Energy Spectra and Fluxes of Buoyancy-Driven Flows

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Rayleigh-Bénard Convection

&

Stably Stratified flow

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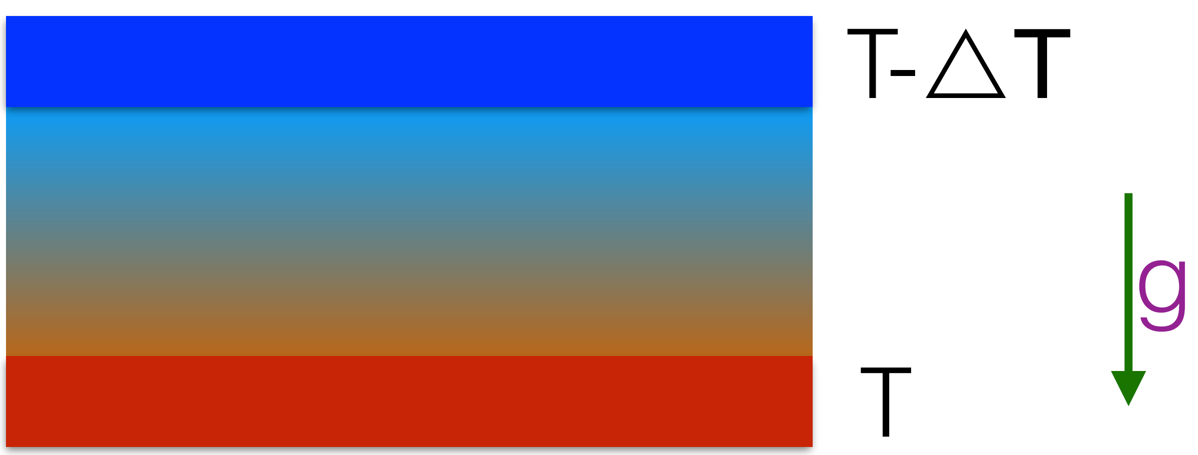
Stably Stratified flow

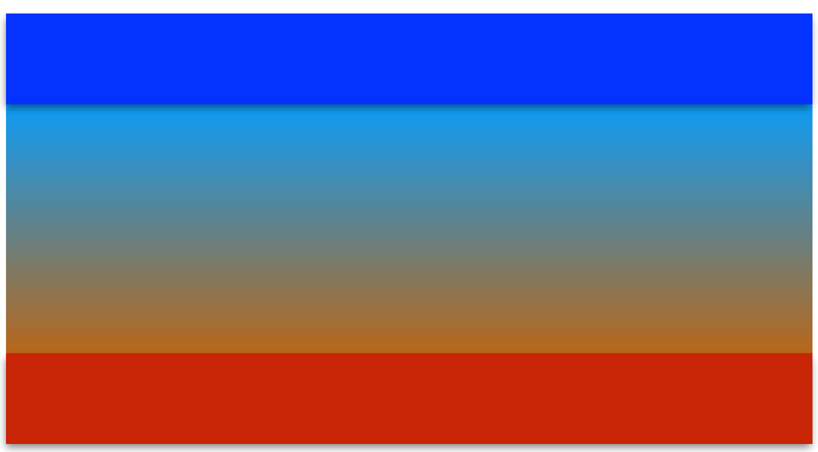


RBC
Unstable

Rayleigh-Bénard Convection

&

Stably Stratified flow



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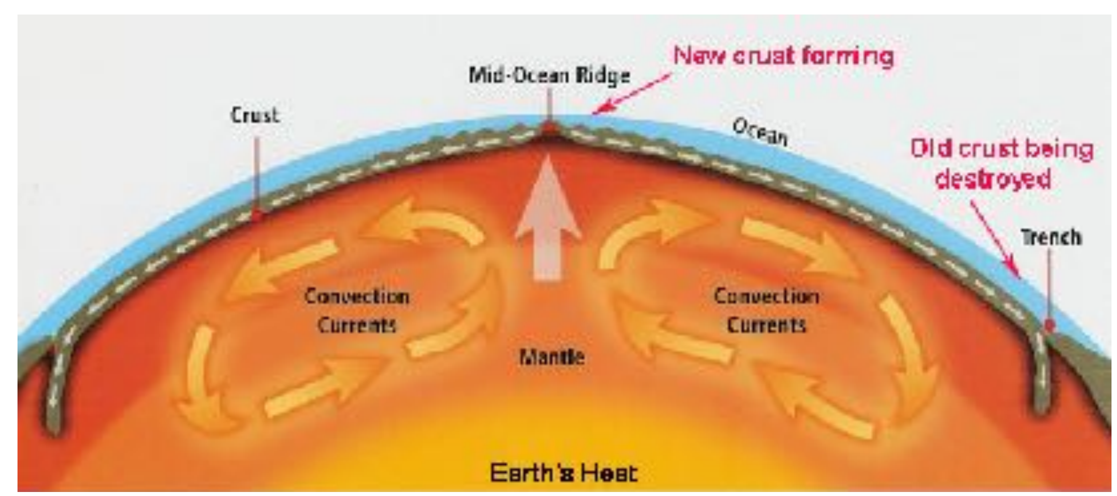

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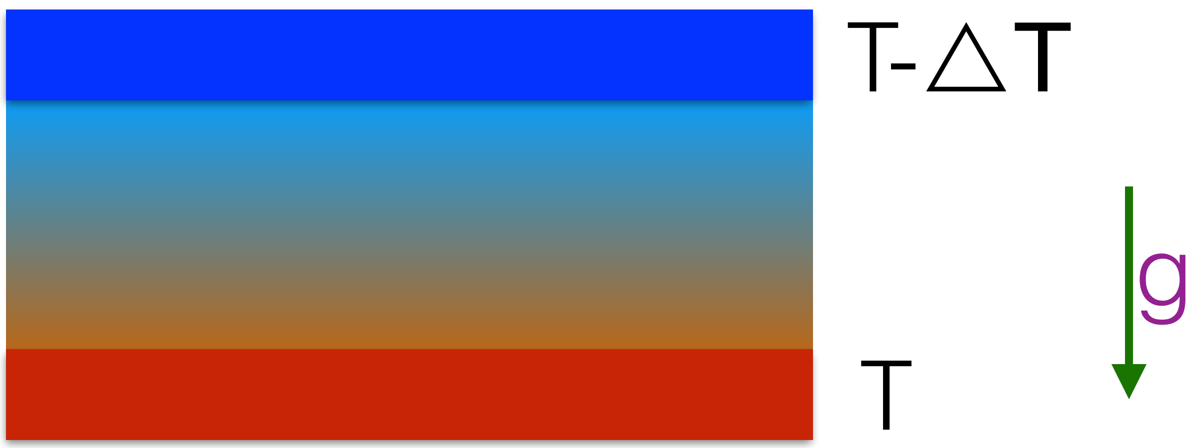

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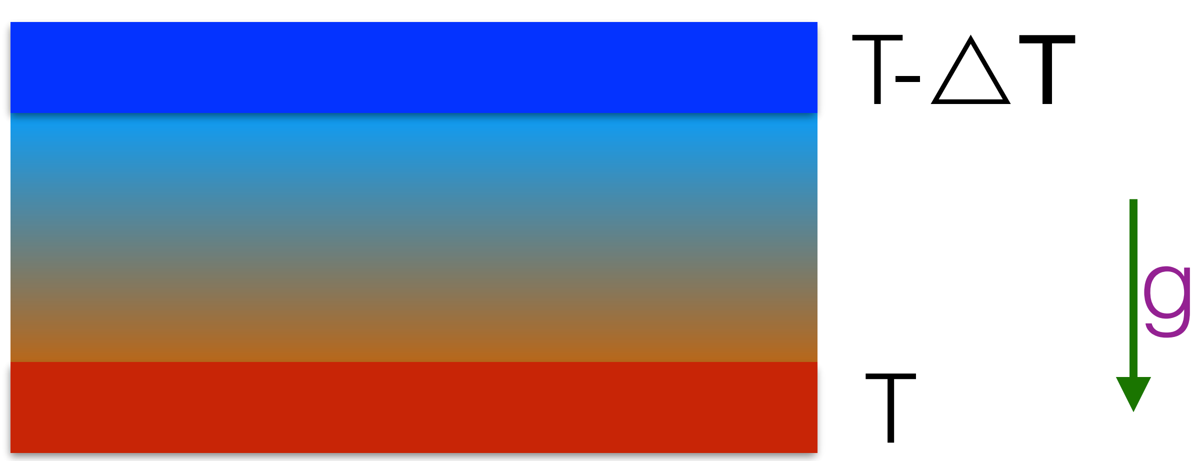

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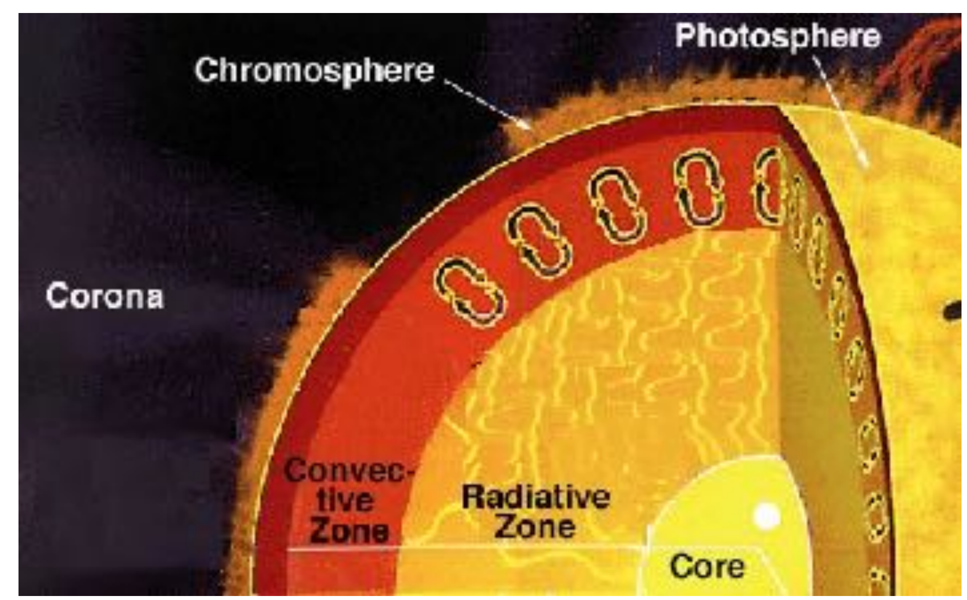

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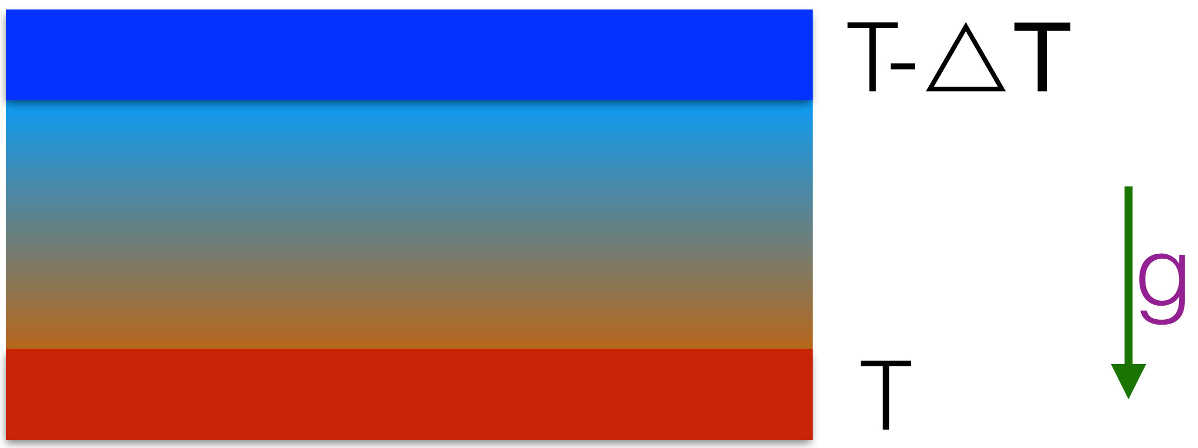

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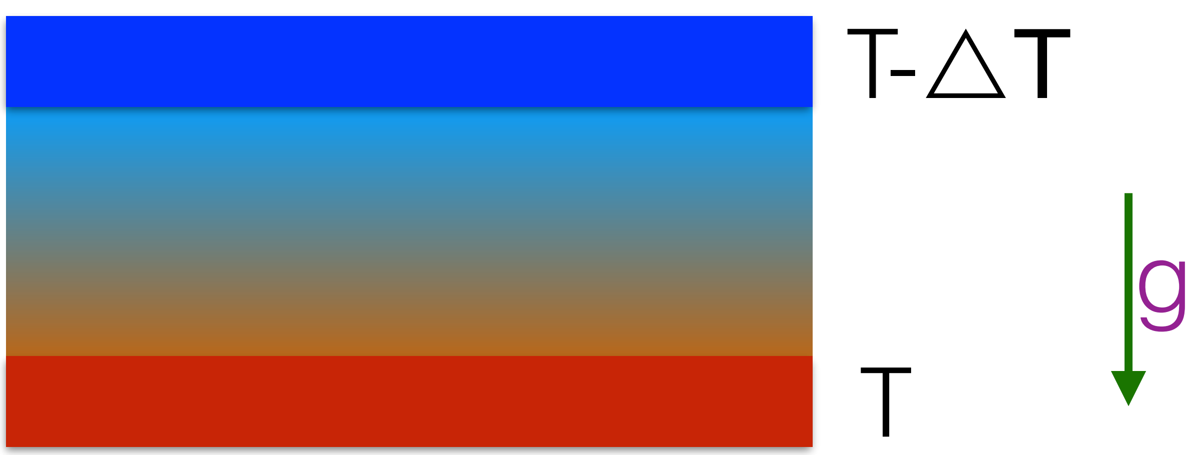

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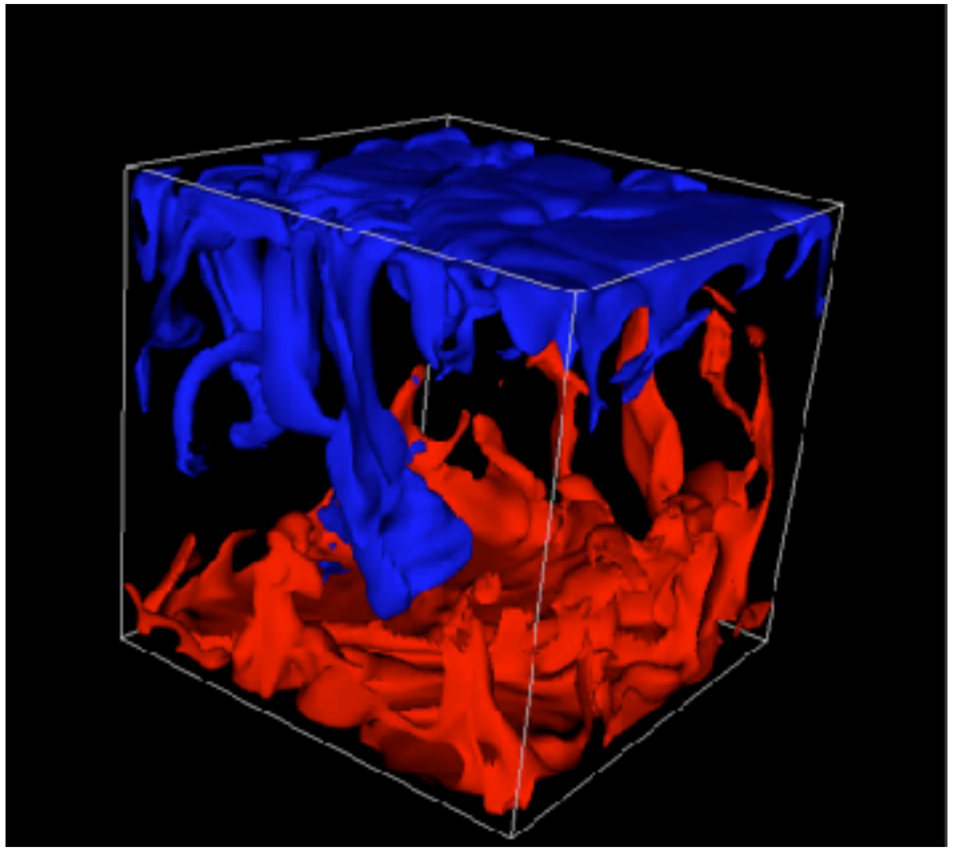

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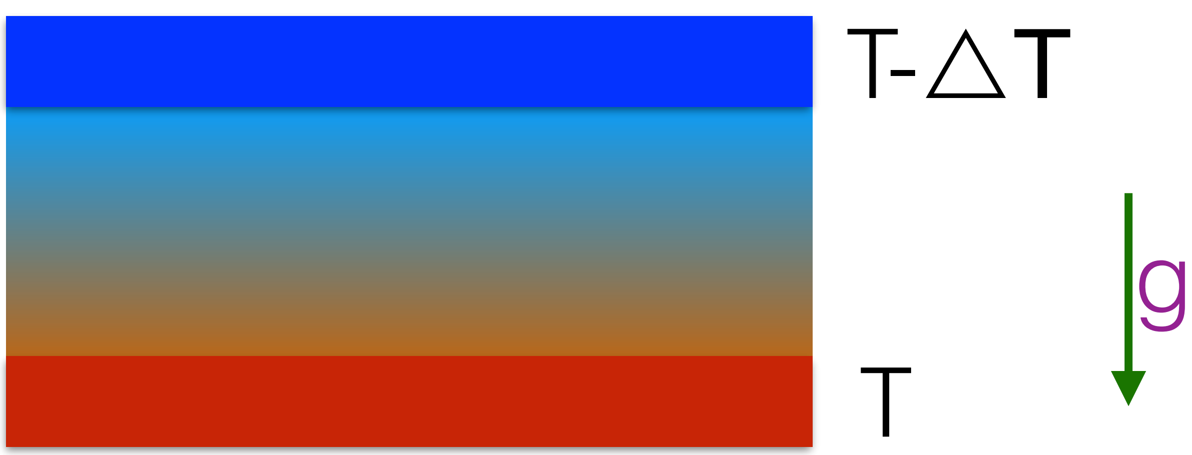

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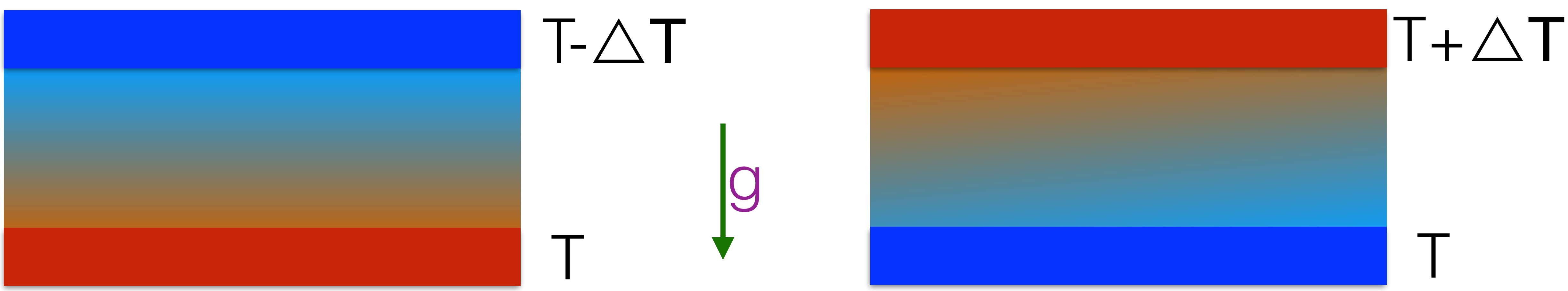

RBC
Unstable

Stably Stratified flow
Stable

Equations

$$\partial_t \mathbf{u} + (\mathbf{u}.\nabla)\mathbf{u} = -\nabla p + \alpha g \theta \hat{z} + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

$$\partial_t \theta + (\mathbf{u}.\nabla)\theta = -\frac{d\overline{T}}{dz} u_z + \kappa \nabla^2 \theta$$

Equations

Velocity field

$$\partial_t \mathbf{u} + (\mathbf{u}.\nabla)\mathbf{u} = -\nabla p + \alpha g \theta \hat{z} + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

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Equations

Velocity field

Pressure

$$\partial_t \mathbf{u} + (\mathbf{u}.\nabla)\mathbf{u} = -\nabla p + \alpha g \theta \hat{z} + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

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Equations

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Pressure

Buoyancy

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Kinematic viscosity

Equations

Velocity field **Pressure** **Buoyancy** **Ext. Force**

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Kinematic viscosity

Equations

Velocity field

Pressure

Buoyancy

Ext. Force

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Kinematic viscosity

Thermal fluctuations

Equations

$$\partial_t \mathbf{u} + (\mathbf{u}.\nabla)\mathbf{u} = -\nabla p + \alpha g \theta \hat{z} + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

Velocity field

Pressure

Buoyancy

Ext. Force

Kinematic viscosity

$$\partial_t \theta + (\mathbf{u}.\nabla)\theta = -\frac{d\overline{T}}{dz} u_z + \kappa \nabla^2 \theta$$

Thermal fluctuations

Temperature stratification

Equations

Velocity field

Pressure

Buoyancy

Ext. Force

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Kinematic viscosity

Thermal diffusivity

Thermal fluctuations

Temperature stratification

Equations

Velocity field

Pressure

Buoyancy

Ext. Force

$$\partial_t \mathbf{u} + (\mathbf{u}.\nabla)\mathbf{u} = -\nabla p + \alpha g \theta \hat{z} + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

Kinematic viscosity

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Thermal fluctuations

Temperature stratification

Thermal diffusivity

Incompressibility

Boussinesq approximation

$$\nabla.\mathbf{u} = 0$$

Non-Dimensional Equations

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}.\nabla)\mathbf{u} = -\nabla p + \theta \hat{z} + \sqrt{\frac{\mathrm{Pr}}{\mathrm{Ra}}} \nabla^2 \mathbf{u} + \mathbf{f}$$

$$\frac{\partial \theta}{\partial t} + (\mathbf{u}.\nabla)\theta = S u_z + \frac{1}{\sqrt{\mathrm{RaPr}}} \nabla^2 \theta$$

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Rayleigh Number

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Rayleigh Number

$$\mathrm{Ra} = \frac{\alpha g d^4}{\nu \kappa}\frac{d\overline{T}}{dz}$$

Non-Dimensional Equations

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Prandtl Number

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Prandtl Number

$$\mathrm{Ra} = \frac{\alpha g d^4}{\nu \kappa}\frac{d\overline{T}}{dz}$$

$$\mathrm{Pr} = \frac{\nu}{\kappa}$$

Non-Dimensional Equations

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}.\nabla)\mathbf{u} = -\nabla p + \theta \hat{z} + \sqrt{\frac{\mathrm{Pr}}{\mathrm{Ra}}}\nabla^2 \mathbf{u} + \mathbf{f}$$

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Rayleigh Number

Prandtl Number

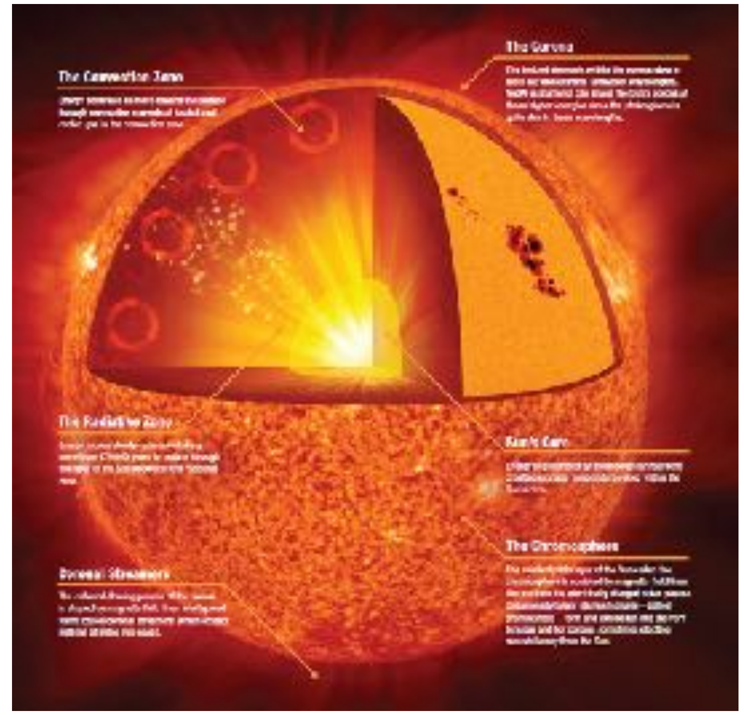
$$\mathrm{Ra} = \frac{\alpha g d^4}{\nu \kappa}\frac{d\overline{T}}{dz}$$

$$\mathrm{Pr} = \frac{\nu}{\kappa}$$

Solar Convection

Ra ~ 10^{20}

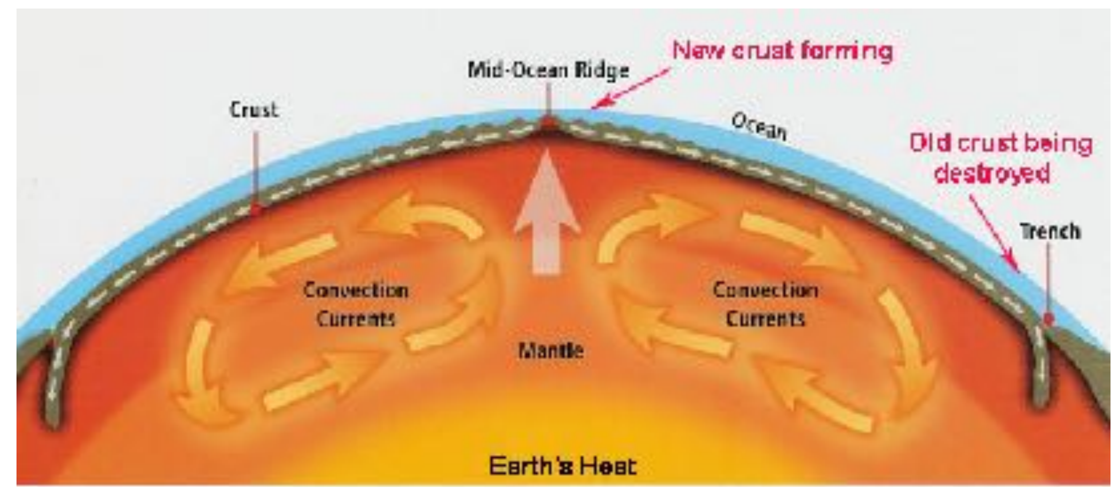
Pr ~ 10^{-6}



Earth's Mantle

Ra ~ 10^{7}

Pr ~ 10^{25}



Jupiter

Ra ~ 10^{24}

Pr ~ 1

Time evolution equation for kinetic energy in Fourier space

$$\frac{\partial}{\partial t}\hat{\mathbf{u}}(\mathbf{k},t) = -i\mathbf{k}\hat{p}(\mathbf{k}) - \widehat{(\mathbf{u}.\nabla)\mathbf{u}} + \hat{\theta}(\mathbf{k},t)\hat{z} - \nu k^2\hat{\mathbf{u}}(\mathbf{k},t)$$

Time evolution equation for kinetic energy in Fourier space

$$\hat{\mathbf{u}}^*(\mathbf{k},t)\cdot\left[\frac{\partial}{\partial t}\hat{\mathbf{u}}(\mathbf{k},t)=-i\mathbf{k}\hat{p}(\mathbf{k})-\widehat{(\mathbf{u}.\nabla)\mathbf{u}}+\hat{\theta}(\mathbf{k},t)\hat{z}-\nu k^2\hat{\mathbf{u}}(\mathbf{k},t)\right]+\mathrm{c.c}$$

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$$\frac{\partial}{\partial t}E(k,t)=T(k)+\mathcal{F}_{B}(k)-D(k)$$

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$$\frac{\partial}{\partial t}E(k,t) = T(k) + \mathcal{F}_B(k) - D(k)$$

$$\sum_k \frac{1}{2}|u(k)|^2$$

Time evolution equation for kinetic energy in Fourier space

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$$\frac{\partial}{\partial t}E(k,t) = T(k) + \mathcal{F}_{B}(k) - D(k)$$

$$\sum_{k}\frac{1}{2}|u(k)|^{2} \qquad -\frac{d\Pi(k)}{dk}$$

Time evolution equation for kinetic energy in Fourier space

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$$\sum_k \frac{1}{2}|u(k)|^2 \qquad -\frac{d\Pi(k)}{dk} \qquad \sum_k \Re\langle\widehat{u_z}(\mathbf{k})\hat{\theta}^*(\mathbf{k})\rangle$$

Time evolution equation for kinetic energy in Fourier space

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Time evolution equation for kinetic energy in Fourier space

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Steady State $\frac{\partial}{\partial t}E(k,t) = T(k) + \mathcal{F}_B(k) - D(k)$

$$\sum_k \frac{1}{2}|u(k)|^2 \qquad -\frac{d\Pi(k)}{dk} \qquad \sum_k \Re\langle\widehat{u_z}(\mathbf{k})\hat{\theta}^*(\mathbf{k})\rangle \qquad \sum_k 2\nu k^2 E(k)$$

Time evolution equation for kinetic energy in Fourier space

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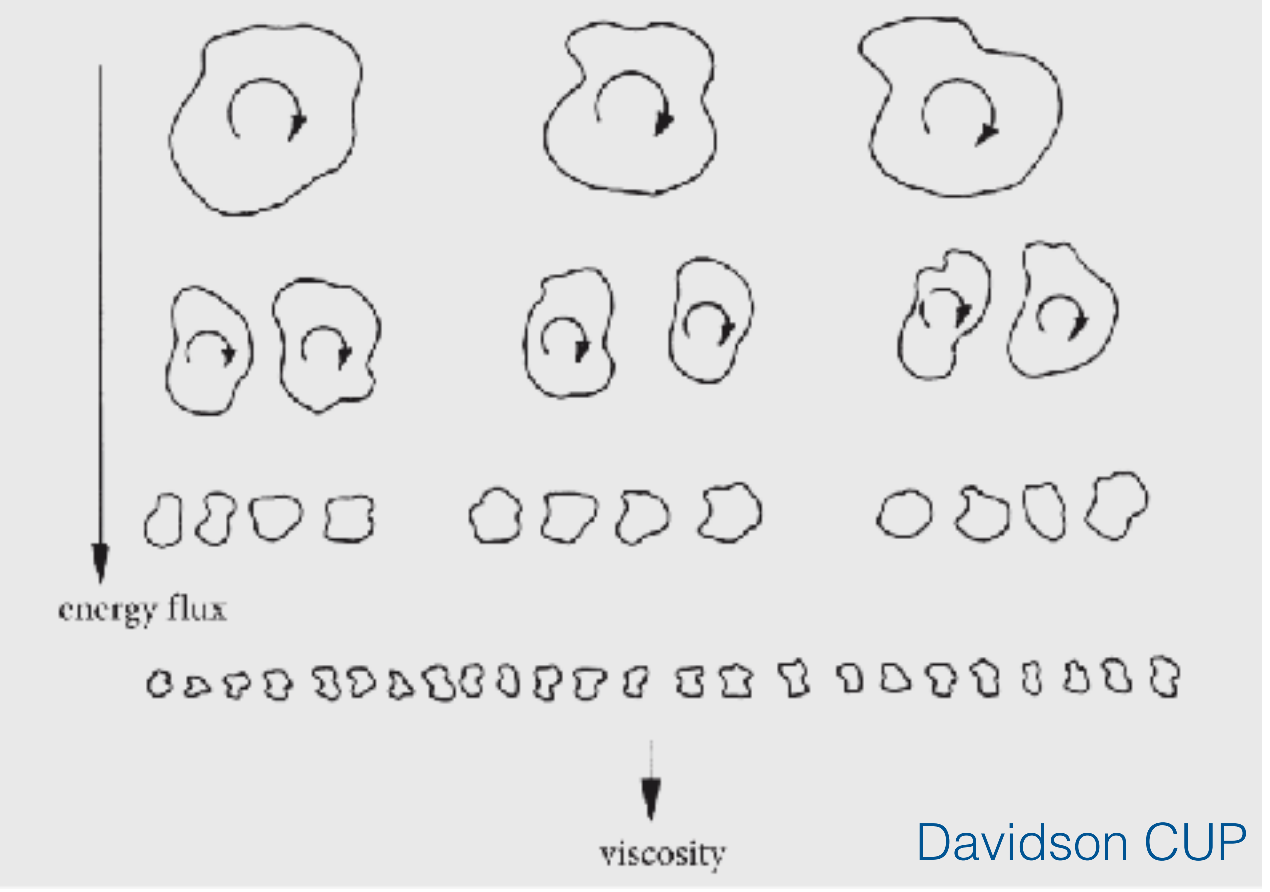
$$\boxed{\frac{d\Pi(k)}{dk} = \mathcal{F}_B(k) - D(k)}$$

Kolmogorov Phenomenology of Hydrodynamic Turbulence

Kolmogorov 1941

Kolmogorov Phenomenology of Hydrodynamic Turbulence

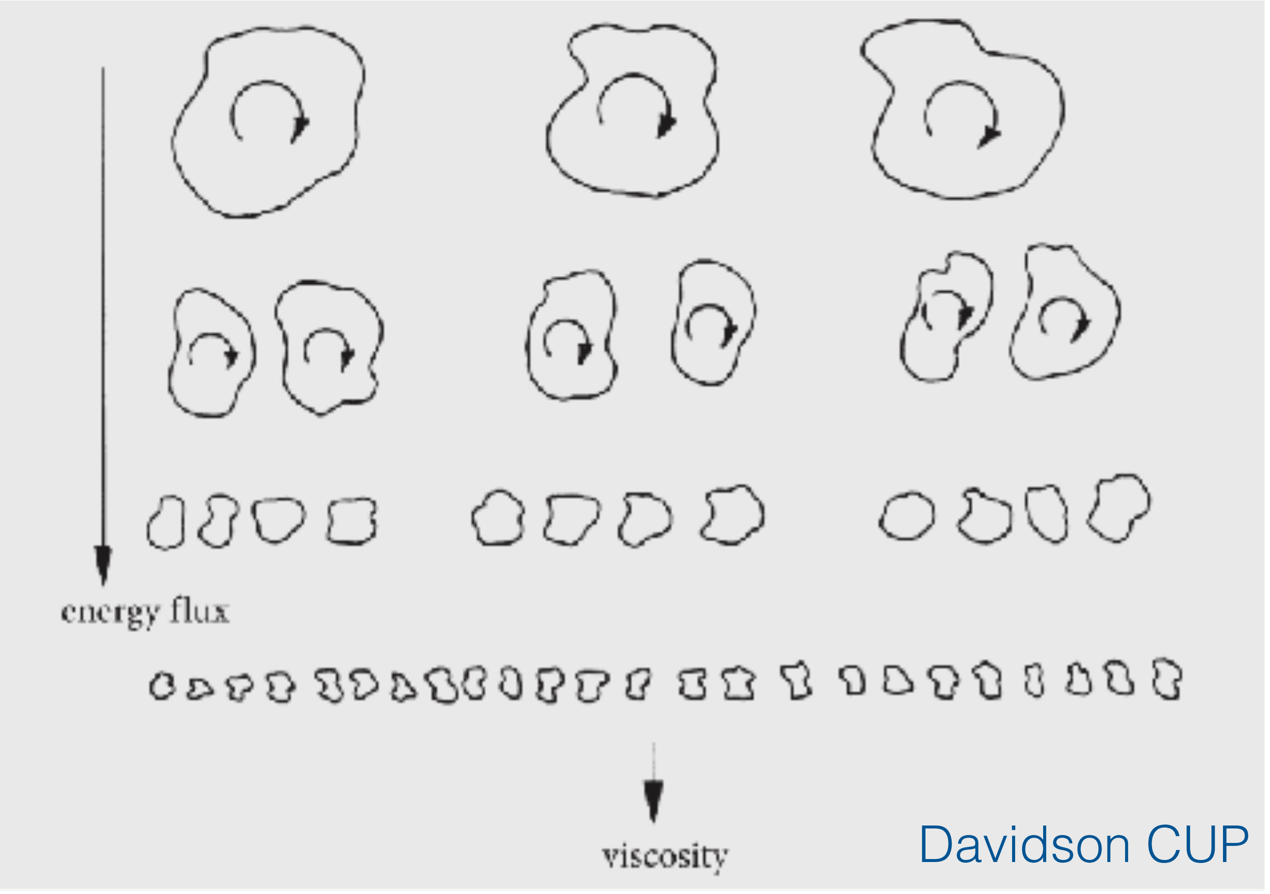


Davidson CUP

Kolmogorov 1941

Kolmogorov Phenomenology of Hydrodynamic Turbulence

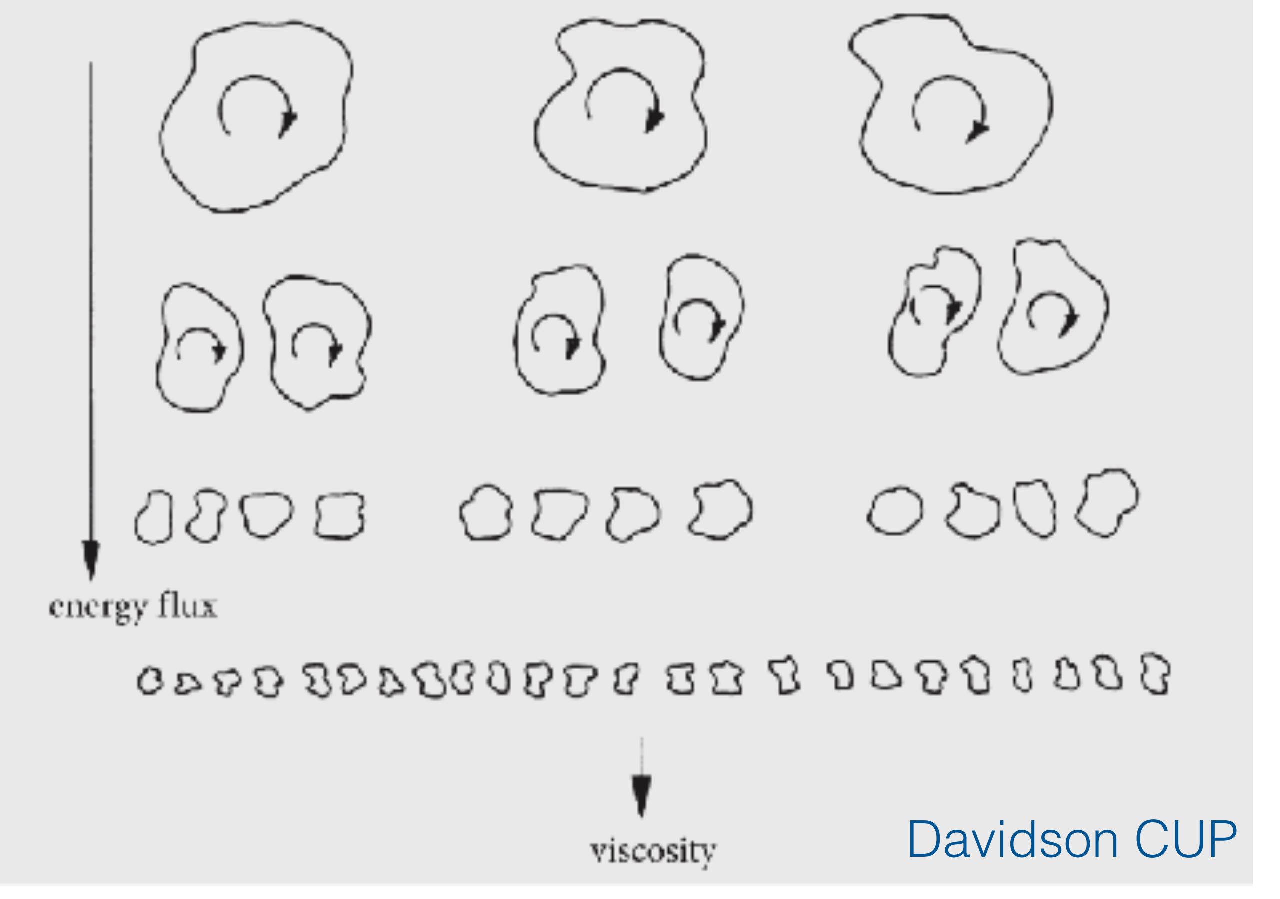
- Energy is injected by the external force at large length scales.



Kolmogorov 1941

Kolmogorov Phenomenology of Hydrodynamic Turbulence

- Energy is injected by the external force at large length scales.
- Large eddies break into smaller eddies and energy flows from large-scale to small-scale.



Kolmogorov 1941

Kolmogorov Phenomenology of Hydrodynamic Turbulence

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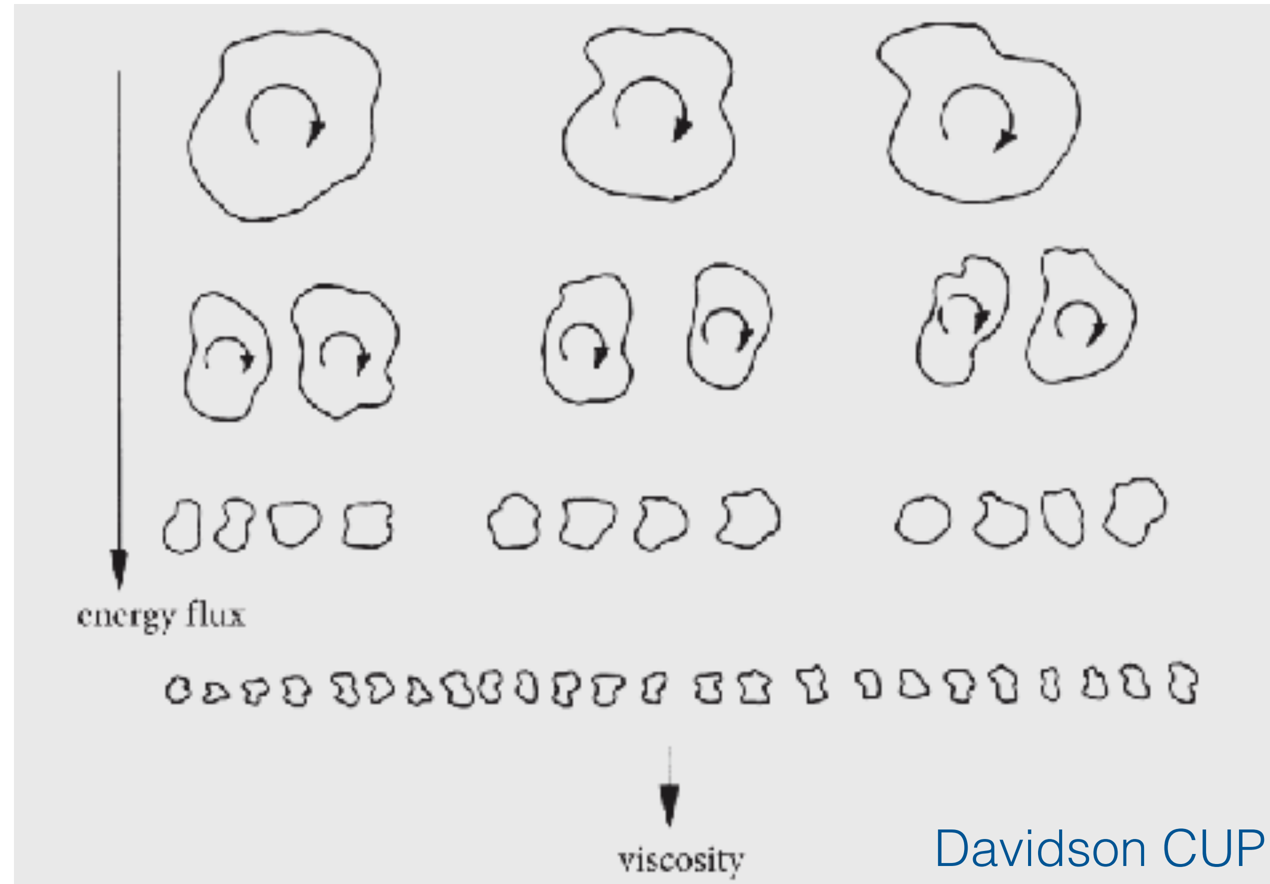
- At sufficiently small scales energy is dissipated by the viscosity.



Davidson CUP

Kolmogorov 1941

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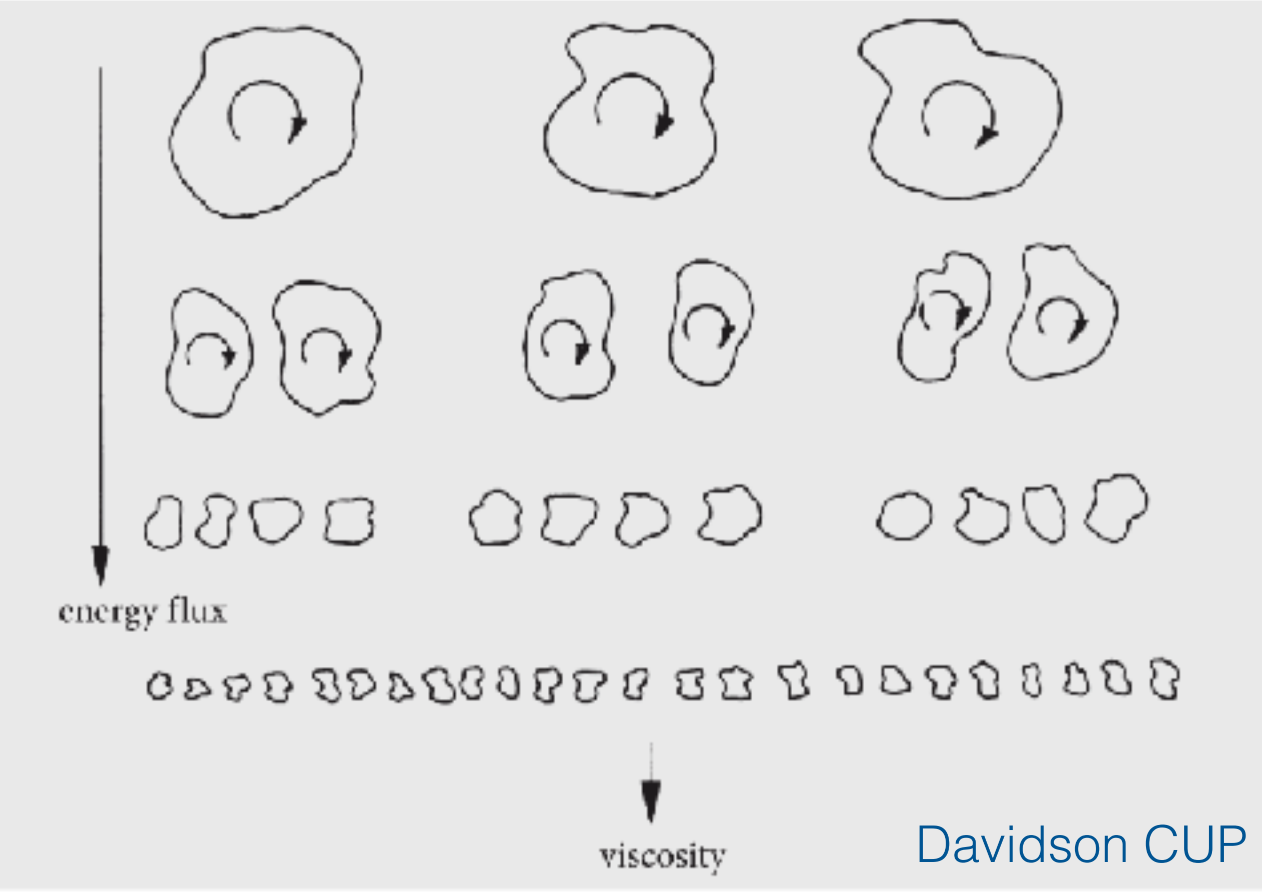
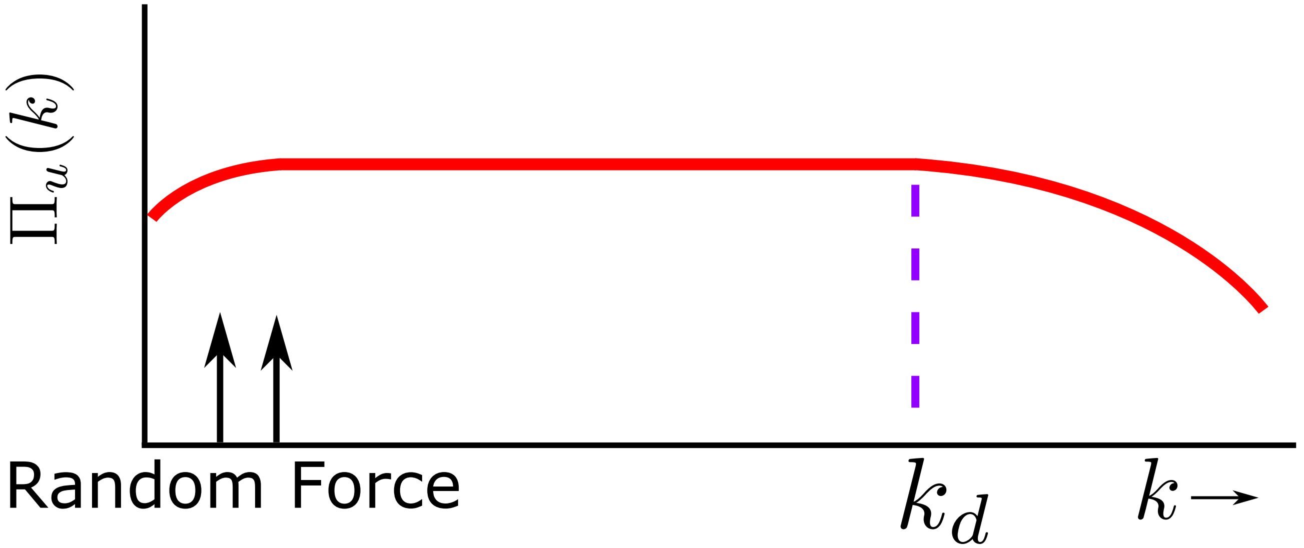
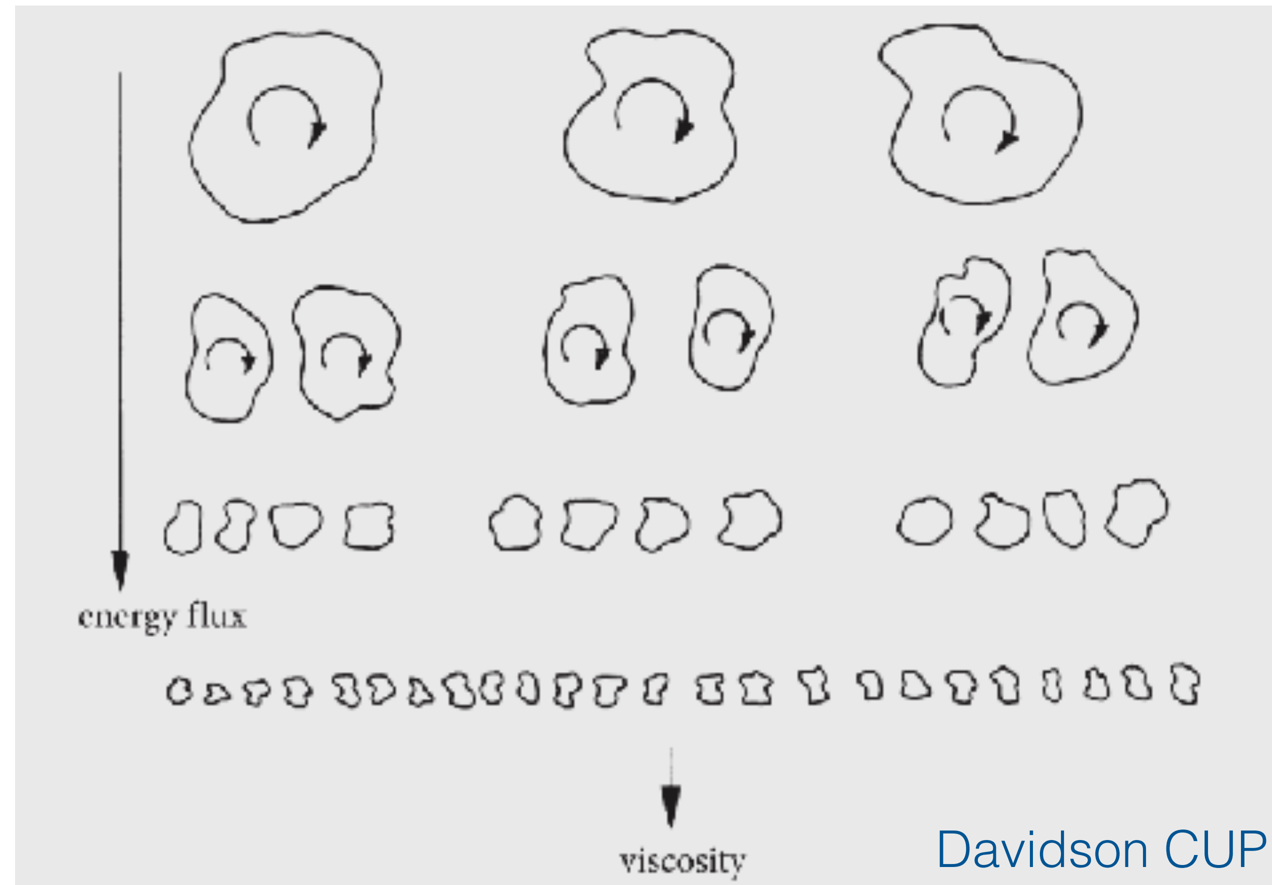
Kolmogorov 1941

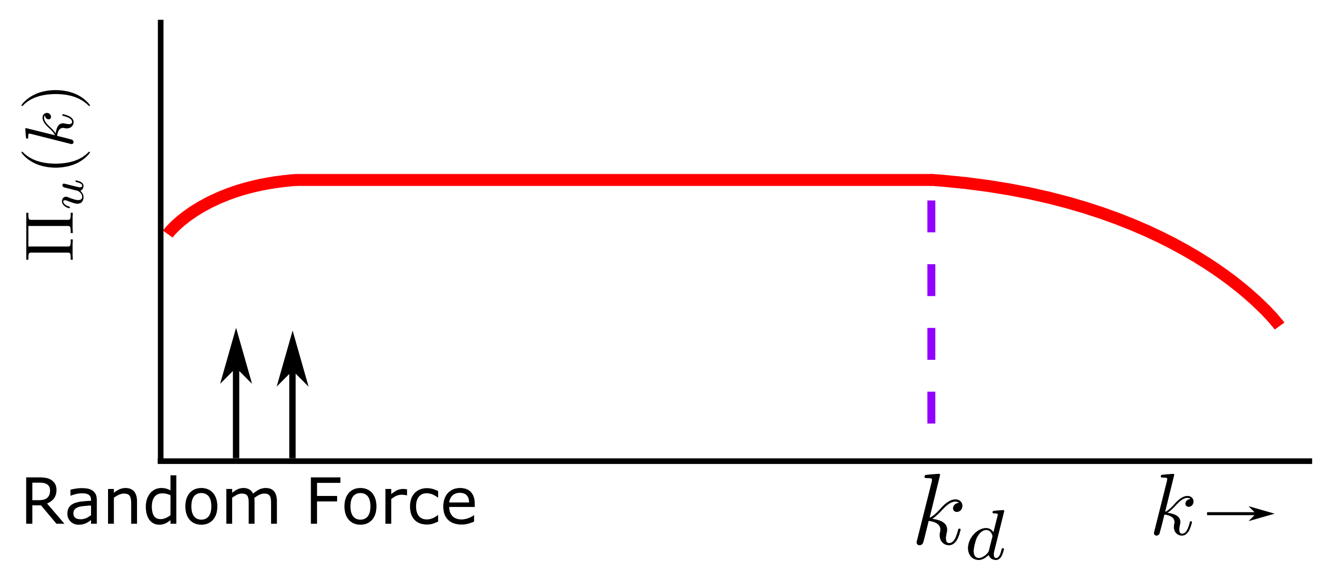




Kolmogorov Phenomenology of Hydrodynamic Turbulence



$$E(k) = K_{Ko}\Pi^{2/3}k^{-5/3}$$



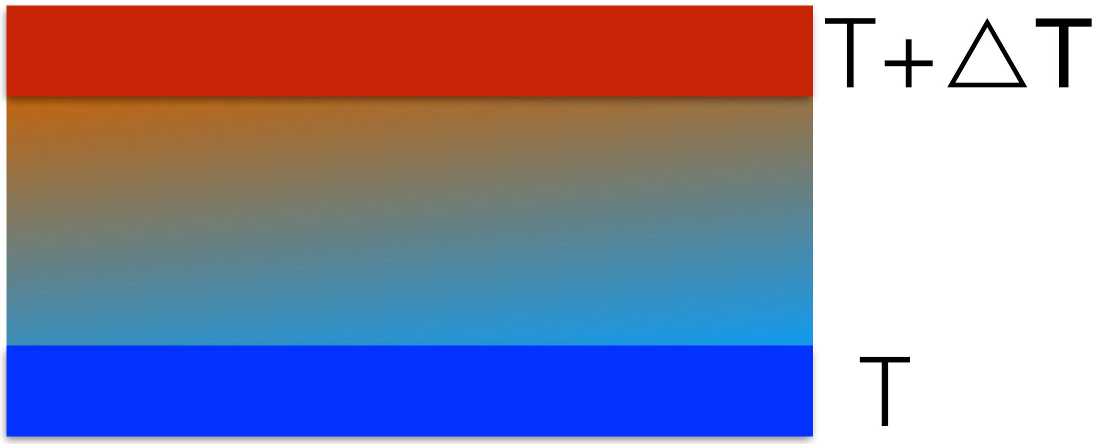
Kolmogorov 1941

Phenomenology of Buoyancy-Driven Turbulence

Bolgiano-Obukhov Phenomenology

Stably Stratified Flow

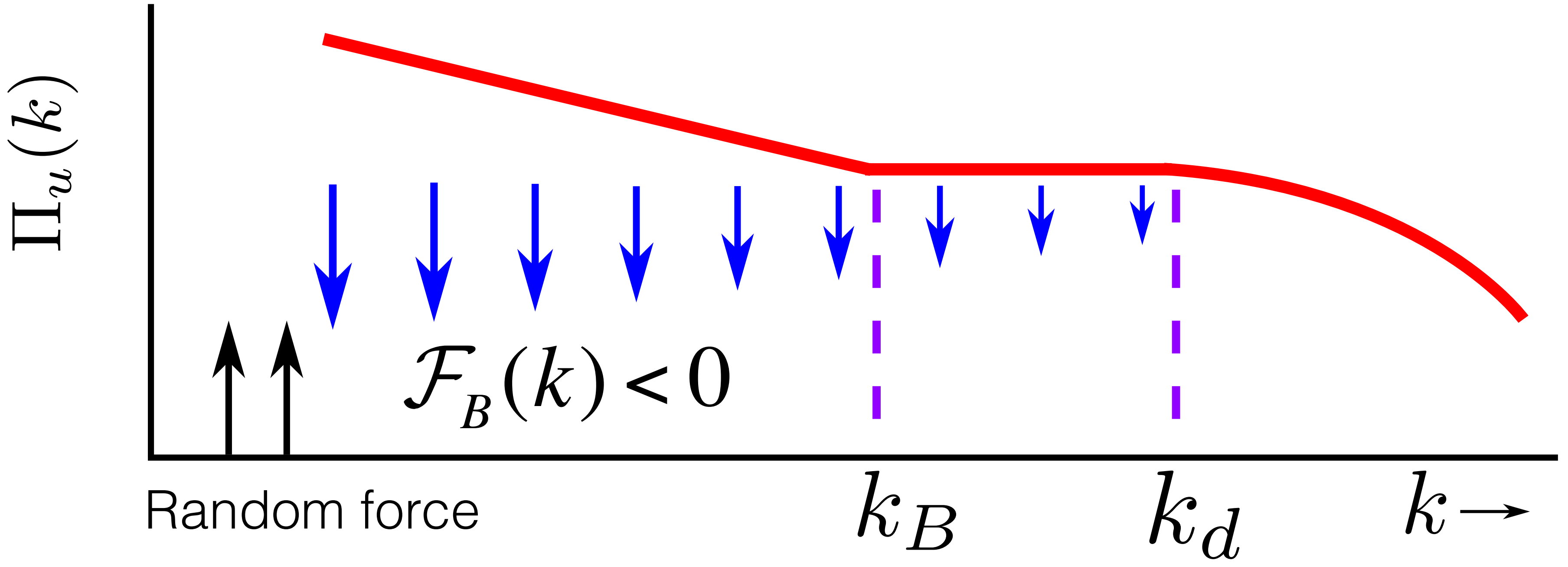
Bolgiano, 1959
Obukhov, 1959



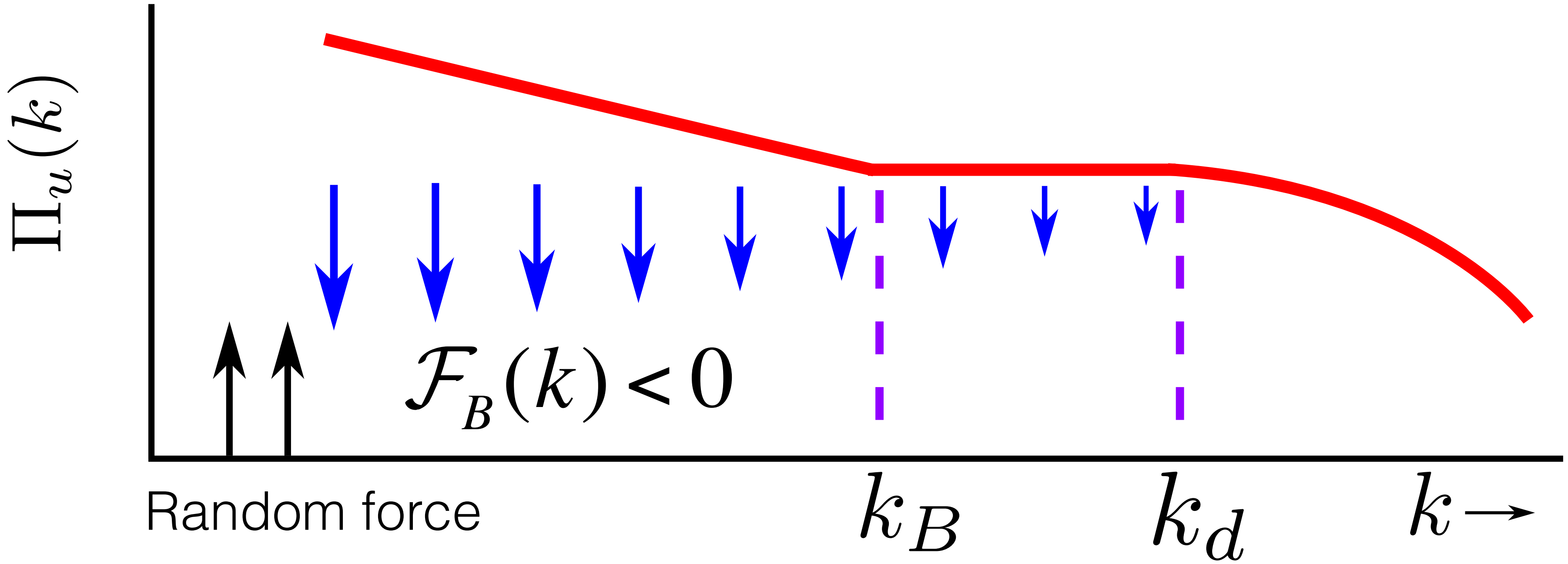
- Turbulent **Re>>1**
- Moderately stratified **Fr~1**

$$\mathrm{Fr} \approx \frac{1}{\sqrt{\mathrm{Ri}}};\ \mathrm{Ri} = \frac{\mathrm{PE}}{\mathrm{KE}}$$

Bolgiano-Obukhov Phenomenology



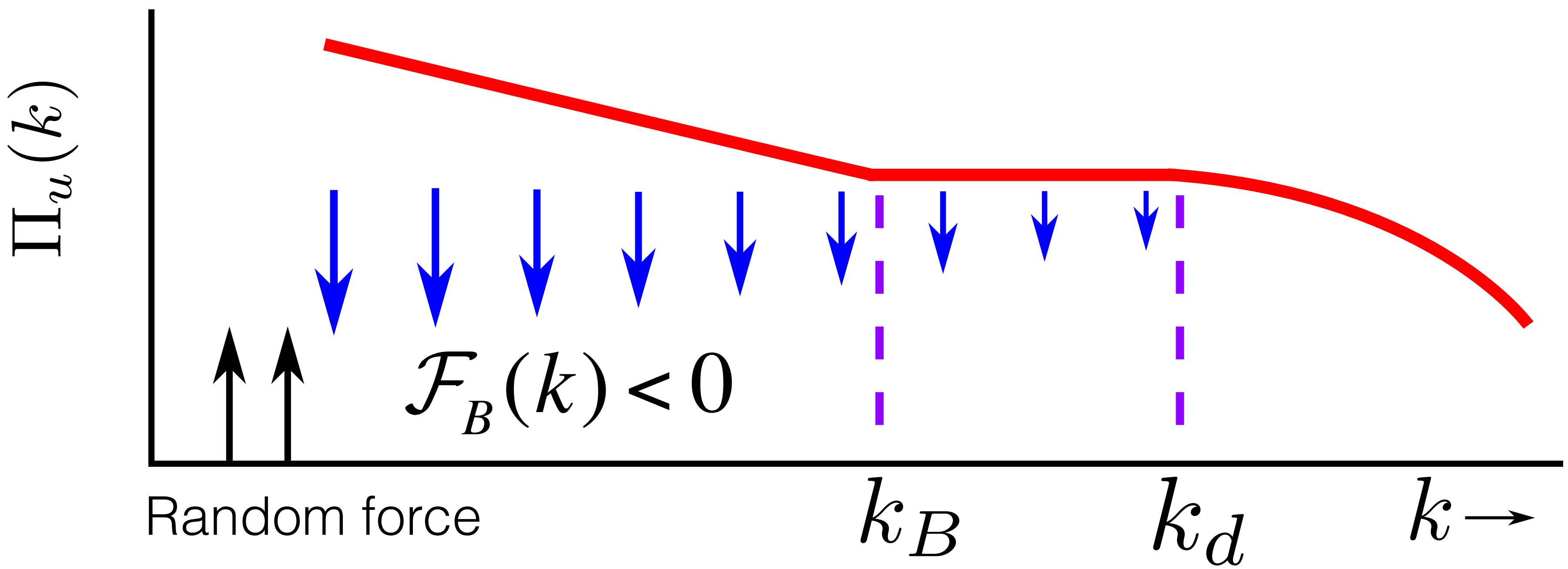
Bolgiano-Obukhov Phenomenology



Advective ~ Buoyancy

$$(\mathbf{u}.\nabla)\mathbf{u} \approx \alpha g \theta \hat{z}$$

Bolgiano-Obukhov Phenomenology



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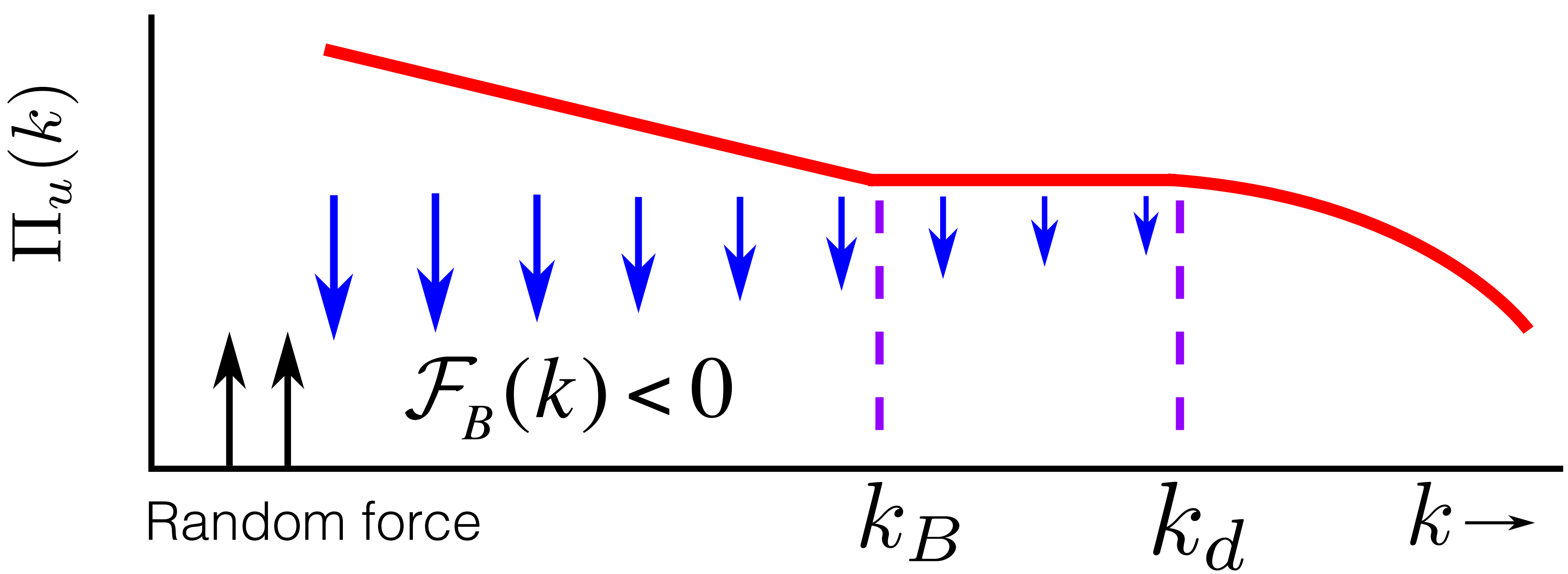
Bolgiano-Obukhov Phenomenology

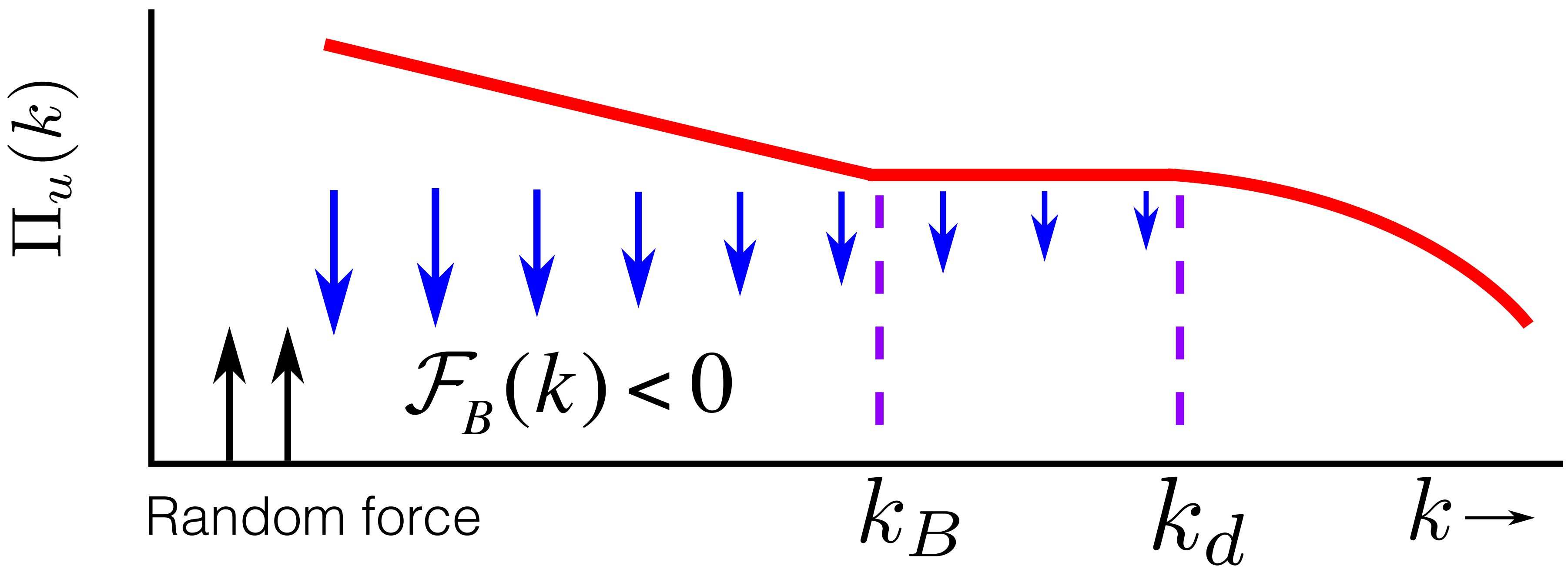


Advective ~ Buoyancy

$$(\mathbf{u}.\nabla)\mathbf{u} \approx \alpha g \theta \hat{z}$$

$$k u_k^2 = \alpha g \theta_k$$

Bolgiano-Obukhov Phenomenology



Advective ~ Buoyancy

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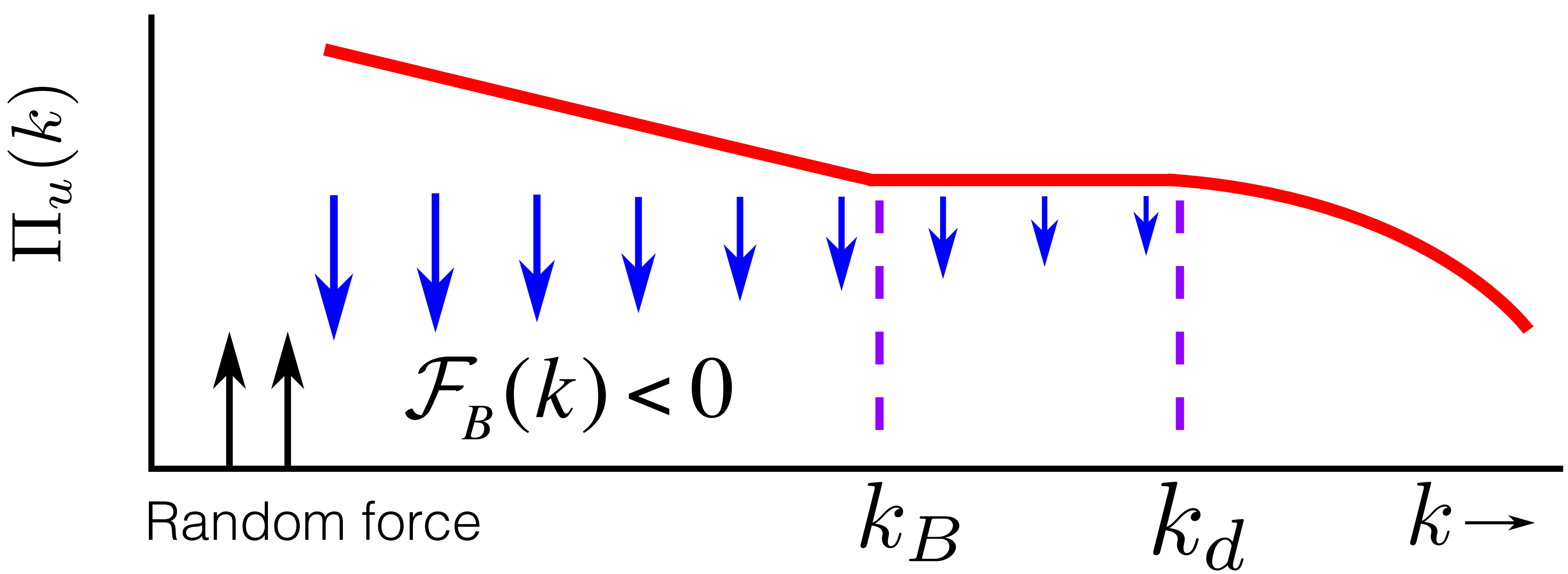

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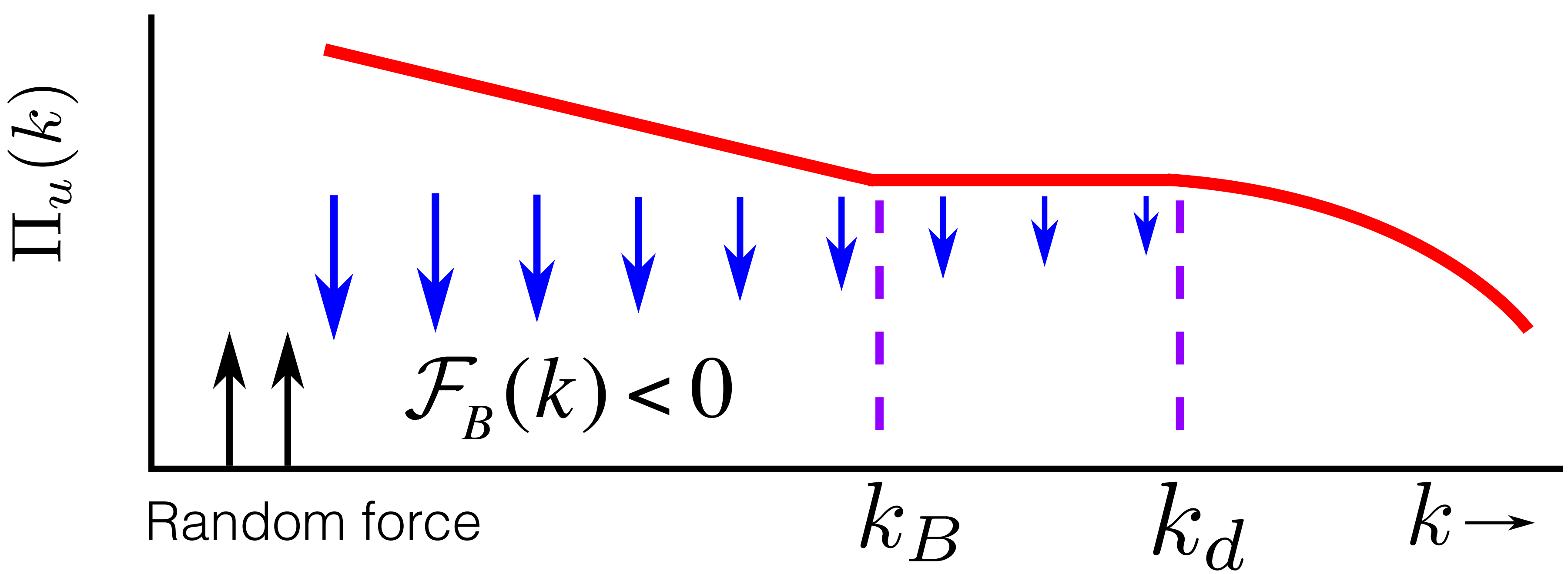

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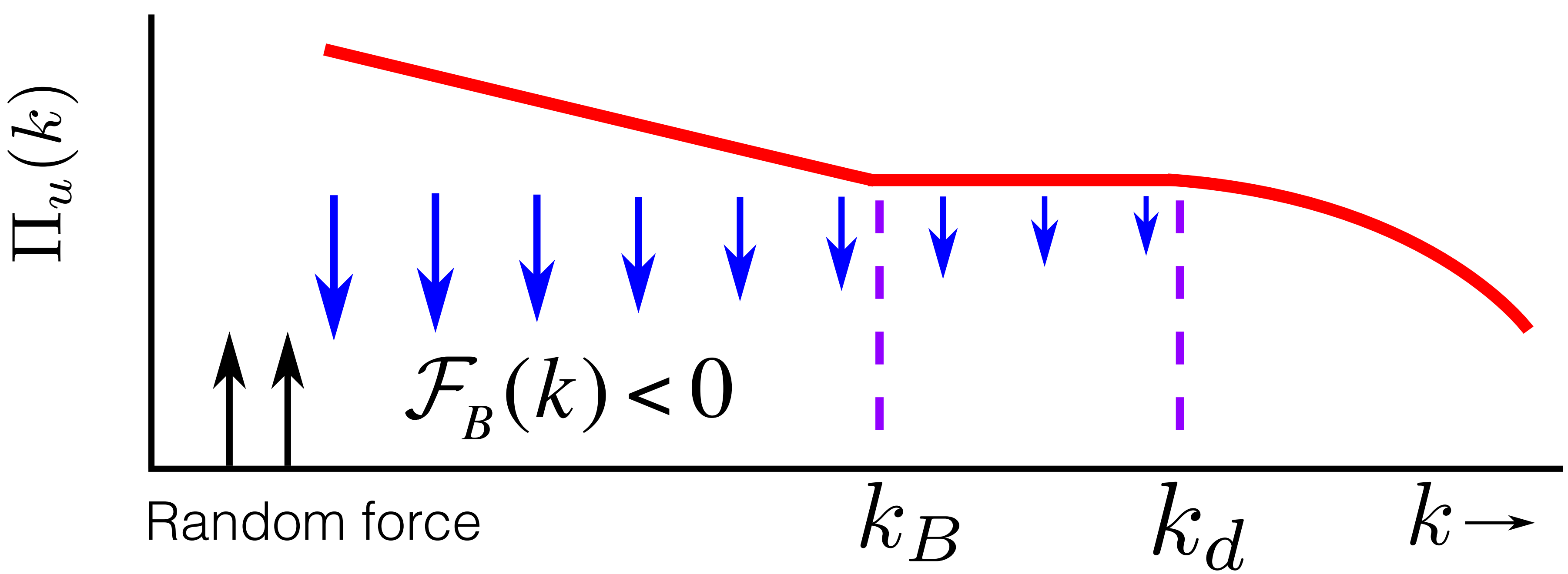

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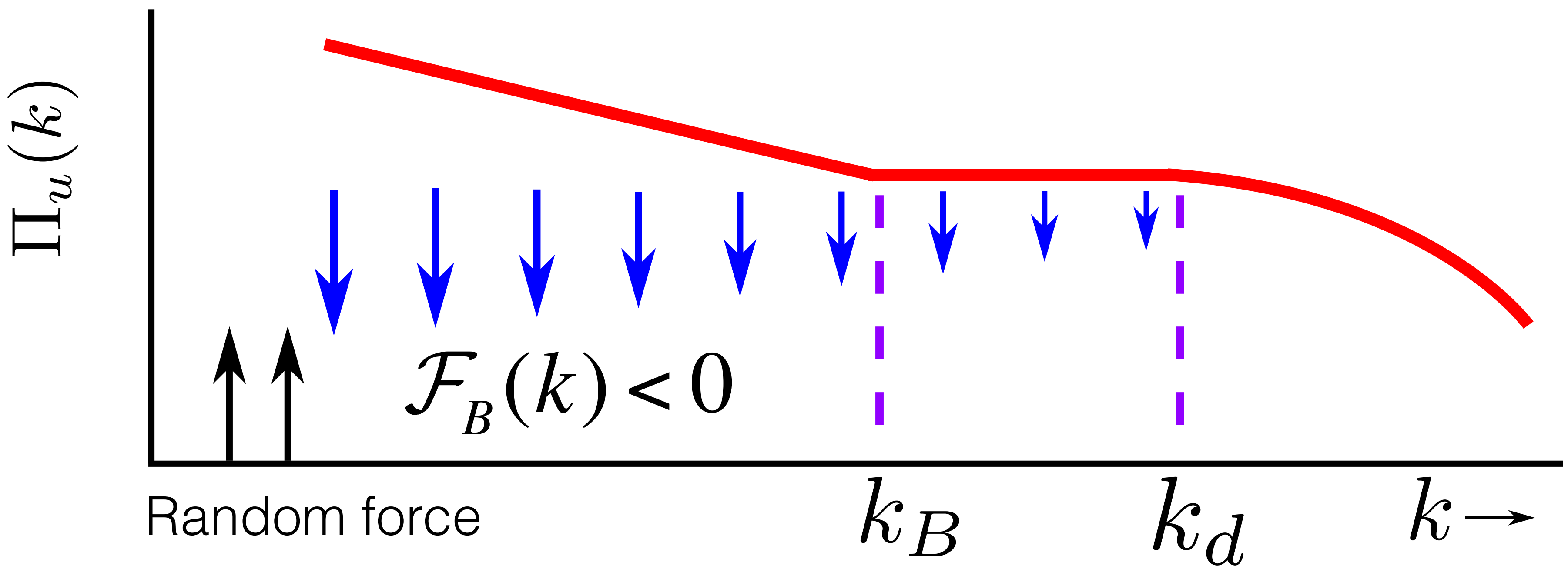

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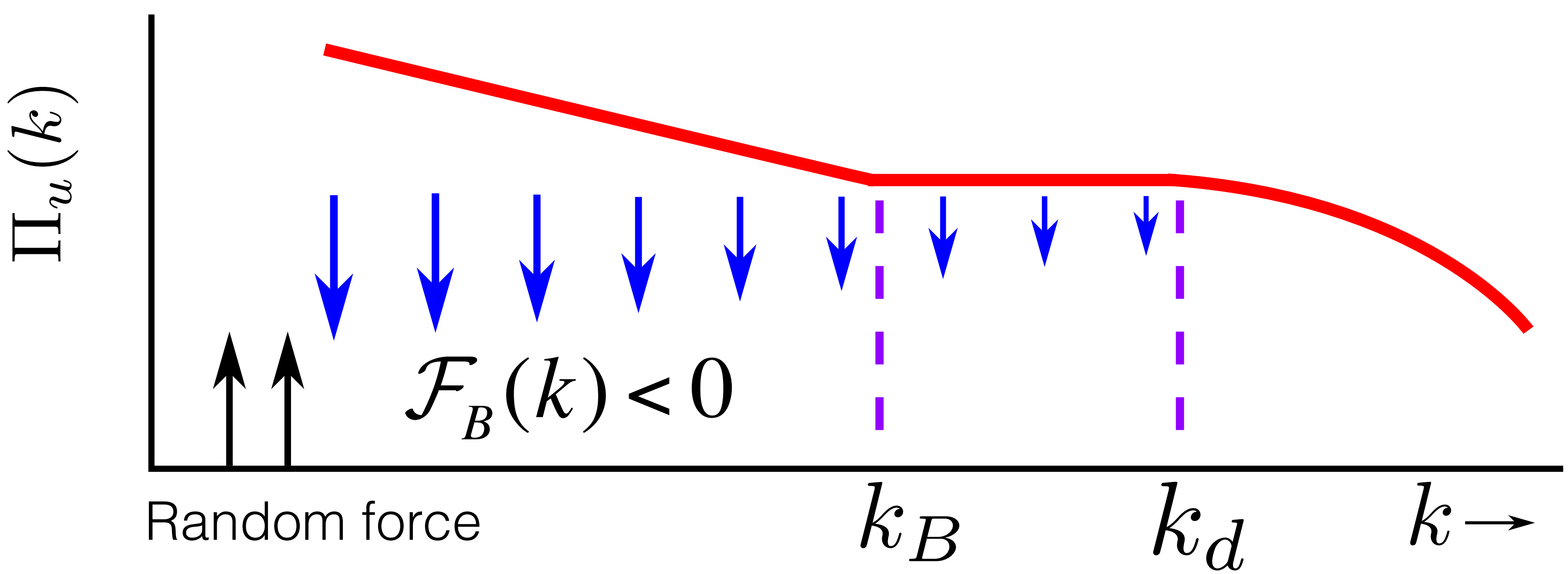

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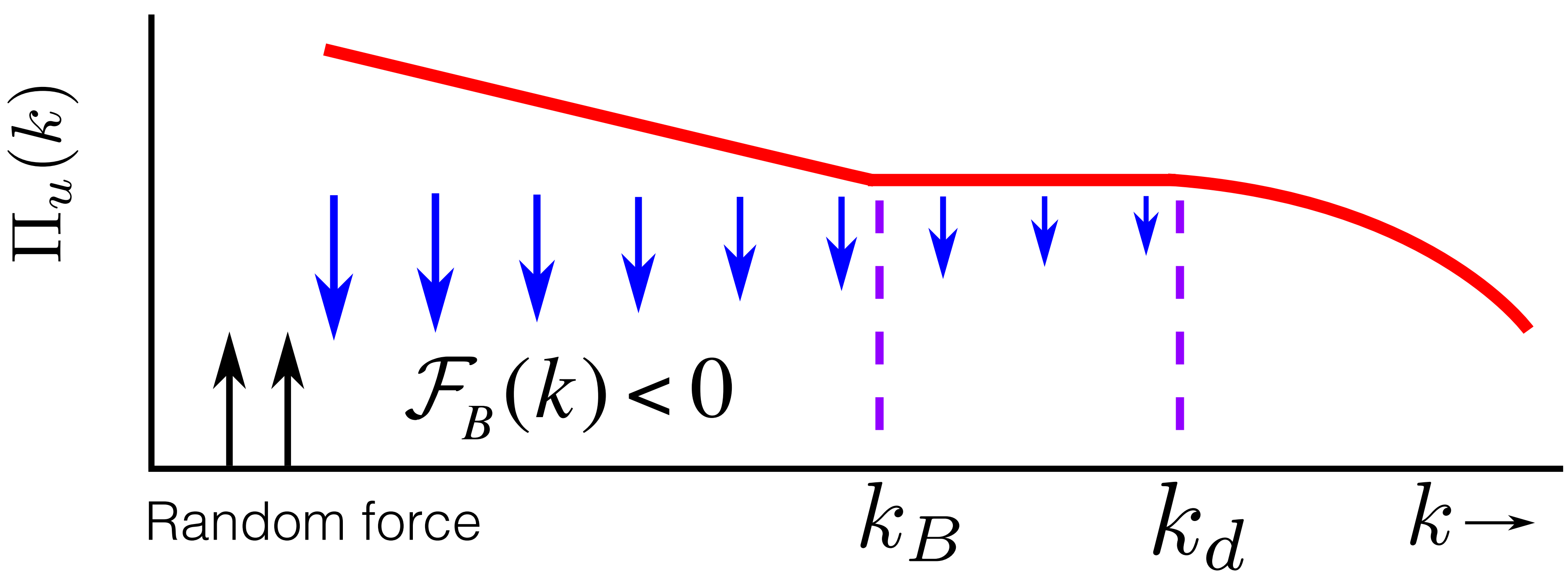

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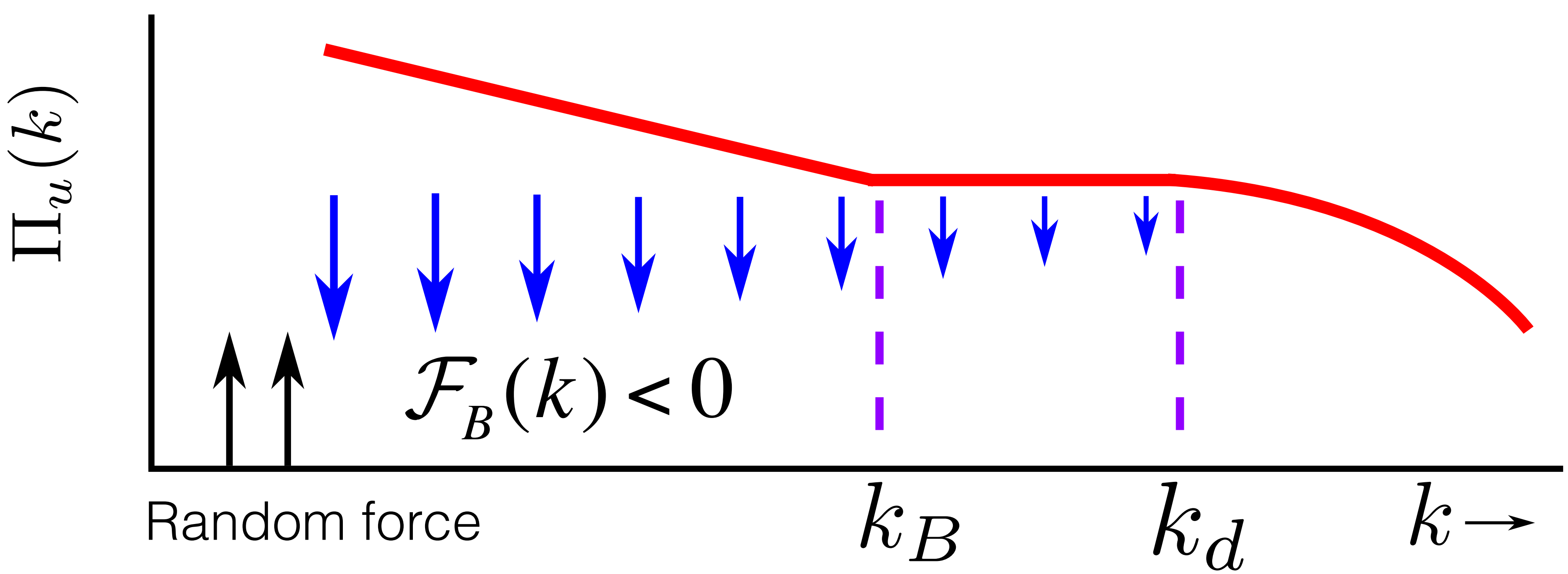

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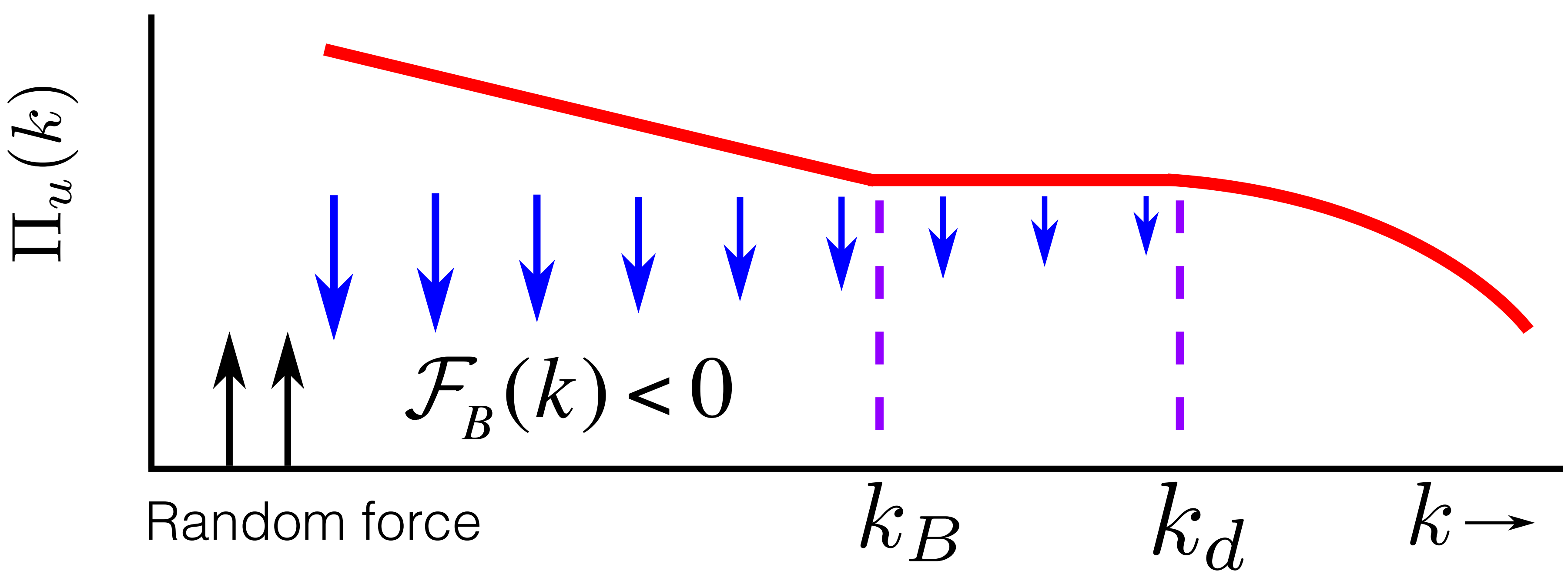

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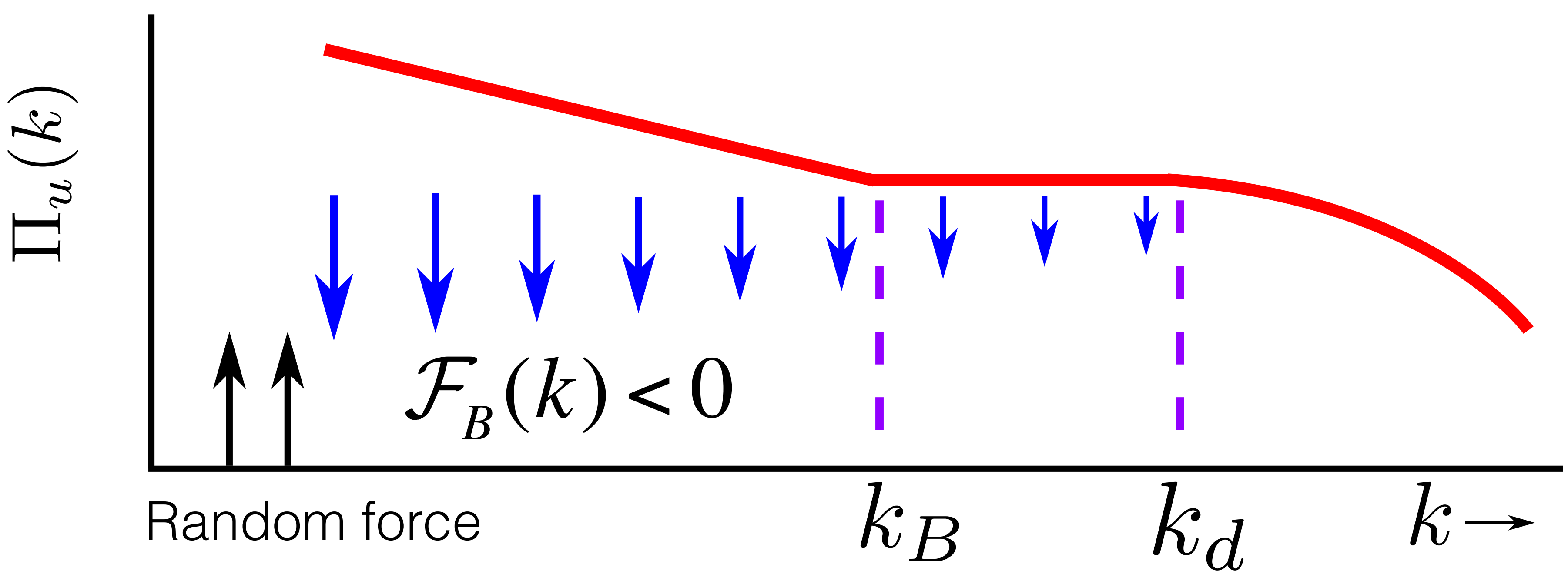
Bolgiano-Obukhov Phenomenology



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Extension to RBC

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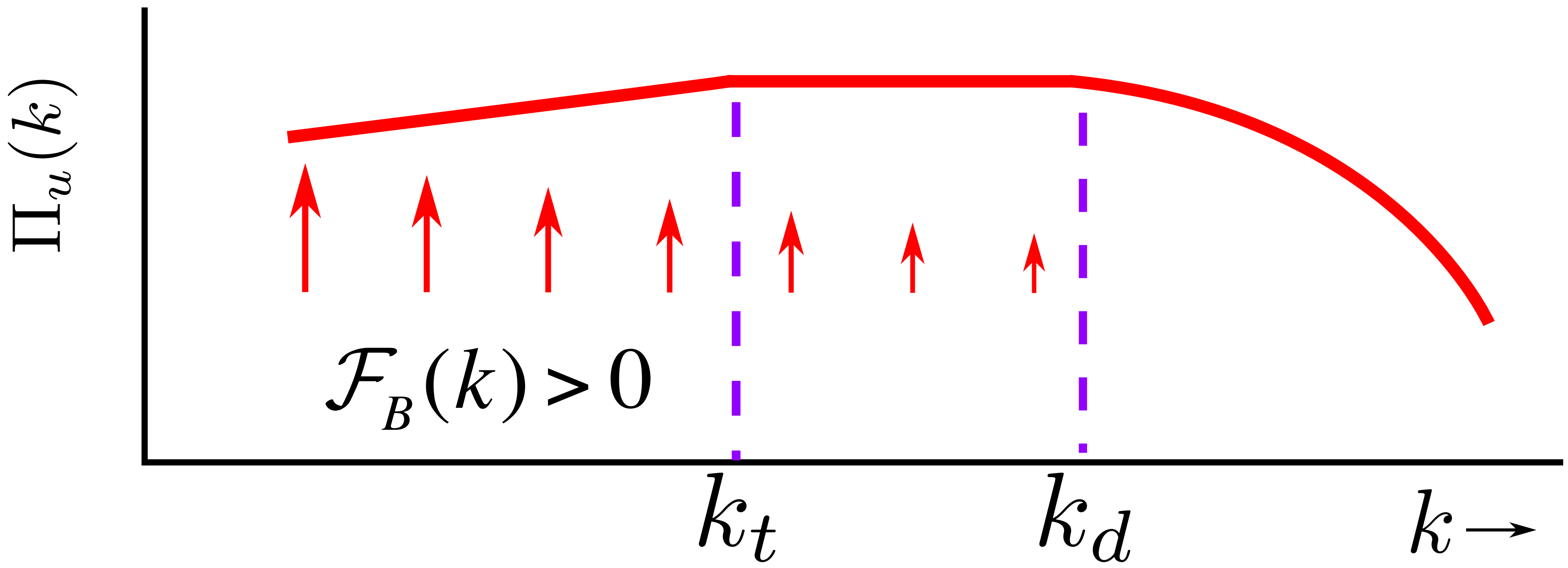
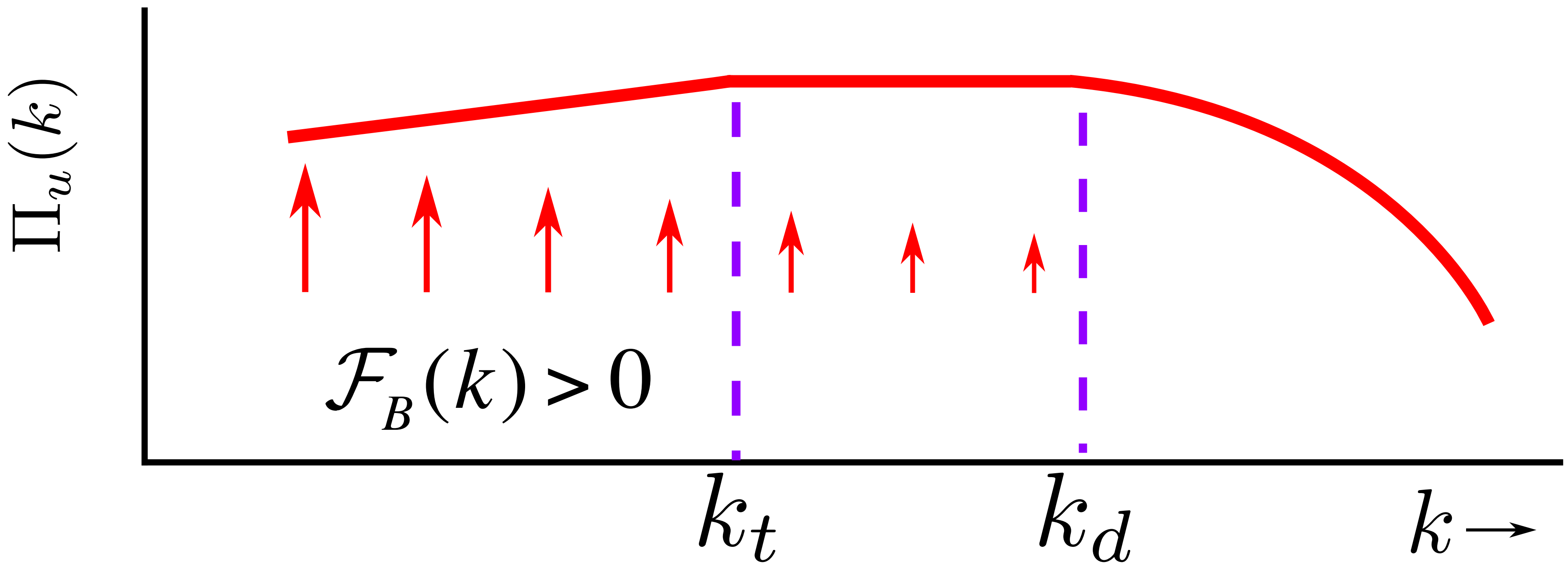
But .. Not so!!



Kumar et al. PRE 2014



For k < k_t

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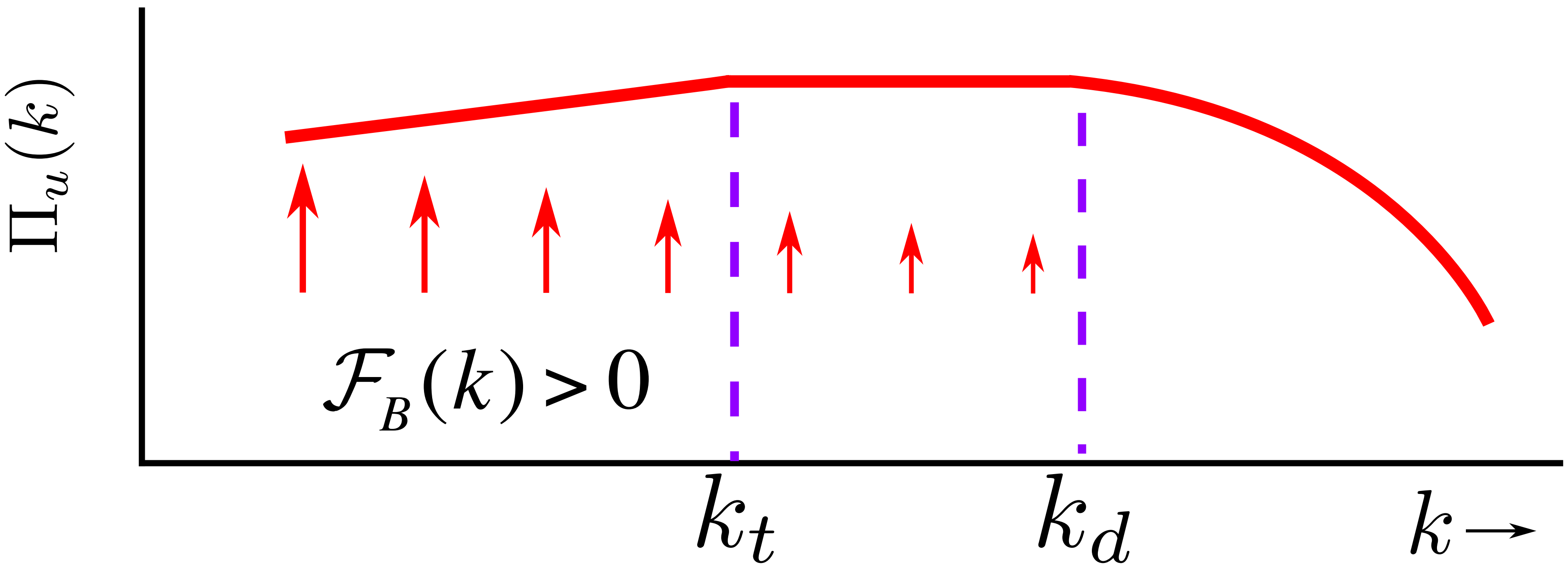
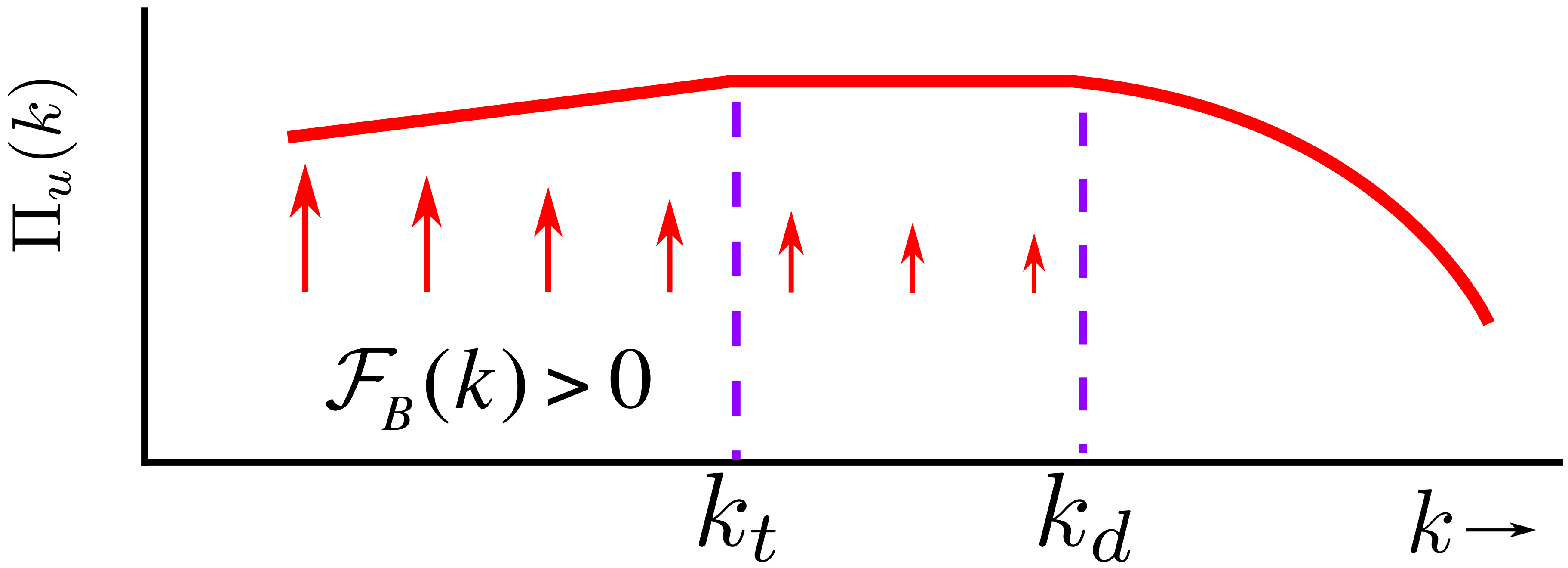


For $k < k_t$

$\Pi_u(k)$ will increase

Kumar et al. PRE 2014



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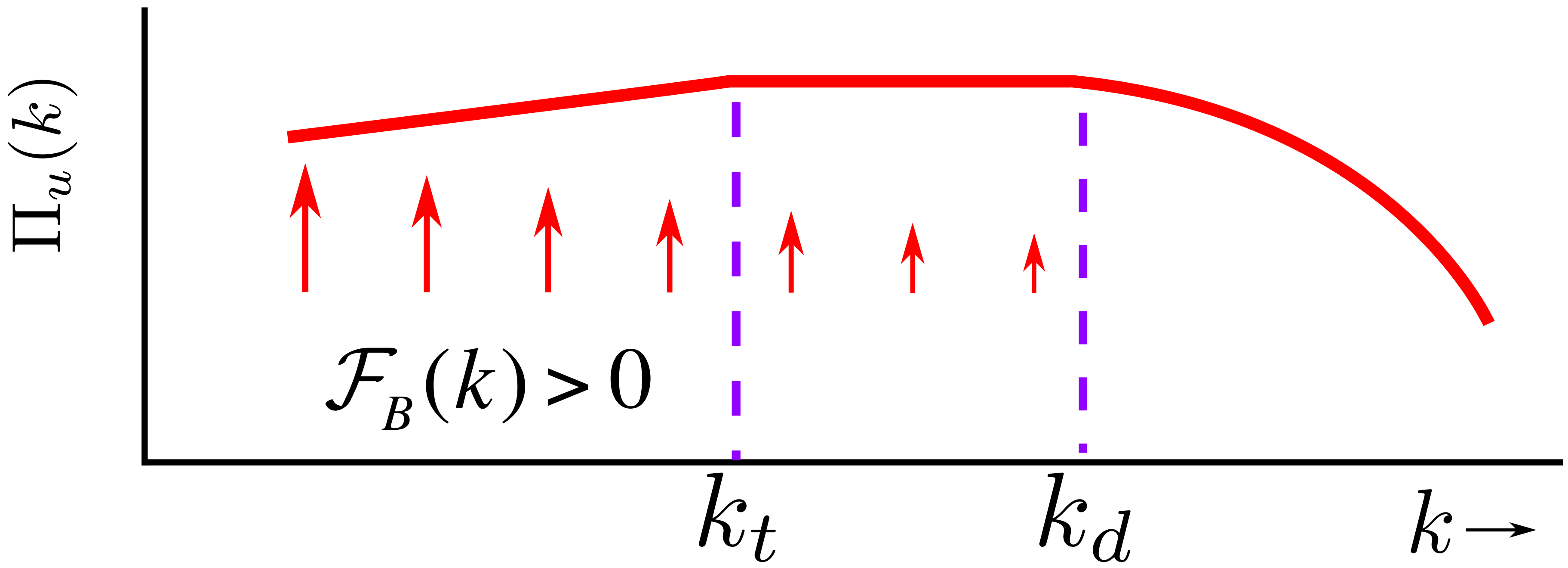
Kumar et al. PRE 2014



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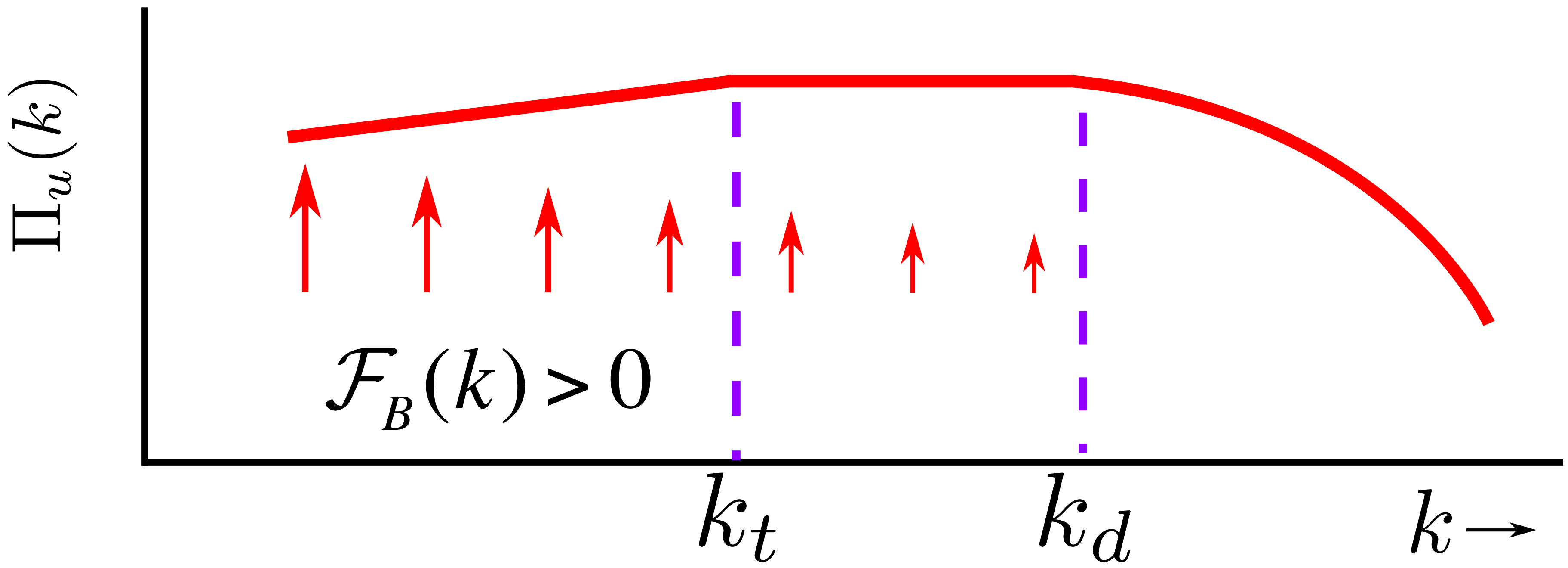
Kumar et al. PRE 2014



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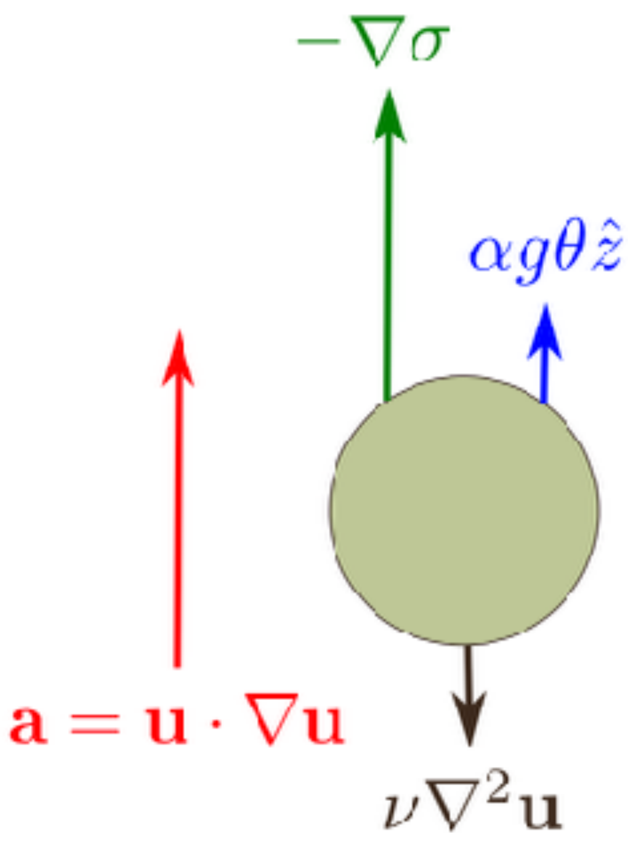
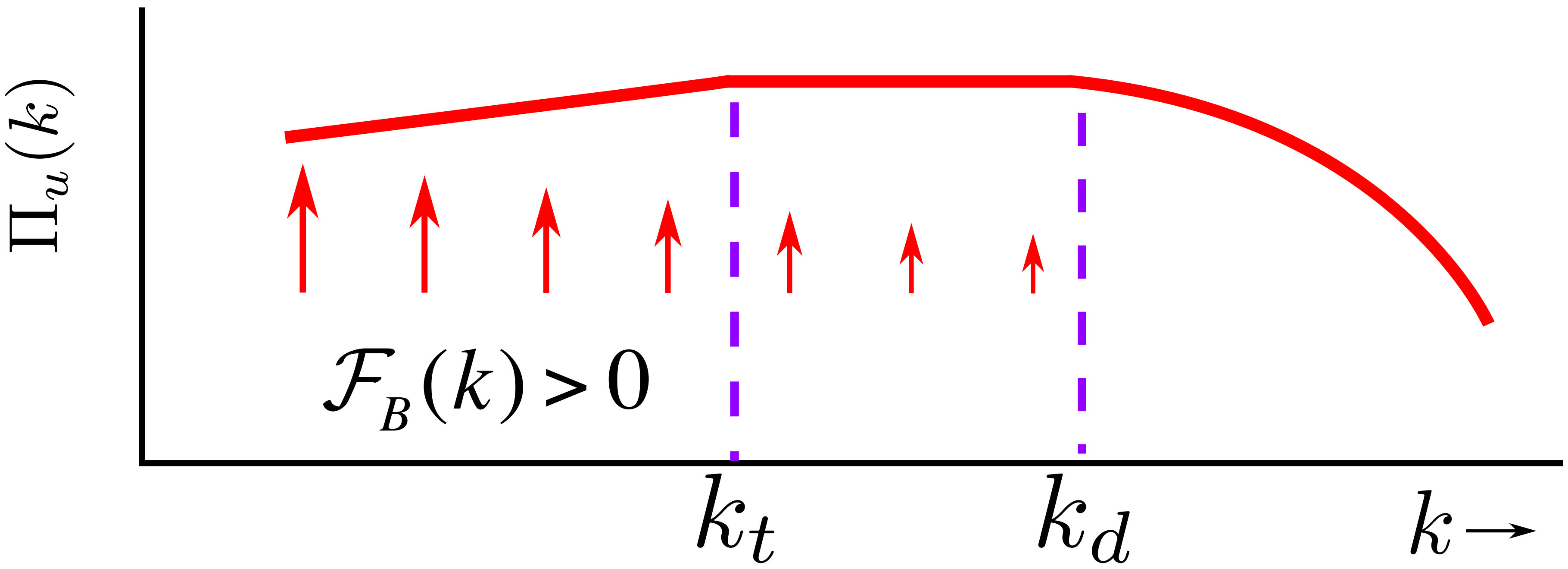


Kumar et al. PRE 2014

Pandey et al. PRE 2017



For $k < k_t$

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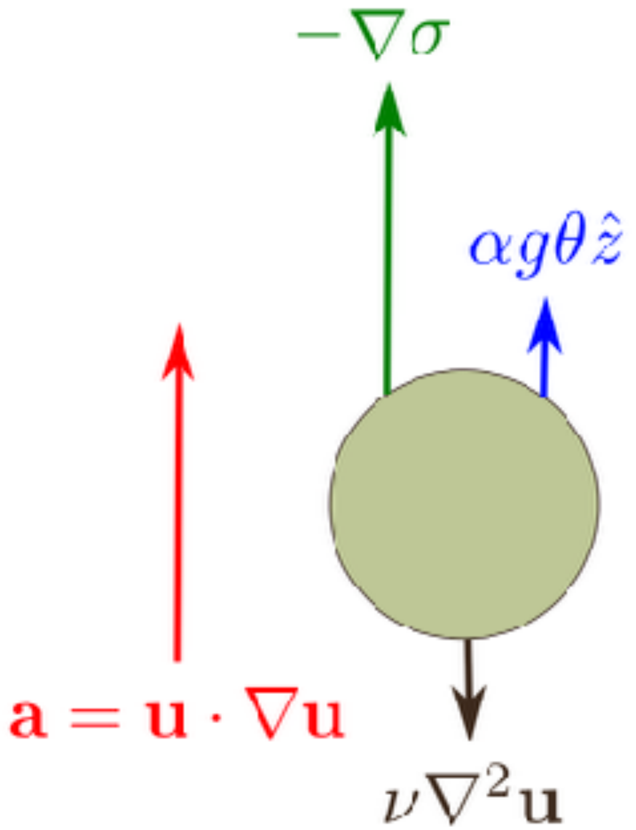

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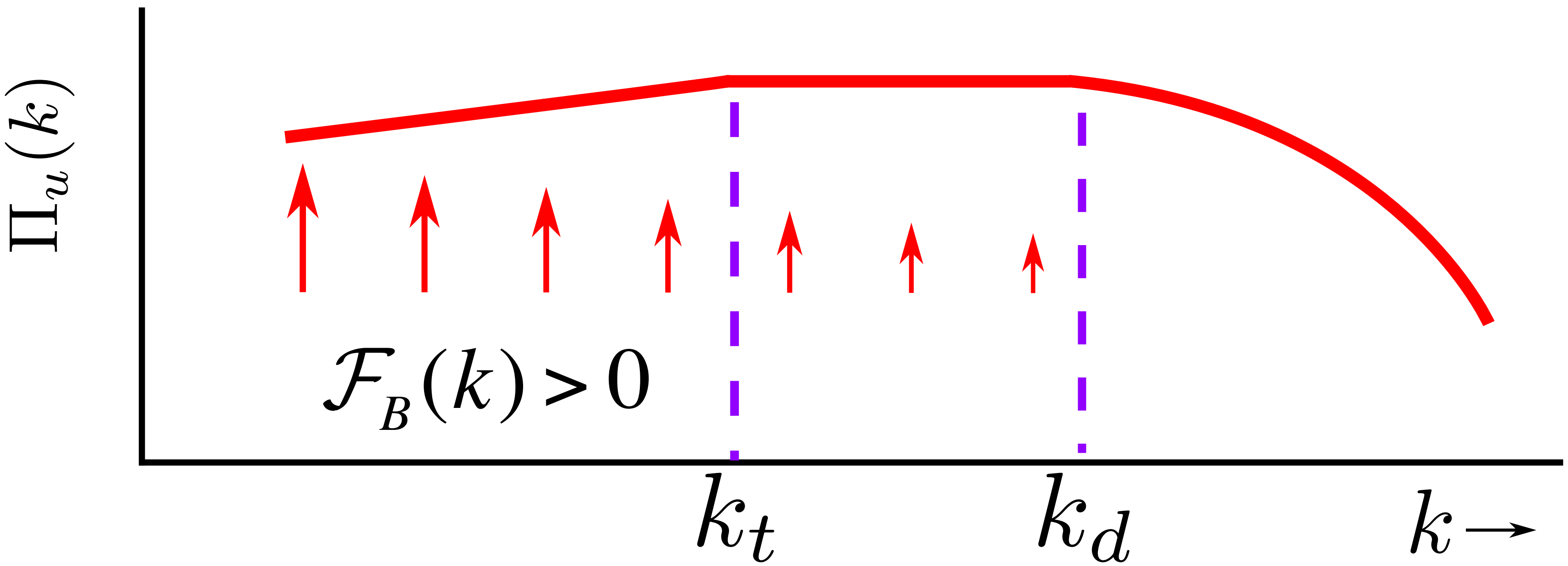

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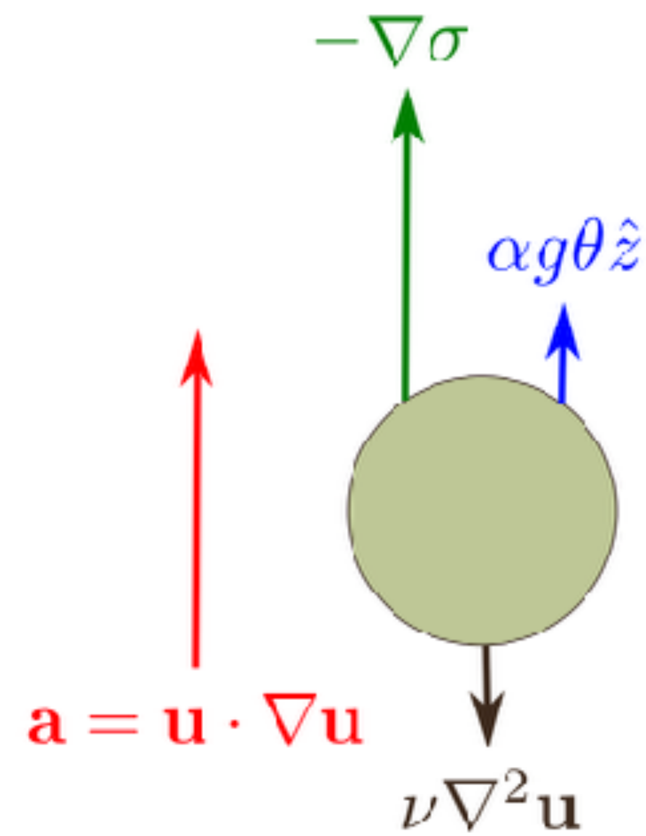

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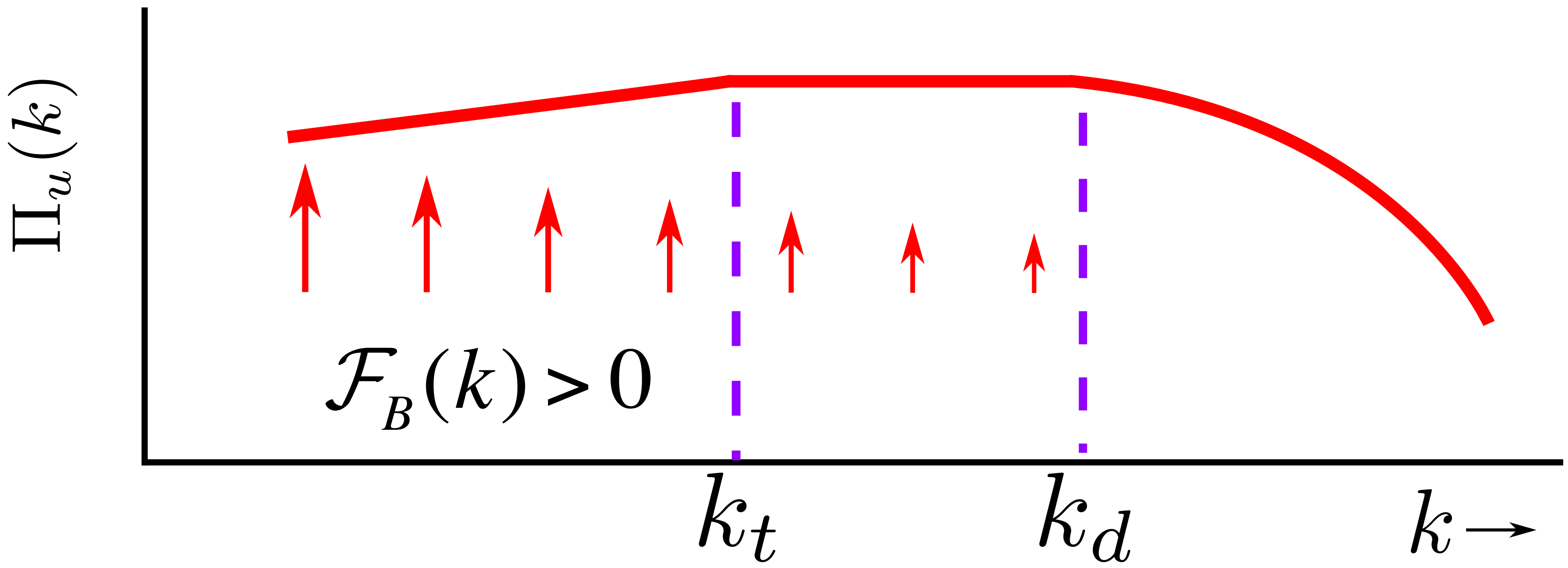

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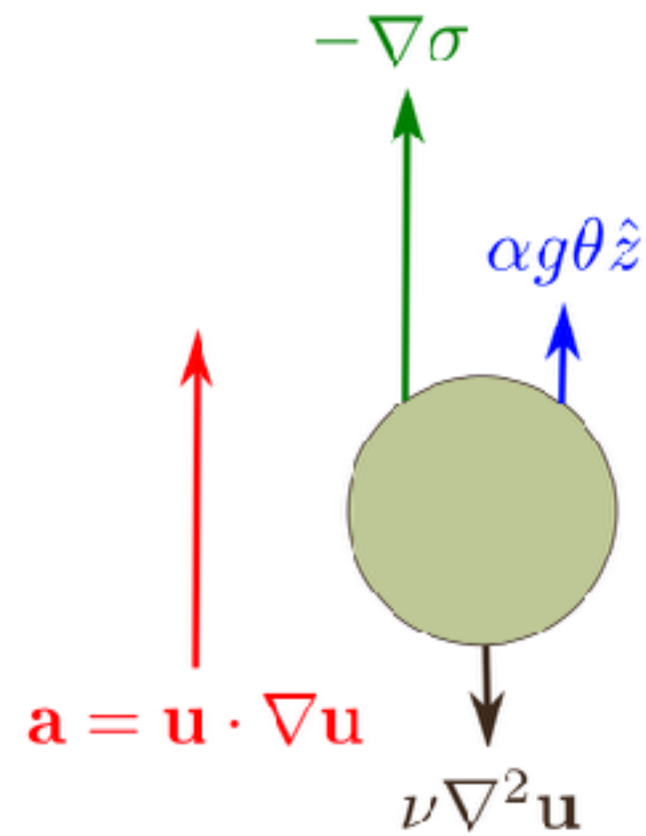

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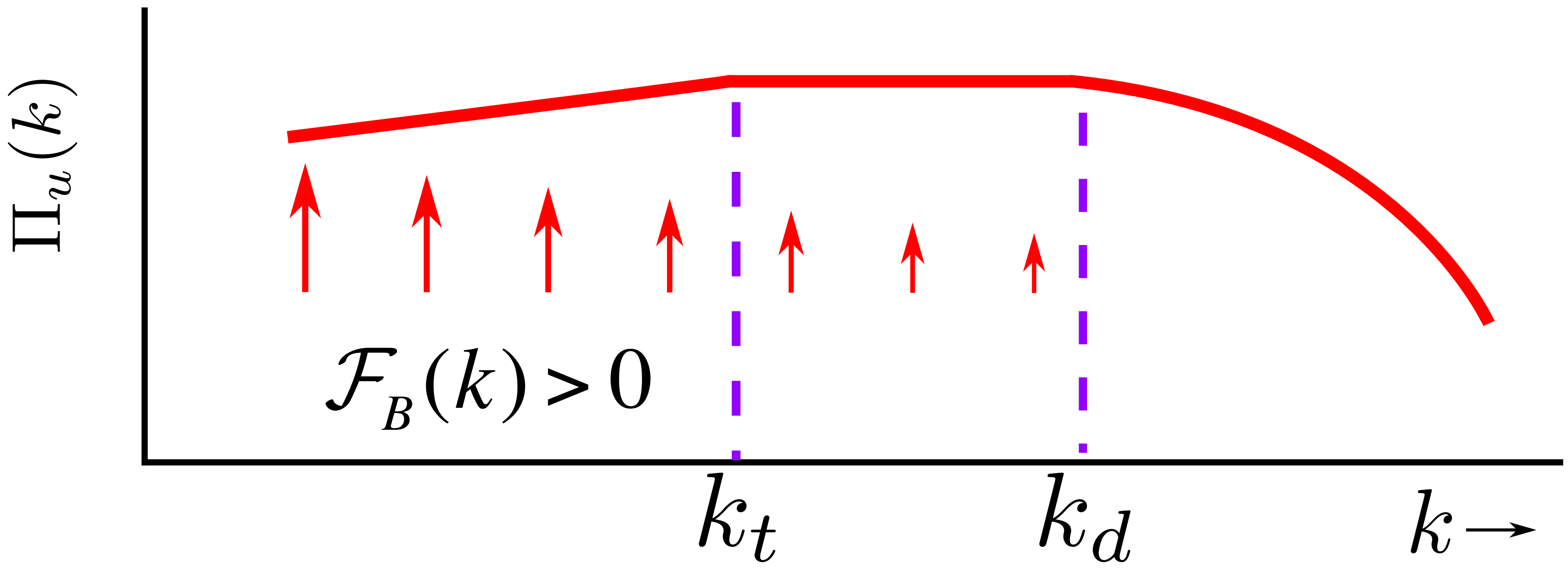

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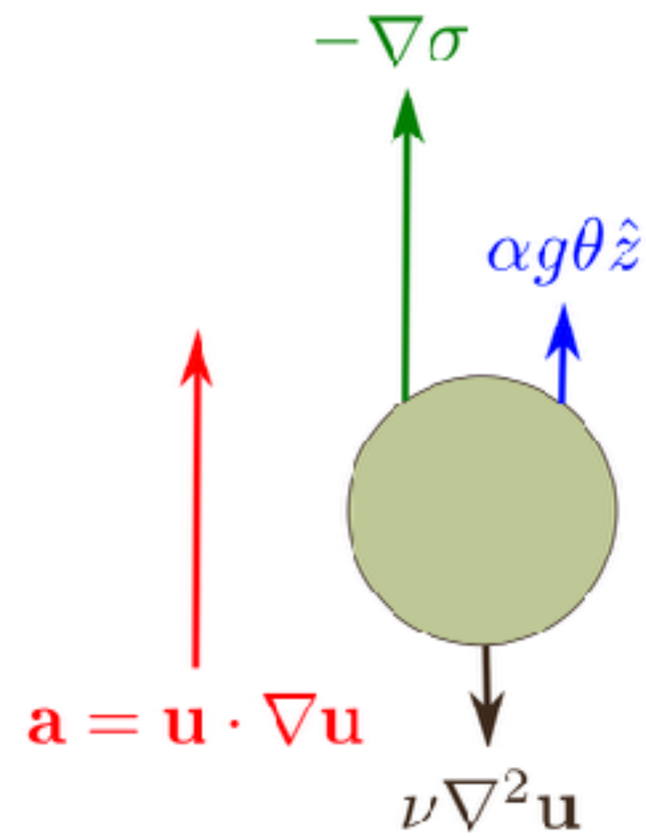

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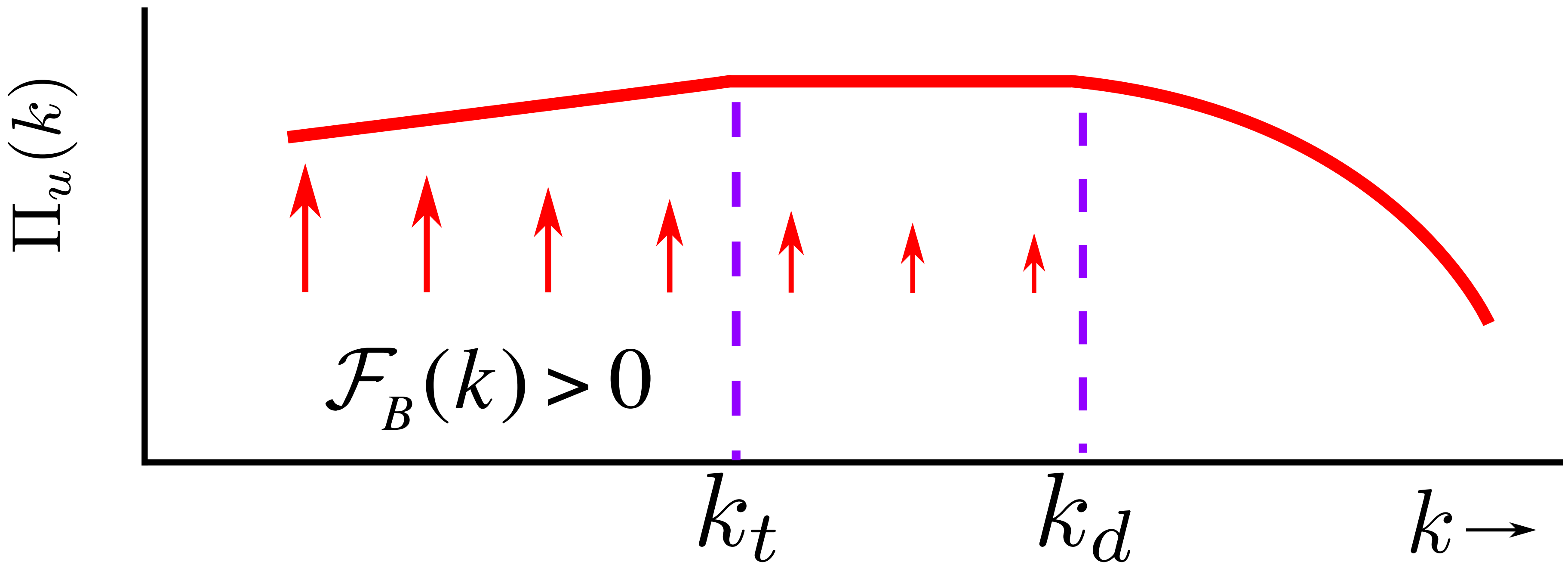

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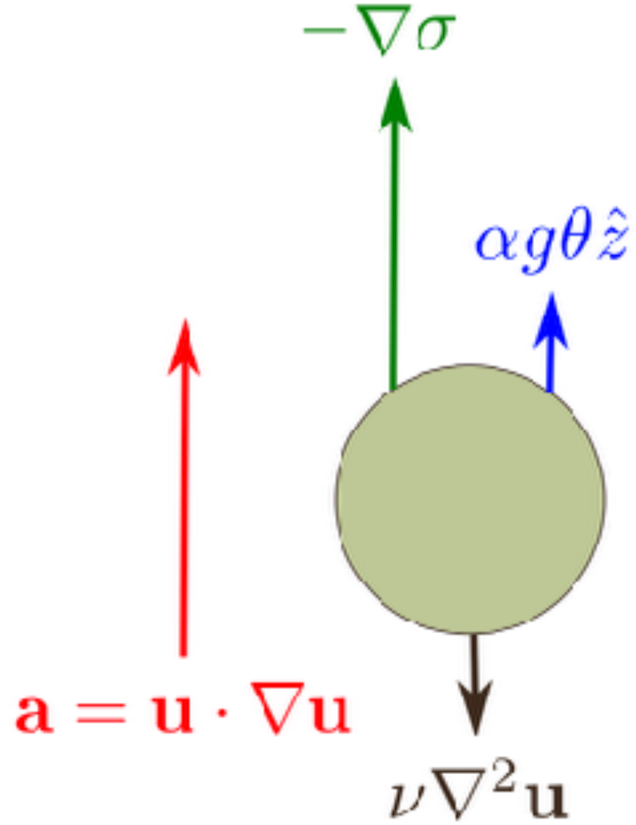

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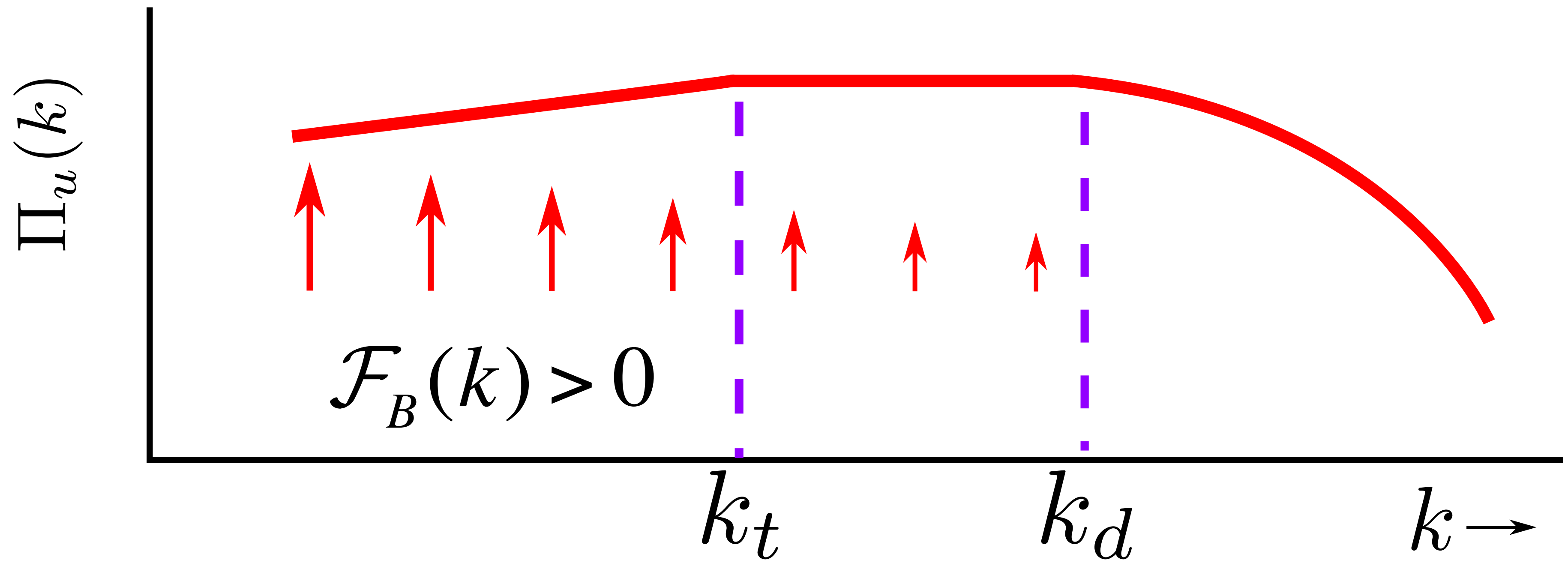

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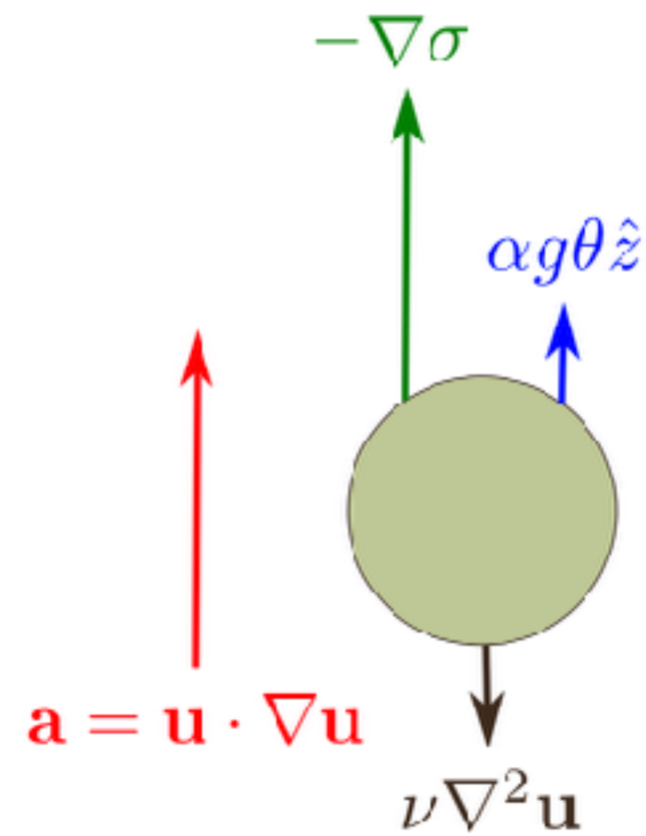

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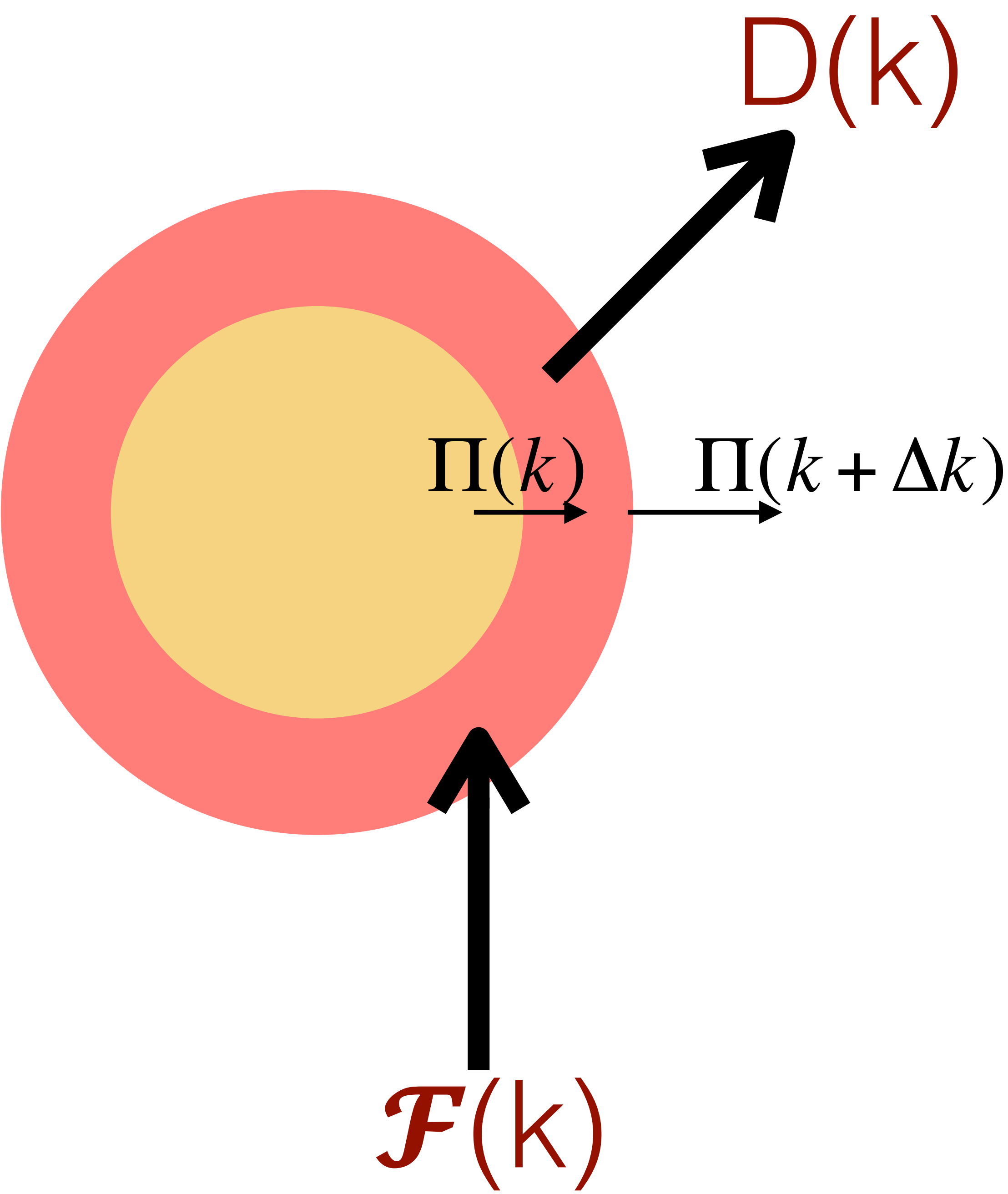
Summary in Fourier space



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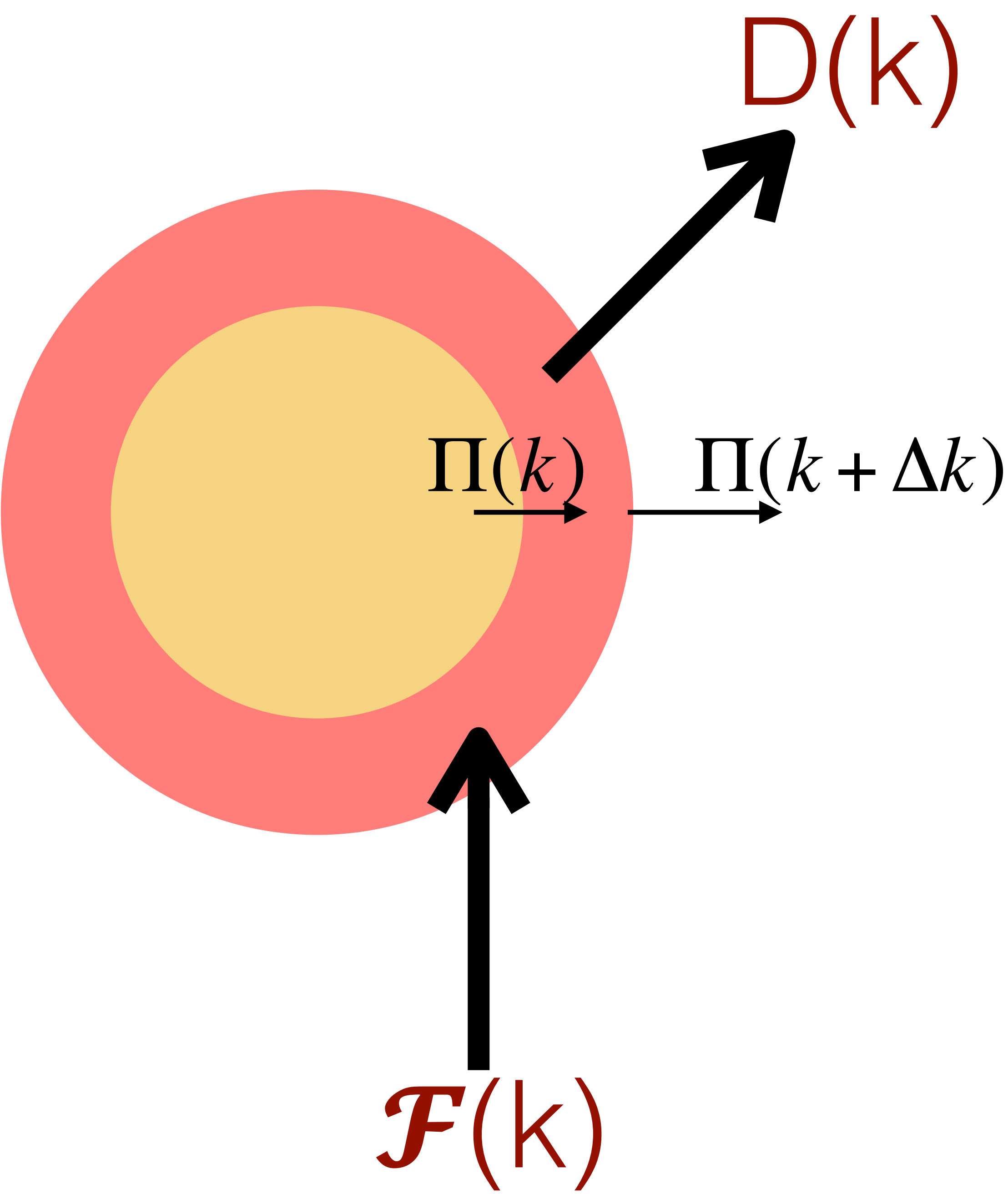

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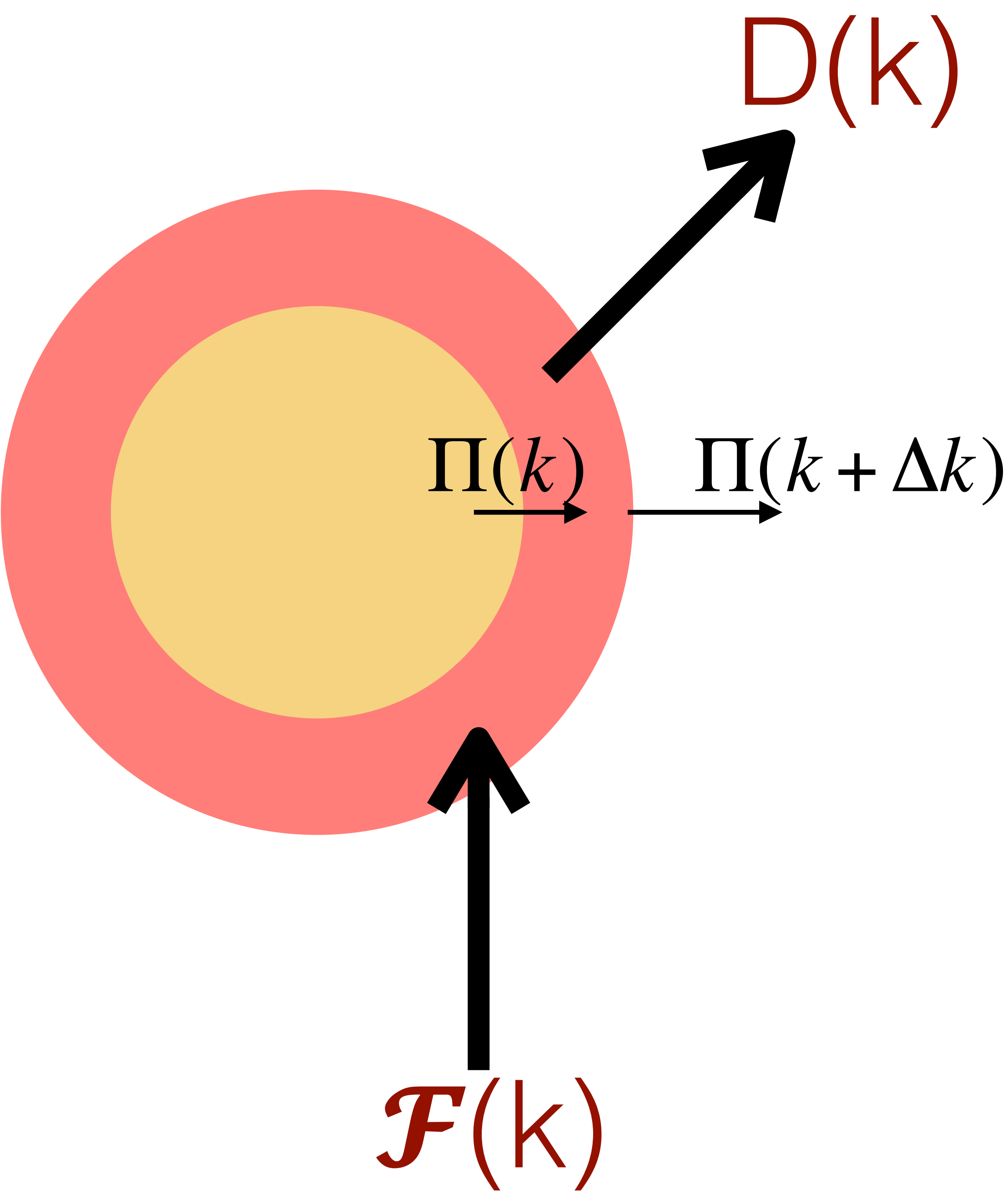
HT

Summary in Fourier space



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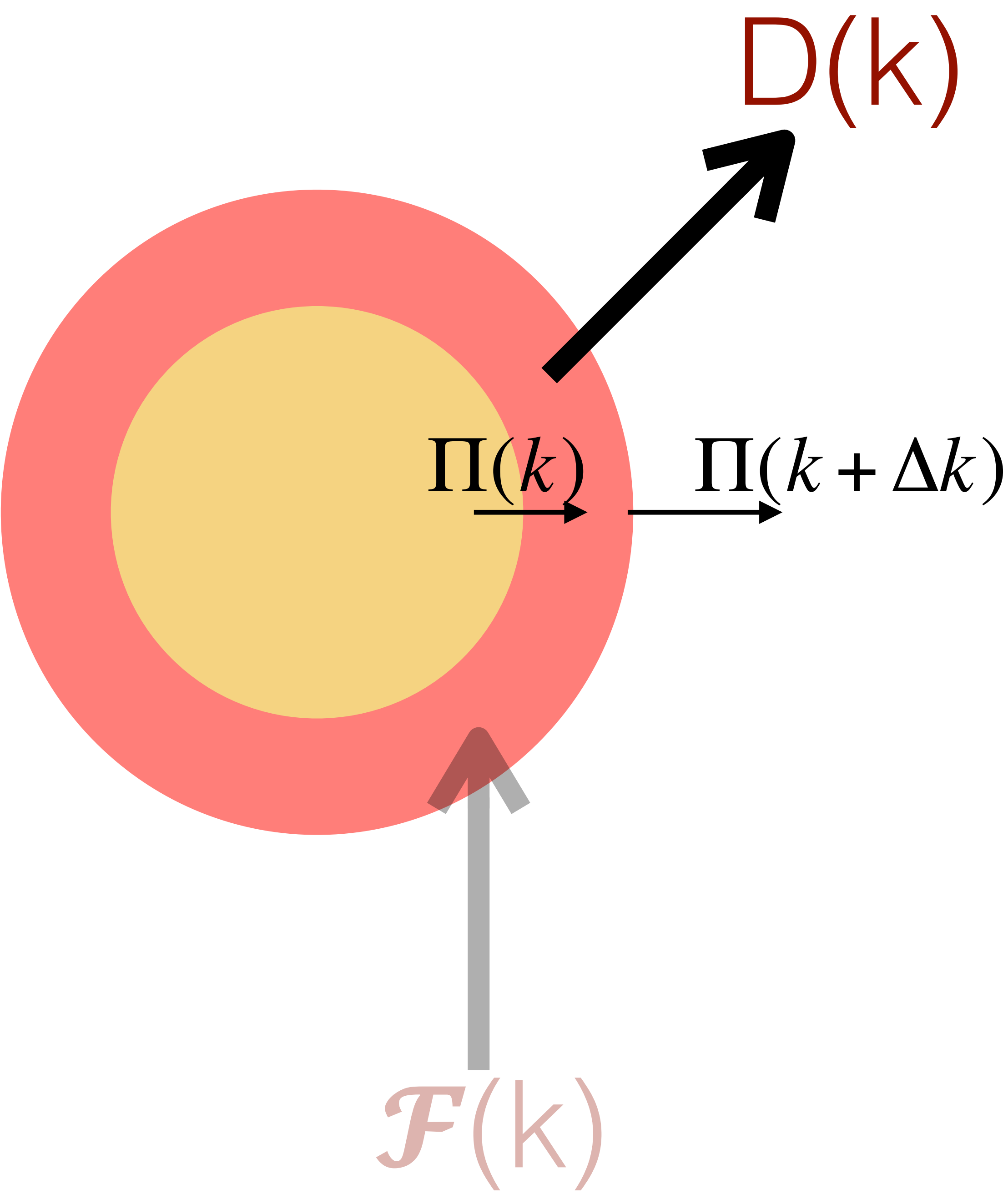

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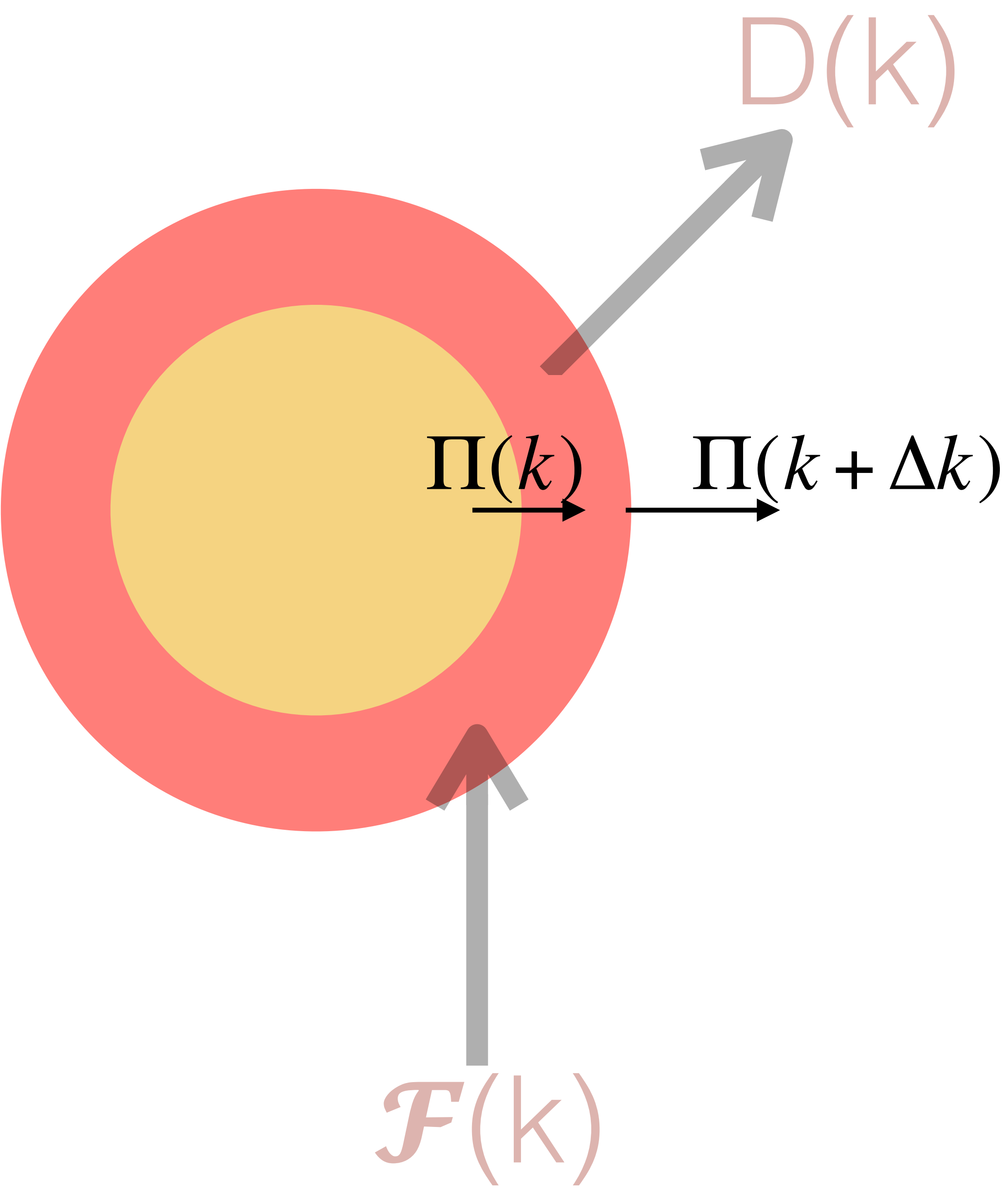

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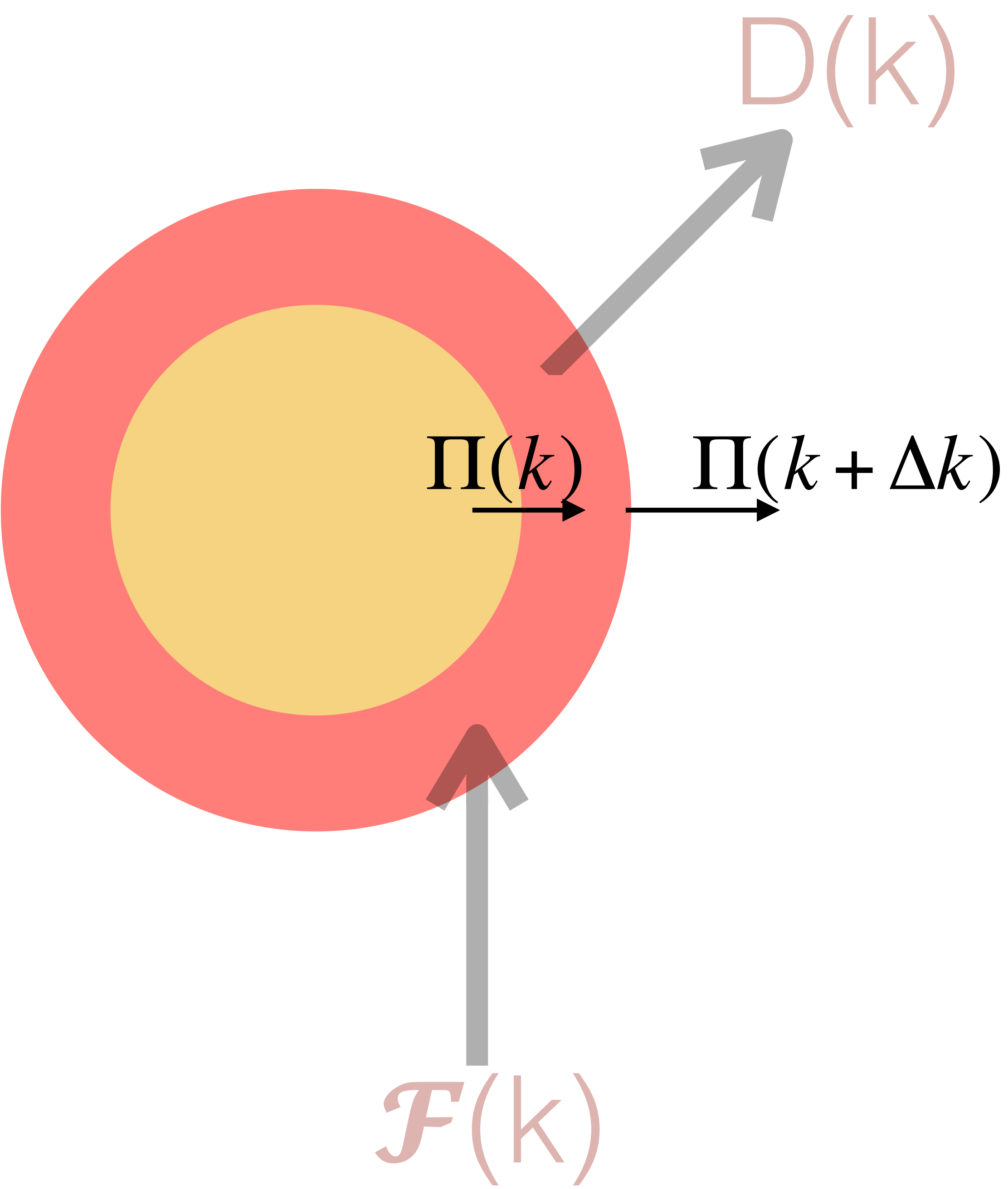

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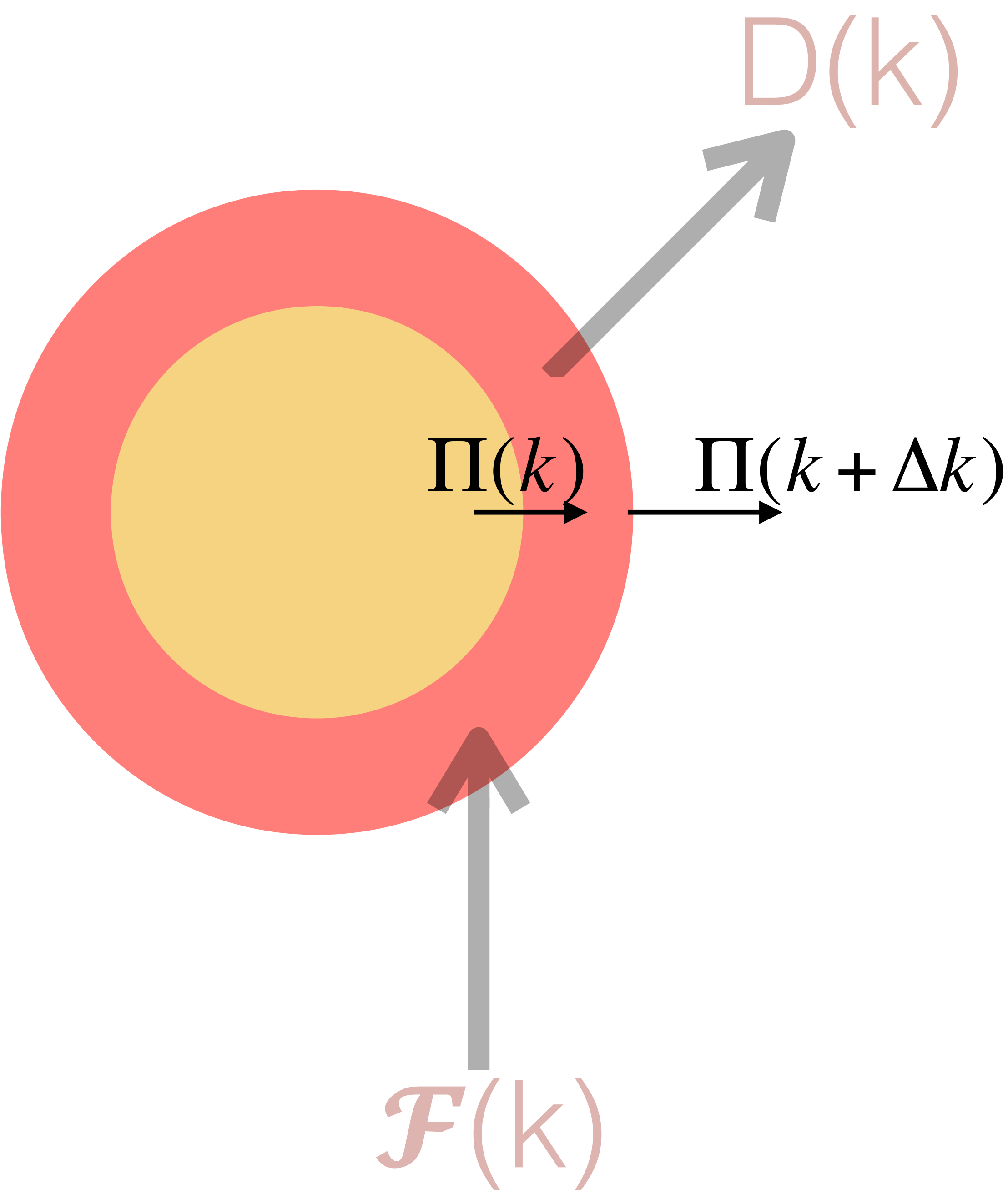

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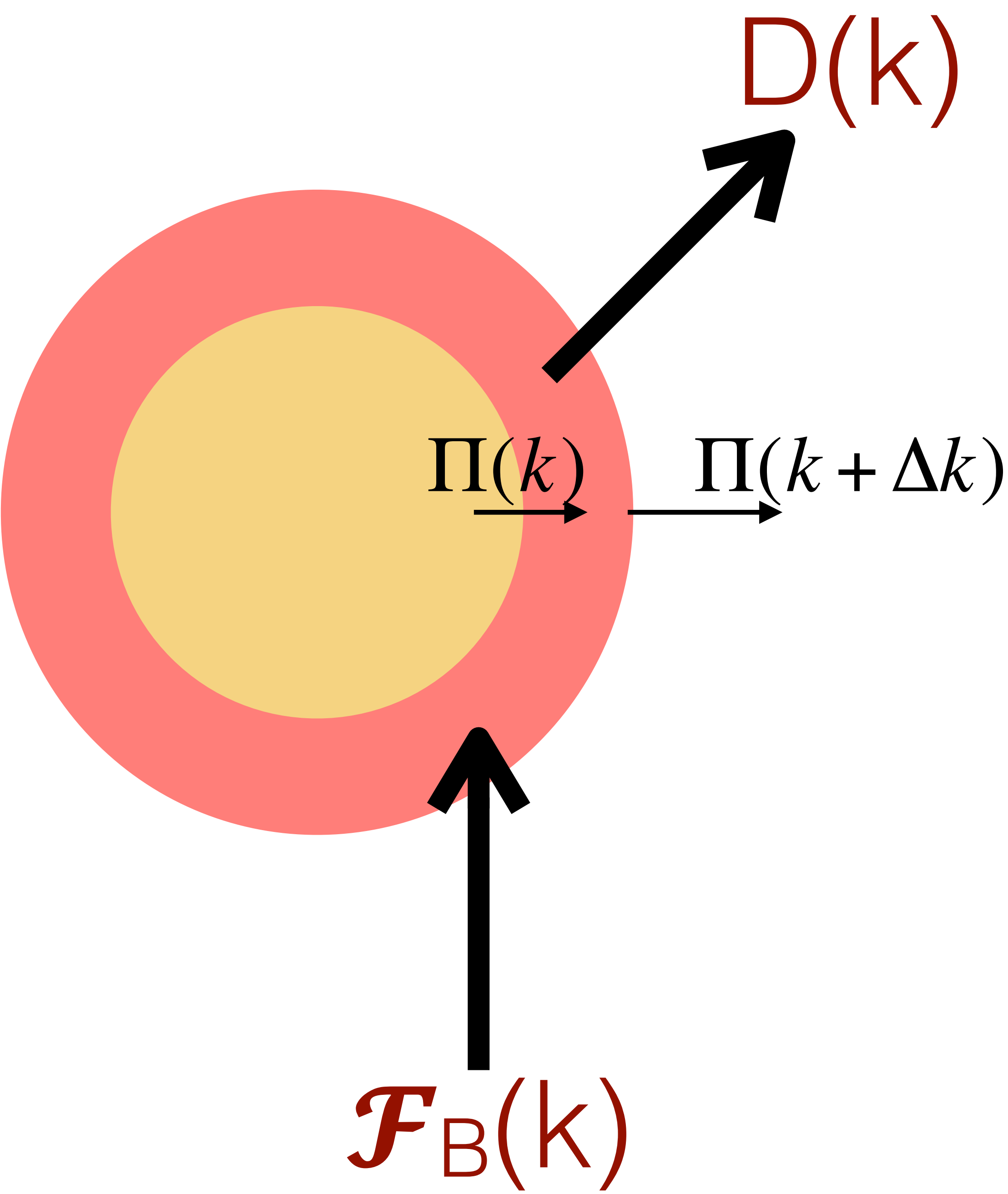

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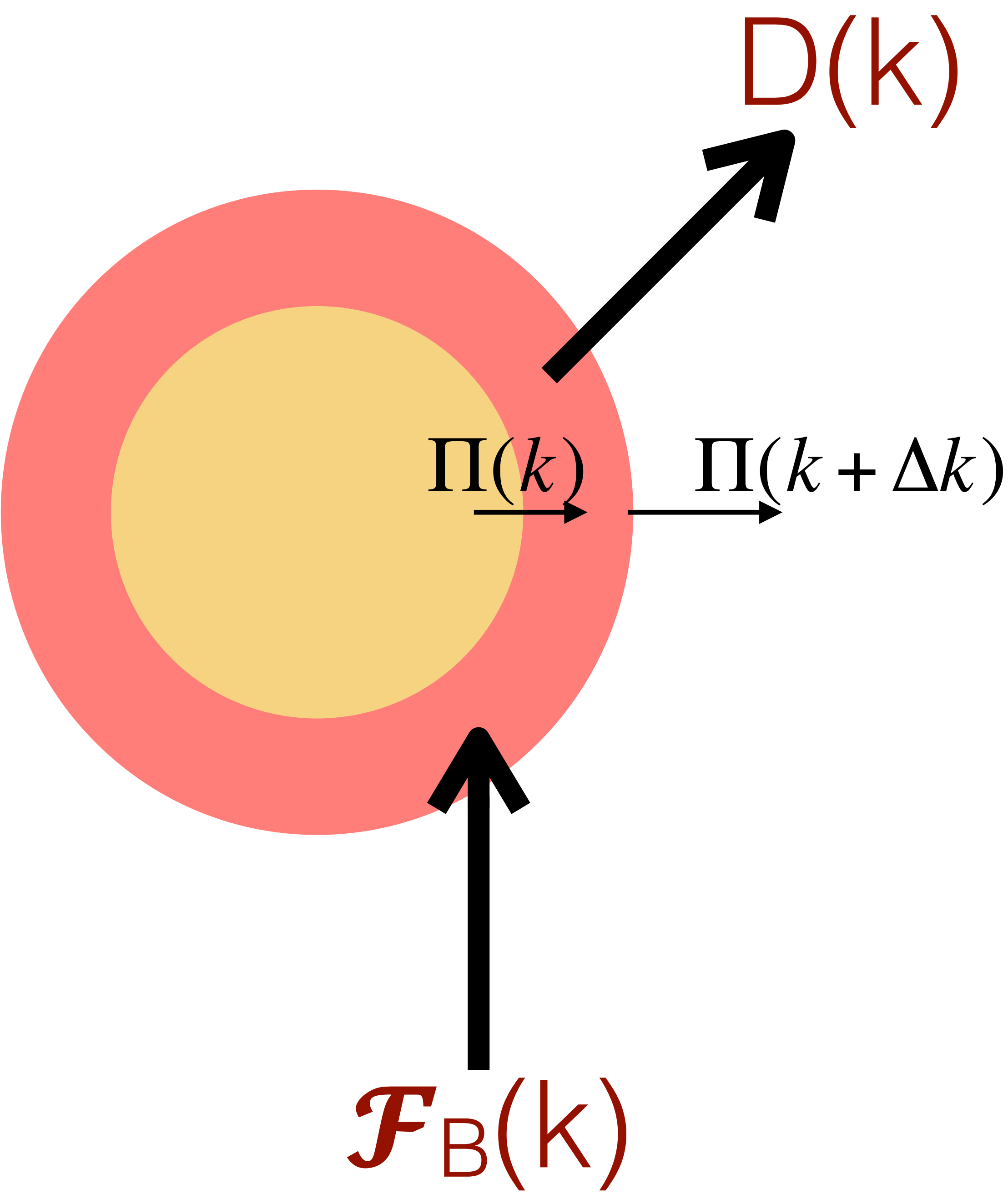
SST

Summary in Fourier space



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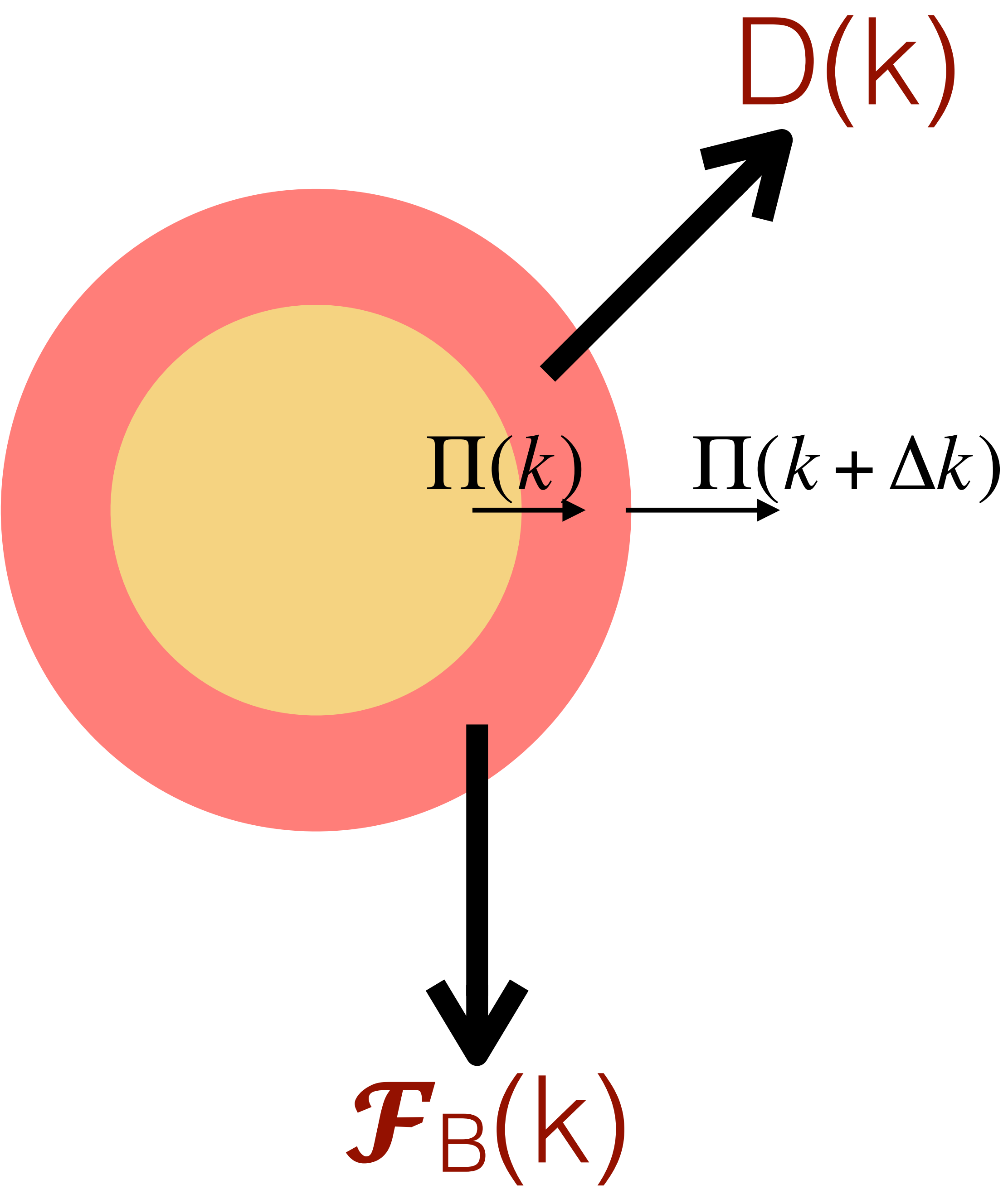
SST $\mathcal{F}_B(k) < 0$

Summary in Fourier space



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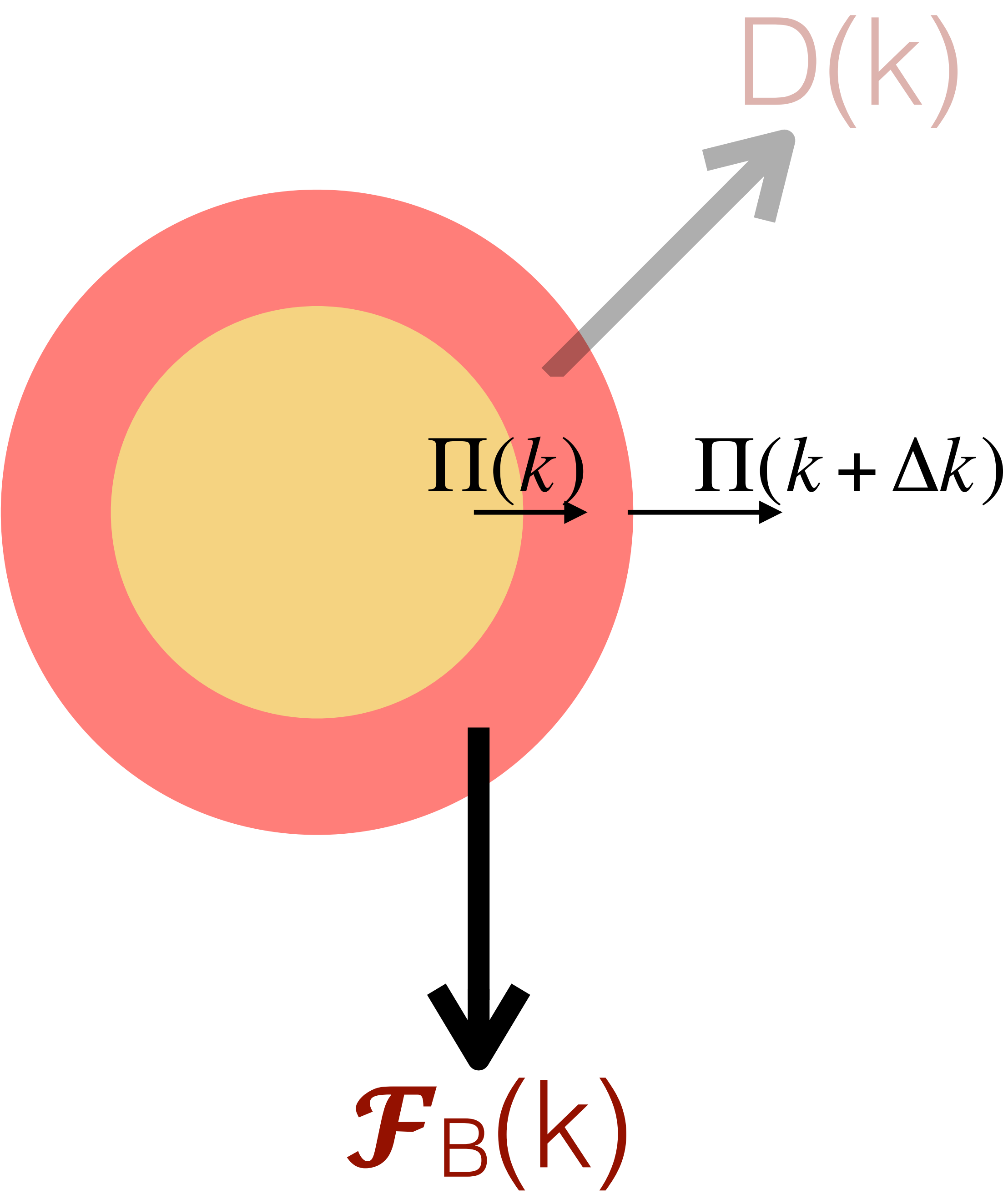

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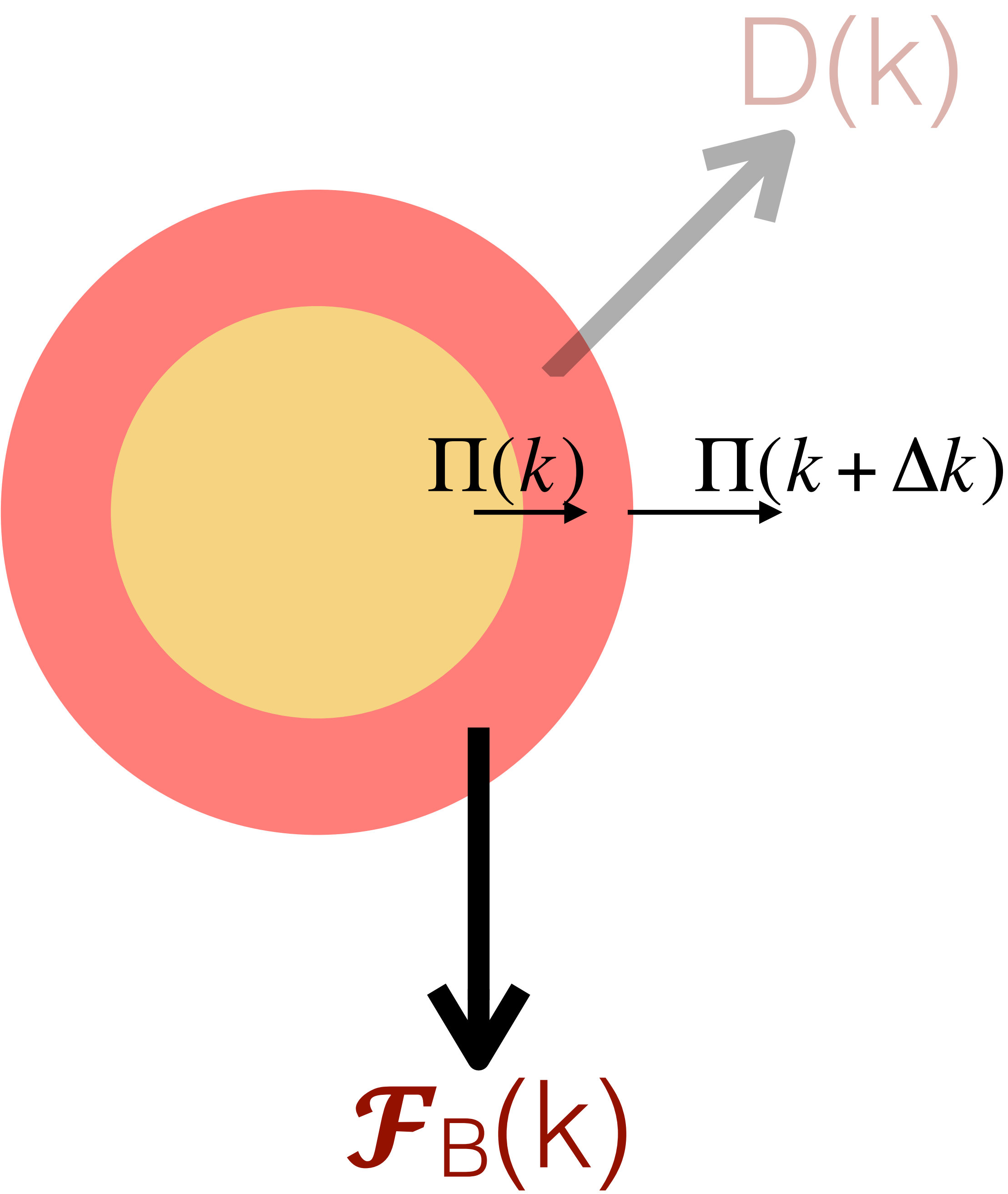

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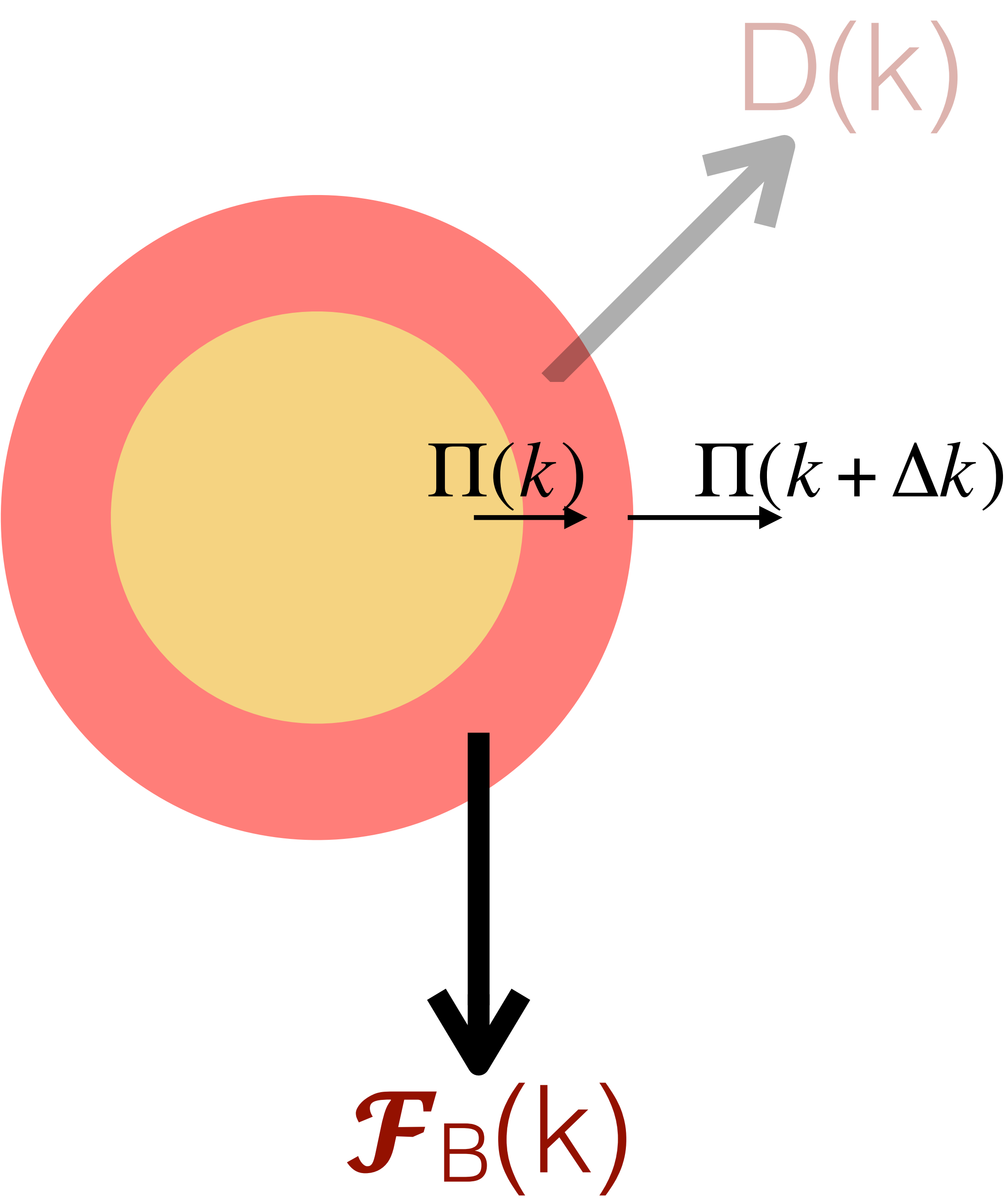
SST $\mathcal{F}_B(k) < 0$ $\Pi(k) \sim k^{-4/5}$

Summary in Fourier space



$$\frac{d\Pi(k)}{dk} = \mathcal{F}(k) - D(k)$$

$$\Pi(k+\Delta k) = \Pi(k) + \left[\mathcal{F}(k) - D(k)\right]\Delta k$$

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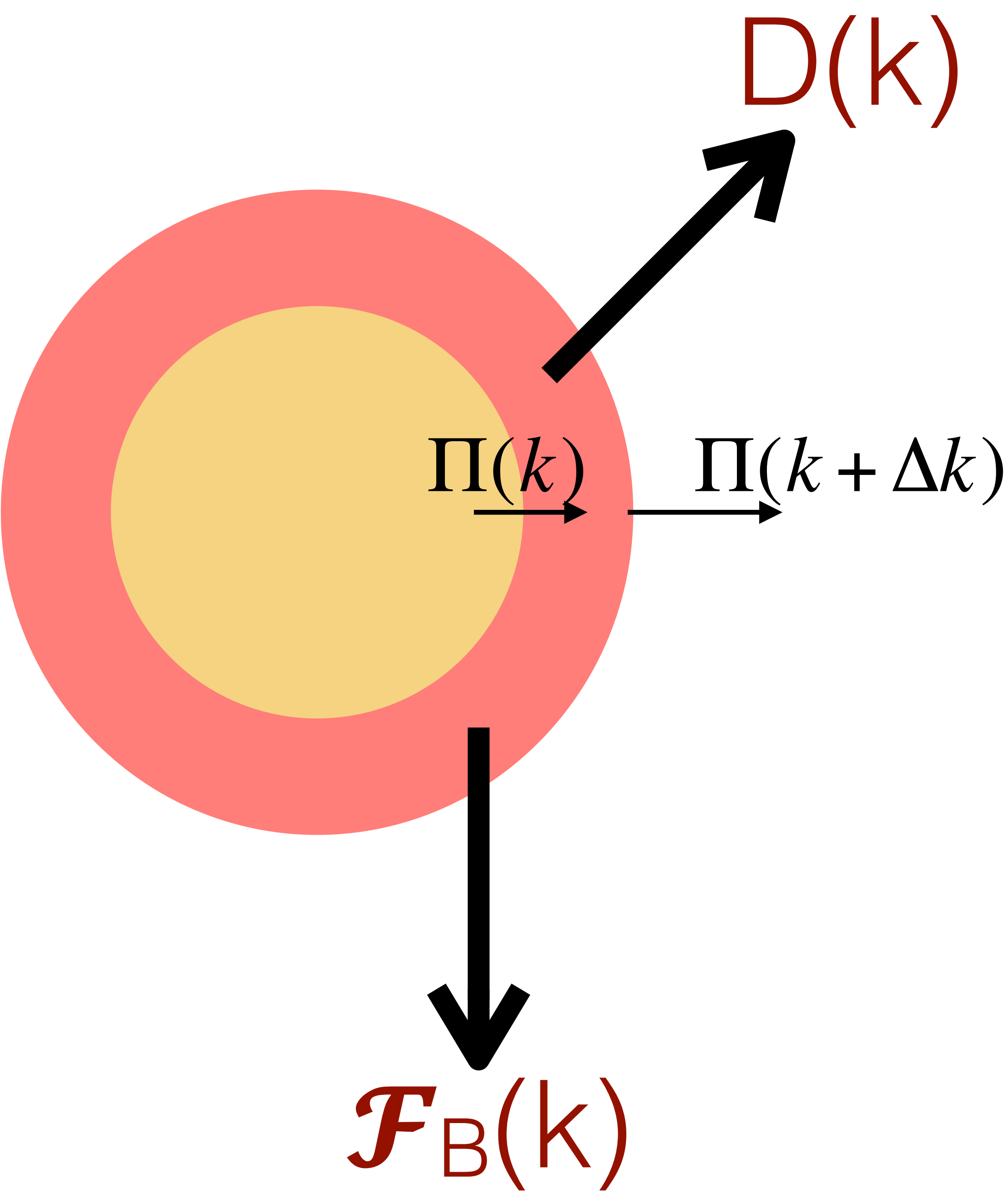
SST	$\mathcal{F}_B(k) < 0$	$\Pi(k) \sim k^{-4/5}$	$E(k) \sim k^{-11/5}$

Summary in Fourier space



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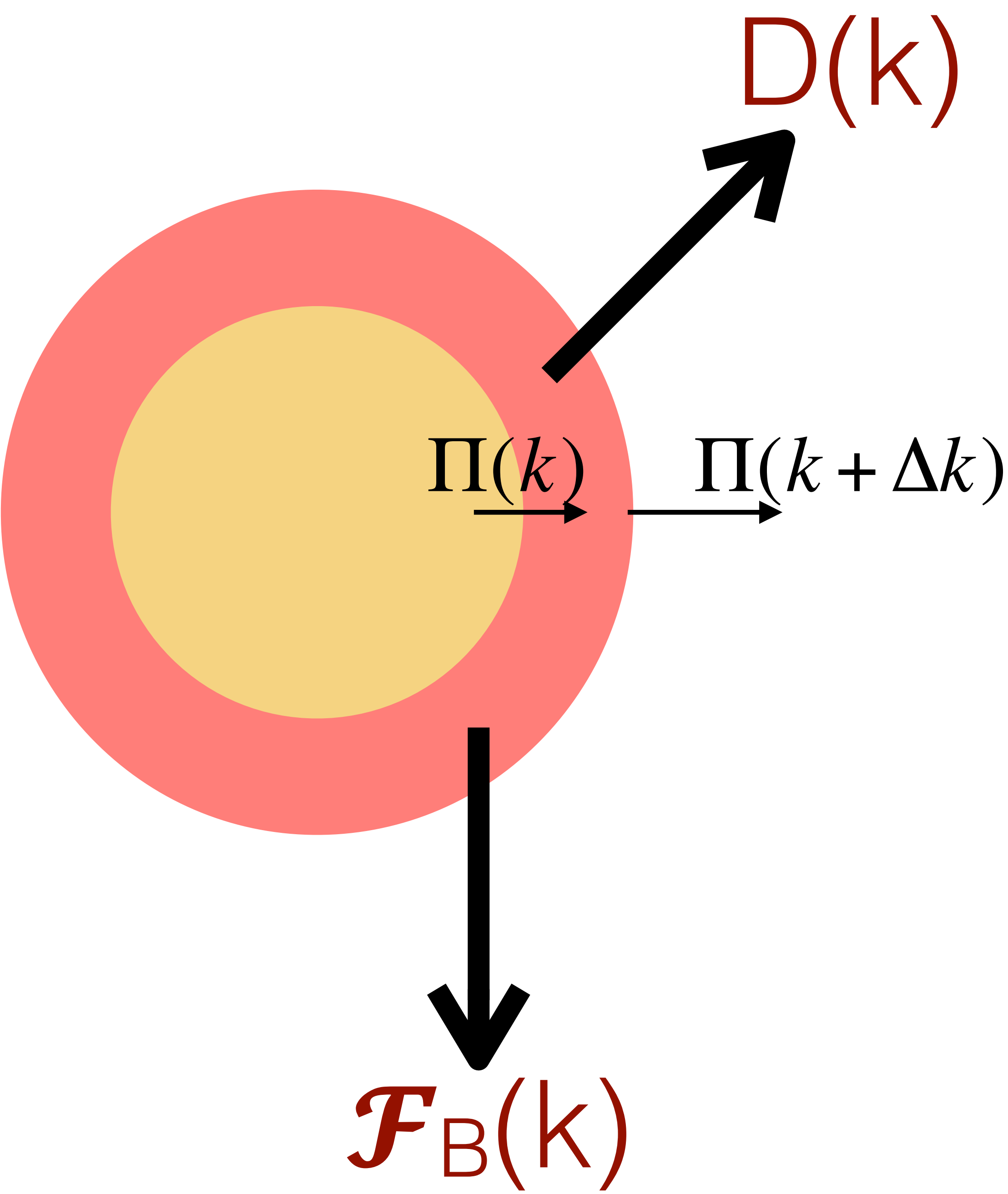

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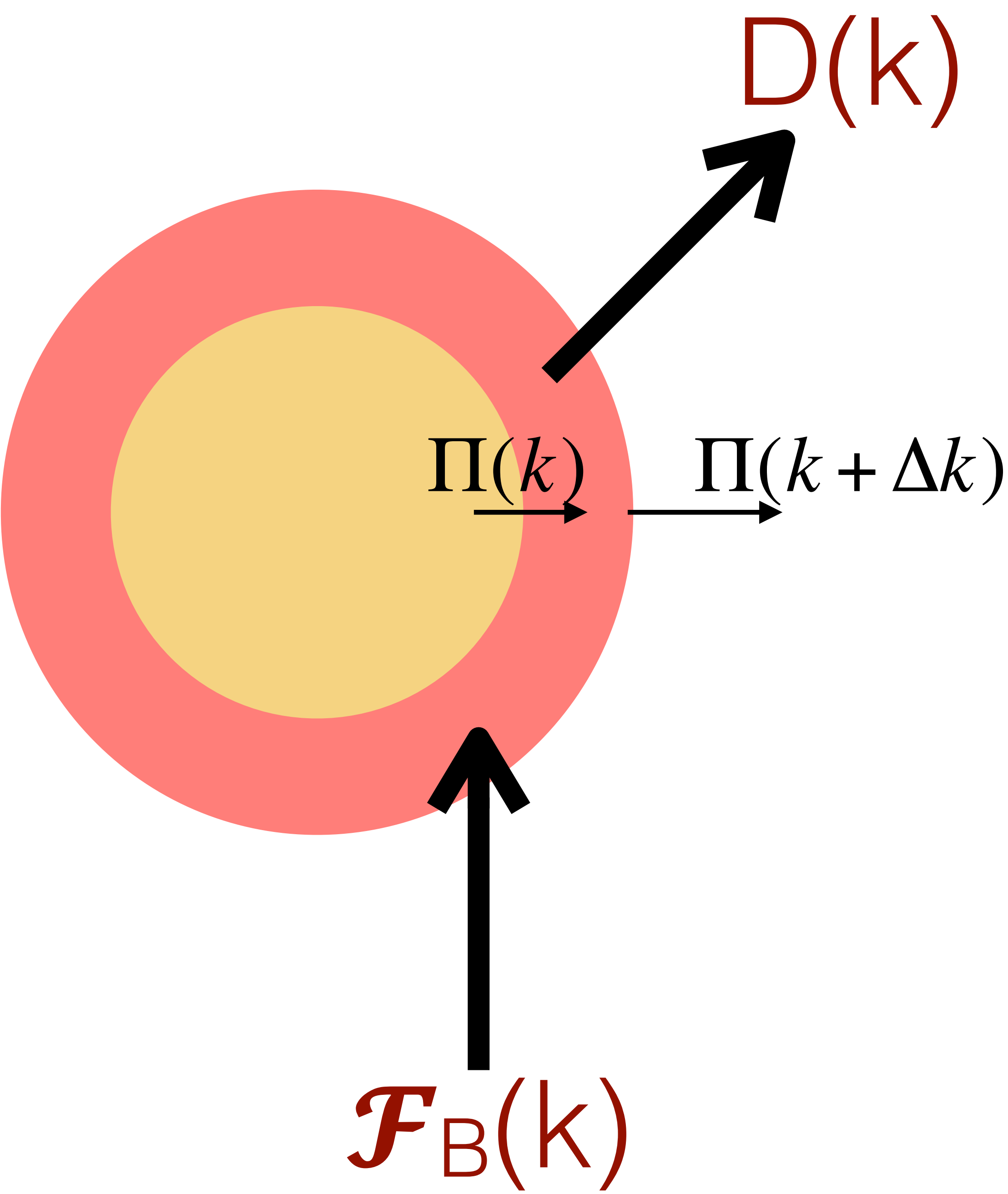

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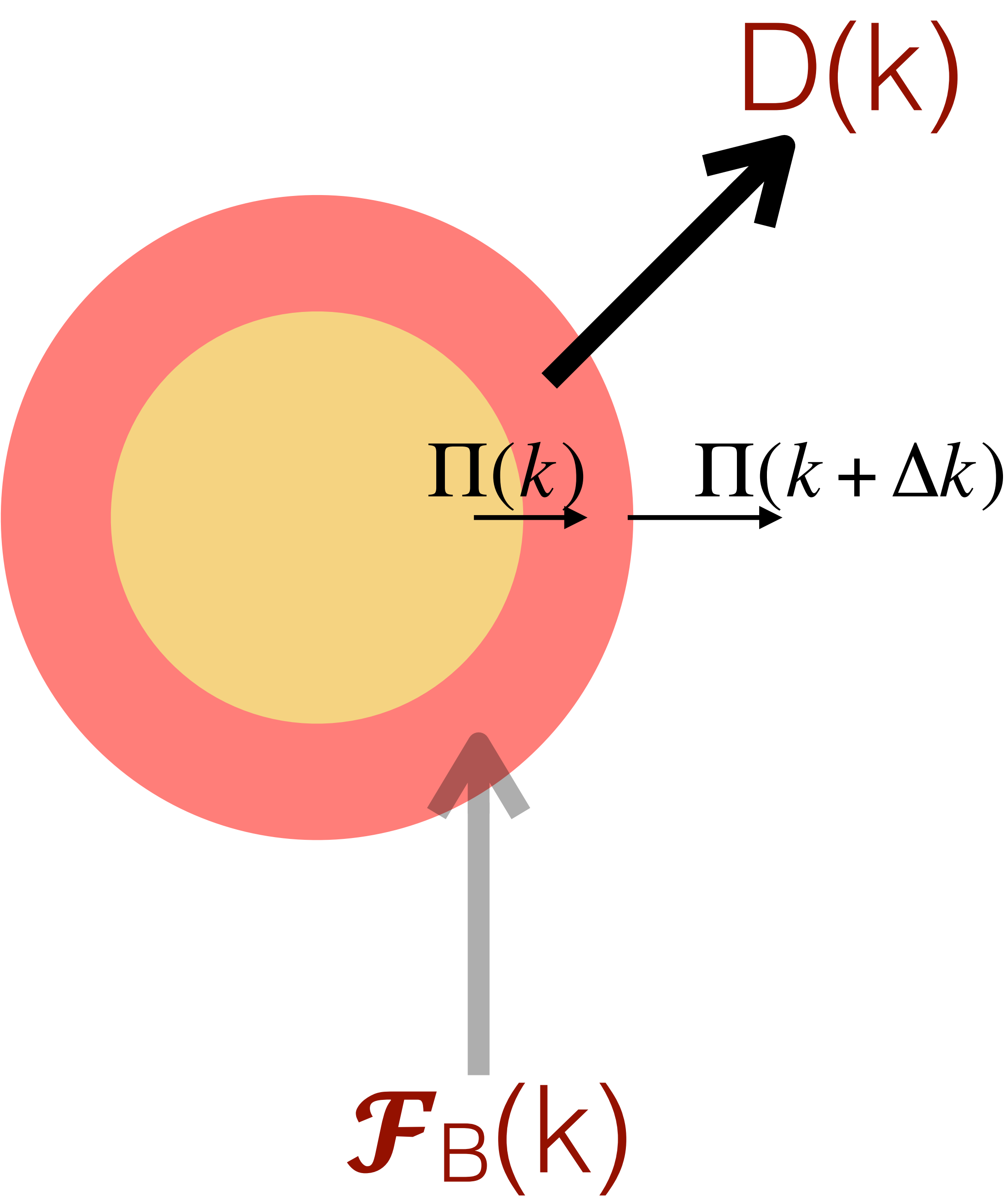

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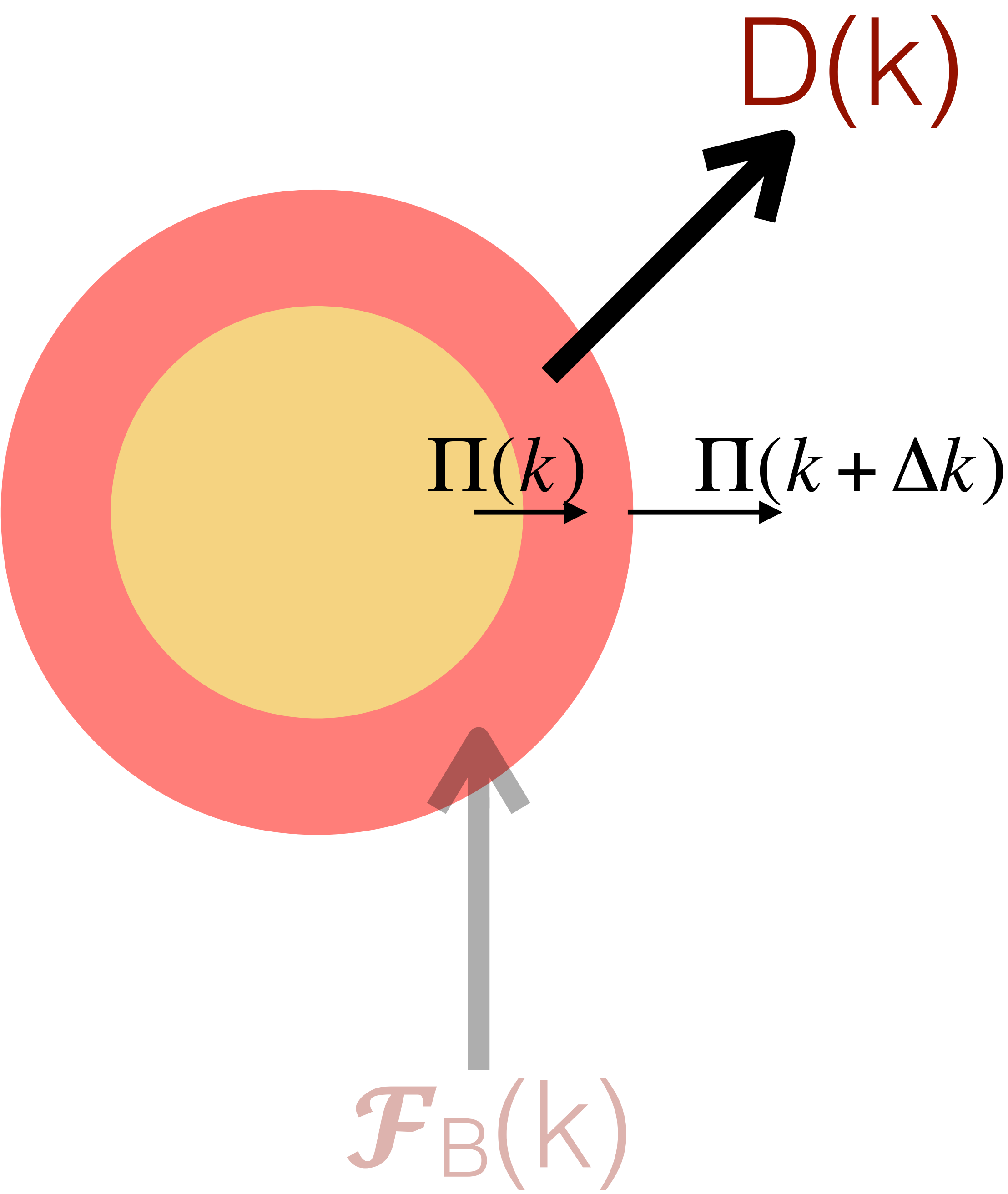

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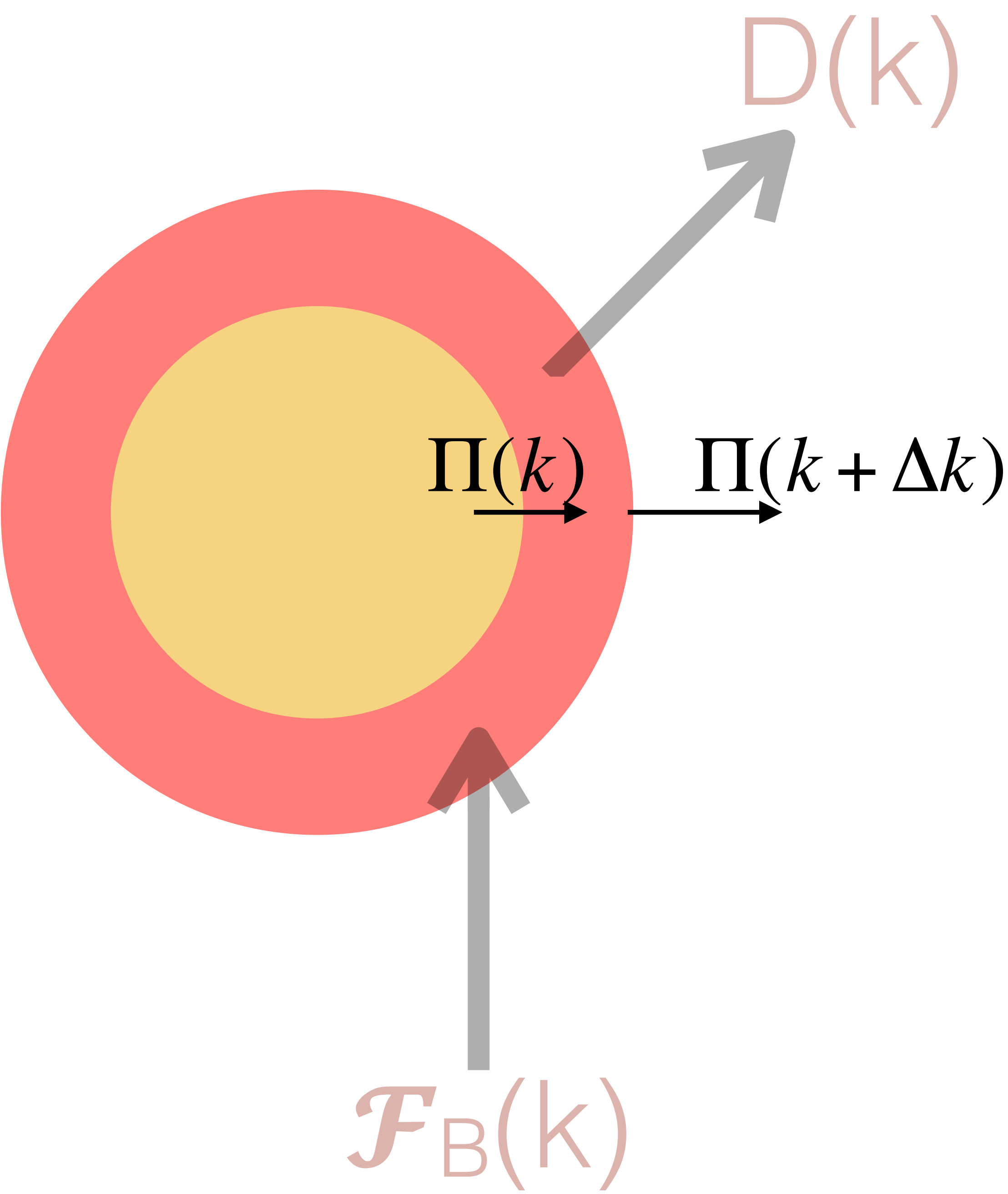

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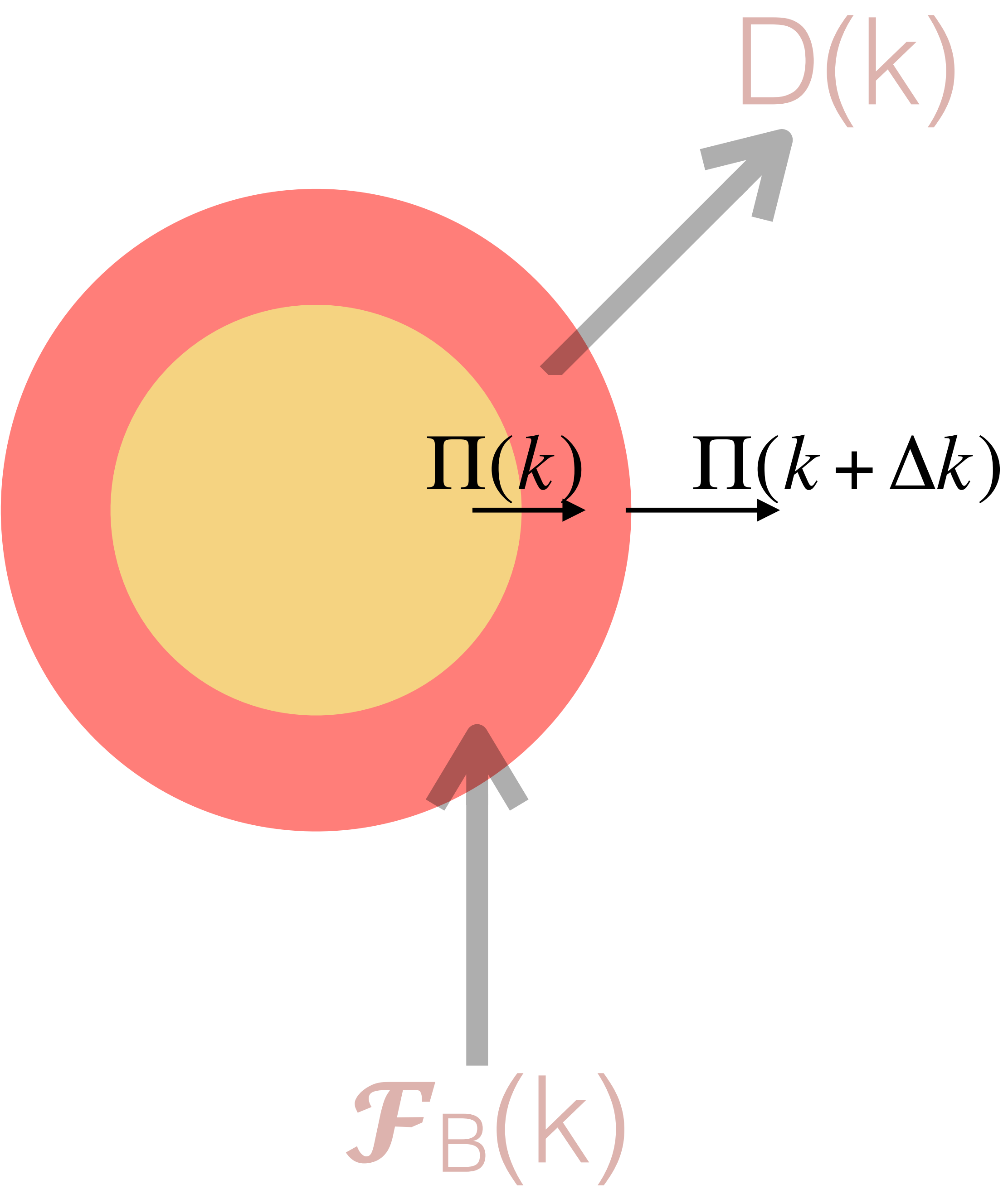

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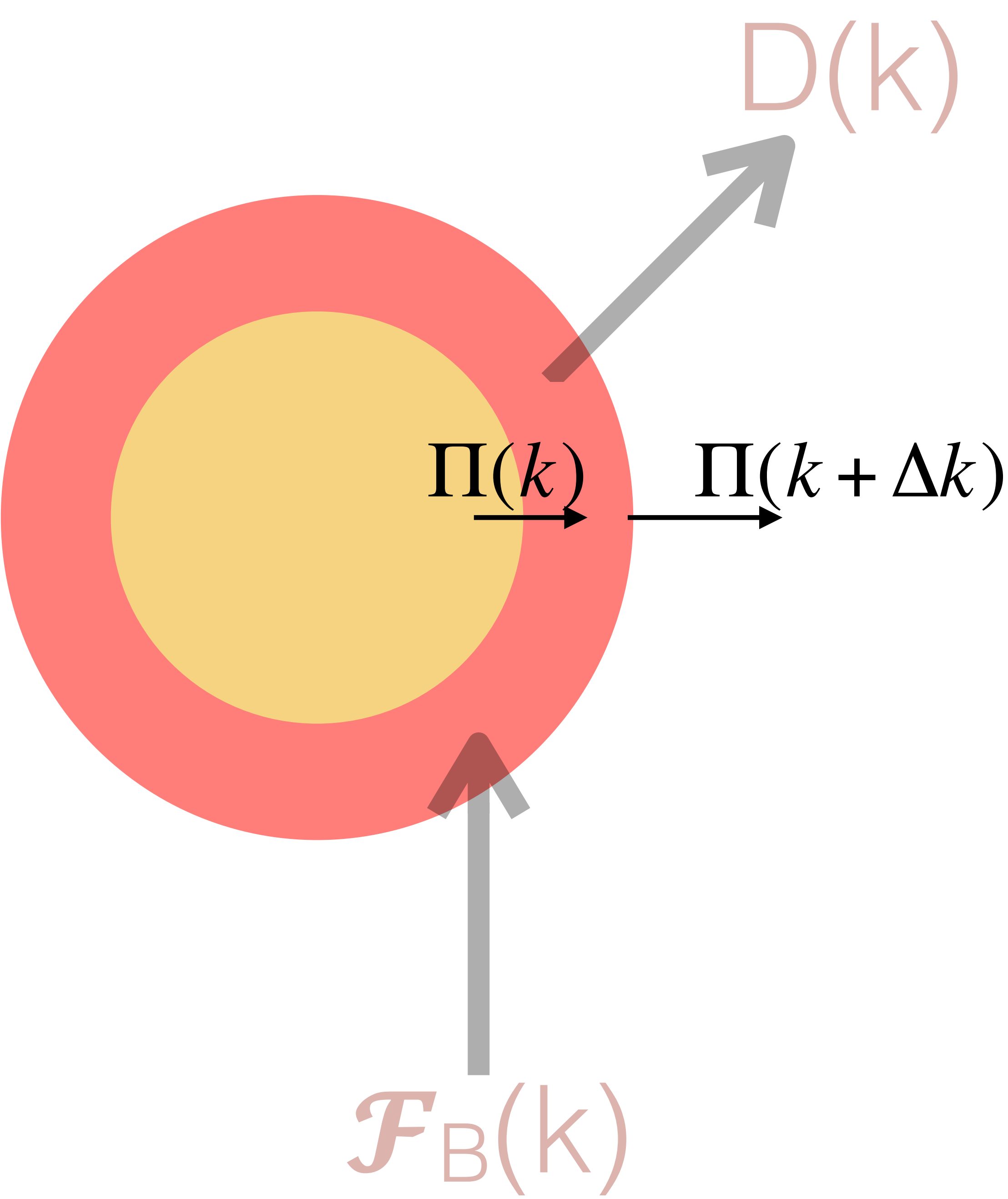

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$\mathcal{F}_B(k) \sim D(k)$

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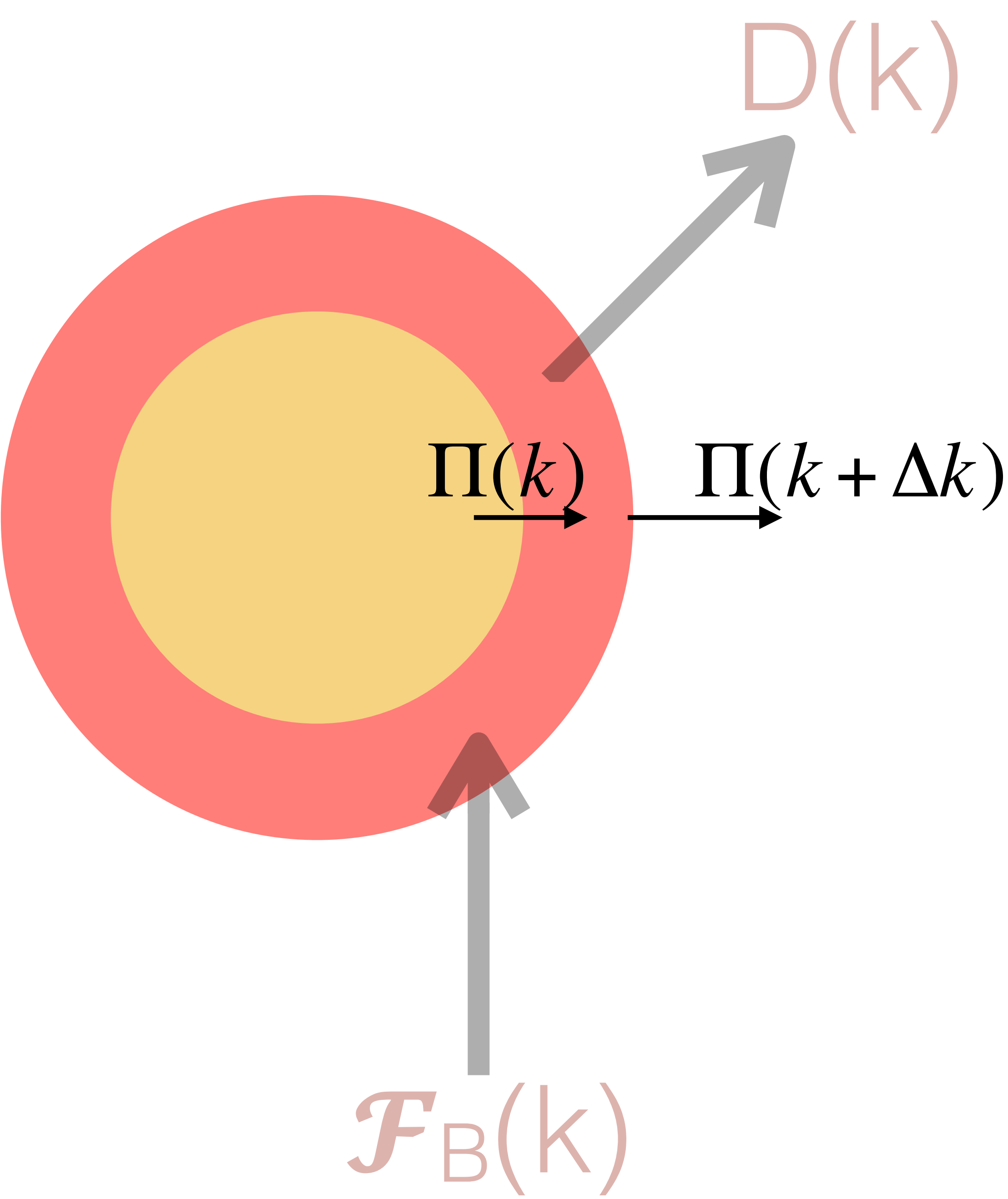

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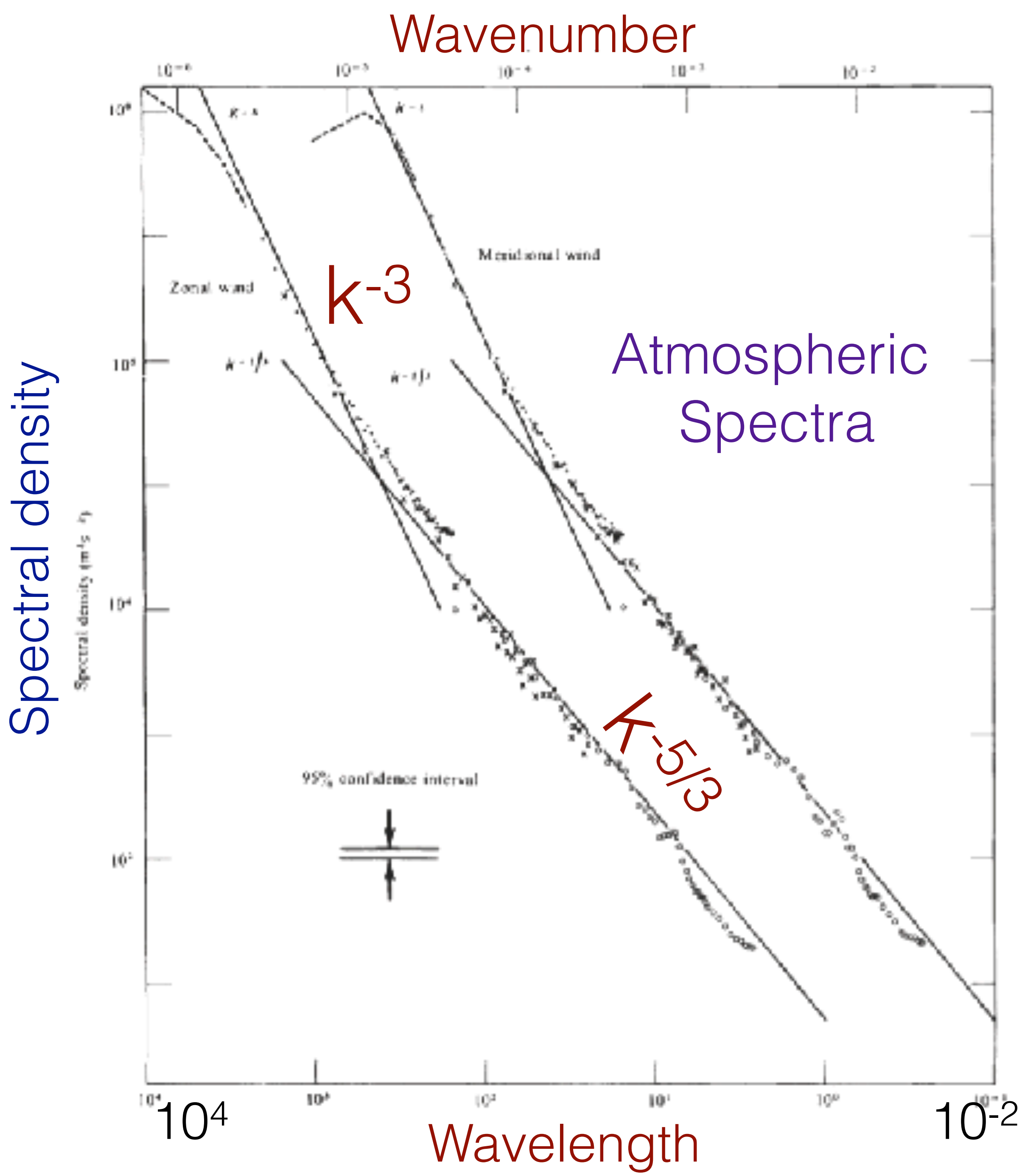
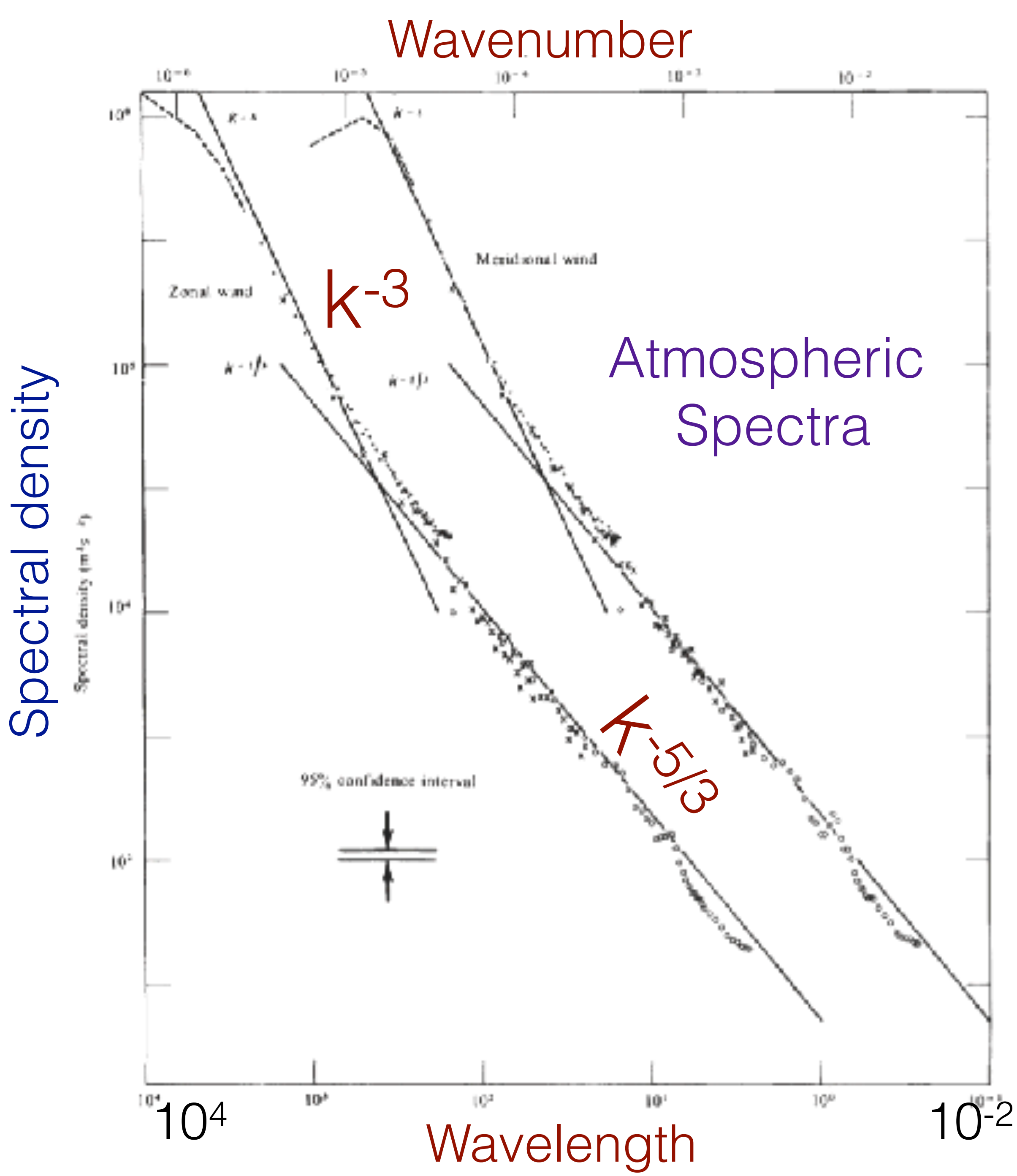
Earlier Work

Stably Stratified Turbulence



Nastrom *et al.* Nature 1985

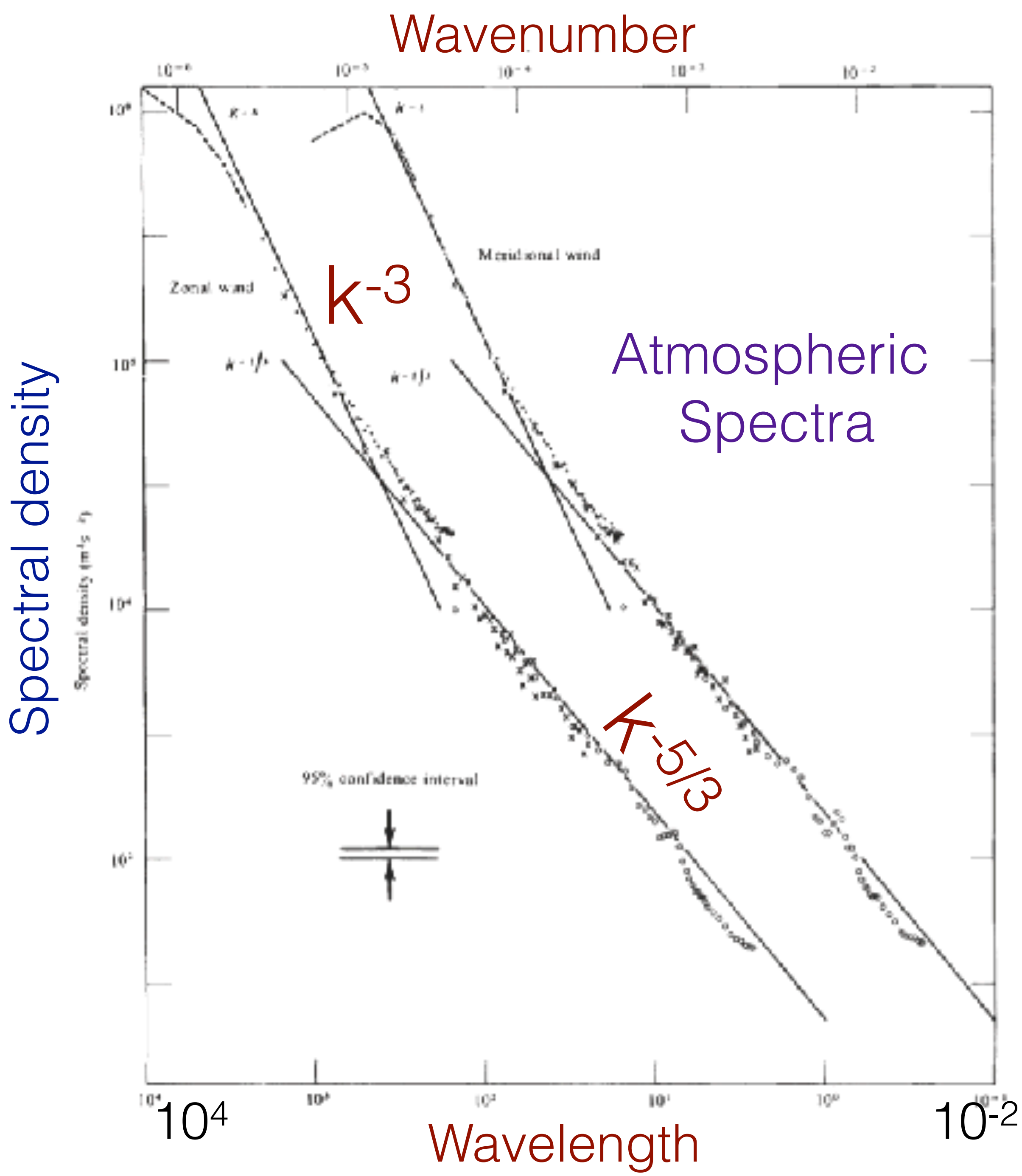
Stably Stratified Turbulence



Nastrom *et al.* Nature 1985

- ~ 500 -2000 Km -> k^{-3}

Stably Stratified Turbulence



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- ~ 500 -2000 Km -> k^{-3}
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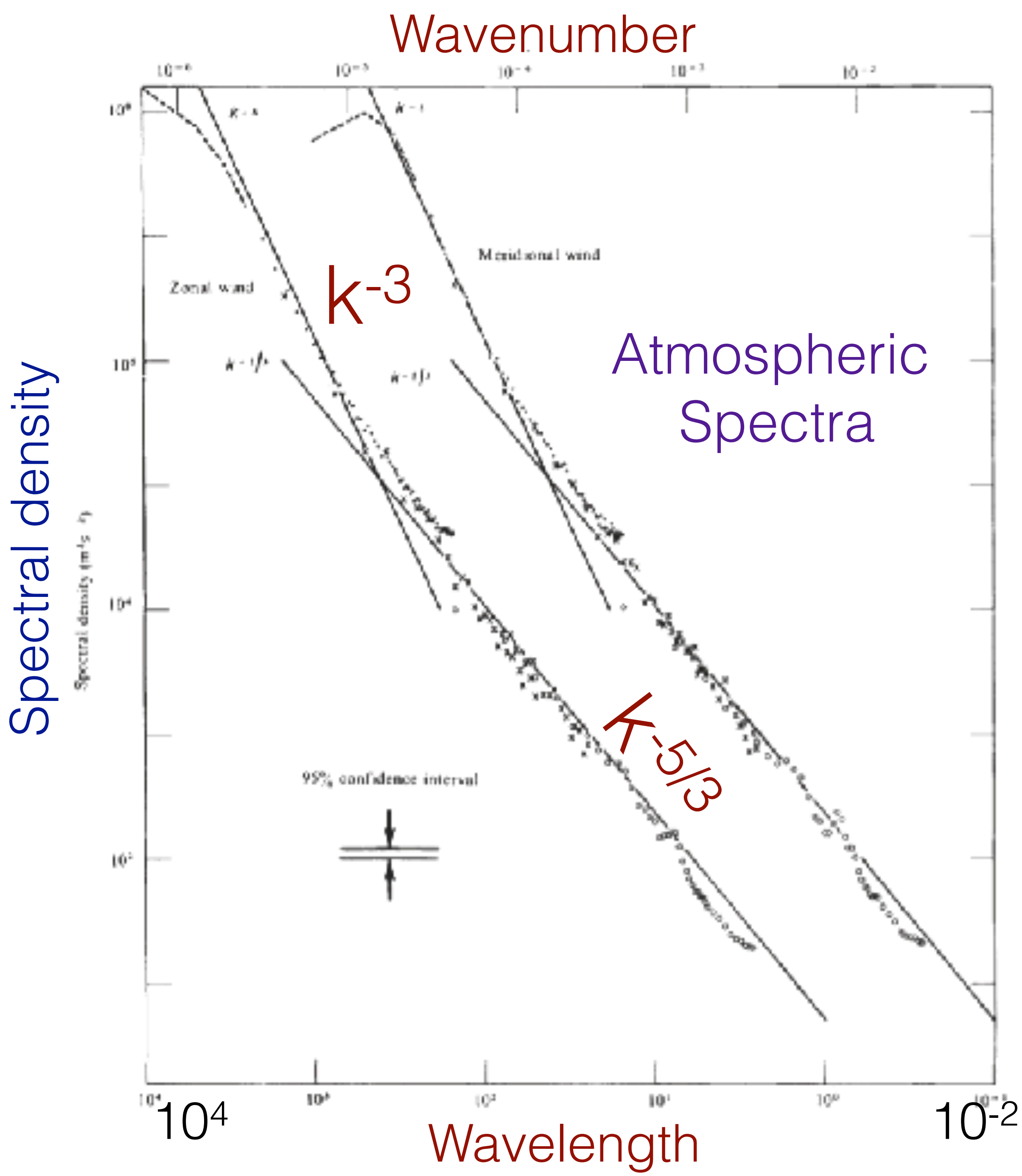
Stably Stratified Turbulence



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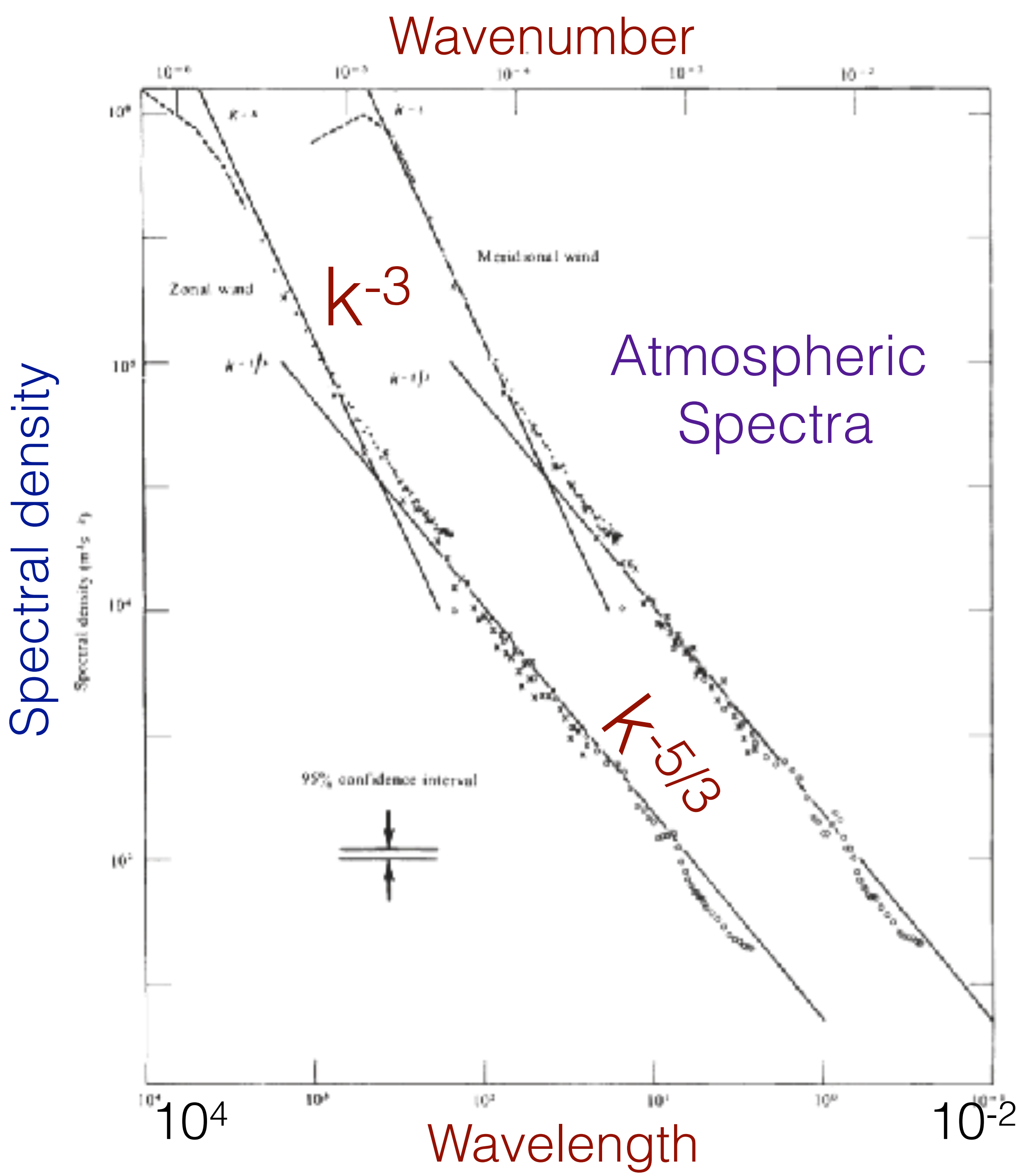
- $k^{-5/3}$ due Backward energy cascade with energy source at kilometer scales. [Falkovich PRL 1992]

Stably Stratified Turbulence



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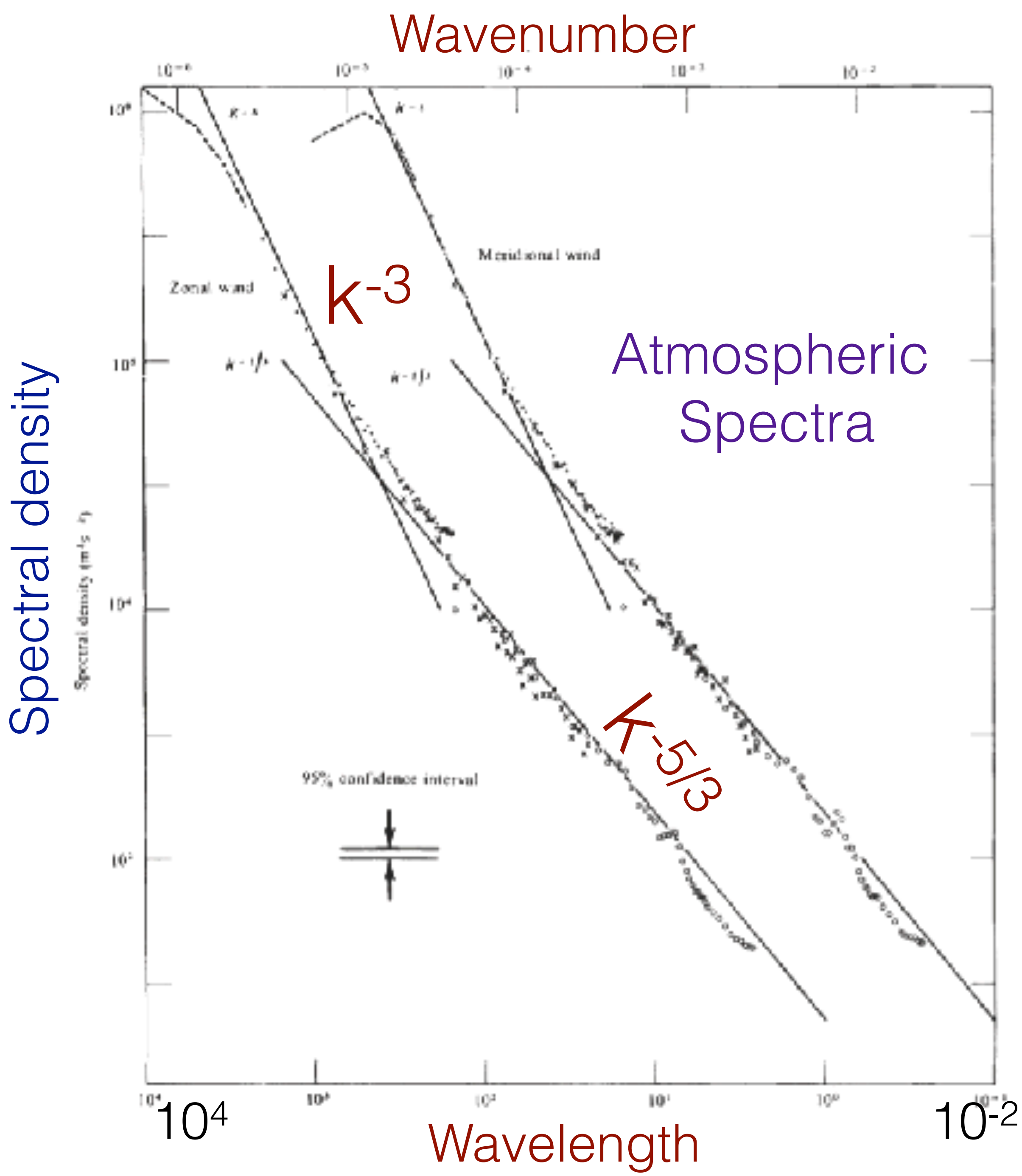
Stably Stratified Turbulence



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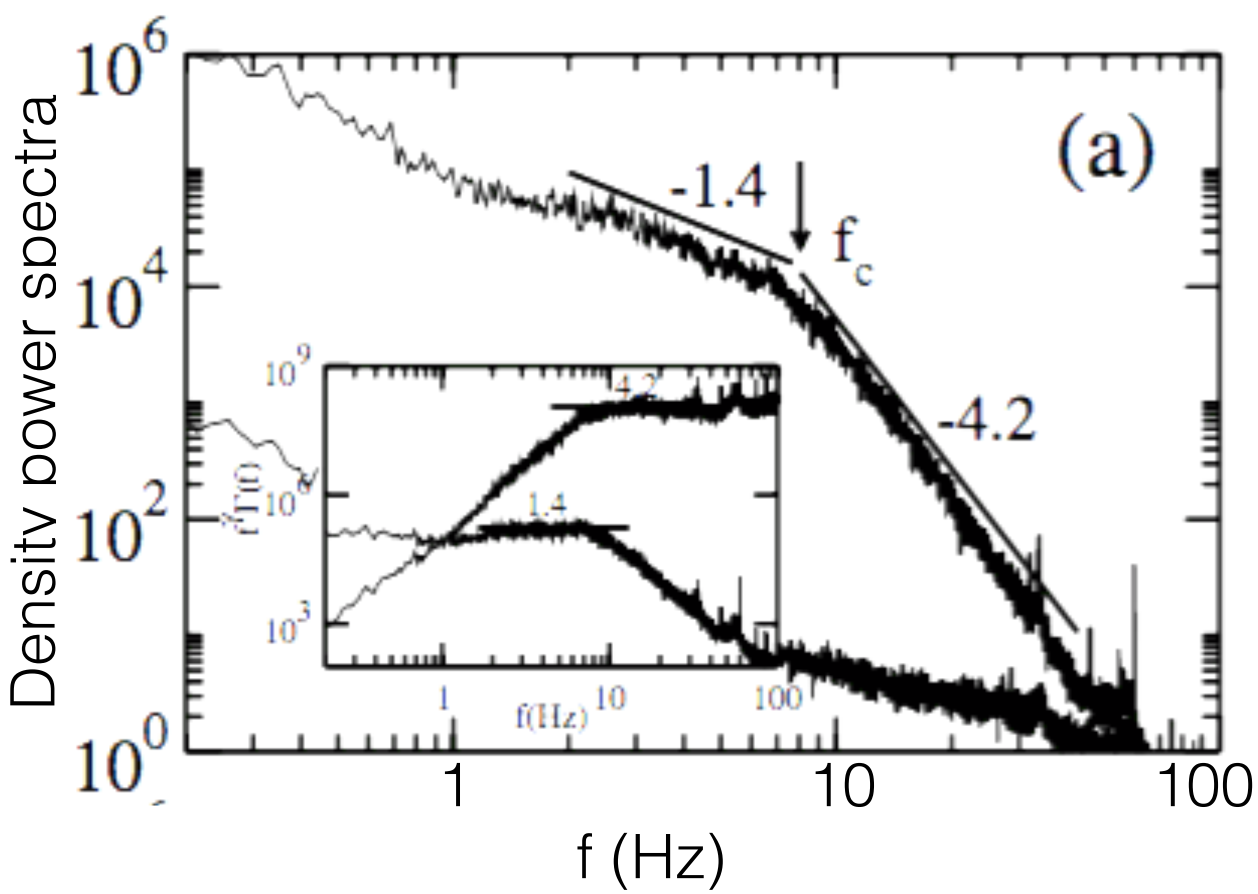
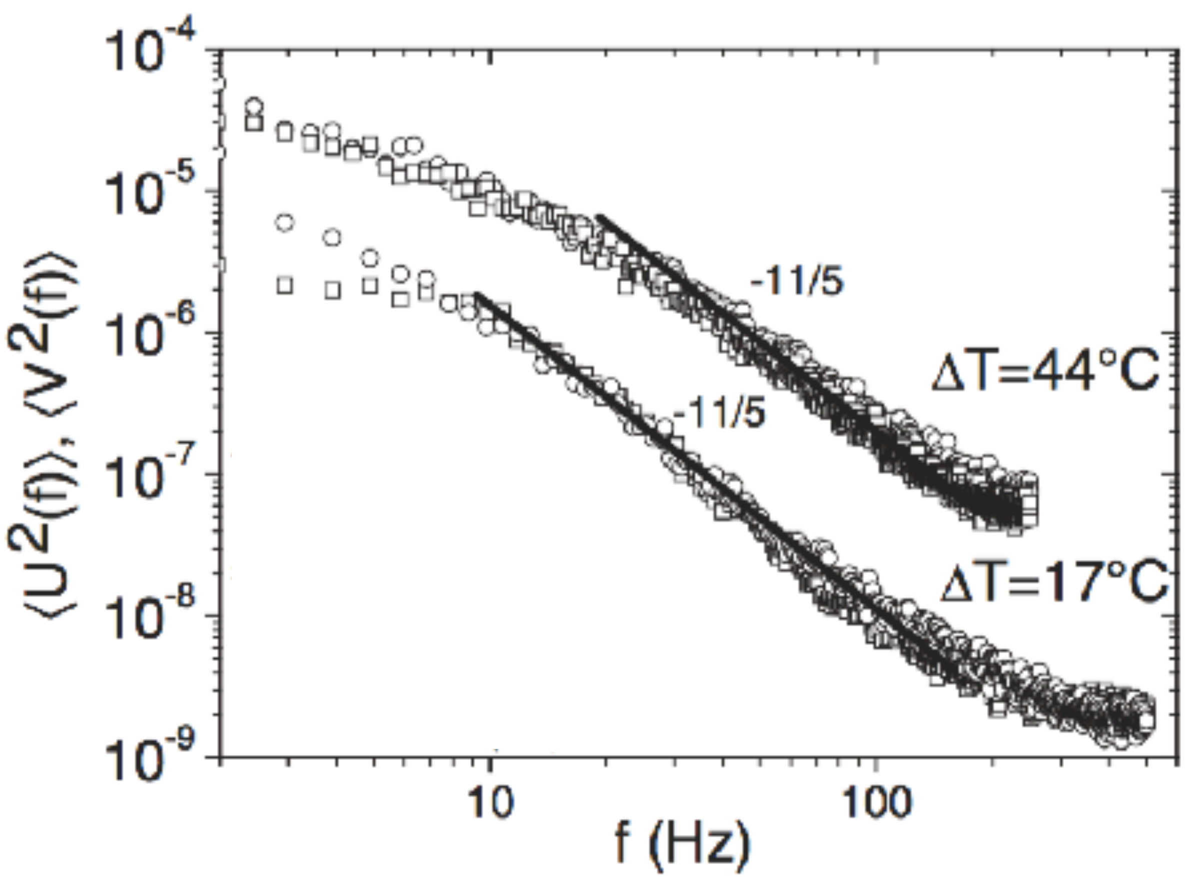
- Lindborg (2005,2006); Brethouwer *et al.* (2007), Vallgren *et al.* (2011), Bartello and Tobias (2013)

Stably Stratified Turbulence

Experiments → BO Spectrum



Zhang, Wu, and Xia PRL(2005)



Seychelles et al. PRL(2008)

Rayleigh-Bénard Convection

Rayleigh-Bénard Convection

- Models

Rayleigh-Bénard Convection

- Models
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Rayleigh-Bénard Convection

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- Experiment: Wu *et al.*(1990), Castaing (1990), Chillà *et al.* (1993), Cioni *et al.* (1995), Niemela *et al.* (2000), Zhou and Xia (2001), Shang and Xia (2001), Mashiko *et al.*(2004), Sun *et al.*(2006) : KO or BO.

Rayleigh-Bénard Convection

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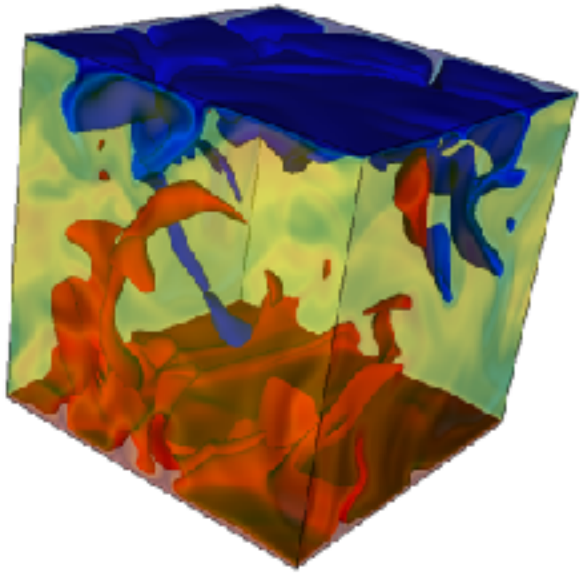
- Review: Lohse and Xia, ARFM (2010)

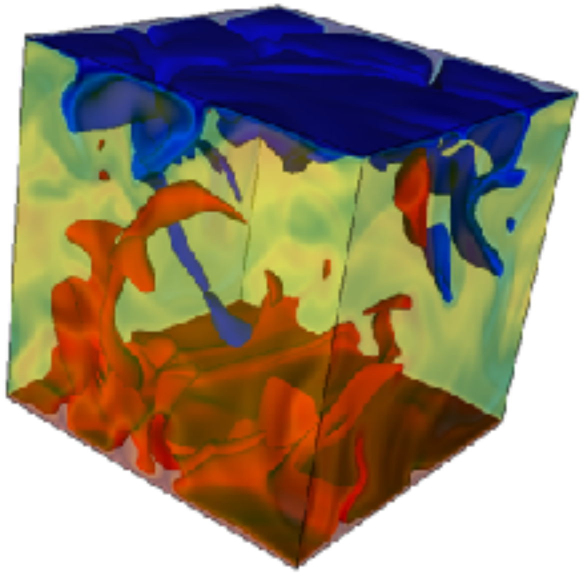
Our Work

Simulations



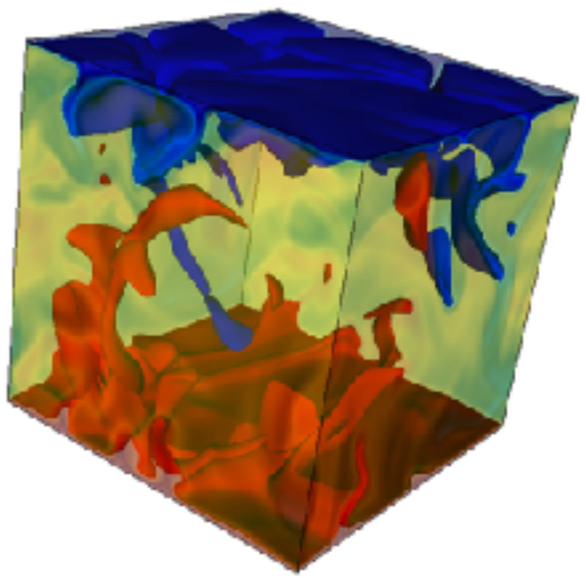
Simulations

- Pseudo-spectral code: Tarang



Simulations

- Pseudo-spectral code: Tarang
- Time stepping: RK4 method



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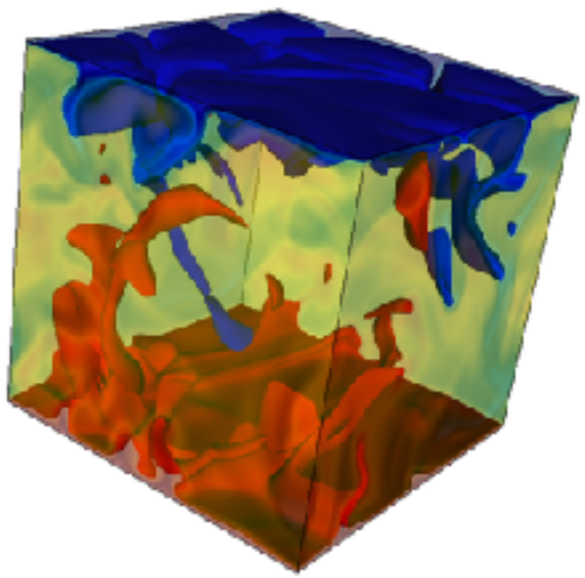
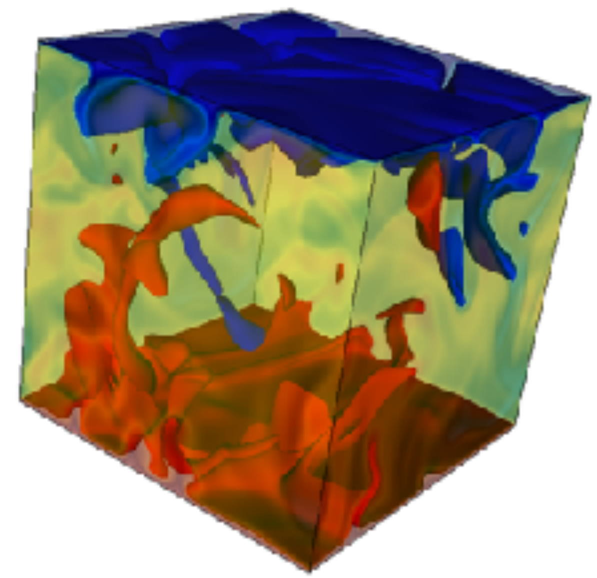


Stably stratified turbulence

Simulations

- Pseudo-spectral code: Tarang
- Time stepping: RK4 method



Periodic BC   ($\mathbf{u}$,θ) ⟵ Stably stratified turbulence

Simulations

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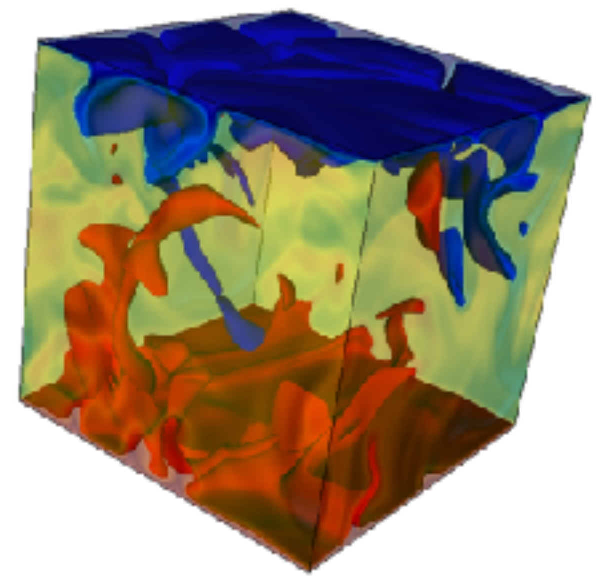
- Time stepping: RK4 method



Periodic BC $(\mathbf{u},\theta)$ ← Stably stratified turbulence

← RBC

Simulations

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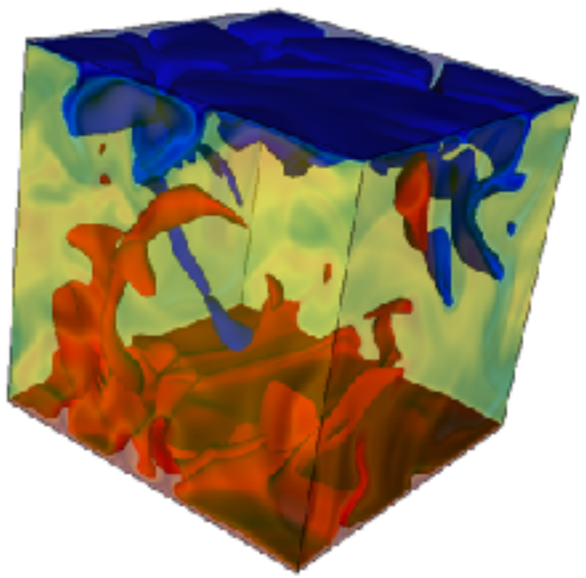
- Time stepping: RK4 method



Periodic BC $(\mathbf{u},\theta)$ ← Stably stratified turbulence

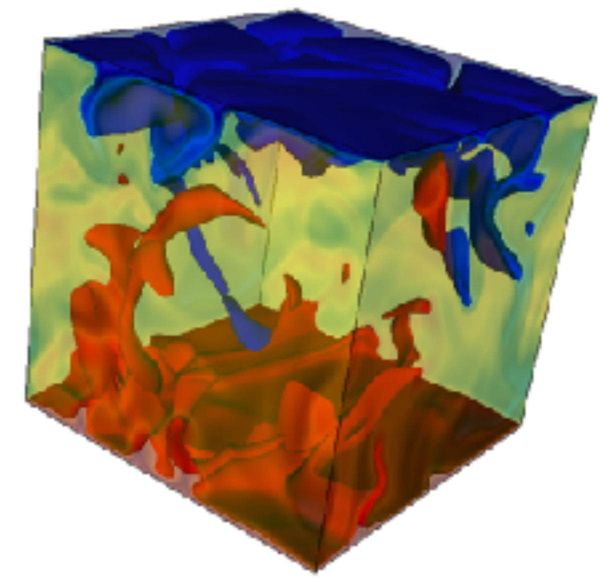
Free-slip BC

$$u_z = 0$$
$$\partial_z u_x = \partial_z u_y = 0$$

} ← RBC

Simulations

- Pseudo-spectral code: Tarang
- Time stepping: RK4 method



Periodic BC $(\mathbf{u},\theta)$ ← Stably stratified turbulence

Free-slip BC
$$u_z = 0$$
$$\partial_z u_x = \partial_z u_y = 0$$

Conducting plates $\theta = 0$

} ← RBC

Stably Stratified Turbulence Simulation Results

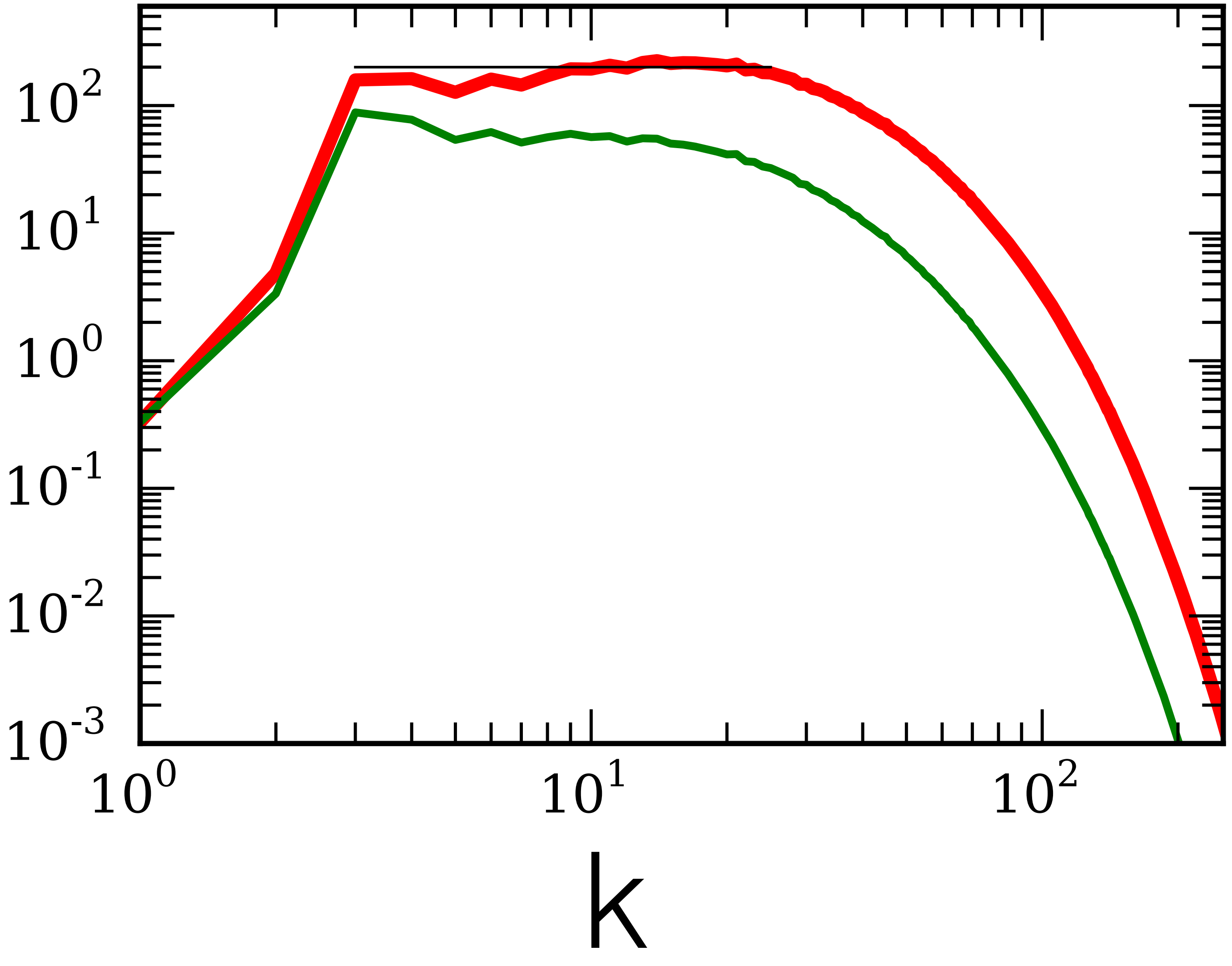
$\text{Grid} = 1024^3$

$\text{Ra} = 5000$

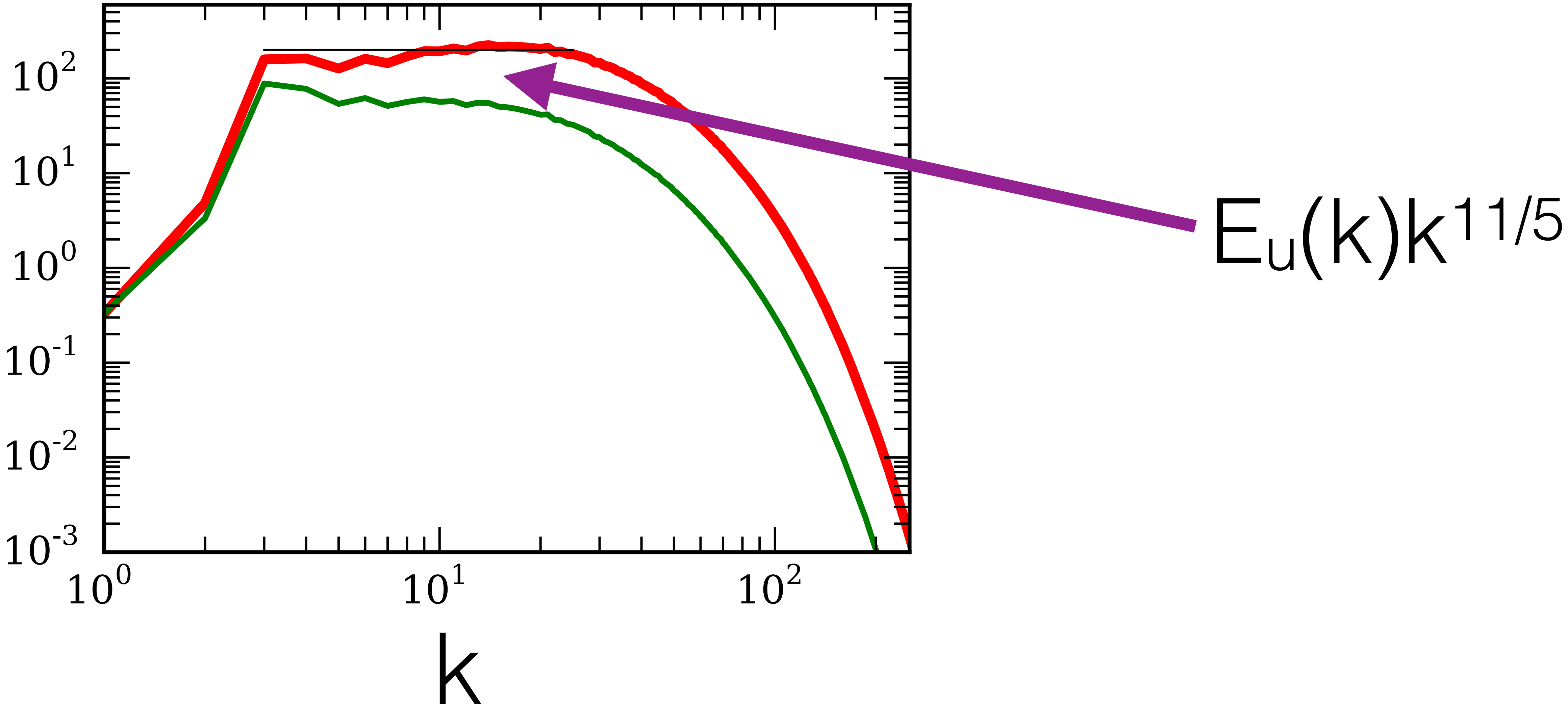
$\text{Pr} = 1$

$\text{Fr} = 10; \quad \text{Re} = 649$

Kumar et al. PRE 2014



Kumar et al. PRE 2014



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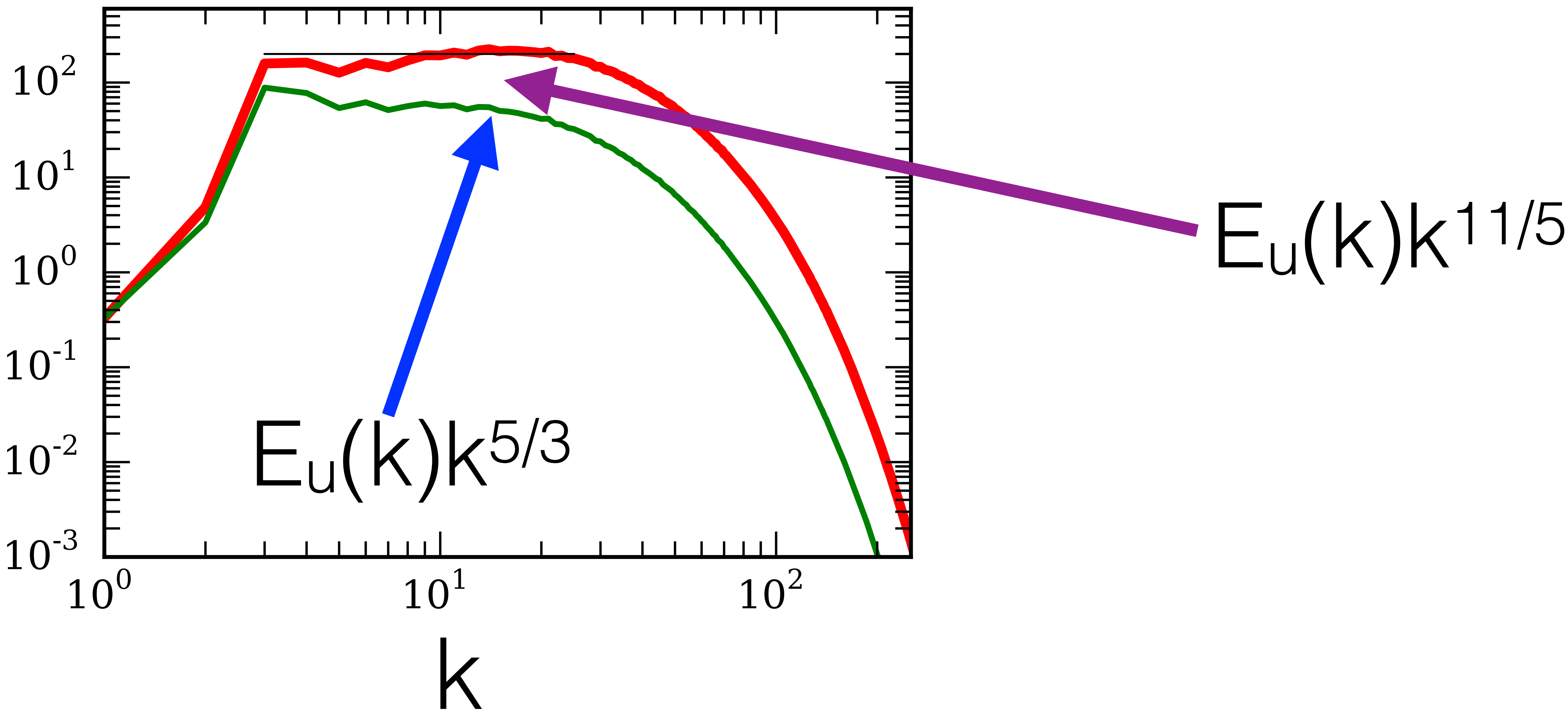

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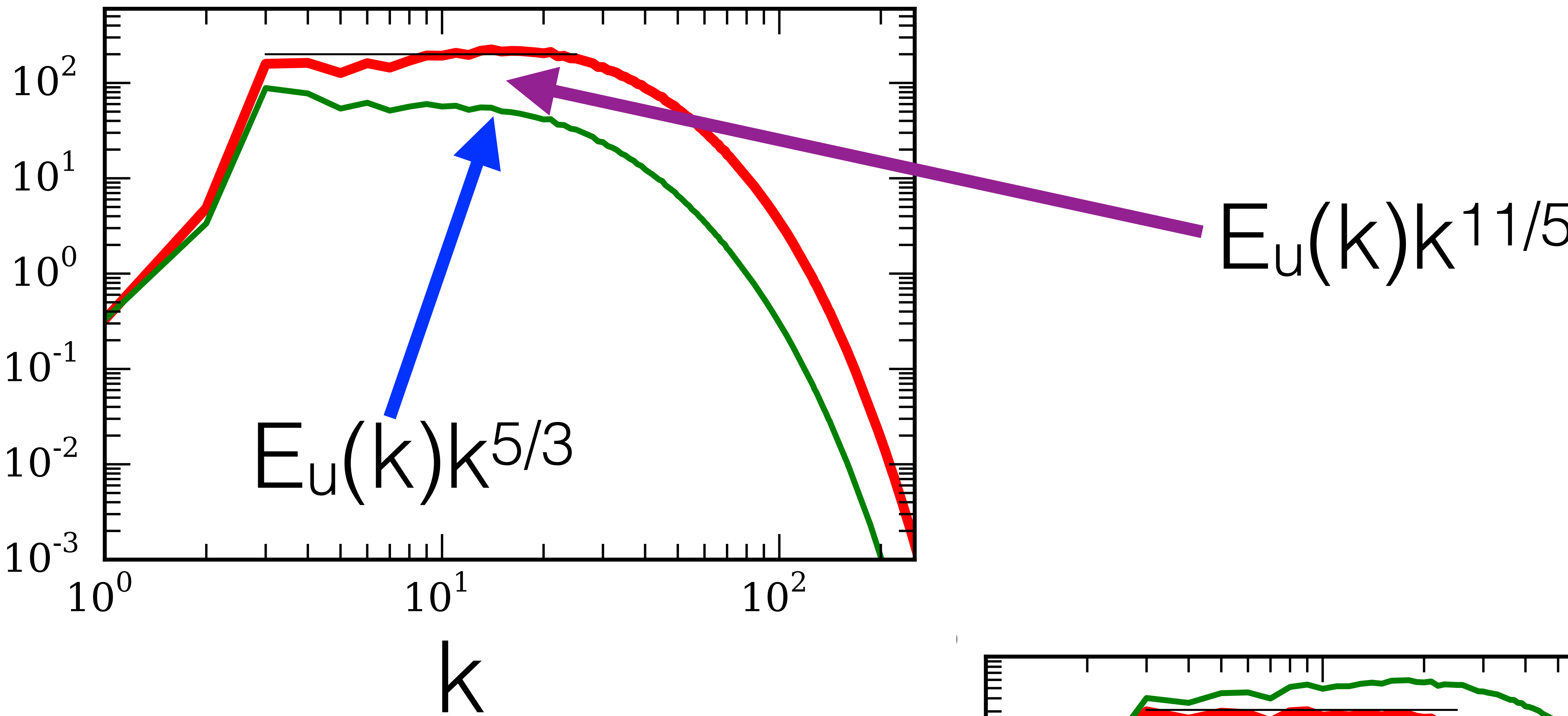
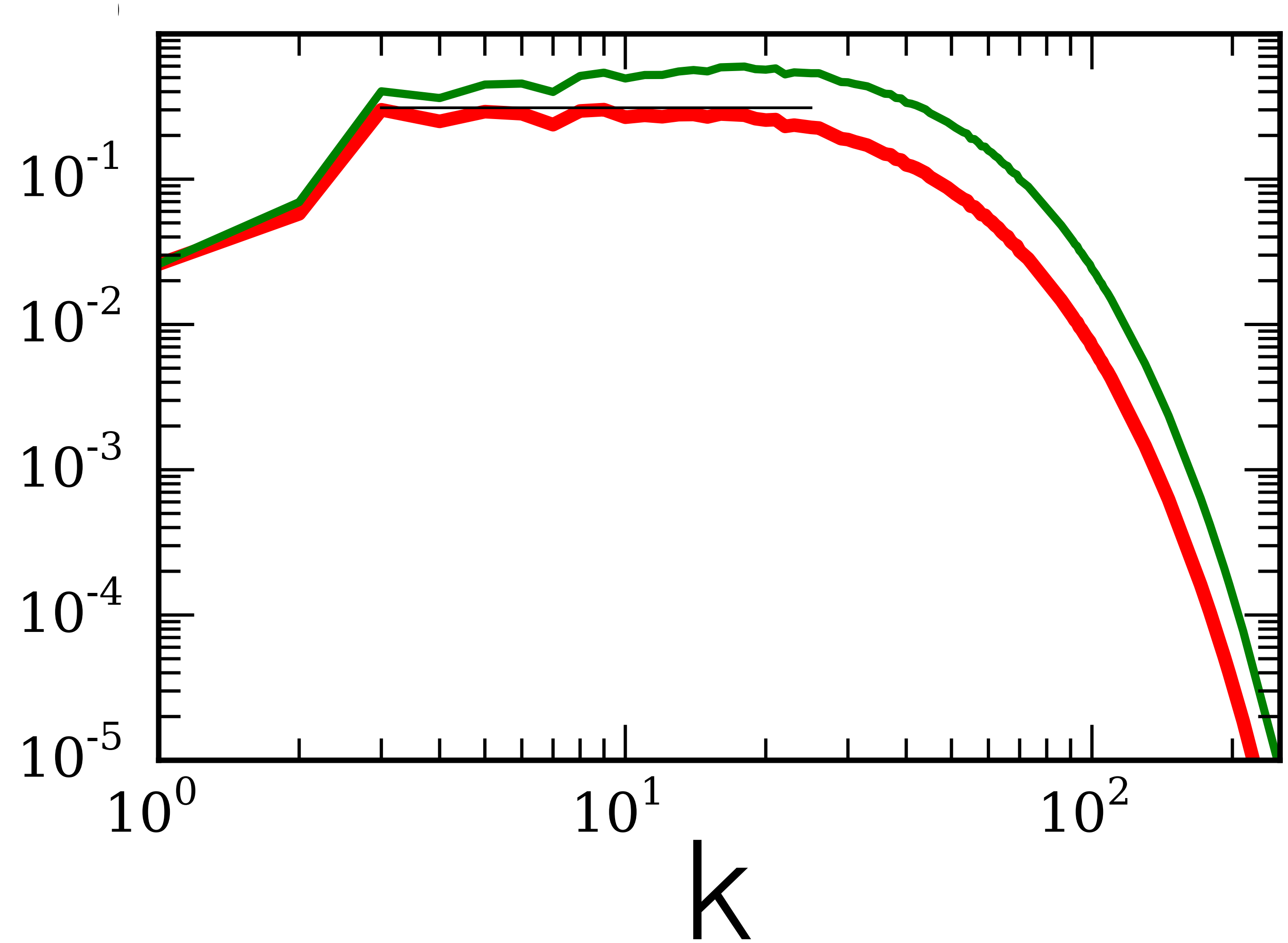
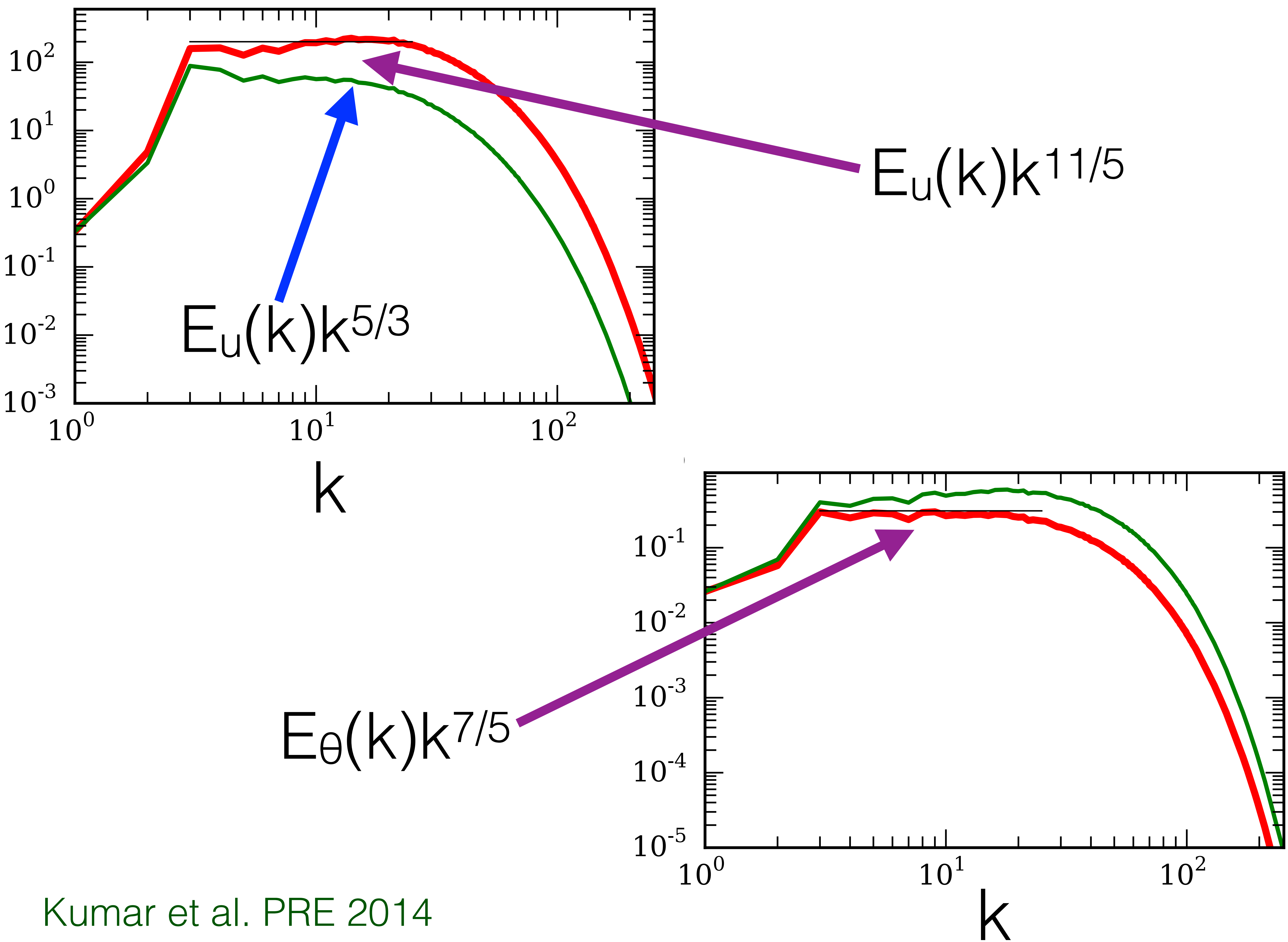
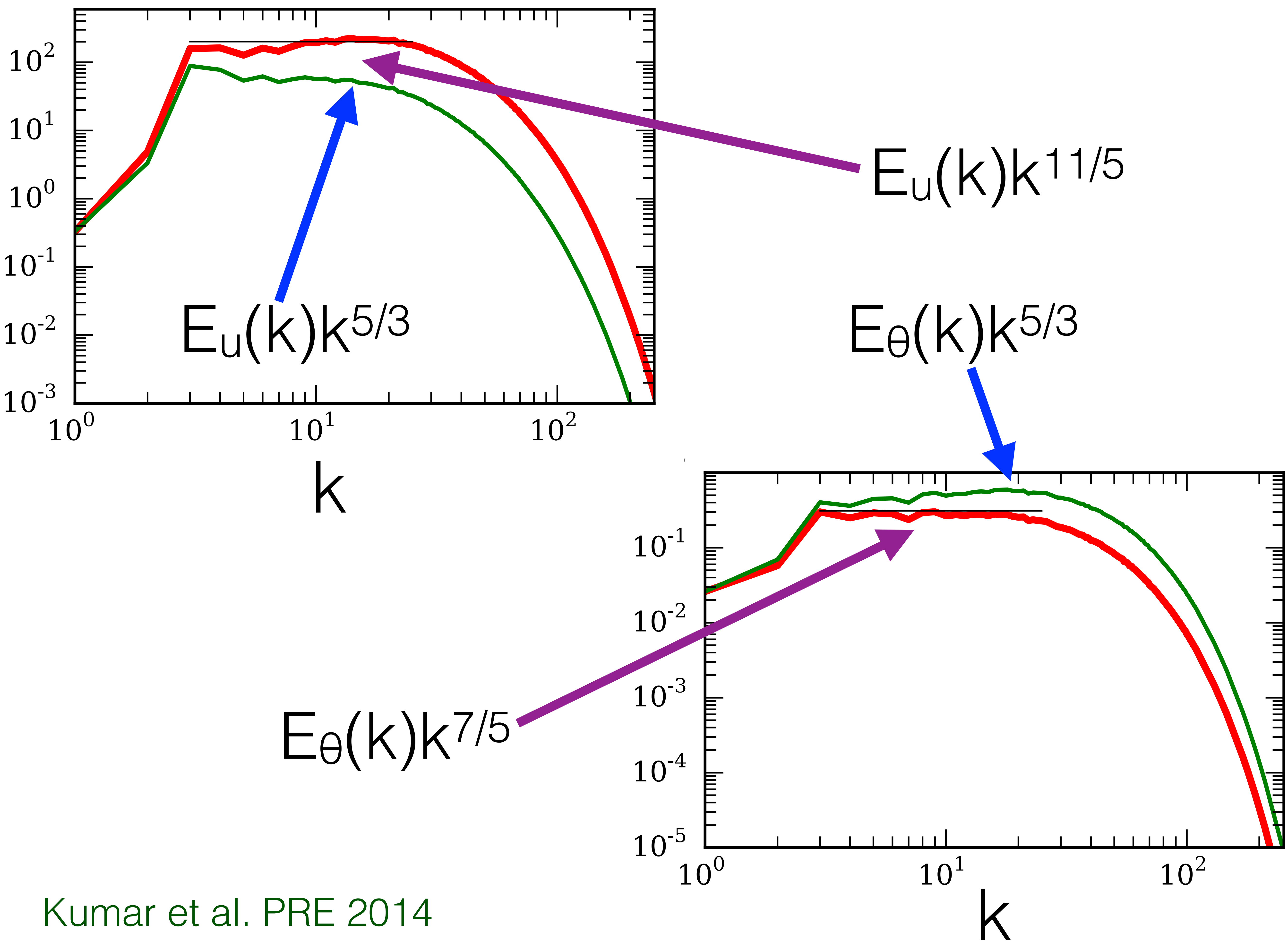




Kumar et al. PRE 2014



Kumar et al. PRE 2014



Kumar et al. PRE 2014

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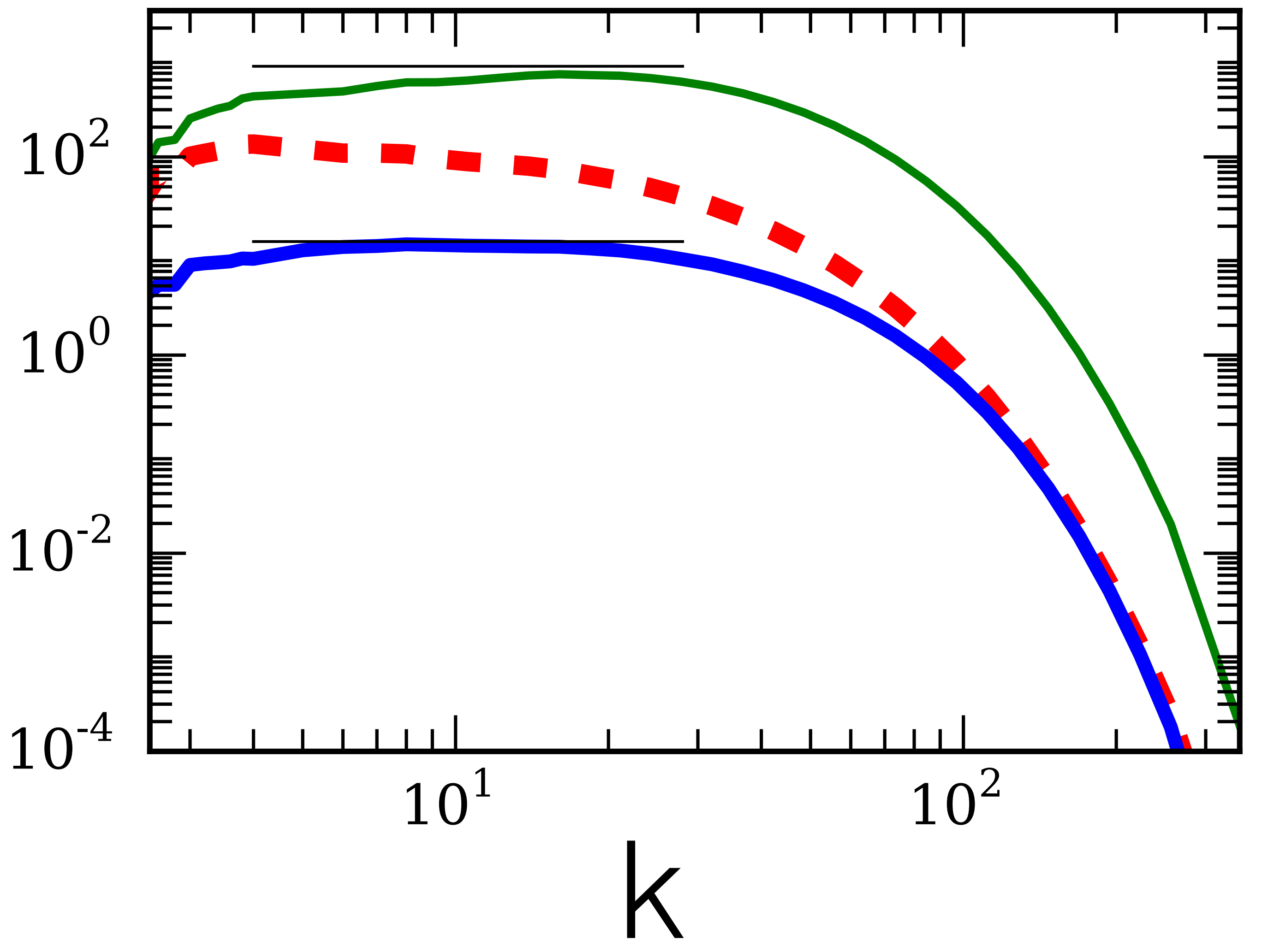

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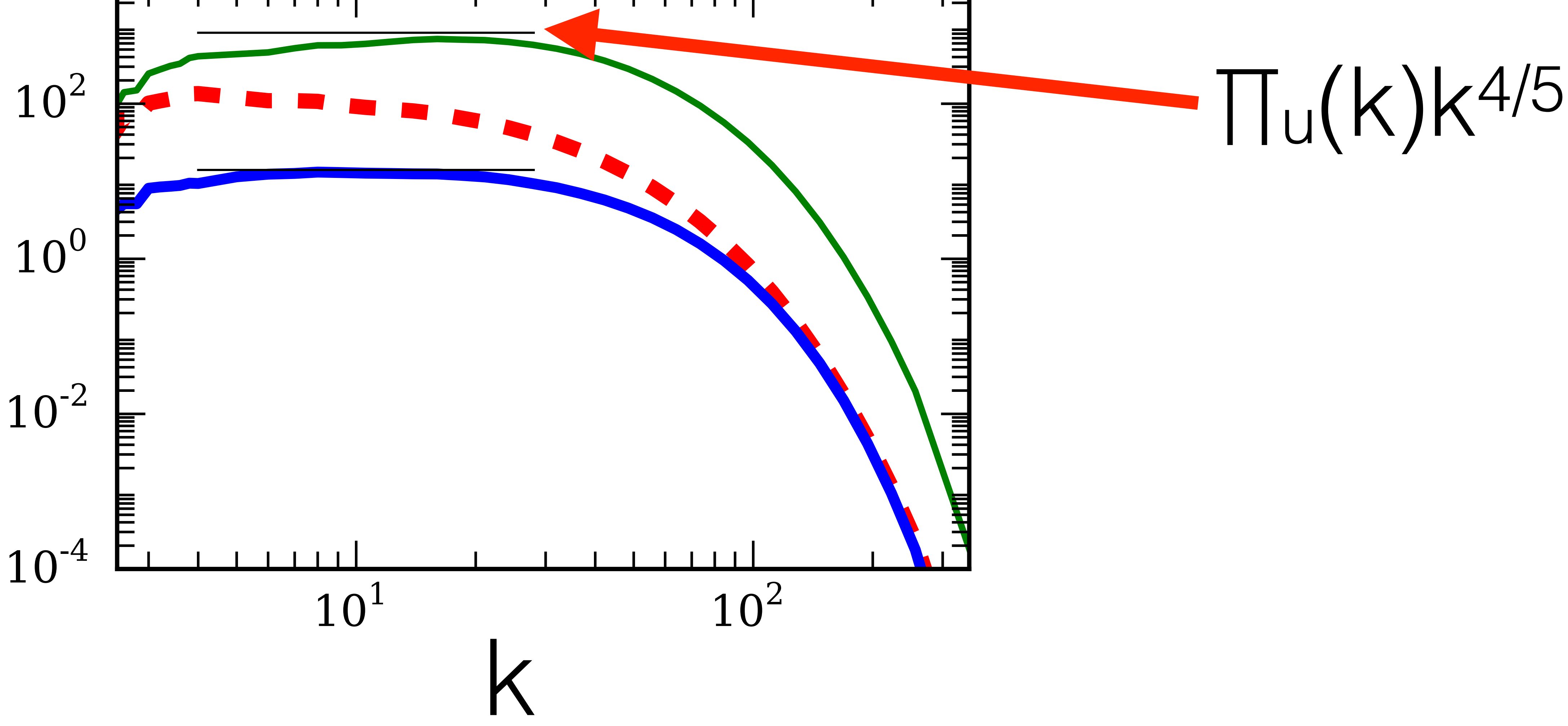

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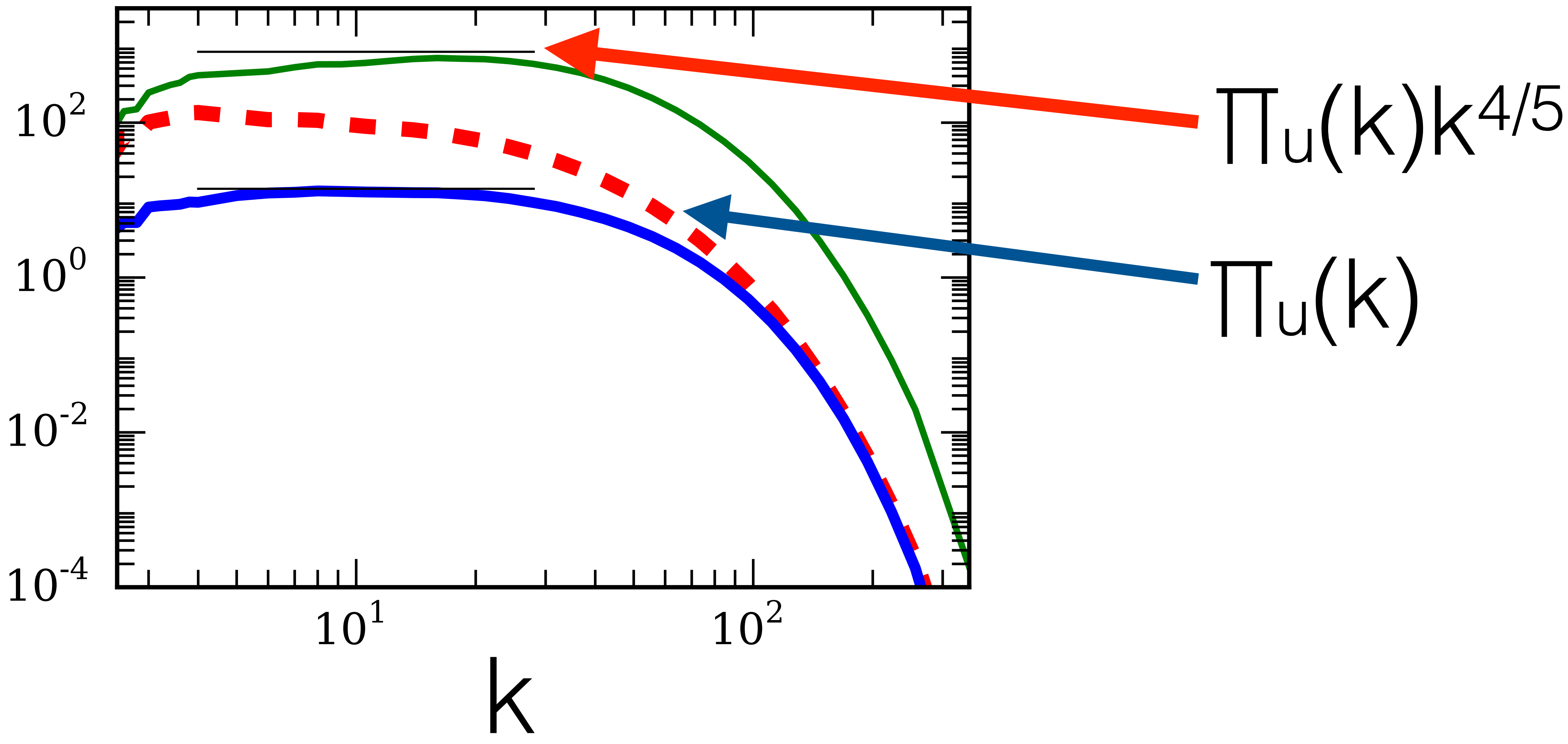

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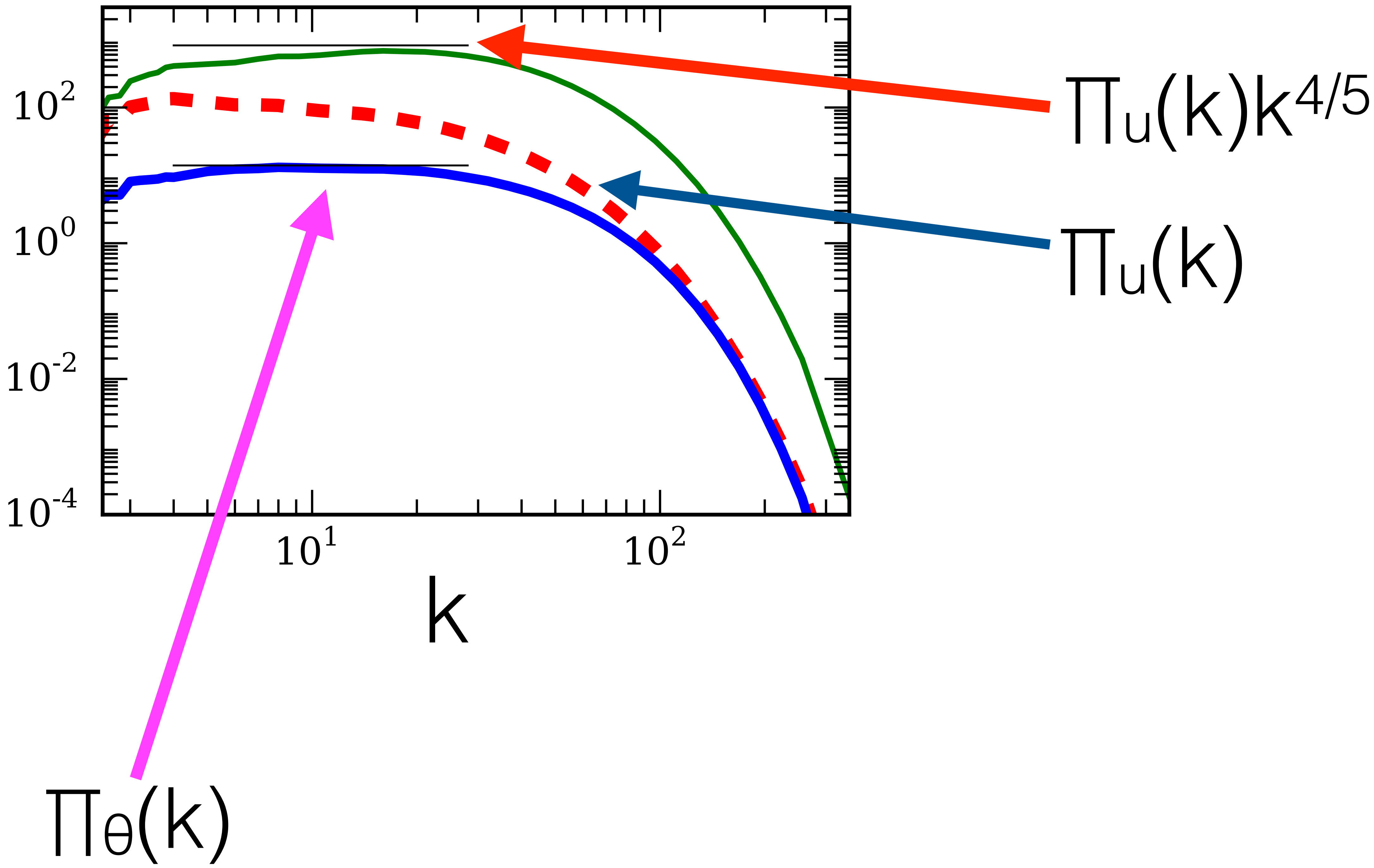

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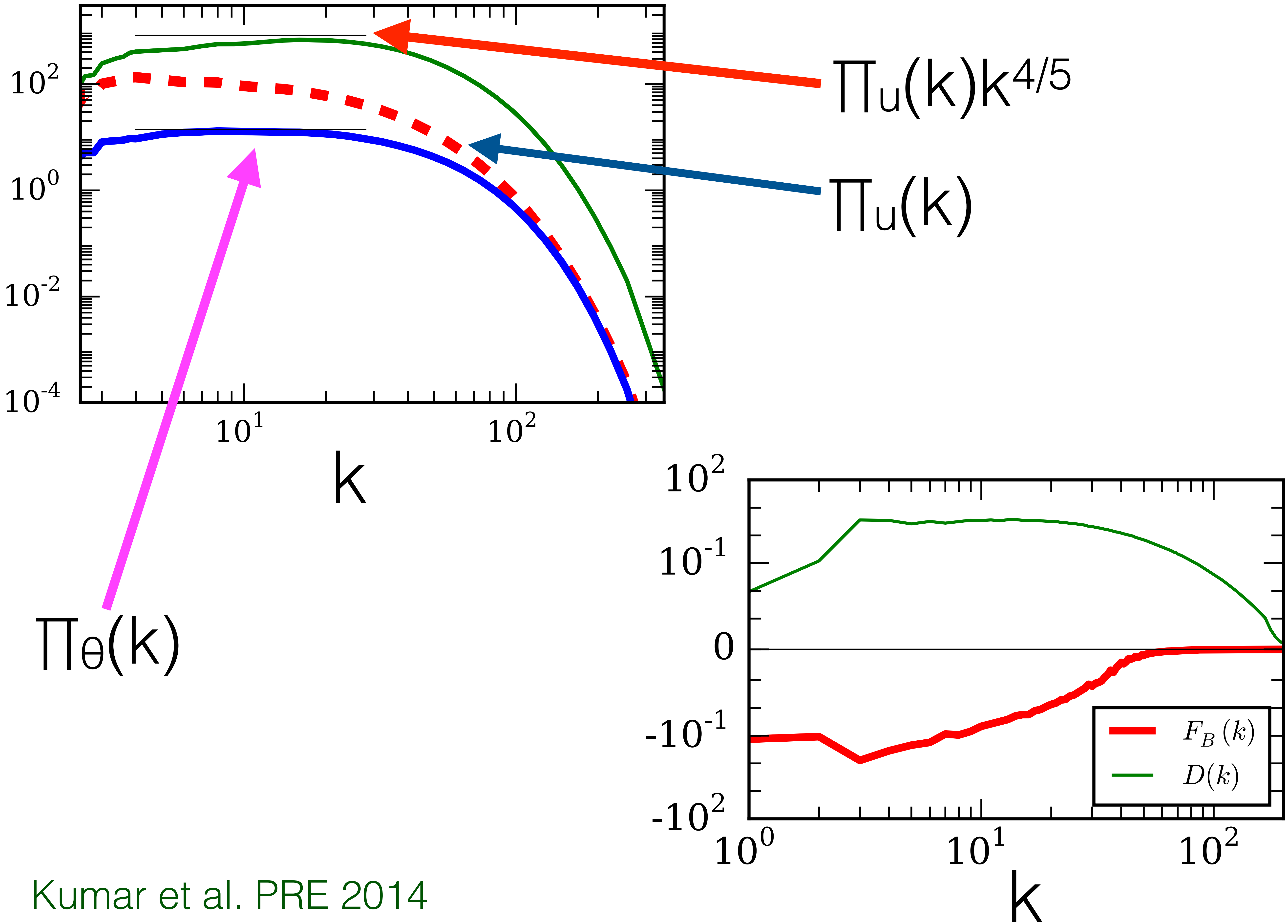

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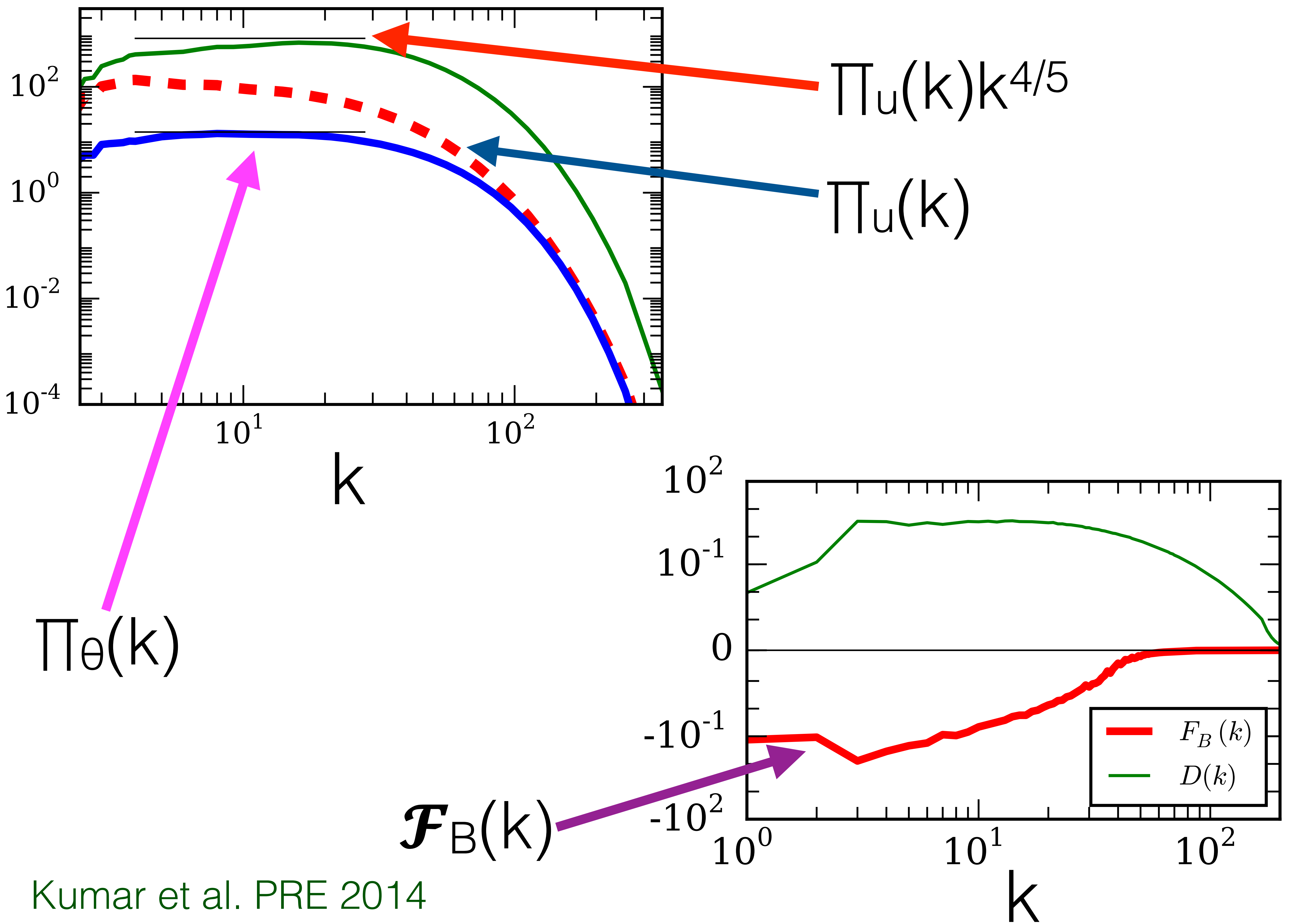
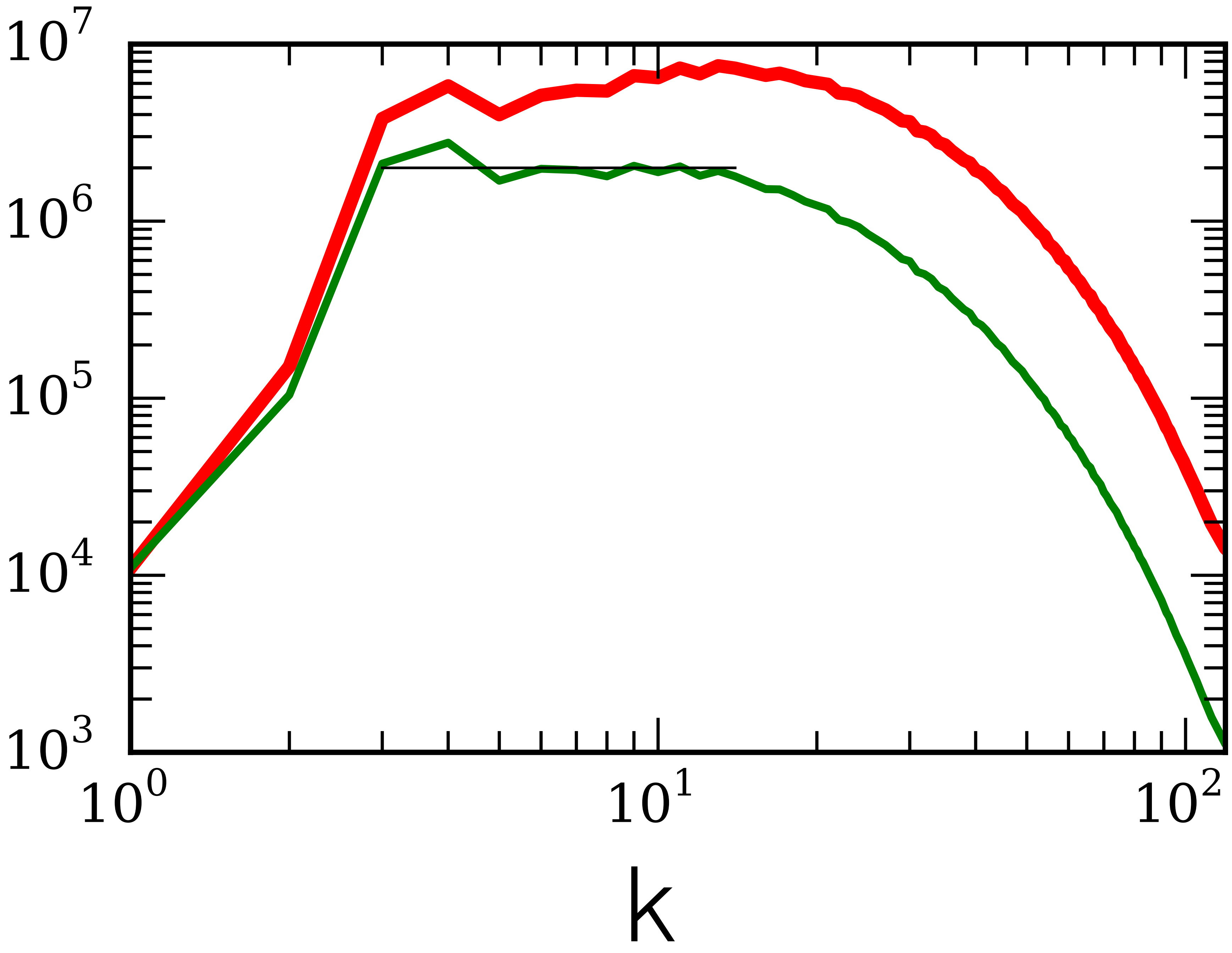


Kumar et al. PRE 2014

Weakly Stratified Flow

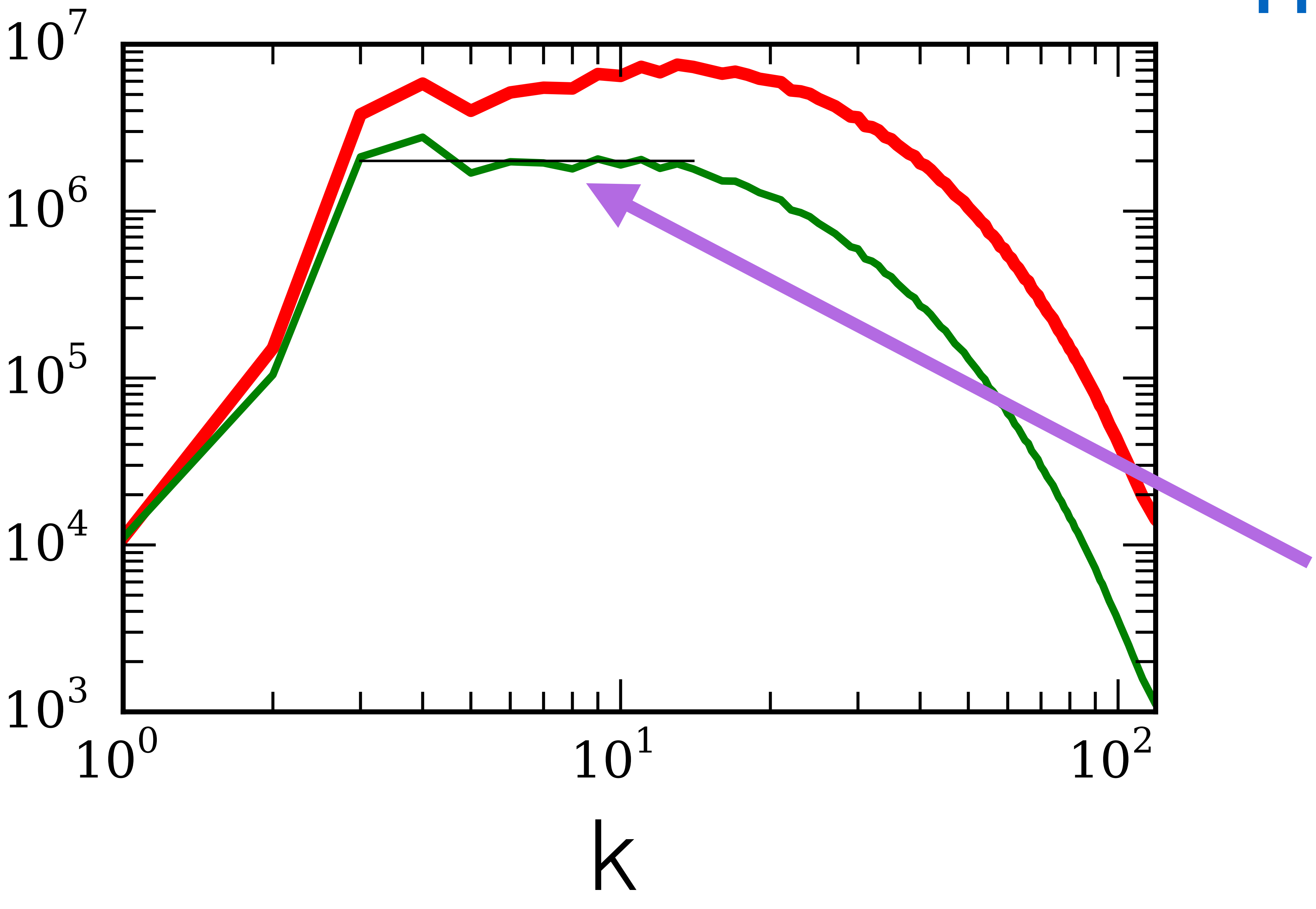
Fr >> 1

Grid = 512^3
Ra = 0.1
Re = 510
Fr = 1.5×10^3



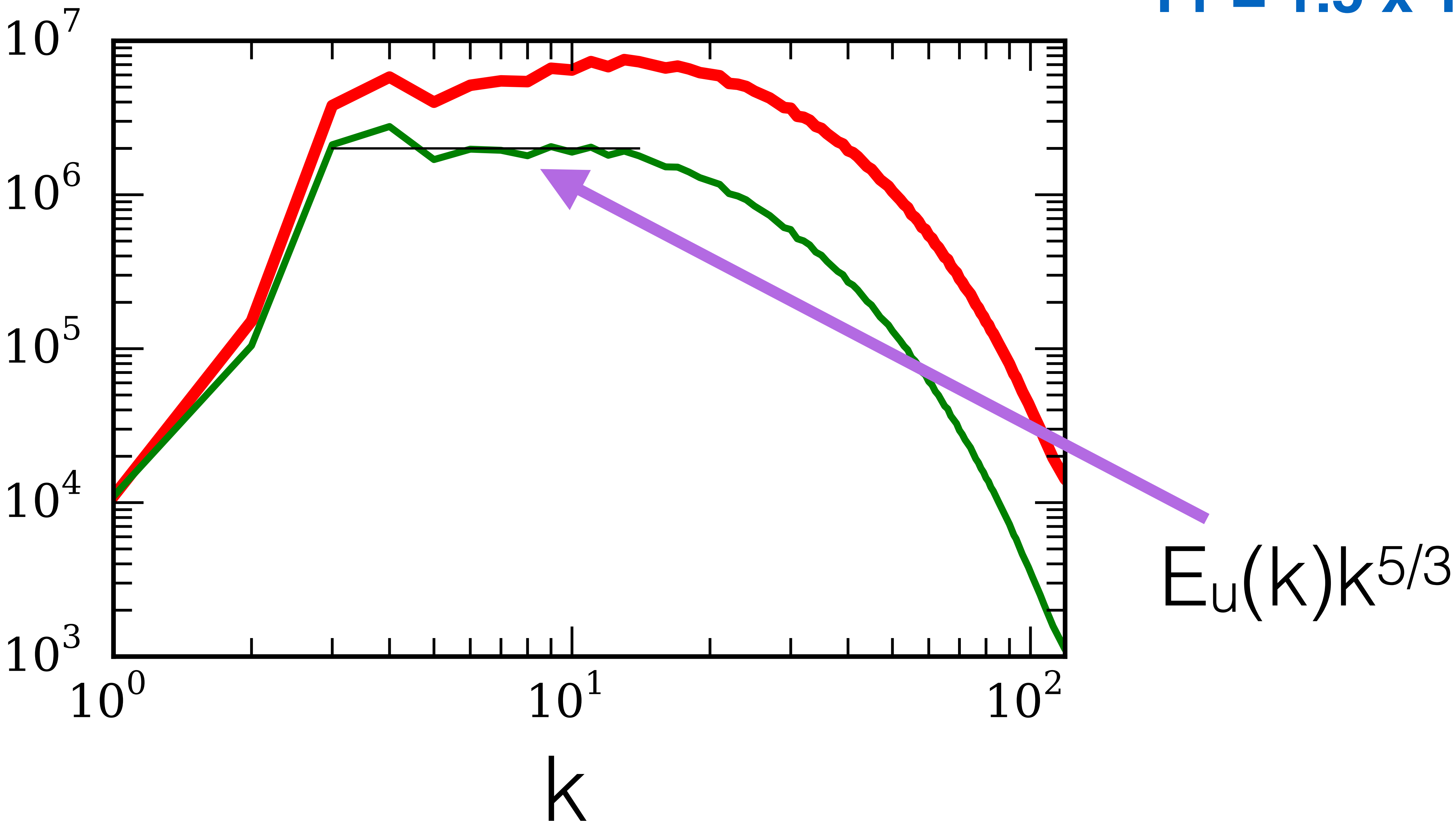
Kumar et al. PRE 2014

Grid = 512^3
Ra = 0.1
Re = 510
Fr = 1.5×10^3



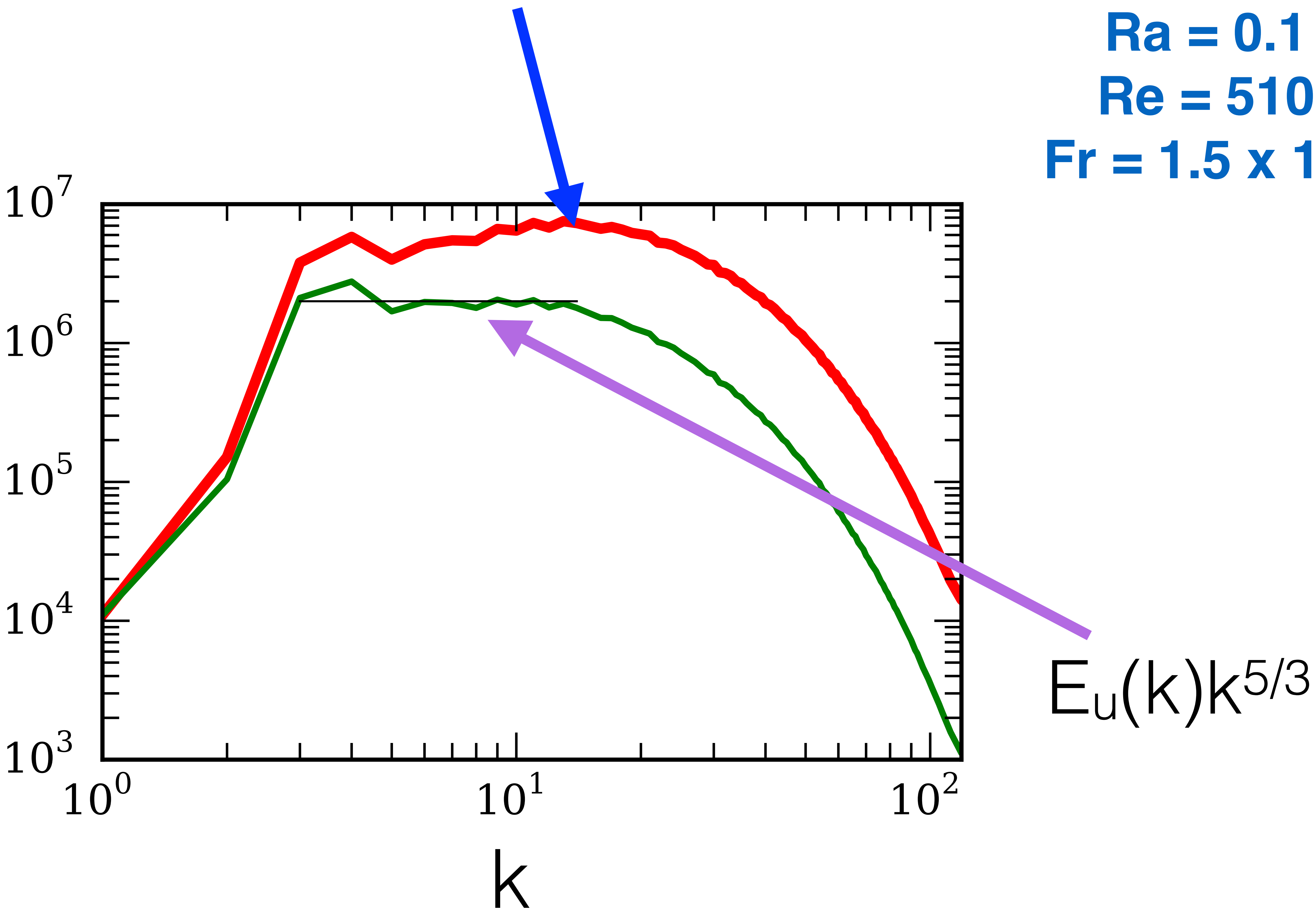
Kumar et al. PRE 2014

Grid = 512^3
Ra = 0.1
Re = 510
Fr = 1.5×10^3

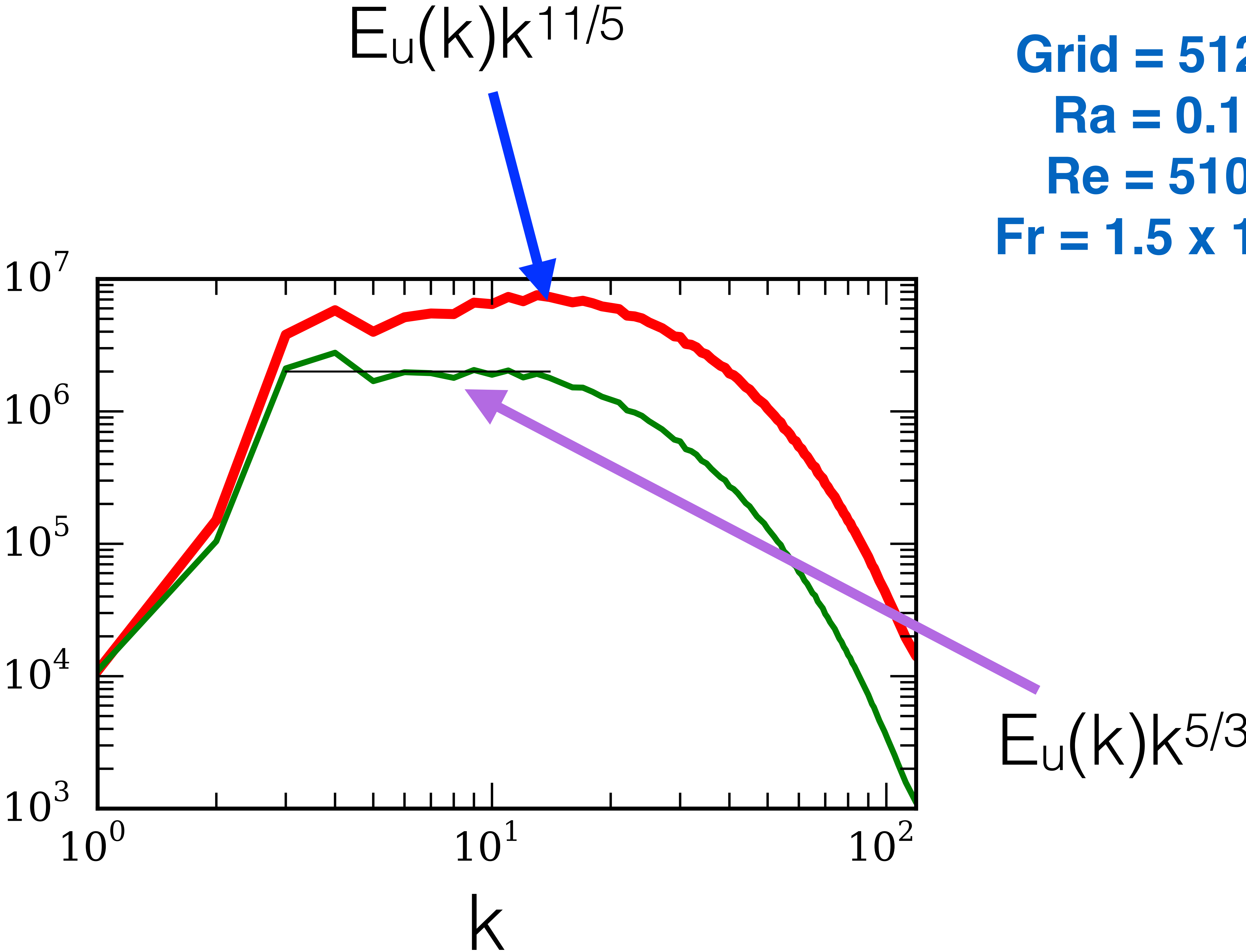


Kumar et al. PRE 2014

Grid = 512^3
Ra = 0.1
Re = 510
Fr = 1.5×10^3



Kumar et al. PRE 2014



Grid = 512^3
Ra = 0.1
Re = 510
Fr = 1.5×10^3

Kumar et al. PRE 2014

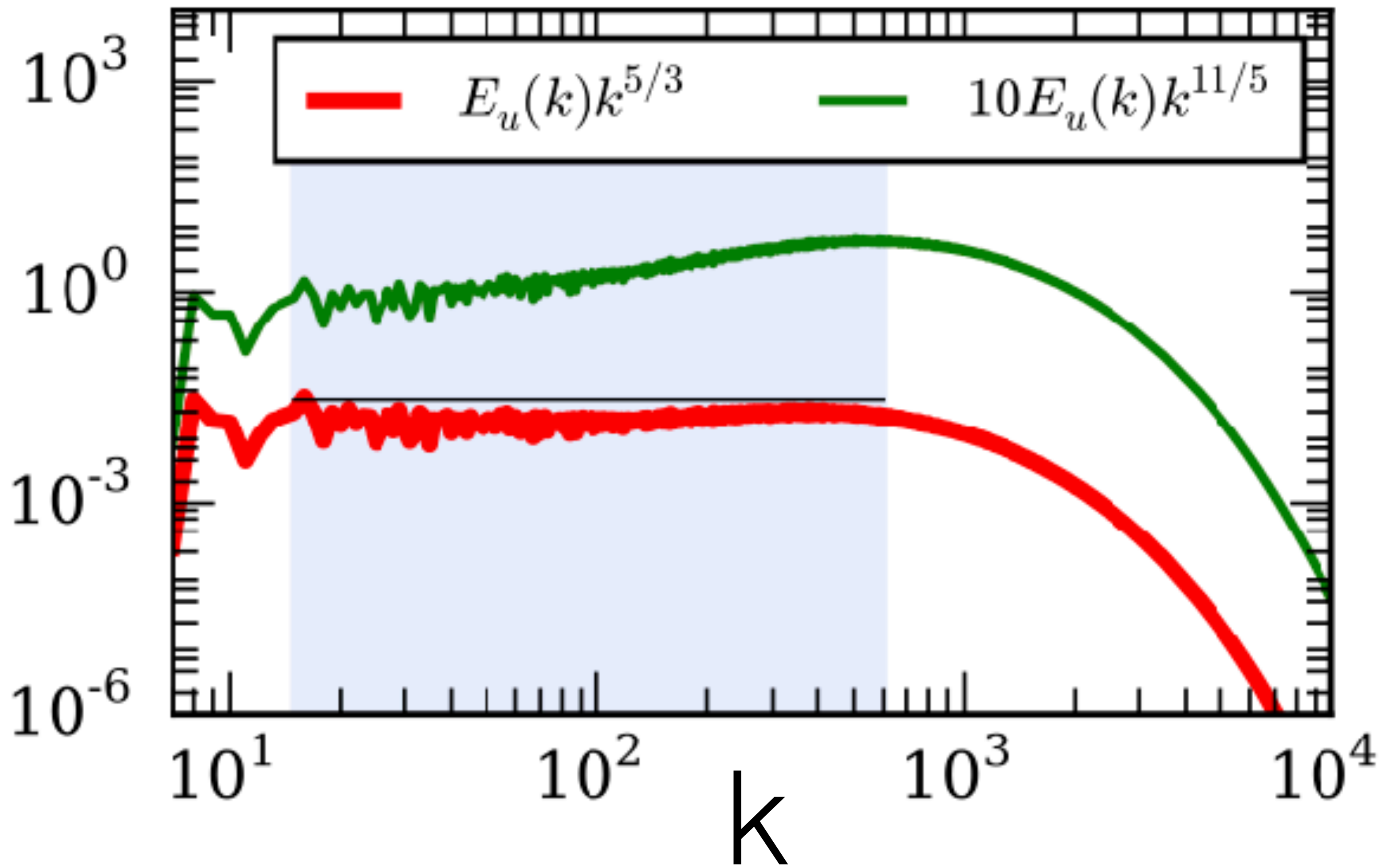
Rayleigh-Bénard Convection

$\text{Grid} = 4096^3$

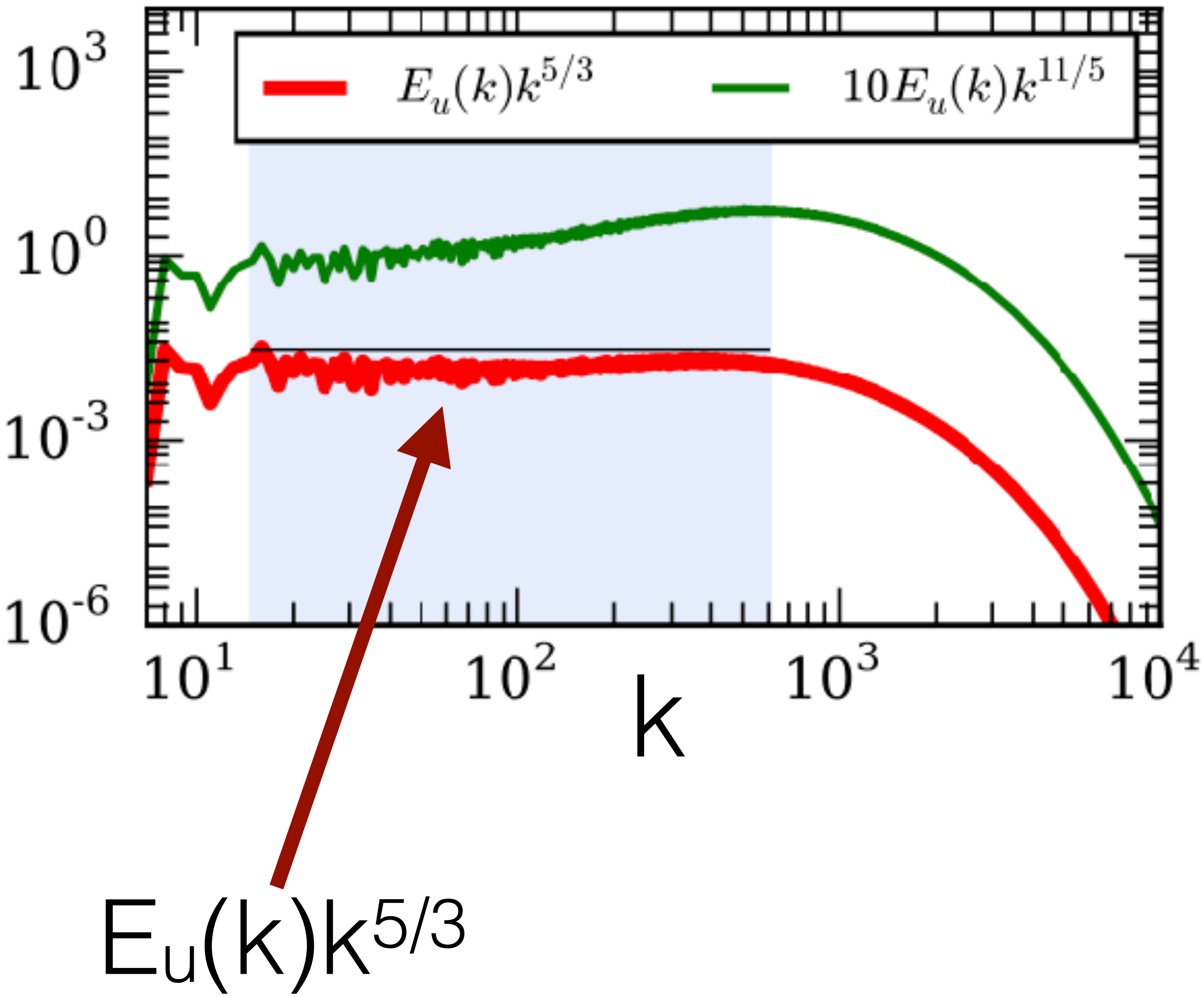
$Ra = 1.1 \times 10^{11}$

$Pr = 1$

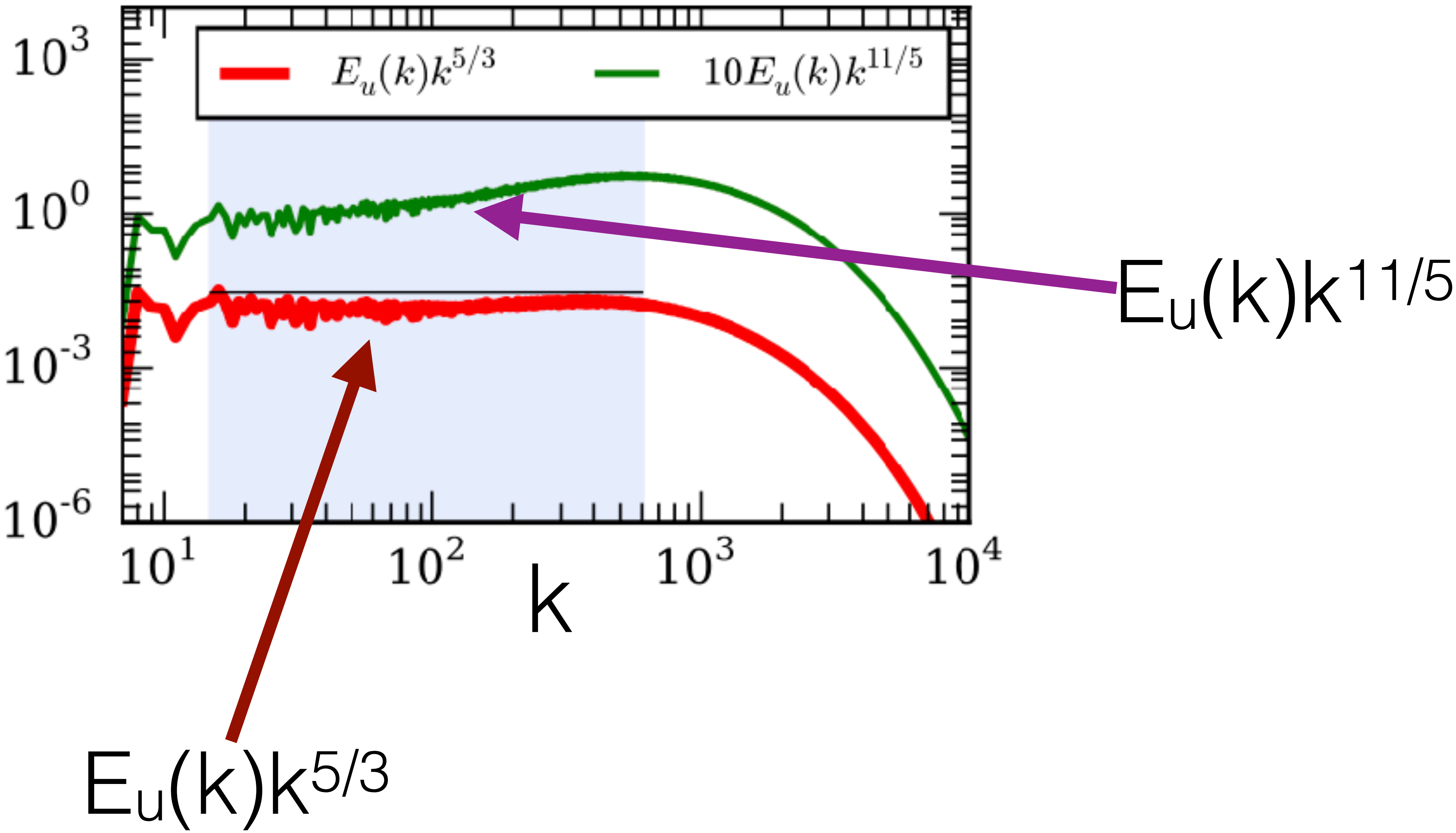
$Re = 4.5 \times 10^4$



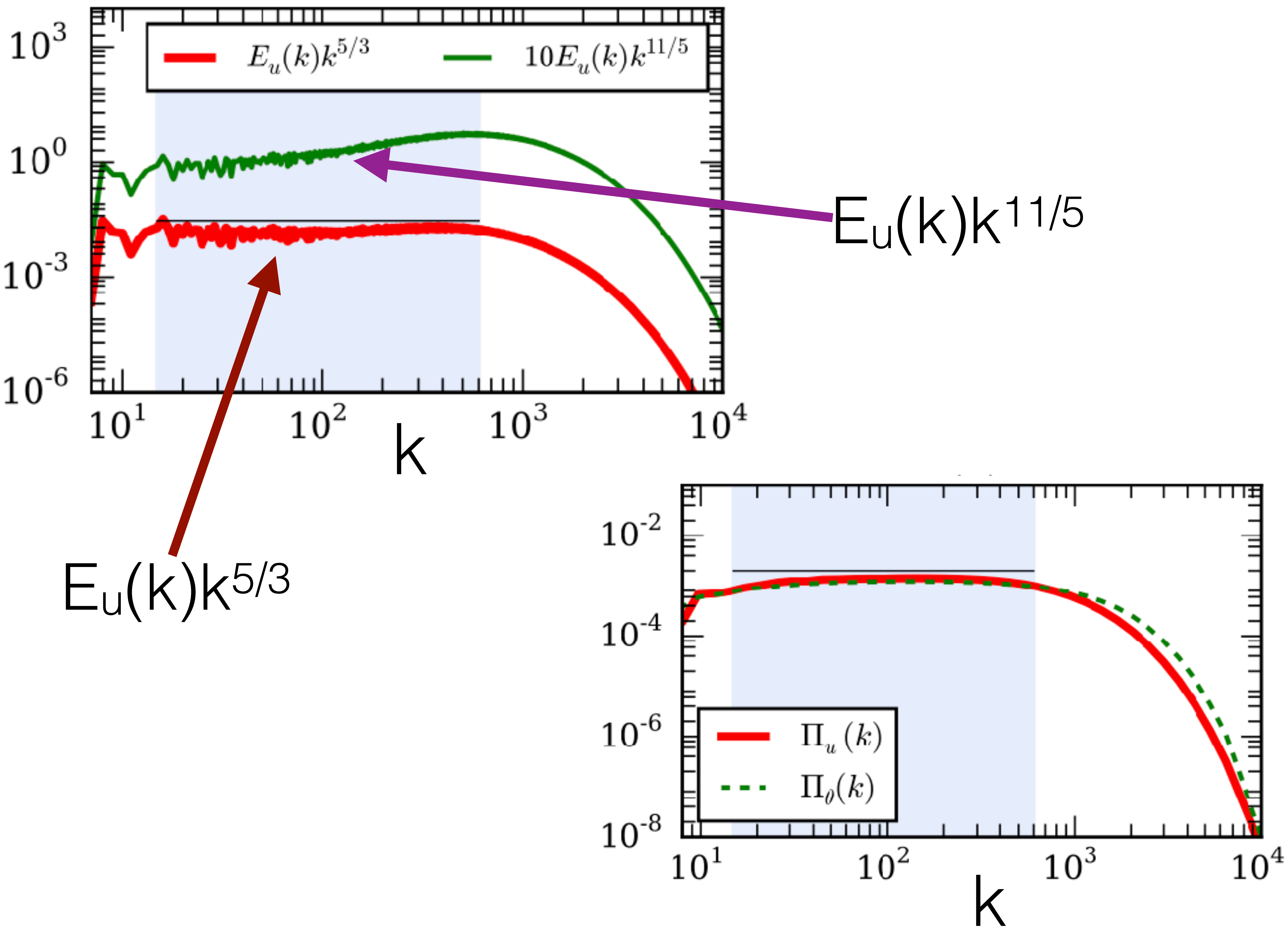
$E_u(k)k^{5/3}$
$10E_u(k)k^{11/5}$
10^3
10^0
10^{-3}
10^{-6}
10^1
10^2
10^3
10^4
k



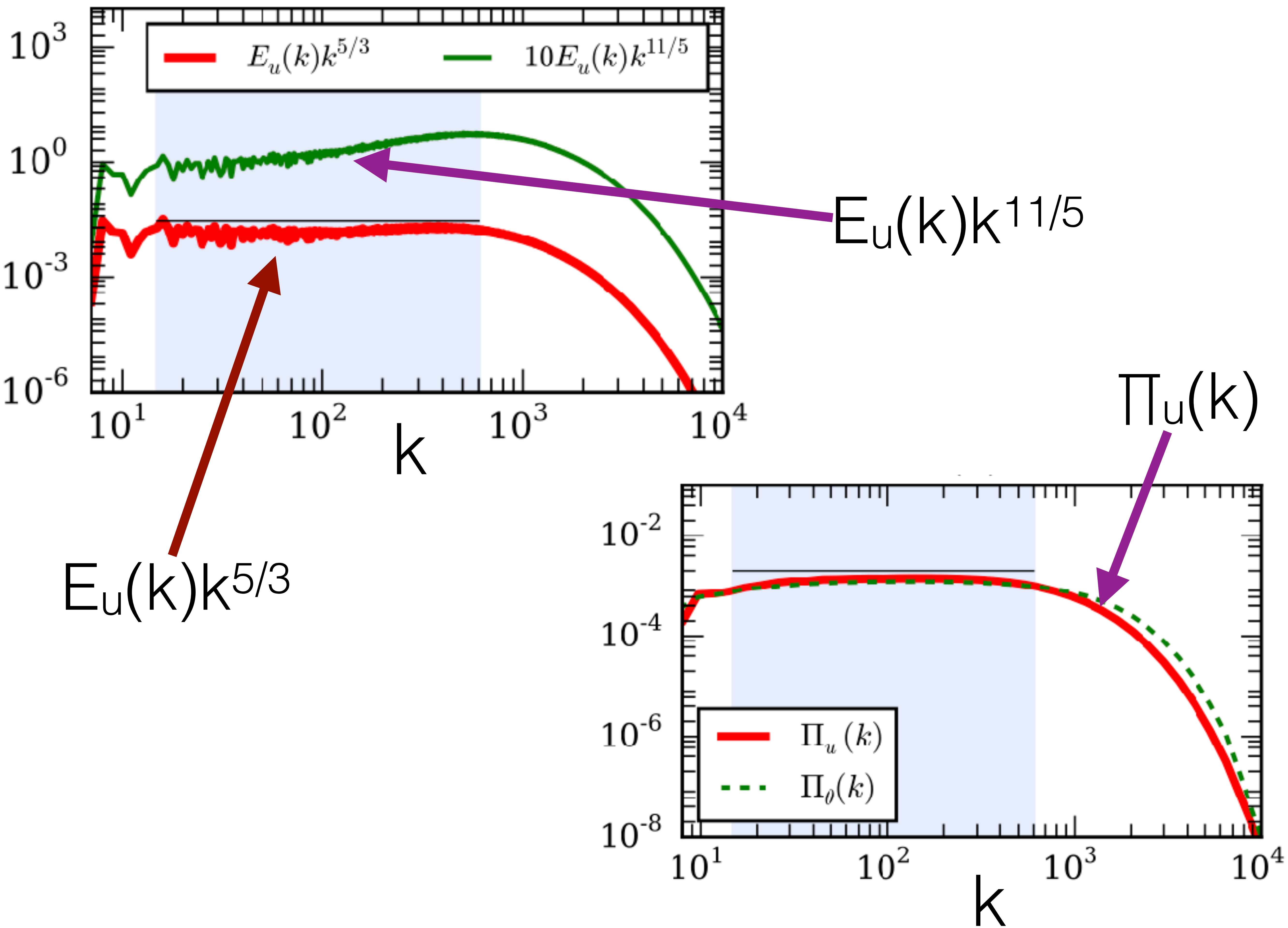
$E_u(k)k^{5/3}$
$10E_u(k)k^{11/5}$
10^3
10^0
10^{-3}
10^{-6}
10^1
10^2
10^3
10^4
k
$E_u(k)k^{5/3}$



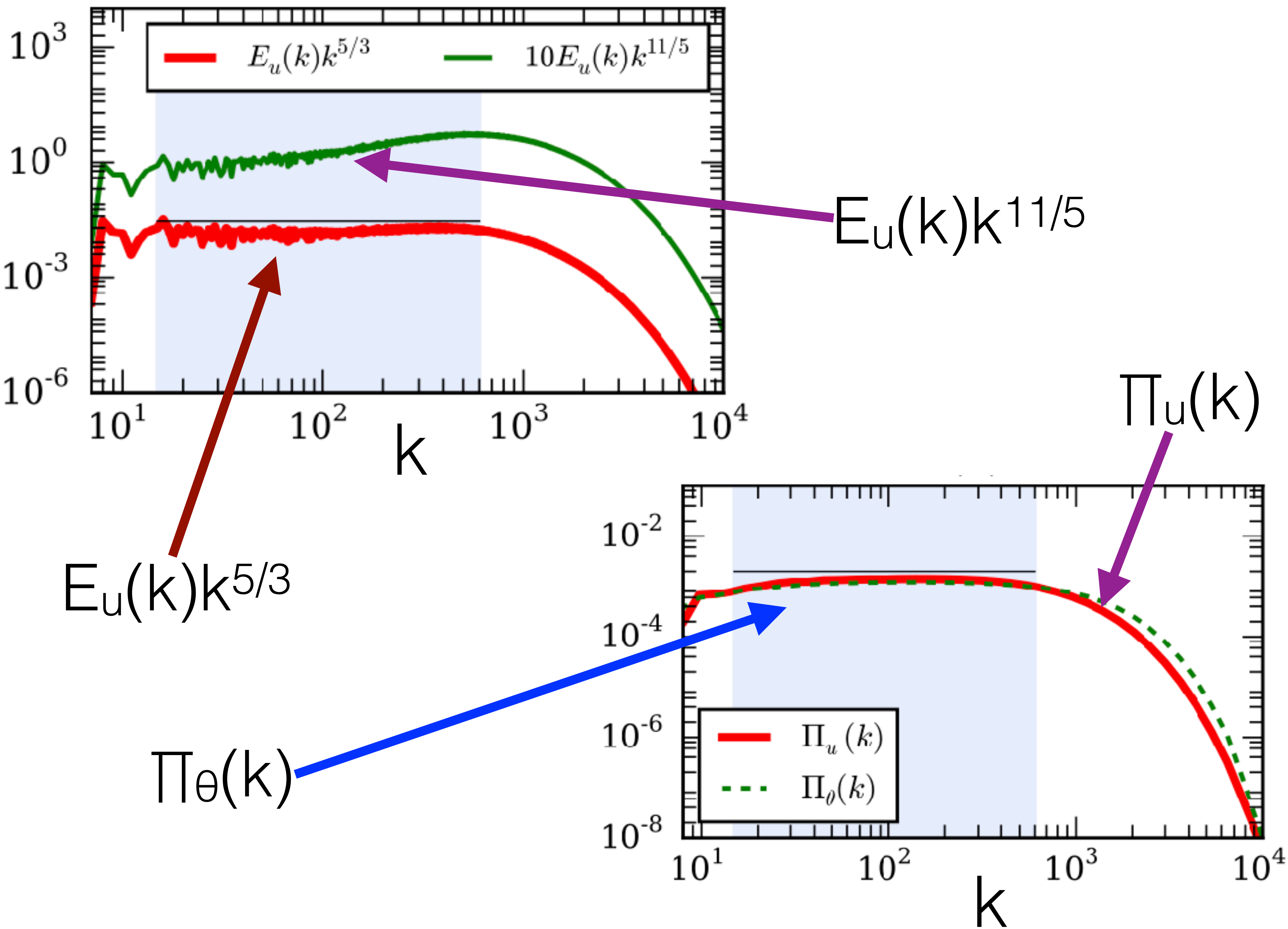
10^3
10^0
10^{-3}
10^{-6}
$E_u(k)k^{5/3}$
$10E_u(k)k^{11/5}$
10^1
10^2
10^3
10^4
k
$E_u(k)k^{11/5}$
$E_u(k)k^{5/3}$



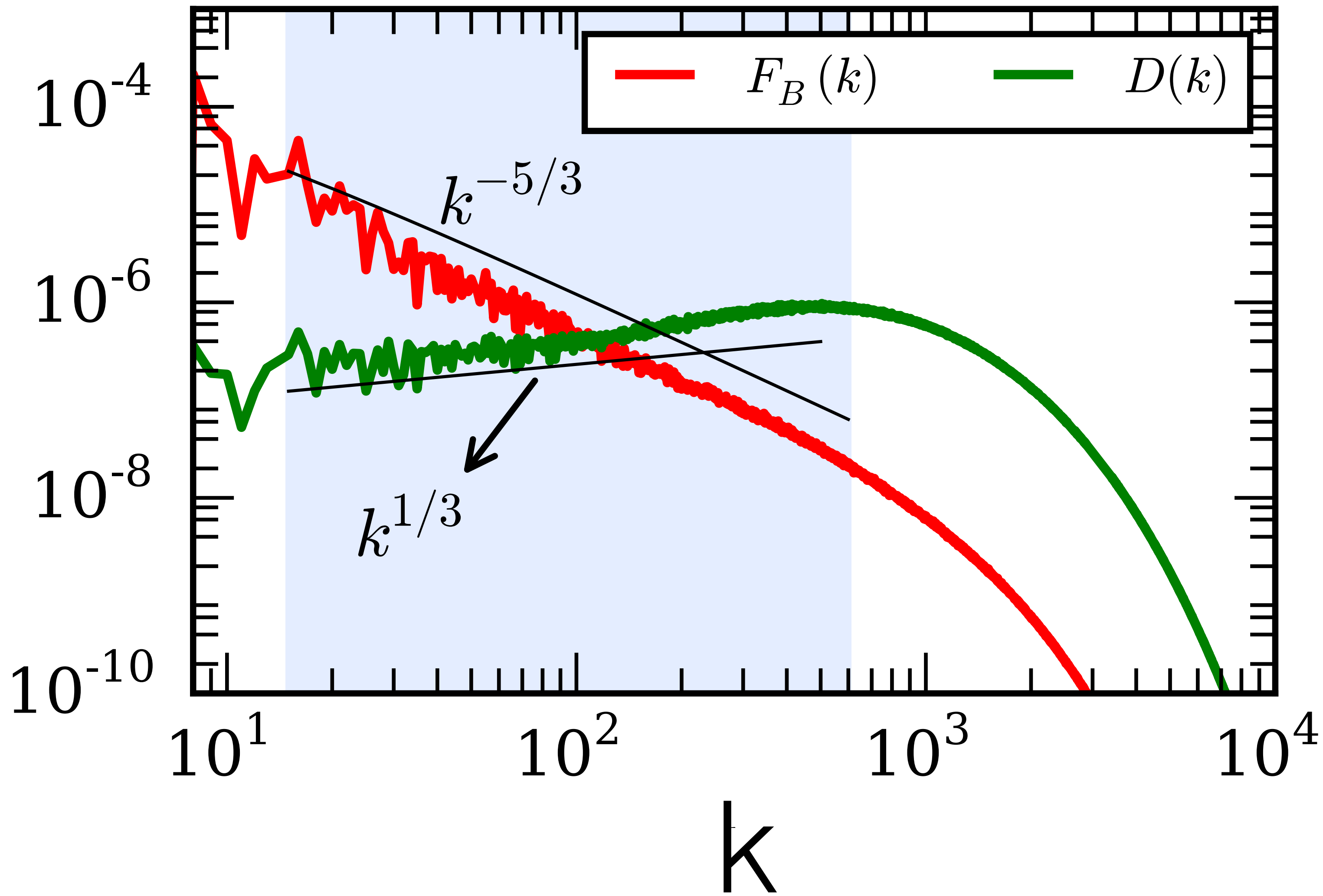
$E_u(k)k^{5/3}$
$10E_u(k)k^{11/5}$
10^3
10^0
10^{-3}
10^{-6}
10^1
10^2
10^3
10^4
k
$E_u(k)k^{11/5}$
$E_u(k)k^{5/3}$
10^{-2}
10^{-4}
10^{-6}
10^{-8}
$\Pi_u(k)$
$\Pi_\theta(k)$
10^1
10^2
10^3
10^4
k



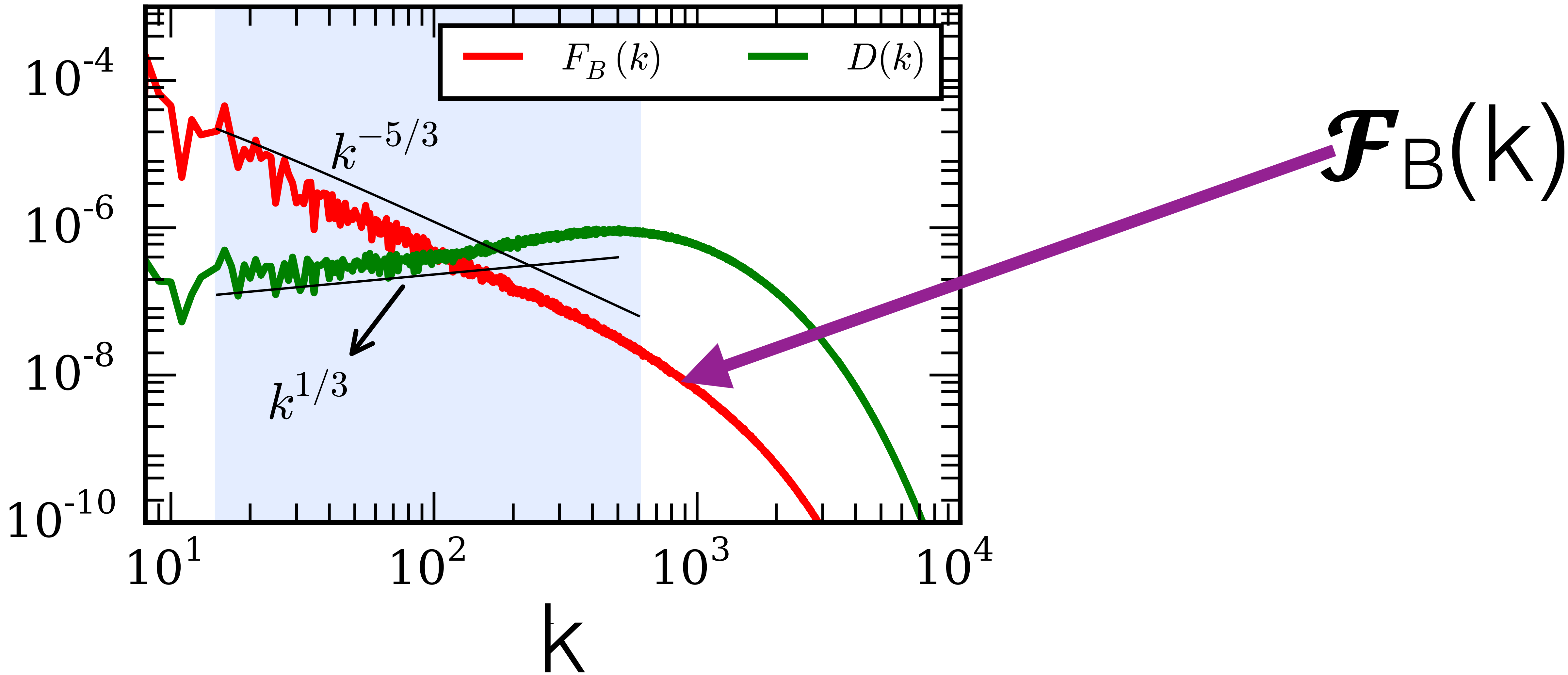
$E_u(k)k^{5/3}$
$10E_u(k)k^{11/5}$
10^3
10^0
10^{-3}
10^{-6}
10^1
10^2
10^3
10^4
k
$E_u(k)k^{11/5}$
$E_u(k)k^{5/3}$
$\Pi_u(k)$
10^{-2}
10^{-4}
10^{-6}
10^{-8}
$\Pi_u(k)$
$\Pi_\theta(k)$
10^1
10^2
10^3
10^4
k



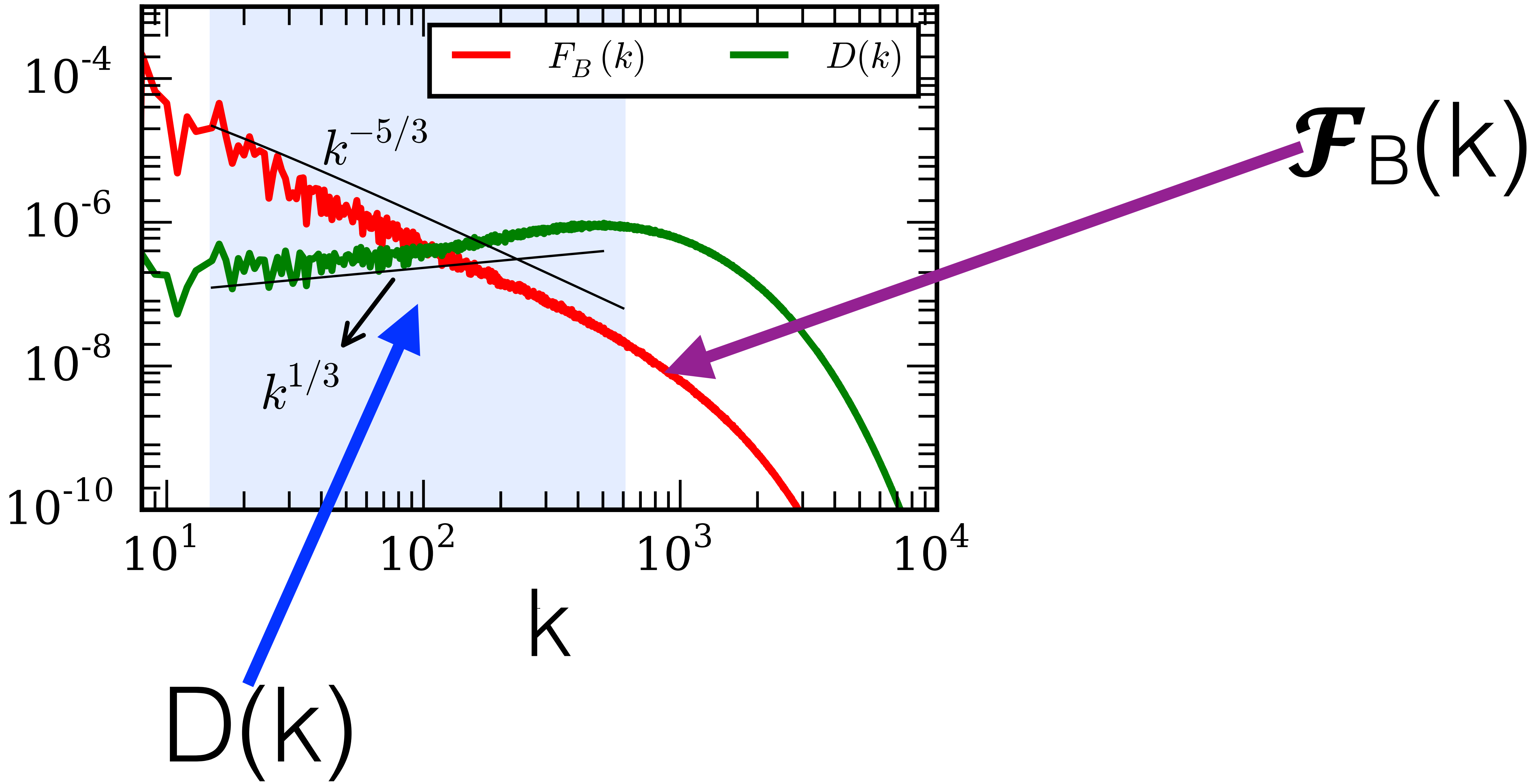
$E_u(k)k^{5/3}$
$10E_u(k)k^{11/5}$
10^3
10^0
10^{-3}
10^{-6}
10^1
10^2
10^3
10^4
k
$E_u(k)k^{11/5}$
$E_u(k)k^{5/3}$
$\Pi_u(k)$
10^{-2}
10^{-4}
10^{-6}
10^{-8}
$\Pi_u(k)$
$\Pi_\theta(k)$
10^1
10^2
10^3
10^4
k
$\Pi_\theta(k)$



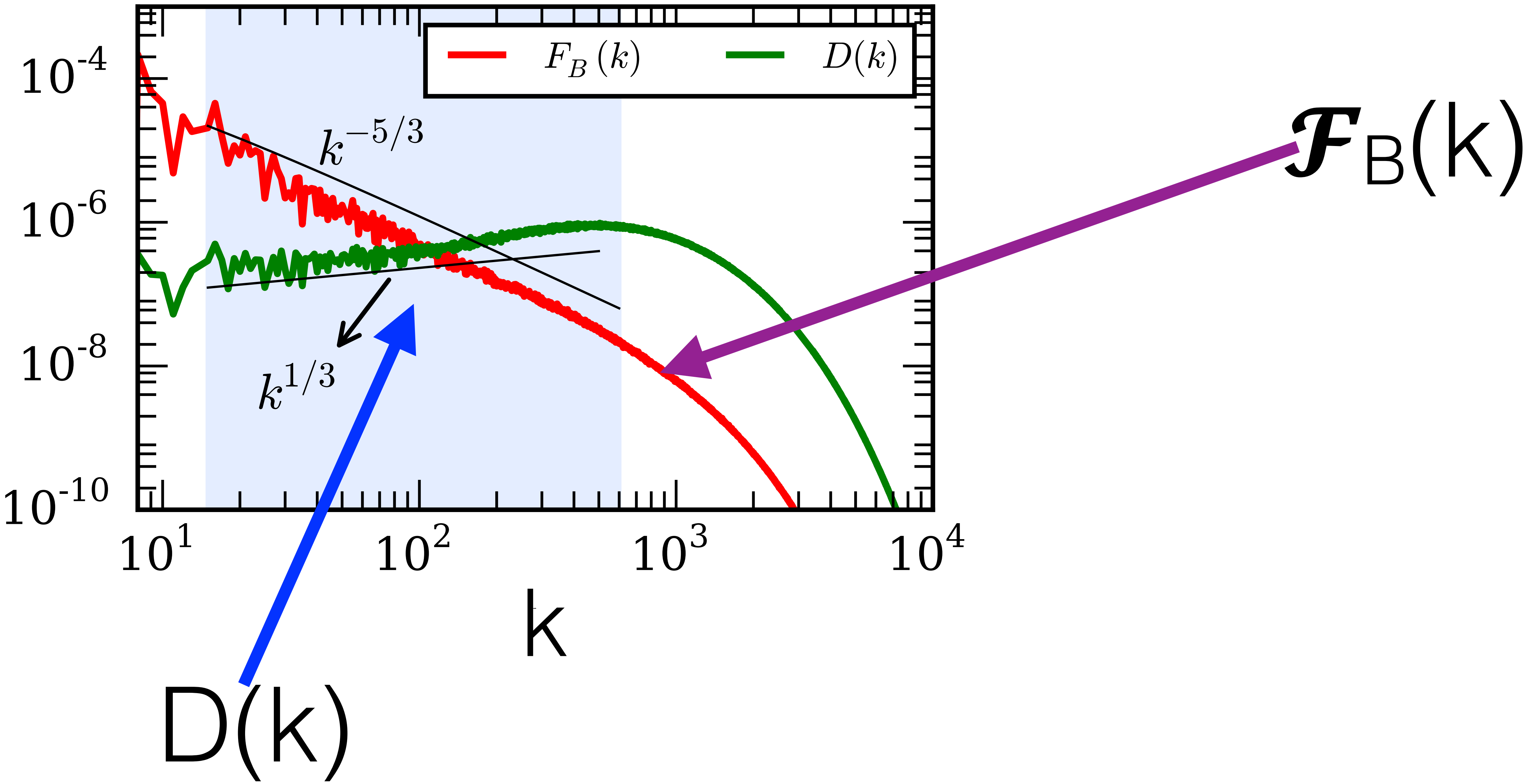
$F_B(k)$
$D(k)$
$k^{-5/3}$
$k^{1/3}$
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^1
10^2
10^3
10^4
k



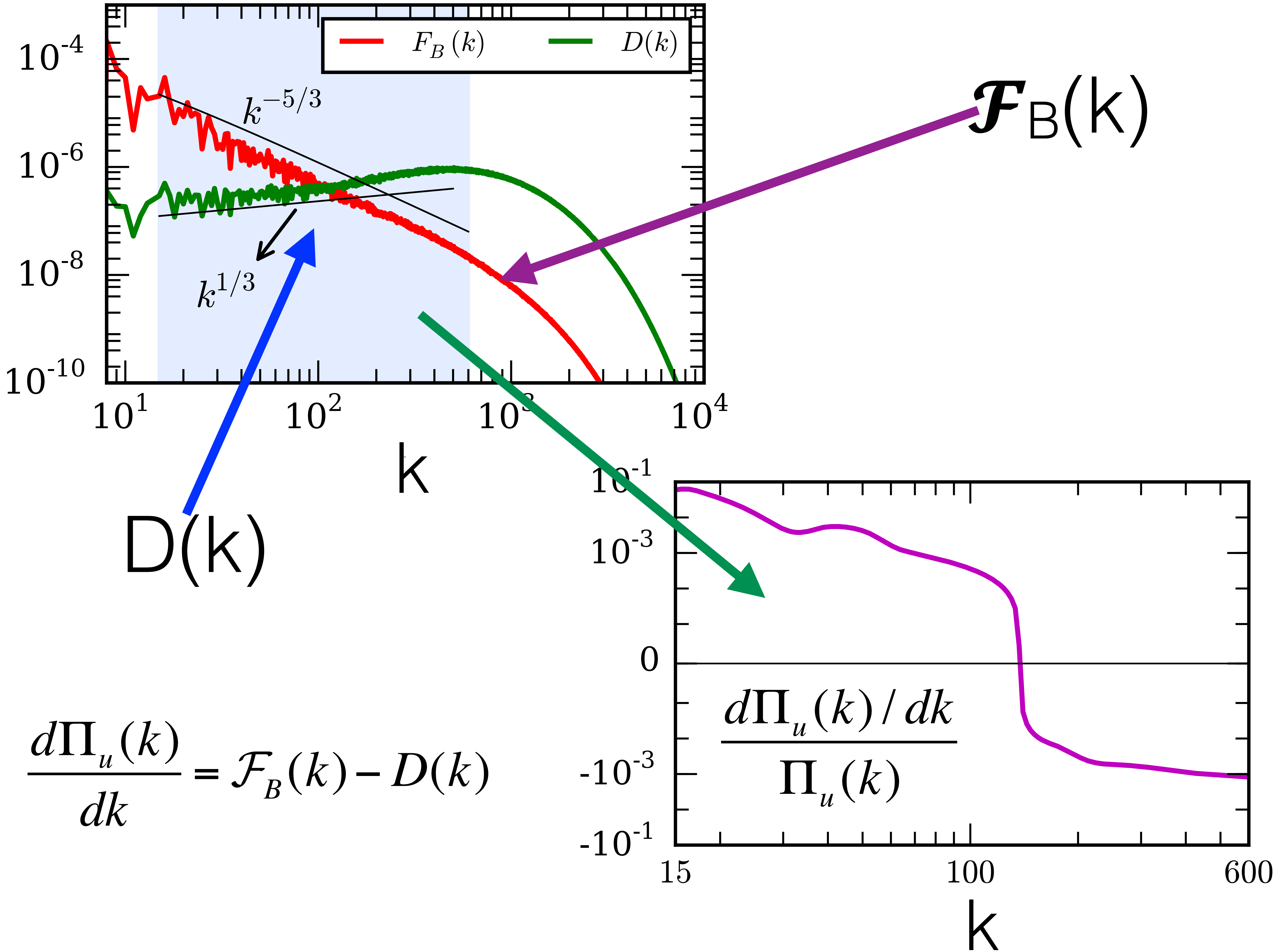
$F_B(k)$
$D(k)$
$k^{-5/3}$
$k^{1/3}$
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{1}
10^{2}
10^{3}
10^{4}
k
$\mathcal{F}_B(k)$



$F_B(k)$
$D(k)$
$k^{-5/3}$
$k^{1/3}$
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^1
10^2
10^3
10^4
k
$\mathcal{F}_B(k)$
D(k)



$$\frac{d\Pi_u(k)}{dk} = \mathcal{F}_B(k) - D(k)$$

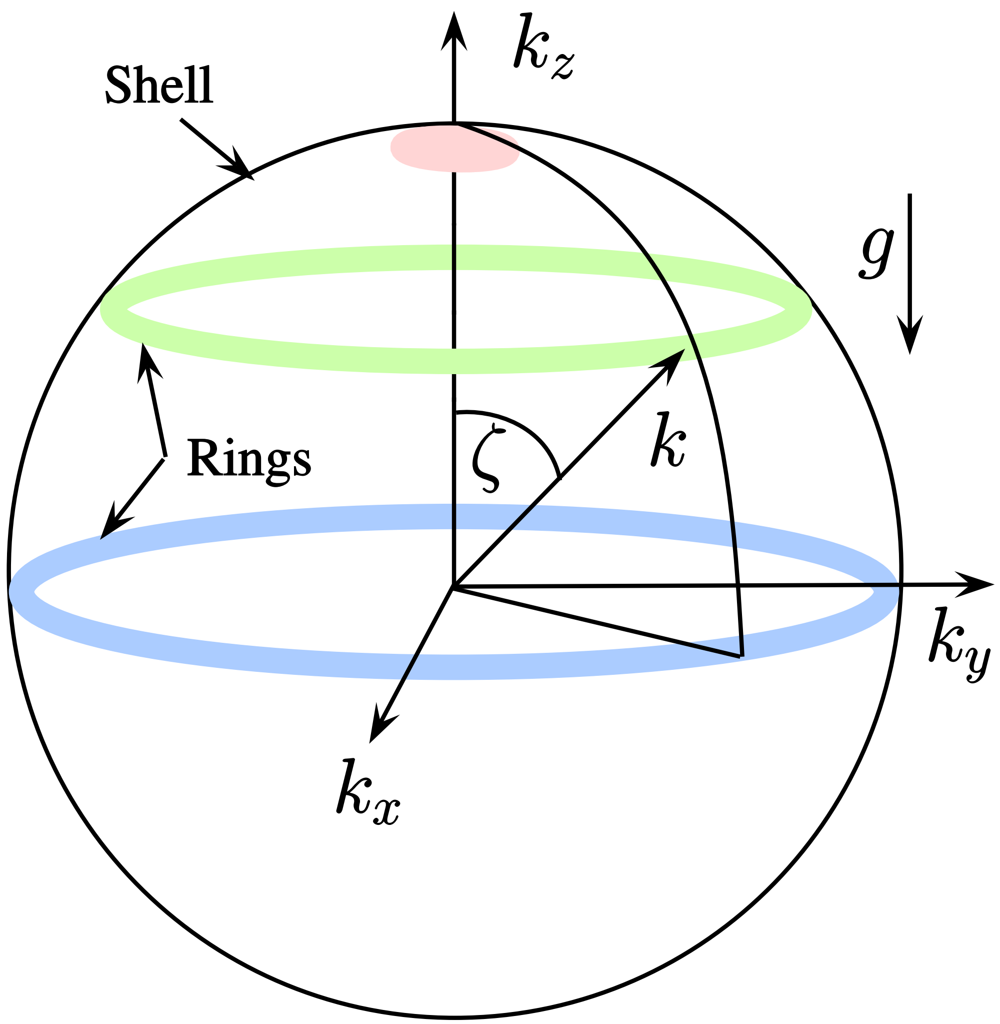


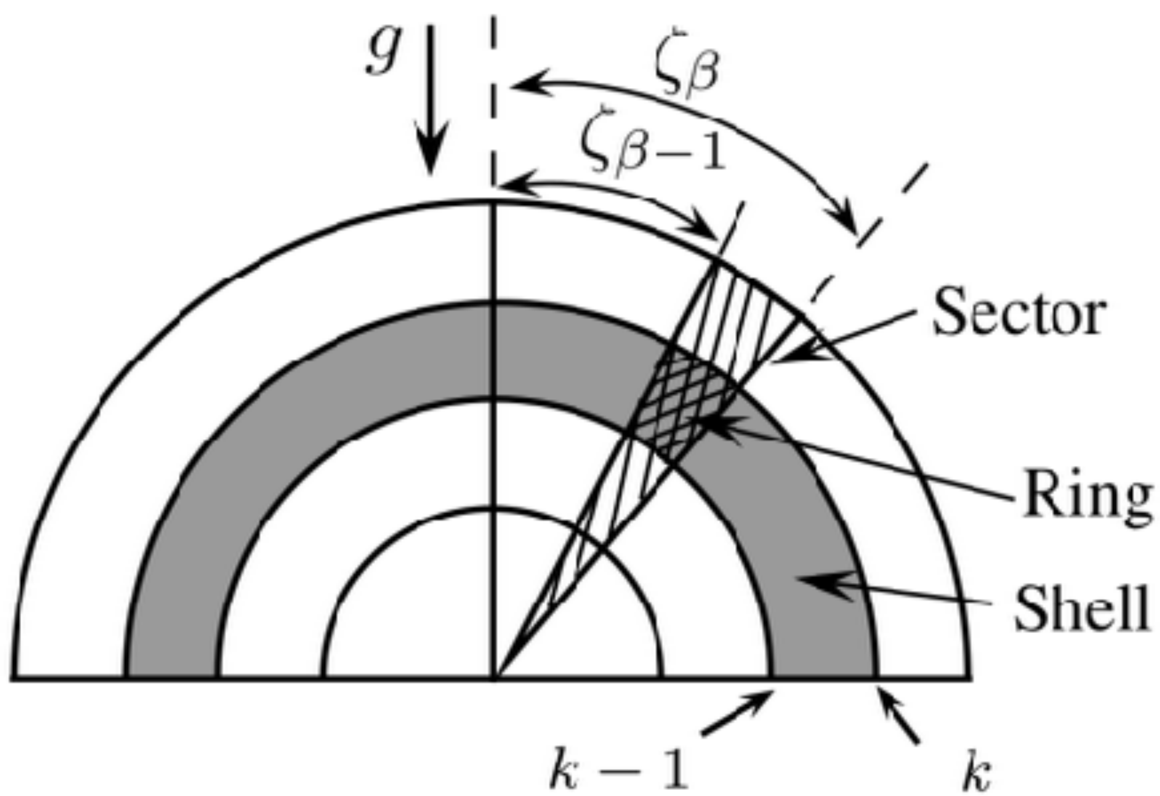
$F_B(k)$
$D(k)$
$k^{-5/3}$
$k^{1/3}$
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{1}
10^{2}
10^{3}
10^{4}
k
$\mathcal{F}_B(k)$
D(k)
10^{-1}
10^{-3}
0
-10^{-3}
-10^{-1}
15
100
600
k
$\frac{d\Pi_u(k)/dk}{\Pi_u(k)}$
$\frac{d\Pi_u(k)}{dk} = \mathcal{F}_B(k) - D(k)$

Ring Spectrum
&
Shell-to-Shell Energy Transfer

Rayleigh-Bénard Convection

Ring Spectrum





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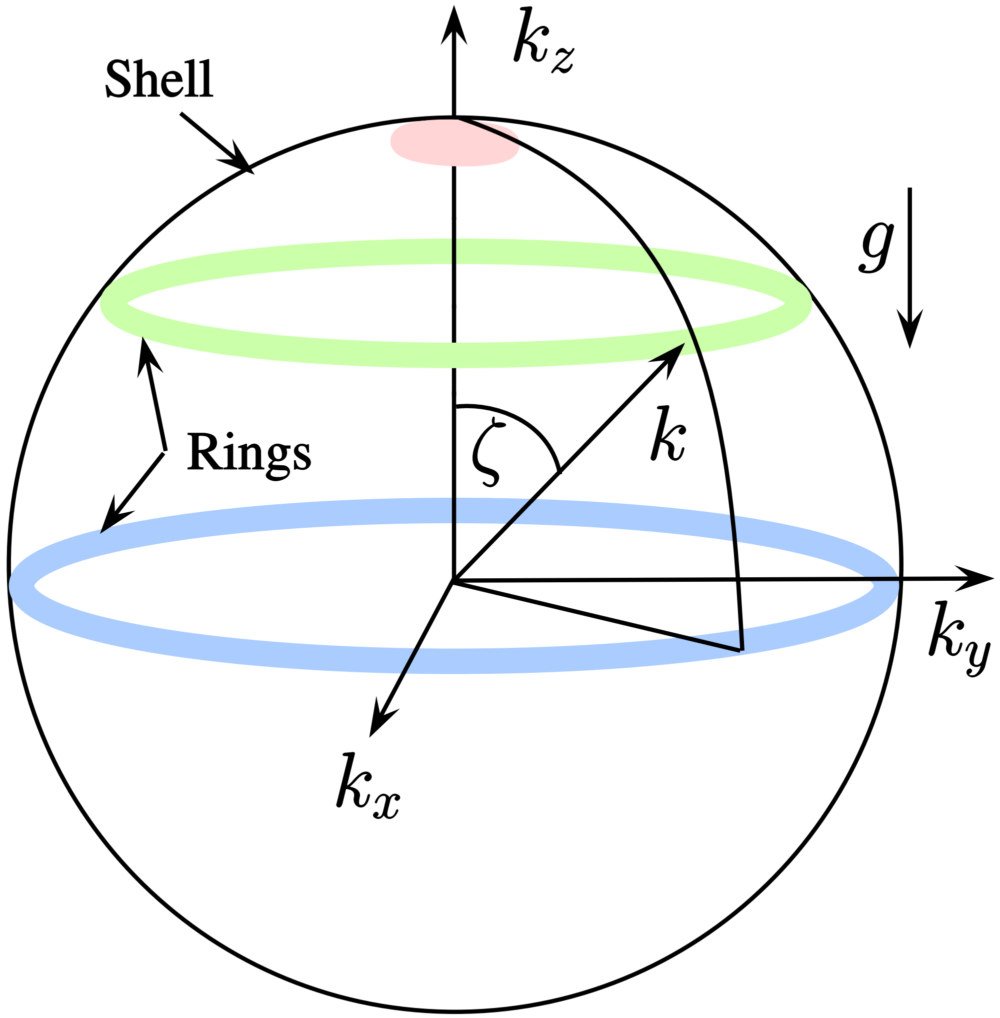
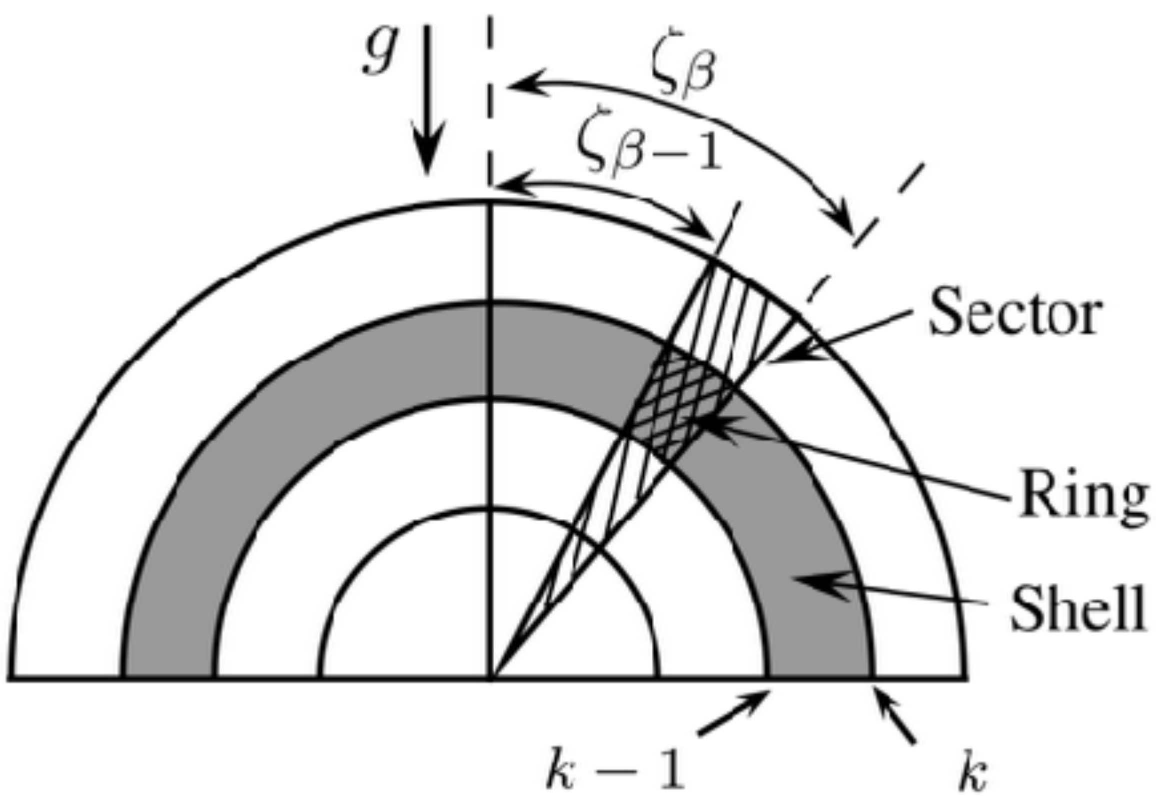
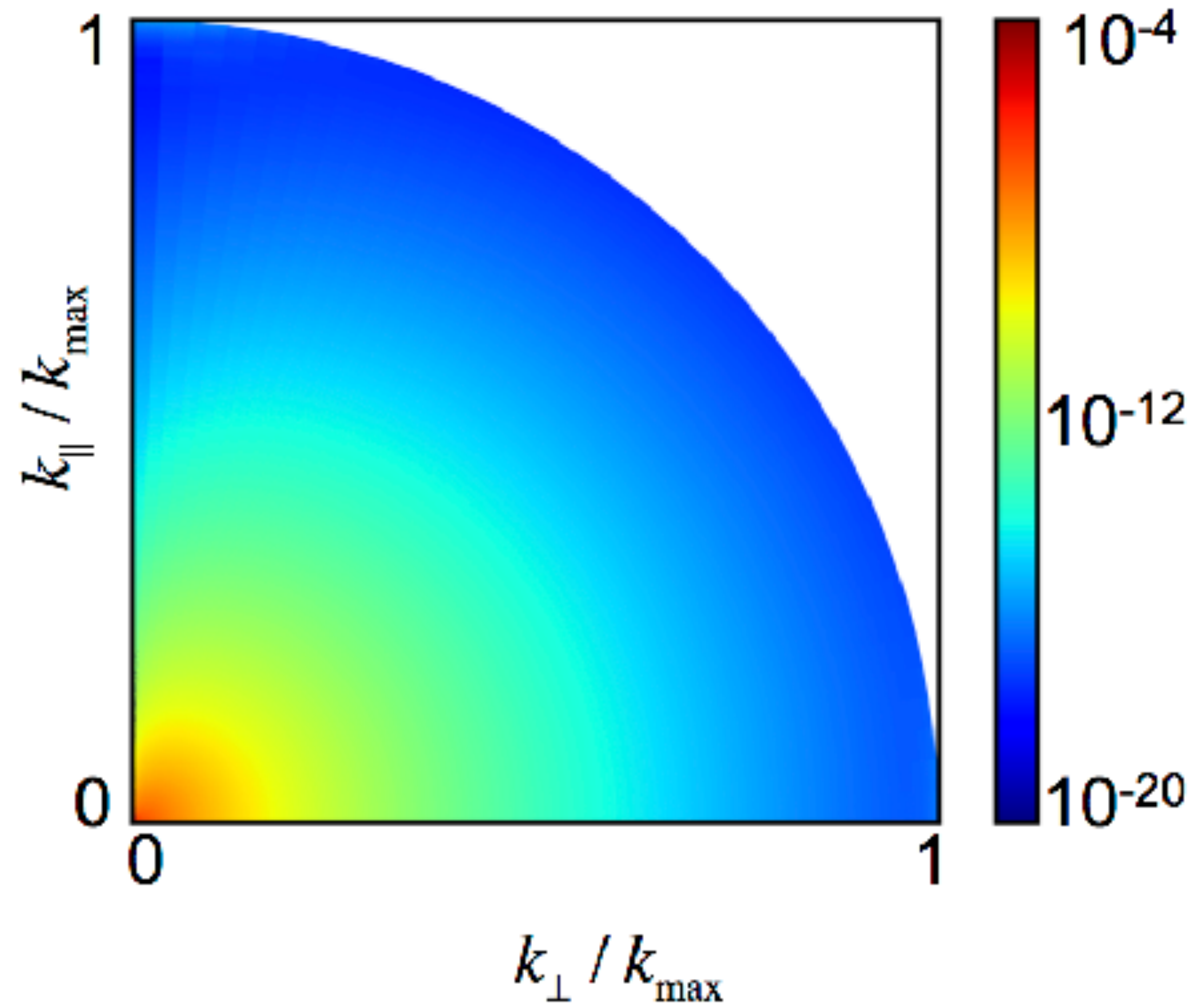
Ring Spectrum







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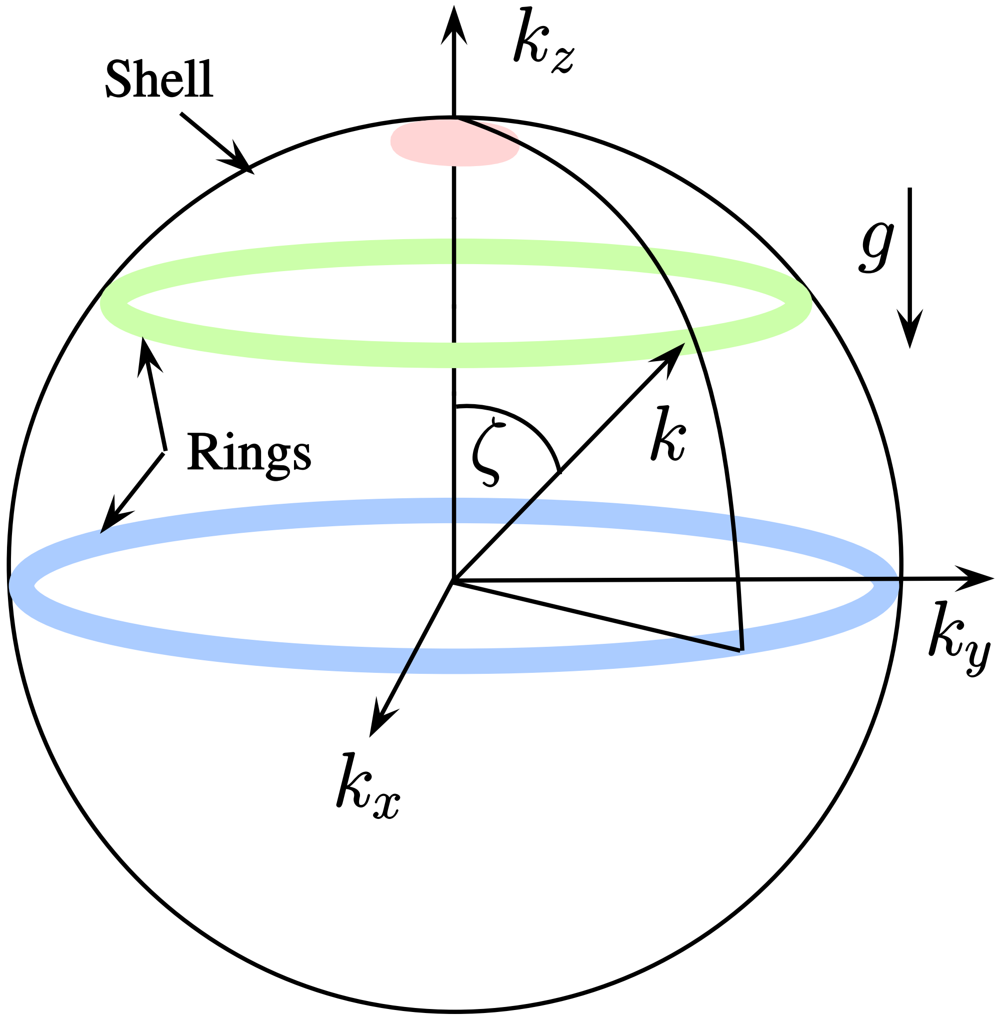
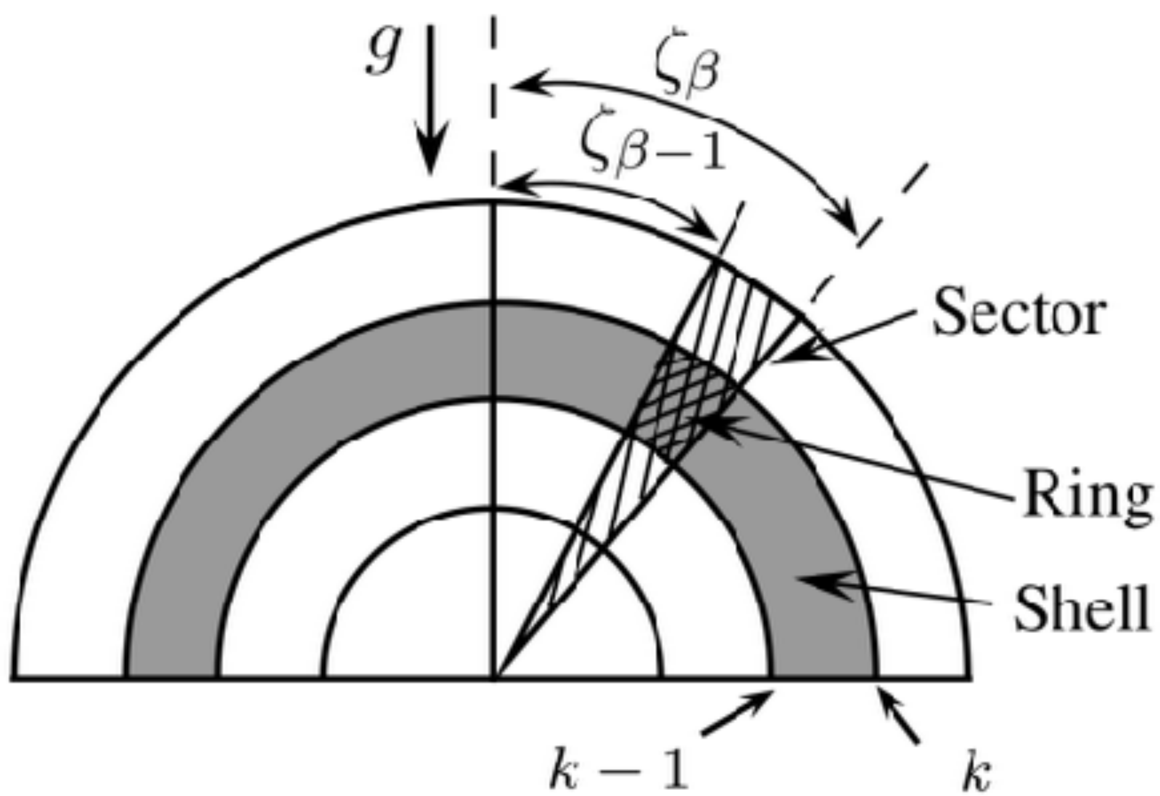
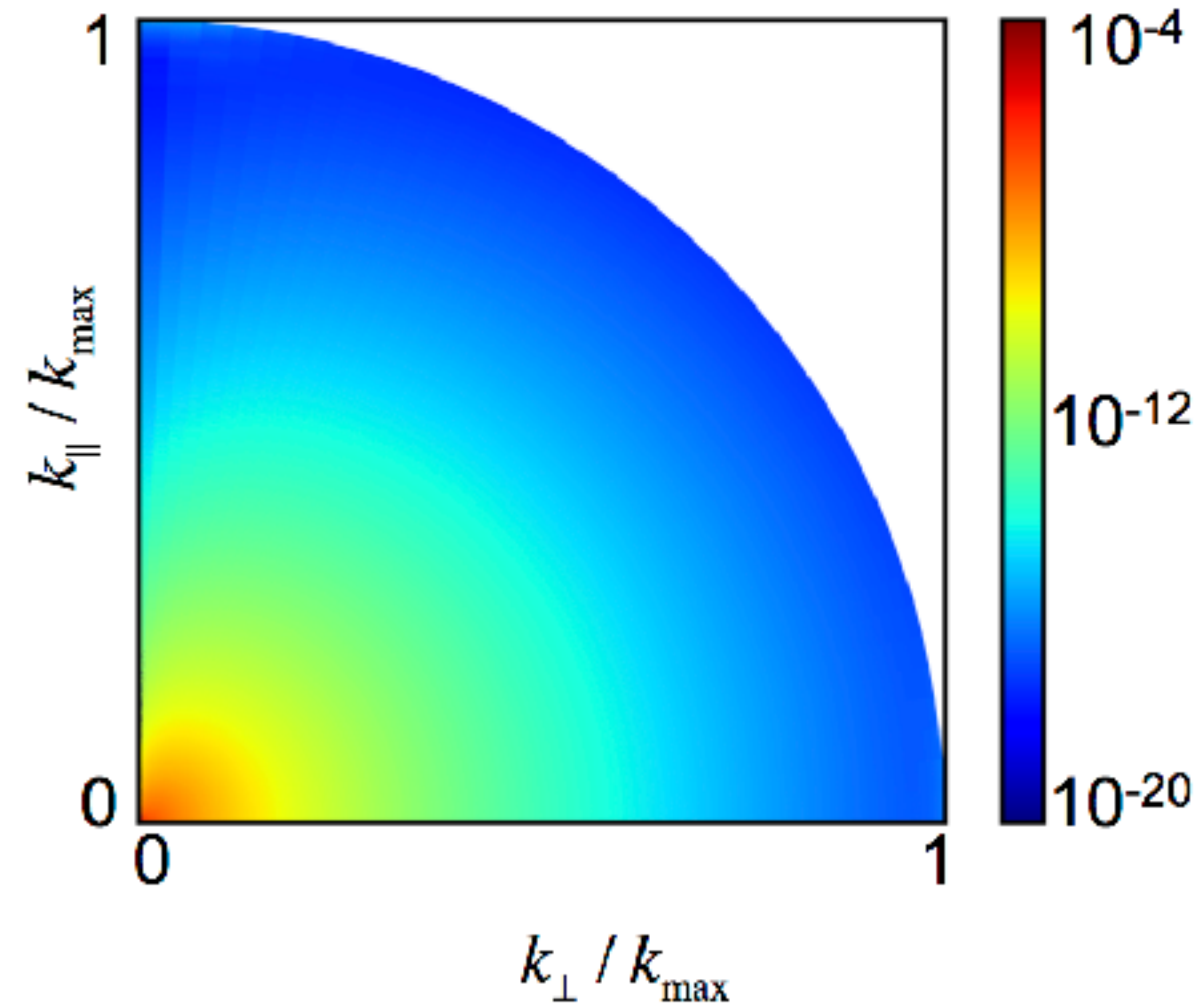
Ring Spectrum







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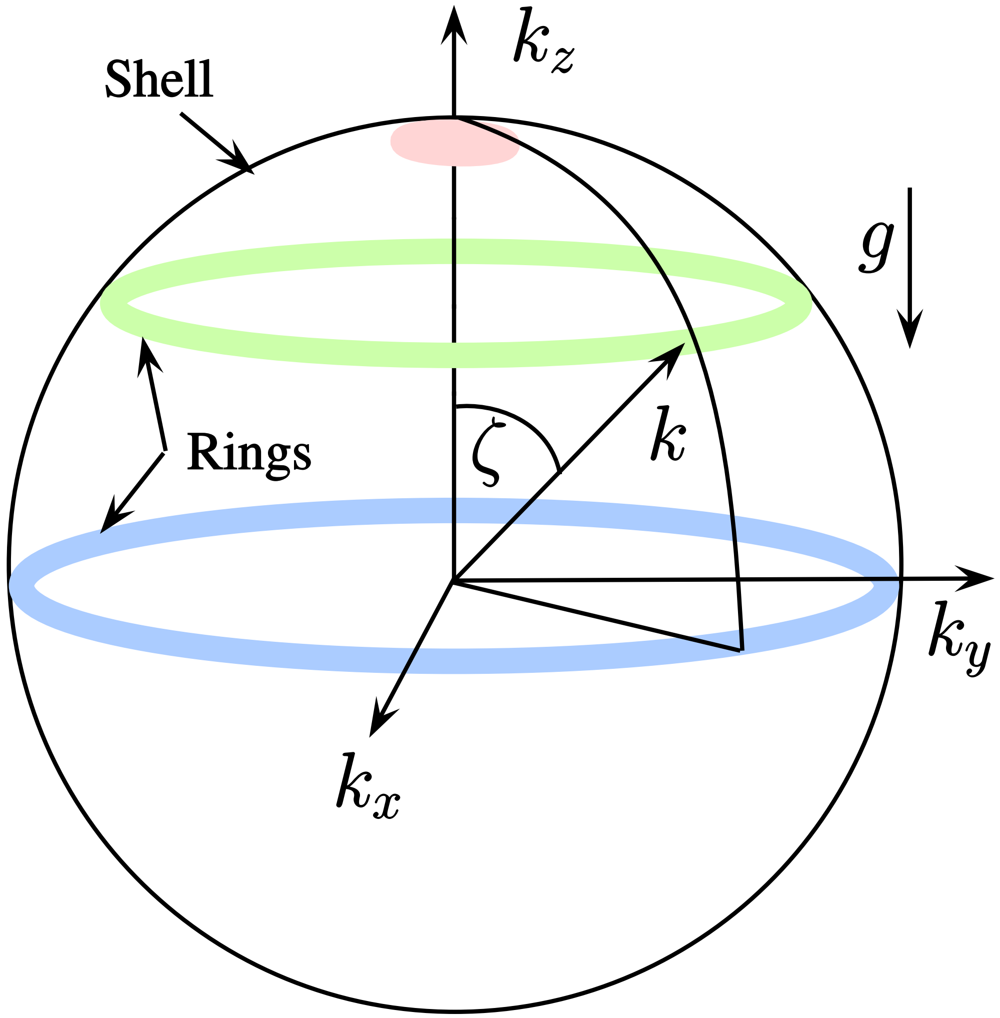
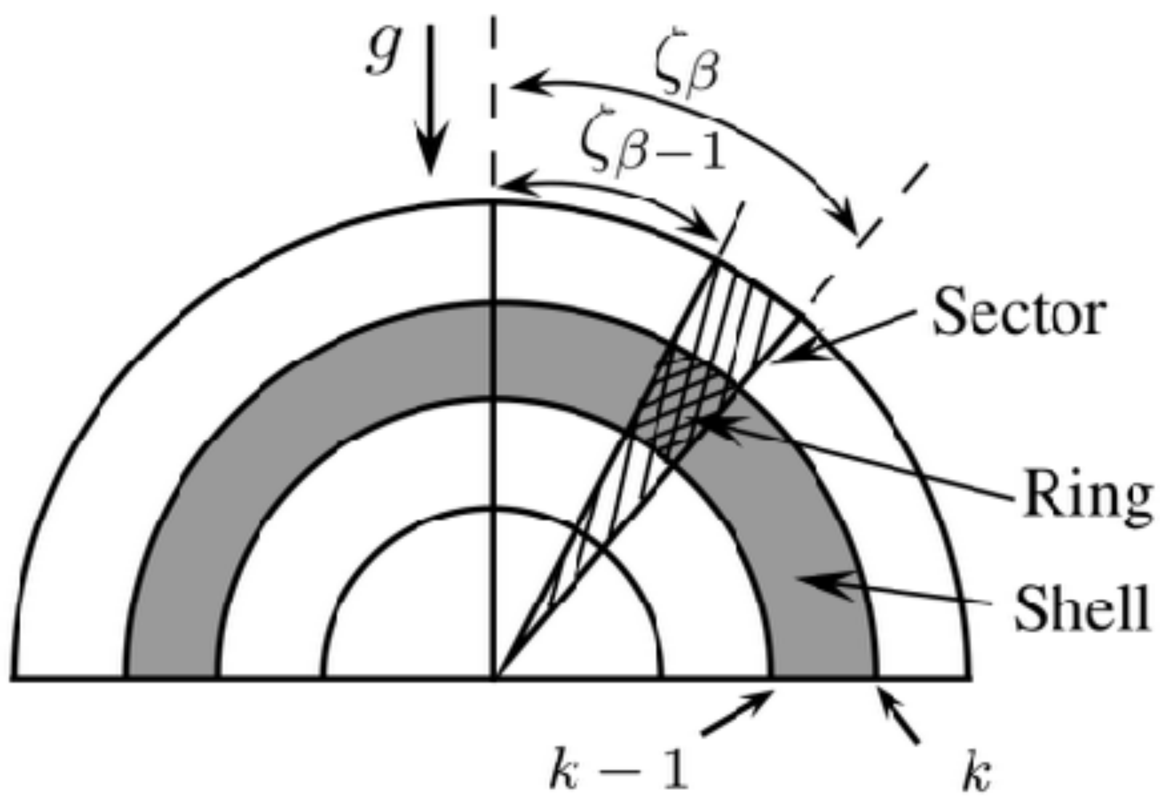
Nearly Isotropic

Shell-to-Shell ET





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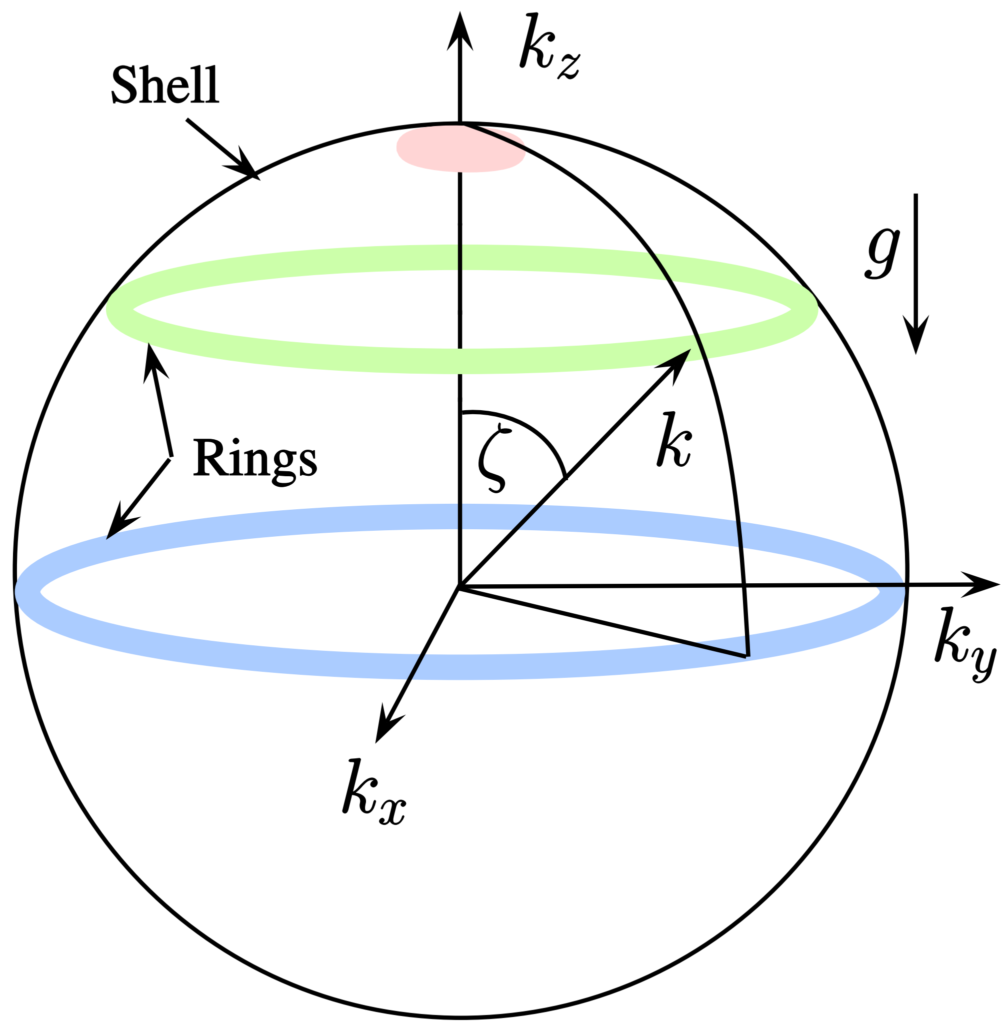
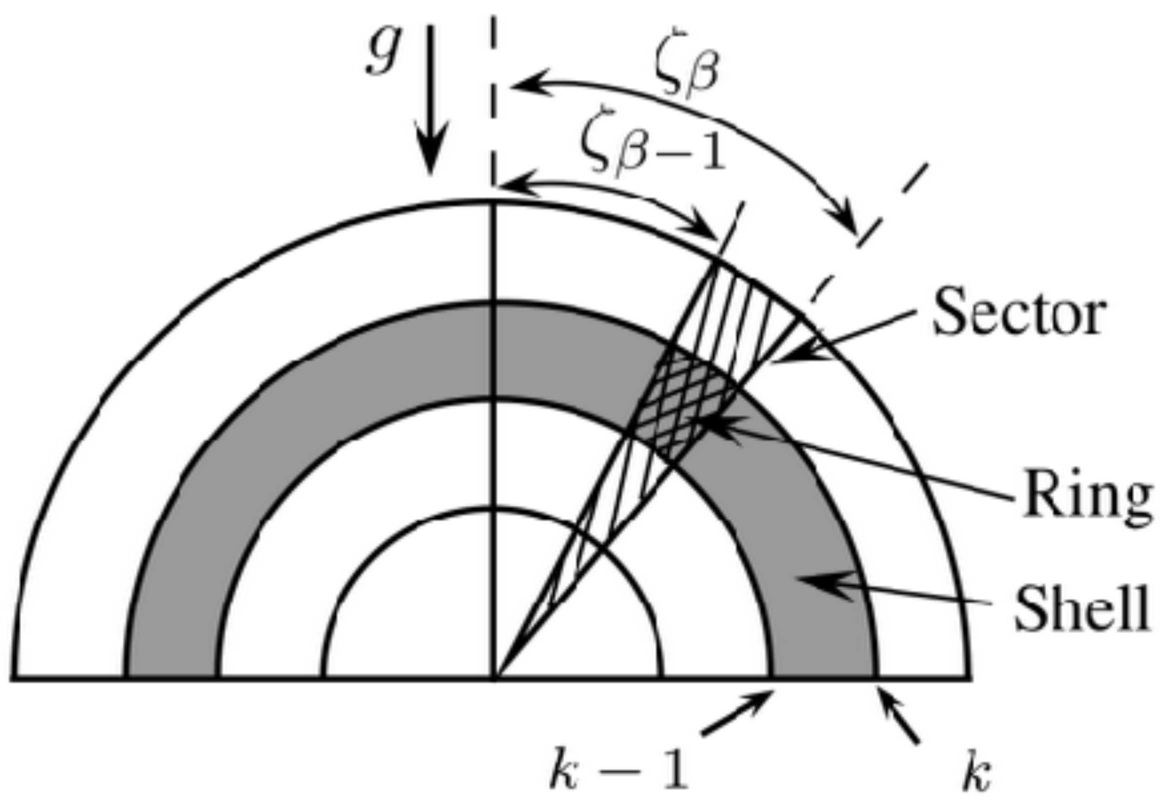
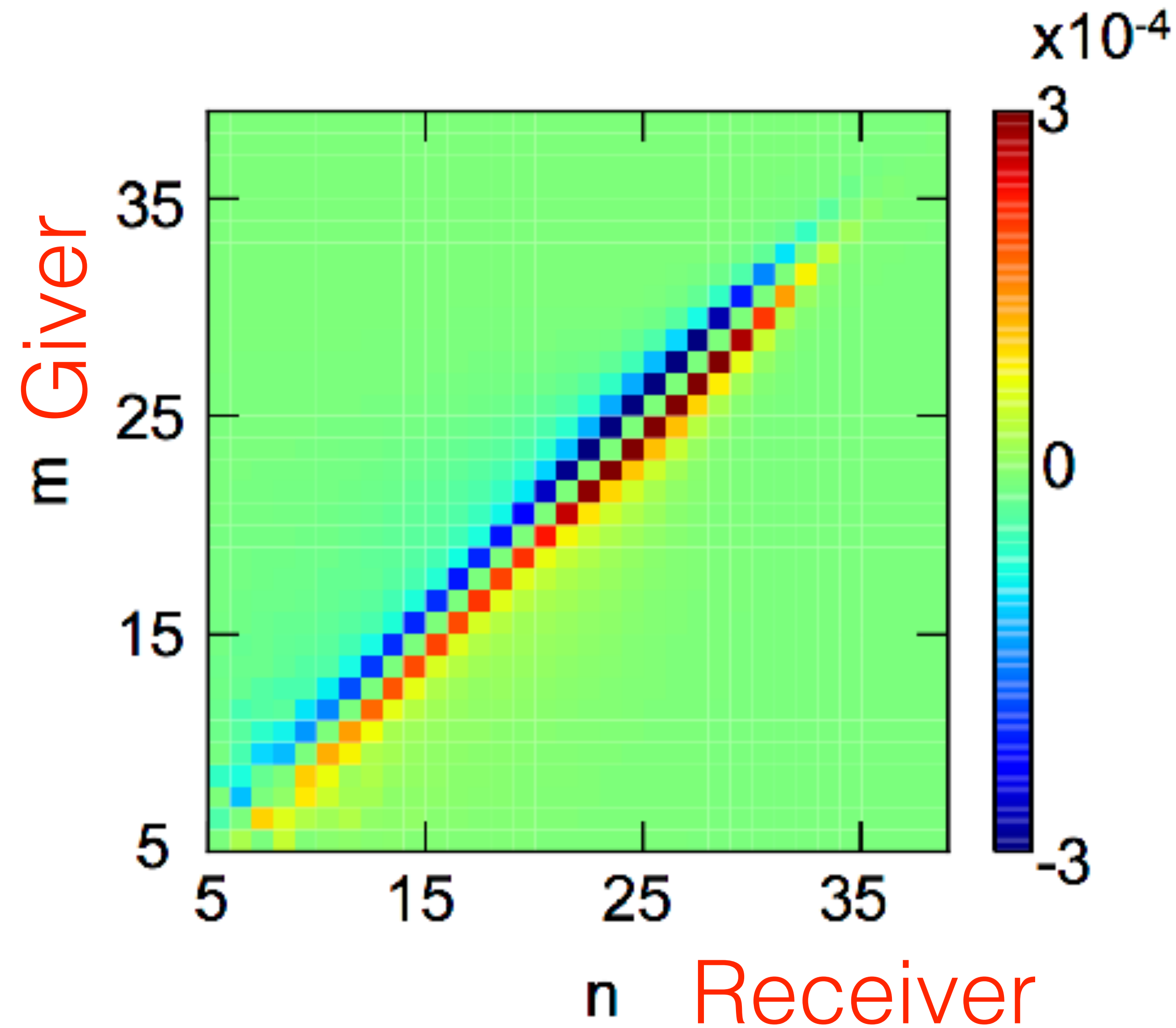




Shell-to-Shell ET



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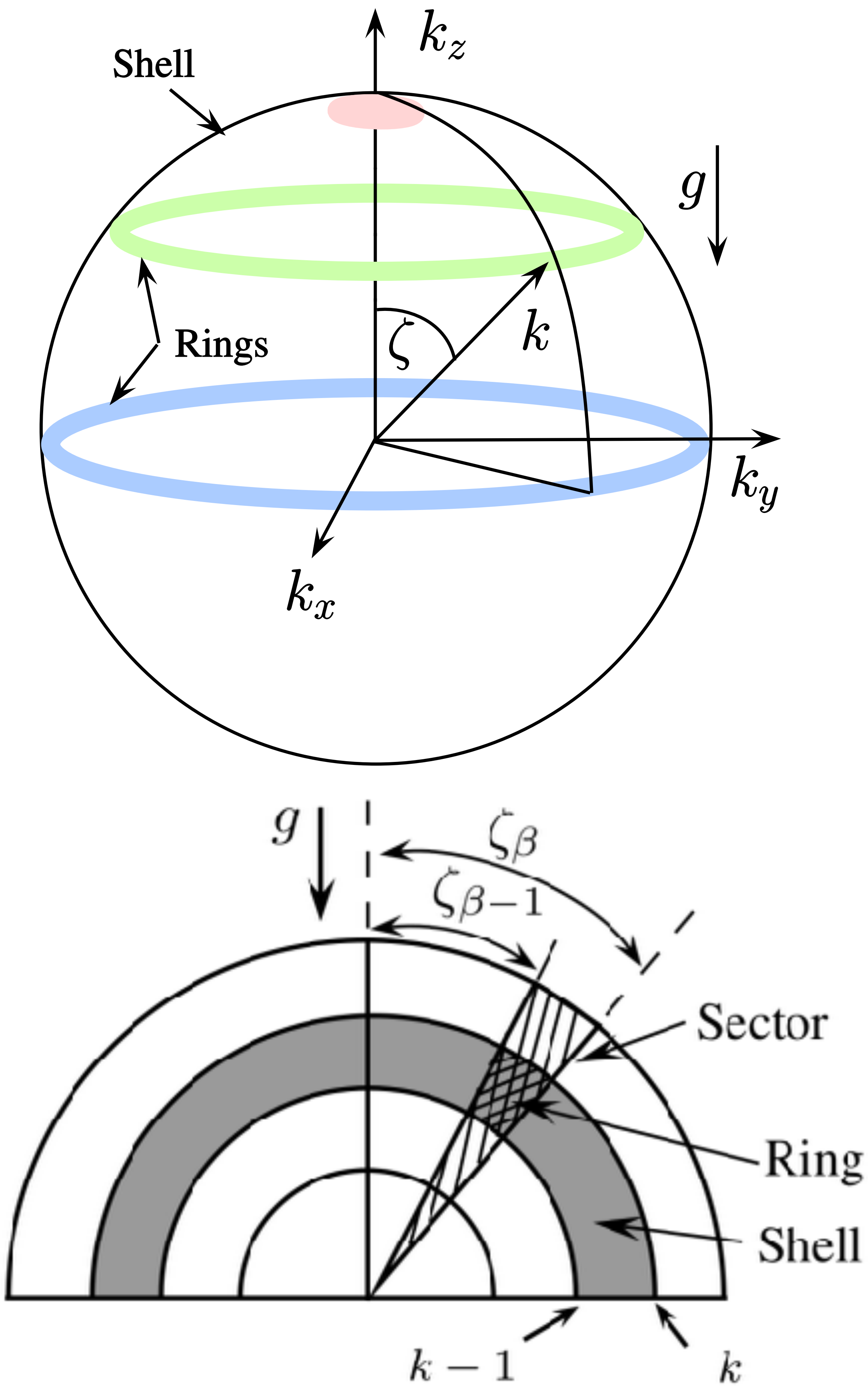
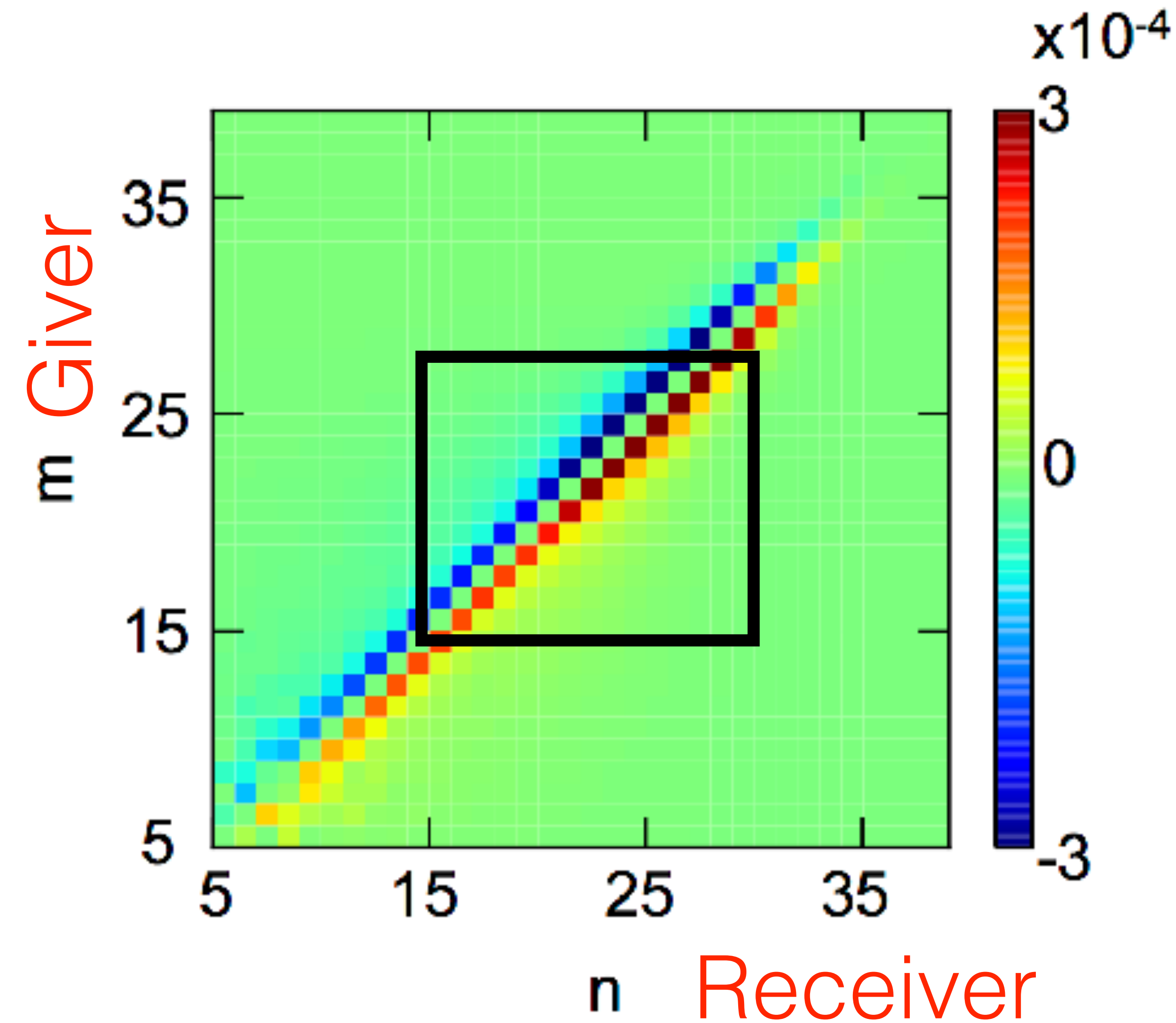
Shell-to-Shell ET





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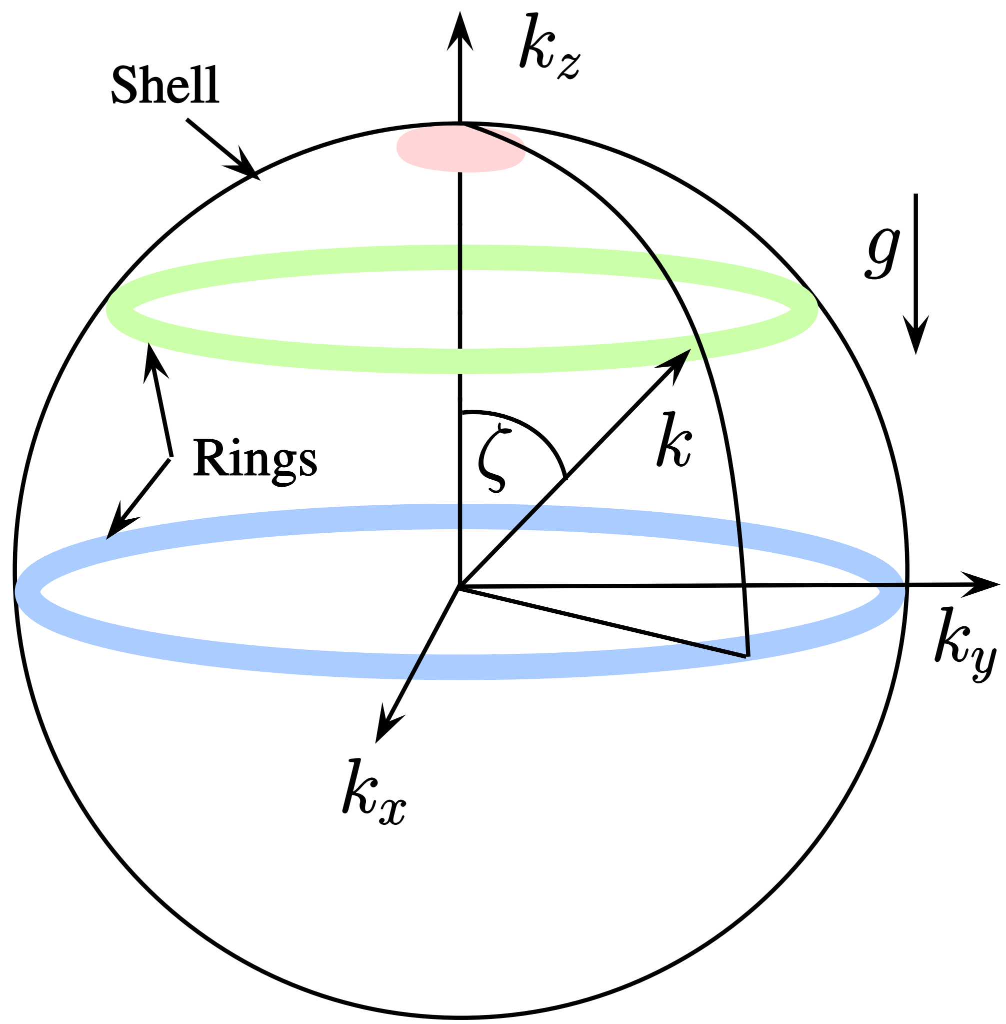
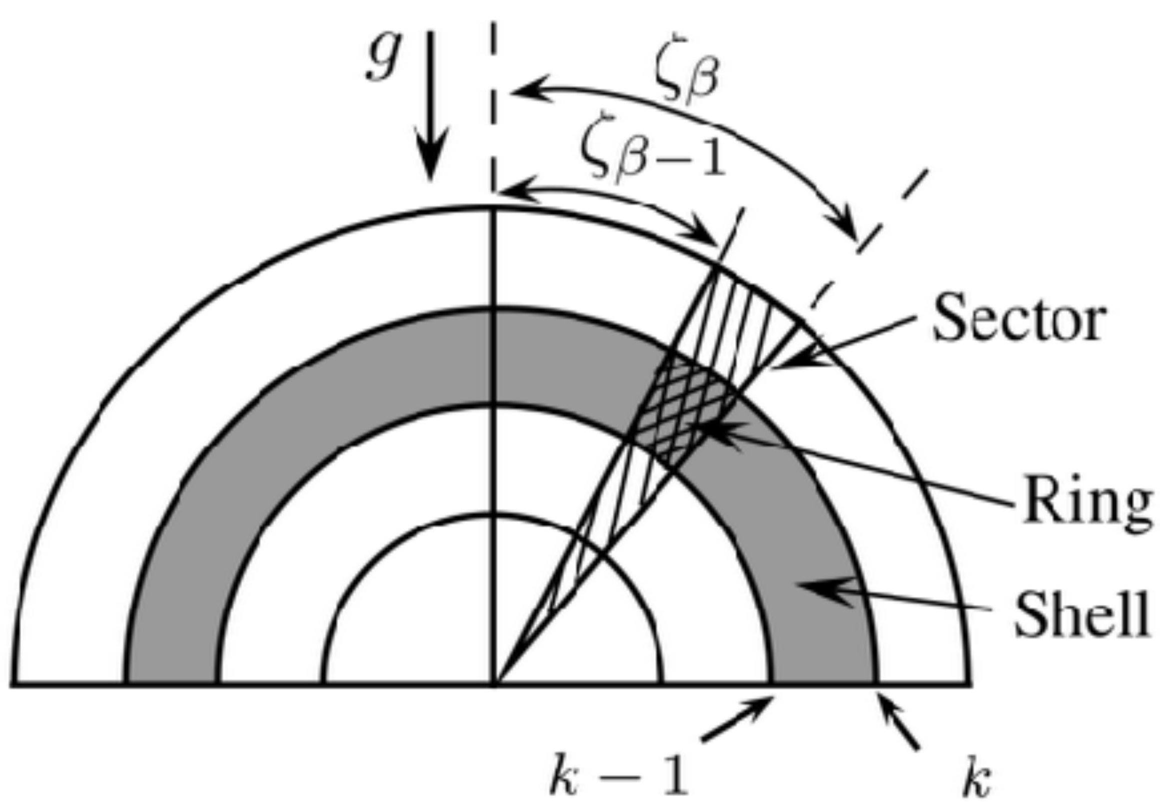
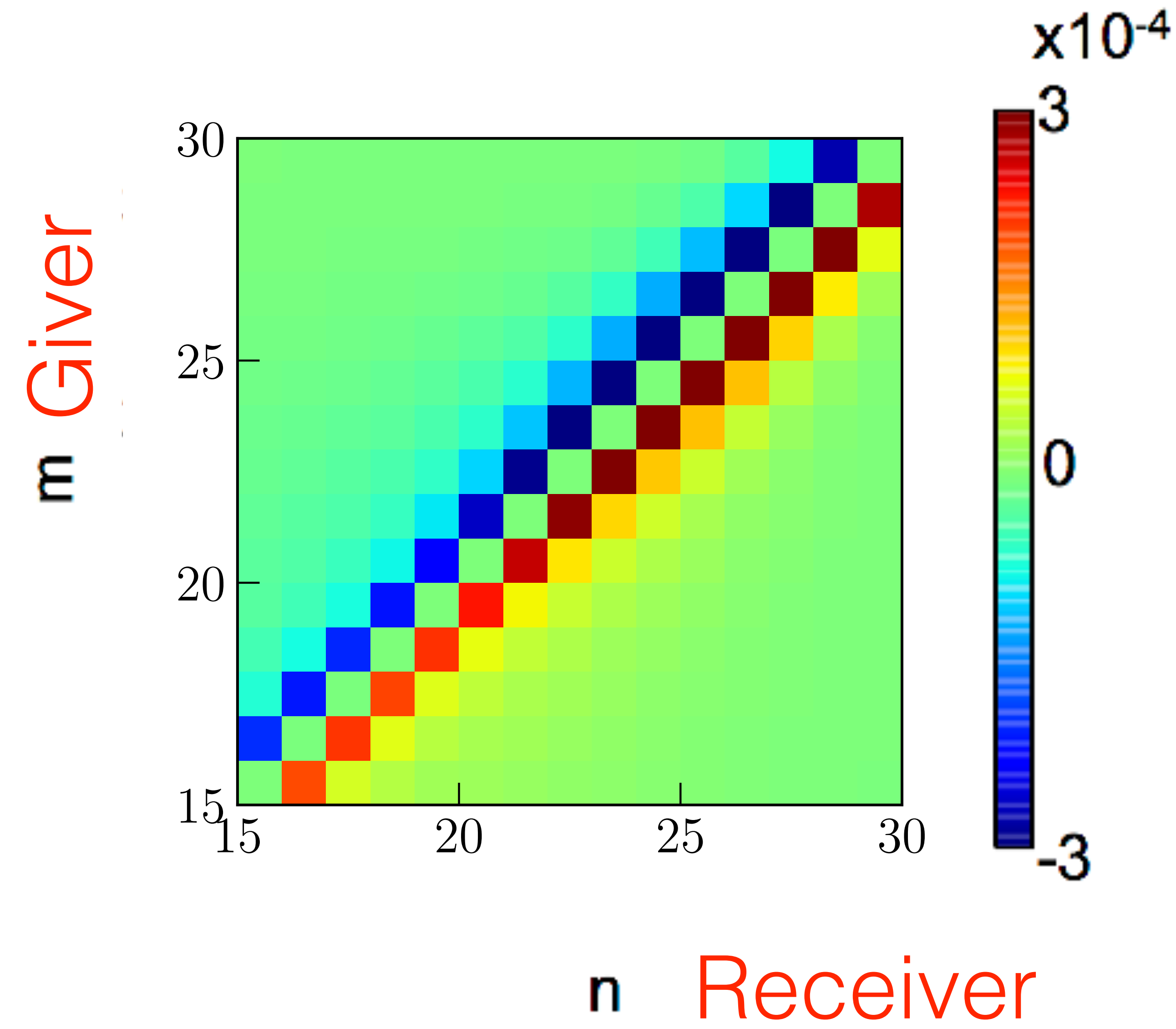
Shell-to-Shell ET







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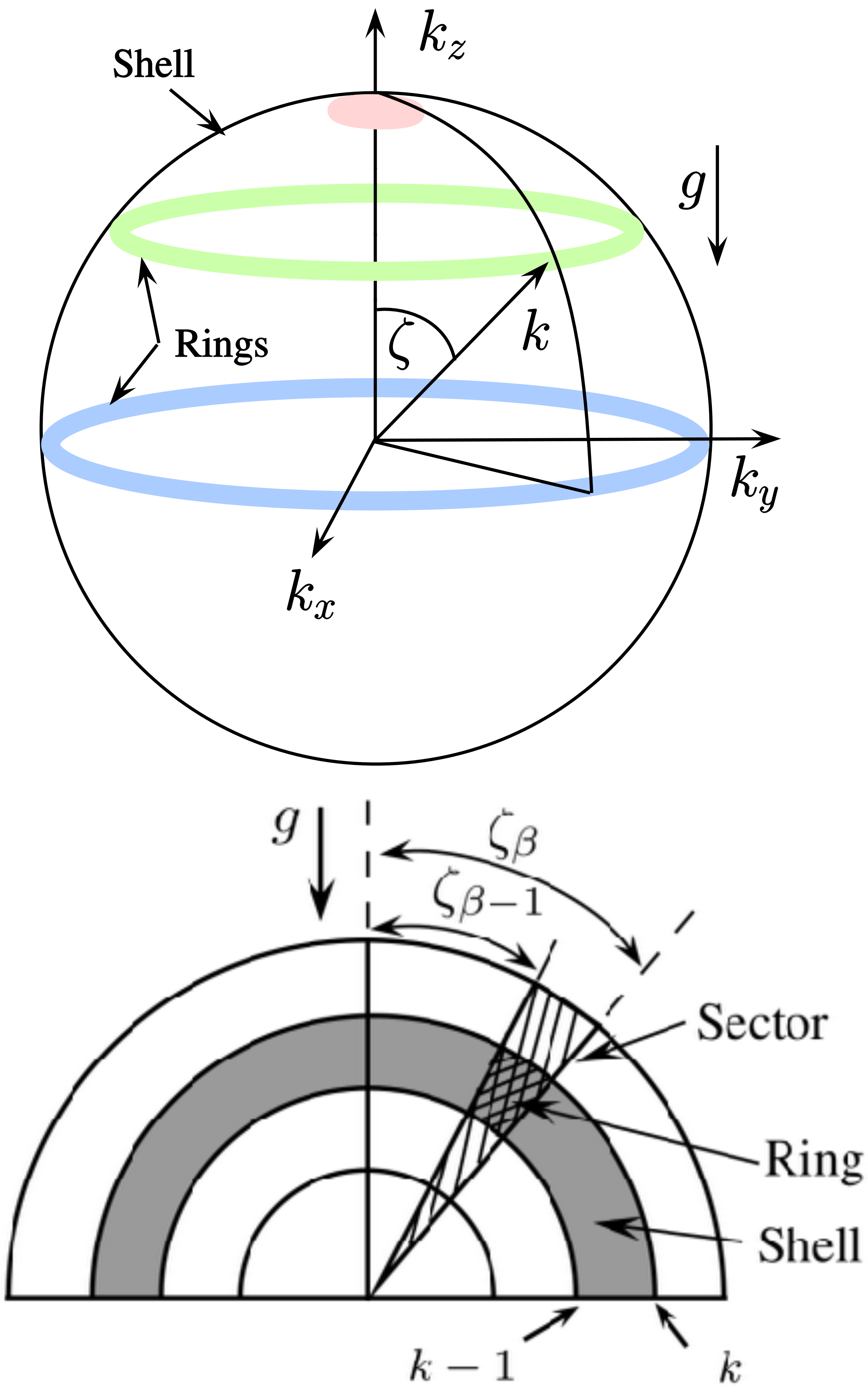

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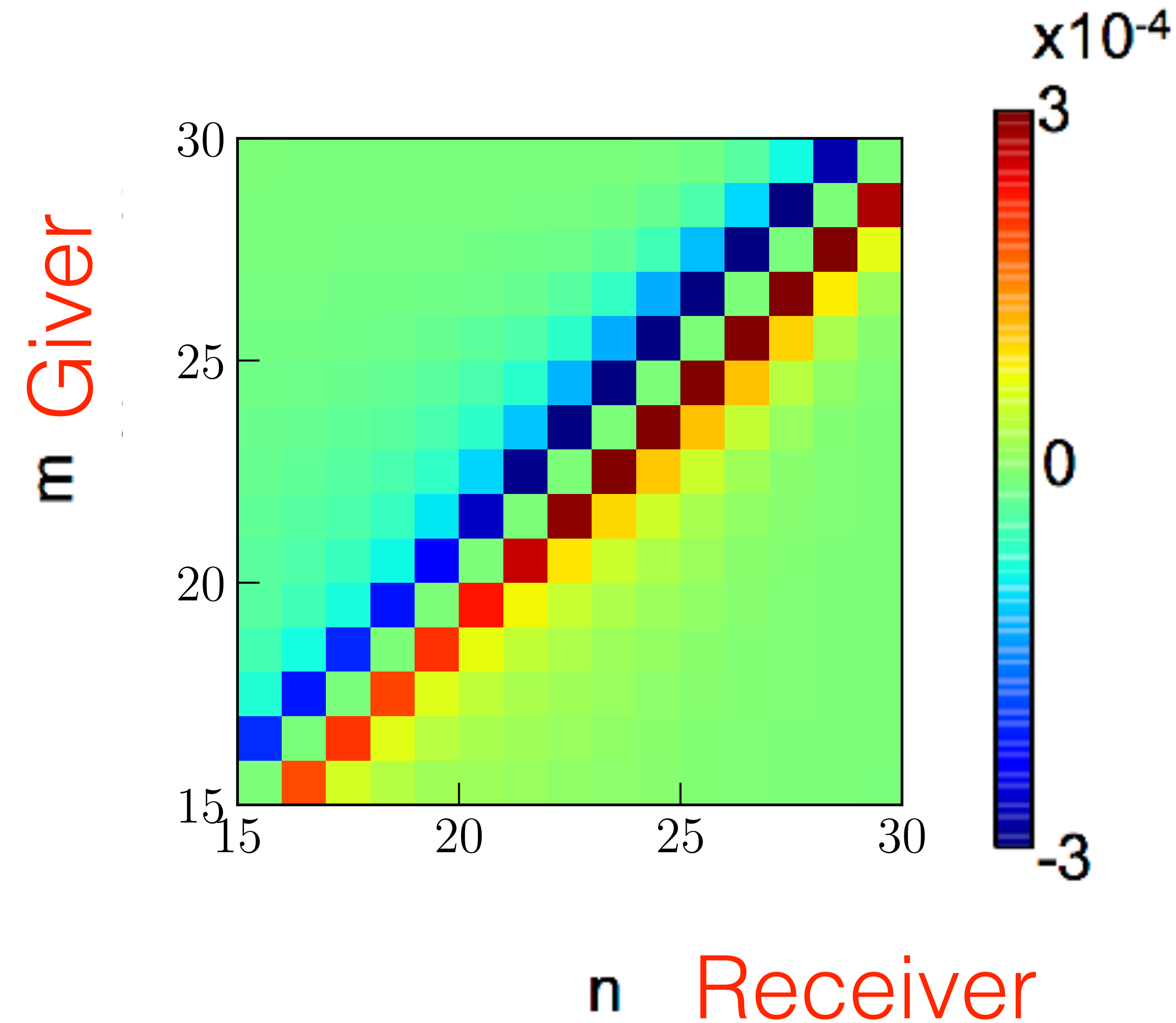

Forward & local energy transfer

Conclusions

Conclusions

Stably Stratified Turbulence

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Stably Stratified Turbulence

- BO scaling for $Fr \sim 1$.

Conclusions

Stably Stratified Turbulence

- BO scaling for Fr ~ 1.
- KO scaling for Fr >> 1.

Conclusions

Stably Stratified Turbulence

- BO scaling for $Fr \sim 1$.
- KO scaling for $Fr \gg 1$.

RBC

Conclusions

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- BO scaling for Fr ~ 1.
- KO scaling for Fr >> 1.

RBC

- KO scaling in three dimension.

Conclusions

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Kumar and Verma, Phys. Rev. E **91**, 043014 (2015)
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Recent Outcome

J. K. Bhattacharjee, *Kolmogorov argument for the scaling of the energy spectrum in a stratified fluid*. Phys. Lett. A (2015)

D. Rosenberg *et al.*, *Evidence for Bolgiano-Obukhov scaling in rotating stratified turbulence using high-resolution direct numerical simulations.* Phys. Fluids (2015)

S. S. Pawar and J. H. Arakeri, *Kinetic energy and scalar spectra in high Rayleigh number axially homogeneous buoyancy driven turbulence*. Phys. Fluids (2016)

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PAPER

Phenomenology of buoyancy-driven turbulence: recent results

Mahendra K Verma, Abhishek Kumar and Ambrish Pandey

Department of Physics, Indian Institute of Technology Kanpur, Kanpur 208016, India

E-mail: mkv@iitk.ac.in

Keywords: buoyancy-driven flows, Rayleigh–Benard convection, stably-stratified turbulence, thermal convection, thermally-driven turbulence

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Thank You