bar{K} \quad \frac{(\delta^{LL}_{12})^2}{M^2_{\tilde{Q}}} \leq \frac{1}{(100\mathrm{TeV})^2} \Longrightarrow$ Naturalness?

Generic Flavor structures strongly constrained

Operator	Bounds on Λ in TeV ($c_{\rm NP}=1$) Re	Im	Bounds on $c_{\rm NP}$ ($\Lambda = 1$ TeV) Re	Im	Observables
$(\bar s_L \gamma^\mu d_L)^2$	9.8×10^2	1.6×10^4	9.0×10^{-7}	3.4×10^{-9}	Δm_K; ϵ_K
$(\bar s_R d_L)(\bar s_L d_R)$	1.8×10^4	3.2×10^5	6.9×10^{-9}	2.6×10^{-11}	
$(\bar c_L \gamma^\mu u_L)^2$	1.2×10^3	2.9×10^3	5.6×10^{-7}	1.0×10^{-7}	Δm_D; $\lvert q/p\rvert_D$, ϕ_D
$(\bar c_R u_L)(\bar c_L u_R)$	6.2×10^3	1.5×10^4	5.7×10^{-8}	1.1×10^{-8}	
$(\bar b_L \gamma^\mu d_L)^2$	6.6×10^2	9.3×10^2	2.3×10^{-6}	1.1×10^{-6}	Δm_{B_d}; $\sin(2\beta)$ from $B_d \to \psi K$
$(\bar b_R d_L)(\bar b_L d_R)$	2.5×10^3	3.6×10^3	3.9×10^{-7}	1.9×10^{-7}	
$(\bar b_L \gamma^\mu s_L)^2$	1.4×10^2	2.5×10^2	5.0×10^{-5}	1.7×10^{-5}	Δm_{B_s}; $\sin(\phi_s)$ from $B_s \to \psi\phi$
$(\bar b_R s_L)(\bar b_L s_R)$	4.8×10^2	8.3×10^2	8.8×10^{-6}	2.9×10^{-6}	

Isidori Nir Perez 10

Problem already known since ’86 technicolour (Chivukula Georgi) susy (Hall Randall) extra dimensions (Rattazzi Zafferoni)

Maybe there is an energy gap between the theory of flavor and the EW scale , ameliorating also a clash from the scale of the bounds in the table above and the requirement of solving the hierarchy problem

SM

$Y_u\,,\ Y_d\,,\ Y_l$

$$\mathcal{L}^Y_{SM} \;=\; \bar{Q}\; Y_D\; D\; H$$

MFV

Flavour scale

$Y_u\,,\ Y_d\,,\ Y_l$

MNP

$$\mathcal{L}^Y_{MFV} \;=\; \mathcal{L}^Y_{SM} + \mathrm{dim}_{-}6$$

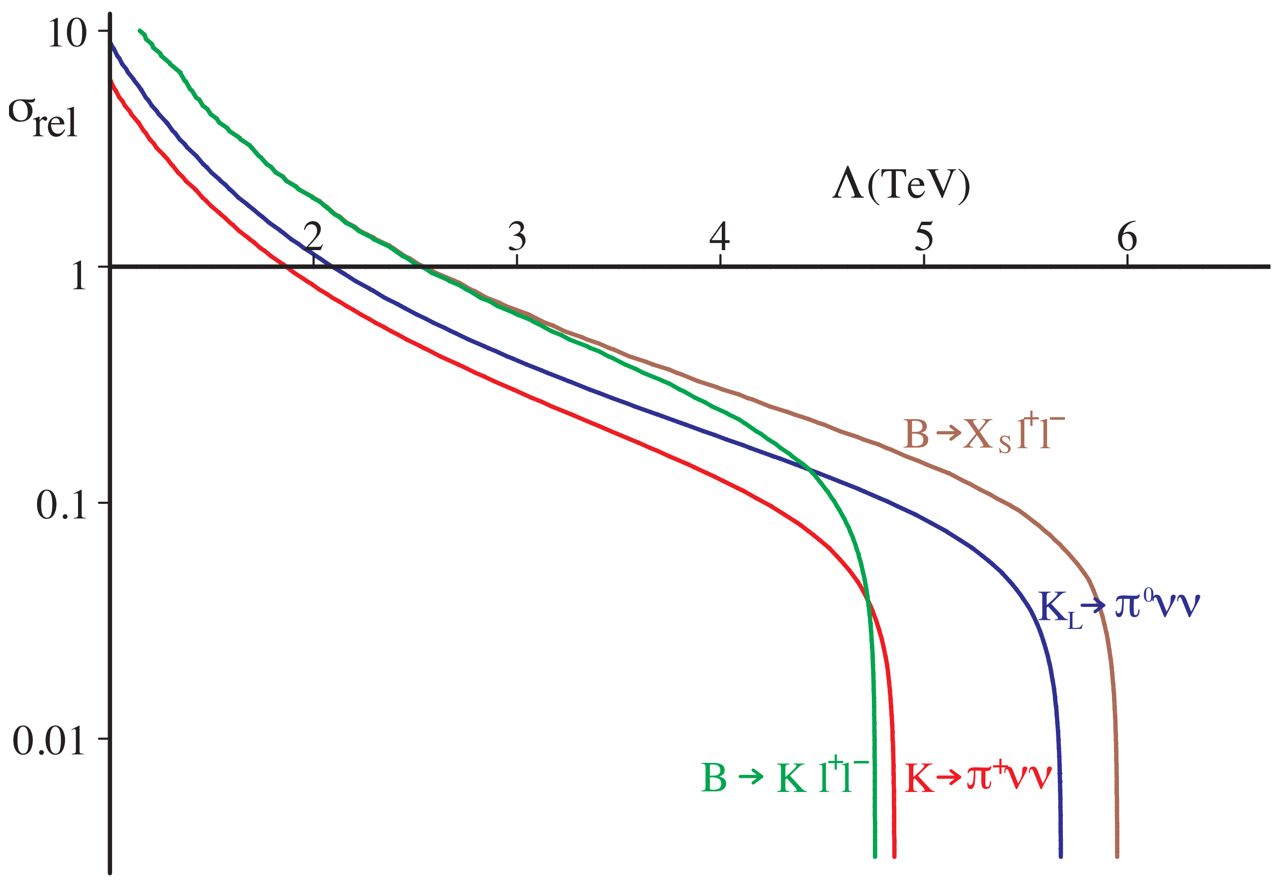
MEW

$$G_F = \overbrace{\mathrm{U}(3)_Q \otimes \mathrm{U}(3)_U \otimes \mathrm{U}(3)_D \otimes \mathrm{U}(3)_L \otimes \mathrm{U}(3)_E}^{\text{global symmetry}} + \overbrace{Y_{U,D,E}}^{\text{spurions}}$$

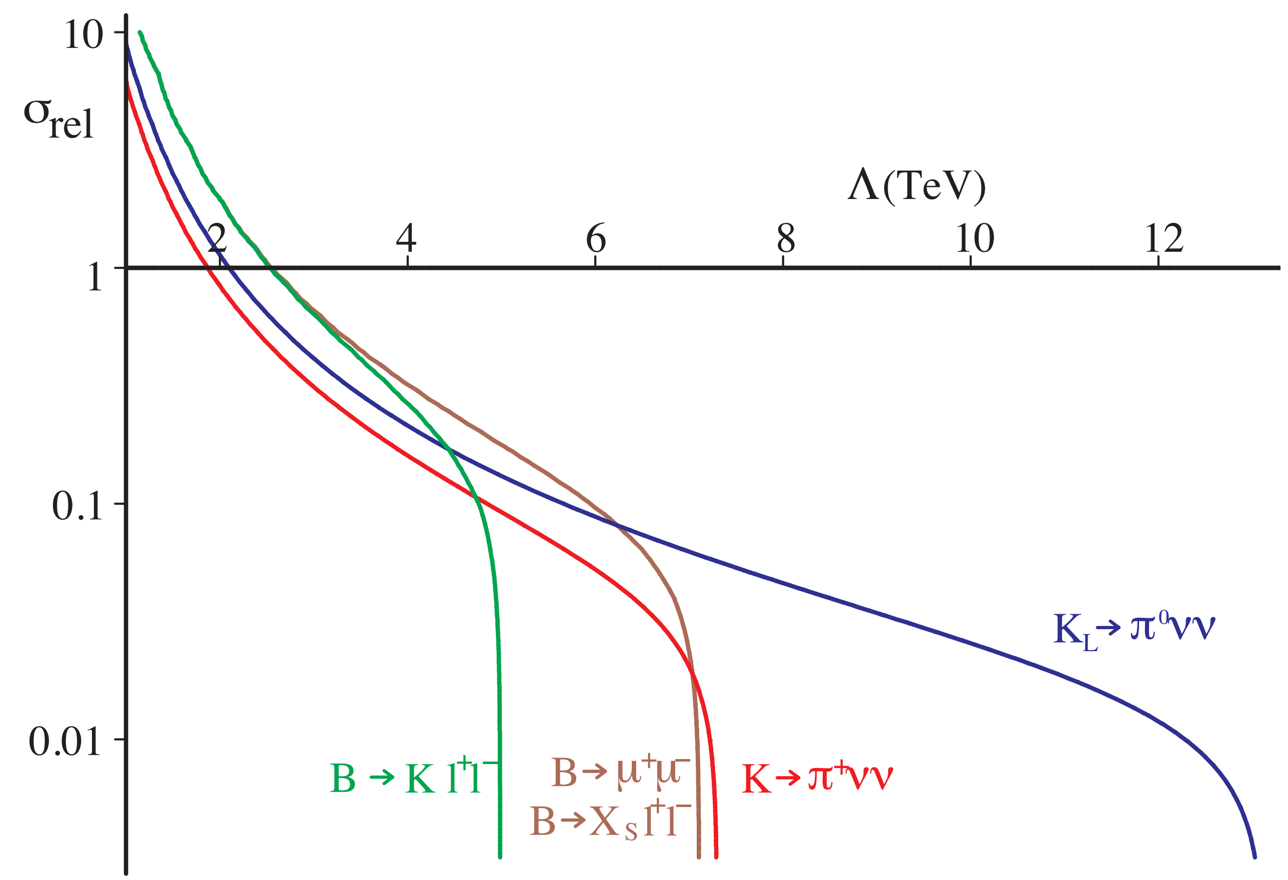
Bounds ameliorated

Minimally flavour violating dimension six operator		main observables	Λ [TeV] $-$	$+$	
$\mathcal{O}_0 =$	$\frac{1}{2}(\bar{Q}_L \lambda_{\rm FC} \gamma_\mu Q_L)^2$	$\epsilon_K, \quad \Delta m_{B_d}$	6.4	5.0	
$\mathcal{O}_{F1} =$	$H^\dagger \left(\bar{D}_R \lambda_d \lambda_{\rm FC} \sigma_{\mu\nu} Q_L\right) F_{\mu\nu}$	$B \to X_s \gamma$	8.3	13.4	
$\mathcal{O}_{G1} =$	$H^\dagger \left(\bar{D}_R \lambda_d \lambda_{\rm FC} \sigma_{\mu\nu} T^a Q_L\right) G^a_{\mu\nu}$	$B \to X_s \gamma$	2.3	3.8	
$\mathcal{O}_{\ell 1} =$	$(\bar{Q}_L \lambda_{\rm FC} \gamma_\mu Q_L)(\bar{L}_L \gamma_\mu L_L)$	$B \to (X)\ell\bar{\ell}, \quad K \to \pi\nu\bar{\nu}, (\pi)\ell\bar{\ell}$	3.1	2.7	$*$
$\mathcal{O}_{\ell 2} =$	$(\bar{Q}_L \lambda_{\rm FC} \gamma_\mu \tau^a Q_L)(\bar{L}_L \gamma_\mu \tau^a L_L)$	$B \to (X)\ell\bar{\ell}, \quad K \to \pi\nu\bar{\nu}, (\pi)\ell\bar{\ell}$	3.4	3.0	$*$
$\mathcal{O}_{H1} =$	$(\bar{Q}_L \lambda_{\rm FC} \gamma_\mu Q_L)(H^\dagger i D_\mu H)$	$B \to (X)\ell\bar{\ell}, \quad K \to \pi\nu\bar{\nu}, (\pi)\ell\bar{\ell}$	1.6	1.6	$*$
$\mathcal{O}_{q5} =$	$(\bar{Q}_L \lambda_{\rm FC} \gamma_\mu Q_L)(\bar{D}_R \gamma_\mu D_R)$	$B \to K\pi, \quad \epsilon'/\epsilon, \ldots$	~ 1		

GD,Giudice,Isidori,Strumia



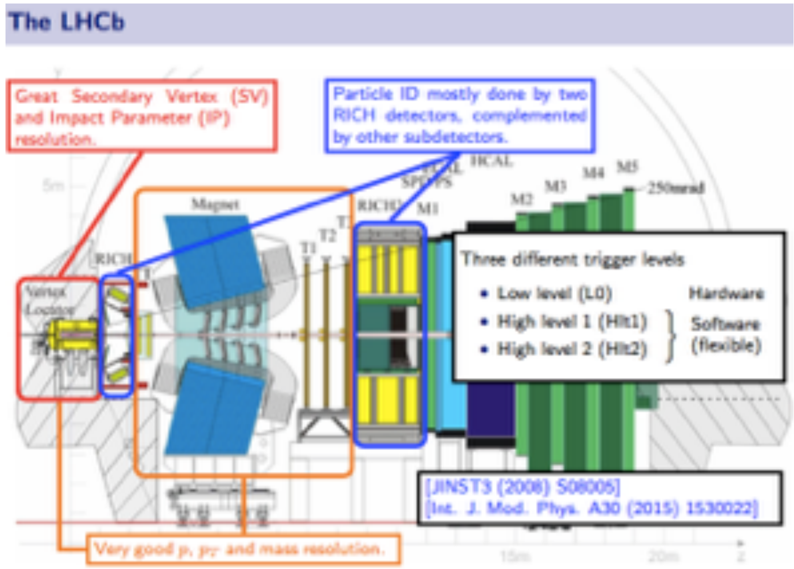
10% Overall CKM error



1% Overall CKM error

Also LHCB is playing a crucial role in Kaons

$K_S \to \mu\overline{\mu}$ LHCB

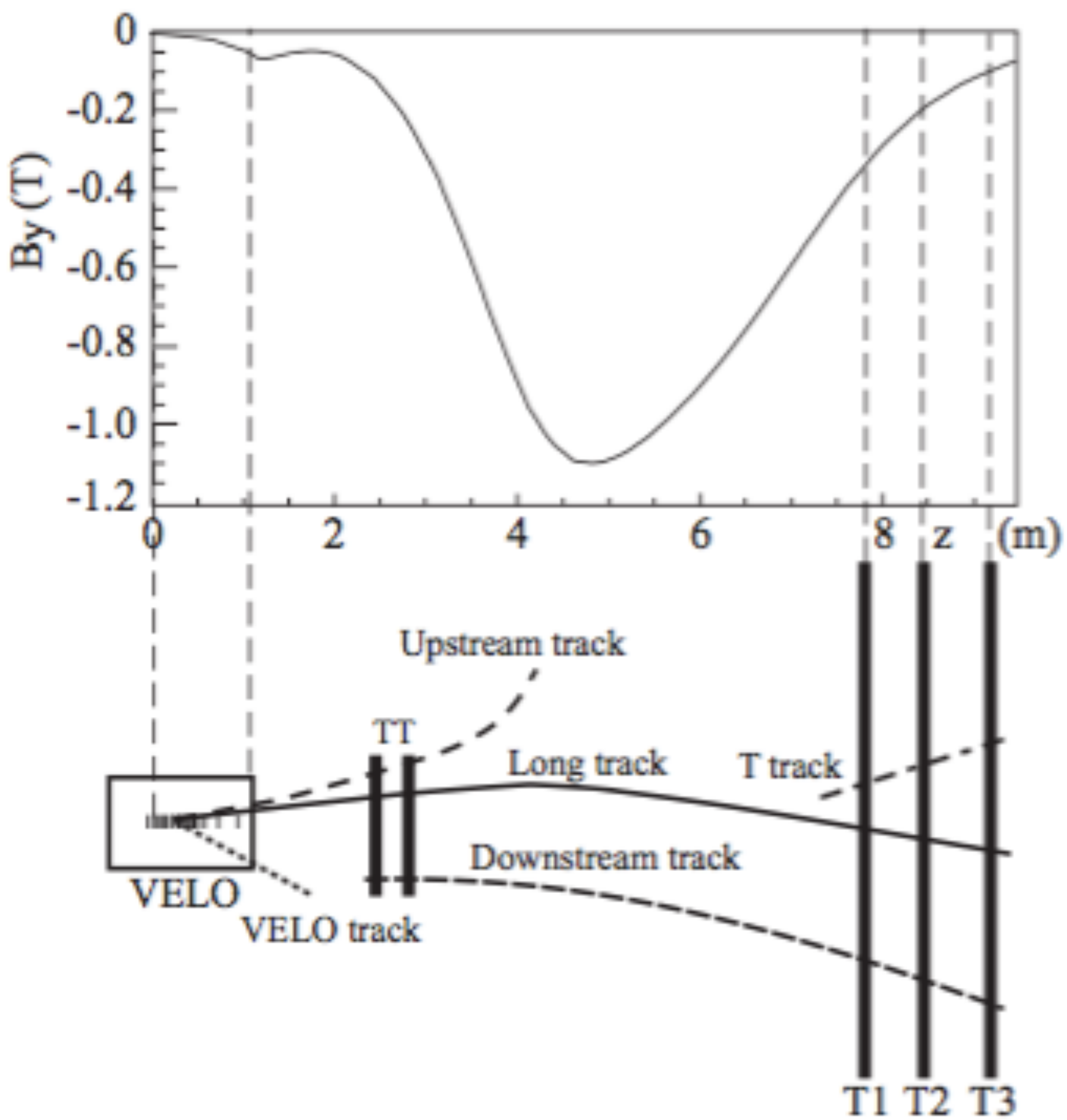


CERN SPS

After 40 years $B(K_S \to \mu^+\mu^-) < 3.1 \times 10^{-7}$at 90%CL

improvement by 3 orders of magnitudes from LHCB

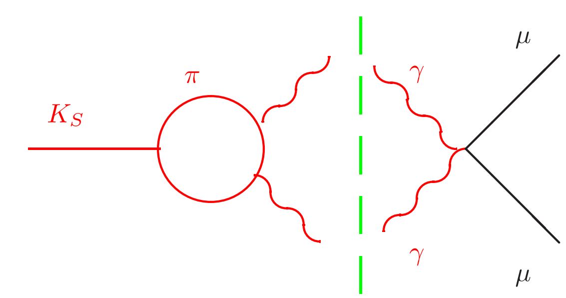
$$B(K_S \to \mu^+\mu^-) < 6.9(5.8) \times 10^{-9} \text{at } 95(90)\%\text{CL}$$



[Int. J. Mod. Phys. A30 (2015) 1530022]

SM

$\sim 5 \times 10^{-12}$

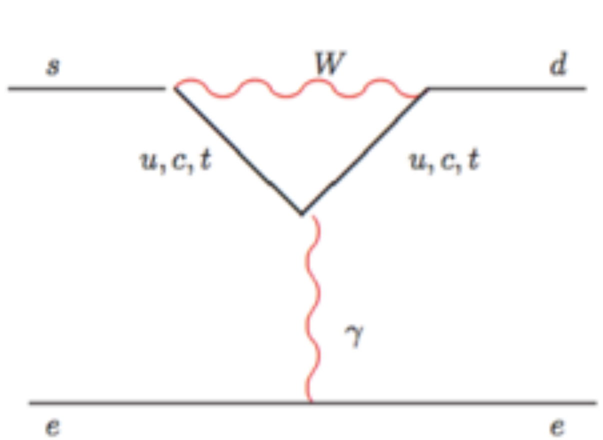


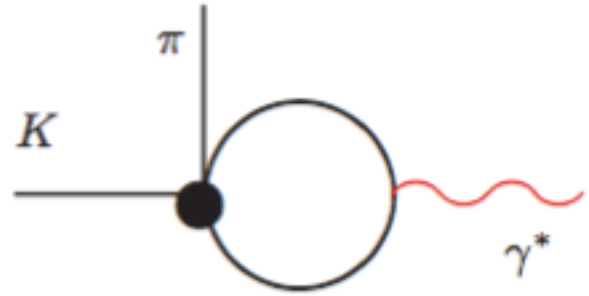
SD 1.5 10^-12

NP 1.5 10^-11
Allowed

NP Limits from
CPviol in $K_L \to \mu\mu$

$K^+ \to \pi^+ e^+ e^-$ $\quad$ $K_S \to \pi^0 e^+ e^-$

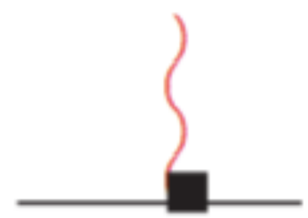




$O(p^4)$ ChPT

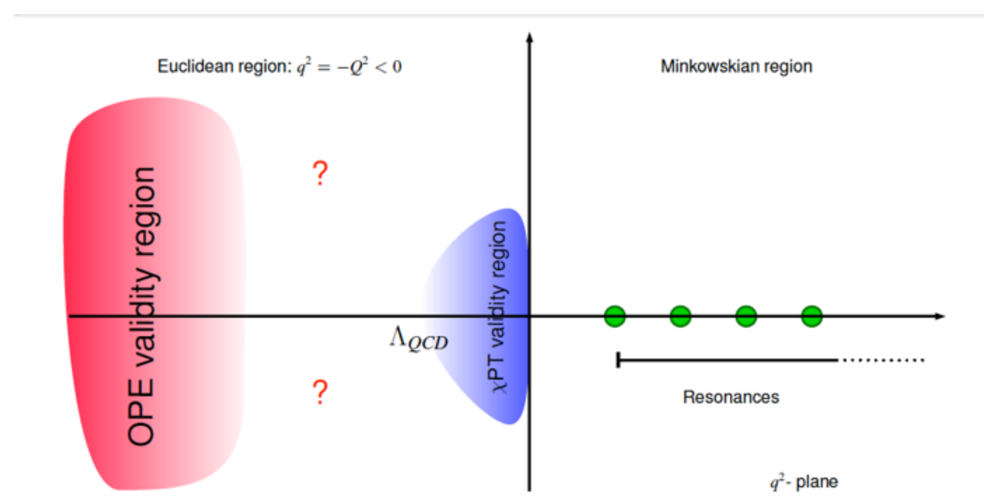
electrons and μ's in the final state

Short distance



constant+ loop

Ecker, Pich, de Rafael



'97 Initial data inconsistency e and μ's: LFV?

$K^+ \to \pi^+ e^+ e^-$ $\quad$ $K_S \to \pi^0 e^+ e^-$

- gauge+Lorentz inv. =>1 ff

$$i \int d^4x e^{iqx} \langle \pi(p)| T\left\{ J^\mu_{\rm elm}(x) \mathcal{L}_{\Delta S=1}(0) \right\} |K(k)\rangle = \frac{W(z)}{(4\pi)^2} \left[z(k+p)^\mu - (1-r_\pi^2) q^\mu \right]$$

$$W^i = G_F m_K^2 (a_i + b_i z) + W^i_{\pi\pi}(z)$$

$$i = \pm, S$$

$$a_i, b_i \sim O(1), \qquad z = \frac{q^2}{m_K^2}$$

- Observables $\Gamma(K^+ \to \pi^+ e^+ e^-)$, $\Gamma(K^+ \to \pi^+ \mu \bar{\mu})$, slopes

- a_i $\quad O(p^4)$ $\qquad a_+ \sim N_{14} - N_{15}$, $\qquad a_S \sim 2N_{14} + N_{15}$ $\qquad$ Ecker, Pich, de Rafael
- b_i $\quad O(p^6)$ $\qquad$ G.D., Ecker, Isidori, Portoles

- $a_+,\ b_+$ in general not related to $a_S,\ b_S$ $\qquad$ Recent lattice determinations Christ et al.

averaging flavour

$$a_+^{\rm exp.} = -0.578 \pm 0.016$$
$$b_+^{\rm exp.} = -0.779 \pm 0.066$$

Collaboration with Crivellin, A Hoferichter, M and Tunstall,
Phys.Rev. D 2016

LFUV: Kaons

Channel	a_+	b_+	Reference
ee	-0.587 ± 0.010	-0.655 ± 0.044	E865
ee	-0.578 ± 0.016	-0.779 ± 0.066	NA48/2
$\mu\mu$	-0.575 ± 0.039	-0.813 ± 0.145	NA48/2

$$a_+^{\rm NP} = \frac{2\pi\sqrt{2}}{\alpha} V_{ud} V_{us}^* * C_{7V}^{\rm NP}$$

$$C_{7V}^{\mu\mu} - C_{7V}^{ee} = \alpha \frac{a_+^{\mu\mu} - a_+^{ee}}{2\pi\sqrt{2} V_{ud} V_{us}^*} \stackrel{MFV}{\Longrightarrow} C_{9V}^{B,\mu\mu} - C_{9V}^{B,ee} = \alpha \frac{a_+^{\mu\mu} - a_+^{ee}}{2\pi\sqrt{2} V_{td} V_{ts}^*} = -19 \pm 79$$

NA62 PLEASE!!

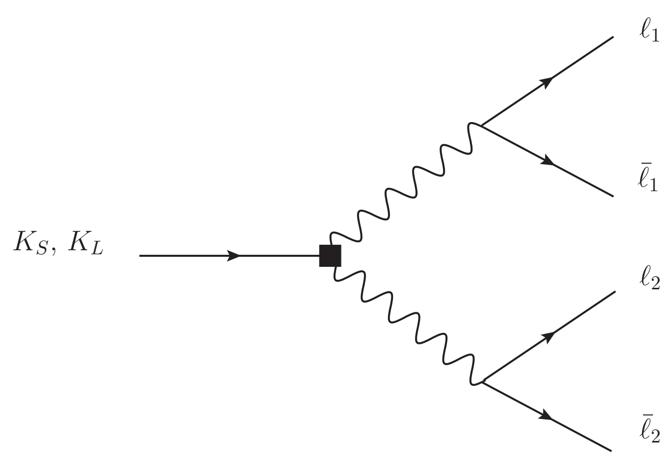
High statistics: nominal # of decays 50 times greater than NA48/2

Also analyzed LFV Kaon decays and $K_L \to \mu\mu \quad (C_{10}^{NP})$

Other interesting channels : discussed with LHCB in SANTIAGO Diego, Marco, Vidal, ..Fredric

$$
\begin{array}{lcl}
K_S \to \mu\mu\mu\mu & - & \mathrm{SM\ LD}\ \sim 2\times 10^{-14} \\
K_S \to ee\mu\mu & - & \sim 10^{-11} \\
K_S \to eeee & - & \sim 10^{-10}
\end{array}
$$

GD,Greynat, Vulvert



AND $\quad K_S \to \pi^0\mu\mu \quad K_S \to \pi^+\pi^-e^+e^-$

Conclusions

- ICTS has a bright future
- QCD tools prepared for the uncover all secrets of flavor including ϵ'
- Many rare channels very clean NA62 and KOTO
- B-anomalies very promising

$K^+ \to \pi^+ e^+ e^-$ $K_S \to \pi^0 e^+ e^-$ form factor calculation

- Bardeen Buras Gerard
- Recent lattice determinations Christ et al.
- epsilon′ ?

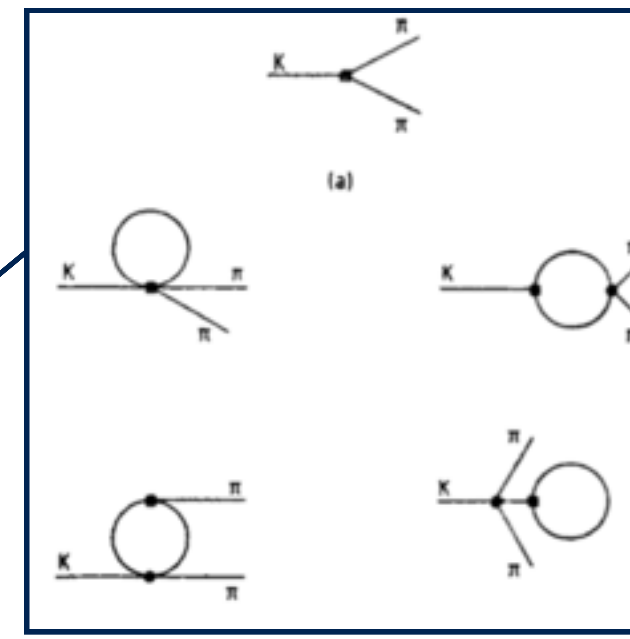
Bardeen Buras Gerard approach to K->ππ

Also evaluated ΔS=2 transitions, epsilon′ (Buras) and $\pi^+ - \pi^0$ mass diff.

Main idea: phys. amplitudes scale independent

Match SD with LD with a precise prescription for CT

CHPT+Large Nc



$$H_{\text{eff}} = \sum_i C_i(\mu)\; Q_i(\mu)$$

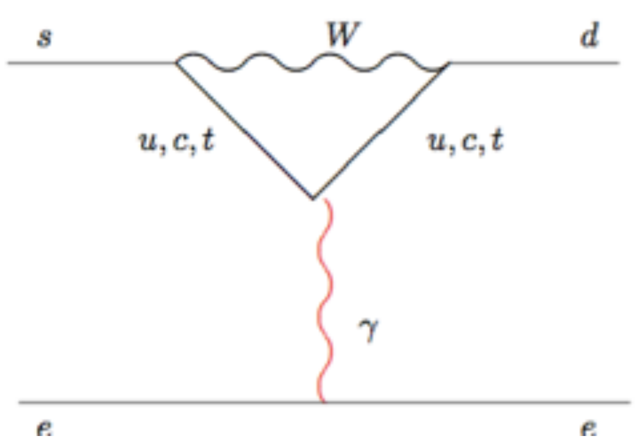
SD

Can we test somewhereelse the Bardeen Buras Gerard (BBG) approach?

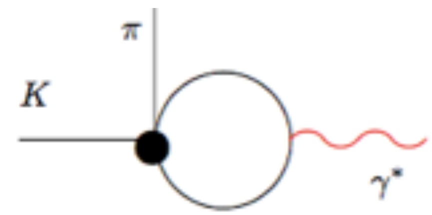
Coluccio-Leskow, Estefania, GD, Greynat,David and Nath,Atanu

$K^+ \to \pi^+ e^+ e^-$ $\quad$ $K_S \to \pi^0 e^+ e^-$

SD



CHPT



CHPT

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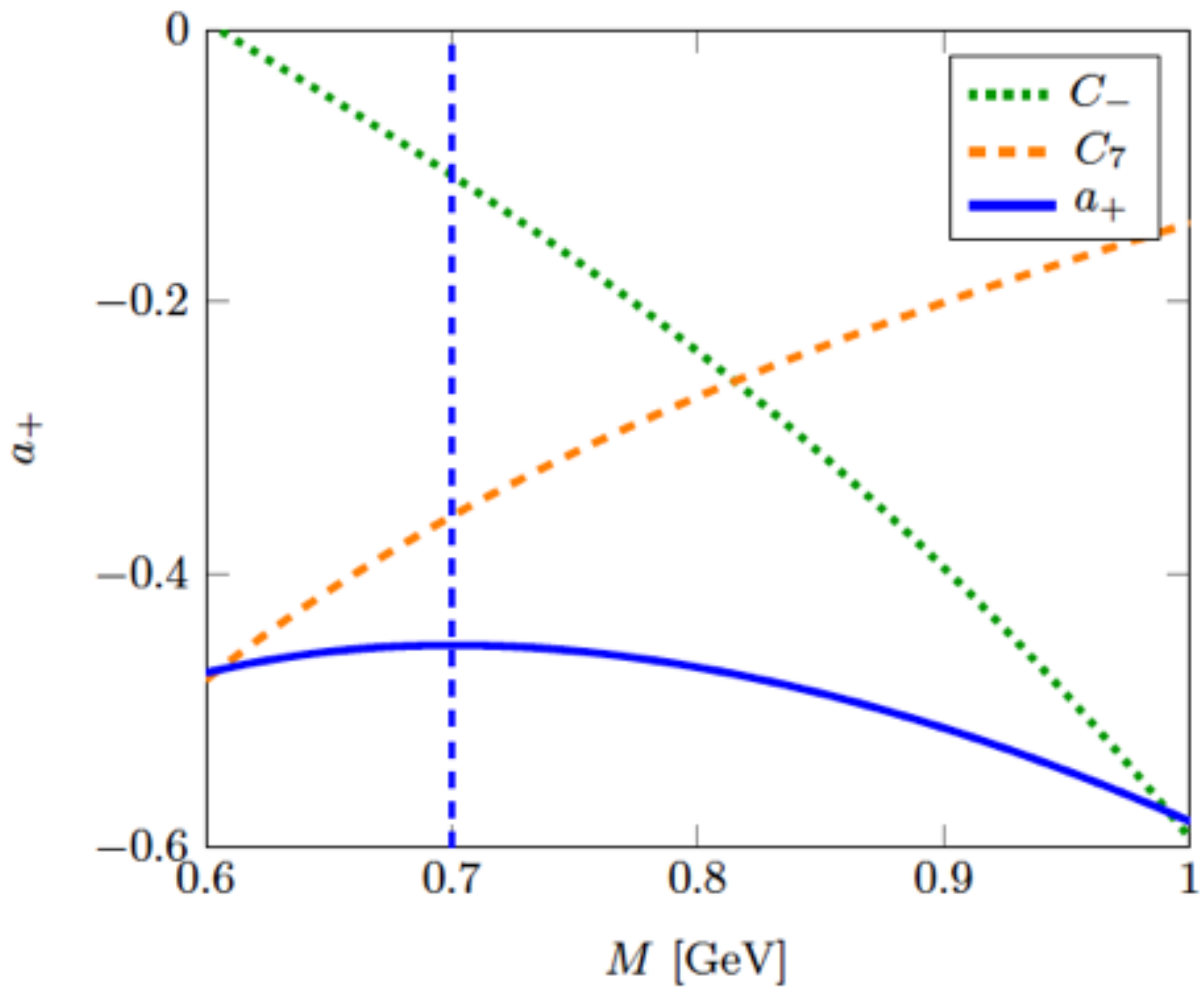
- $a_+,\ b_+$ in general not related to $a_S,\ b_S$

averaging flavour

$$a_+^{\text{exp.}} = -0.578 \pm 0.016$$
$$b_+^{\text{exp.}} = -0.779 \pm 0.066$$

Matching a la BBG for $K^+ \to \pi^+ e^+ e^-$

Coluccio-Leskow,E. G.D ,Greynat, D and Nath, A



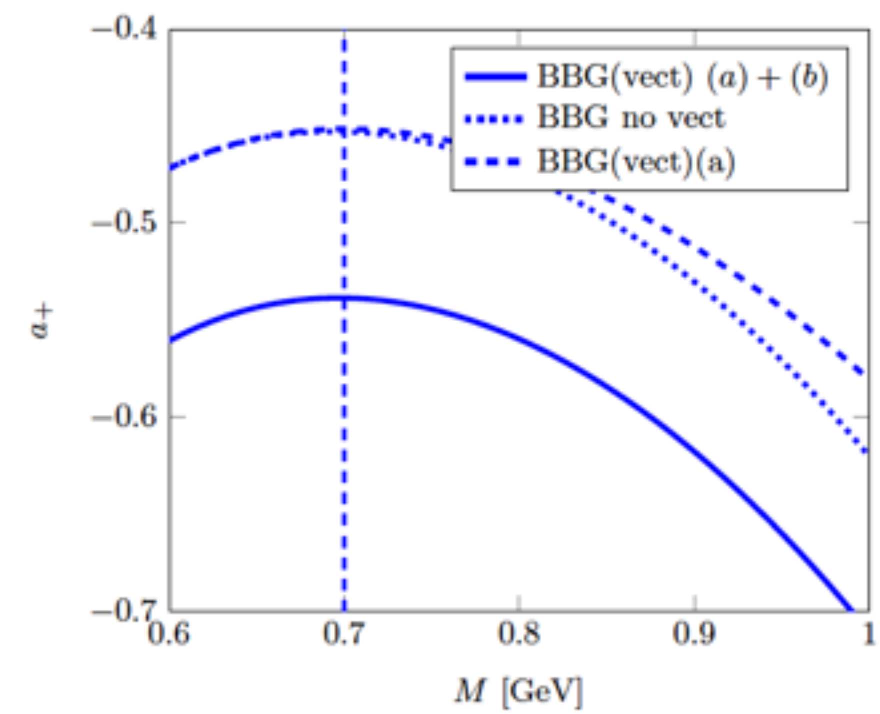


FIG. 5. a_+ as a function of M in the three different frameworks:'BBG no vect.' where vectors are not included, 'BBG(vect)(a)' represents the contribution coming only from diagrams (a) in Fig. 4 and 'BBG(vect) $(a)+(b)$' is the case where both (a) and (b) diagrams were included. The vertical line indicates the value $M = 0.7$ GeV.

Modified Z-coupling scenario cont.

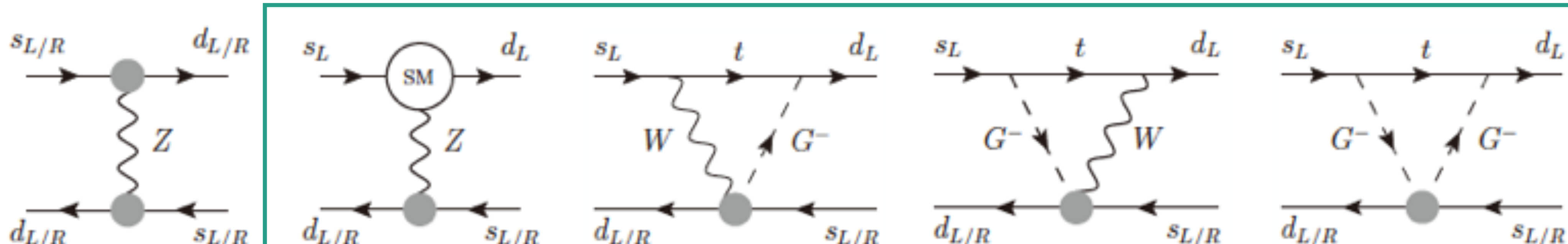
[Buras, De Fazio, Girrbach, '13, '14]
[Buras, Buttazzo, Knegjens, '15]
[Buras, '16]
[Endo, **TK**, Mishima, Yamamoto, '16]
[Bobeth, Buras, Celis, Jung,'17]

- SM + dim-6 eff. operators

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{\rm SM} + \frac{c_L}{\Lambda^2} i(H^\dagger \overleftrightarrow{D_\mu} H)(\overline{Q}_L \gamma^\mu Q'_L) + \frac{c_R}{\Lambda^2} i(H^\dagger \overleftrightarrow{D_\mu} H)(\overline{d}_R \gamma^\mu d'_R), \\ &= \mathcal{L}_{\rm SM} - \frac{\sqrt{2} v M_Z}{\Lambda^2} (c_L \overline{s} \gamma^\mu Z_\mu P_L d + c_R \overline{s} \gamma^\mu Z_\mu P_R d) + \ldots\end{aligned}$$

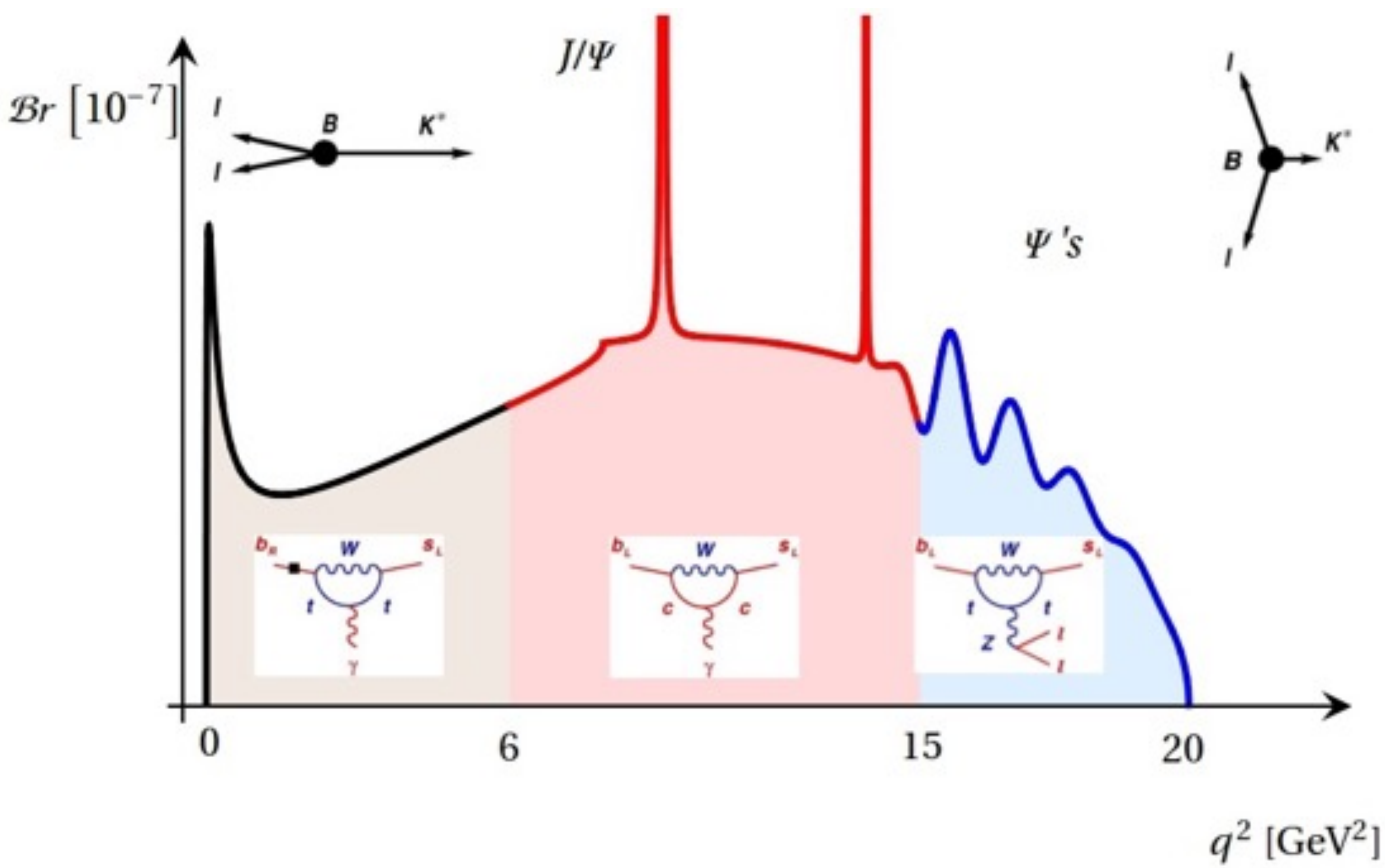
→ modified Z-couplings (FCNC) emerge

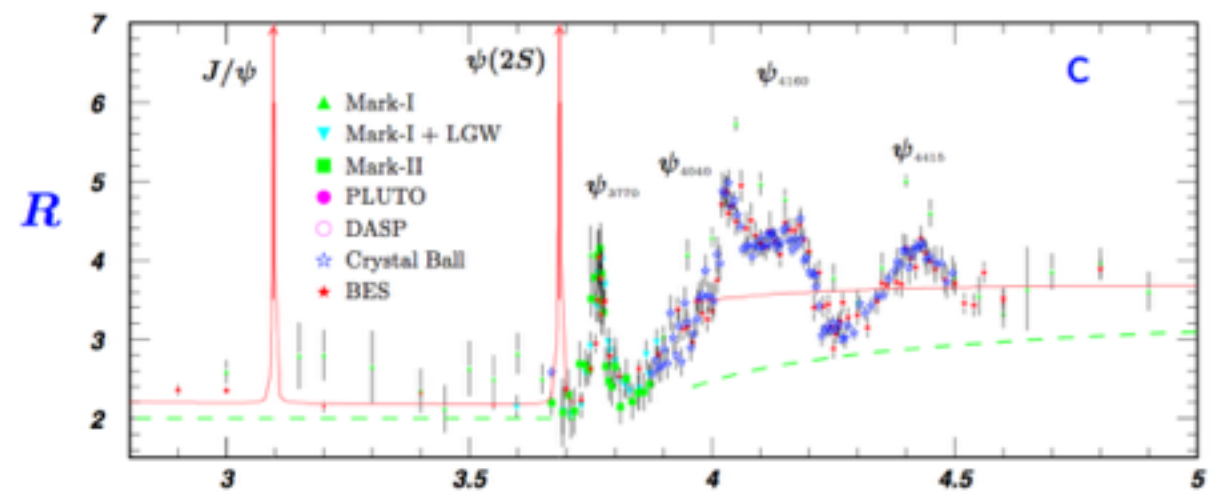
- Constraint comes from ΔS=2 process : ϵ_K (Assumption: NP ΔS=2 (sd)^2 operator is suppressed)



Interference terms

we included these contributions: **Novel point**





Large-recoil region (low q^2)

Heavy to collinear light quark ⇒ QCDf or SCET (power-corrections)

Dominant effect of the photon pole

Charmonium region

Dominated by long-distance (hadronic) effects

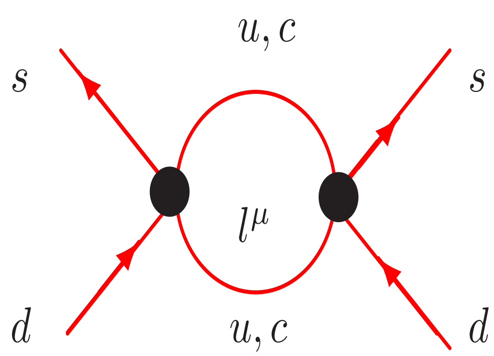
Starting at the perturbative $c\bar{c}$ threshold $q^2 \simeq 6-7$ GeV^2 Low-recoil region (high q^2)

Heavy quark EFT + Operator Product Expansion (OPE) (duality violation)

Dominated by semileptonic operators

Some thoughts how did we get here, in particular FCNC, GIM

$$\mathcal{H}_{eff}^{|\Delta S|=1} = -\frac{4G_F}{\sqrt{2}} \left[\lambda_u (\bar{s}_L \gamma^\mu u_L)(\bar{u}_L \gamma^\mu d_L) + \overbrace{\lambda_c (\bar{s}_L \gamma^\mu c_L)(\bar{c}_L \gamma^\mu d_L)}^{\text{missing piece}} \right] \qquad \lambda_q = V_{qs}^* V_{qd}$$



$$(\bar{s}_L \gamma_\mu d_L)^2 (V_{us}^* V_{ud})^2 \int_0^{M_W} \frac{d^4 l}{(2\pi)^4} < \frac{i}{\not{l} - m_u} \gamma^\nu \frac{i}{\not{l} - m_u} \gamma_\nu >$$

$$(\bar{s}_L \gamma_\mu d_L)^2 (V_{us}^* V_{ud})^2 \ (\ M_W^2 \ + \ \mathcal{O}(m_u^2) \ )$$

expts. $K^0 \bar{K}^0$ (and $K_L \to \mu\mu$) M_W^2-term 10^4 too large!

charm invented to implement unitarity $V_{us}^* V_{ud} + V_{cs}^* V_{cd} = 0$

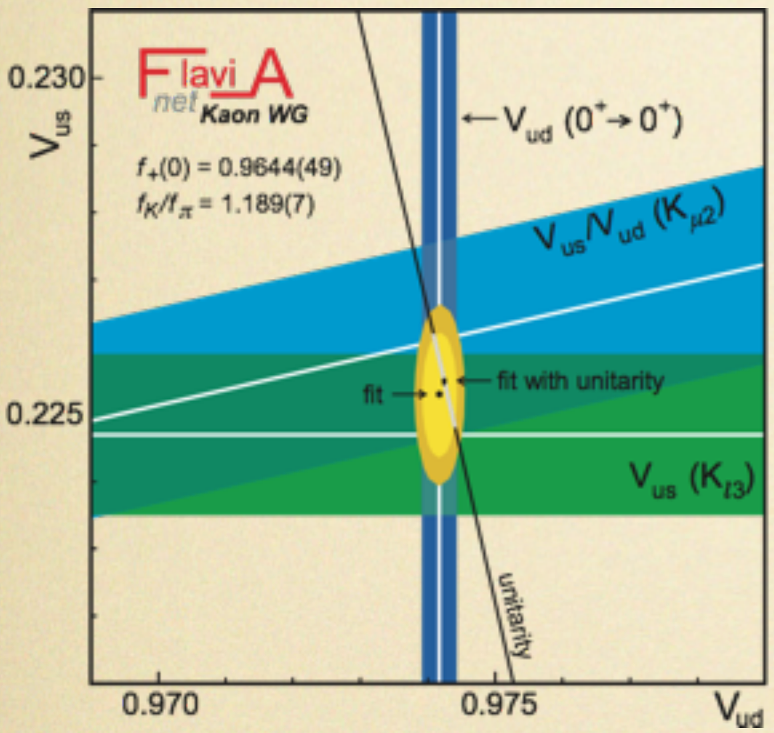
$\Longrightarrow$ cancellation u, c-loops

$$\mathcal{H}_{eff}^{|\Delta S|=2} = \frac{G_F^2}{4\pi^2} M_W^2 (\bar{s}_L \gamma^\mu d_L)^2 [\lambda_c^2 \eta_1 F(x_c) + \lambda_t^2 \eta_2 F(x_t) + 2\lambda_c \lambda_t \eta_3 F(x_c, x_t)]$$

- CP violating and CP conserving
- dominance of *top* contribution $V_{ts}^* V_{td}\ m_t^2 \sim \lambda^5 m_t^2$
- SM suppression very strong: loop factor + CKM structure

$$\mathcal{M}_{\Delta F=2}^{\rm SM} \approx \frac{G_F^2 m_t^2}{16\pi^2} V_{3i}^* V_{3j}\ \langle \bar{M} | (\bar{d}_L^i \gamma^\mu d_L^j)^2 | M \rangle \times F\left(\frac{m_t^2}{m_W^2}\right) \qquad [M = K^0, B_d, B_s]$$

Some impressive KLOE results



Cabibbo's angle

Vus at 0.5 %

M. Antonelli, GD

$\Re(\epsilon)$ $\Im(\delta)$

Review Bell-Steinberger relations: unitarity determines CP and CPT violating in terms of $A_L(f)A_S^*(f)$

$$\left[\frac{\Gamma_S+\Gamma_L}{\Gamma_S-\Gamma_L}+i\tan\phi_{\rm SW}\right]\left[\frac{\Re(\epsilon)}{1+|\epsilon|^2}-i\Im(\delta)\right]=\frac{1}{\Gamma_S-\Gamma_L}\sum_f A_L(f)A_S^*(f)$$

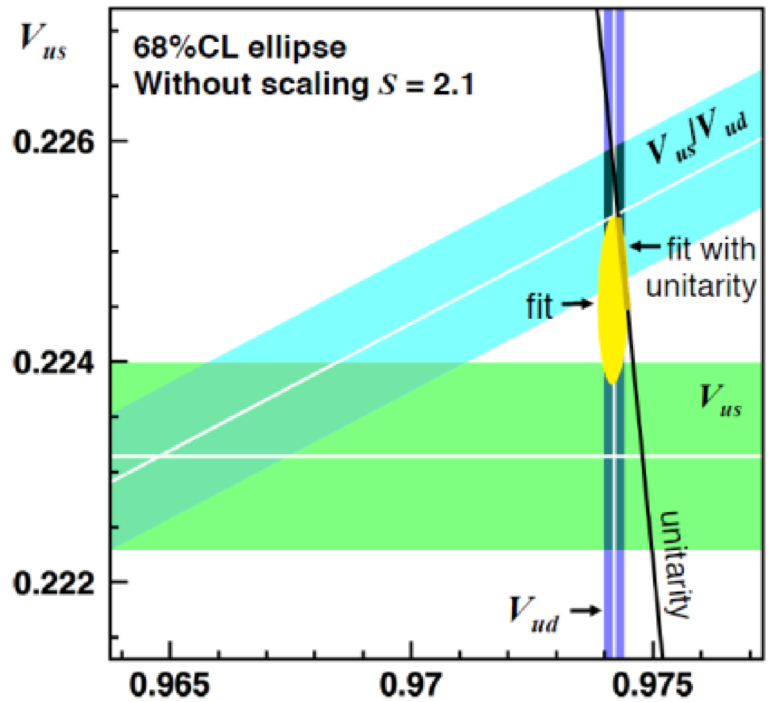
CPLEAR, NA48, KLOE, PDGfit, KTEV

$$\alpha_i \equiv \frac{1}{\Gamma_S}\langle A_L(i)A_S^*(i)\rangle = \eta_i\, B(K_S\to i)\,, \qquad i=\pi^0\,,\ \pi^+(\gamma),3\pi^0\,,\ \pi^0\pi^+\pi^-(\gamma),$$

$|V_{us}|$: kaon decays (ex) M. Moulson

N_f = 2+1+1: Fit to results for $|V_{ud}|$, $|V_{us}|$, $|V_{us}|/|V_{ud}|$
$f_+(0) = 0.9704(32)$, $f_K/f_\pi = 1.1933(27)$

$|V_{ud}| = 0.97420(21)$
$|V_{us}| = 0.2231(9)$
$|V_{us}|/|V_{ud}| = 0.2308(6)$



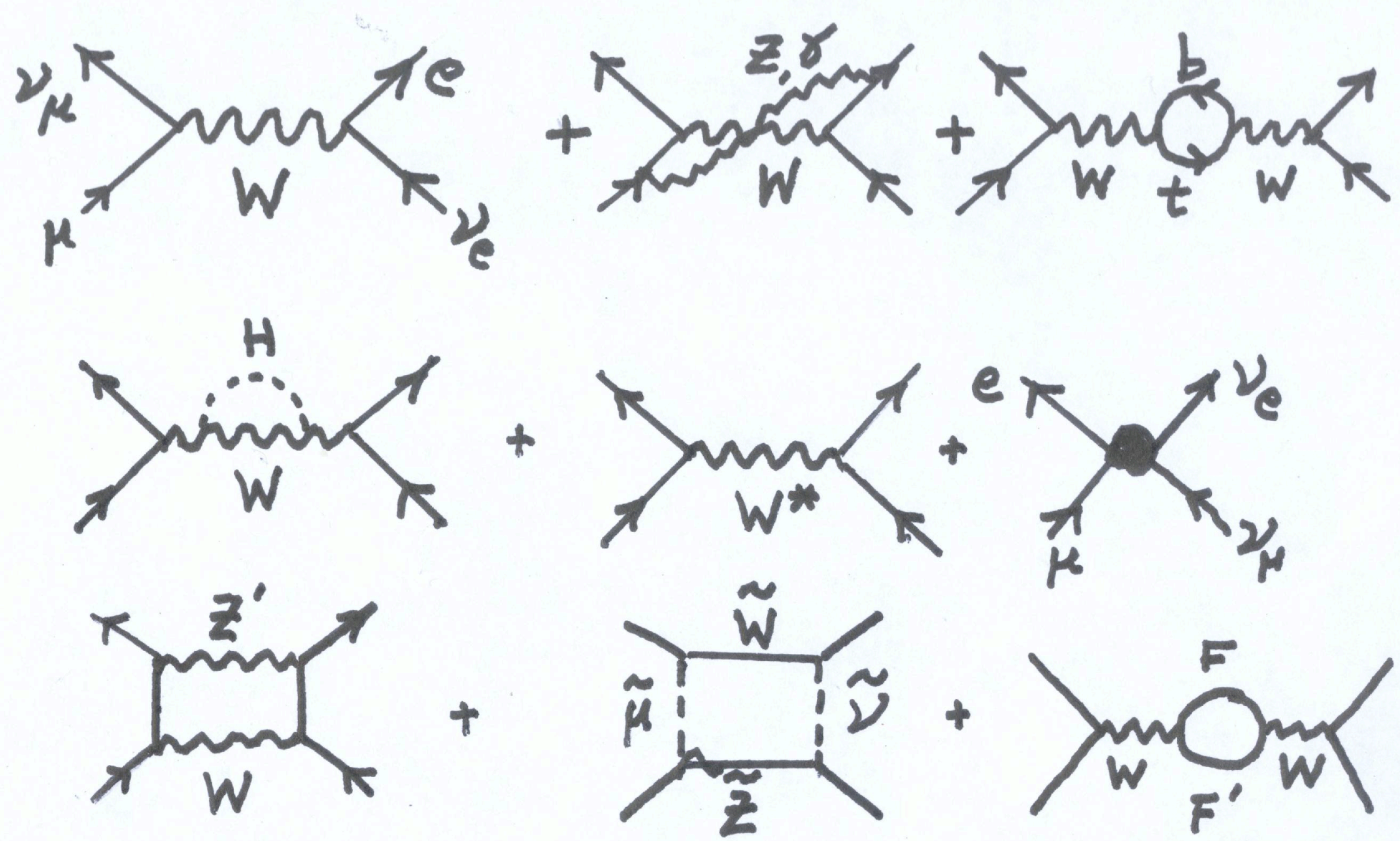
Fit results, no constraint

$V_{ud} = 0.97418(21)$
$V_{us} = 0.2246(5)$
χ^2/ndf = 4.2/1 (3.9%)
$\Delta_{CKM} = -0.0007(5)$
-1.1σ

With scale factor $S = 2.1$

$V_{ud} = 0.97418(43)$
$V_{us} = 0.2246(10)$

Loop and Tree Level Corrections to Muon Decay



Z' Boson

SUSY

Technicolor

$+ \cdot \cdot \cdot$

$|V_{ud}|$ and $|V_{us}|$: summary

- $|V_{ud}|$
  - $0^+ \to 0^+$: stable 0.02% accuracy ; need to consider transition independent correction for significant improvement
  - neutron : can differently probe NP; good motivation to improve; many on-going measurements

- $|V_{us}|$
  - accuracy from kaon decays : < 1%
  - SU(2), QED corrections are becoming relevant
  - resolution to τ $|V_{us}|$ puzzle (3 σ discrepancy from K decays)

Energy Scale	Fields	Effective Theory
M_W	W, Z, γ, g τ, μ, e, ν_i t, b, c, s, d, u	Standard Model
	$\Big\downarrow$ OPE	
$\lesssim m_c$	$\gamma, g\ ;\ \mu, e, \nu_i$ s, d, u	$\mathcal{L}_{\mathrm{QCD}}^{(n_f=3)}$, $\mathcal{L}_{\mathrm{eff}}^{\Delta S=1,2}$
	$\Big\downarrow$ $N_C \to \infty$	
M_K	$\gamma\ ;\ \mu, e, \nu_i$ π, K, η	χPT

IR Chiral Perturbation theory

Chiral sym. breaking through dim. parameter

F_{π}, χ related to $\langle 0|J_{5\mu}|\pi\rangle$, $\langle 0|\overline{q_L}\, q_R|0\rangle$

$F_{\pi} \approx 93$ MeV

$$\mathcal{L}_{\Delta S=0} = \mathcal{L}^2_{\Delta S=0} + \mathcal{L}^4_{\Delta S=0} + \cdots = \frac{F_\pi^{\ 2}}{4} \overbrace{\langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + U \chi^\dagger \rangle}^{\pi\to l\nu,\ \pi\pi\to\pi\pi,\ K\to\pi..} \quad + \quad \overbrace{\sum_i L_i O_i}^{K\to\pi..} + \cdots$$

L_i Gasser Leutwyler coeff determined from expts.

O_i p^4 operator

Hard Wall model

QCD

$$\Pi_V(Q^2) = \left[\beta \ln \frac{\mu^2}{Q^2} + \frac{\gamma}{Q^4} + \frac{\delta}{Q^6} + \cdots\right]$$

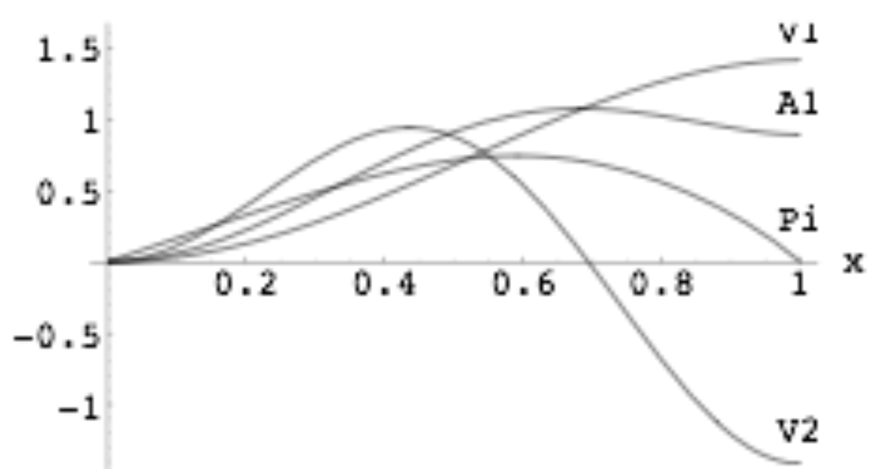
$\beta \simeq N_c/(12\pi^2)$ $\gamma \simeq \alpha_s \langle G^2_{\mu\nu}\rangle/12\pi$ $\delta \simeq -28\pi\alpha_s\langle\bar{q}q\rangle^2/9$

$$\int d^4x\, e^{ip\cdot x}\langle J_\mu(x)J_\nu(0)\rangle = \left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\Pi(p^2)$$

$$\Pi_V(p^2) = \sum_{n=1}^{\infty} \frac{f_n^2}{p^2 - m_n^2}$$

HW

$$\Pi_V(q^2) = -\frac{4}{3}\frac{N_c}{(4\pi)^2}\left[\log\frac{q^2}{\mu^2} - \pi\frac{Y_0\left(qz_0\right)}{J_0\left(qz_0\right)}\right]$$



$$m_n = \frac{\gamma_{0,n}}{z_0}$$

$$J_0(\gamma_{0,n}) = 0$$

$$\varphi_n(z) = \frac{\sqrt{2}}{\left|J_1\left(\gamma_{0,n}\right)\right|}\frac{z}{L_1}J_1\left(\gamma_{0,n}\frac{z}{L_1}\right)$$

HW:CHPT predictions

Hirn Sanz

$$\frac{F_\pi{}^2}{4}\overbrace{\langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + U\chi^\dagger\rangle}^{\pi\to l\nu,\ \pi\pi\to\pi\pi,\ K\to\pi..} + \overbrace{\sum_i L_i O_i}^{K\to\pi..} + \cdots$$

$$\langle F_{5\mu}\rangle^2 \to \mathcal{L}_2^{\chi PT} + ..$$

The 5D gauge action will generate all

$$\langle F_{\mu\nu}\rangle^2 \to \mathcal{L}_4^{\chi PT} + ..$$

$$2\,\langle L_{5\mu}L_{5\nu} + R_{5\mu}R_{5\nu}\rangle = (\partial_5\alpha)^2\,\langle D_\mu U D_\nu U^\dagger\rangle +$$

$$f_\pi = \frac{\sqrt{N_c}}{\sqrt{6}\pi z_0}$$

$$L_1 = \frac{11N_c}{9216\pi^2}$$

$$L_9 = \frac{N_c}{64\pi^2}$$

G-L coeff.	AdS_5 HW	Experiment
$10^3 L_1$	0.52	0.4 ± 0.3
$10^3 L_2$	1.04	1.35 ± 0.3
$10^3 L_3$	-3.12	-3.5 ± 1.1
$10^3 L_9$	6.8	6.9 ± 0.7
$10^3 L_{10}$	-6.8	-5.5 ± 0.7

$$L_2 = 2L_1,\ L_3 = -6L_1,\ L_{10} = -L_9$$