Rising bubbles in a liquid column

SUMMER SCHOOL
AND
DISCUSSION MEETING ON BUOYANCY-DRIVEN FLOWS

by
Rajaram Lakkaraju
Department of Mechanical Engineering
Indian Institute of Technology Kharagpur

Acknowledgement
INSPIRE-DST, Govt. India

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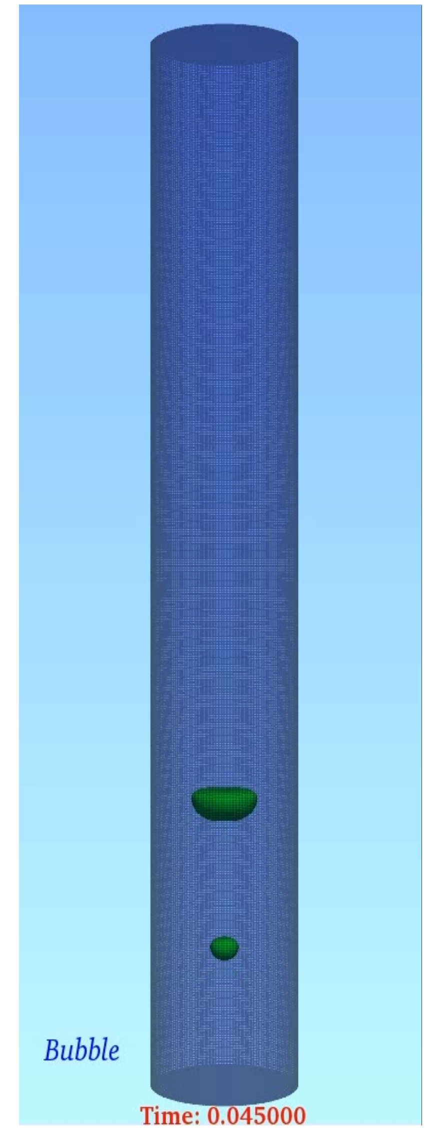

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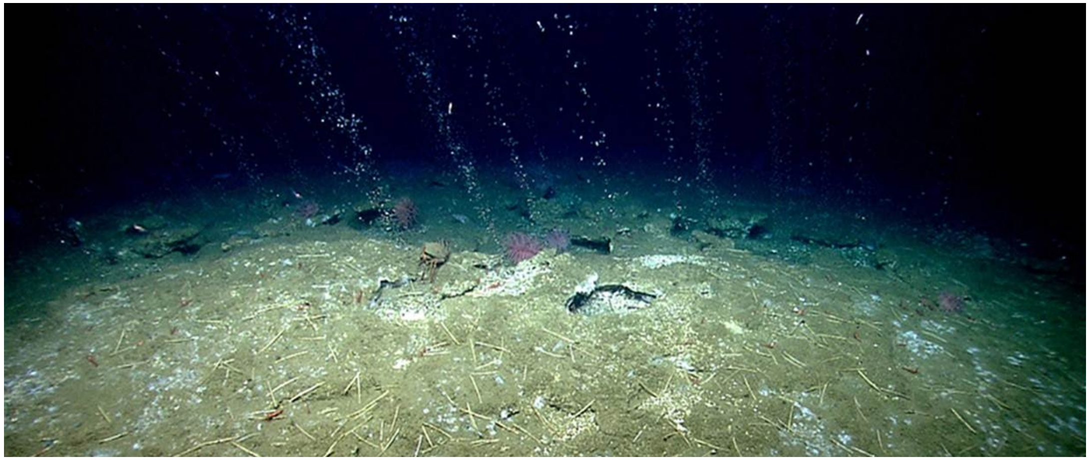
Where do we see bubbles?



Methane streams in oceans-climatic change

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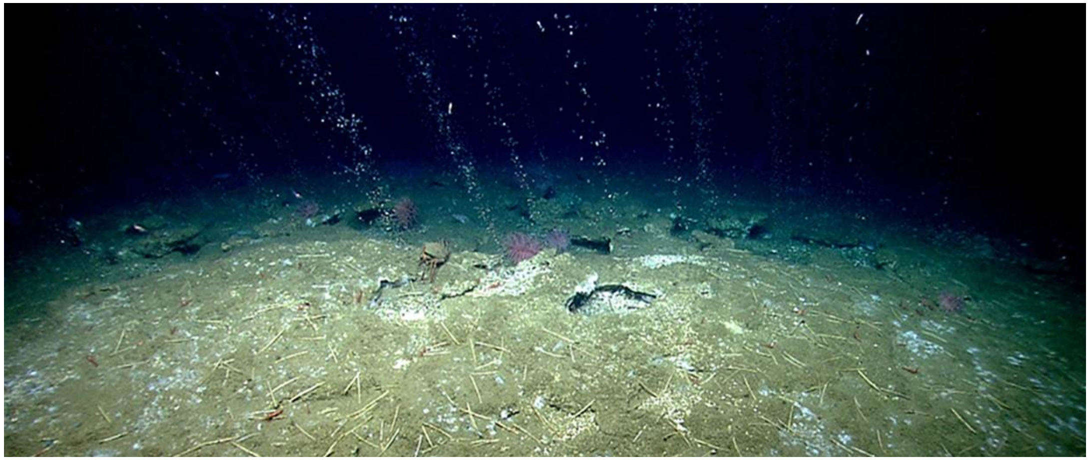

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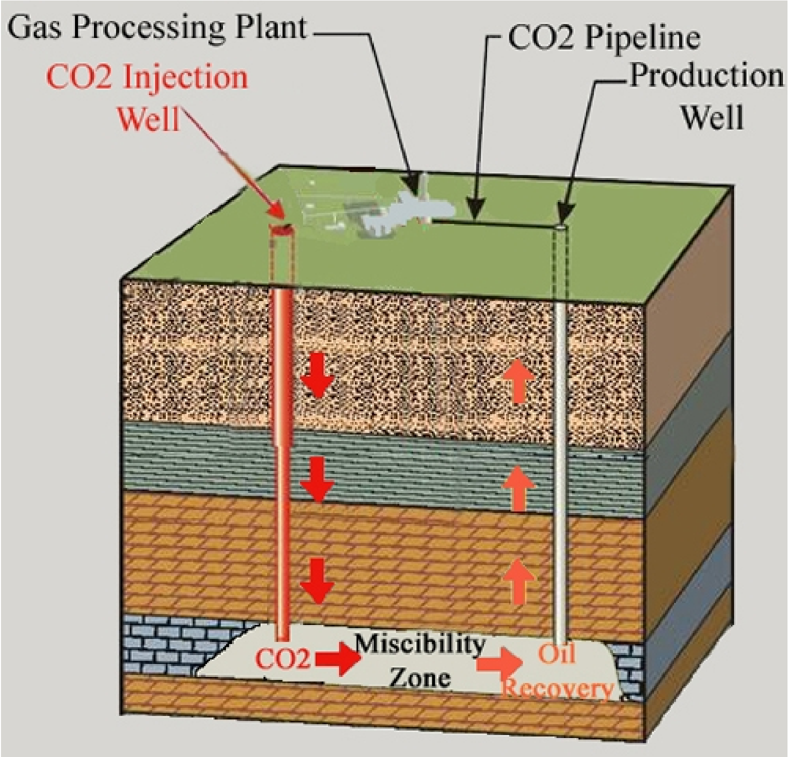

Enhanced oil recovery

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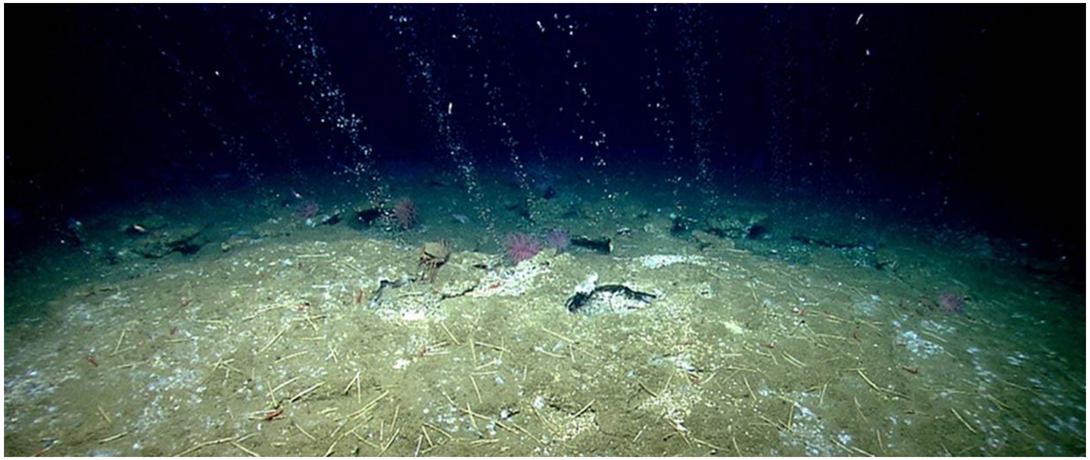

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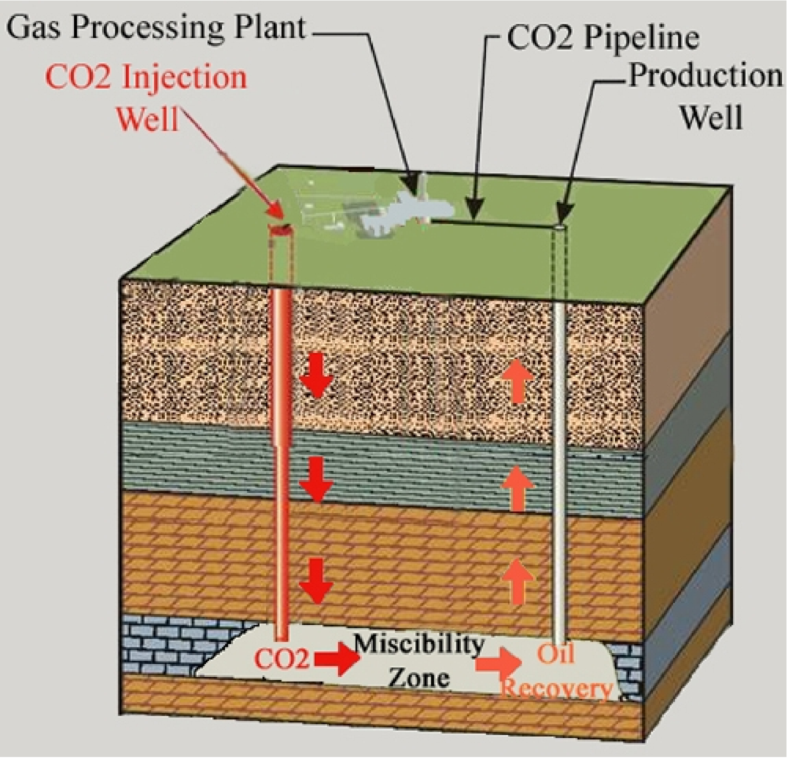

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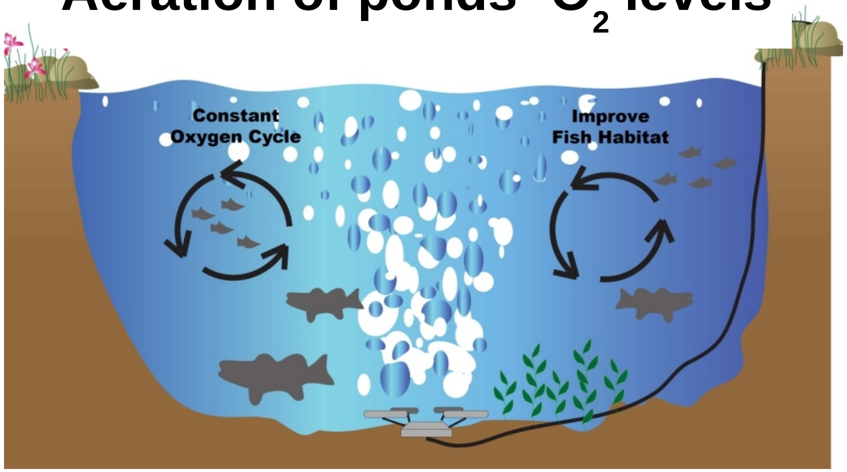
Aeration of ponds- O_2 levels



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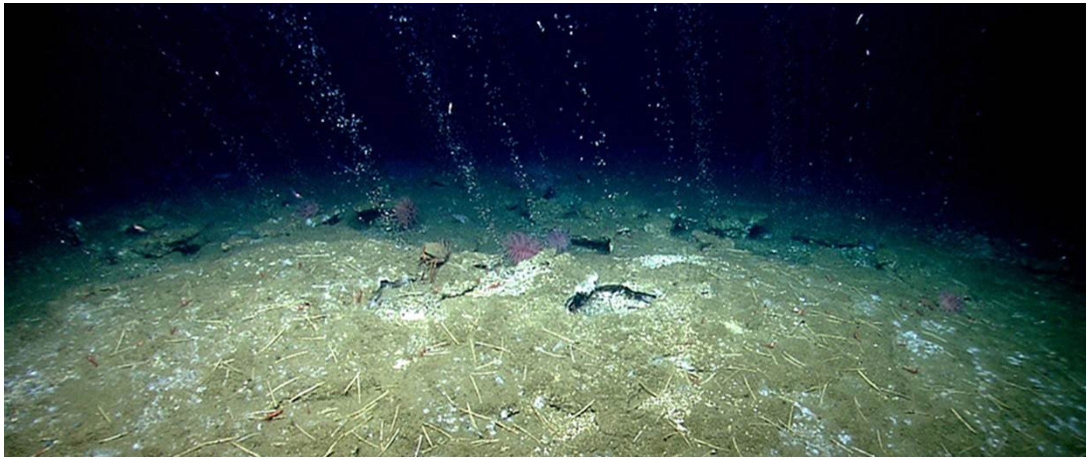

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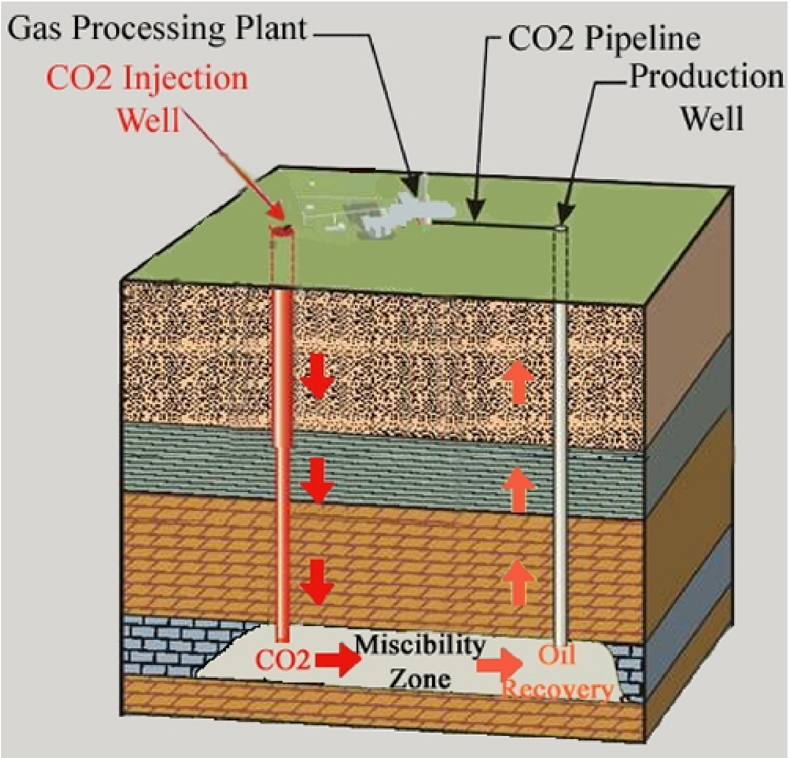

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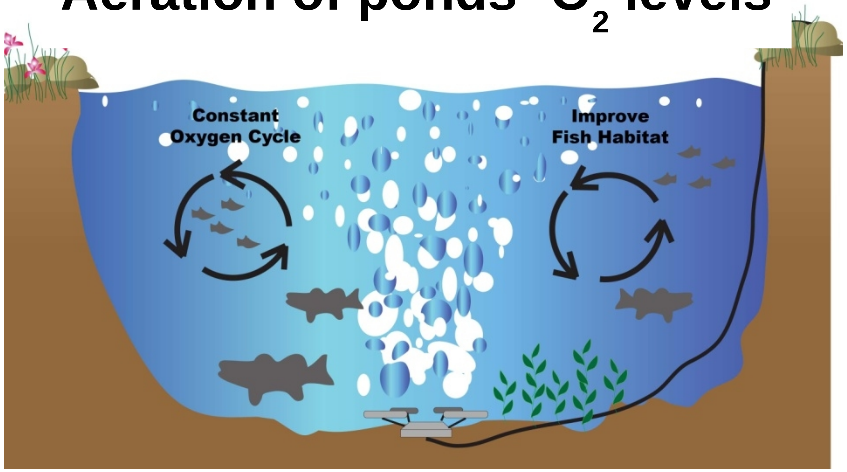
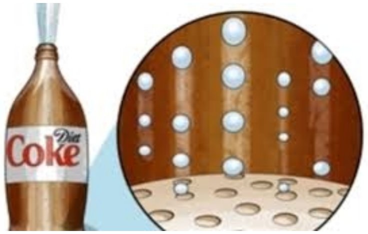


Drinks



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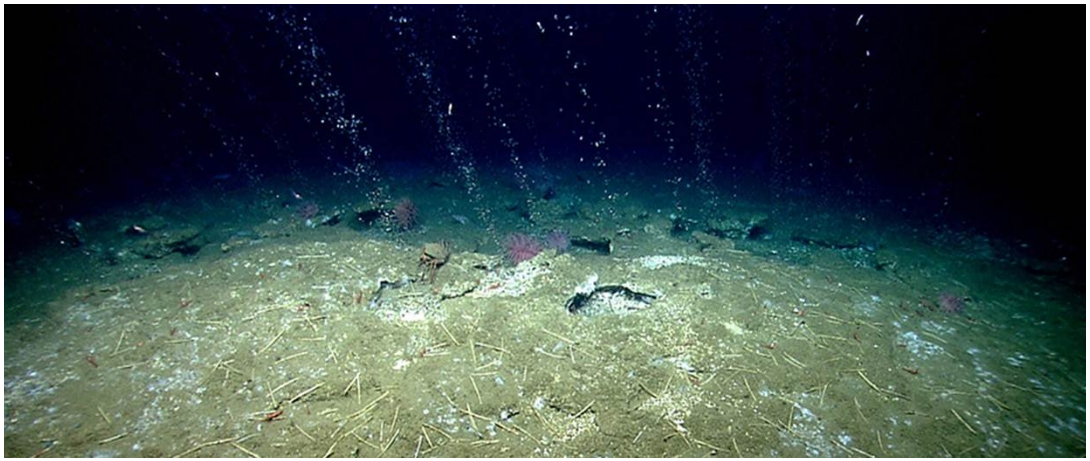

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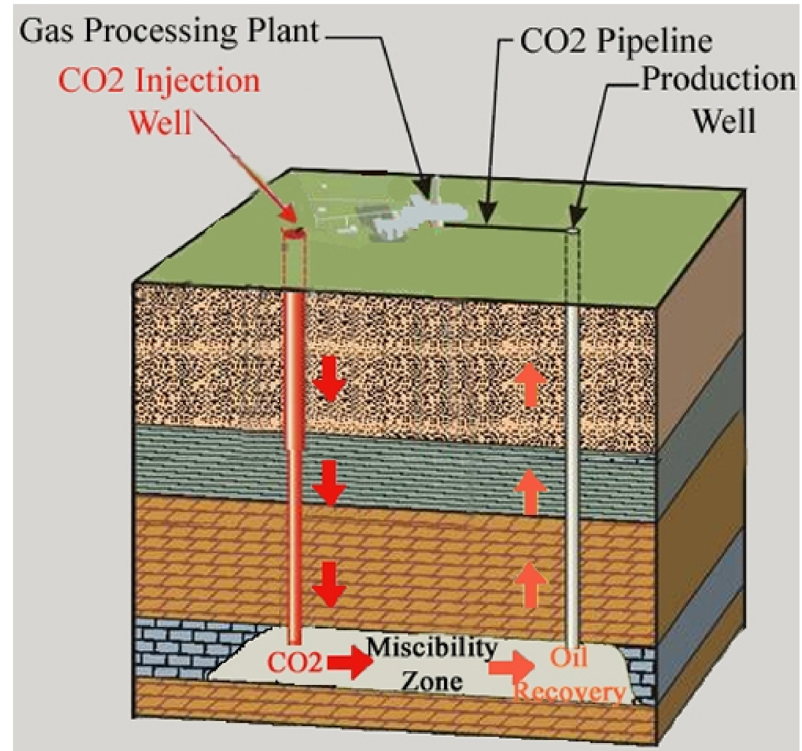

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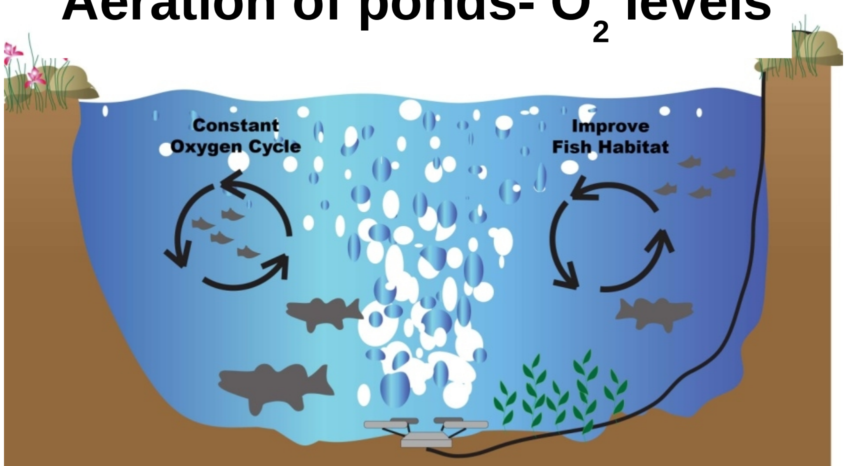

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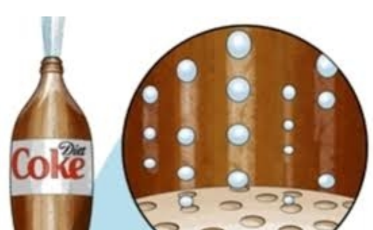

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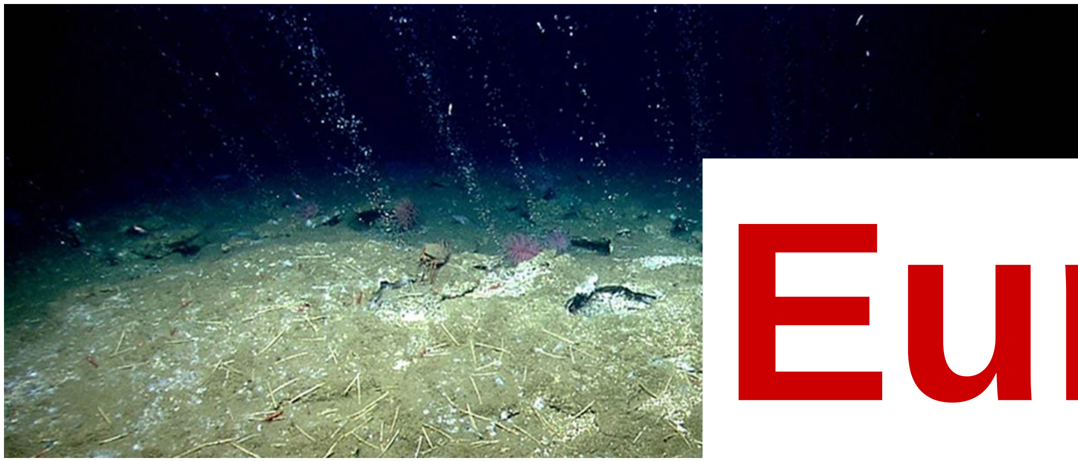

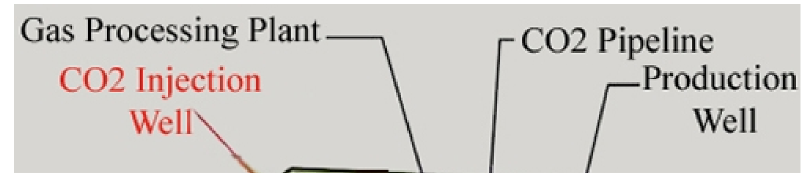

Eureka!
It is RBC...

Methane streams in ocea

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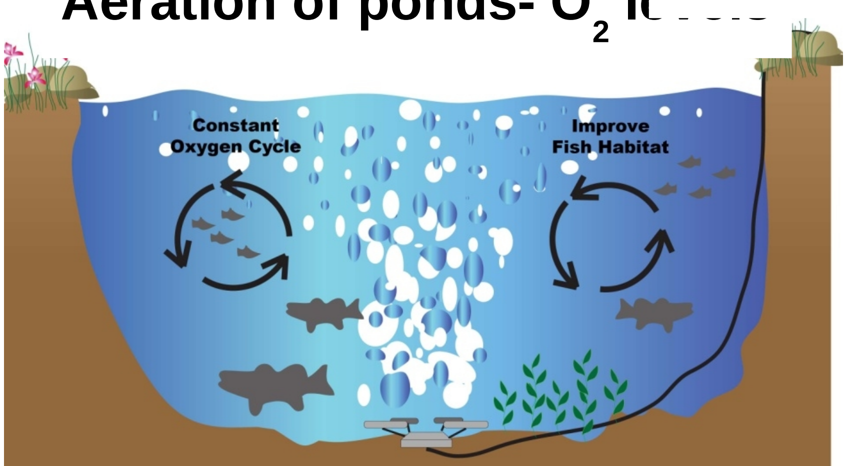

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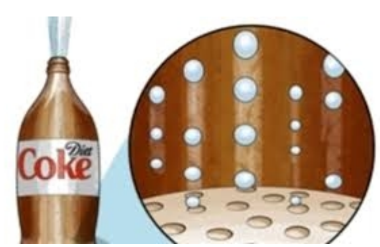

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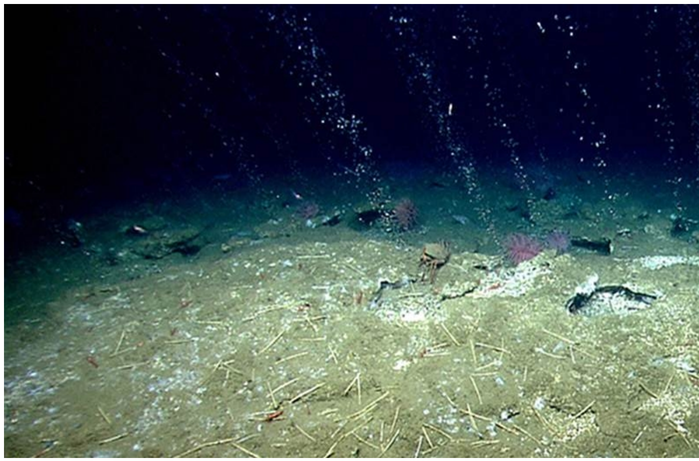

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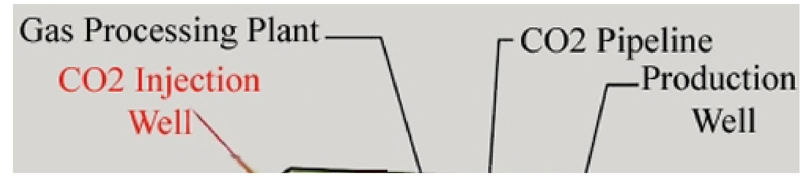

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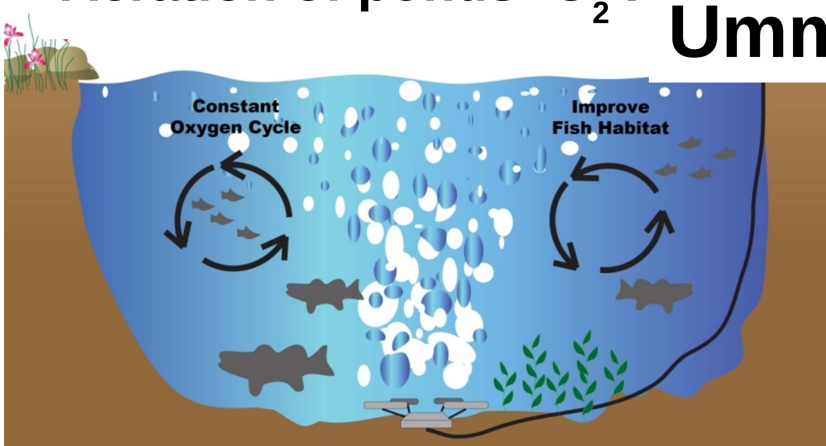

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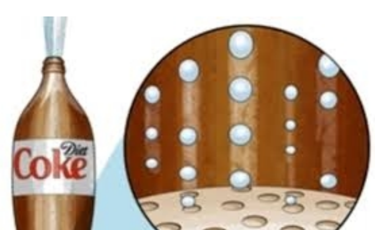

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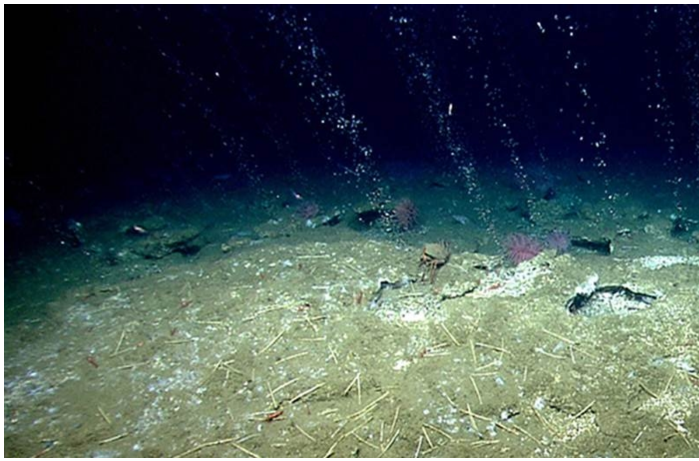

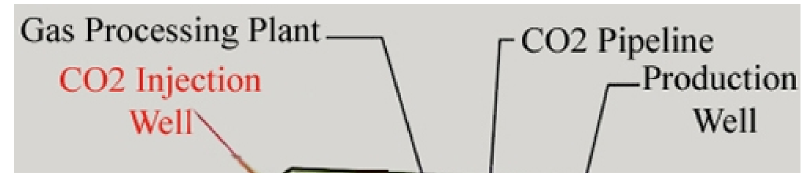

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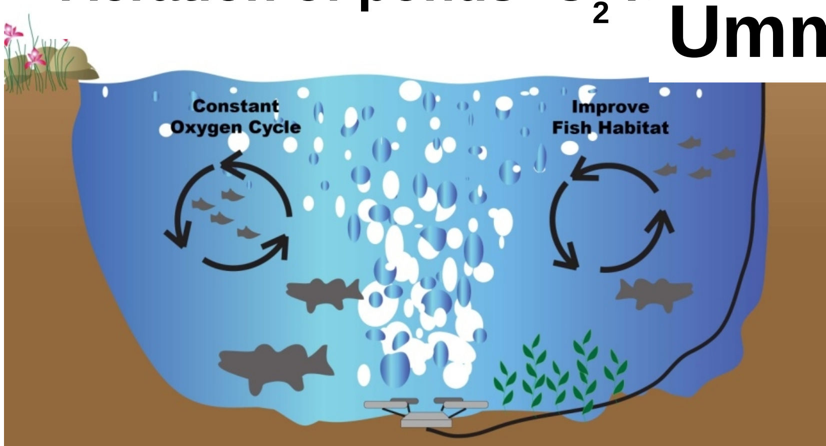

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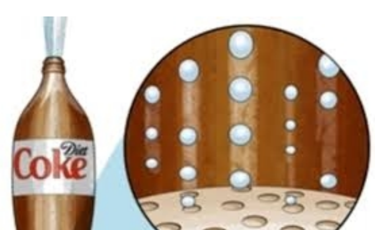

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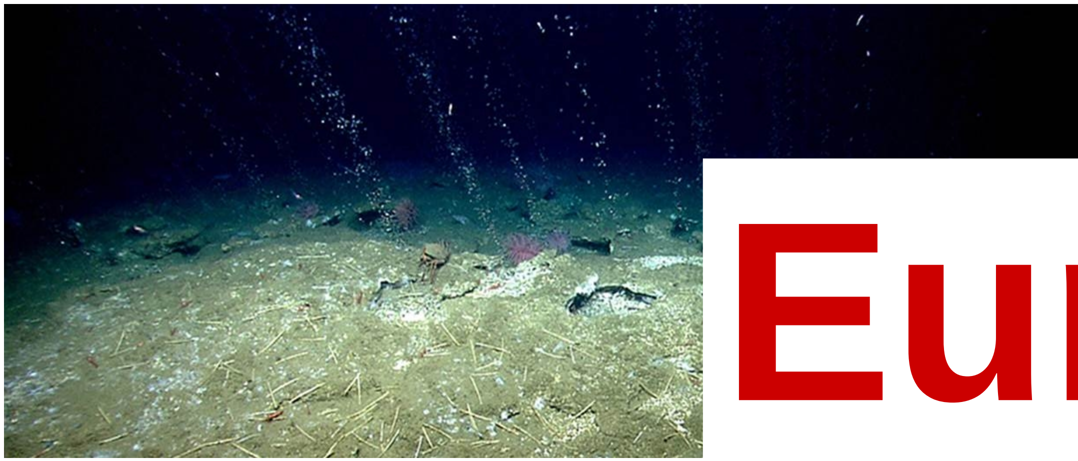

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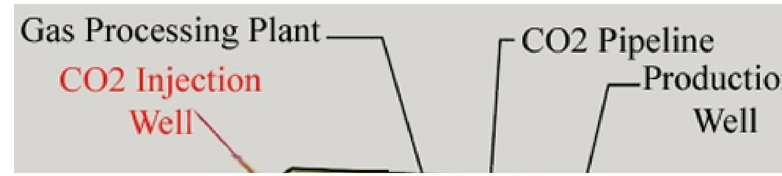

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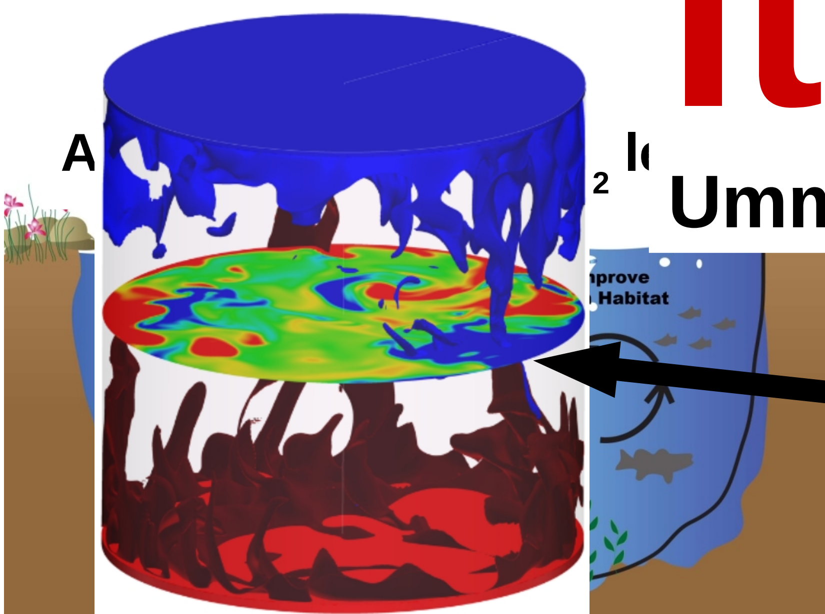
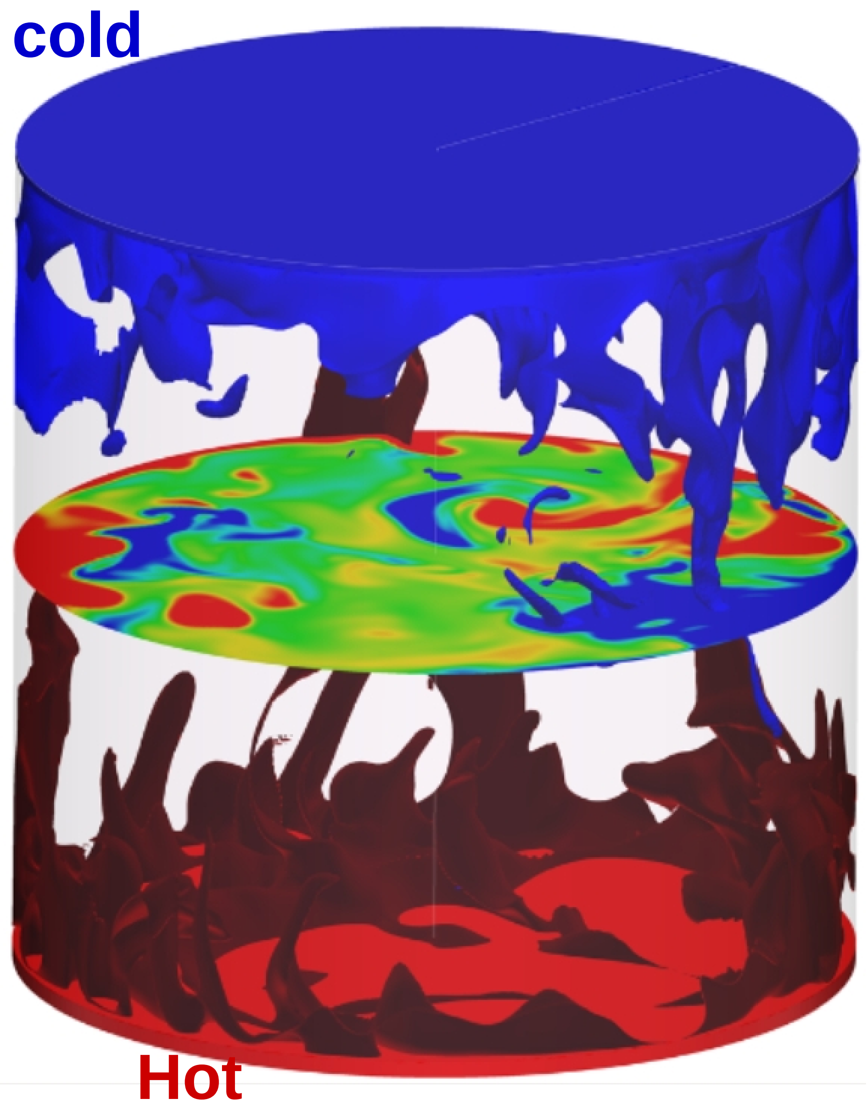


Drinks

How we started into this

Initially began with traditional RBC



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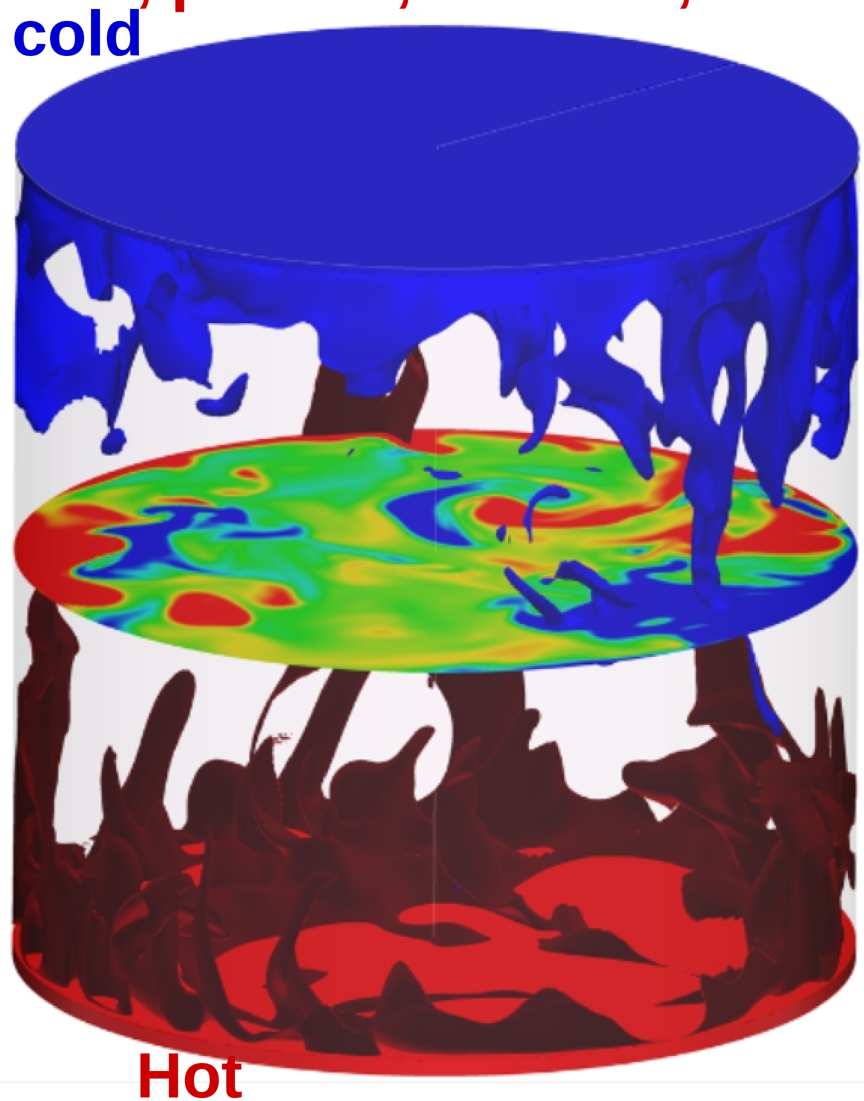
Initially began with traditional RBC...

LSC, plumes, heatflux, structure functions...what not...



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How about boiling RBC?

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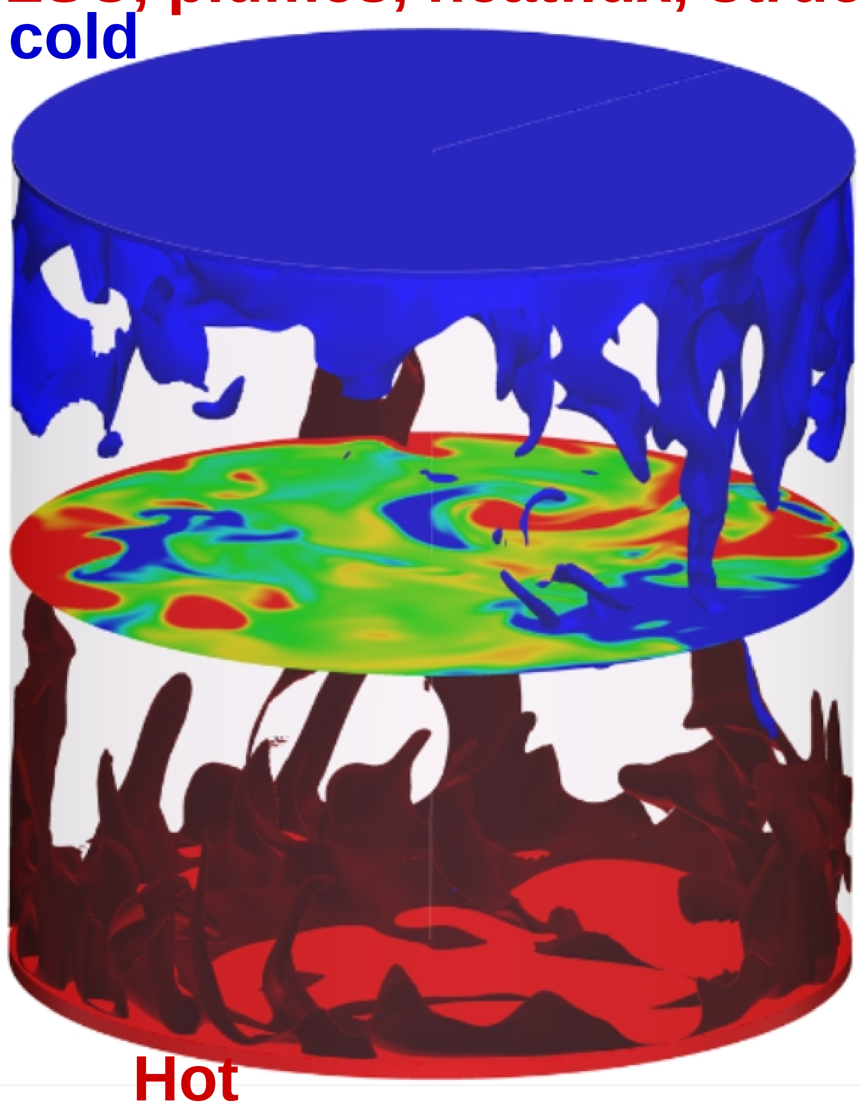

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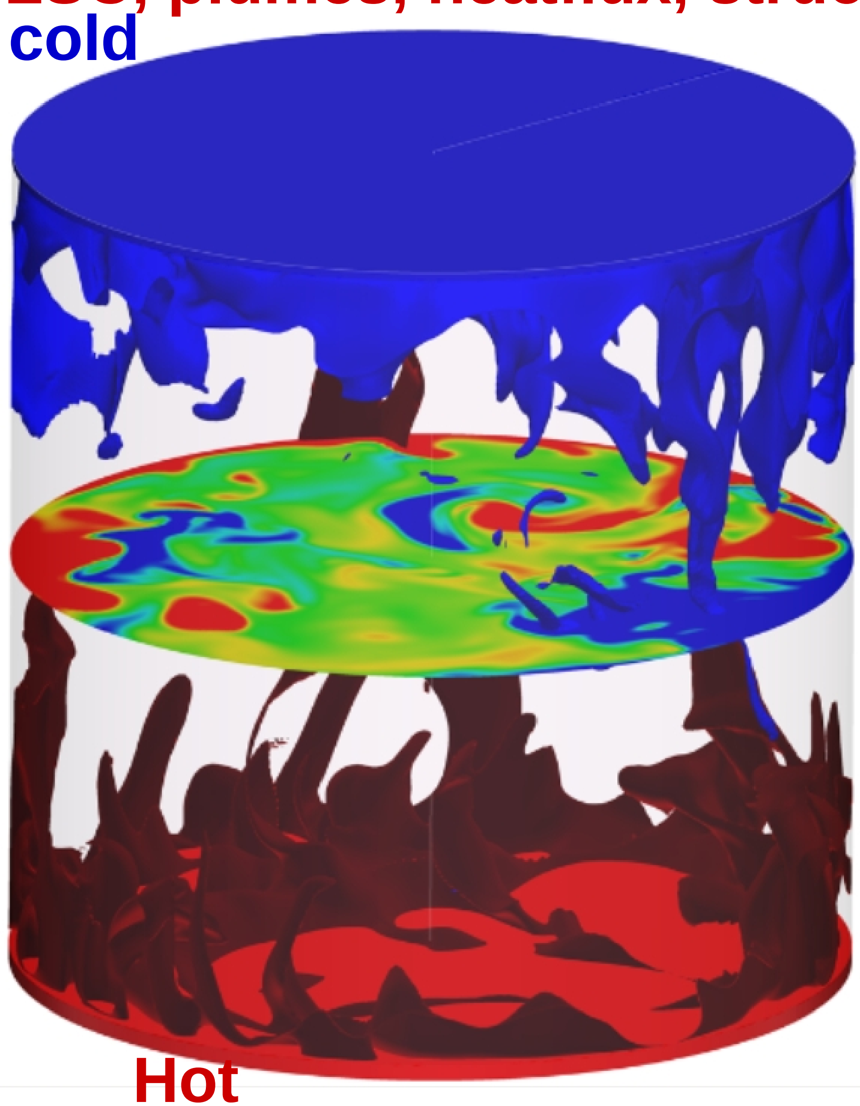

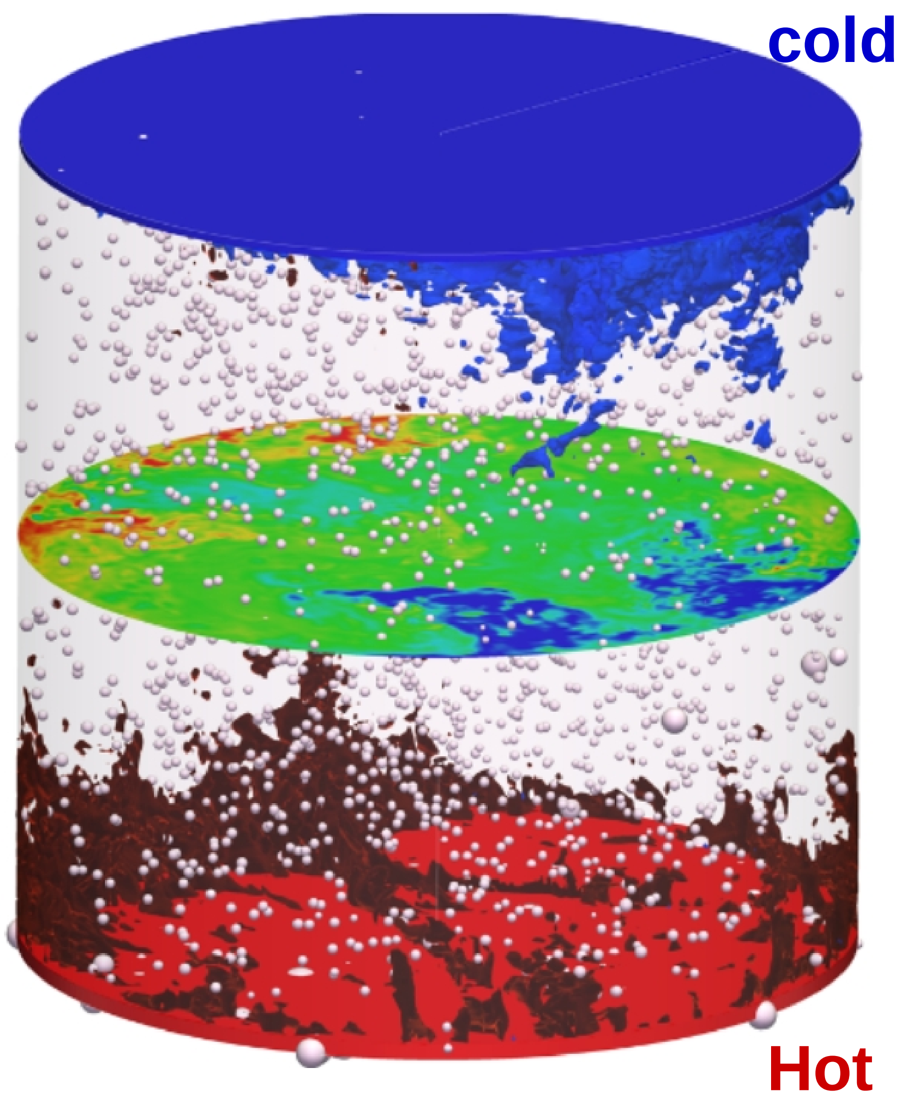

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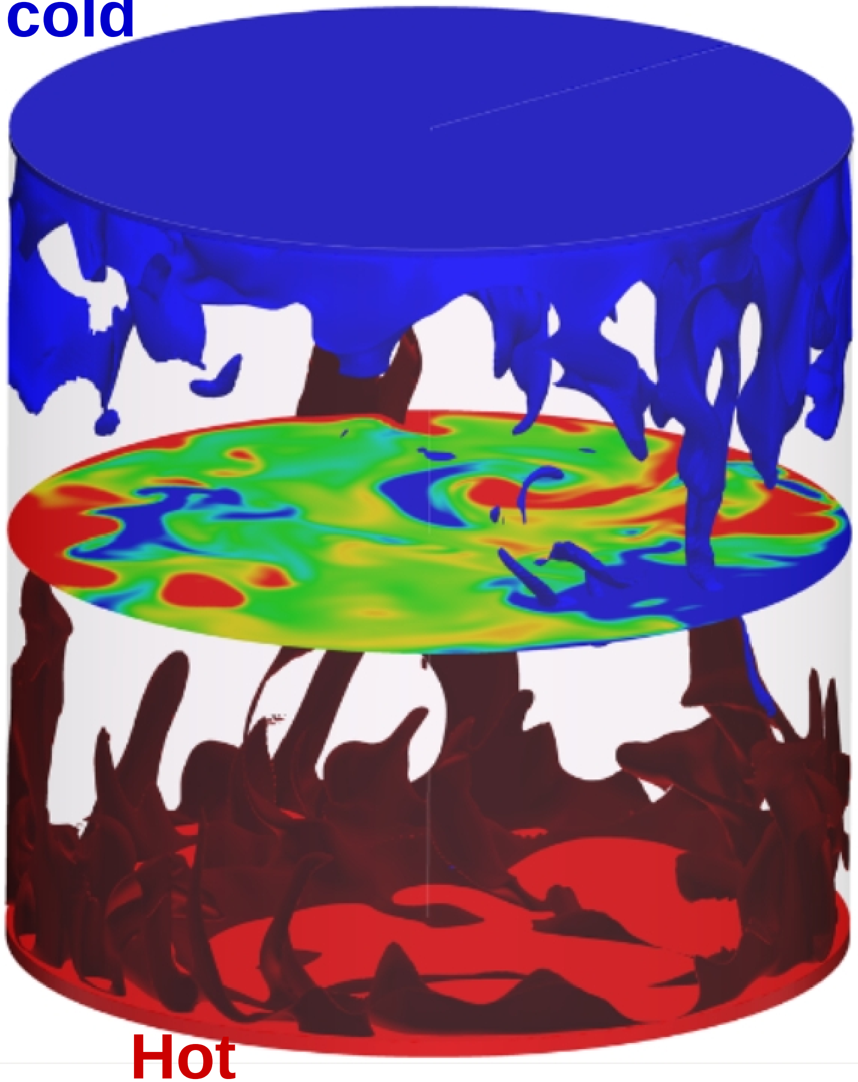

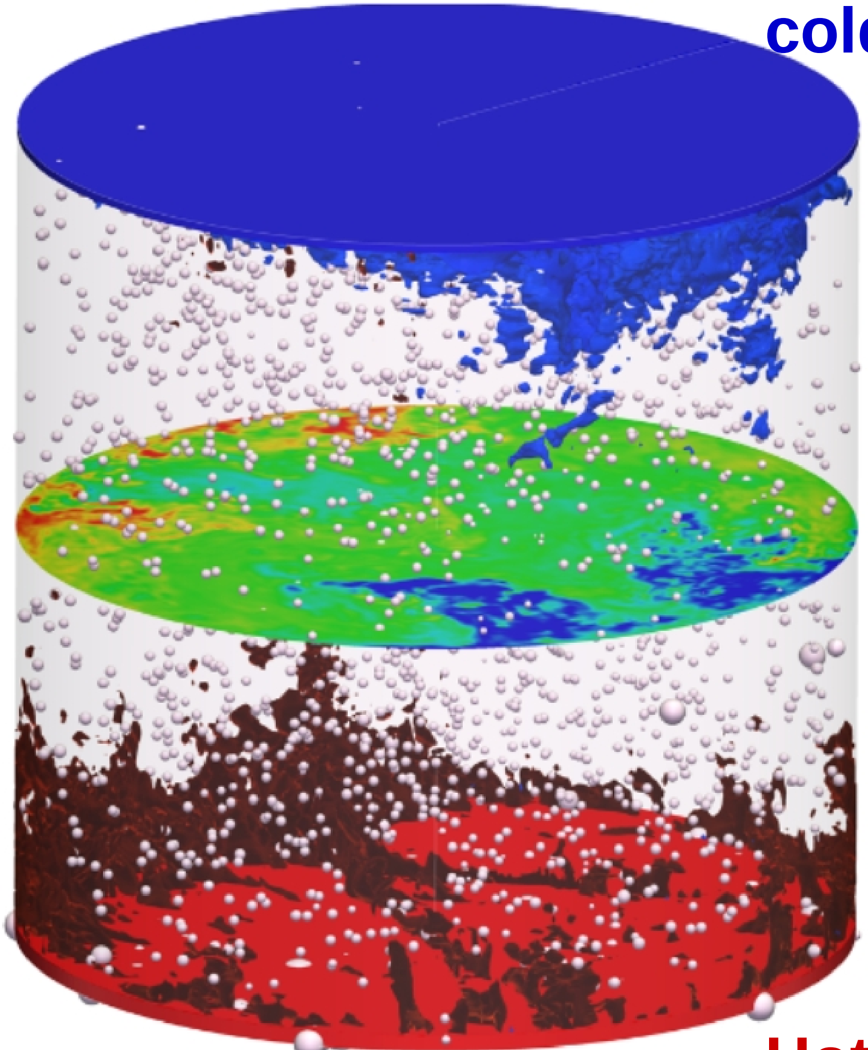

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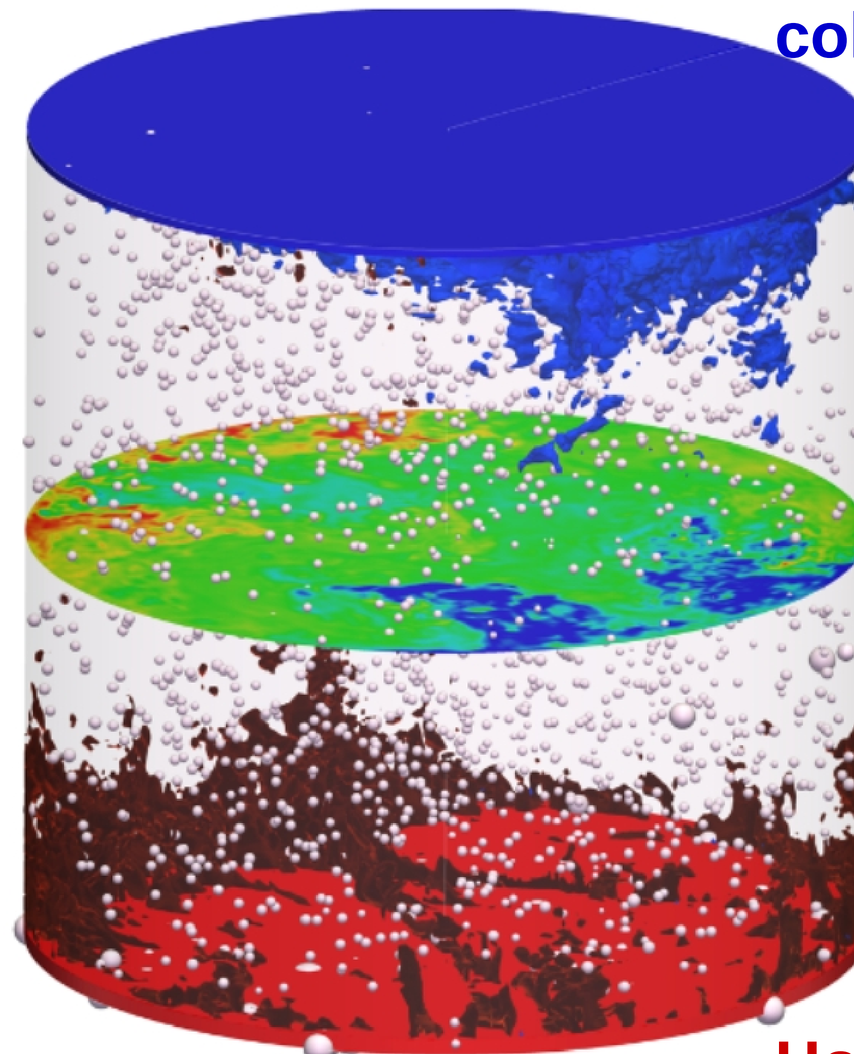
<u>A beautiful mathematical framework</u>



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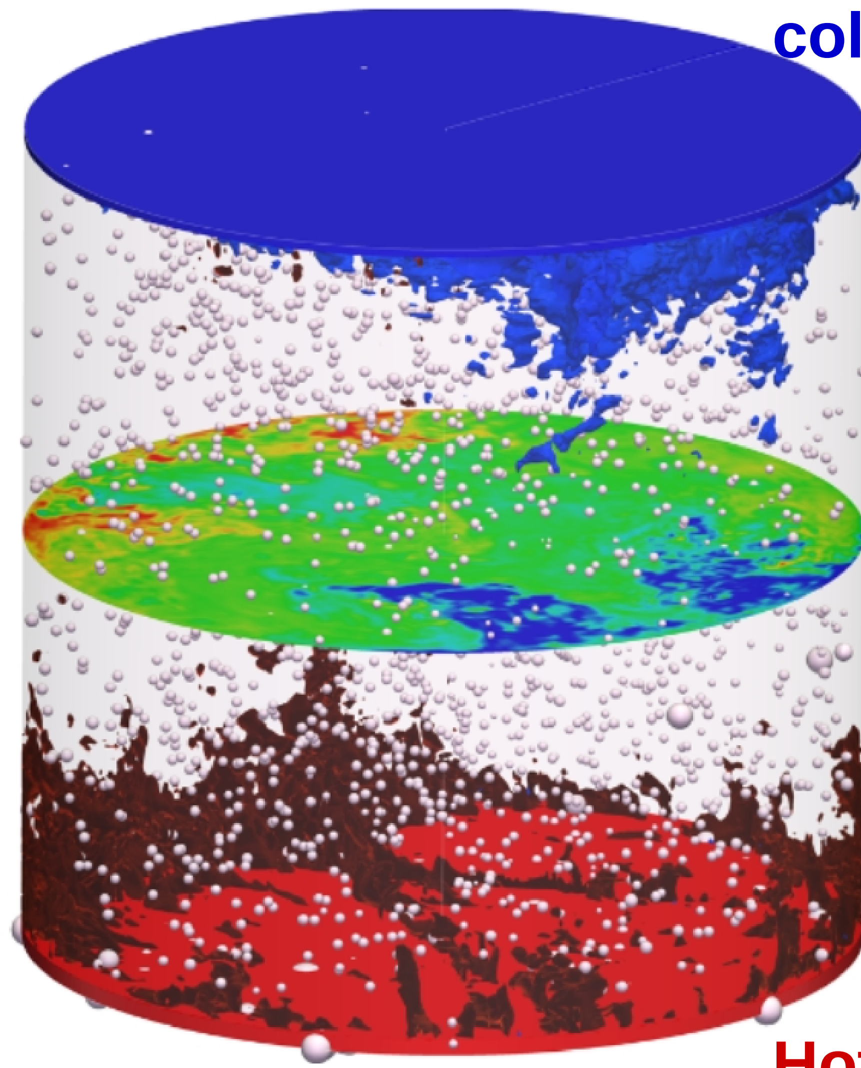
Incompressible flow for liquid - Euler
+
Point size – Lagrange
+
Growth/Condensation



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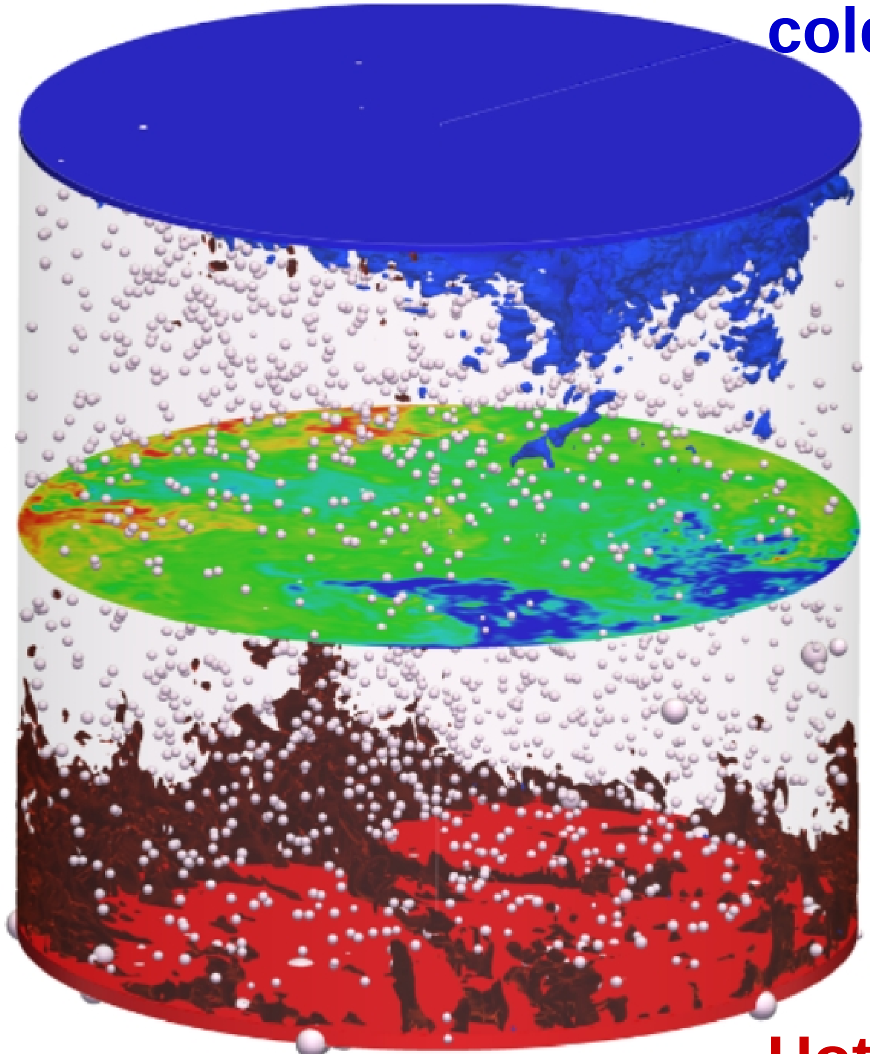
Wait a minute...



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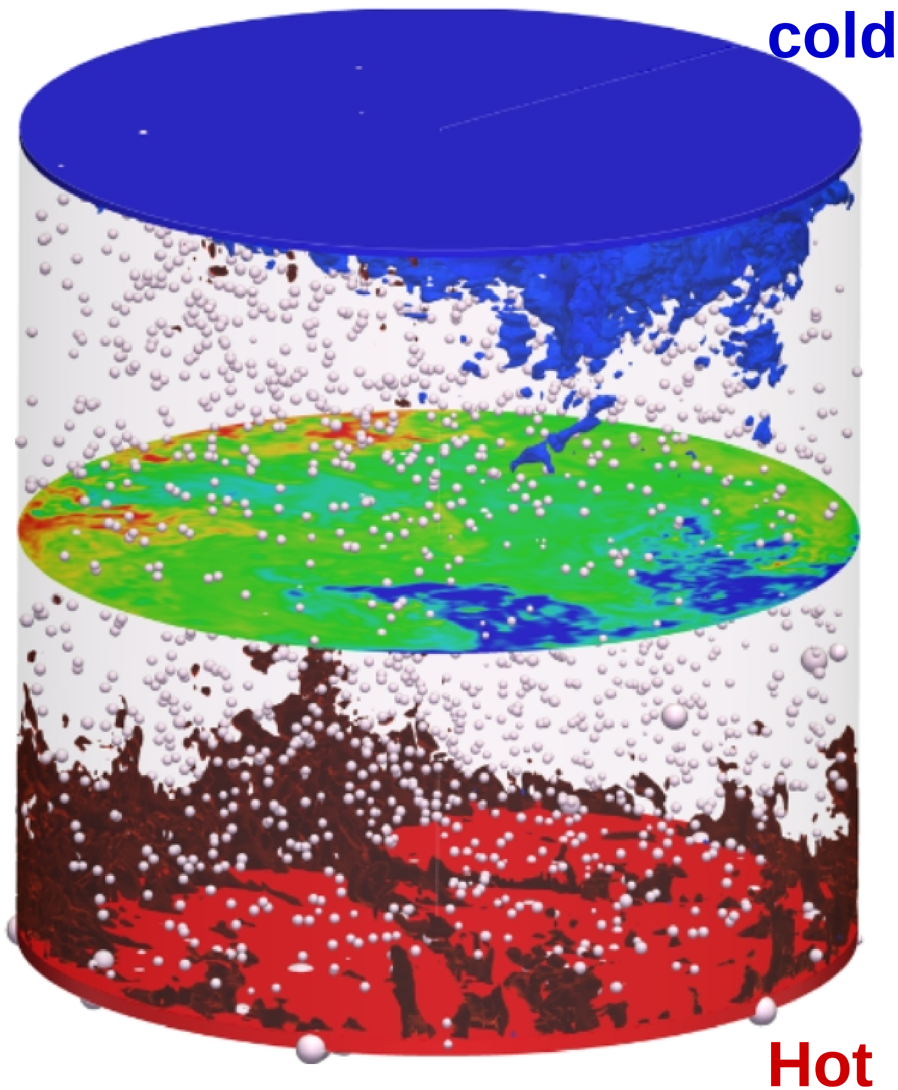
Wait a minute...
Are these real bubbles?
Can they rotate, coalesce, bounce, tumble,
Oscillate, create wake?



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Are these real bubbles?
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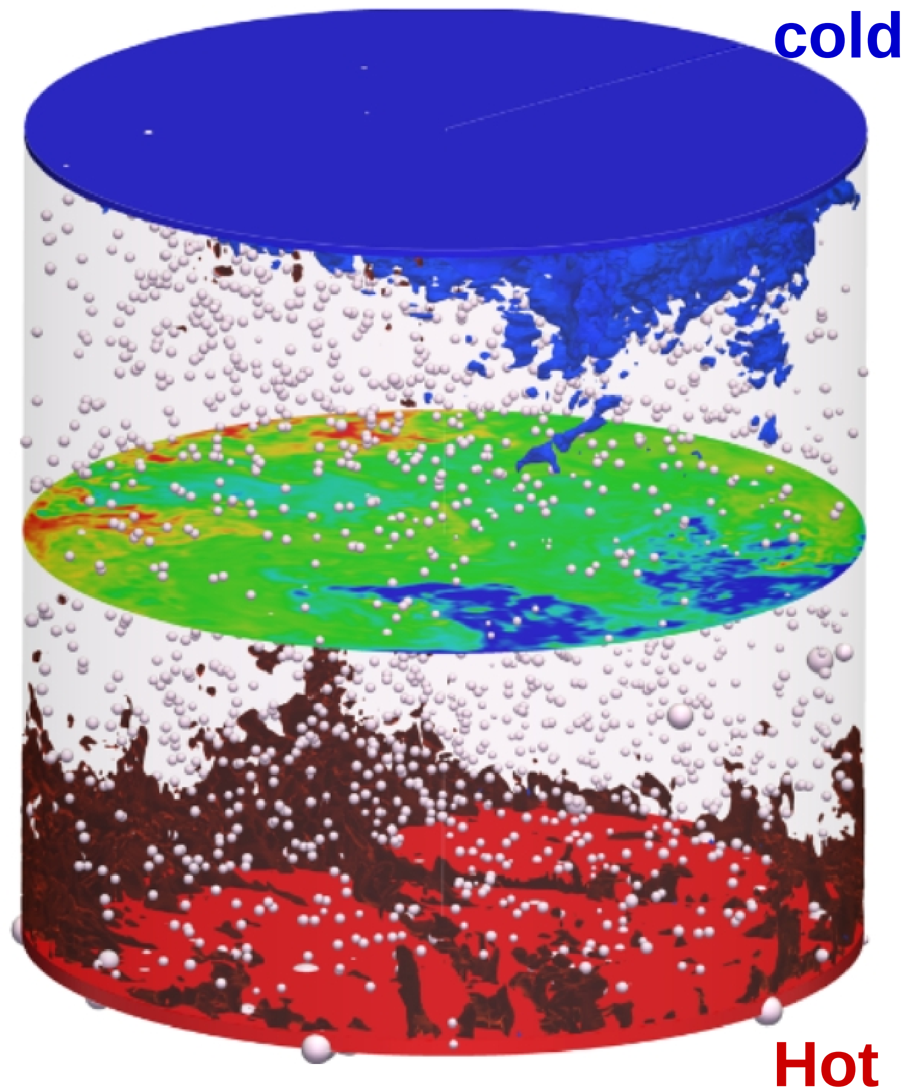
Oscillate, create wake?

NO!

But, see we can publish papers... hmm...



How we started into this

Then we started to work on real bubbles

Which can deform, rotate, coalesce, bounce, tumble...blah blah blah.

How we started into this

Then we started to work on real bubbles

Which can deform, rotate, coalesce, bounce, tumble...blah blah blah.

Raja, we are waiting for colorful dynamics

Show us some fancy animation

How we started into this

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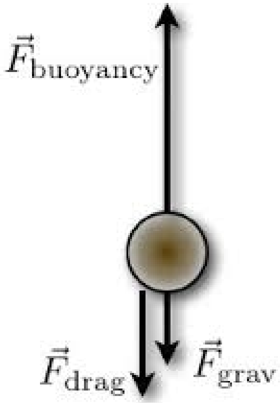
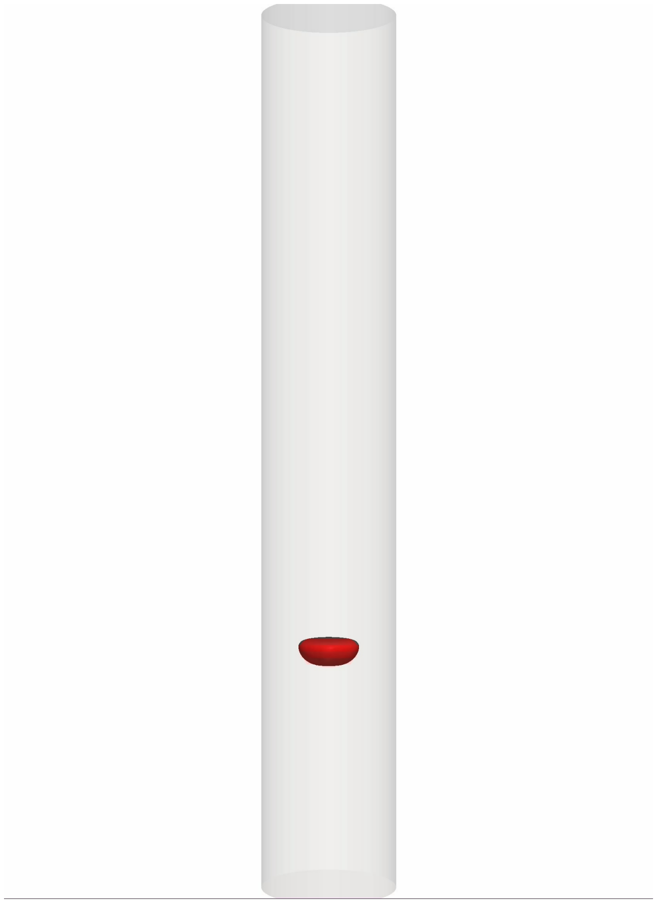
How about rising bubble in a quiscent liquid?





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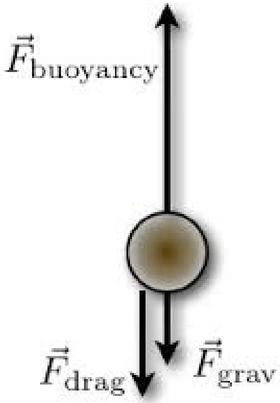
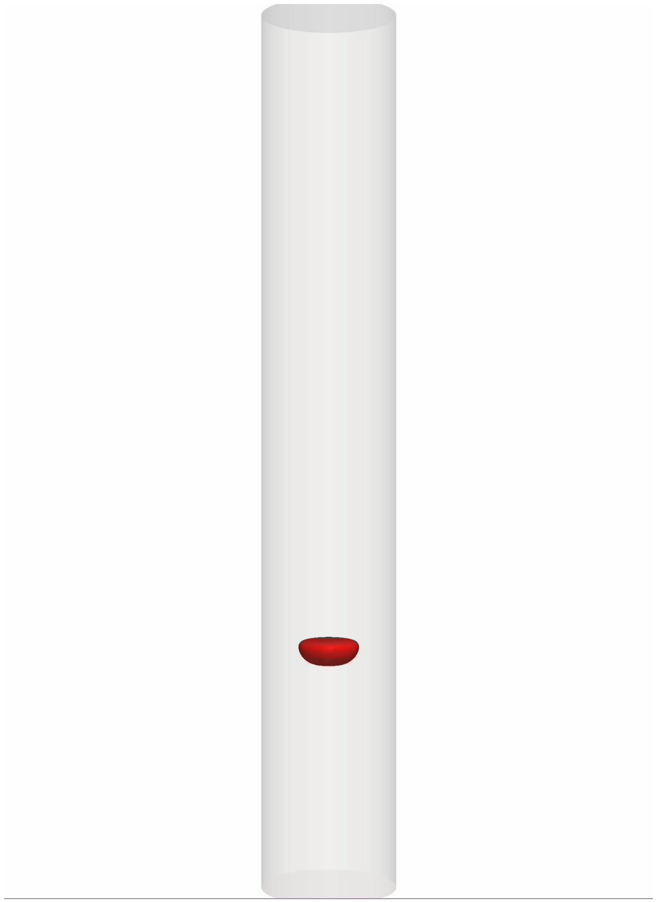
Size ~ 5 mm
cylinder of 16 cm x 2 cm
Rise velocity ~ 20 cm/s





Rising bubbles in a liquid column

Let us see if a small bubble trails a leading big one

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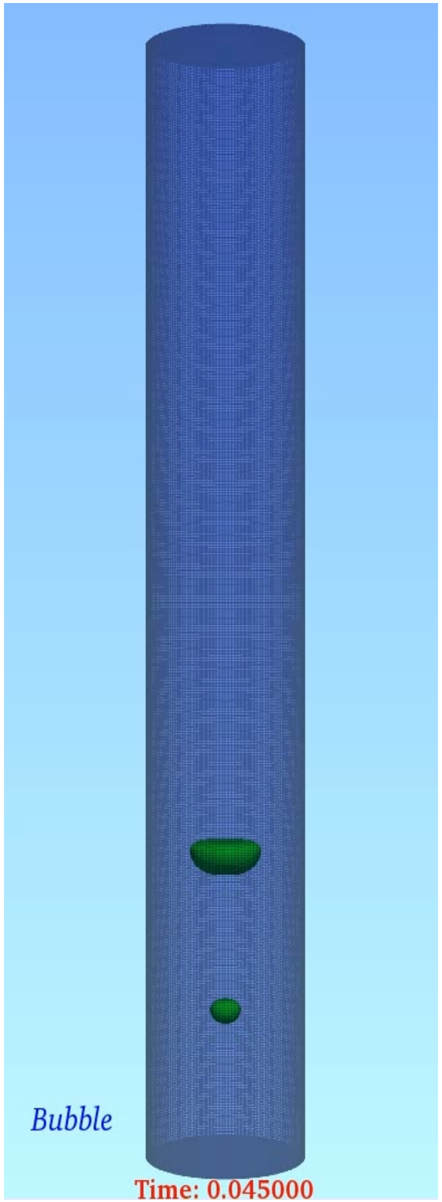
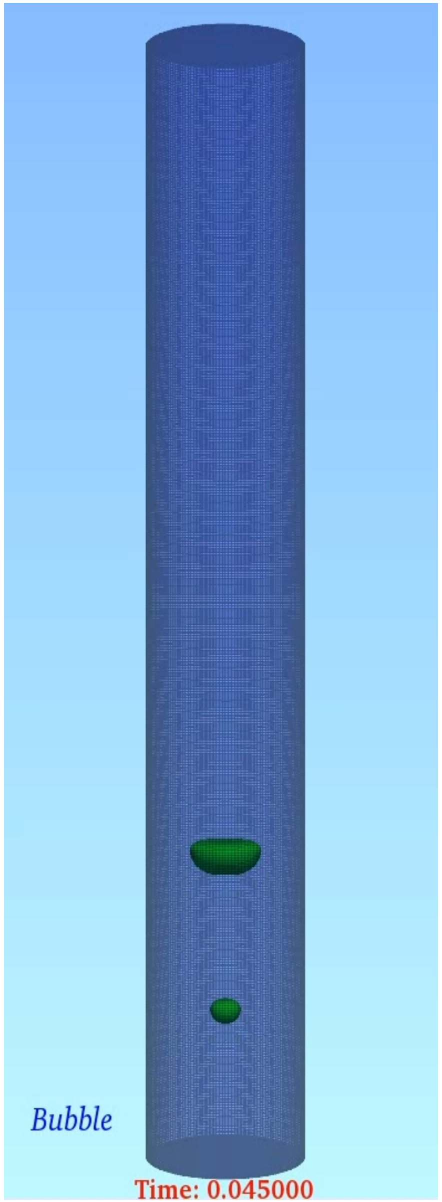


Rising bubbles in a liquid column

Let us see if a small bubble trails a leading big one

Small trailing bubble rises fast and approaches big leading bubble



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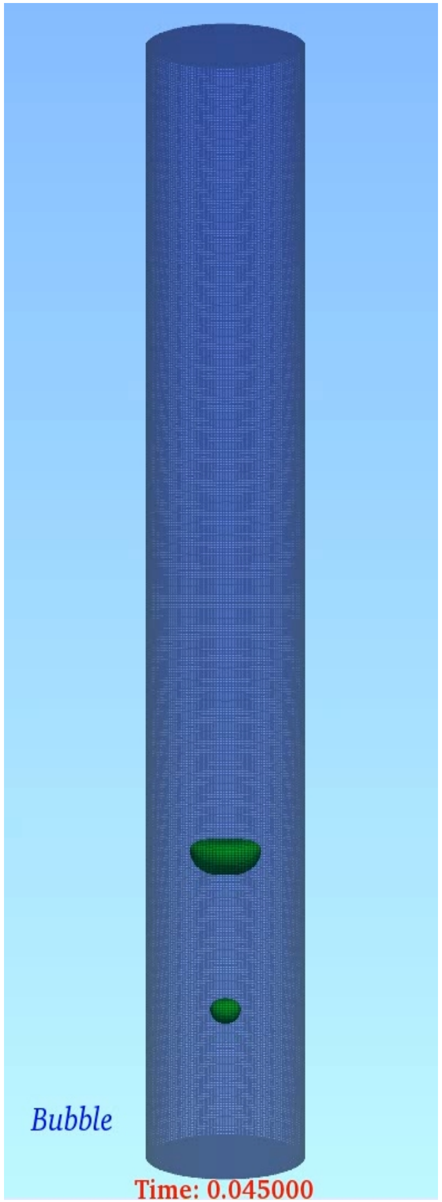

Rising bubbles in a liquid column

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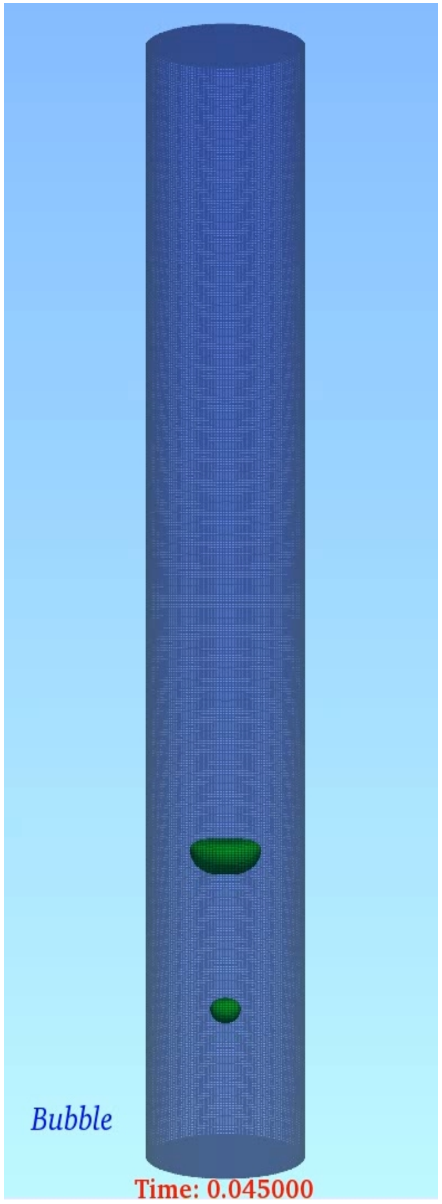
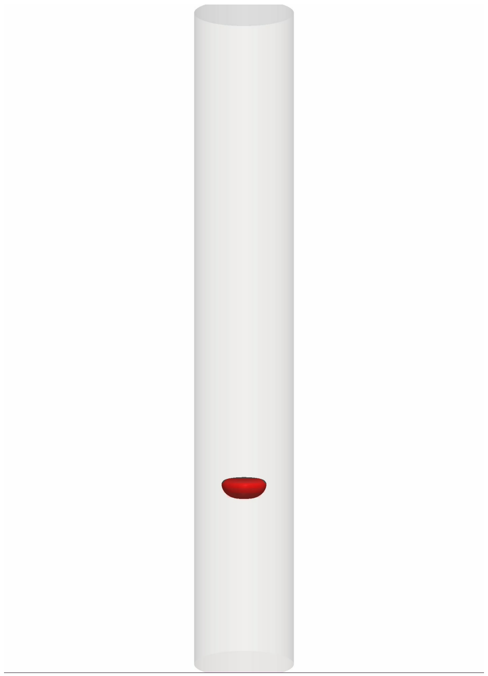
Why?

In next 15 min., we will see why...



A single bubble rising in a column

bubble rises due to gravity and based on
the bubble size they may reach steady rise



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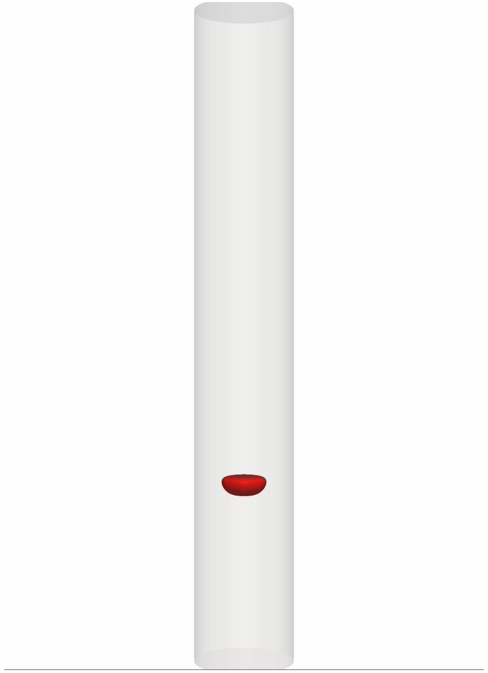
Based on parameters they show various shapes



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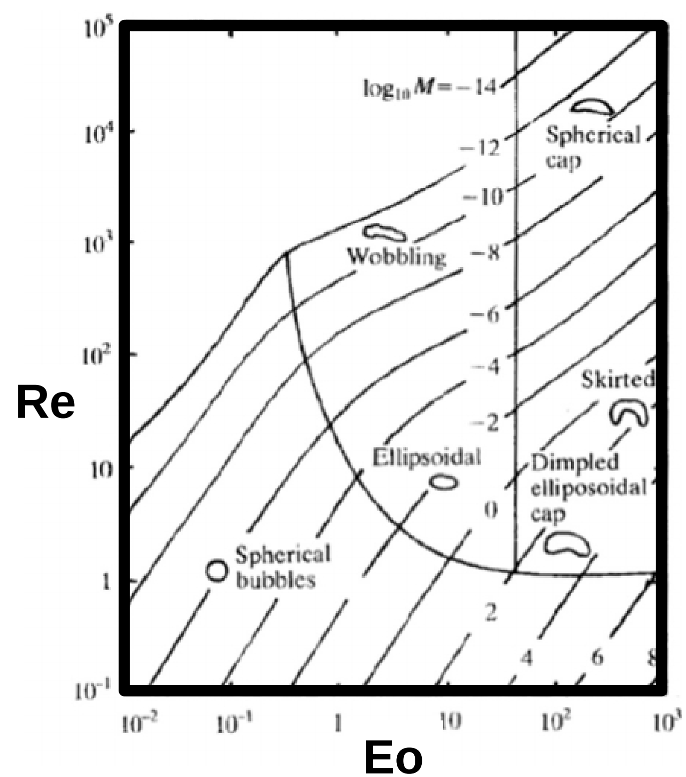
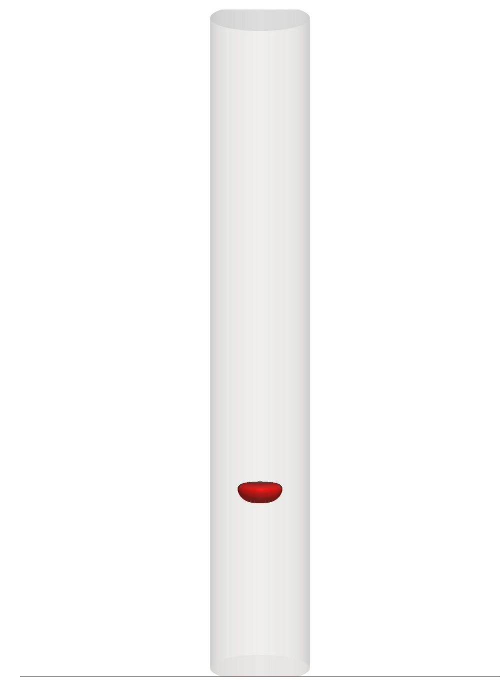
Bhaga & Weber (1981)





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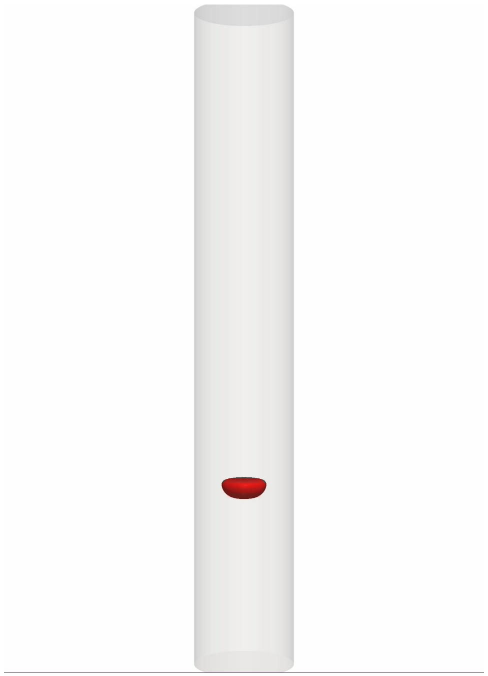

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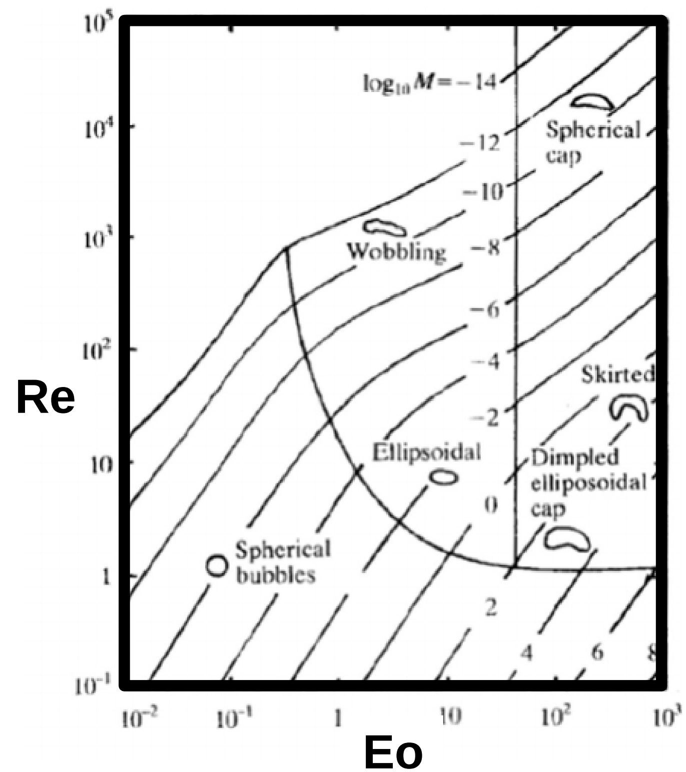
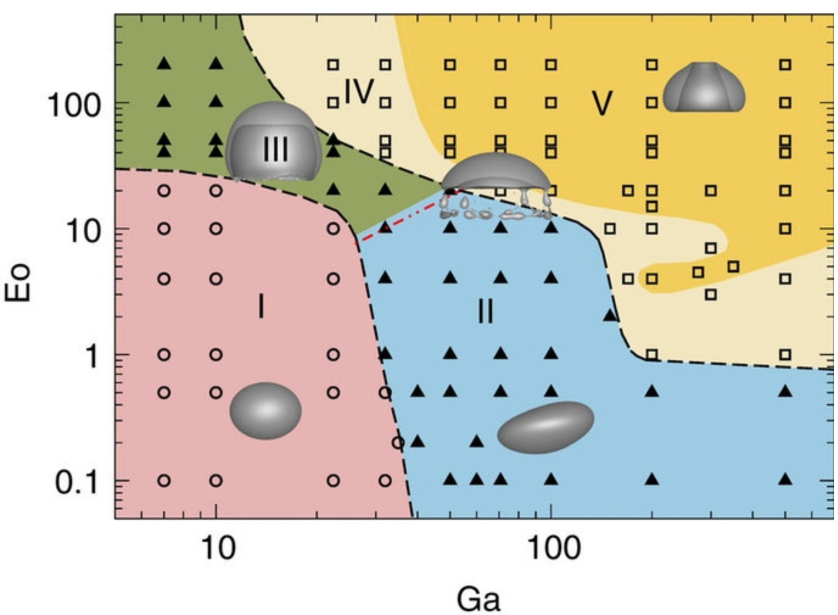


Tripathi et al. (2015)



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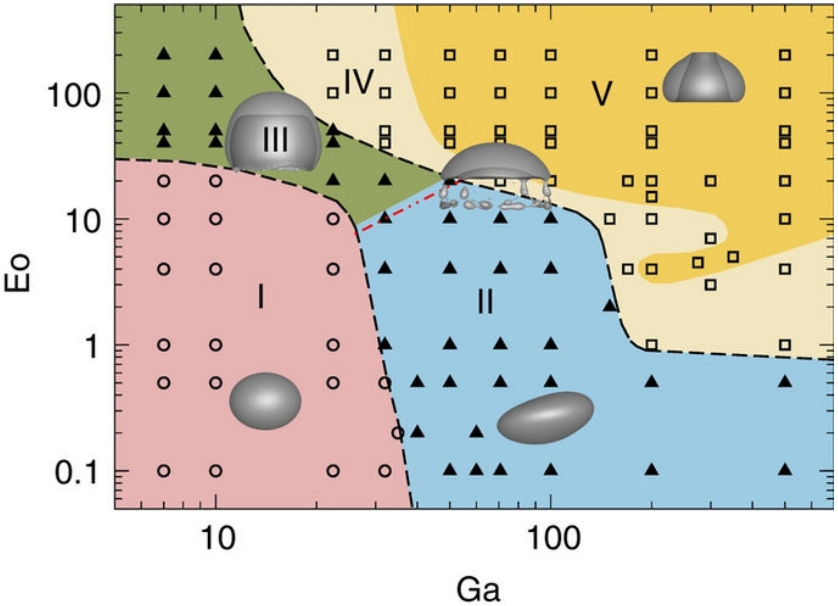

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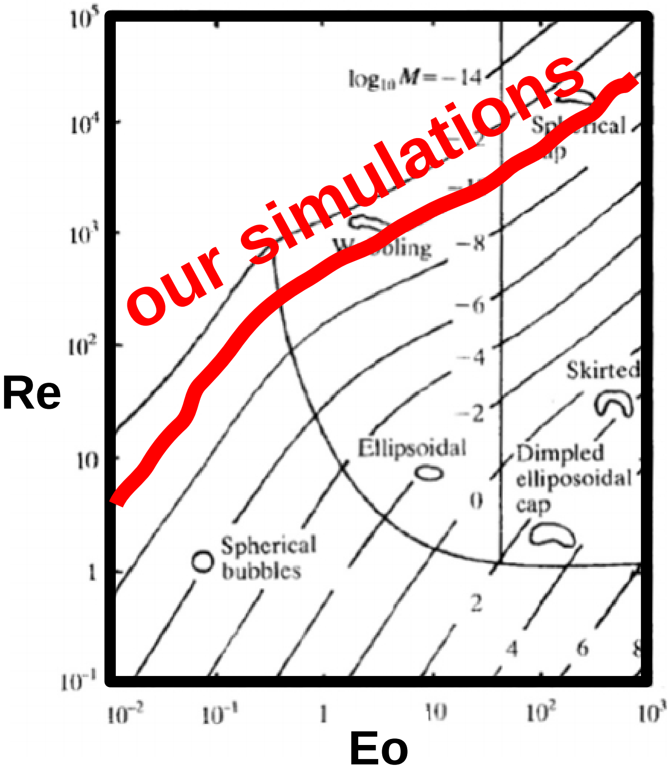
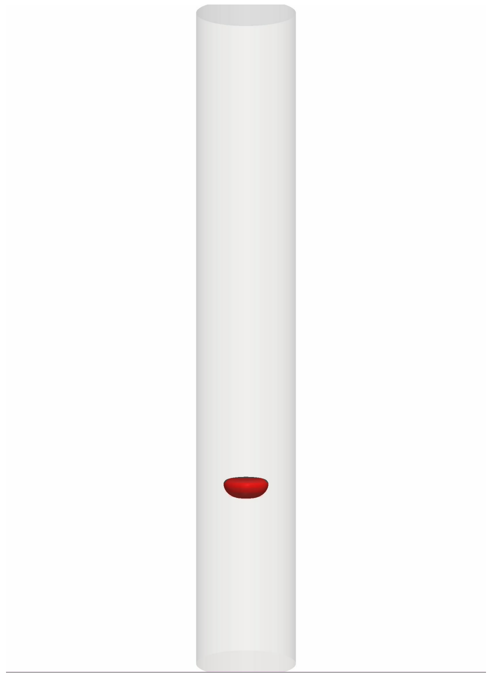




A single bubble rising in a column

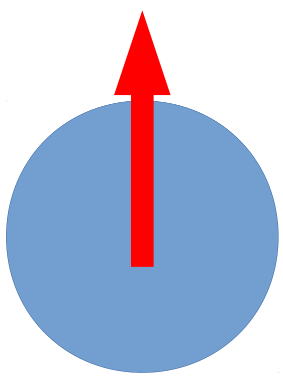
These parameters are

Archimedes number = buoyancy / viscous
Eotvos number = buoyancy / surface tension

A single bubble rising in a column

These parameters are

Archimedes number = buoyancy / viscous

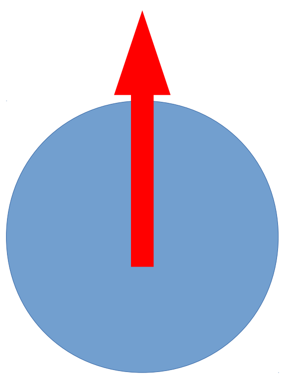


High Archimedes
Quick rise

A single bubble rising in a column

These parameters are

Archimedes number = buoyancy / viscous
Eotvos number = buoyancy / surface tension



High Archimedes
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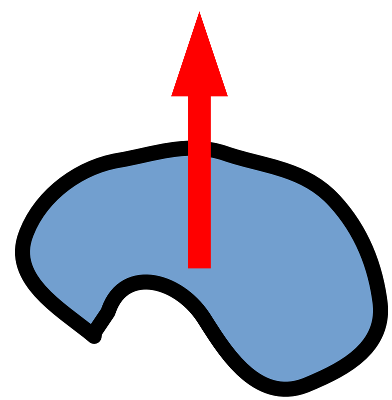

High Eotvos
Large deformation

A single bubble rising in a column

These parameters are

Archimedes number = buoyancy / viscous
Eotvos number = buoyancy / surface tension

Rise velocity: bubble Reynolds number

A single bubble rising in a column

These parameters are

Archimedes number = buoyancy / viscous
Eotvos number = buoyancy / surface tension

Rise velocity: bubble Reynolds number

Shape dynamics?

A single bubble rising in a column

These parameters are

Archimedes number = buoyancy / viscous
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Rise velocity: bubble Reynolds number

Shape dynamics?

} Known in scientific world

A single bubble rising in a column

These parameters are

Archimedes number = buoyancy / viscous
Eotvos number = buoyancy / surface tension

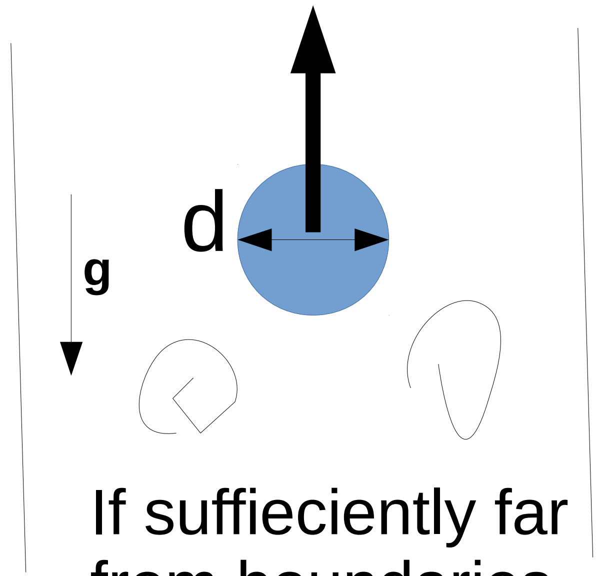
Rise velocity: bubble Reynolds number

Shape dynamics?

} Known in scientific world

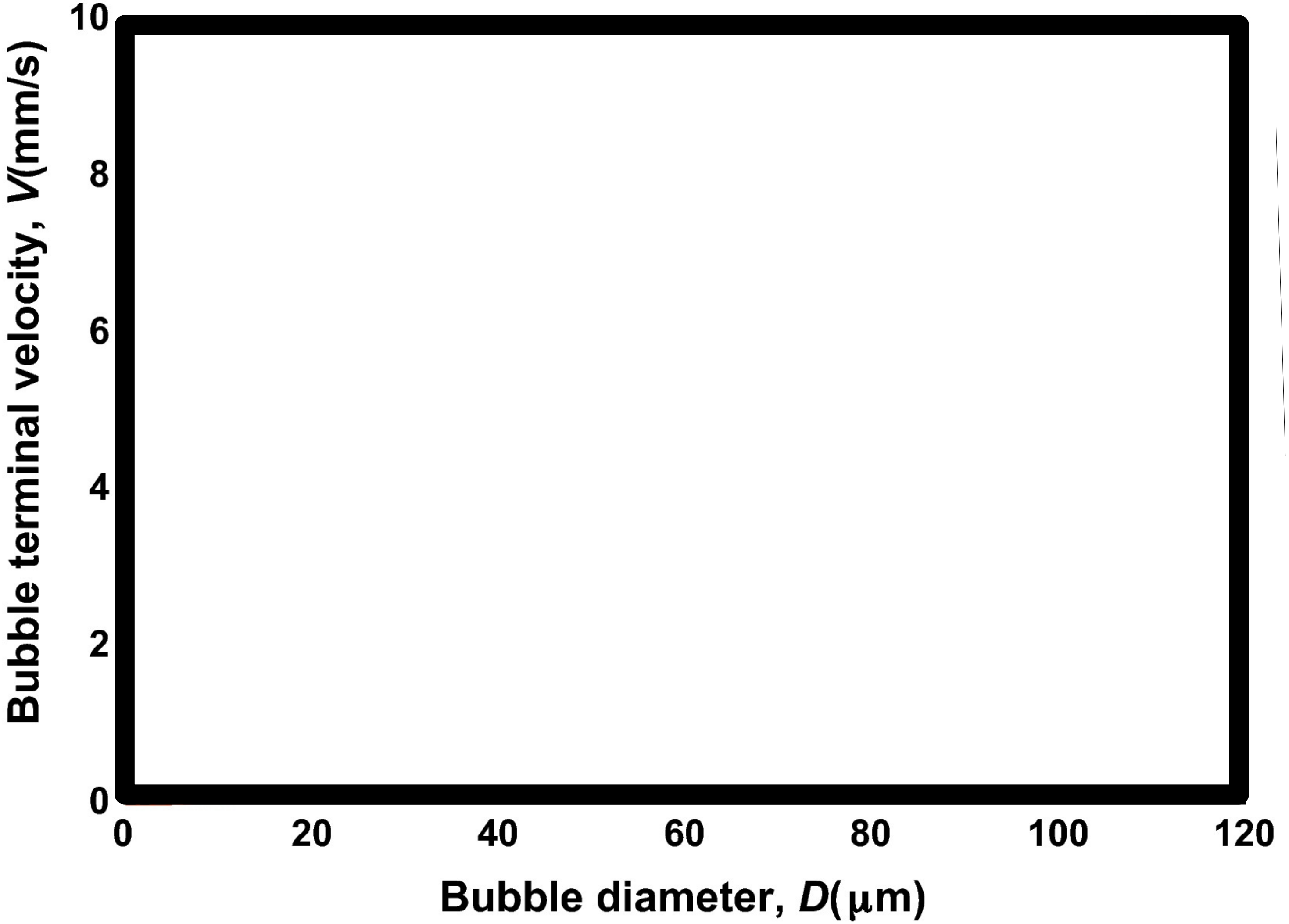
Flow features in the wake- Not much known

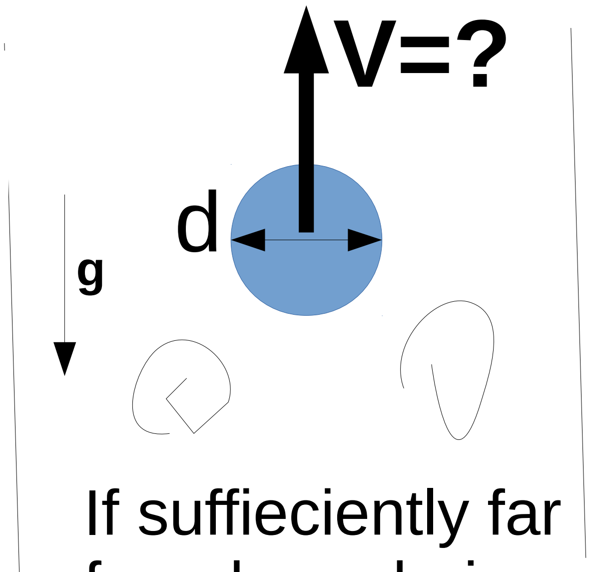
Single bubble rise velocity



If suffieciently far
from boundaries

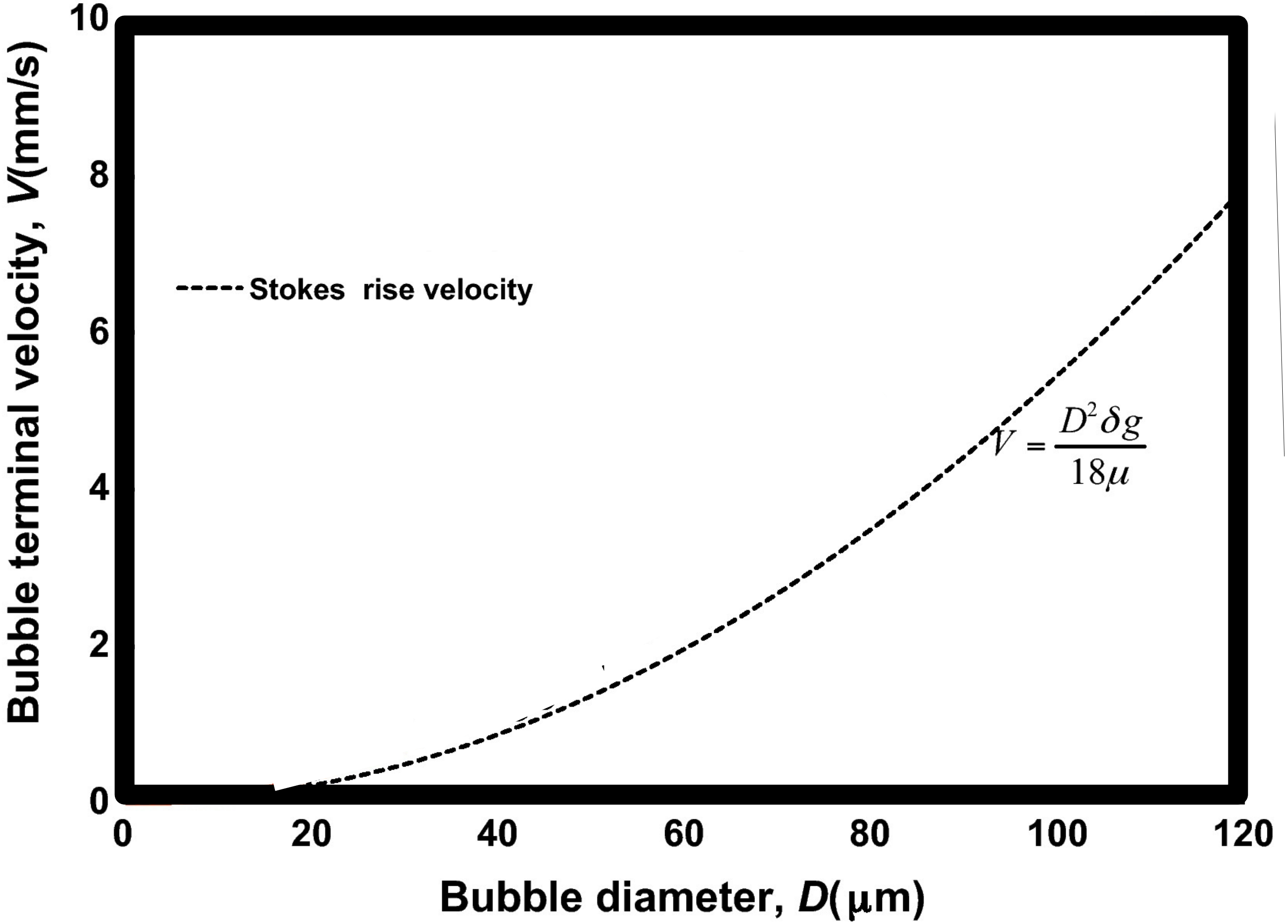
Single bubble rise velocity

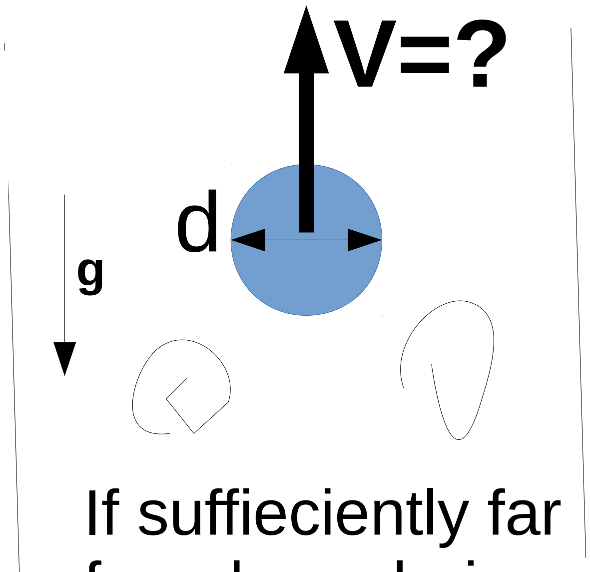




If suffieciently far from boundaries

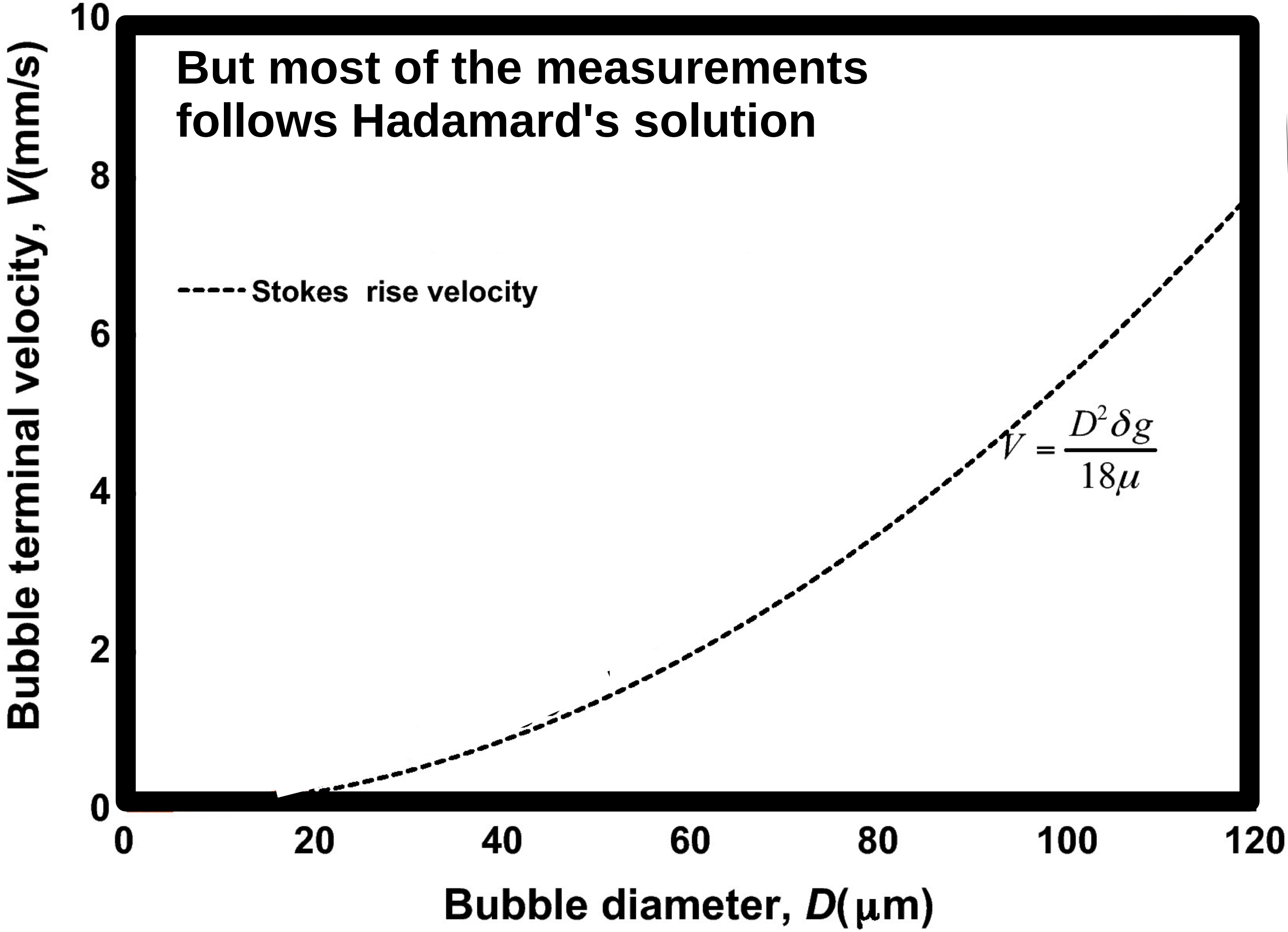
Single bubble rise velocity

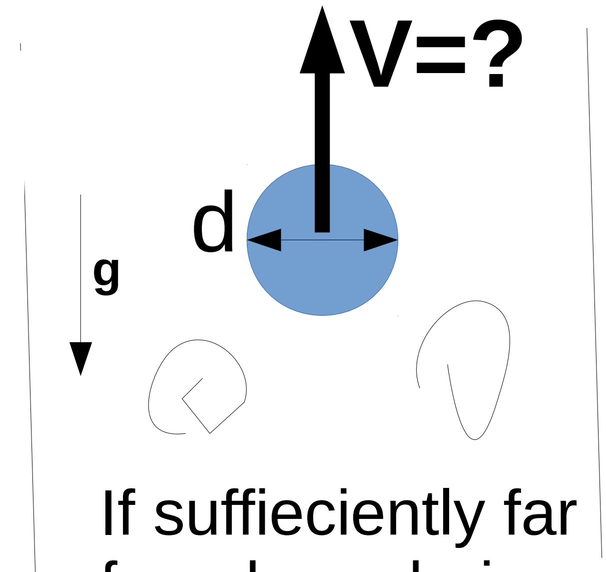




If suffieciently far from boundaries

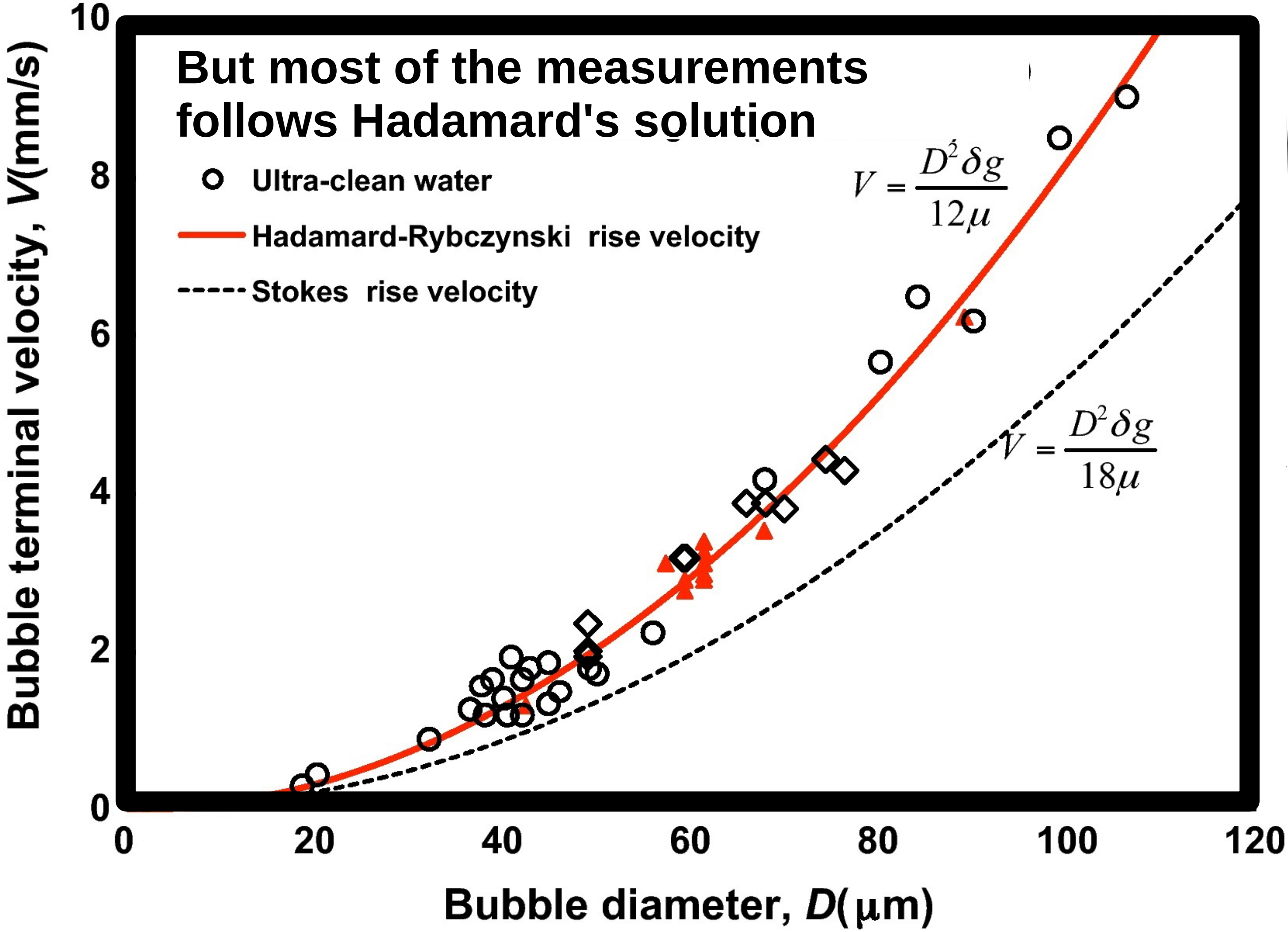
Single bubble rise velocity

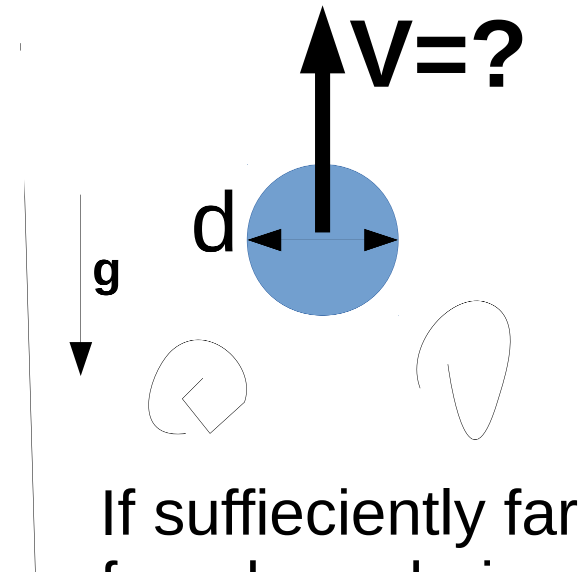




If suffieciently far from boundaries

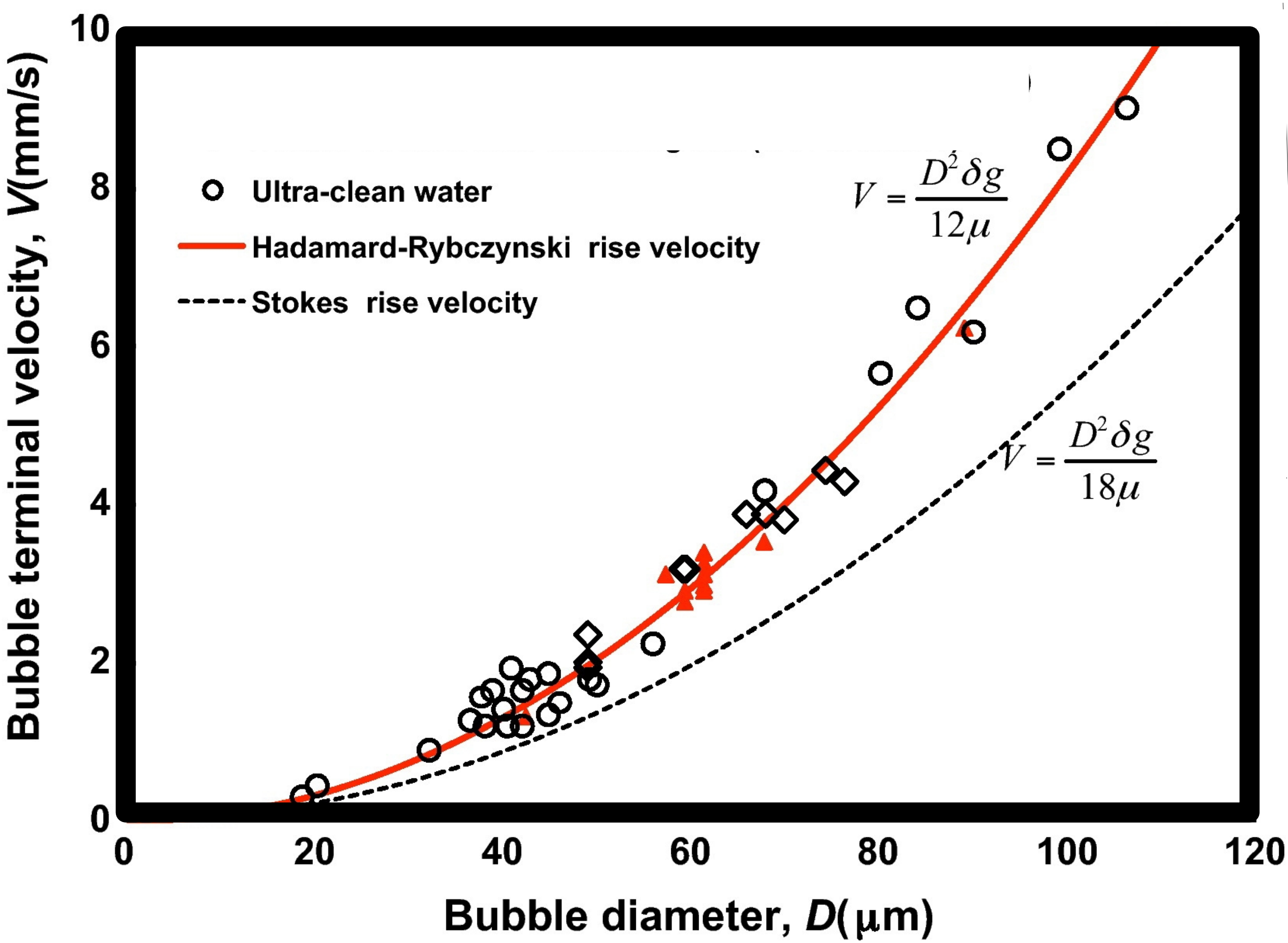
Single bubble rise velocity

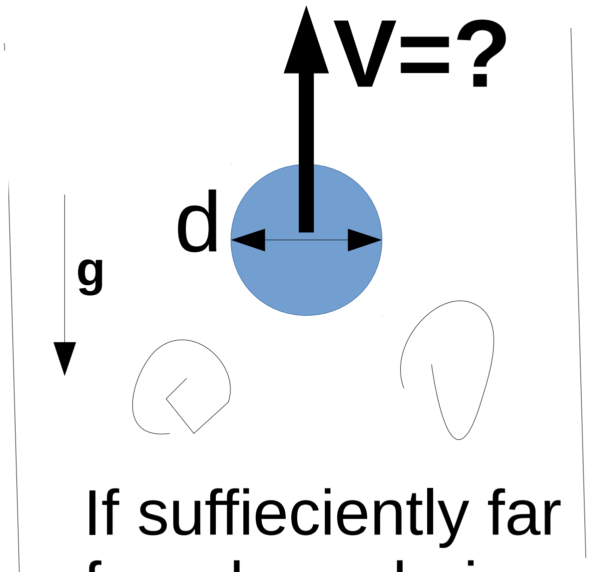




If suffieciently far from boundaries

Single bubble rise velocity





If suffieciently far from boundaries

For larger bubbles (mm or cm) the solution is different

Single bubble rise velocity

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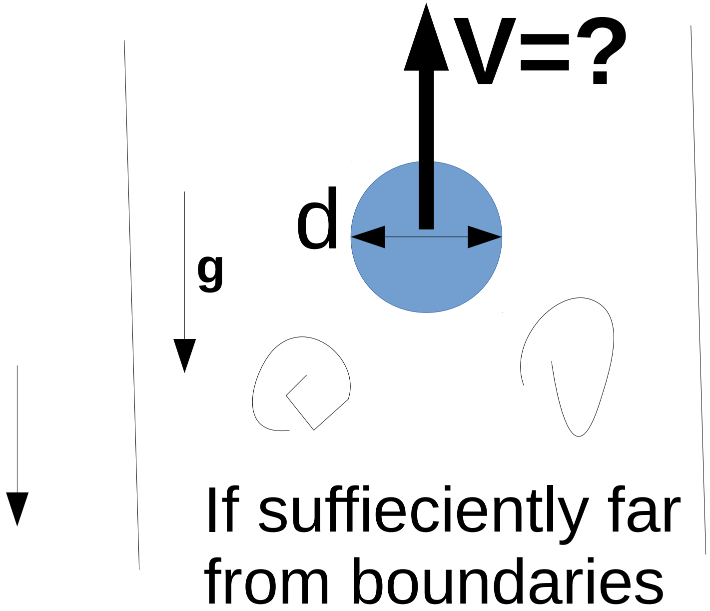
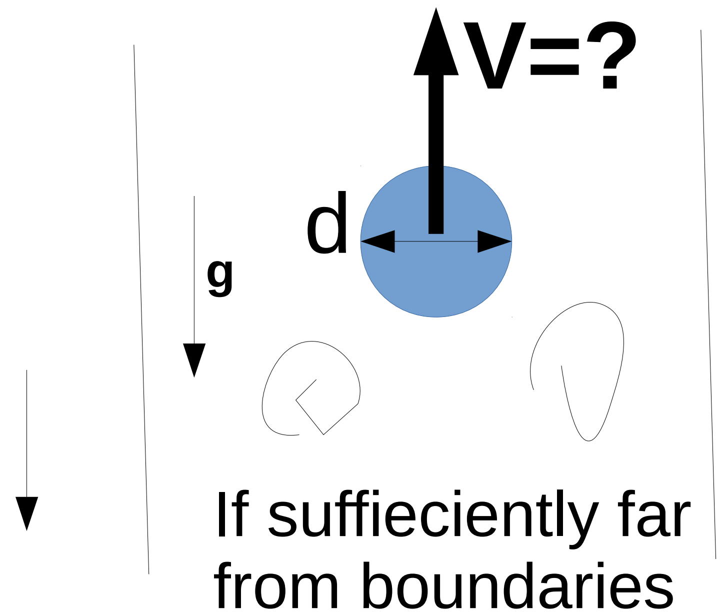


Single bubble rise velocity

For larger bubbles (mm or cm) the solution is different

**Because of wakes, deformability,
3-dimensionality, contamination**

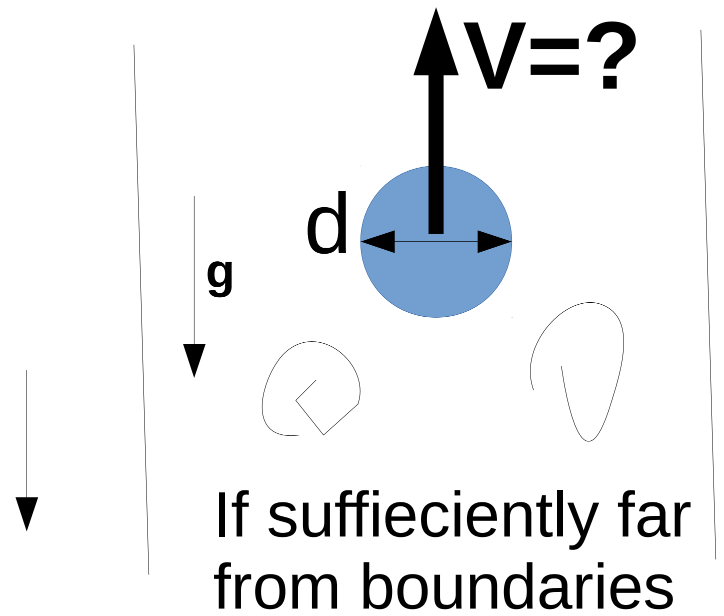


Single bubble rise velocity

For larger bubbles (mm or cm) the solution is different

Because of wakes, deformability,
3-dimensionality, contamination

Increase/decrease in rise velocity
may happen with increase in bubble size



Single bubble rise velocity

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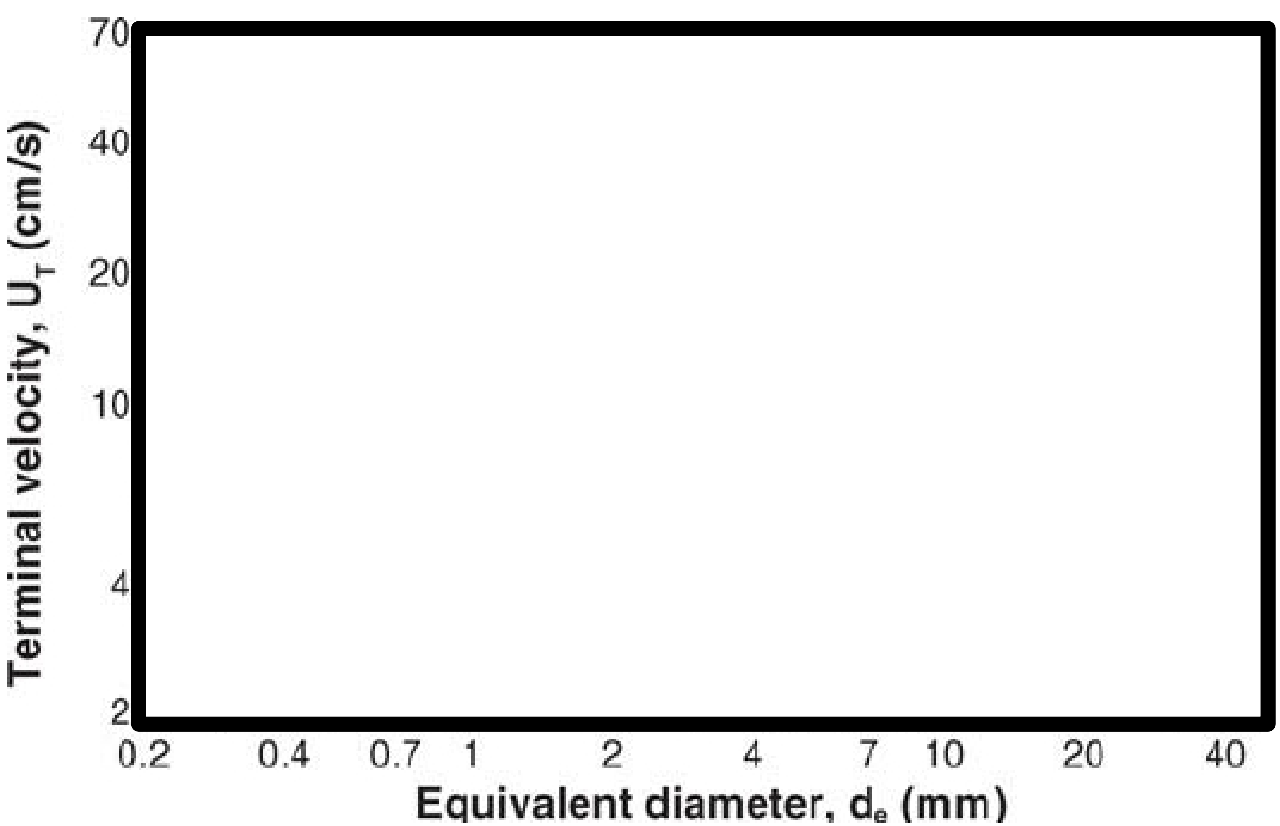

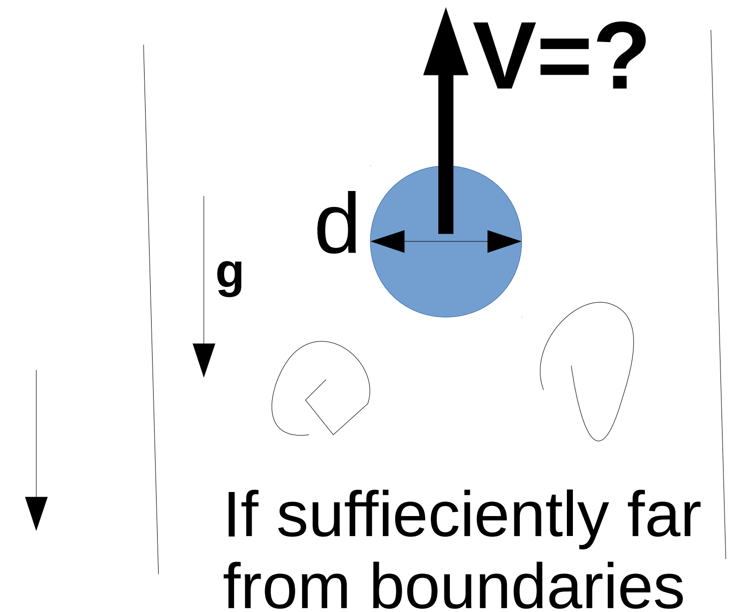

Single bubble rise velocity

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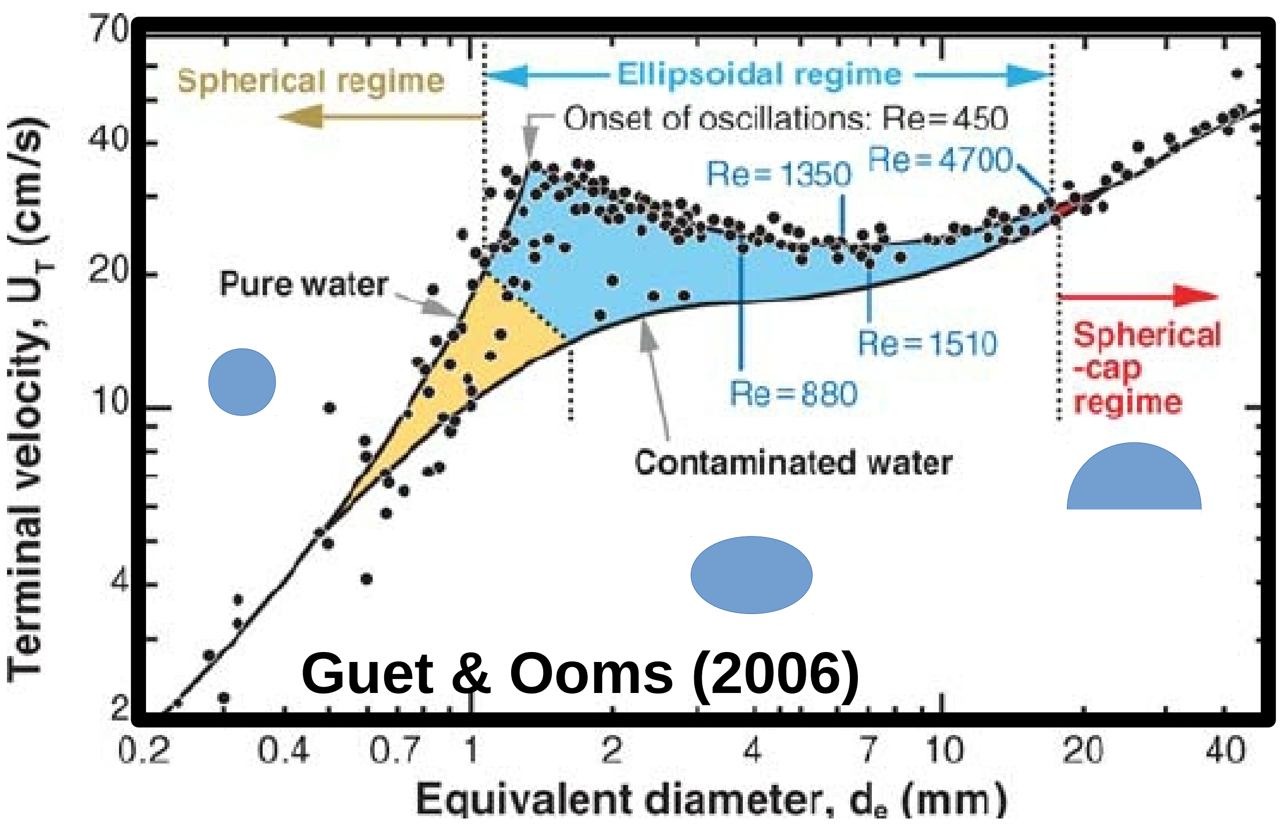
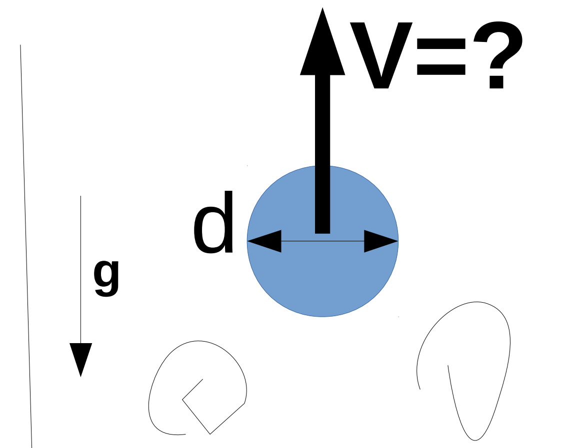
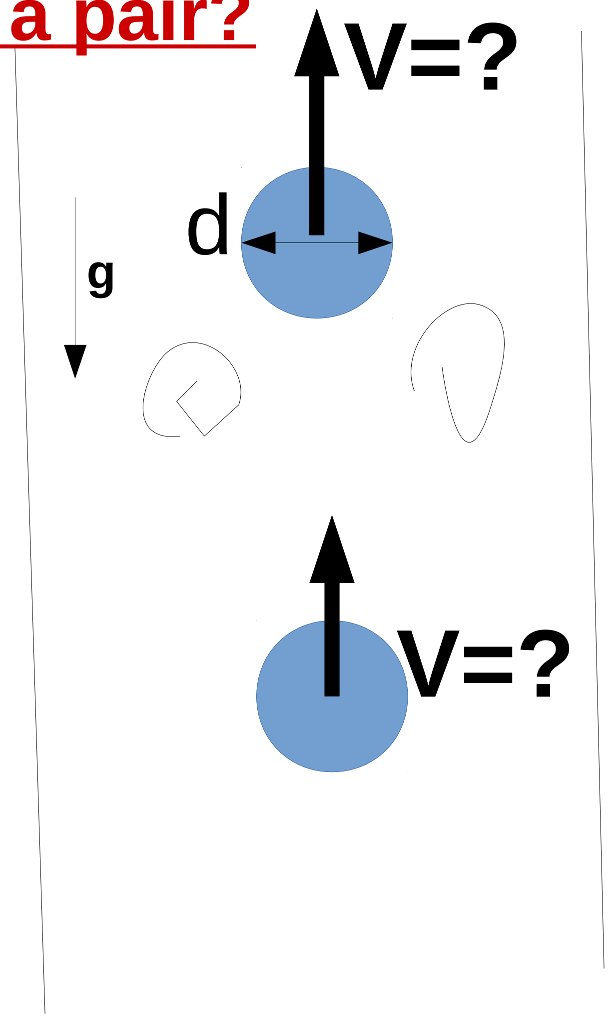




If suffieciently far from boundaries

In case of pair of bubbles

Do we see any steady rise velocities for a pair?



In case of pair of bubbles

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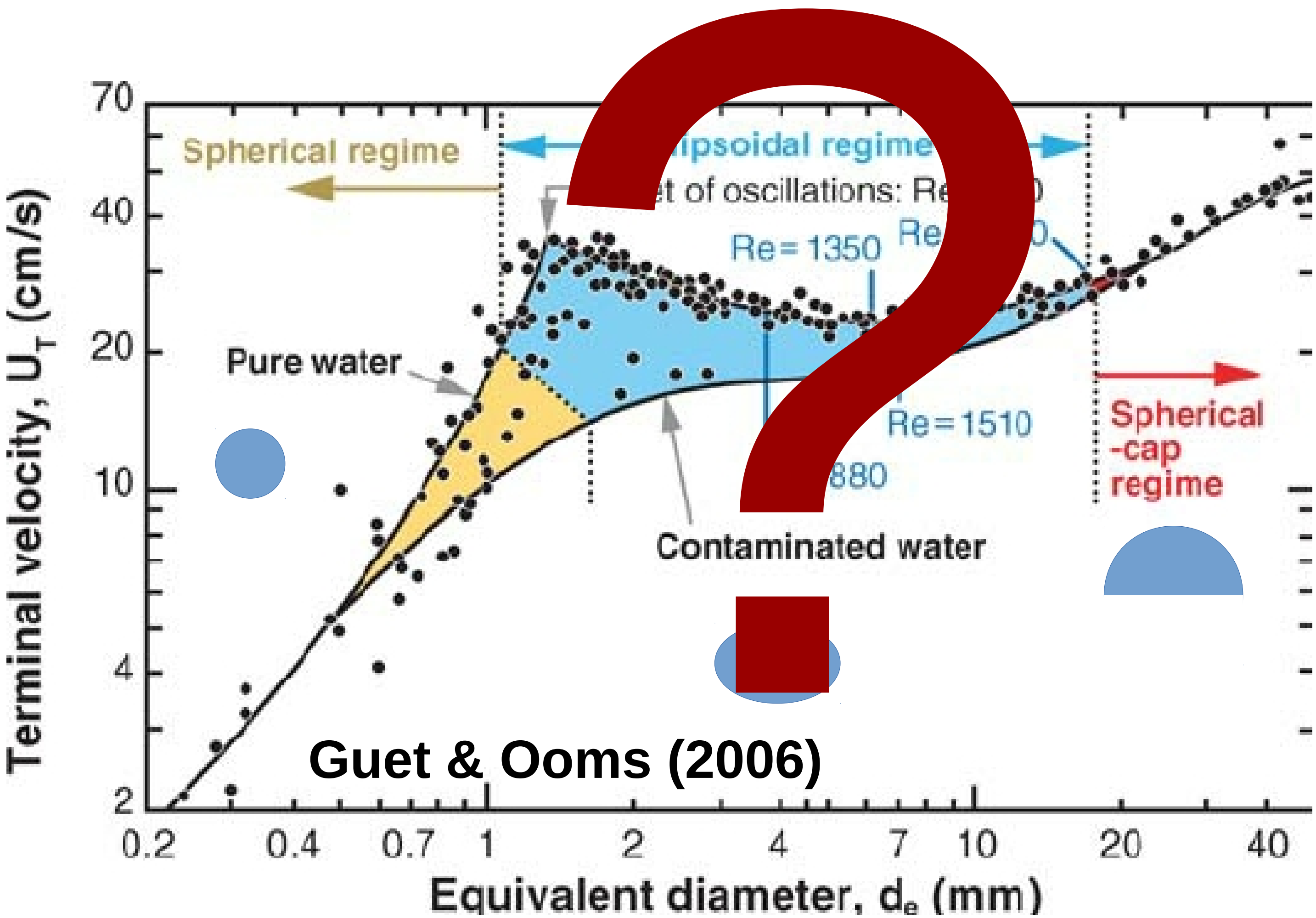
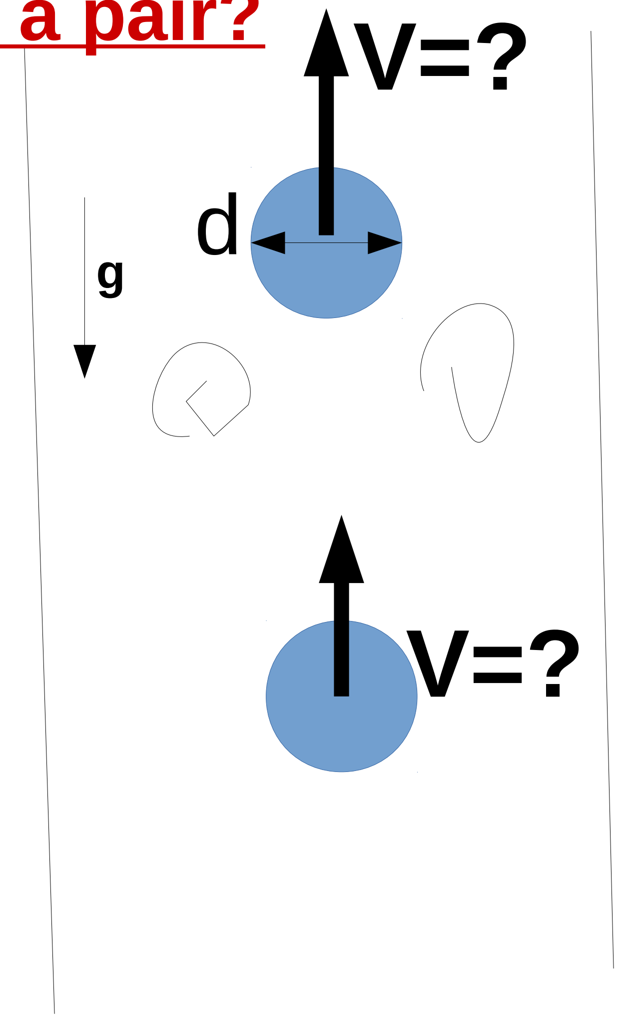
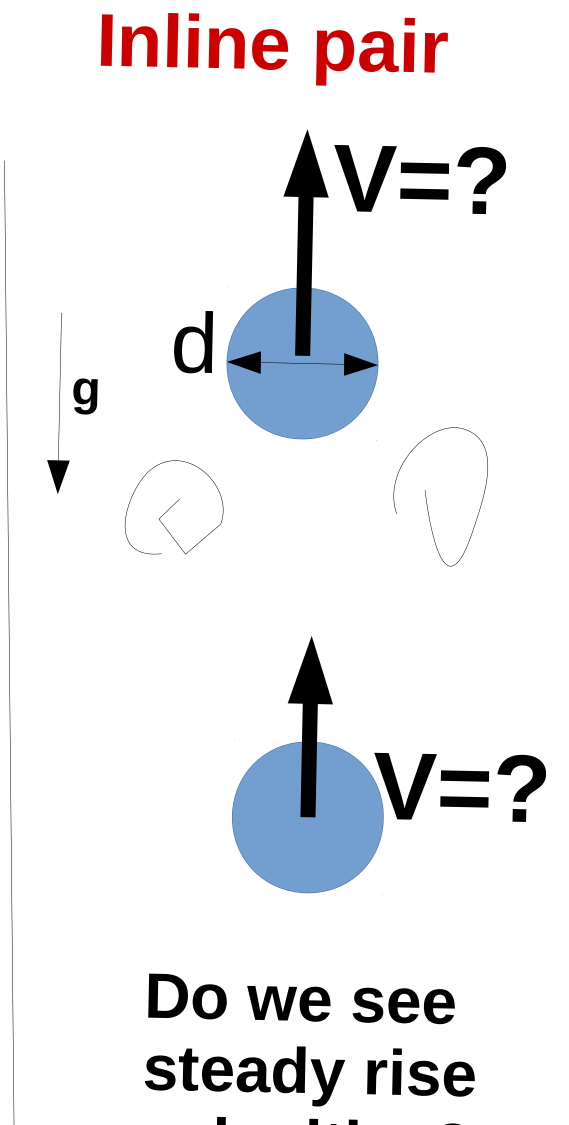




In case of pair of bubbles

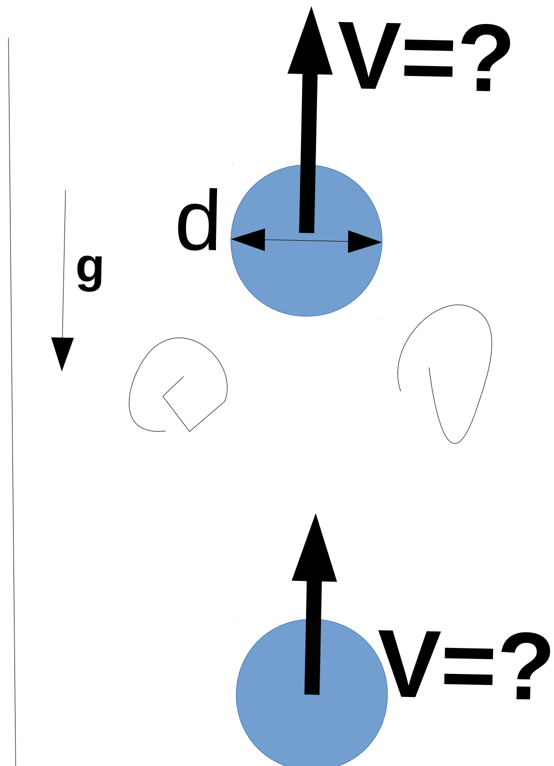


Do we see steady rise velocities?

Long outstanding problem

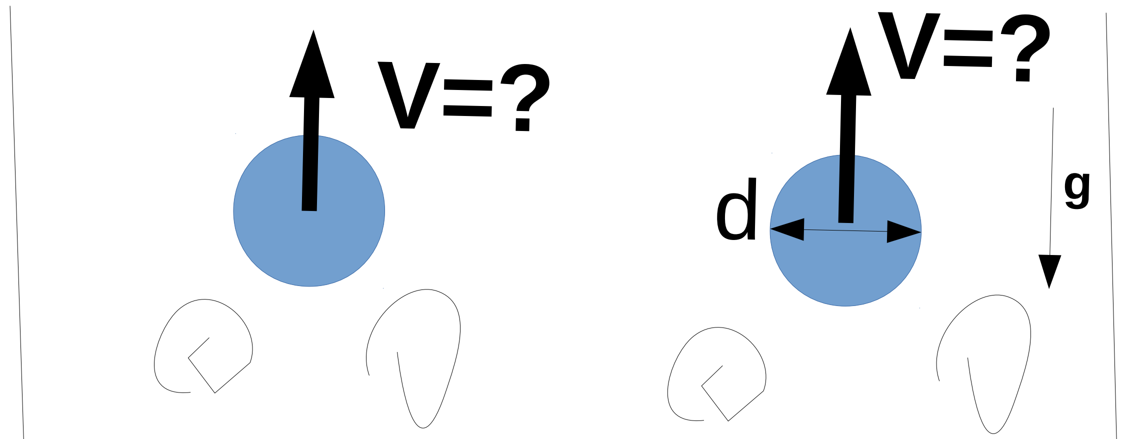
In case of pair of bubbles

Inline pair



Do we see steady rise velocities?

Side by side pair



Do we see steady rise velocities?

Long outstanding problems

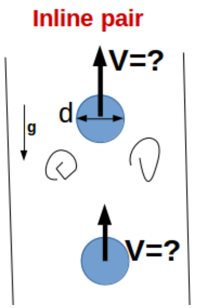
In case of pair of bubbles

Stokes flow
(Re~0)

Potential flow
(Re very large)

Various theories counter-intutive results

Moderate Re



In case of pair of bubbles

Stokes flow
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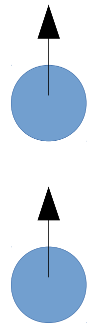

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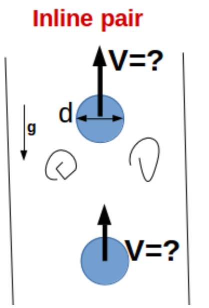
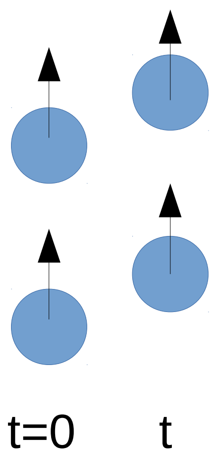
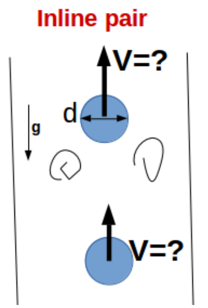


In case of pair of bubbles

Stokes flow
(Re~0)

Steady rise
never approach





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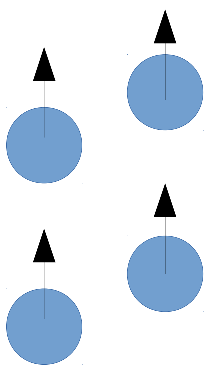
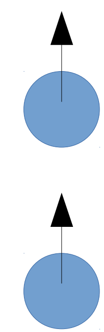


t=0 t

Potential flow
(Re very large)
& no wake



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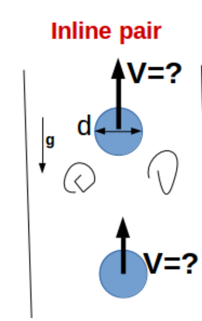

In case of pair of bubbles

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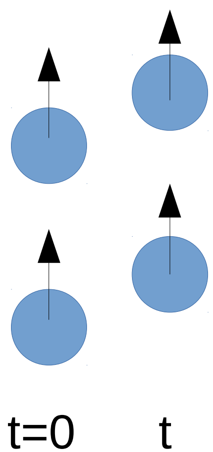
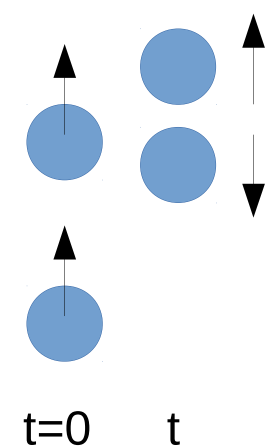
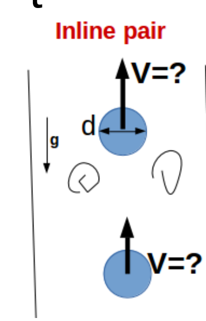


Potential flow
(Re very large)
& no wake

They repel vertically
(van Wijngaarden 1982)

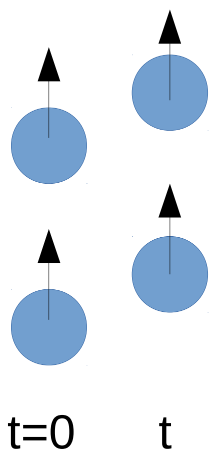




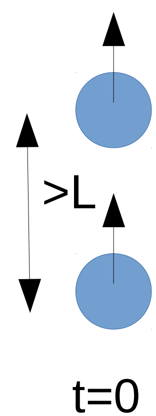
In case of pair of bubbles

Stokes flow (Re~0)

Steady rise
never approach



Potential flow & thin wake



Potential flow (Re very large) & no wake

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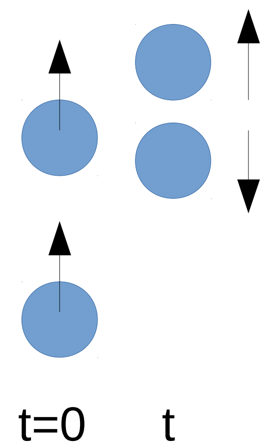
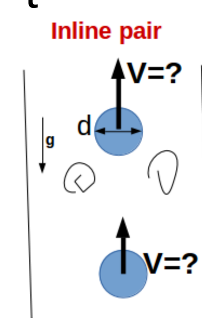
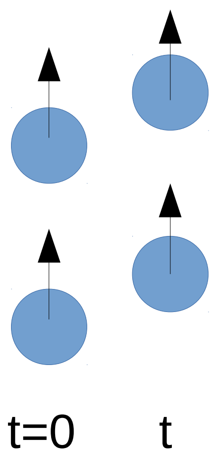




In case of pair of bubbles

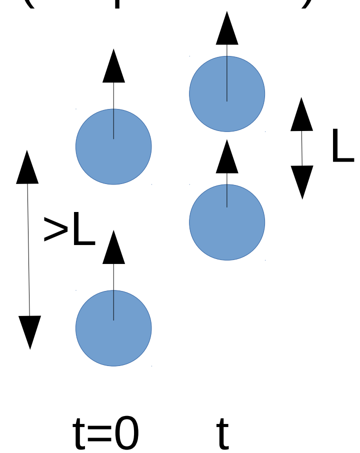
Stokes flow (Re~0)

Steady rise never approach



Potential flow & thin wake

Equilibrium distance
(Harper 1970)



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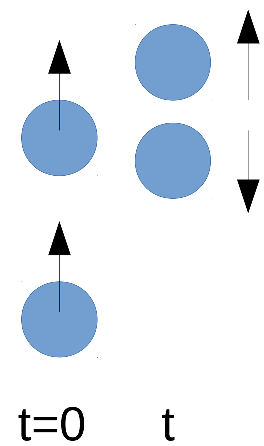

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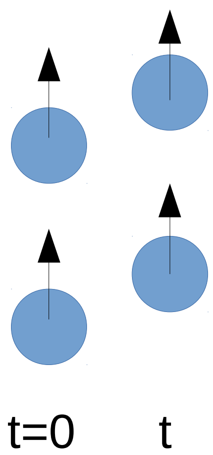

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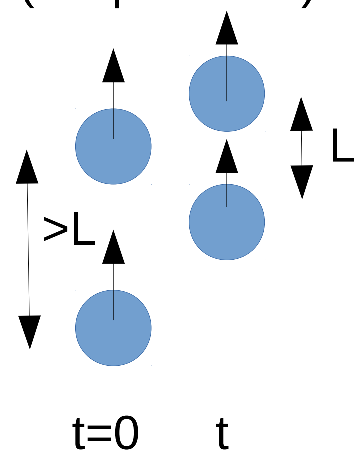

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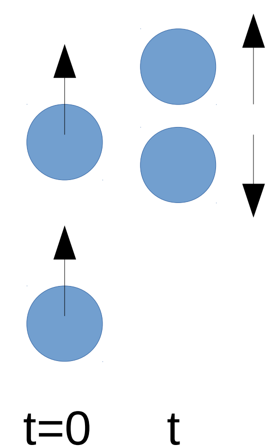
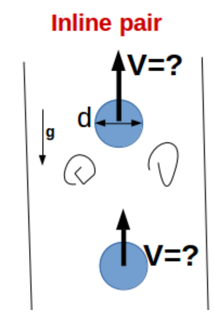


Moderate Re



In case of pair of bubbles

Stokes flow
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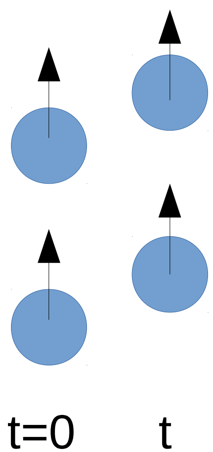
Steady rise
never approach



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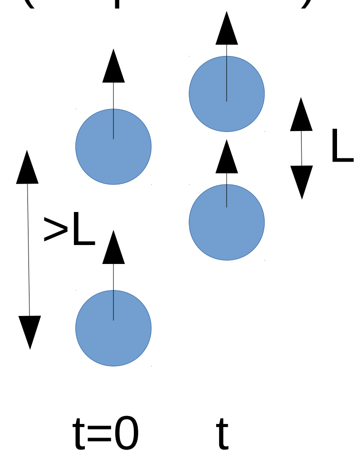

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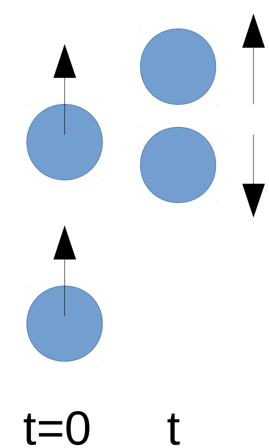
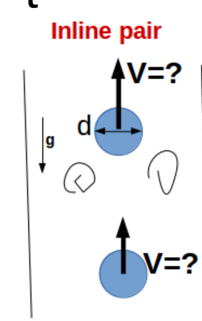


Moderate Re

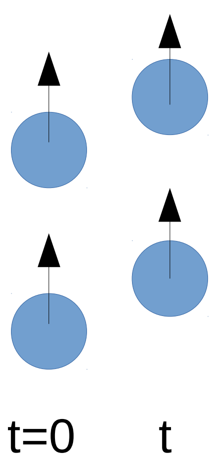


Axisymmetric bubbles
Equilibrium distance
(Prosperetti 1994)

In case of pair of bubbles

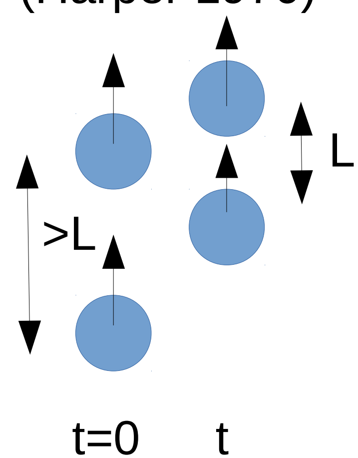
Stokes flow
(Re~0)

Steady rise
never approach



Potential flow
& thin wake

Equilibrium distance
(Harper 1970)



Potential flow
(Re very large)
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They repel vertically
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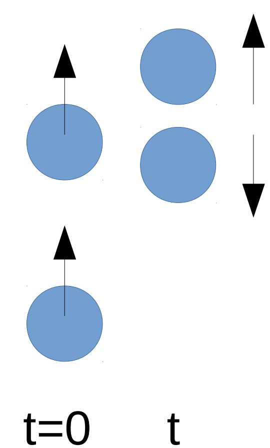
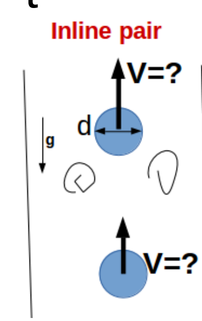




Moderate Re

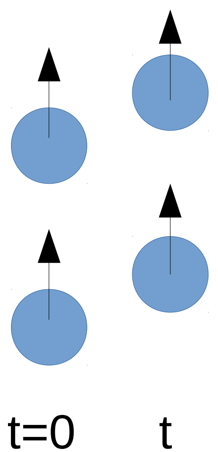
Axisymmetric bubbles
Equilibrium distance
(Prosperetti 1994)

Experiments
coalescence
(Meneveau 1996)

In case of pair of bubbles

Stokes flow
(Re~0)

Steady rise
never approach



Potential flow
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Equilibrium distance
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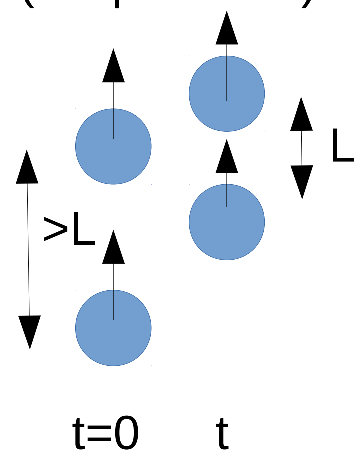

Potential flow
(Re very large)
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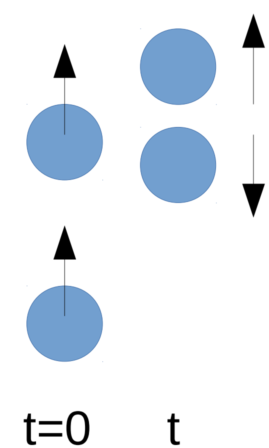


Moderate Re

Axisymmetric bubbles
Equilibrium distance
(Prosperetti 1994)

Experiments
coalescence
(Meneveau 1996)

Why this discrepancy?

Because of wakes, deformability, 3-dimensionality, contamination

In case of pair of bubbles

Let us ask a question... <u>**Moderate Re**</u>

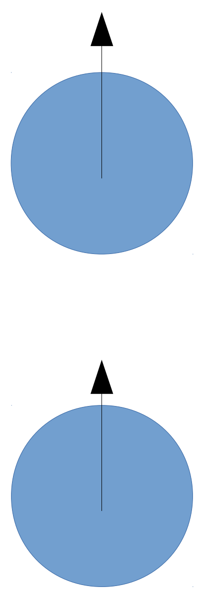
Instead an equal size pair if an unequal size pair rises then?

In case of pair of bubbles

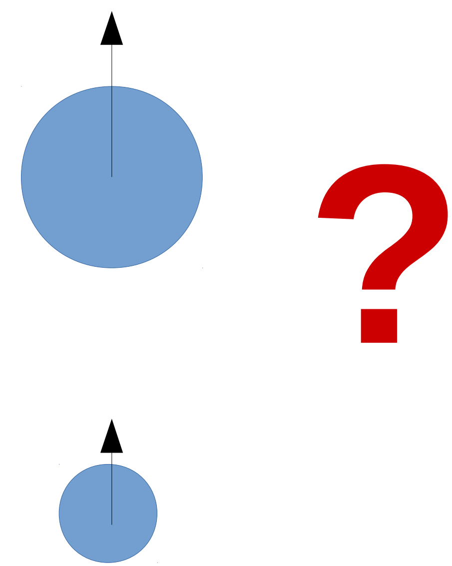
Let us ask a question...

Moderate Re

Instead an equal size pair if an unequal size pair rises then?



coalesce
bounce
slide



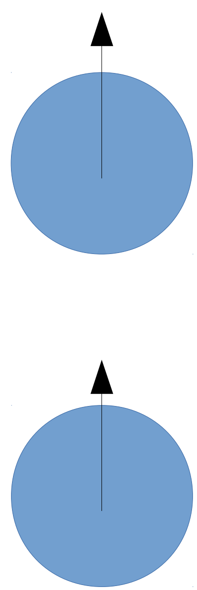
Experiments are right

In case of pair of bubbles

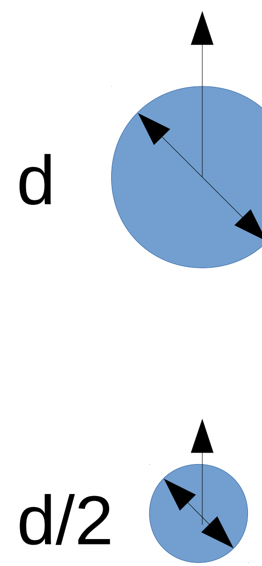
Let us ask a question...

Moderate Re

Instead an equal size pair if an unequal size pair rises then?



coalesce
bounce
slide



?

Experiments are right

In case of pair of bubbles

Let us ask a question...

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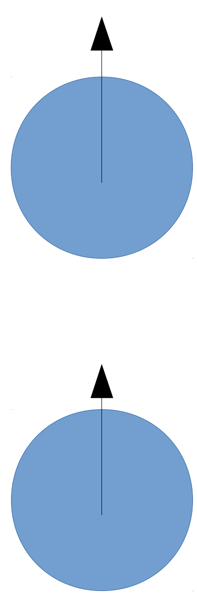

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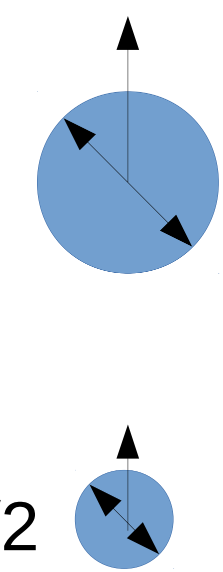
Experiments are right

Buoyancy force ~ d^3

d



?

d/2

Buoyancy force ~ $(1/8)d^3$

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Let us ask a question...

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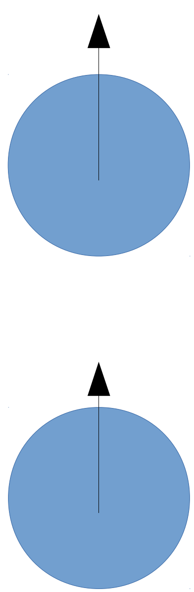

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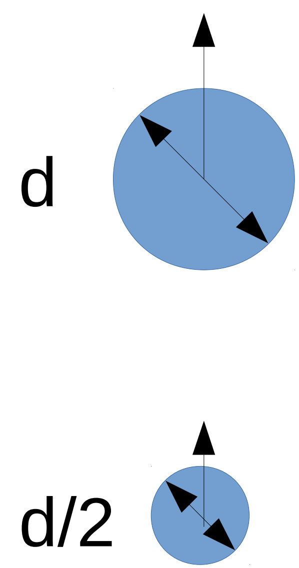
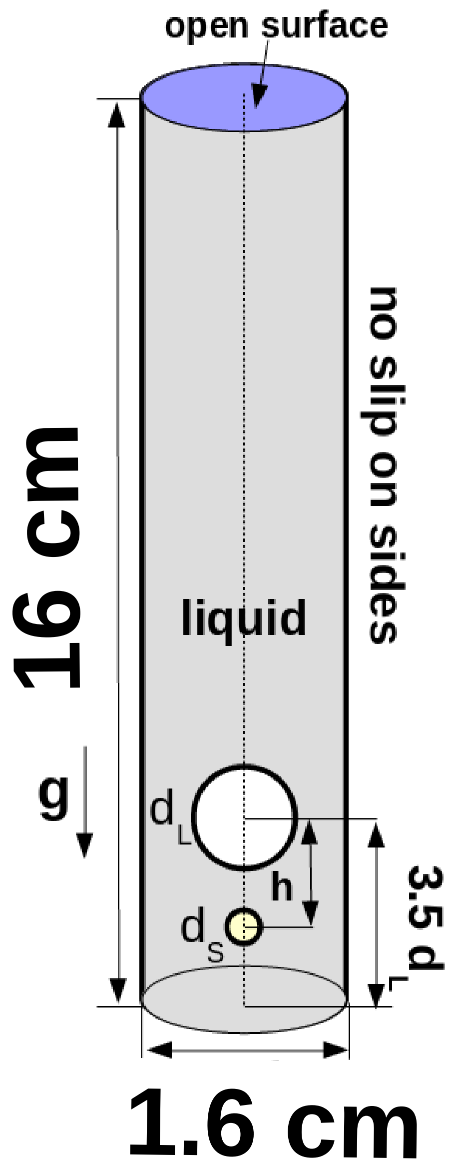


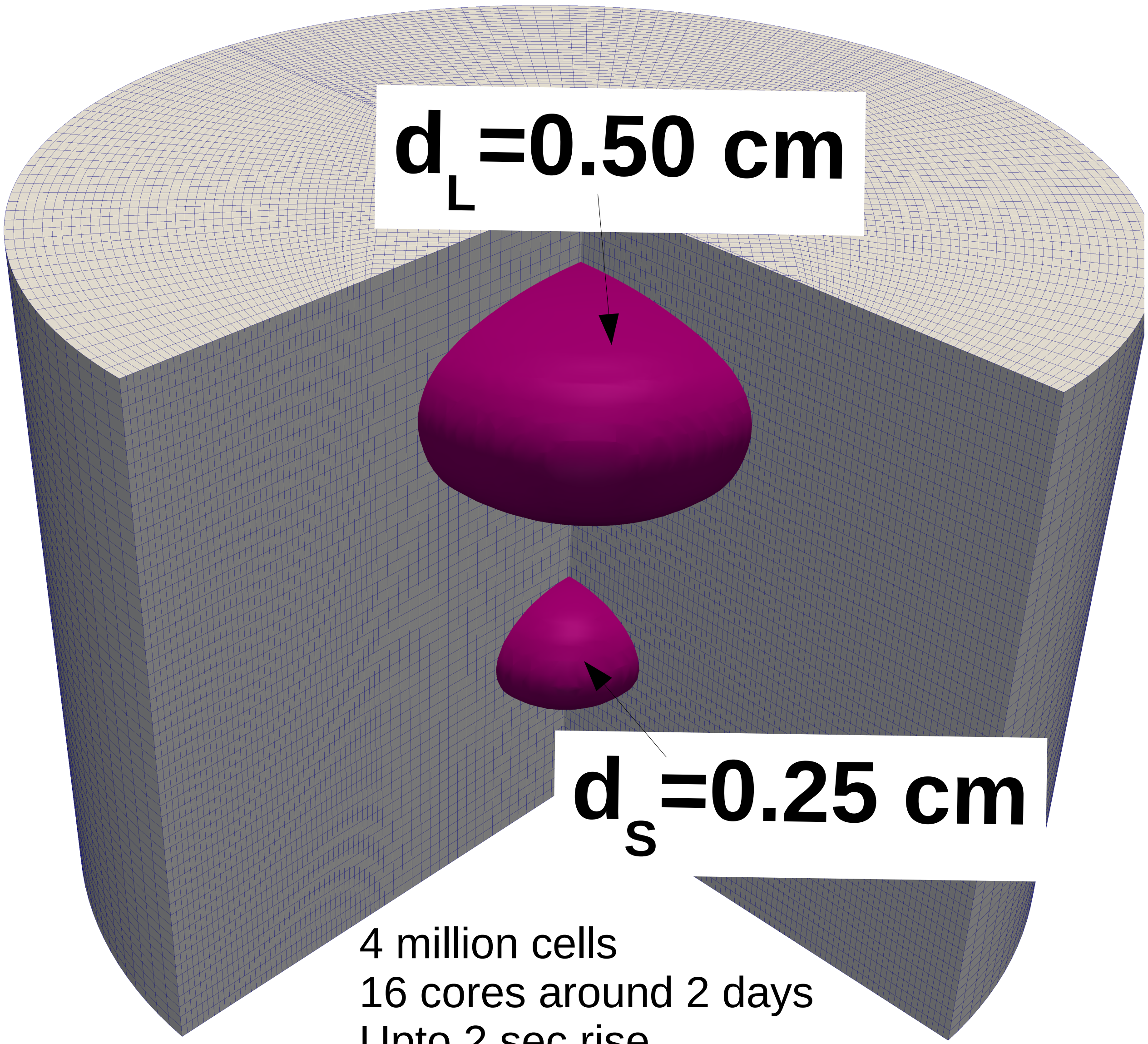
?

Buoyancy force ~ $(1/8)d^3$

Can this small bubble catches-up?

Inline pair - Simulations





4 million cells
16 cores around 2 days
Upto 2 sec rise

Inline pair – Simulations

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial(\rho_m \mathbf{u})}{\partial t} + \nabla \cdot (\rho_m \mathbf{u}\mathbf{u}) = -\nabla p + \nabla \cdot [\mu_m(\nabla \mathbf{u} + \nabla \mathbf{u}^T)] + \rho_m \mathbf{g} + \mathbf{f}_s$$

$$\rho_m = \rho_g \alpha + \rho_l (1 - \alpha)$$

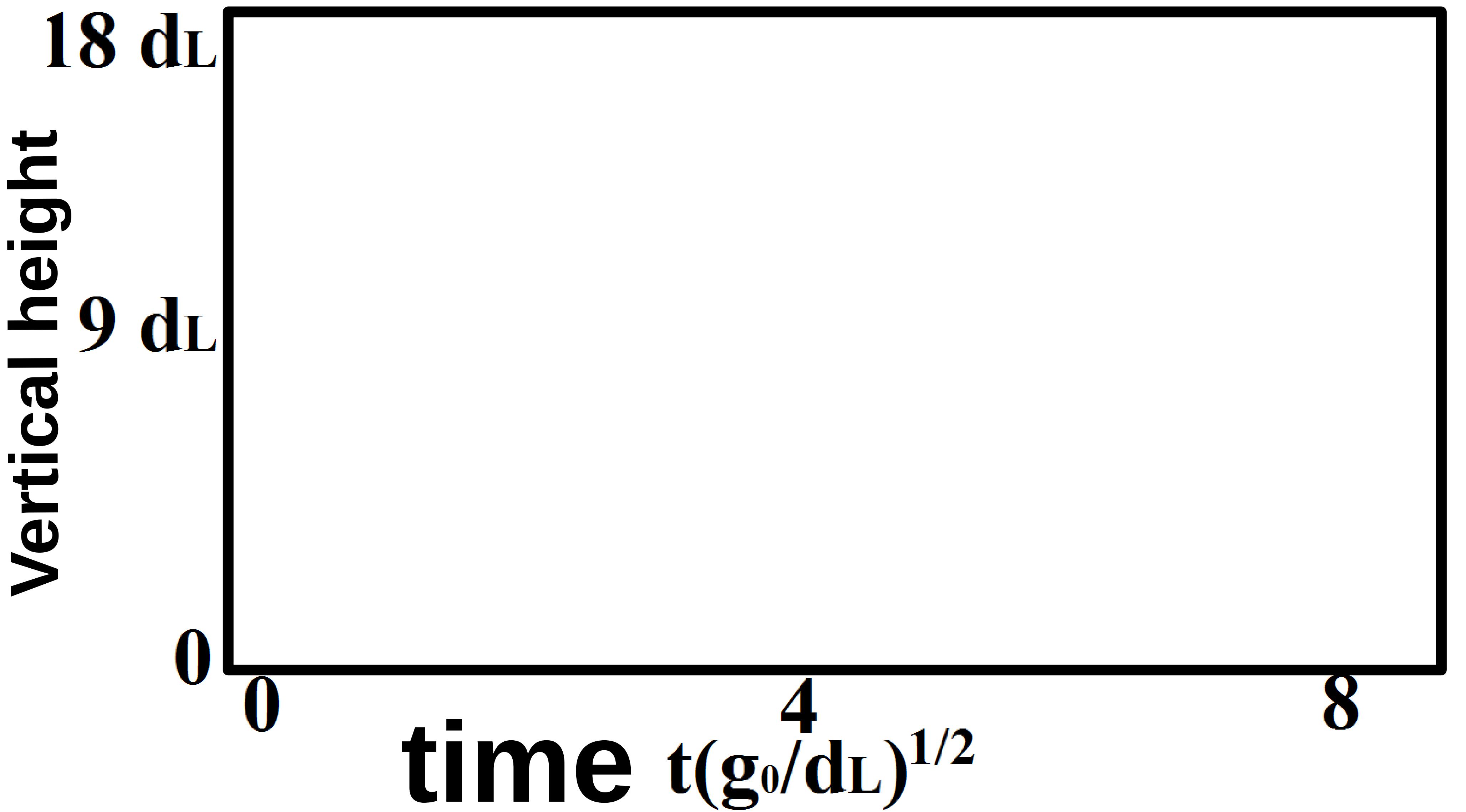
$$\mathbf{f}_s = \gamma \kappa \nabla \alpha$$

$$\mu_m = \mu_g \alpha + \mu_l (1 - \alpha)$$

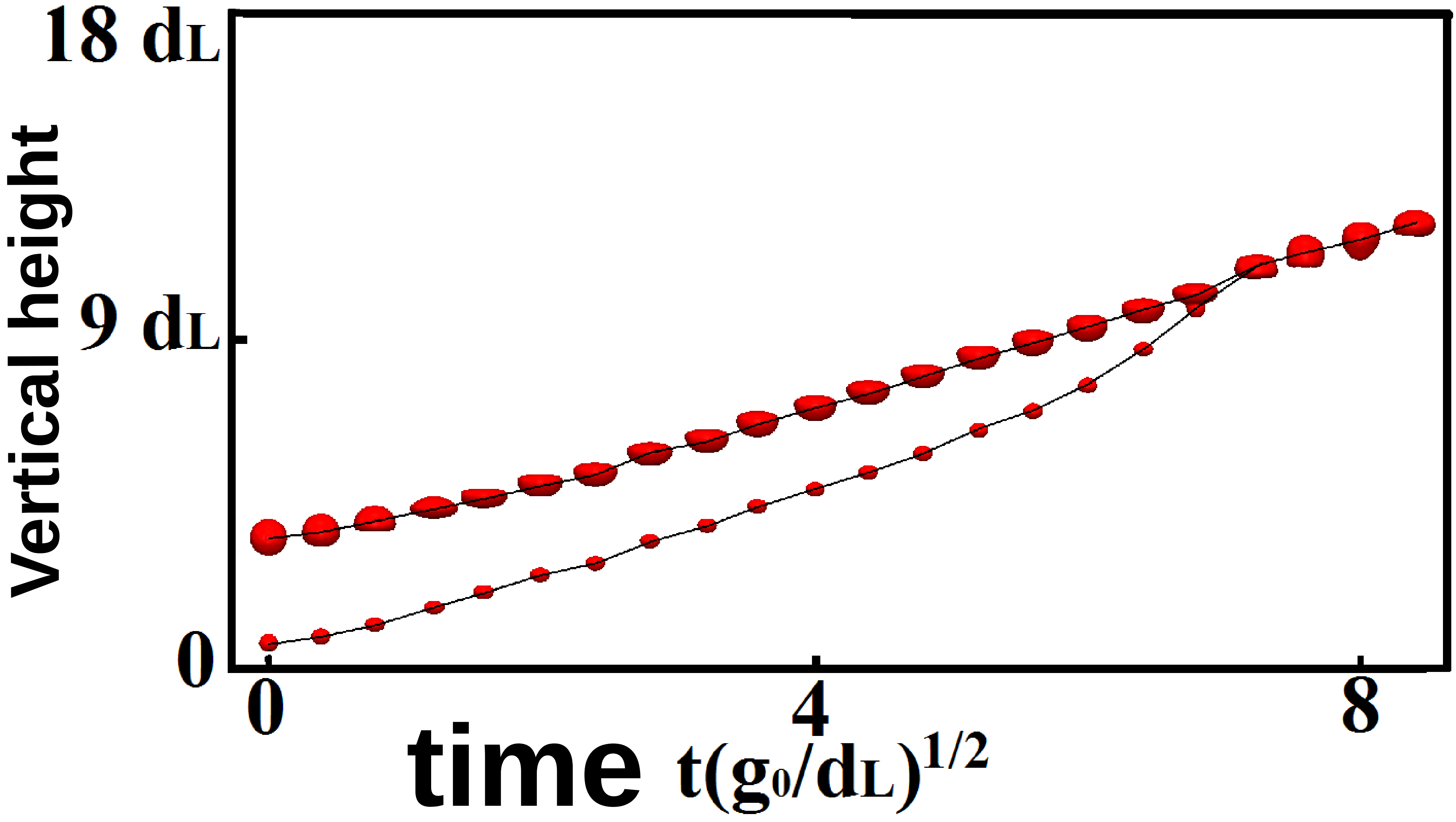
$$\frac{\partial \alpha}{\partial t} + \nabla \cdot (\alpha \mathbf{u}) - \nabla \cdot [\alpha(1 - \alpha)\mathbf{u}_r)] = 0$$

$$\mathbf{u_r} = \mathbf{n} \, \min(c_\gamma \frac{|\phi|}{|S|}, max(\frac{|\phi|}{|S|}))$$

Our observation

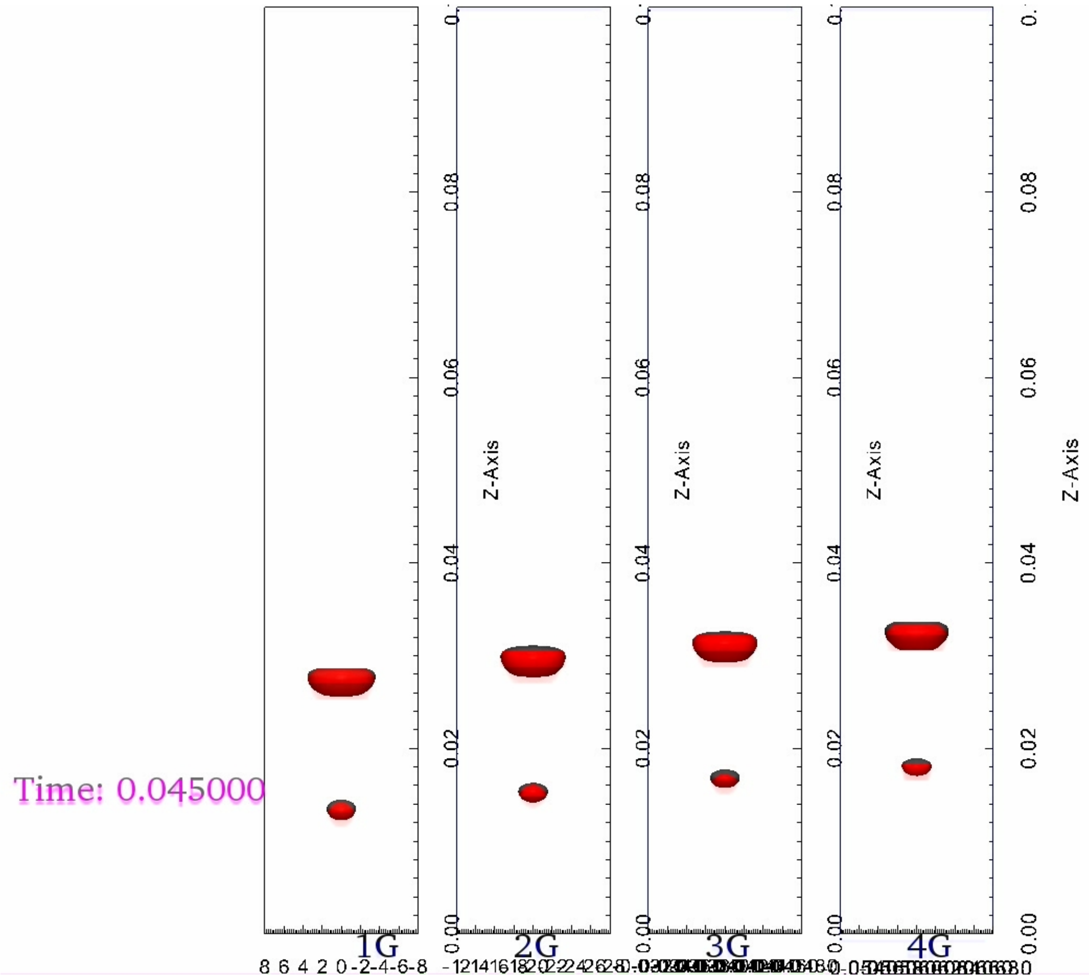


Our observation



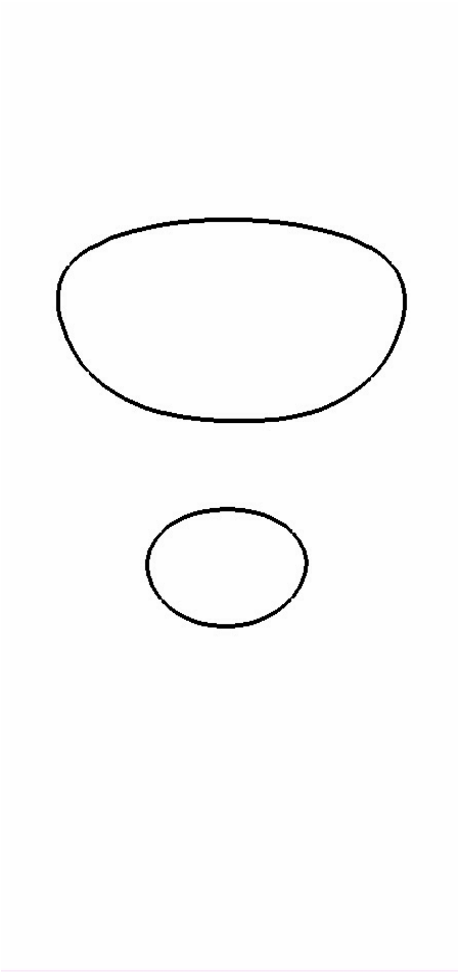
Increase in buoyancy

Increase in buoyancy



Shape dynamics

Shape oscillations during coalescence

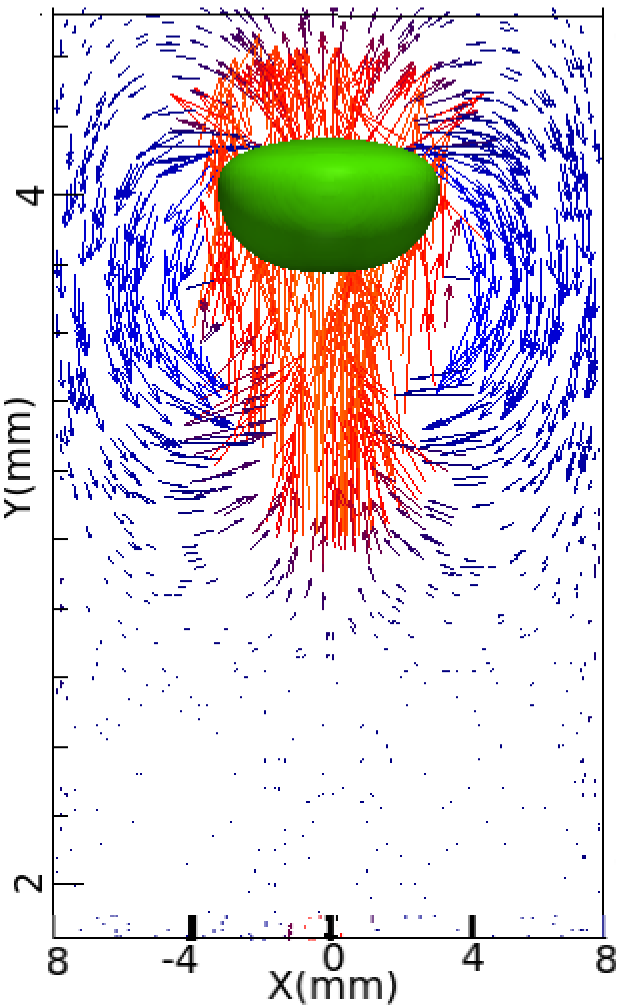


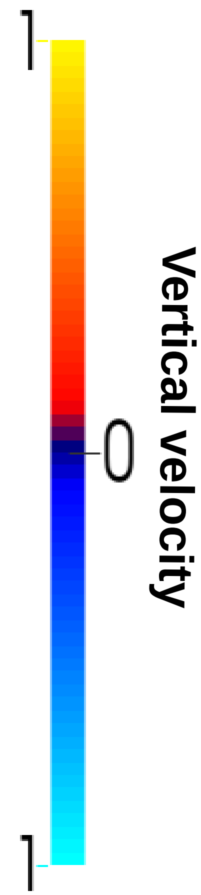
Let us look at velocity vectors

Let us look at velocity vectors

Leading bubble's wake

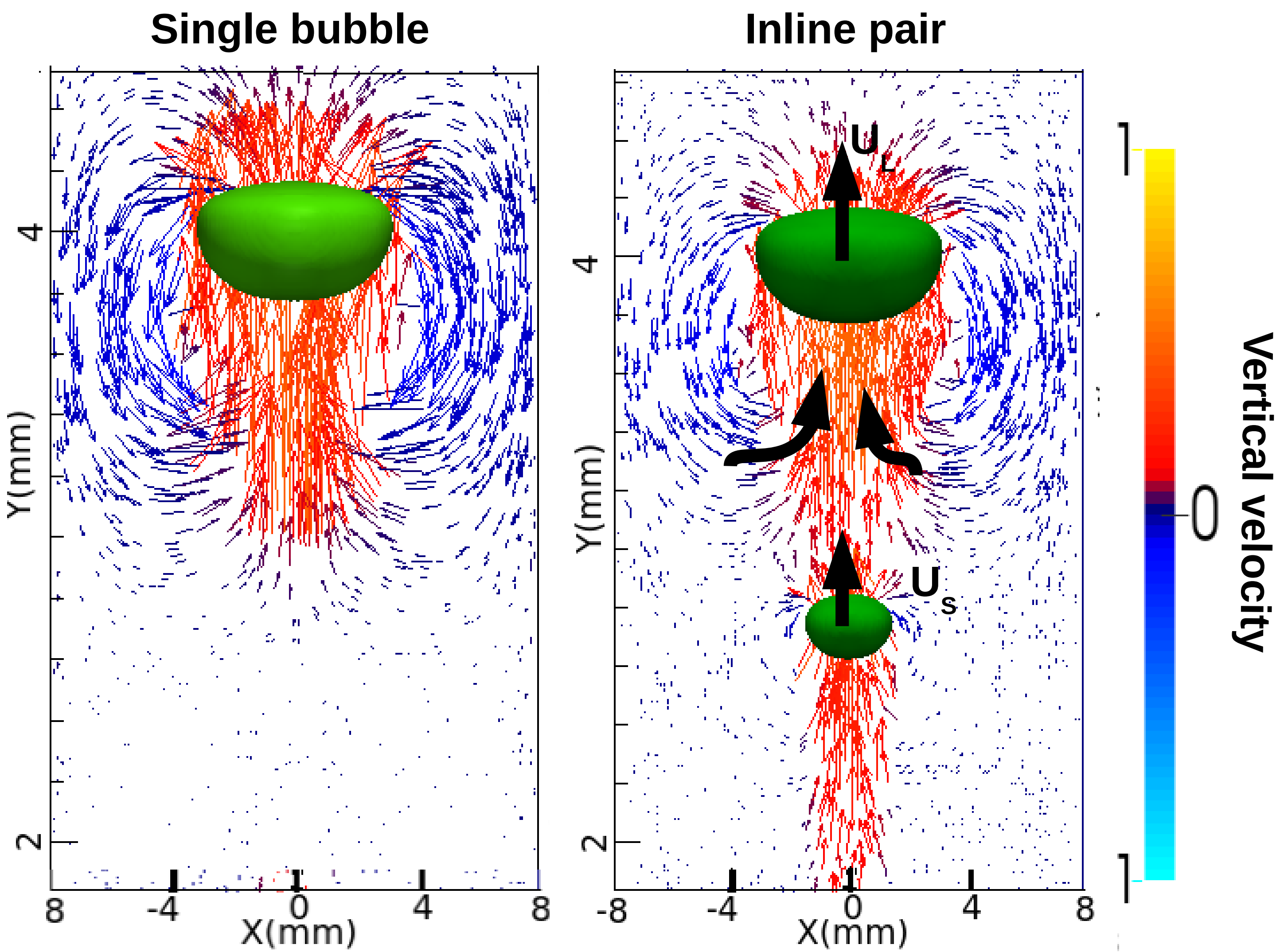
Single bubble





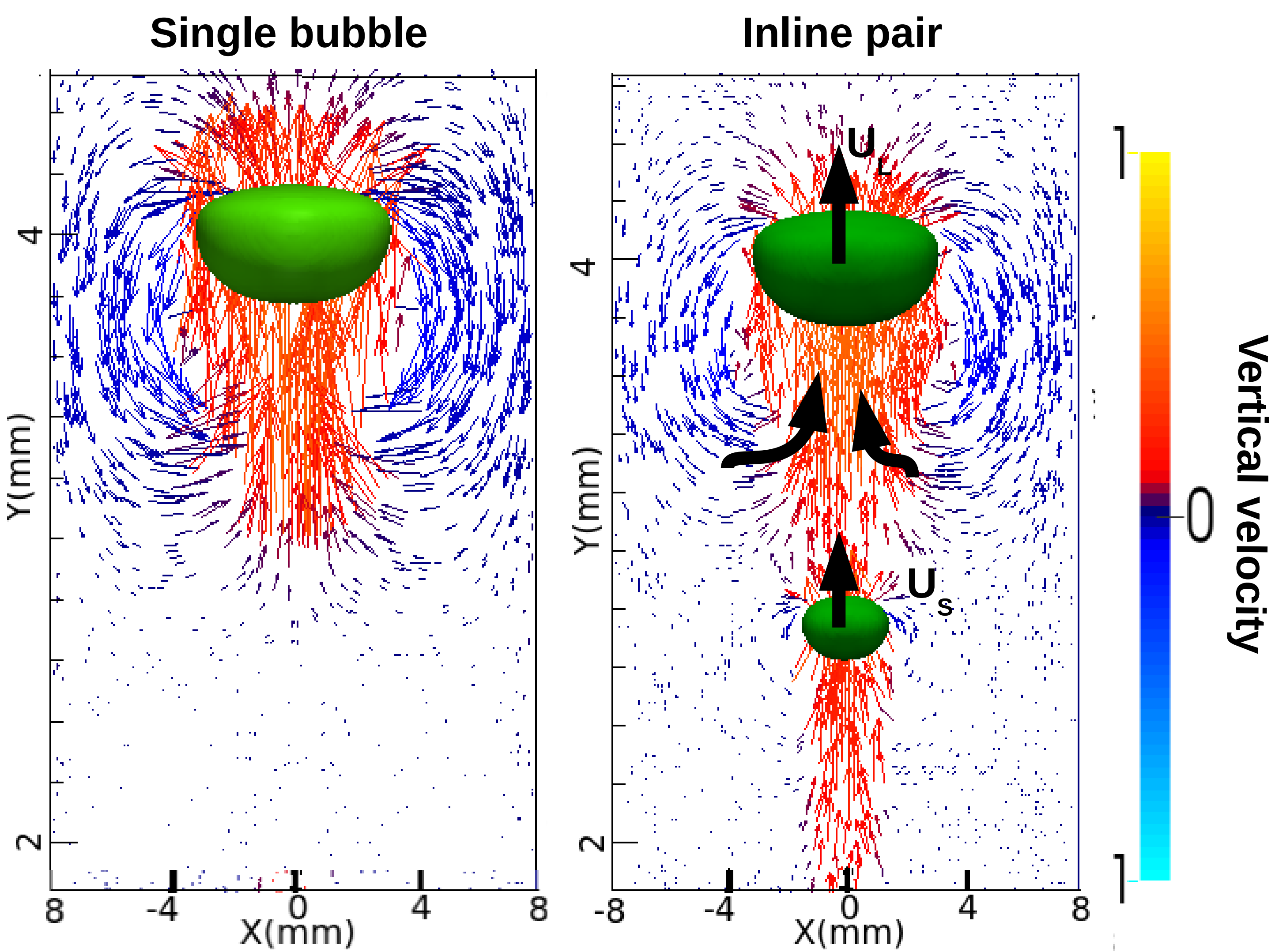
Let us look at velocity vectors

Leading bubble's wake



Leading bubble's wake plays a role

Leading bubble's wake play a big role- so trailing bubble speeds-up



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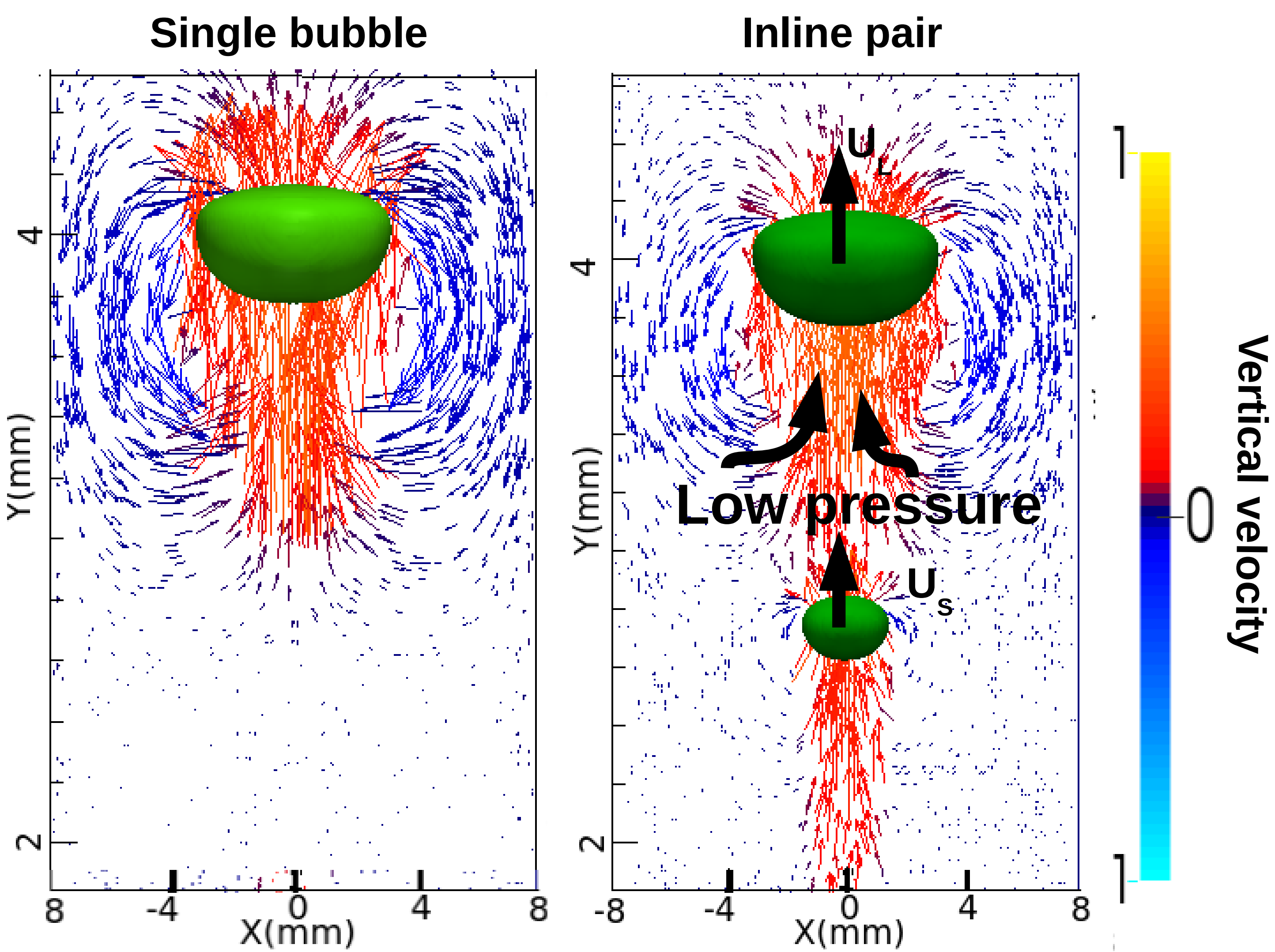

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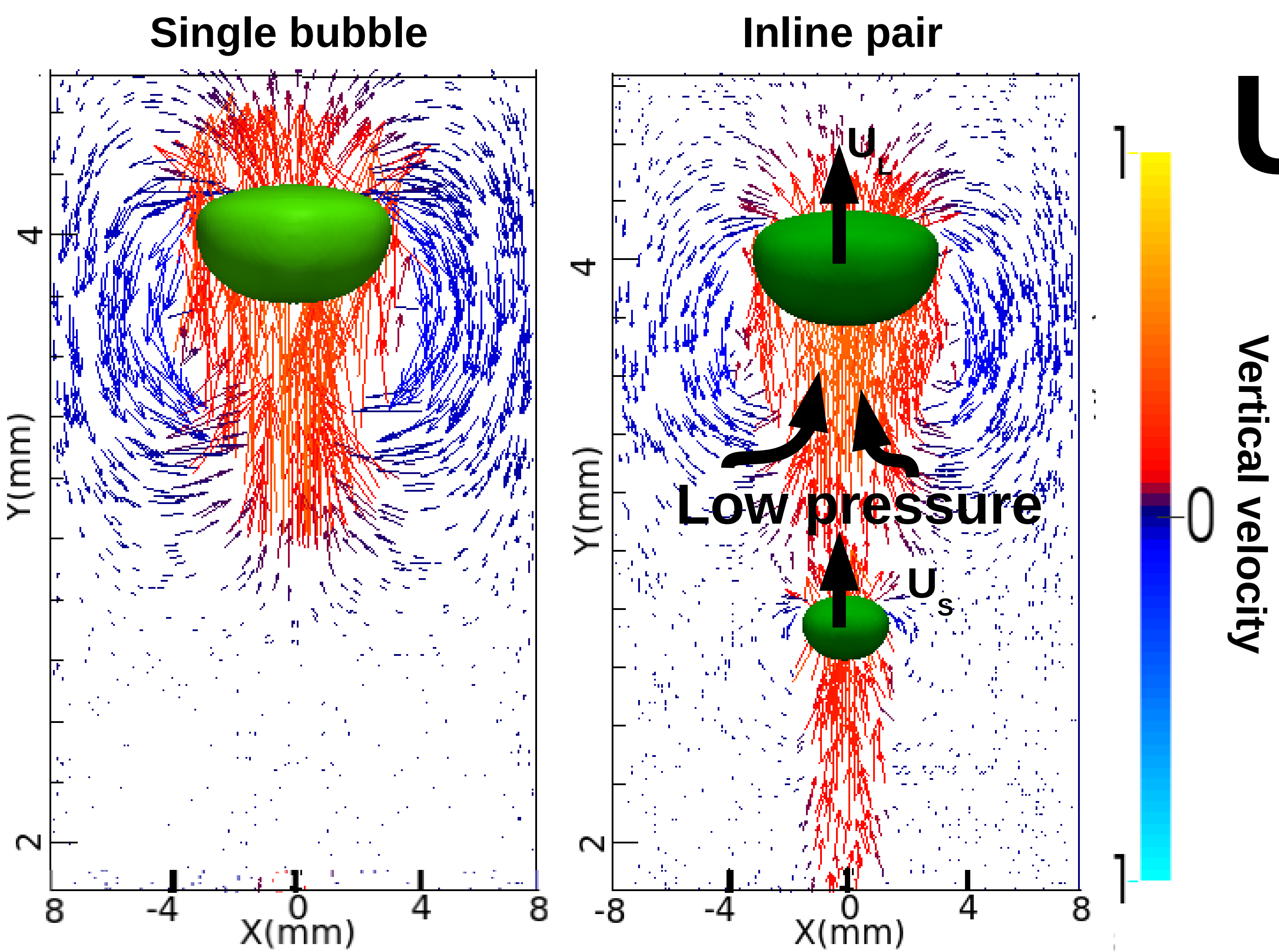

$$U_S/U_L = ?$$

Wake is very complex

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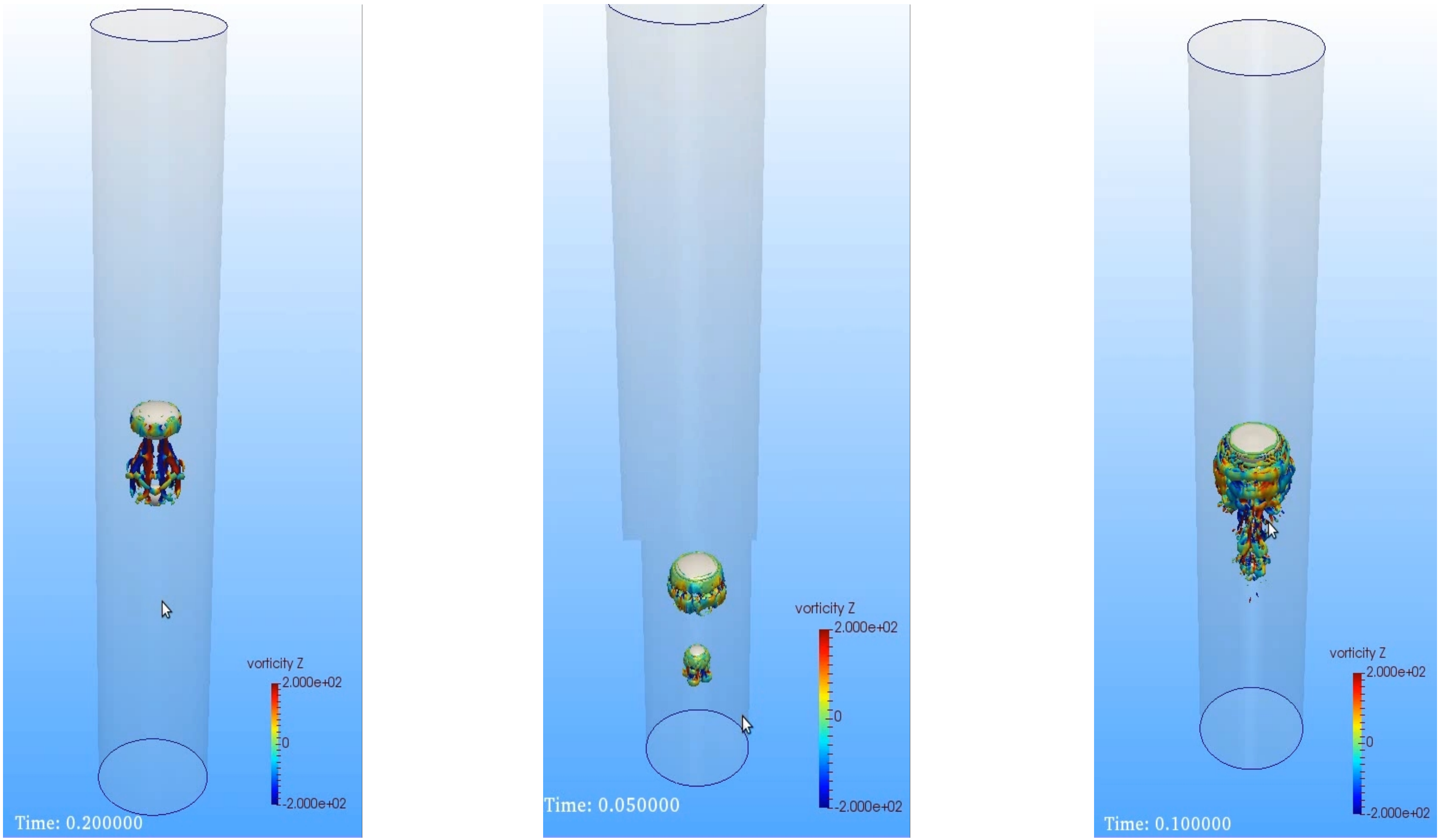
Q-criterion based vortex structures
Color mapped by streamwise vorticity

- Hairpin like structures during rise
- Jet ejections away from bubble
- Toroidal roll development at coalescence

Wake is very complex

Buoyancy increases →



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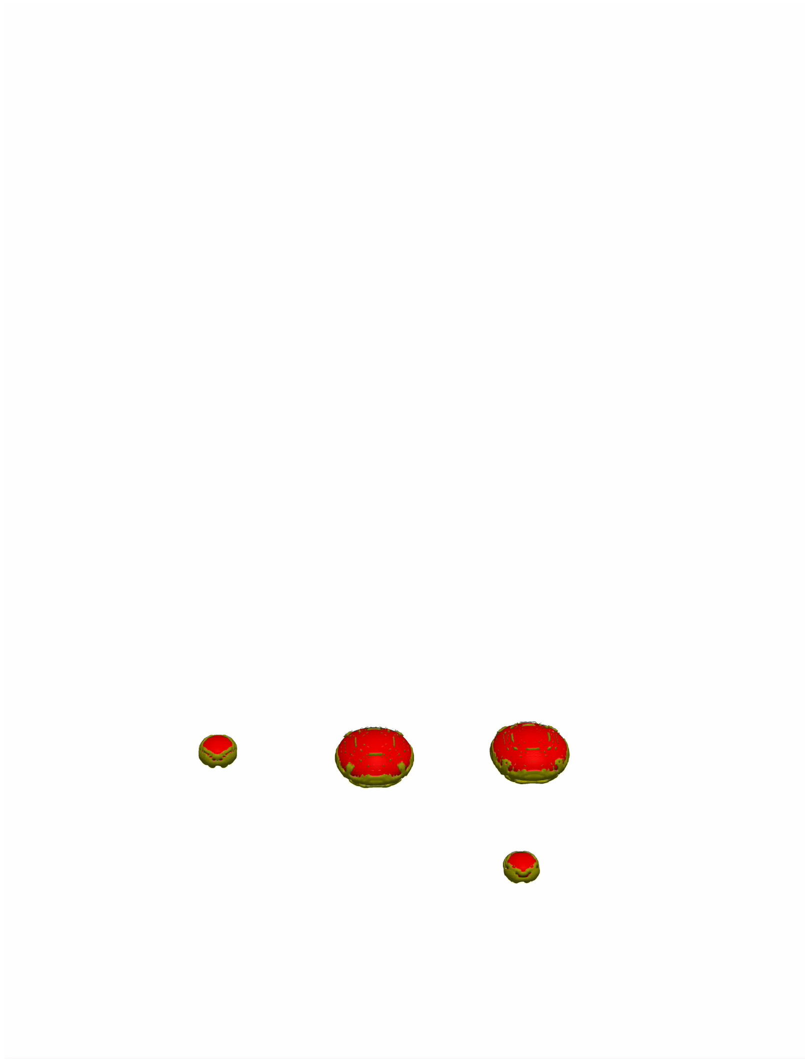

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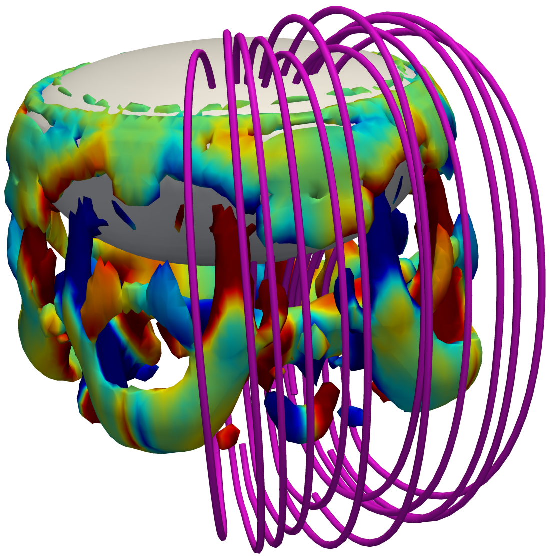
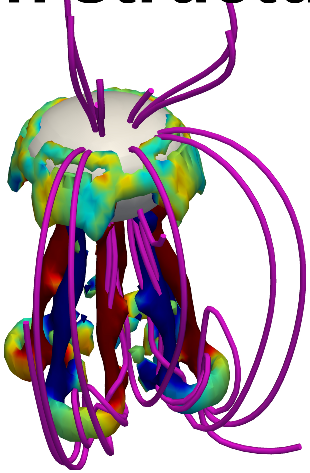
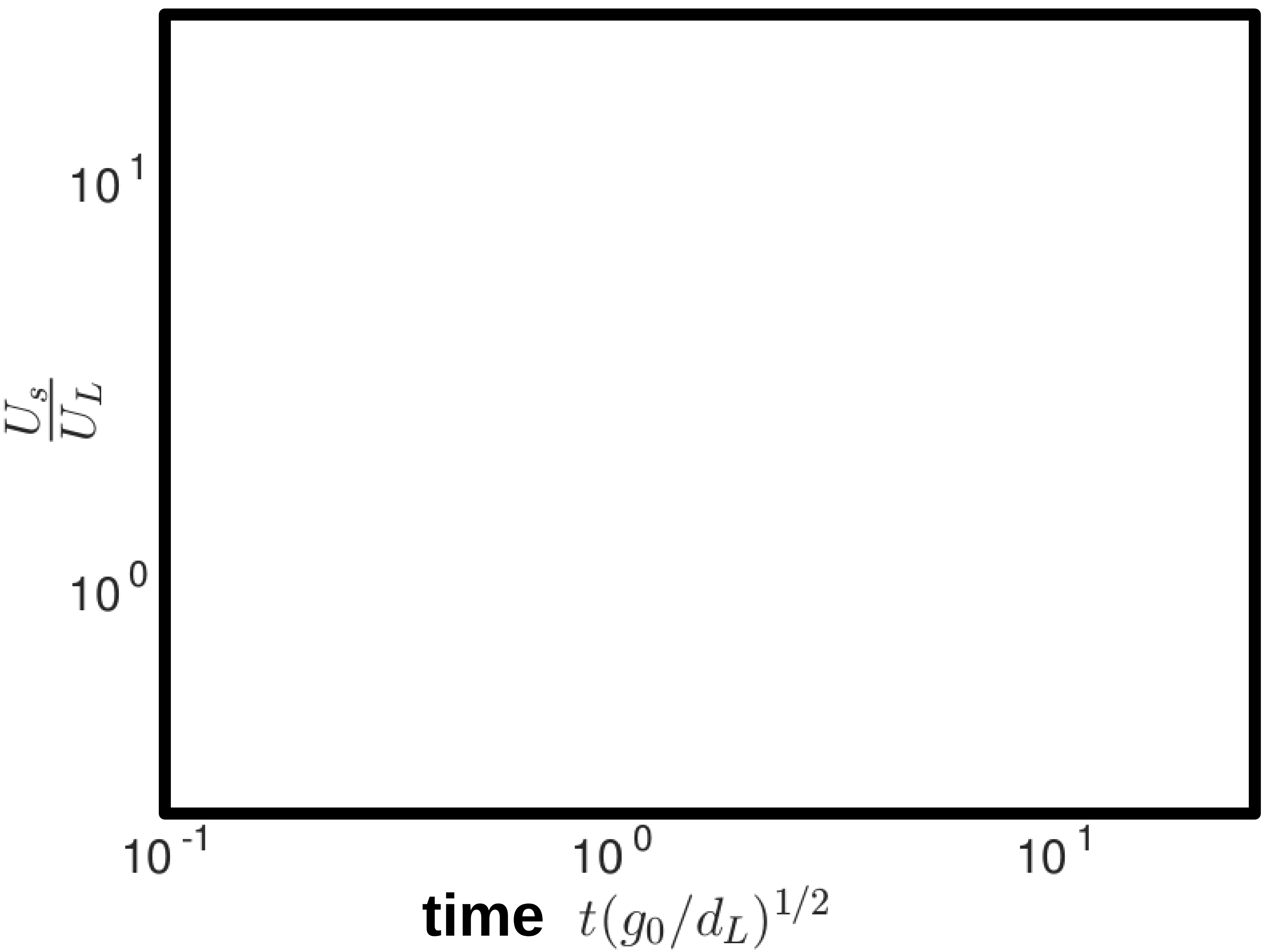


Streamlines twined on hairpin structures



Trailing bubble speed: U_S/U_L ratio



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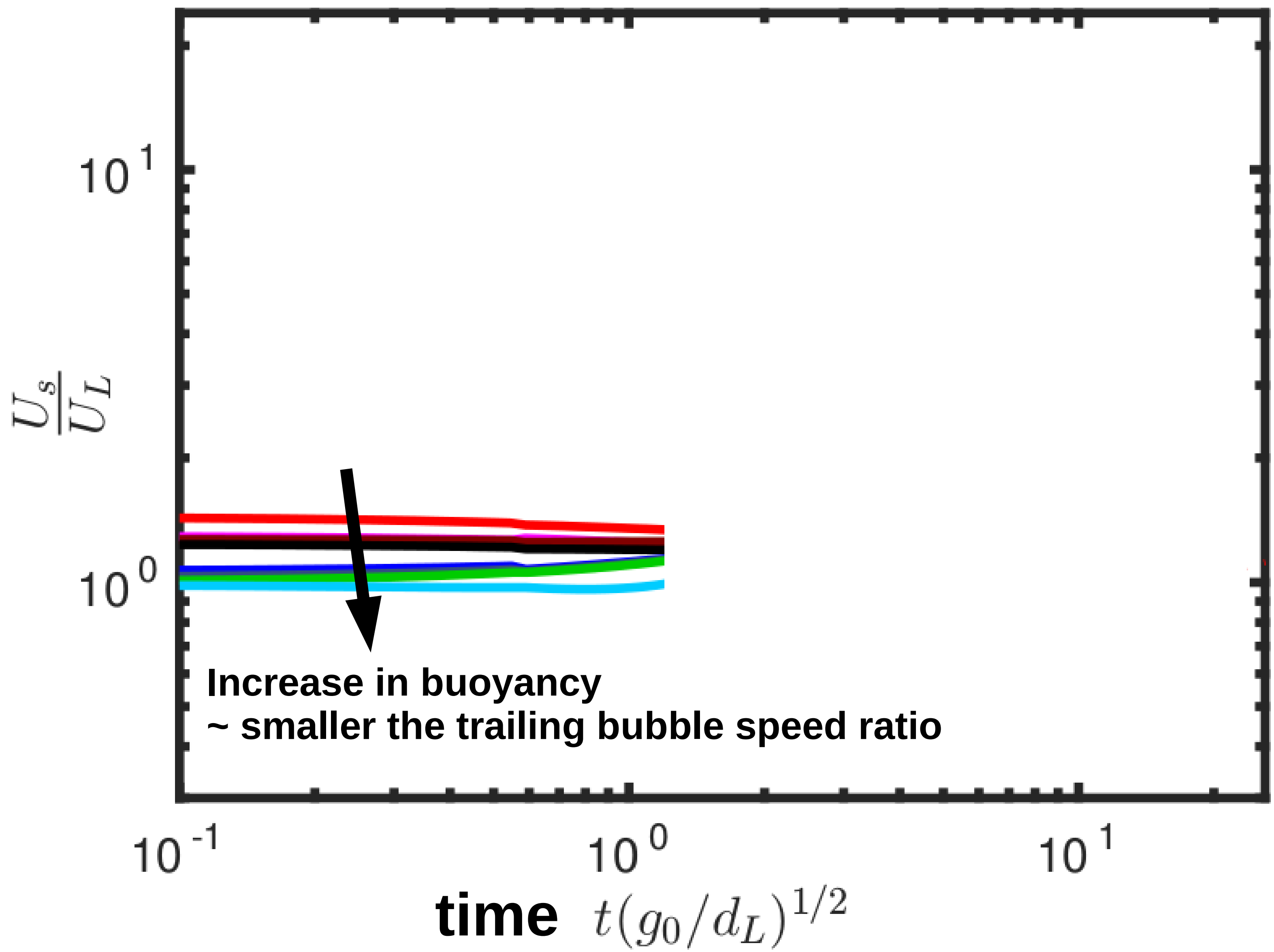

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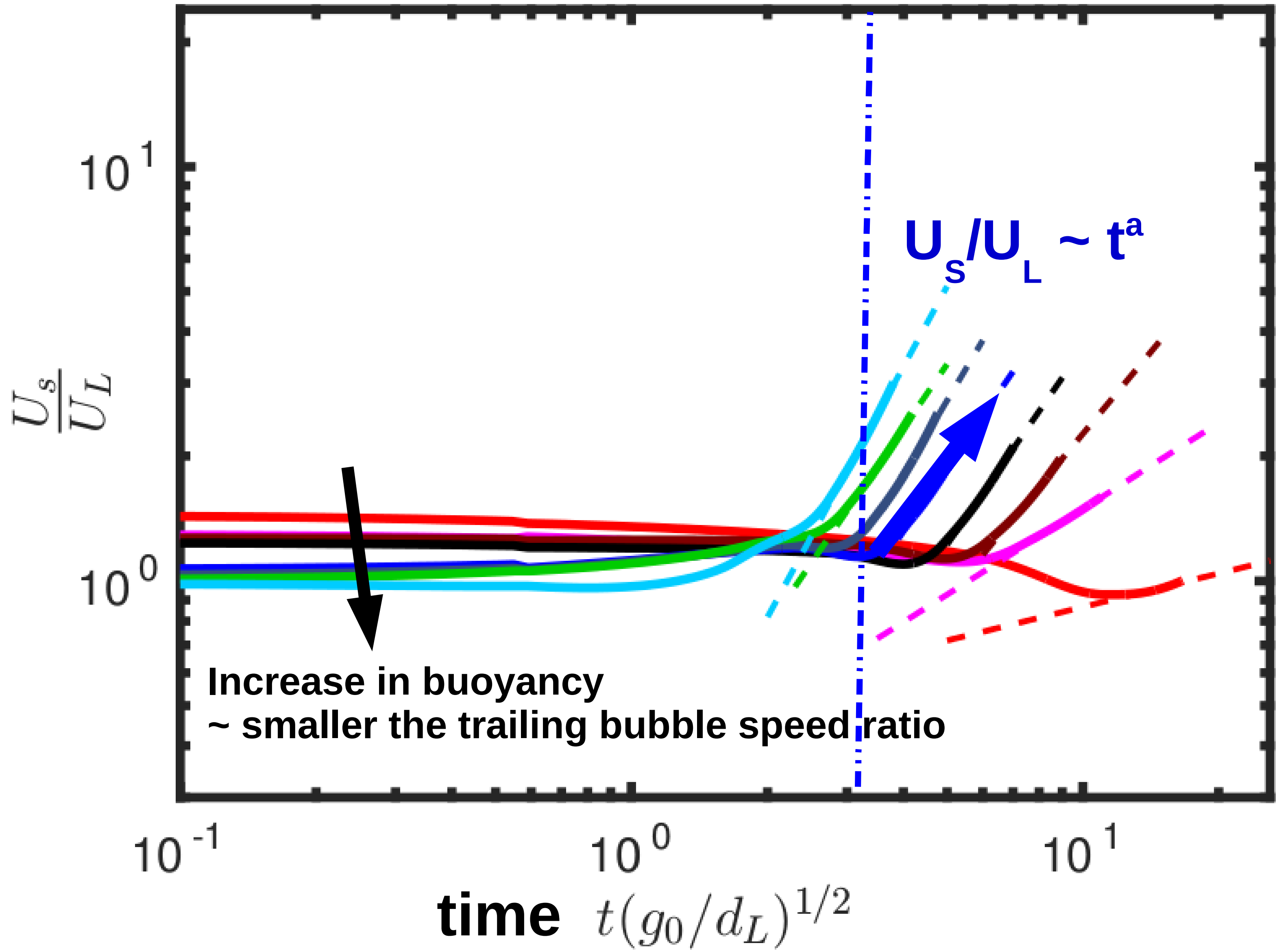

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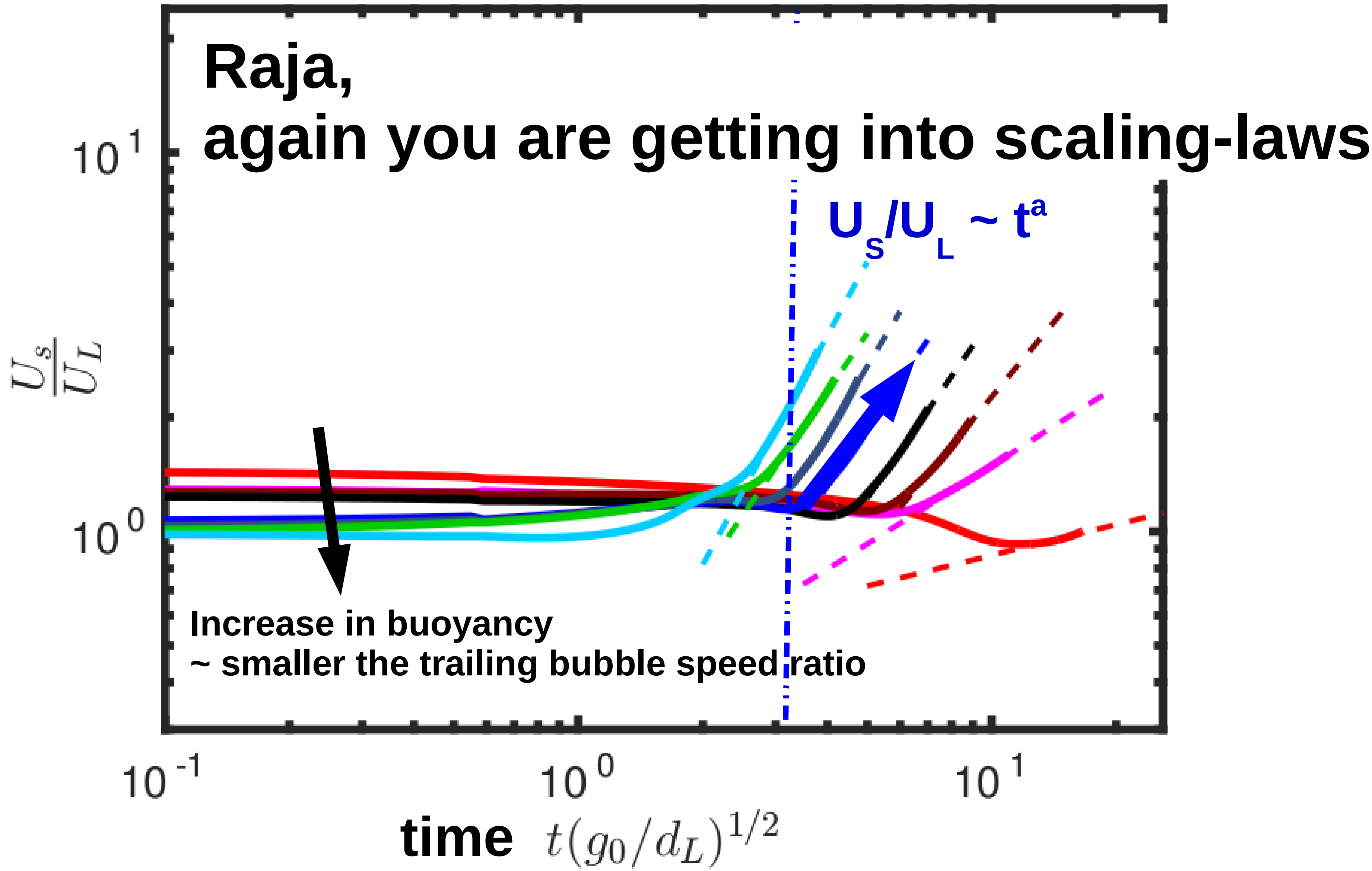

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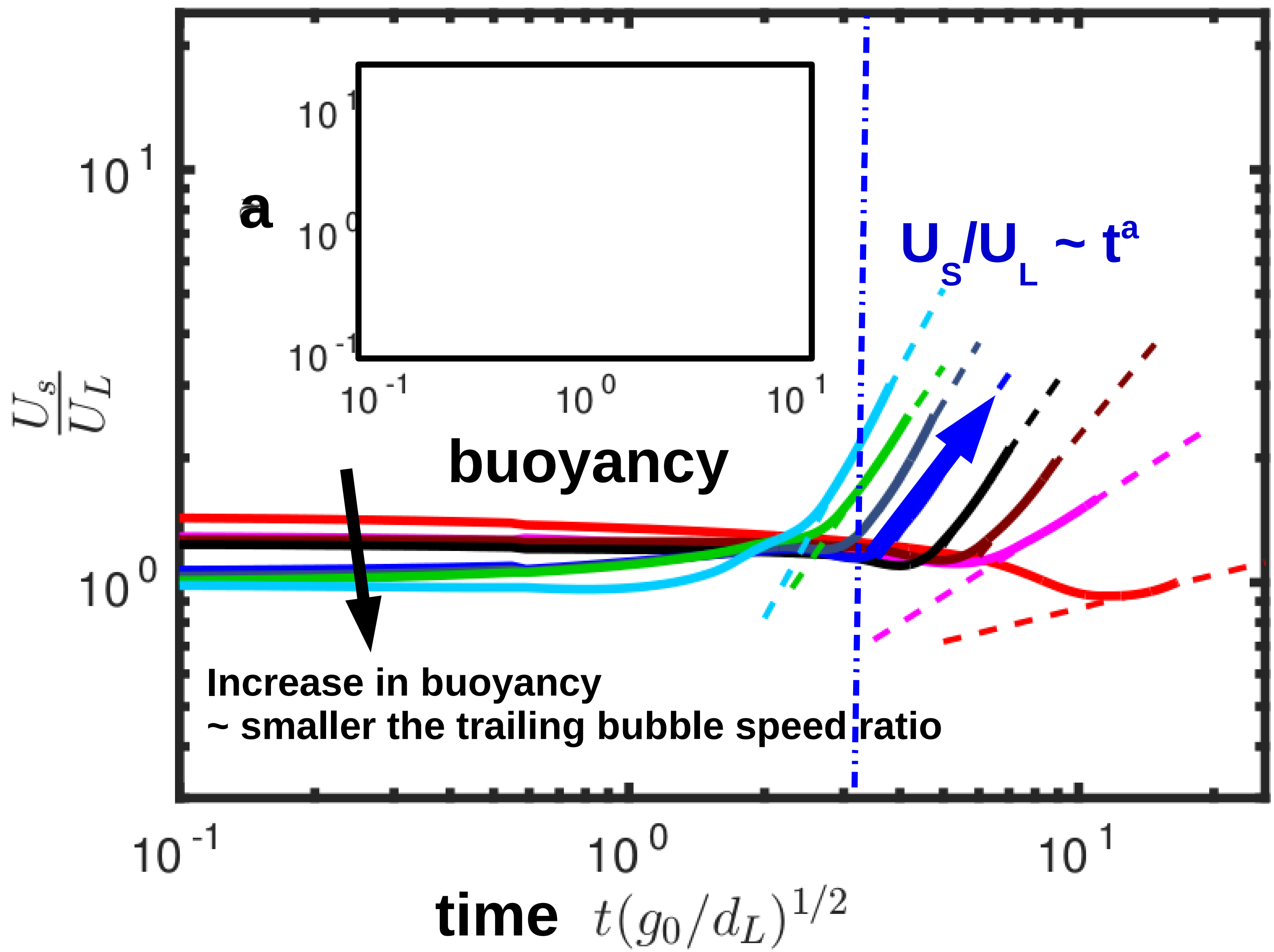

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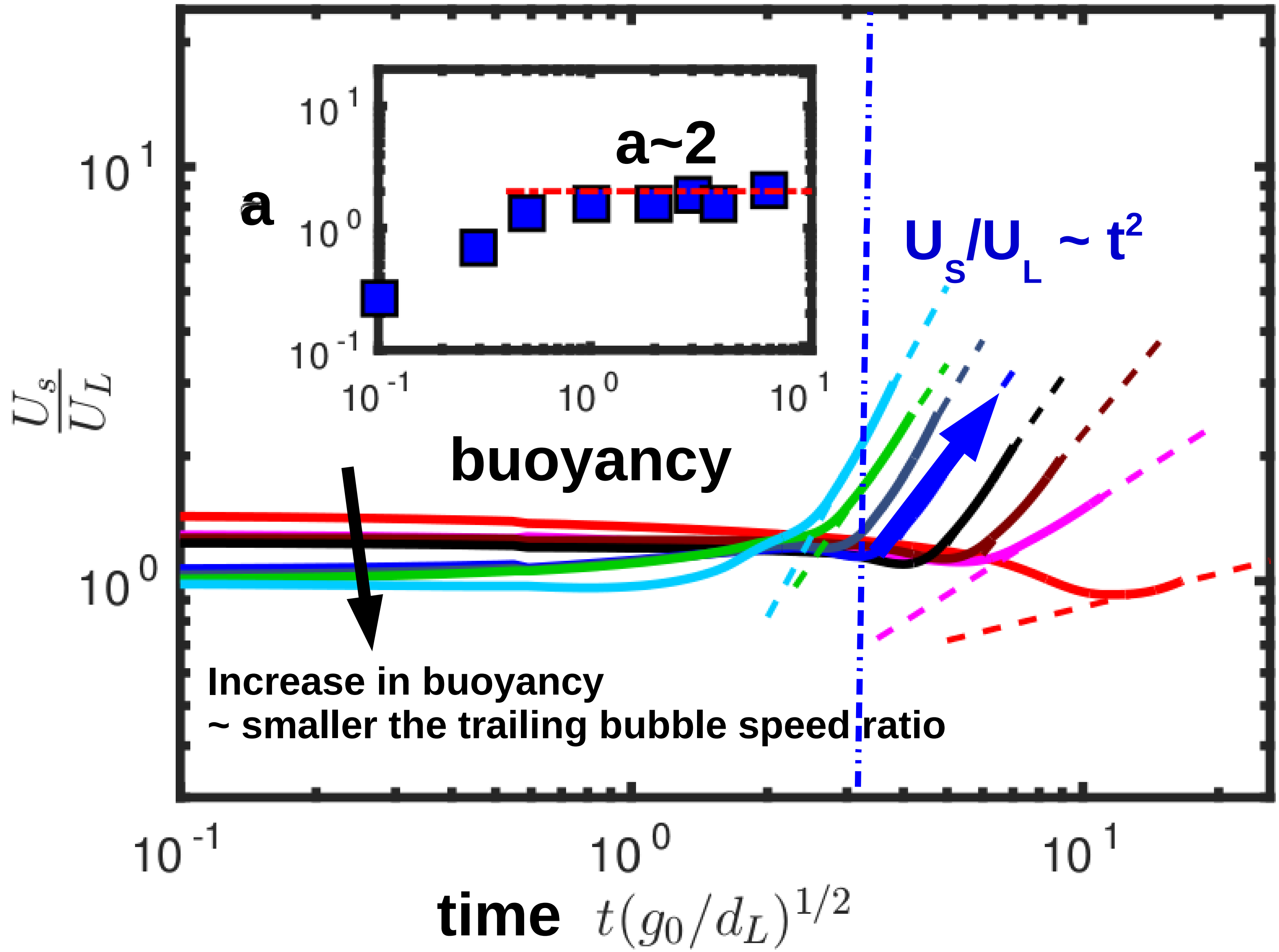

Liquid velocity fluctuations?

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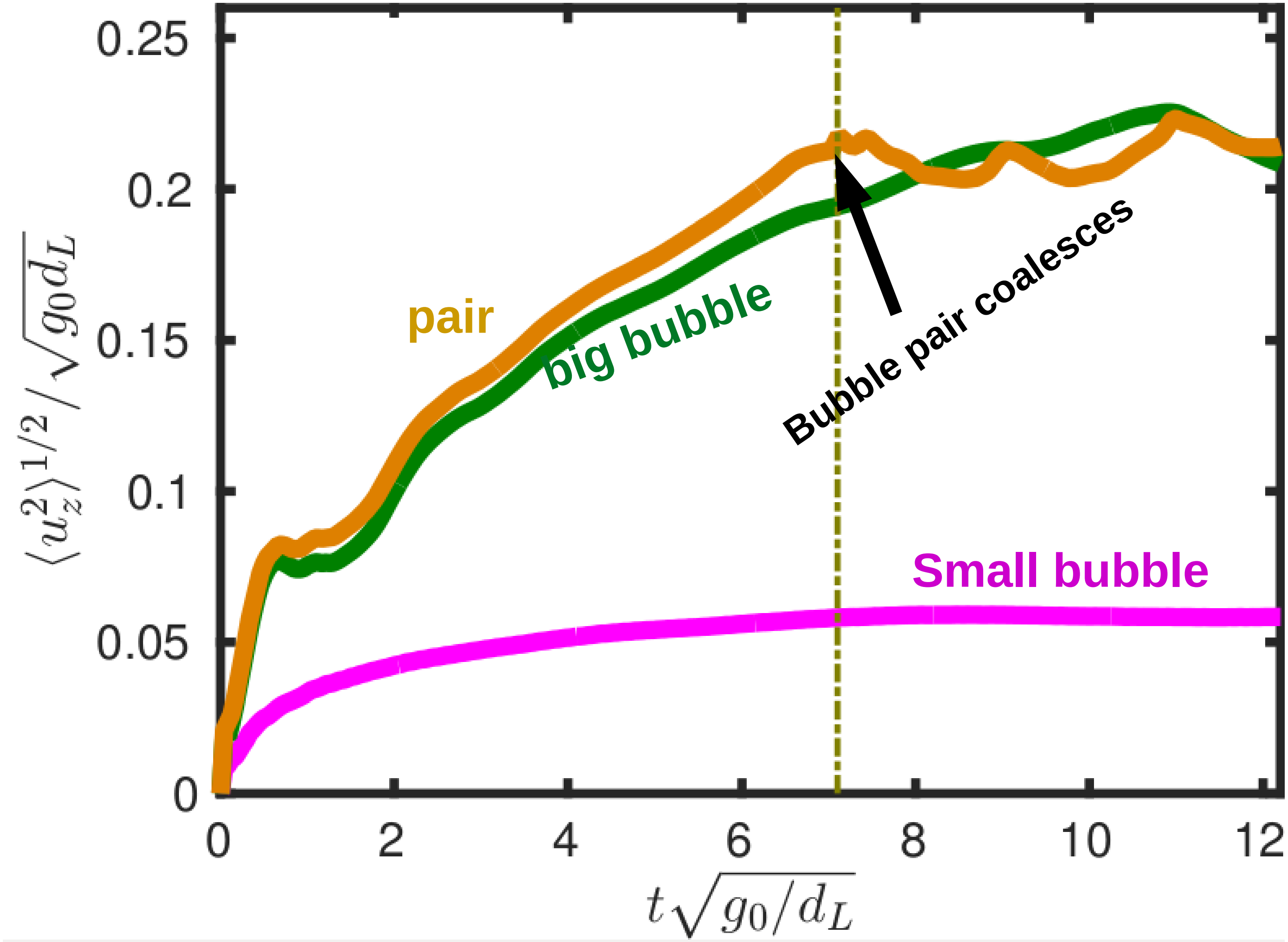
Vertical component is dominant



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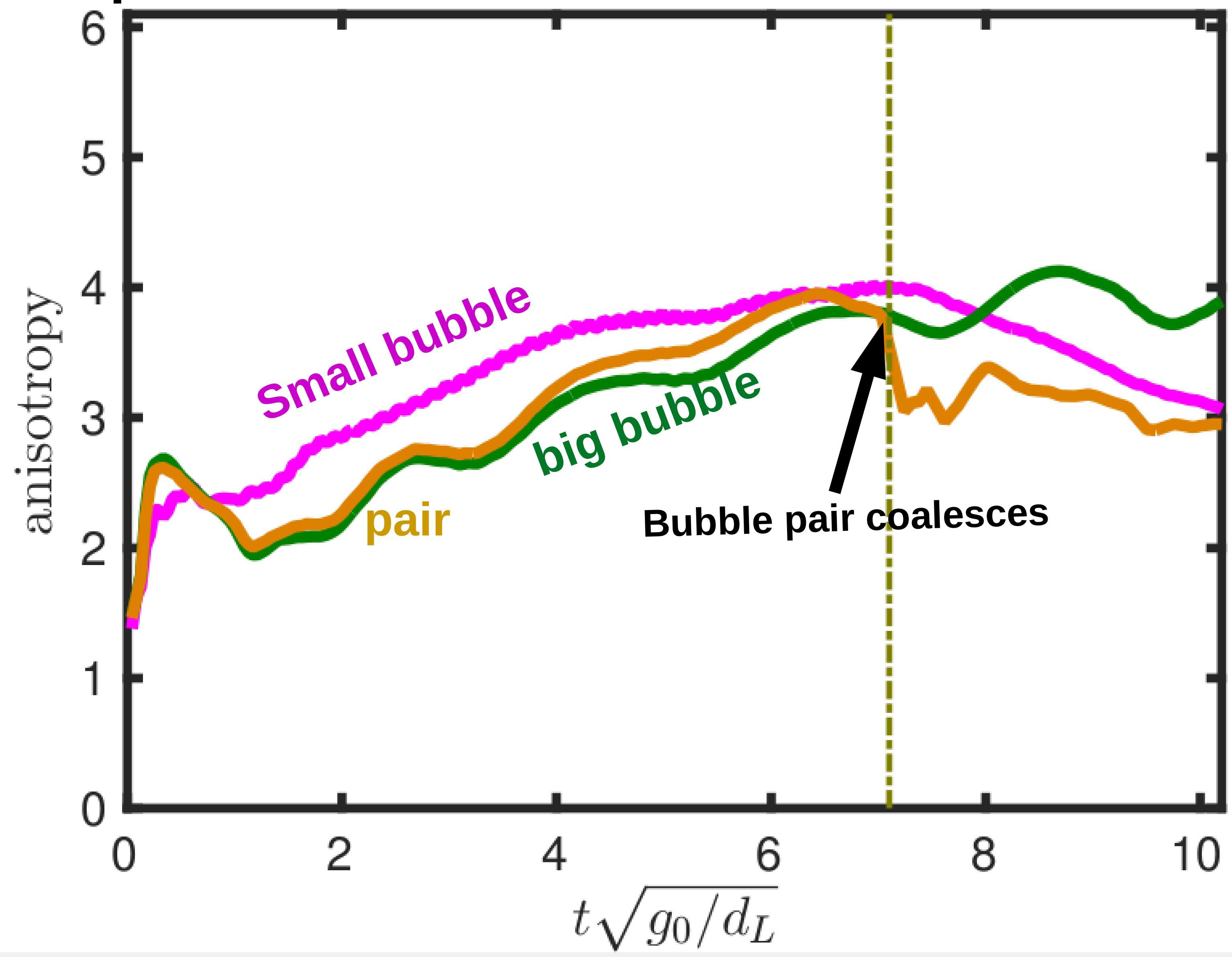

KE increases in liquid as bubbles rise

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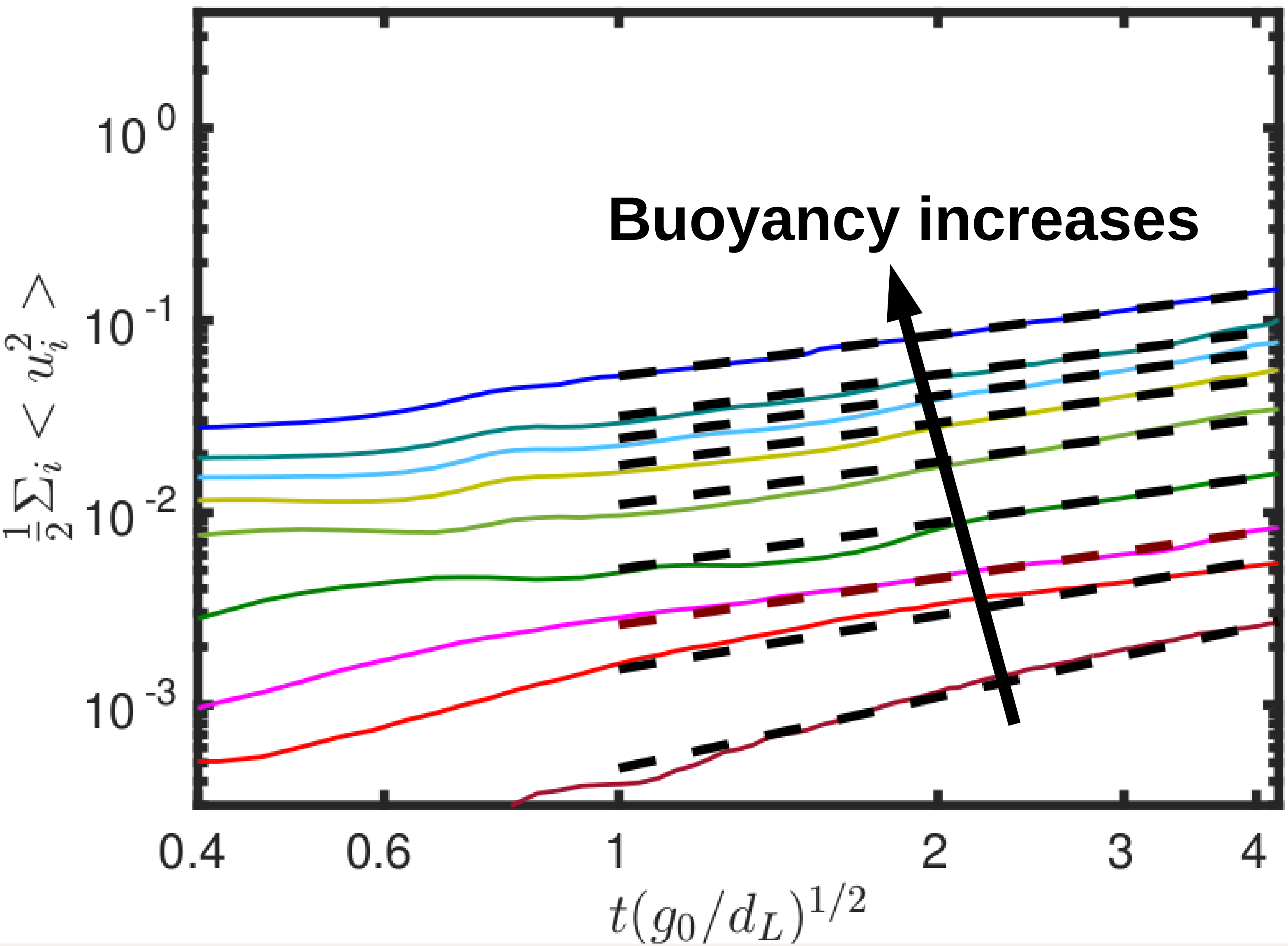
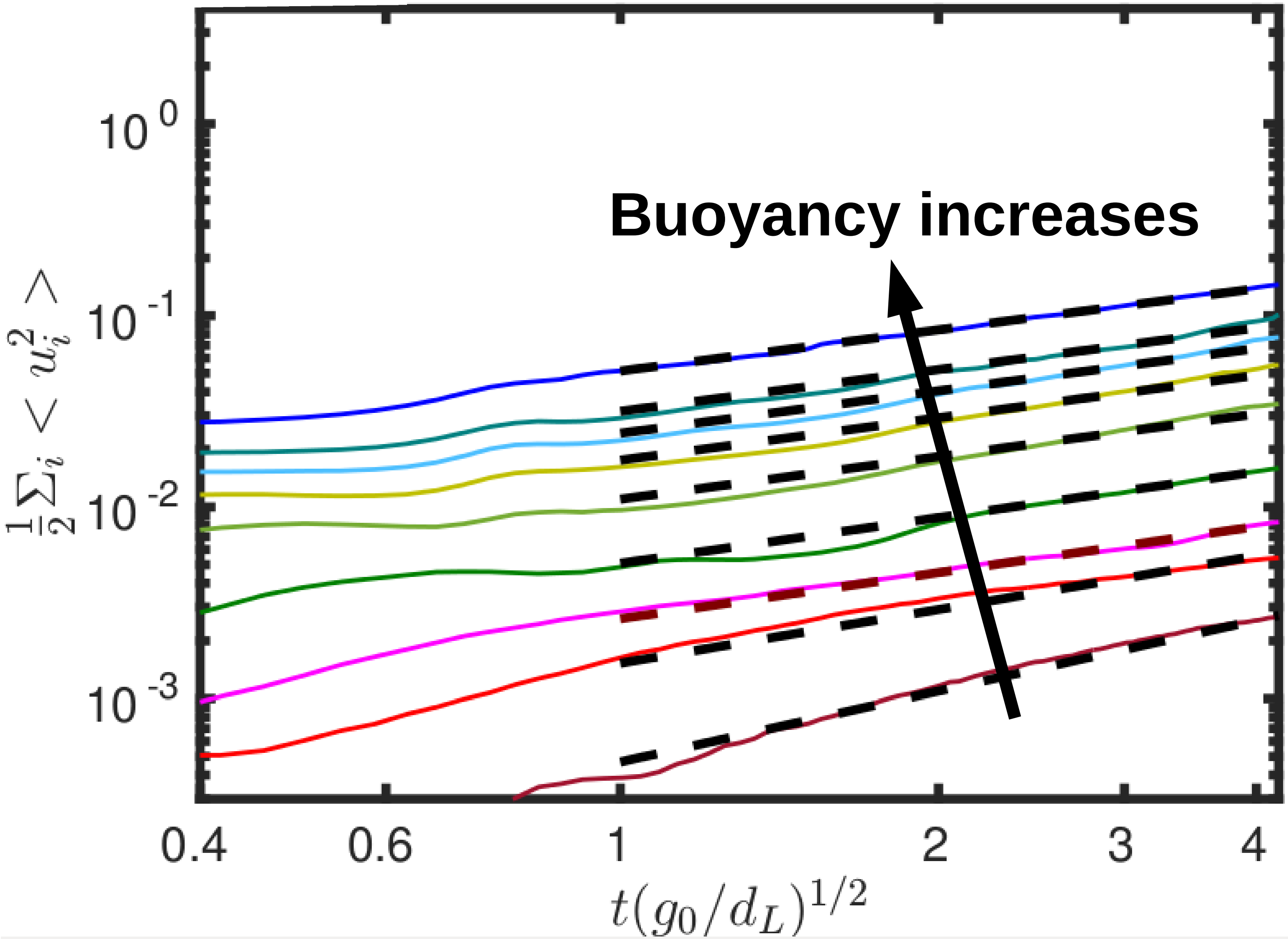


KE increases in liquid as bubbles rise

$$\frac{1}{2}\Sigma_i < u_i^2 > \propto t^\gamma$$



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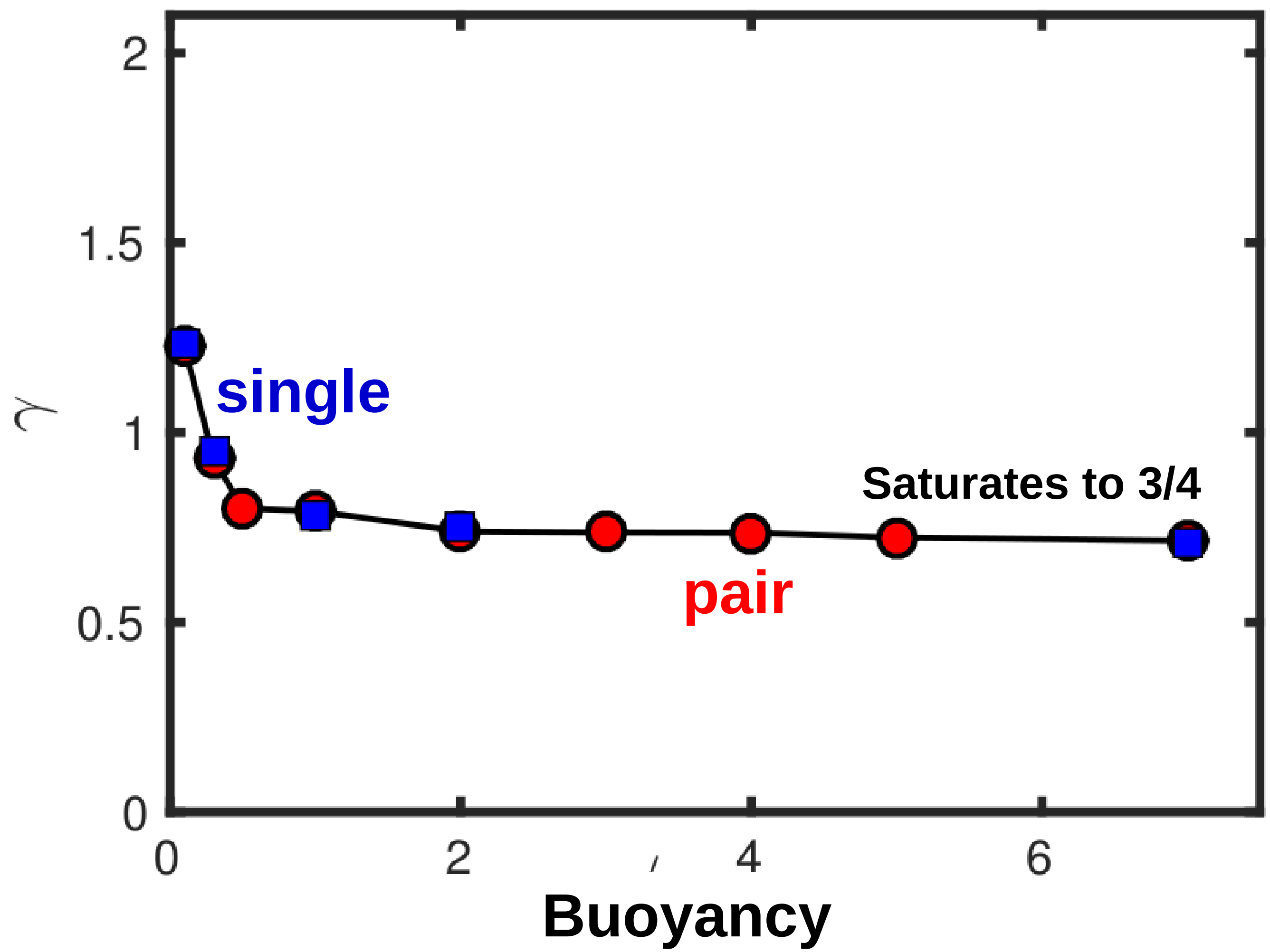

Summary

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- **Small trailing bubbles speeds-up in the wake, $U_S/U_L \sim t^2$**
- **Due to bubble rise- liquid vertical velocity component is dominant**
- **Liquid KE increases with time** $\frac{1}{2}\Sigma_i < u_i^2 > \propto t^\gamma$

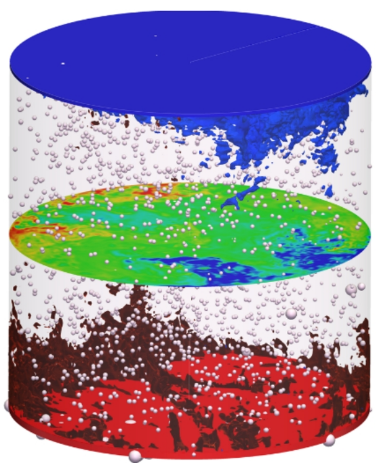
What happend with boiling...

What happend with boiling...

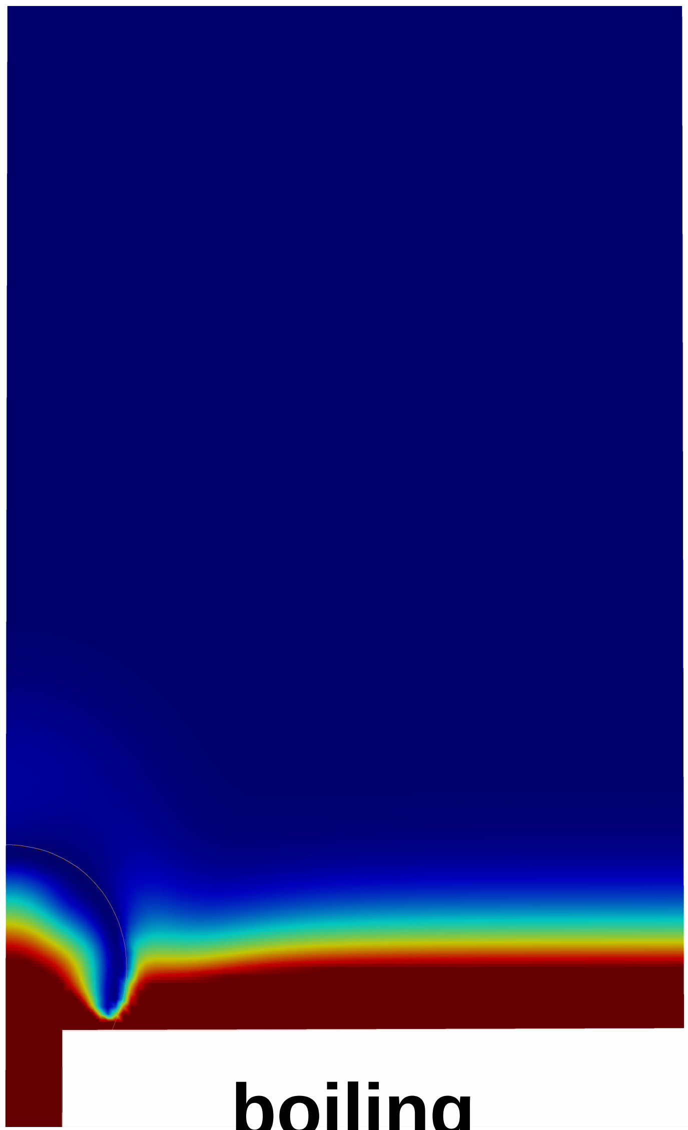
I think we are still on track

What happend with boiling...

I think we are still on track



Are these real bubbles?
Can they rotate, coalesce, bounce, tumble,
Oscillate, create wake?



boiling