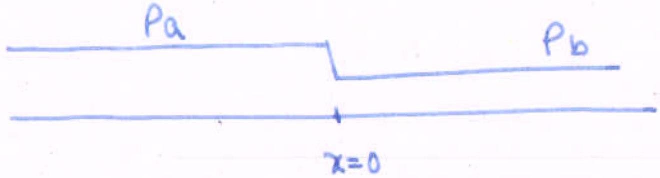
System Approaching equilibrium



Statistics of current

Average: $\langle Q_T \rangle \sim \sqrt{T}$ (net flow)

[How to see:

$$\langle Q_T\rangle = \int_0^T d\tau\, j(0,\tau)$$

$$= -\int_0^T d\tau\, D(n(0,\tau))\,\partial_x n(0,\tau)$$

$$\xrightarrow[t=\tau/T]{x=\frac{X}{\sqrt{T}}} \sqrt{T} \quad]$$

How about full distribution?

$$P\left(\frac{Q_T}{\sqrt{T}} = j\right) \asymp e^{-\sqrt{T}\,\phi(j)}$$

$\Rightarrow$ All cumulants $\langle Q_T^n\rangle_c \sim \sqrt{T}$

and $\boxed{a(\lambda) = \ln\langle e^{\lambda Q_T}\rangle \sim \sqrt{T}}$

Long memory effect: ϕ, a depends on initial STATE ~~distribution of ptls~~.

Example

Reason is: in this case a unlikely fluctuation of initial density may give very large current Q_T.

For a formal analysis, we define (in parallel to disordered system)

<u>Anneal</u>

$$G(\lambda) = \log \left\langle e^{\lambda Q_T} \right\rangle_{\text{hist+Init}}$$

<u>Quench</u>

$$G(\lambda) = \left\langle \log \left\langle e^{\lambda Q_T} \right\rangle_{\text{hist}} \right\rangle_{\text{Init}}$$

<u>Some results:</u>

* <u>Non-interacting particles:</u>

<u>anneal:</u> $G(\lambda) \simeq \sqrt{T} \cdot \frac{1}{\sqrt{\pi}} \left[\rho_a (e^{\lambda}-1) + \rho_b (e^{-\lambda}-1) \right]$

<u>quench:</u>

$$G(\lambda) \simeq \sqrt{T} \left\{ \rho_a \int_0^\infty dx \log\left[1 + (e^{\lambda}-1)\tfrac{1}{2}\text{Erfc}\left(\tfrac{-x}{2}\right)\right] + \rho_b \int_0^\infty dx \ln\left[1 + (e^{-\lambda}-1)\tfrac{1}{2}\text{Erfc}\left(\tfrac{x}{2}\right)\right] \right\}$$

* <u>SEP</u>

<u>Anneal:</u>

$$G(\lambda) \simeq \frac{\sqrt{T}}{\pi} \int_{-\infty}^{\infty} dx \ln\left[1 + \omega e^{-x^2}\right]$$

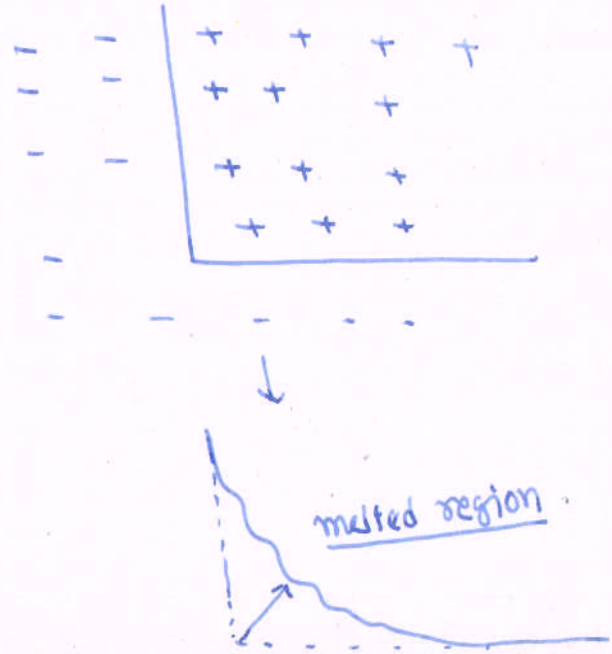
$$\omega = \rho_a (e^{\lambda}-1) + \rho_b (e^{-\lambda}-1) + \rho_a \rho_b (e^{\lambda}-1)(e^{-\lambda}-1)$$

* can be expressed as Fredholm determinant

$$G(\lambda) = \frac{\sqrt{T}}{\pi} \log \det\left[1 + \omega \cdot K\right] \quad \text{where} \quad \int dz\, K(y,z)\,\psi_x(z) = e^{-x^2}\psi_x(y)$$

* **Similar observables:**

① Total activity upto time T



$$P\left(\frac{A}{T}=a\right) \asymp e^{-\sqrt{T}\,\phi(a)}$$

② Tagged particle in single file

$$P\left(\frac{Y_T}{\sqrt{T}}=y\right) \asymp e^{-\sqrt{T}\,\phi(y)}$$

Explicit calculation using Macroscopic Fluctuation Theory

⊕ Calculation of current large-deviation

Equation:

$$\boxed{\frac{d}{d\tau} n(x,\tau) = -\partial_x \mathcal{J}(x,\tau)}$$

⊕ As we want fluctuations $\sim \sqrt{T}$

Do carse graining with $l = \sqrt{T}$

$$(x,t) \cong \left(\frac{X}{\sqrt{T}}, \frac{\tau}{T}\right)$$

$$\rho(x,t) \cong n(x,\tau)$$

$$\frac{1}{}\, j(x,t) \sim \mathcal{J}(x,\tau)$$

Integrated current:

$$Q_T = \int_0^\infty dx \left\{ n(x,T) - n(x,0) \right\}$$

$$\simeq \sqrt{T} \int_{-\infty}^{\infty} dx \cdot \Theta(x) \left(q(x,1) - q(x,0) \right) = \sqrt{T} \cdot J$$

Generating function

$$\langle e^{\lambda Q_T} \rangle = \int \mathcal{D}[p,q] \, e^{-\sqrt{T} \cdot S[p,q] + \sqrt{T} \, J\lambda}$$

$$e^{-\sqrt{T} \, F[q(x,0)]}$$

• ~~For general [illegible]~~

For Quench: $F = 0$ [* equivalent to $\langle \ln Z \rangle$ as log is a slow varying fn^{e} and distn of $q(x,0)$ is sharply peake around (ρ_a, ρ_b).]

For Anneal: $F = \Theta(-x) \, F_{\rho_a} + \Theta(x) \, F_{\rho_b}$

$$F_\rho[r] = \int dx \left\{ f(r) - f(\rho) - f'(\rho)(r-\rho) \right\}$$

$$= \int dx \cdot \int_\rho^r ds \, f'(s) + \int dx \cdot \int_\rho^r ds \, \frac{d}{ds} \left[f'(s)(r-s) \right]$$

$$f''(s)(r-s) - f'(s)$$

cancels

$$\Rightarrow \boxed{F_\rho(r) = \int dx \int_\rho^r ds \, f''(s)(r-s) = \int dx \int_\rho^r ds \cdot \frac{2D(s)}{\sigma(s)} \cdot (r-s)}$$

$\Rightarrow$

$\Rightarrow$ Saddle point

$$\frac{G(\lambda)}{\sqrt{T}} \simeq \max_{p,q} \left\{ \lambda J \left[q(x,1), q(x,0) \right] - F[q(x,0)] - S[p,q] \right\} \equiv W$$

Remind:
$$S[p,q] = \int_0^1 dt \left\{ \int dx\, p\dot{q} - H \right\}$$

$$H = \frac{\sigma(q)}{2} (\partial_x p)^2 - D(q)\cdot(\partial_x q)\cdot(\partial_x p)$$

Variational Calculus:

$$\delta W = \int_0^1 dt \int dx \left\{ \left(\dot{q} - \frac{\delta H}{\delta p}\right) \delta p - \left(\dot{p} + \frac{\delta H}{\delta q}\right) \delta q \right\}$$
$$+ \int dx\, \delta q(x,1) \left\{ -p(x,1) + \frac{\delta Q}{\delta q(x,1)} \right\}$$
$$+ \int dx \cdot \delta q(x,0) \left\{ + p(x,0) + \frac{\delta Q}{\delta q(x,0)} - \frac{\delta F}{\delta q(x,0)} \right\}$$
$$= 0$$

Optimal paths:

$$\dot{q} = \frac{\delta H}{\delta p} \Rightarrow \boxed{\dot{q} - \partial_x (D\, \partial_x q) = -\partial_x \left(\sigma(q) \cdot \partial_x p \right)}$$

$$\dot{p} = -\frac{\delta H}{\delta q} \Rightarrow \boxed{\dot{p} + D\, \partial_{xx} q = -\frac{\sigma'(q)}{2} (\partial_x p)^2}$$

Boundary condition

$$p(x,1) = \frac{\delta J}{\delta q(x,1)} \lambda$$

Quench: $q(x,0) = \rho(x) =$ [step profile]

Anneal: $$p(x,0) = -\frac{\delta J}{\delta q(x,0)} \lambda + \frac{\delta F}{\delta q(x,0)}$$

Few simplification:

$$\int_0^1 dt \left\{ \int dx\, \hat{p}\,\dot{q} \right\} = \int dt \int dx \left\{ p\,\partial_x(D\,\partial_x q) - p\,\partial_x(\sigma\,\partial_x p) \right\}$$

$$= \int dt \int dx \cdot \left\{ - D \cdot \partial_x q \cdot \partial_x p + \sigma \cdot (\partial_x p)^2 \right\}$$

$$\Rightarrow \int dt \int dx \left\{ p\dot{q} - H \right\} = \int dt \int dx \cdot \frac{\sigma(q)}{2} (\partial_x p)^2$$

leads to

Quench:

$$\boxed{\frac{G(\lambda)}{\sqrt{T}} = \lambda J_0 [q(x,1), q(x,0)] - \int dt \int dx\, \frac{\sigma(q)}{2} (\partial_x p)^2}$$

Anneal

$$\frac{G(\lambda)}{\sqrt{T}} = \;\;'' \;\; - F[q(x,0)]$$

Excercise: Prove a symmetry for Annealed case

$$\boxed{G(\lambda) = G\left(-\lambda - \int_{\rho_b}^{\rho_a} ds\, \frac{2D(s)}{\sigma(s)}\right)}$$

Remark: untill now the analysis is same for all observables.

Difficult to solve for general case.

Non-Interacting particles $D=1$; $\sigma = 2q$

$$\partial_t p + \partial_{xx} p = -(\partial_x p)^2$$

$$\partial_t q - \partial_{xx} q = -\partial_x (2q\,\partial_x p)$$

Canonical transformation

$$\left.\begin{aligned} p &\to \ln R \\ q &\to G\cdot R \end{aligned}\right\} \Rightarrow \begin{aligned} \partial_t R + \partial_{xx} R &= 0 \\ \partial_t G - \partial_{xx} G &= 0 \end{aligned}$$

General solution

$$p(x,t) = \log\left[\int_{-\infty}^{\infty} dz\; e^{p(z,1)}\, g(z,1\,|\,x,t)\right]$$

$$q(x,t) = e^{p(x,t)} \int_{-\infty}^{\infty} dz\; e^{-p(z,0)}\, g(x,t\,|\,z,0)$$

Where $\quad g(x,t\,|\,y,t') = \dfrac{e^{-\frac{(x-y)^2}{4(t-t')}}}{\sqrt{4\pi(t-t')}}$ Green's function

use boundary condition $\quad p(x,1) = \dfrac{\delta J}{\delta q(x,1)}\lambda = \lambda\,\theta(x)$

$$p(x,t) = \log\left[1 + (e^{\lambda}-1)\,\tfrac{1}{2}\,\mathrm{Erfc}\left(\frac{-x}{2\sqrt{1-t}}\right)\right]$$

Quenched:

$$\frac{G(\lambda)}{\sqrt{T}} = \lambda\cdot\int_0^{\infty} dx\,[q(x,1) - q(x,0)] - \int_0^1 dt \int dx\; q\,(\partial_x p)^2$$

Annealed:

$$\frac{G(\lambda)}{\sqrt{T}} = \;''\;\; - F[q(x,0)]$$

$$\hookrightarrow \int dx \left\{ q(x,0)\ln\frac{q(x,0)}{P(x)} - q(x,0) + P(x)\right\}$$

Using the optimal equations show

$$q(\partial_x p)^2 = \partial_t (pq) - \partial_x \left(p\,\partial_x q - q\,\partial_x p - 2pq\,\partial_x p \right)$$

leads to

cancels

Quench:

$$\frac{G(\lambda)}{\sqrt{T}} = \lambda \int_0^\infty dx \left[q(x,1) - q(x,0) \right] - \int_{-\infty}^{\infty} dx \left[\underbrace{P(x,1)}_{\lambda\theta(x)}\, q(x,1) - P(x,0)\, q(x,0) \right]$$

$$= \int_0^\infty dx\, q(x,0) \left[p(x,0) - \lambda \right] + \int_{-\infty}^{0} dx \cdot p(x,0)\, q(x,0)$$

$$= \rho_b \int_0^\infty dx \cdot \ln \left[1 + (e^{-\lambda} - 1) \tfrac{1}{2} \mathrm{Ec}\left(\tfrac{x}{2}\right) \right]$$

$$+ \rho_a \int_0^\infty dx \ln \left[1 + (e^{\lambda} - 1) \tfrac{1}{2} \mathrm{Ec}\left(\tfrac{x}{2}\right) \right]$$

Annealed:

$$\frac{G(\lambda)}{\sqrt{T}} = \int_{-\infty}^{\infty} dx\, p(x,0)\, q(x,0) - \lambda \int_0^\infty dx\, q(x,0)$$

$$- \int_{-\infty}^{\infty} dx\, q(x,0) \ln \frac{q(x,0)}{P(x)} + \int_{-\infty}^{\infty} dx \left[q(x,0) - P(x) \right]$$

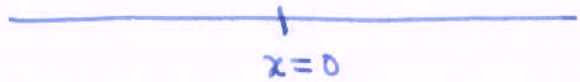
use the boundary condition

$$p(x,0) = + \lambda \theta(x) + \ln \frac{q(x,0)}{P(x)}$$

$$\Rightarrow \frac{G(\lambda)}{\sqrt{T}} = \int_{-\infty}^{\infty} dx \left[q(x,0) - P(x) \right] = \int_{-\infty}^{\infty} dx \cdot P(x) \left[e^{p(x,0) - \lambda\theta(x)} - 1 \right]$$

$$= \frac{1}{\sqrt{\pi}} \left[\rho_a (e^{\lambda} - 1) + \rho_b (e^{-\lambda} - 1) \right]$$

$\boxed{\text{Leave for Tutorial}}$: Microscopic derivation



Step 1

$$\langle e^{\lambda Q_T}\rangle = \prod_i \left\langle e^{\lambda\,\Theta(x_i(T))}\right\rangle_{Y_i<0} \prod_j \left\langle e^{-\lambda\,\Theta(-x_j(T))}\right\rangle_{Y_j<0}$$

Step 2

$$\left\langle e^{\lambda\,\Theta(x_i(T))}\right\rangle_{Y_i<0} = e^{\lambda}\int_0^{\infty} dx\, \frac{e^{-\frac{(x-Y_i)^2}{4T}}}{\sqrt{4\pi T}} + \int_{-\infty}^{0} dx \,\cancel{*} = \frac{1}{2}\,\mathrm{Erfc}\left(\frac{-Y_i}{2\sqrt{T}}\right)e^{\lambda} + \frac{1}{2}\,\mathrm{Erfc}\left(\frac{Y_i}{2\sqrt{T}}\right)$$

$$= \left[1+(e^{\lambda}-1)\tfrac{1}{2}\,\mathrm{Erfc}\left(\frac{-Y_i}{2\sqrt{T}}\right)\right]$$

$$\left\langle e^{-\lambda\,\Theta(-x_i(T))}\right\rangle_{Y_i>0} = \left[1+(e^{-\lambda}-1)\tfrac{1}{2}\,\mathrm{Erfc}\left(\frac{Y_i}{2\sqrt{T}}\right)\right]$$

Step 3:

$$\log\langle e^{\lambda Q_T}\rangle = \sum_i \log\left[1+(e^{\lambda}-1)\tfrac{1}{2}\,\mathrm{Ec}\left(\frac{-Y_i}{2\sqrt{T}}\right)\right] + \sum_i \ln\left[1+(e^{-\lambda}-1)\tfrac{1}{2}\,\mathrm{Ec}\left(\frac{Y_i}{2\sqrt{T}}\right)\right]$$

Quenched:

$$= \int_{-\infty}^{0} dY\, P(Y)\log\left[1+(e^{\lambda}-1)\tfrac{1}{2}\,\mathrm{Ec}\left(\frac{-Y}{2\sqrt{T}}\right)\right] + \int_0^{\infty} dY\, P(Y)\log\left[\qquad\right]$$

$$\Rightarrow \frac{G(\lambda)}{\sqrt{T}} = \rho_a \int_0^\infty dy \ln\left[1+(e^{\lambda}-1)\tfrac{1}{2}\mathrm{Ec}(y/2)\right] + \rho_b \int_0^\infty dy \ln\left[1+(e^{-\lambda}-1)\tfrac{1}{2}\mathrm{Ec}(y/2)\right]$$

<u>Annealed</u>: Prob of $\begin{cases} \text{1 part} & \rho\, dy \\ 0\ " & (1-\rho\, dy) \end{cases}$

$$\left\langle \left\langle e^{\lambda\theta(x_i)} \right\rangle_{Y_i<0} \right\rangle = 1-\rho(y)dy + \rho(y)dy\left[1+(e^{\lambda}-1)\tfrac{1}{2}\mathrm{Erfc}\left(\frac{-Y_i}{2\sqrt{T}}\right)\right]$$

$$= 1 + dy\cdot\rho(y)(e^{\lambda}-1)\tfrac{1}{2}\mathrm{Ec}(\ )$$

$$\simeq e^{dy\cdot\rho(y)(e^{\lambda}-1)\frac{1}{2}\mathrm{Ec}(\ )}$$

$$\Rightarrow \frac{G(\lambda)}{\sqrt{T}} = \int_{-\infty}^{0} dy\, \rho(y)(e^{\lambda}-1)\tfrac{1}{2}\mathrm{Ec}\left(\frac{-y}{2}\right) + \int_0^\infty dy\, \rho(y)(e^{-\lambda}-1)\tfrac{1}{2}\mathrm{Ec}(y/2)$$

Tagged particle in single-file

* An example where MFT led to results directly tested in experiment.

* Observable

$$\int_0^\infty dx\, q(x,0) = \int_{Y_T}^\infty dx\, q(x,T)$$

$$\Rightarrow Y_T = Y_T\left[q(x,T), q(x,0)\right]$$

* Very similar formalism, little more detailed algebra.

Results:

$$P\left(\frac{Y_T}{\sqrt{T}} = y\right) \asymp e^{-\sqrt{T}\,\phi(y)} \equiv \ln\langle e^{\lambda Y_T}\rangle \sim \sqrt{T}\, G(\lambda)$$

Where: Quench:

$$G(\lambda) = 2\rho \int_0^\infty dy \cdot \ln\left[1 + \sinh^2\left(\frac{\lambda}{2\rho}\right) Ec(y)\, Ec(-y)\right]$$

Anneal:

$$G(\lambda) = \frac{\rho}{4}\left(\sqrt{h(y)} - \sqrt{h(-y)}\right)^2 \cdot \left[Erf(y)\sqrt{\frac{h(-y)}{h(y)}} + E(-y) \right]$$

$$\lambda =$$

Ref: Krapivsky, Mallick, Sadhu (2015), 160, 885

Microscopic solution

Solution for general system

A series expansion:

By definition:

$$\frac{G(\lambda)}{\sqrt{T}} = \lambda \langle Q_T \rangle + \frac{\lambda^2}{2!} \langle Q_T^2 \rangle_c + \frac{\lambda^3}{3!} \langle Q^3 \rangle_c + \cdots$$

Quench:

$$\frac{G(\lambda)}{\sqrt{T}} = \lambda \int_0^\infty dx \left\{ q_\lambda(x,1) - q_\lambda(x,0) \right\} - \int_0^1 dt \int dx \, \frac{\sigma(q_\lambda)}{2} (\partial_x p_\lambda)$$

optimal equation

$$\dot{q}_\lambda - \partial_x \left[D(q_\lambda) \partial_x q_\lambda \right] = - \partial_x \left[\sigma(q_\lambda) \partial_x p_\lambda \right]$$

$$\dot{p}_\lambda + D(q_\lambda) \partial_{xx} p_\lambda = - \frac{\sigma'(q_\lambda)}{2} (\partial_x p_\lambda)^2$$

Boundary

$$q_\lambda(x,0) = \rho(x) \quad \text{with} \quad p_\lambda(x,1) = \lambda \, \theta(x)$$

- For $\lambda = 0$:

$$\dot{q}_0 - \partial_x \left[D(q_0) \partial_x q_0 \right] = 0$$

$$p_0 = 0$$

Expansion around $\lambda = 0$

$$q_\lambda = q_0 + \lambda q_1 + \lambda^2 q_2 + \cdots$$

$$p_\lambda = \lambda p_1 + \lambda^2 p_2 + \cdots$$

Strategy: Solve the optimal equations order by order.

Simple case : $\rho_a = \rho_b = \rho$

$$\Rightarrow \boxed{q_0(x,t) = \rho}$$

The Action

$$\frac{G(\lambda)}{\sqrt{T}} = \underbrace{\lambda \cdot \int_0^\infty dx \left[q_0(x,1) - q_0(x,0) \right]}_{\substack{0 \\ (\text{symmetry})}}$$

$$+ \lambda^2 \left\{ \int_0^1 dx \left[q_1(x,1) - q_1(x,0) \right] - \int_0^1 dt \int dx \, \frac{\sigma(\rho)}{2} (\partial_x P_1) \right\}$$

$$+ \cdots$$

$$\Rightarrow \frac{\langle Q_T^2 \rangle}{\sqrt{T}} = 2 \left[\int_0^1 dx \left[q_1(x,1) - \underbrace{q_1(x,0)}_{0} \right] - \int_0^1 dt \int dx \, \frac{\sigma(q)}{2} (\partial_x P_1) \right]$$

Explicit calculation :

$$\dot{q}_1 - D(\rho)\, \partial_{xx} q_1 = - \partial_x \left[\sigma(\rho)\, \partial_x P_1 \right]$$

$$\dot{P}_1 + D(\rho)\, \partial_{xx} P_1 = 0$$

Boundary $\quad q_1(x,0) = 0 \quad ; \quad P_1(x,1) = \Theta(x)$

Solution :

$$P_1(x,t) = \int dz \, g(z,1|x,t)\, \Theta(z)$$

where $\quad g(z,1|x,t) = \dfrac{e^{-\frac{(z-x)^2}{4(1-t)D}}}{\sqrt{4\pi D(1-t)}}$

$$\Rightarrow \boxed{P_1(x,t) = \frac{1}{2} \, \mathrm{Erfc}\left(\frac{-x}{2\sqrt{D(1-t)}} \right)}$$

Solution for $q_1(x,t)$: (A trick)

$$q_1(x,t) = -\partial_x \Psi(x,t)$$

If we know Ψ we know q_1

$$\dot{\Psi} - D(\rho)\,\partial_{xx}\Psi = \sigma(\rho)\,\partial_x P_1$$

Boundary $\Psi(x,0) = \text{constant}$

$= 0$ (choice)

$\partial_x P_1$ is $\neq 0$ only at $x \to \infty$ $\Rightarrow \Psi = 0$ at $x \to \infty$

Solution:

$$\Psi(x,t) = \int_0^t dt' \int dz\; g(x,t\,|\,z,t')\,\sigma(\rho)\,\partial_z P_1(z,t')$$

Step 1:

$$\int_0^\infty dx\, q_1(x,1) = -\int_0^\infty dx\, \partial_x \Psi(x,1) = \Psi(0,1) - \underbrace{\Psi(\infty,1)}_{0}$$

$$\frac{\sigma(\rho)}{2}\int_0^1 dt \int dx\,(\partial_x P_1)^2 = \frac{\sigma(\rho)}{2}\int_0^1 dt$$

$$= \frac{\Psi(0,1)}{2}$$

leading to

$$\frac{\langle Q_T^2\rangle}{\sqrt{T}} = 2\cdot\left[\Psi(0,1) - \frac{\Psi(0,1)}{2}\right]$$

$$= \Psi(0,1)$$

Step 3: $\Psi(0,1) = \sigma(\rho)\int_0^1 dt \int dz\; g(0,t\,|\,z,t)\cdot \partial_z P_1(z,t)$

$$\Rightarrow \boxed{\frac{\langle Q_T^2\rangle}{\sqrt{T}} = \frac{\sigma(\rho)}{\sqrt{2\pi D(\rho)}}}$$

For Anneal:

$$\frac{\langle Q_T^2 \rangle}{\sqrt{T}} = 2 \cdot \frac{\sigma(\rho)}{\sqrt{2\pi D(\rho)}}$$

Similarly one can solve for higher cumulants.

Remark: Tagged particle problem

Quench: $\langle X_T^2 \rangle = \frac{\sqrt{T}}{\rho^2} \cdot \frac{\sigma(\rho)}{\sqrt{2\pi D(\rho)}}$

$$= \frac{\sqrt{T} \cdot \sqrt{D}}{\sqrt{2\pi}} \cdot \left[\frac{\sigma(\rho)}{\rho^2 D} \right] \hookrightarrow \frac{2}{\beta} \chi(\rho)$$

$$\Rightarrow \langle Y_T^2 \rangle = \underbrace{\left[\sqrt{\frac{2 D(\rho)}{\pi} \cdot \frac{\chi(\rho)}{\beta}} \right]}_{\mathcal{D}} \sqrt{T}$$

[obtained using Mode coupling]

Kollman

Anneal: $\boxed{\langle Y_T^2 \rangle = 2 \mathcal{D} \sqrt{T}}$

Open problem full solution not known.

SEP: Krapivsky, Mallick, Sadhu.

$\boxed{\text{Multitime statistics}}$:

Other problems (open)

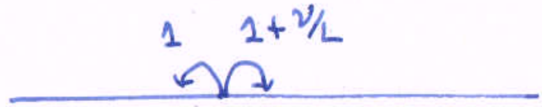
⊛ Multi-time correlation :

$$\left\langle e^{\int dx \int dt\, \lambda(x,t)\, \Phi(x,t)} \right\rangle$$

Tagged particle

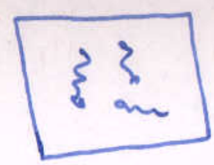
⊛ KPZ equation at short-time

⊛ Bulk-driven systems : what is the fluctuating hydrodynamics?



$$j(x,t) = -D\,\partial_x q + \sigma(q)\,E + \eta$$ not-fully developed.

⊛ Active matter : Large-deviation.



Activated Brownian particle.

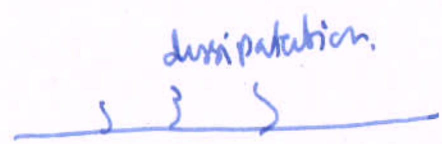
$$P[\rho(x)] \asymp e^{-L\,\phi[\rho(x)]}$$

⊛ Higher-dimension.

⊛ Phase-transitions in terms of large-deviations !

⊛ More-symmetry properties?

⊛ Non-conserved models



⊛ Multiple conserved quantity : ABC model.

⊛ Non-linear fluctuating hydrodynamics and anomalous transport.

Many other problems

1 KPZ equation:

$$\boxed{\partial_t h = D\,\partial_{xx} h + \frac{\lambda}{2}(\partial_x h)^2 + \eta(x,t)}$$

where $\langle \eta(x,t)\,\eta(y,t')\rangle = 2\sigma\,\delta(t-t')\,\delta(x-y)$

Exact solution: 1 Discrete models in the same universality class [Sasamoto, Spohn

2 Directed polymer in random medium

[Calabrese, Le Doussal]

What can MFT do? Small parameter!

$$P\left(h(0,T)\right) = ?$$

Rescale

$$t \to t\cdot T \;;\; x \to x\sqrt{TD} \;;\; h \to \frac{D}{\lambda} h$$

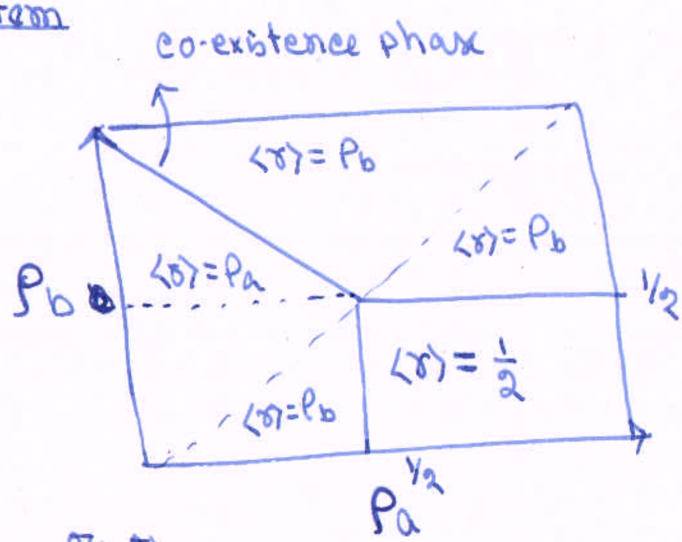
$$\Rightarrow \frac{1}{T}\cdot\frac{D}{\lambda}\cdot\partial_t h = D\cdot\frac{1}{TD}\cdot\frac{D}{\lambda}\cdot\partial_{xx} h + \frac{\lambda}{2}\cdot\frac{1}{TD}\cdot\frac{D^2}{\lambda^2}\cdot(\partial_x h)^2 + \eta$$

$$\Rightarrow \partial_t h = \partial_{xx} h + \frac{1}{2}(\partial_x h)^2 + \underbrace{\left[\frac{T\lambda}{D}\,\eta\right]}_{\xi}$$

$$\left[\quad \frac{D^2}{T^2\lambda^2}\cdot\langle \xi(x,t)\,\xi(y,t')\rangle = 2\sigma\cdot\frac{1}{T}\cdot\frac{1}{\sqrt{DT}}\cdot\delta(x-x')\,\delta(t-t')\right.$$

$$\Rightarrow \langle \xi(x,t)\,\xi(y,t')\rangle = \left[\frac{2\sigma\cdot\lambda^2}{D^{5/2}}\cdot\sqrt{T}\right]\delta(x-x')\,\delta(t-t')$$

Non-equilibrium biased system



$$h(\gamma, F, \rho) = \gamma \ln \frac{\gamma}{F} + (1-\gamma)\ln\frac{1-\gamma}{1-F} + \ln\frac{F(1-F)}{\rho(1-\rho)}$$

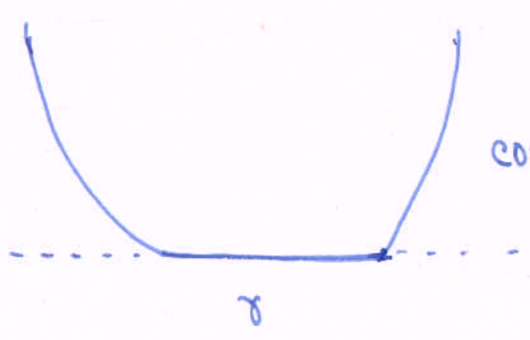
For $\rho_a > \rho_b$:

$$\phi[\gamma] = \max_{F(x)} \int_0^1 dx\, h(\gamma(x), F(x), \rho)$$

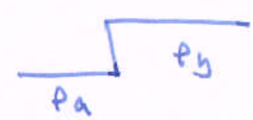
For $\rho_b > \rho_a$:

$$\phi[\gamma] = \min_{0\le y\le 1} \left\{ \int_0^y dx\, h(\gamma(x), \rho_a, \rho) + \int_y^1 dx\, h(\gamma(x), \rho_b, \rho) \right\}$$

Along shock:



coexistence



Ref: Derrida, Lebowitz, Speer (2002). PRL, 89, 030601.