$\mu \to e\gamma$ in a supersymmetric radiative neutrino mass model

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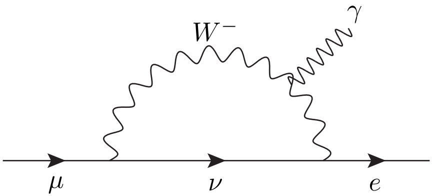
$\mu \to e\gamma$

- The decay $\mu \to e\gamma$ is flavour violating.
- In the standard model,



The branching ratio of this decay is $Br(\mu \to e\gamma) \sim 10^{-48}$.

- The experimental group MEG have been looking for this decay. But it has never been observed so far, and we have an upper bound on its branching ratio.

$$Br(\mu \to e\gamma) < 4.2 \times 10^{-13}$$

Beyond standard model

- There are several reasons for beyond standard model.
  - Fermion mass hierarchy.
  - Existence of dark matter.
  - Matter-anti-matter asymmetry.
- A popular candidate for beyond standard model is supersymmetry.

$$\text{boson} \leftrightarrow \text{fermion}$$

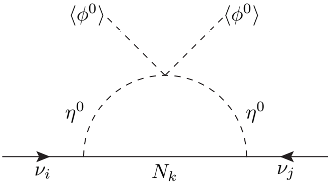
- In models of supersymmetry, branching ratio of $\mu \to e\gamma$ may exceed the experimental limit.

Inert Higgs doublet model

- The idea in this model is proposed by E. Ma (2006) & R. Barbieri, L.J. Hall and V.S. Rychkov (2006).
- This model contains Z_2 symmetry and the following additional fields, apart from SM fields.

$$\eta = (\eta^+, \eta^0), \quad N_i$$

- Z_2 symmetry forbids the Yukawa couplings $\bar{L}_i \tilde{\phi} N_j$, hence neutrinos are massless at tree level.
- Neutrinos acquire masses through the following 1-loop diagram.



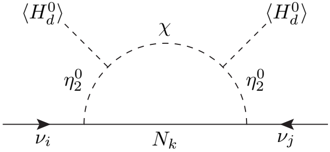
The supersymmetric model by Ernest Ma

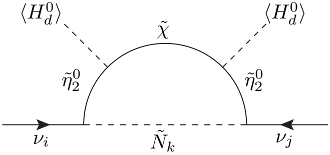
- This model has a discrete symmetry $Z_2 \times Z_2'$, whose purpouse is to forbid the Yukawa term $\hat{L}_i \hat{H}_u \hat{N}_j$ at tree level.
- Apart from MSSM, the additional fields are:
  - two electroweak doublets: $\hat{\eta}_1 = (\hat{\eta}_1^0, \hat{\eta}_1^-)$, $\hat{\eta}_2 = (\hat{\eta}_2^+, \hat{\eta}_2^0)$.
  - three right-handed neutrino fields, $\hat{N}_i, i = 1, 2, 3$.
  - a singlet field $\hat{\chi}$.
- The superpotential of this model containing additional fields is

$$
\begin{aligned}
W &= (Y_\nu)_{ij} \hat{L}_i \hat{\eta}_2 \hat{N}_j + \lambda_1 \hat{H}_d \hat{\eta}_2 \hat{\chi} + \lambda_2 \hat{H}_u \hat{\eta}_1 \hat{\chi} + \\
&\quad \mu_\eta \hat{\eta}_2 \hat{\eta}_1 + \frac{1}{2} \mu_\chi \hat{\chi} \hat{\chi} + \frac{1}{2} M_{ij} \hat{N}_i \hat{N}_j
\end{aligned}
$$

Neutrino masses

- Neutrinos in this model acquire masses at 1-loop level.





- The expression for neutrino masses are

$$
\begin{aligned}
(m_\nu)_{ij} \quad = \quad & \sum_{k,l=1}^{3} \frac{(Y_\nu)_{ik}(Y_\nu)_{jk}}{16\pi^2} M_k \left[[U_R(2,l)]^2 \frac{m^2_{\eta Rl}}{m^2_{\eta Rl} - M_k^2} \ln \frac{m^2_{\eta Rl}}{M_k^2} - [U_I(2,l)]^2 \frac{m^2_{\eta Il}}{m^2_{\eta Il} - M_k^2} \ln \frac{m^2_{\eta Il}}{M_k^2} \right] \\
& + \sum_{k,l=1}^{3} \frac{(Y_\nu)_{ik}(Y_\nu)_{jk}}{16\pi^2} [U_\eta(2,l)]^2 m_{\tilde{\eta}_l} \left[\frac{m^2_{Rk}}{m^2_{Rk} - m^2_{\tilde{\eta}_l}} \ln \frac{m^2_{Rk}}{m^2_{\tilde{\eta}_l}} - \frac{m^2_{Ik}}{m^2_{Ik} - m^2_{\tilde{\eta}_l}} \ln \frac{m^2_{Ik}}{m^2_{\tilde{\eta}_l}} \right]
\end{aligned}
$$

Fitting to neutrino masses

- Since the neutrino mass scale is 0.1 eV, if we take all the supersymmetric masses to be at few 100 GeV, from naive order of estimation, we get

$$(Y_\nu)_{ij} \sim 10^{-5}$$

- For these small Yukawa couplings, flavour violating decays such as $\mu \to e\gamma$ are automatically suppressed.
- Alternatively, in order to have $\mathcal{O}(1)$ Yukawa couplings and to satisfy neutrino masses, we can fine-tune some soft parameters of the model.

Fine-tuning of parameters

- The masses of real and imaginary components of right-handed sneutrinos are

$$m_{Ri}^2 = M_i^2 + (m_N^2)_i + (b_M)_i, \quad m_{Ii}^2 = M_i^2 + (m_N^2)_i - (b_M)_i$$

- The mixing mass matrices for real and imaginary parts of neutral scalar fields $(\eta_1^0, \eta_2^0, \chi)$ can be written as

$$m^2(\epsilon) = m_0^2 + \epsilon \begin{pmatrix} 0 & -b_\eta & -[(A\lambda)_2 v_2 - \mu\lambda_2 v_1] \\ \times & 0 & [(A\lambda)_1 v_1 - \mu\lambda_1 v_2] \\ \times & \times & b_\chi \end{pmatrix}$$

  Here $\epsilon = +1$ or -1 for real and imaginary fields.

- If we fine-tune the following parameters

$$(b_M)_i, \quad b_\eta, \quad b_\chi, \quad (A\lambda)_1, \quad (A\lambda)_2$$

  we can fit the neutrino mass scale with $\mathcal{O}(1)$ Yukawa couplings.

Analysis on neutrino masses

- To solve the neutrino mass relation, we have assumed degenerate masses for right-handed neutrinos and right-handed sneutrinos. Then,

$$
\begin{aligned}
(m_\nu)_{ij} &= \frac{S_{ij}}{16\pi^2}\sum_{l=1}^{3}\Bigg\{M\left[[U_R(2,l)]^2\frac{m^2_{\eta_{Rl}}}{m^2_{\eta_{Rl}}-M^2}\ln\frac{m^2_{\eta_{Rl}}}{M^2}-[U_I(2,l)]^2\frac{m^2_{\eta_{Il}}}{m^2_{\eta_{Il}}-M^2}\ln\frac{m^2_{\eta_{Il}}}{M^2}\right]+\\
&\quad [U_\eta(2,l)]^2 m_{\tilde{\eta}_l}\left[\frac{m_R^2}{m_R^2-m^2_{\tilde{\eta}_l}}\ln\frac{m_R^2}{m^2_{\tilde{\eta}_l}}-\frac{m_I^2}{m_I^2-m^2_{\tilde{\eta}_l}}\ln\frac{m_I^2}{m^2_{\tilde{\eta}_l}}\right]\Bigg\},\\
S_{ij} &= \sum_{k=1}^{3}(Y_\nu)_{ik}(Y_\nu)_{jk}
\end{aligned}
$$

- We can calculate the elements of the neutrino mass matrix from the following.

$$m_\nu = U^*_{\rm PMNS}\cdot {\rm diag}(m_1,m_2,m_3)\cdot U^\dagger_{\rm PMNS}$$

From neutrino oscillation data

- In the normal or inverted mass hierarchy, we can the mass eigenvalues as

$$m_1 = 0, \quad m_2 = \sqrt{\Delta m^2_{sol}}, \quad m_3 = \sqrt{\Delta m^2_{atm}}$$
$$m_3 = 0, \quad m_1 = \sqrt{\Delta m^2_{atm}}, \quad m_2 = \sqrt{\Delta m^2_{sol} + m_1^2}$$
$$\Delta m^2_{sol} = 7.6 \times 10^{-5}\ \mathrm{eV}^2, \quad \Delta m^2_{atm} = 2.48 \times 10^{-3}\ \mathrm{eV}^2$$

- The $U_{\rm PMNS}$ matrix depends on three mixing angels and a CP-violating phase, which we can take as follows.

$$\sin\theta_{12} = \frac{1}{\sqrt{3}}, \quad \sin\theta_{23} = \frac{1}{\sqrt{2}}, \quad \sin\theta_{13} = 0.15, \quad \delta_{\rm CP} = 0$$

SUSY parameters in the analysis

- In our analysis, we have chosen the required SUSY parameters as follows.

$$\mu_\chi = 600 \text{ GeV}, \quad m_{\eta_1} = 400 \text{ GeV}, \quad m_{\eta_2} = 500 \text{ GeV}, \quad m_\chi = 600 \text{ GeV},$$
$$m_N = 700 \text{ GeV}, \quad \lambda_1 = 0.5, \quad \lambda_2 = 0.6, \quad \tan\beta = 10$$

- Regarding the fine-tuning parameters, we have taken them as

$$(A\lambda)_1 = \lambda_1 \mu v_2/v_1, \quad (A\lambda)_2 = \lambda_2 \mu v_1/v_2$$
$$b_M = b_\eta = b_\chi = b_{\rm susy}$$

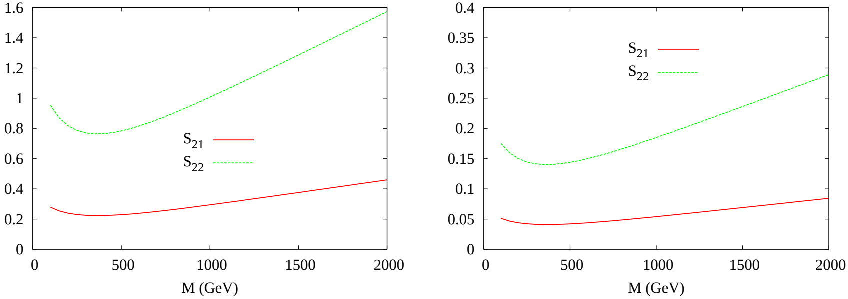
- After fixing these, the free parameters in our model are

$$M, \quad b_{\rm susy}, \quad \mu_\eta$$

Results on Yukawa couplings

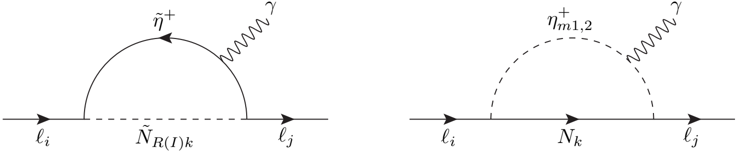
$\mu_\eta = 1$ TeV

$b_{\rm susy} = (3 \times 10^{-2})^2$ GeV^2 $(7 \times 10^{-2})^2$ GeV^2



I have also computed $S_{11}, S_{13}, \cdots$, and they found to be of $\mathcal{O}(1)$.

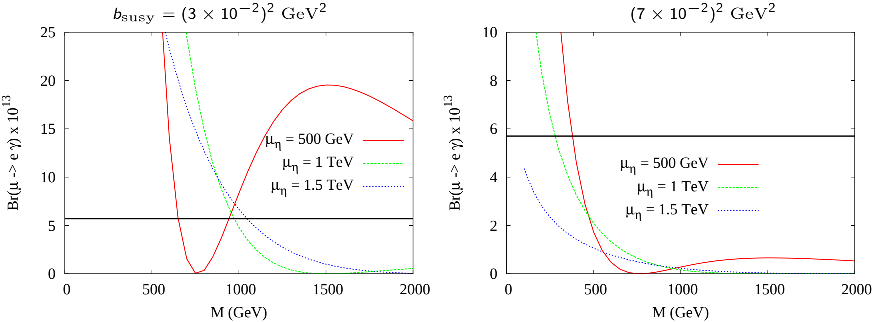
- Feynman diagrams for decays of the form $\ell_i \to \ell_j \gamma$ are



- The branching ratio of $\mu \to e\gamma$ is

$$
\begin{aligned}
Br(\mu \to e\gamma) &= \frac{3\alpha}{16\pi G_F^2} \left| \sum_{k=1}^{3} (Y_\nu)_{1k} (Y_\nu)_{2k} \times \right. \\
& \left. \left\{ \frac{1}{4\mu_\eta^2} [f_2(x_{Rk}) + f_2(x_{Ik})] - \left[\cos^2\theta \frac{f_2(x_{k2})}{2m_{2+}^2} + \sin^2\theta \frac{f_2(x_{k1})}{2m_{1+}^2} \right] \right\} \right|^2 \\
x_{Rk} &= \frac{m_{Rk}^2}{\mu_\eta^2}, \quad x_{Ik} = \frac{m_{Ik}^2}{\mu_\eta^2}, \quad x_{k2} = \frac{M_k^2}{m_{2+}^2}, \quad x_{k1} = \frac{M_k^2}{m_{1+}^2}, \\
f_2(x) &= \frac{1}{(1-x)^4} \left[\frac{1}{6} - x + \frac{1}{2}x^2 + \frac{1}{3}x^3 - x^2 \ln(x) \right]
\end{aligned}
$$

Results on $\mu \to e\gamma$



muon $g-2$

- For a Dirac particle of mass m, charge e, and spin $\vec{S}$, the spin magnetic moment is
$$\vec{\mu} = g\frac{e}{2m}\vec{S}$$
- The g-factor is 2 at tree level. However, radiative corrections can shift the value of g-factor away from 2.
- For muon, we can define
$$a_\mu = \frac{g-2}{2}$$
- At the moment, there seems to be a discrepancy between experiment and theoretical value of a_μ in the standard model.
$$\Delta a_\mu = a_\mu^{\mathrm{Exp}} - a_\mu^{\mathrm{SM}} = (29 \pm 9) \times 10^{-10}$$

muon $g-2$ in our model

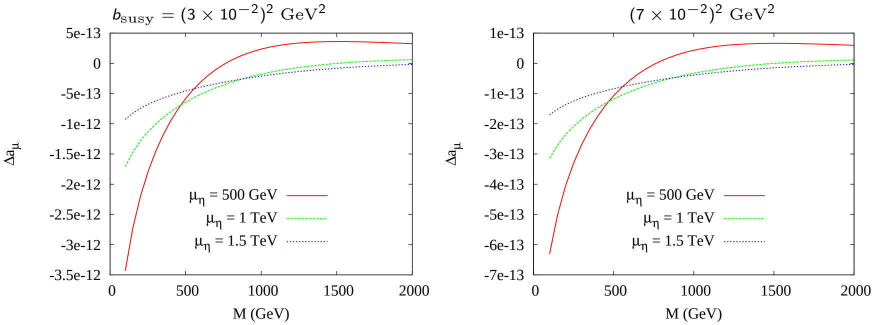
- In our model, Δa_μ gets contribution from MSSM and also from the additional fields of this model.

$$\Delta a_\mu = \Delta a_\mu^{\rm MSSM} + \Delta a_\mu^{\rm add}$$

- $\Delta a_\mu^{\rm MSSM}$ is computed by Moroi, and it is known that $\Delta a_\mu^{\rm MSSM}$ fits the discrepancy in muon $g-2$.
- In this model the additional contribution to muon $g-2$ is found out to be

$$\Delta a_\mu^{\rm add} = \frac{m_\mu^2}{16\pi^2}\sum_{k=1}^{3}[(Y_\nu)_{2k}]^2\left\{\frac{1}{2\mu_\eta^2}[f_2(x_{Rk})+f_2(x_{Ik})] - \left[\cos^2\theta\frac{f_2(x_{k2})}{m_{2+}^2} + \sin^2\theta\frac{f_2(x_{k1})}{m_{1+}^2}\right]\right\}$$

Results on muon $g-2$



The additional contribution to muon $g-2$ is insignificant.

Summary

- In this talk, I focused on a supersymmetric model, where neutrinos acquire masses at loop level.
- We can fit the neutrino oscillation data with $\mathcal{O}(1)$ Yukawa couplings, but SUSY breaking soft parameters need to be fine-tuned.
- For $\mathcal{O}(1)$ Yukawa couplings, $Br(\mu \to e\gamma)$ can be substantial. Current upper bound on $Br(\mu \to e\gamma)$ gives lower bound on right-handed neutrino mass. This lower bound can be about 1 TeV, depending on the parameter choice.
- In this model, muon $g - 2$ due to additional fields is found be insignificant.

Thank you