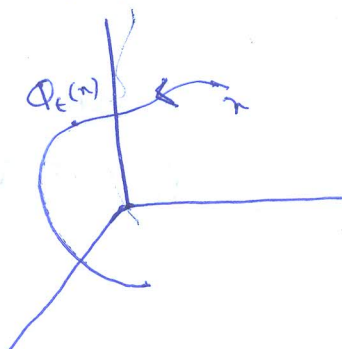


Classical System  
(N particles obeying Hamiltonian Dynamics)

Microstates  $\rightarrow$  sets  $\{ \vec{r}_i, \vec{p}_i \}_{i=1 \text{ to } N}$

dN dimensional phase space. Microstate  $x$ , point in space

$\Phi_t(x)$  - trajectory.  $\Phi_0(x) = x$



~~Ergodicity~~ Some function (property)  $f$   
 $f^+(x) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T f(\Phi_t(x)) dt$  - infinite time average

Ensemble average

$$\langle f \rangle = \int_{\Omega} f(x) dx / \int_{\Omega} dx ; \Omega = \text{constant energy shell}$$

Ergodic Hypothesis

$f^+(x) = \langle f \rangle$   $\forall$  integrable  $f$  and "almost all"  $x$ .

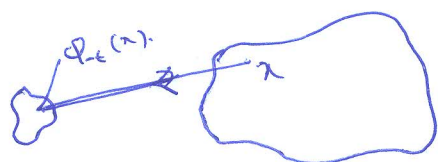
$\int_M dx = 0$ ,  $\forall x \in M$  for which the Hypothesis is not true.

Ergodic Hypothesis: Amount of time spent in a region  $\propto$  volume of region

Exercise: 1D Harmonic Oscillator  $H = \frac{\omega}{2} (p^2 + q^2)$  is ergodic.

Liouville's theorem: Phase space density

$$p_t(x) = p_0[\Phi_{-t}(x)] \quad \forall t \text{ and } \forall x \in \Omega.$$



Why are  $f^*$  and  $\langle f \rangle$  useful?  $\langle f \rangle$  can be calculated easily

$f^* \rightarrow$  time average. Measurements take finite time

$$f_{obs}^{(x)} = \frac{1}{\tau} \int_0^{\tau} f(\Phi_t(x)) dt \quad \text{Is } f_{obs}^{(x)} = f^*(x)? \quad \tau \neq \infty$$

How large should  $\tau$  be to effectively be  $\infty$ , i.e.

How much time does  $x$  take to sample all  $\Omega$ ?

(Poincaré recurrence time).

$N$  particles  $\tau \sim e^N \tau_0$ .  $\tau_0$  - microscopic time

$N = 10^{23} \Rightarrow \tau \sim e^{10^{23}} \tau_0 \gg$  age of the universe

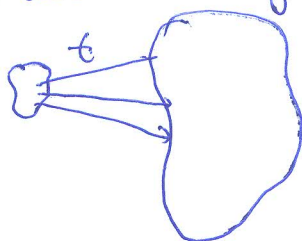
for any reasonable  $\tau_0$

We need  $\lim_{t \rightarrow \infty} f[\Phi_t(x)] \rightarrow \langle f \rangle \quad \forall$  almost all  $x$ .

How can we justify this?

Approach 1: Gibbsian or Ensembleist approach

Start with density  $p_0(x)$  and not single point



and say " $f[\Phi_t(x)] \rightarrow f^{mc}$ " where  $f^{mc} = \frac{1}{\int_{\Omega} dx}$

