Vortices driving droplets driving buoyancy driving vortices

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Work of S Ravichandran aka Croor Singh

tifr

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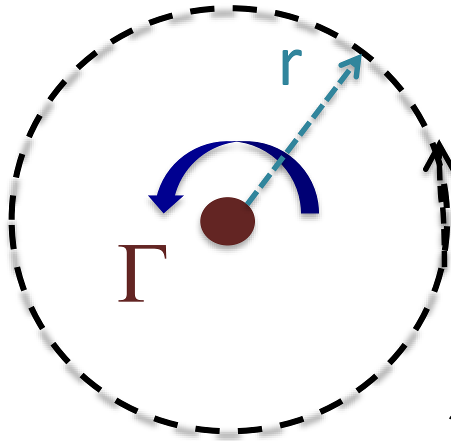
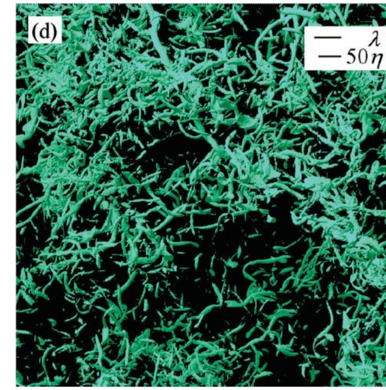
Harish Dixit (IIT Hyderabad)

19 June 2017
Buoyancy-Driven Flows

$$\Gamma = \frac{1}{2\pi}\iint \omega \cdot dA$$

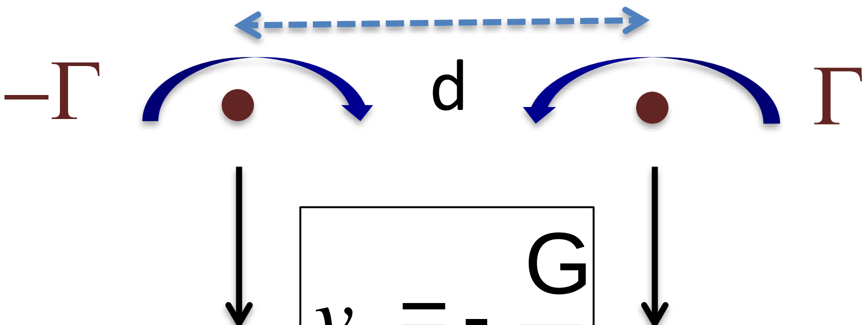


$$u_\theta = \frac{\Gamma}{r}$$



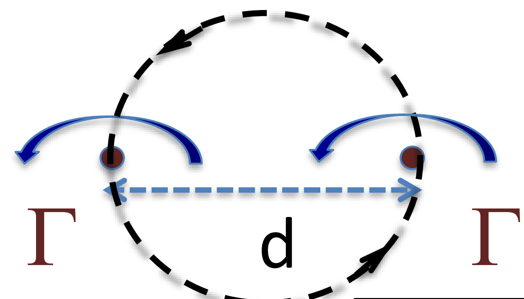
Okamoto et al, 2007

Counter-rotating vortex pair



$$v_z = -\frac{\Gamma}{d}$$

Co-rotating vortex pair

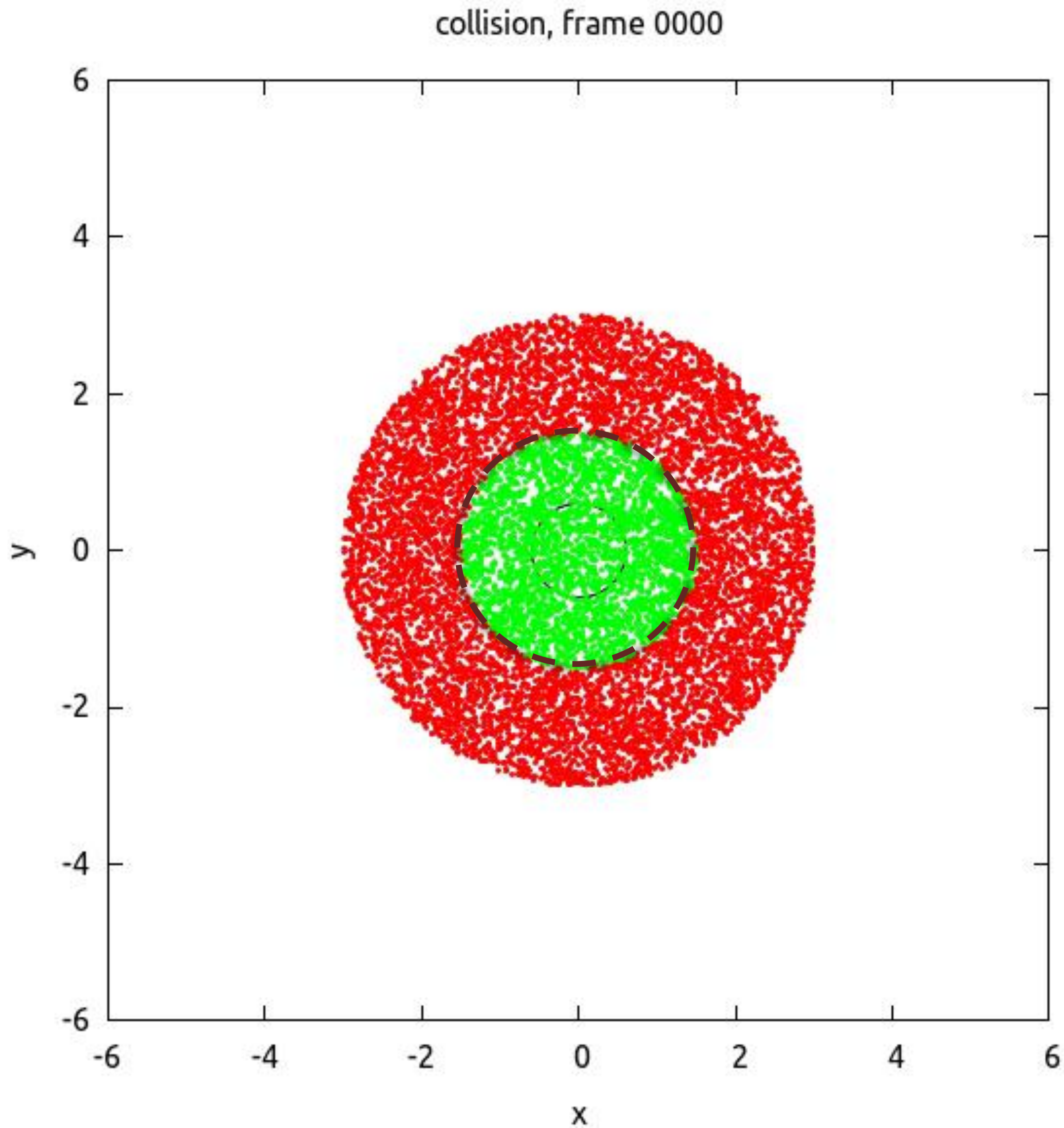


$$v_\theta = \frac{\Gamma}{d}$$

Two regions of different particle dynamics near a vortex

Particles/ droplets with inertia

$$R_v = 0.6\sqrt{\Gamma t}$$



Deepu, Ravichandran & RG, Phys. Rev. Fluids 2017

Small, heavy inertial particles

Maxey & Riley, 1983, Phys. Fluids

$$\dot{x} = v$$

$$\dot{v} = -\frac{1}{\tau}\left(v - u\right)$$

$$\tau = \frac{2}{9}\frac{a^2}{\nu}\frac{\rho_p}{\rho_f}$$

For particles near a vortex

$$L = \left(t\mathrm{G}\right)^{1/2}$$

$$\ddot{r} + b(t)\dot{r} = \frac{\zeta^2}{r^3}$$

$$\dot{\zeta} + b(t)\zeta = b(t)$$

Boundary layer structure

$r >> 1, t >> 1$

$R = dr, T = et$

$e = d^4$

Inner solution

$r << 1, t << 1$

$R = \frac{r}{d}, T = \frac{t}{e}$

$e = d^2$ or $e = d$

r_c » 0.5

Outer solution

In the inner layer

$$\nabla \cdot v = \frac{1}{r_i \sqrt{r^2 - r_i^2}}$$

LARGE AND POSITIVE

Ballistic, vicinity evacuated quickly

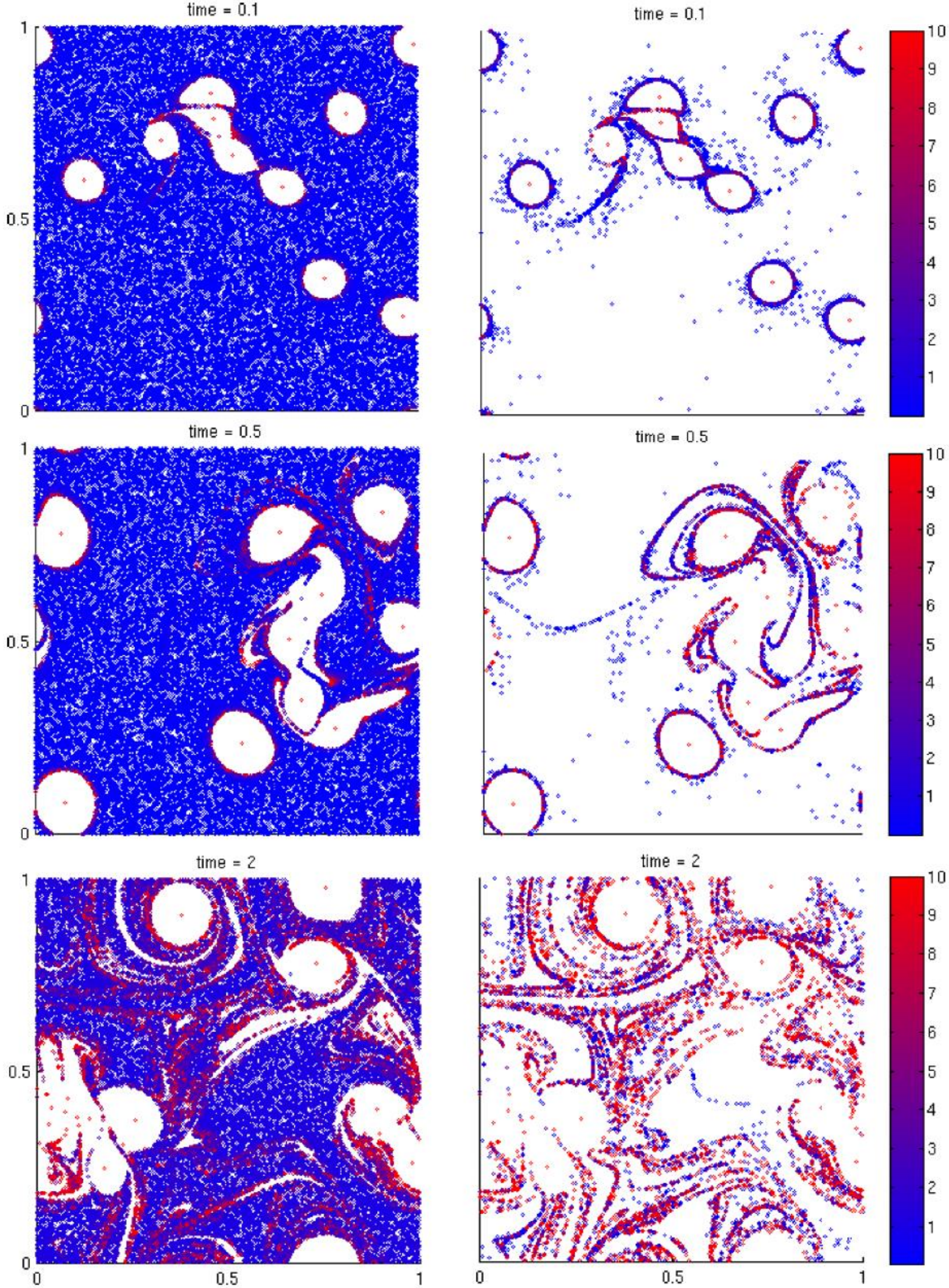
In the outer layer

$$\nabla \cdot v = -\frac{1}{r_0^2}$$

SMALL AND NEGATIVE

Slow centrifuging, nothing much happens

Excluding inner region

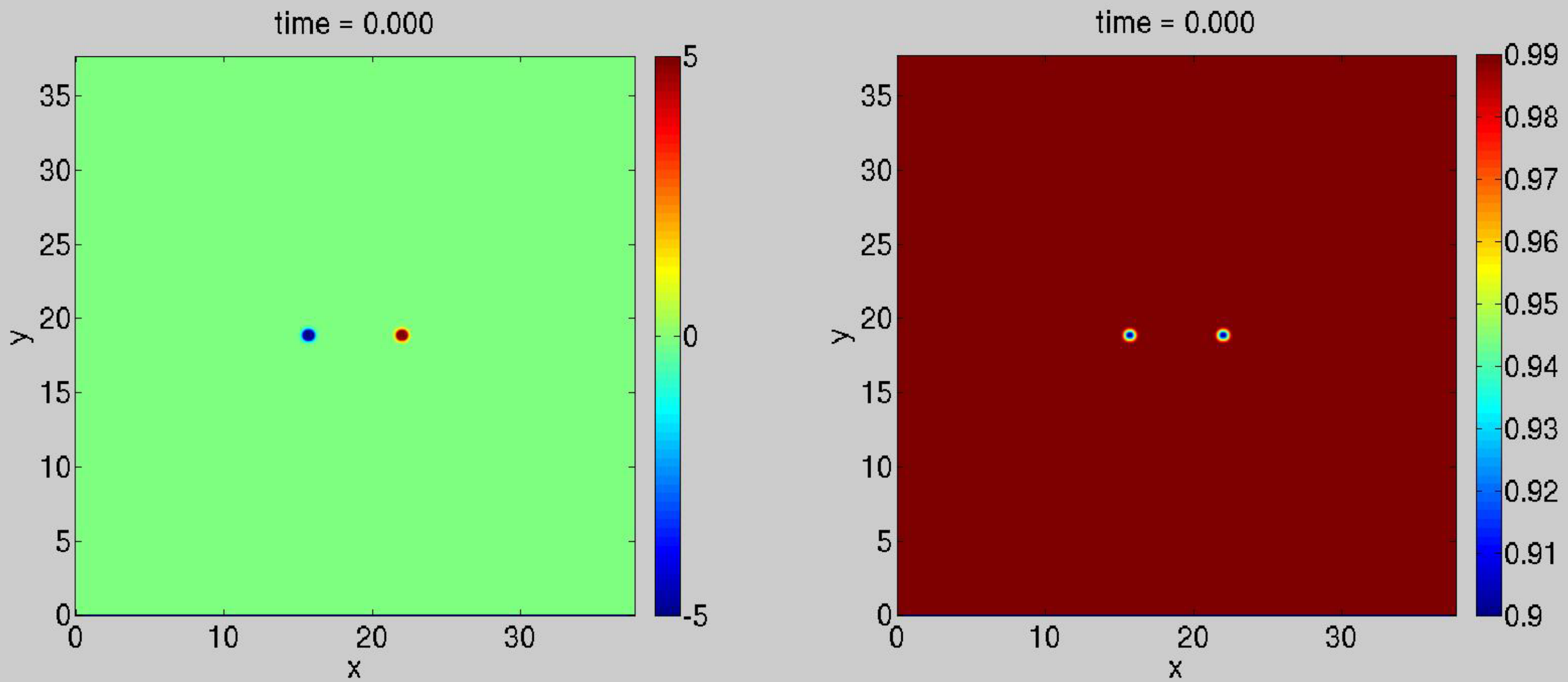


Started life in inner region

Ravichandran & RG
Phys. Fluids 2015

Dynamics of light or heavy vortices

Re=5000, no buoyancy

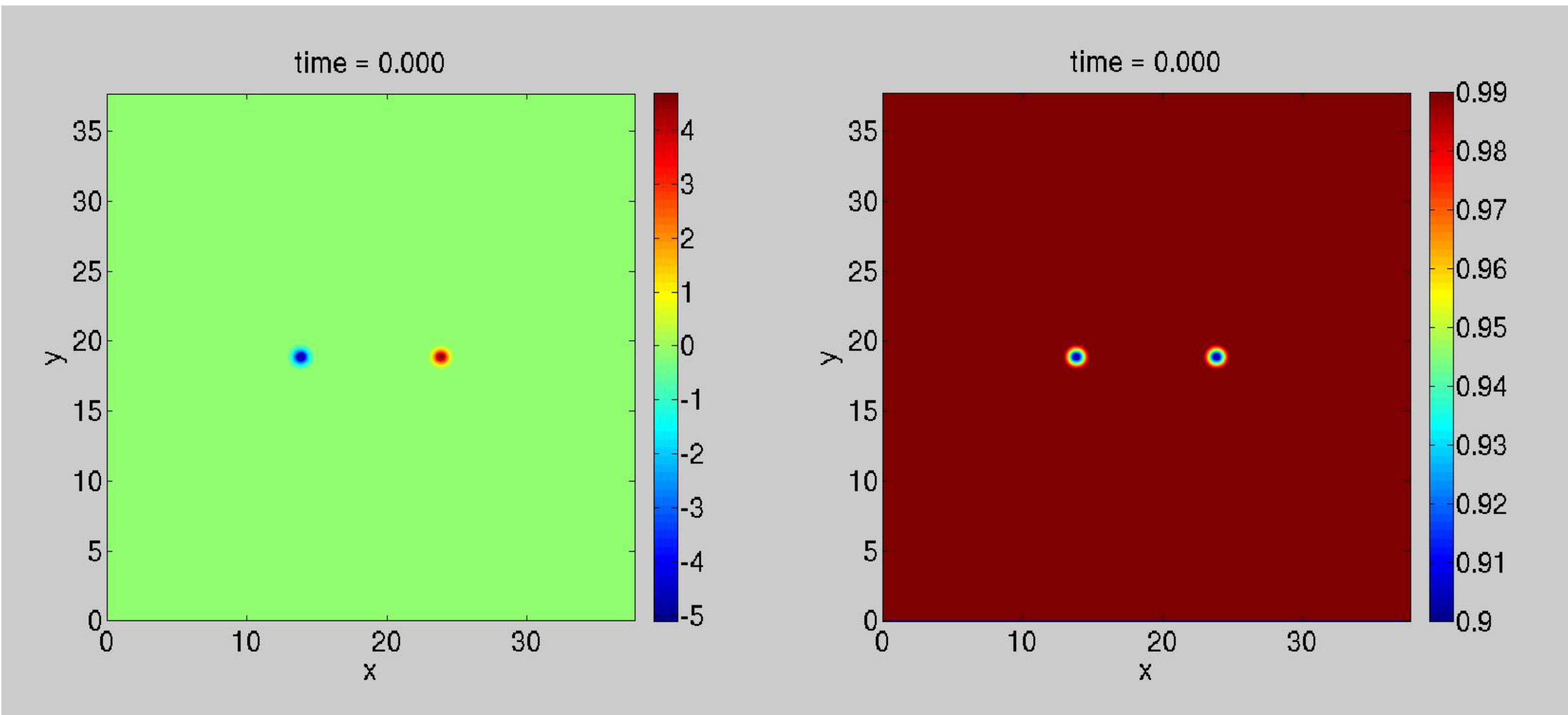


$$\mathrm{Re} = \frac{\Omega_0 d_0^2}{\nu}, \quad \mathrm{Fr}^2 = \frac{\Omega_0^2 d_0}{g A}$$

Re=5000, Fr=100

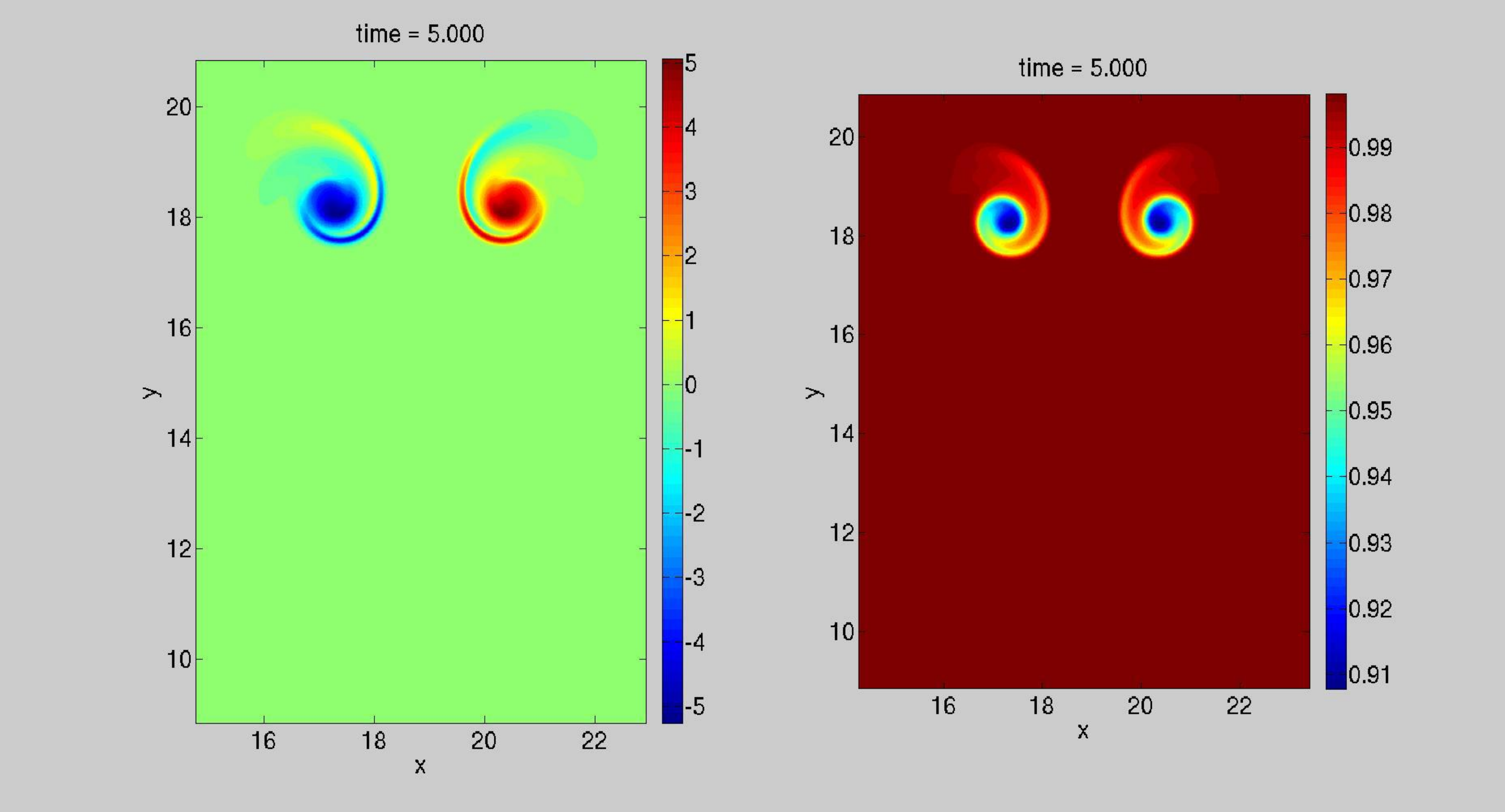
Vorticity

Density



Ravichandran, Dixit & RG Phys. Rev. Fluids 2017

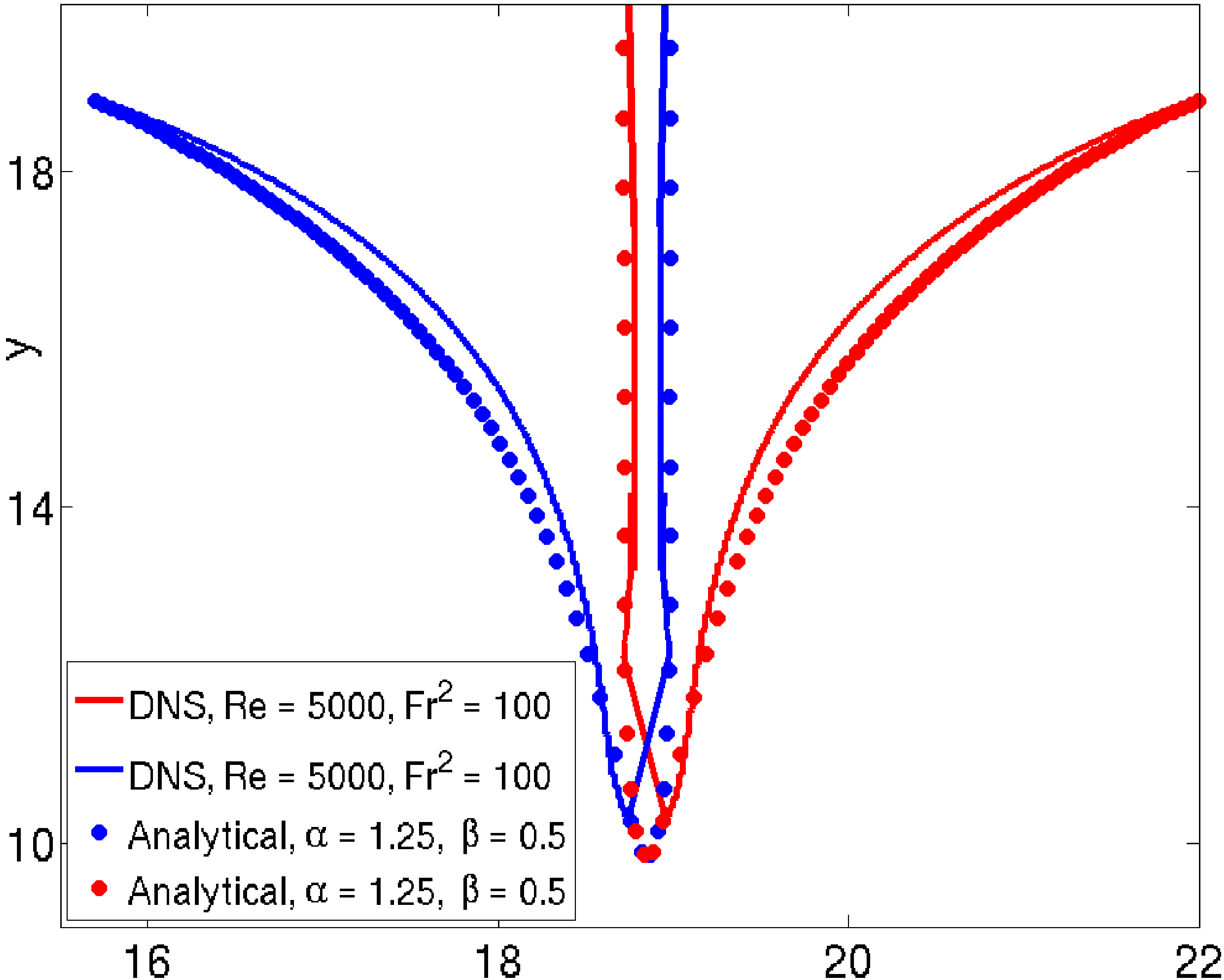
Slomo, close-up



“Theory”

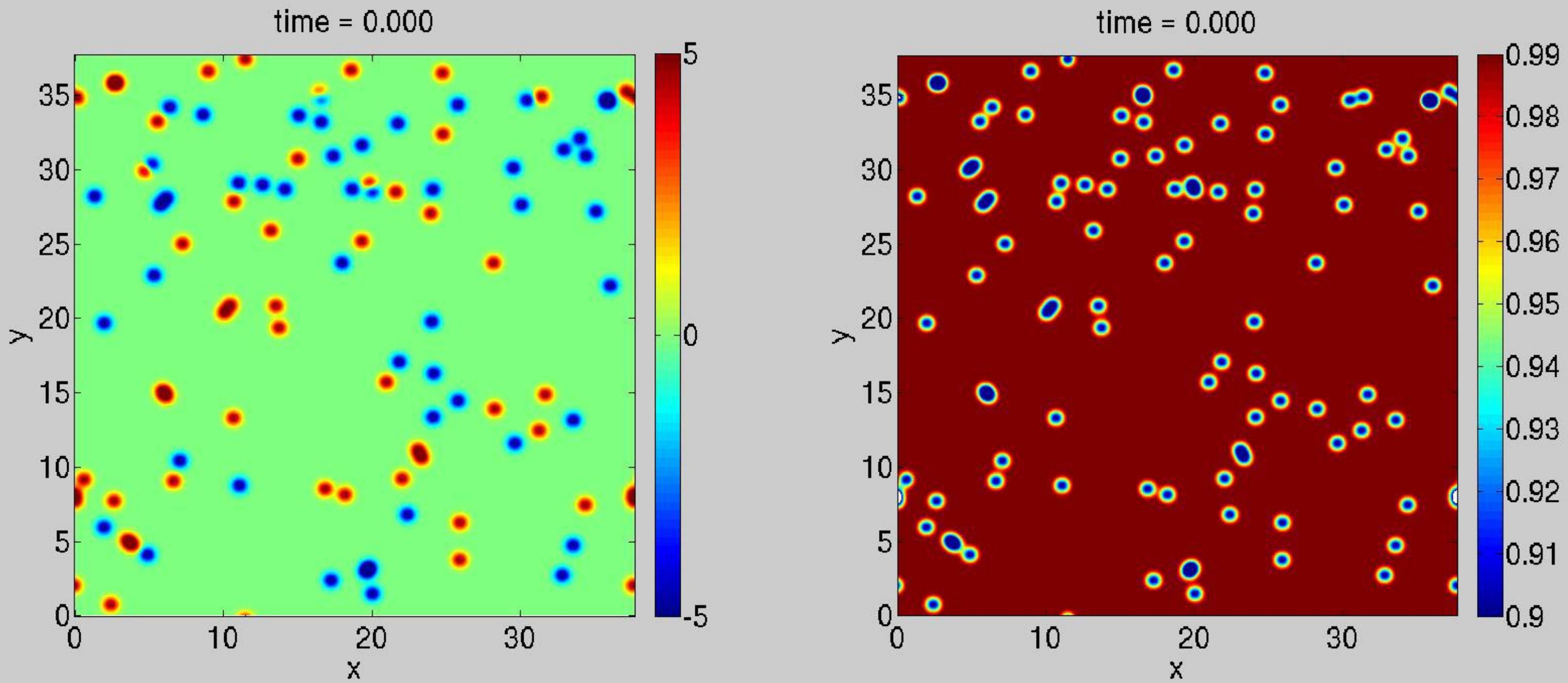
Relative velocity of a spinning ball

$$\frac{dv}{dt} = \frac{1}{Fr^2}\hat{j} + a\left[\hat{k} \times v\right] - b|v|v$$

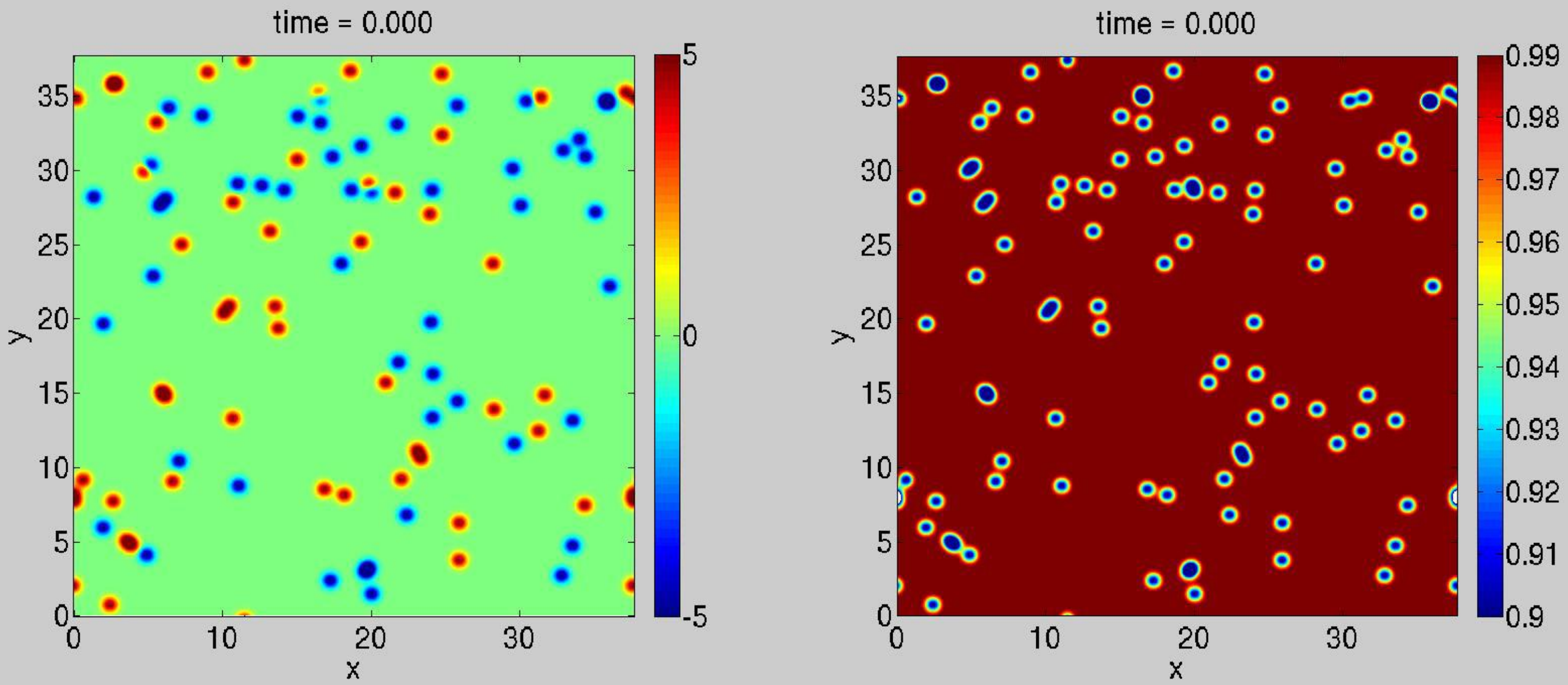


18
14
10
y
16
18
20
22
DNS, Re = 5000, Fr^2 = 100
DNS, Re = 5000, Fr^2 = 100
Analytical, α = 1.25, β = 0.5
Analytical, α = 1.25, β = 0.5

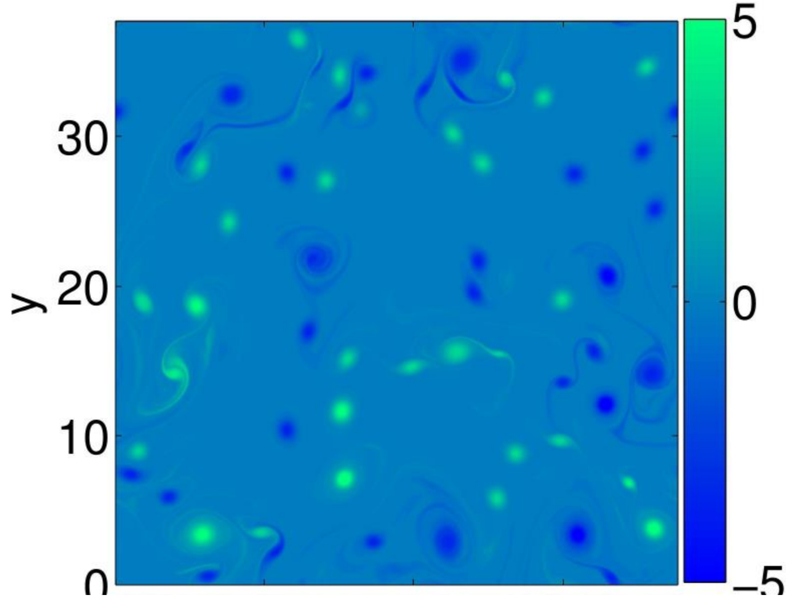
No gravity

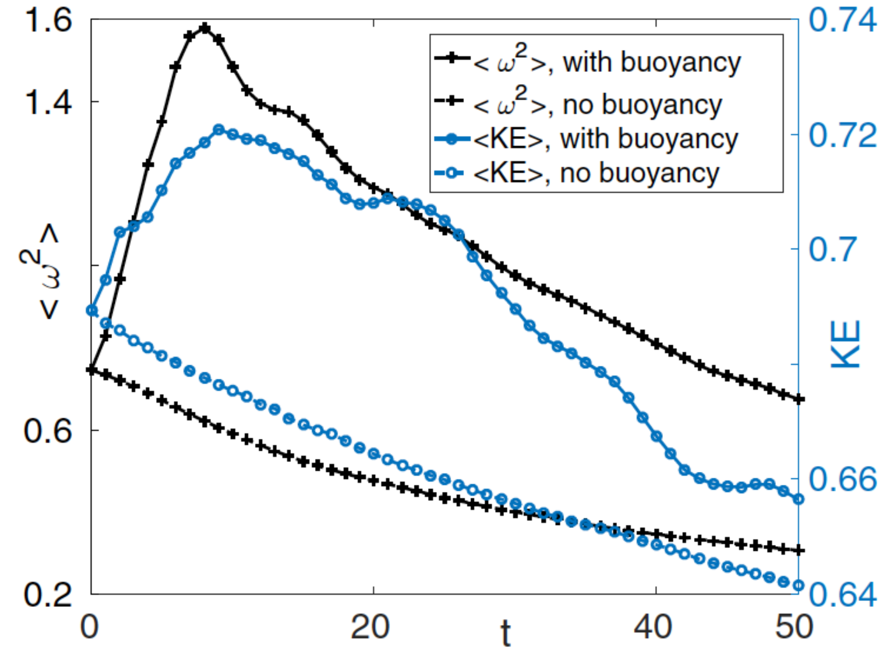

time = 0.000
y
x
0
10
20
30
35
30
25
20
15
10
5
0
5
0
-5
time = 0.000
y
x
0.99
0.98
0.97
0.96
0.95
0.94
0.93
0.92
0.91
0.9

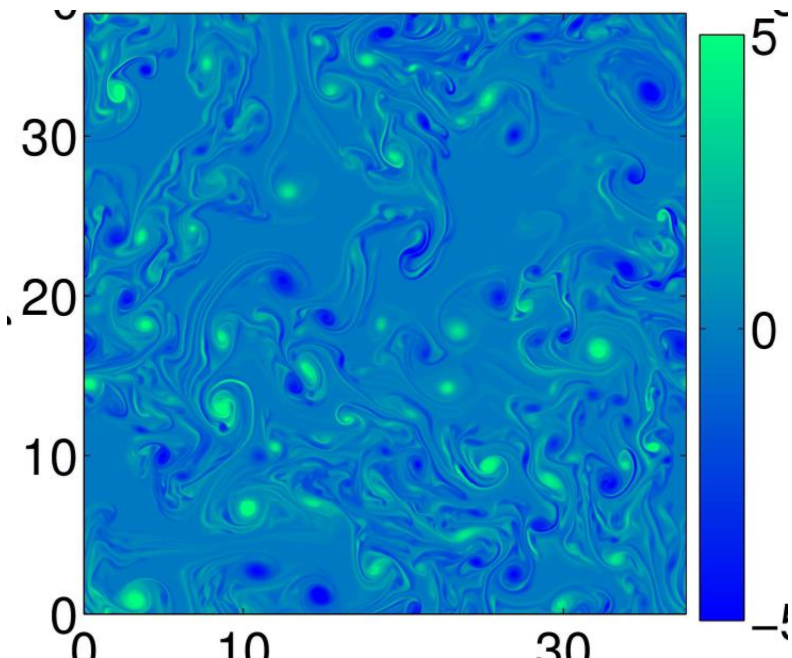
Heavy vortices

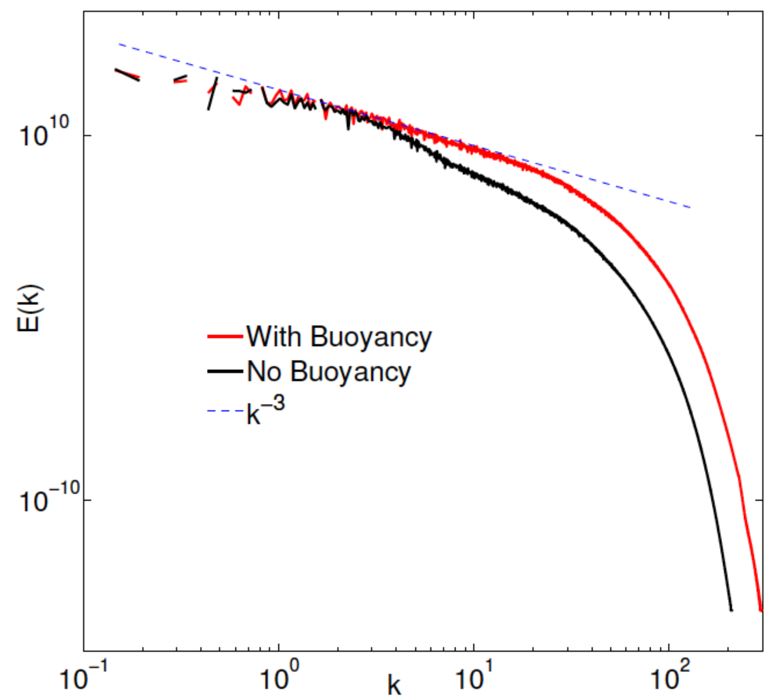


No buoyancy

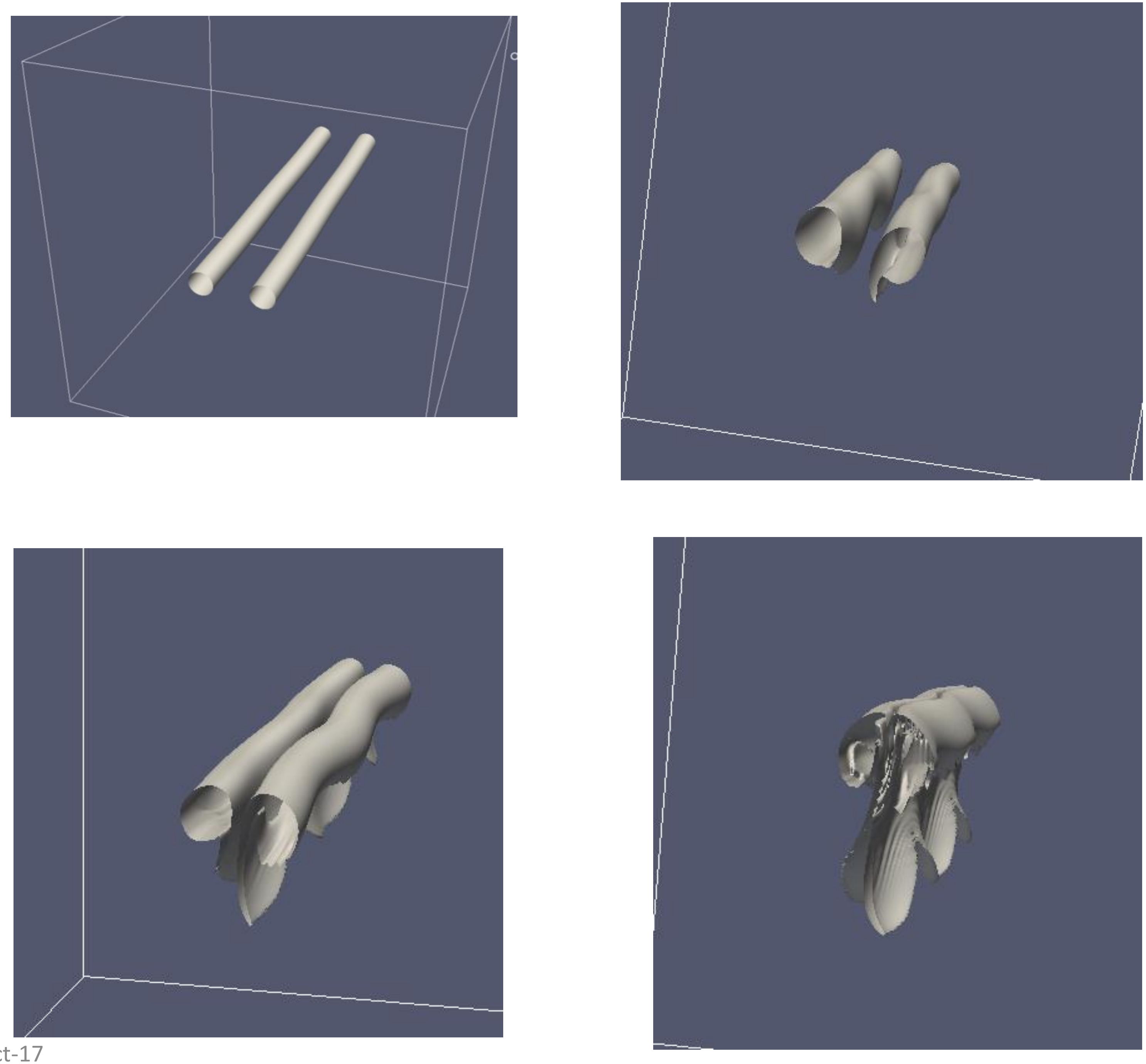








With buoyancy



In 3D

Inertial droplets with phase change

Evolution with phase change

$$\frac{Dw}{Dt} = \frac{1}{\mathrm{Re}}\nabla^2 w + \frac{1}{Fr^2}\nabla \times \left[q\hat{e}_y \right]$$

$$\frac{Dq}{Dt} = \frac{1}{Pe_q}\nabla^2 q + H\left[a\left(\frac{r_v - r_s}{r_s St_s} \right) \right]$$

$$\frac{Dr_v}{Dt} = \frac{1}{Pe_v}\nabla^2 r_v - H\left[\left(\frac{r_v - r_s}{r_s St_s} \right) \right]$$

$$\frac{\partial r_v}{\partial t} + \nabla \cdot \left(v r_l \right) = \frac{1}{Pe_l}\nabla^2 r_l + H\left[\left(\frac{r_v - r_s}{r_s St_s} \right) \right]$$

$$v = u - St_p \frac{Du}{Dt}$$

$$H = 1 \text{ if } r_v > r_s \text{ or if } r_v < r_s, r_l > 0$$

$$\mathrm{Re} = \frac{W_0 d_0^2}{n}$$

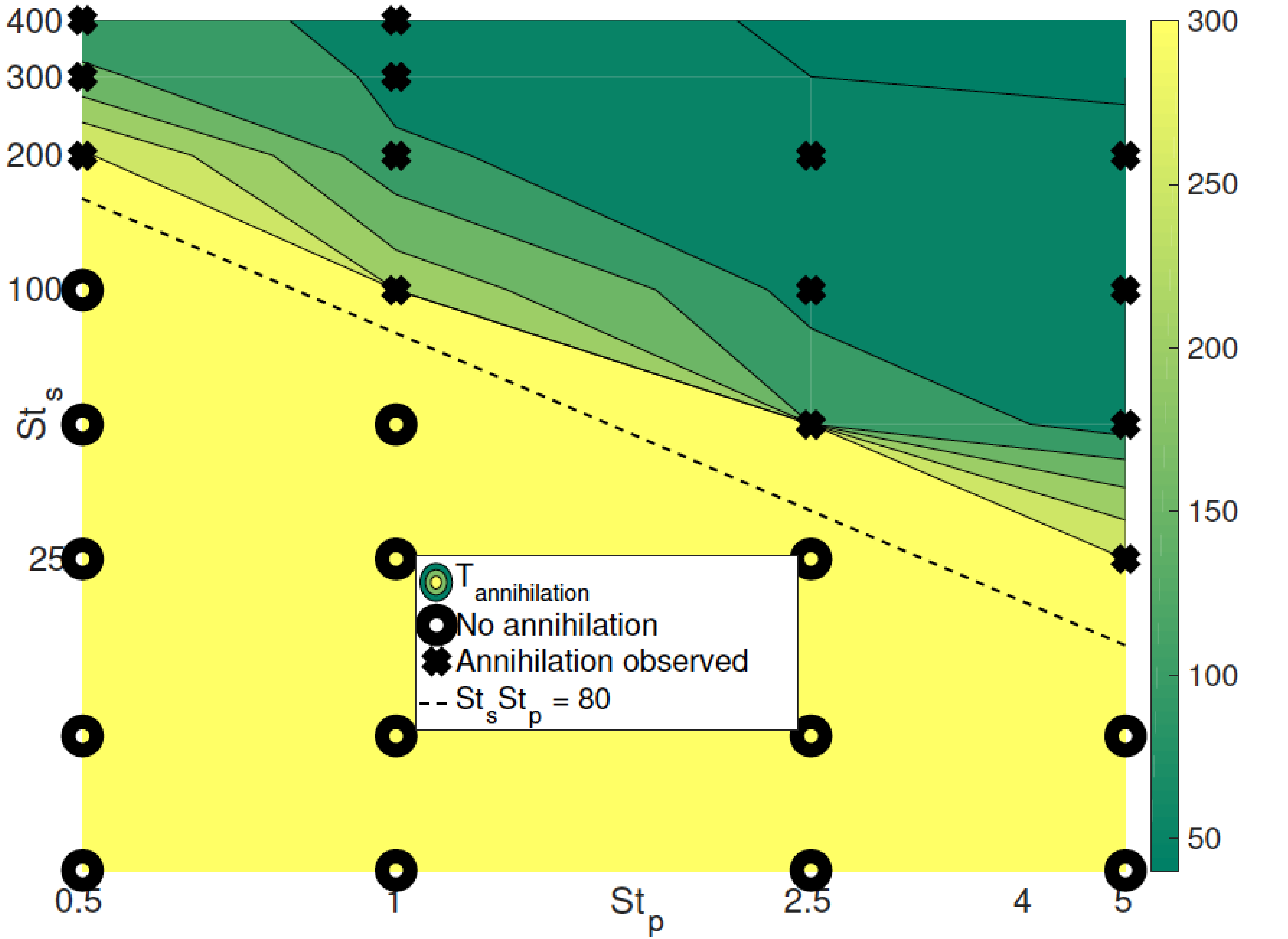
$$Fr^2 = \frac{W_0^2 d_0}{gAt}$$

$$a = \frac{L_v r_s^0}{C_p DT}$$

$$St_p = W_0 t_p$$

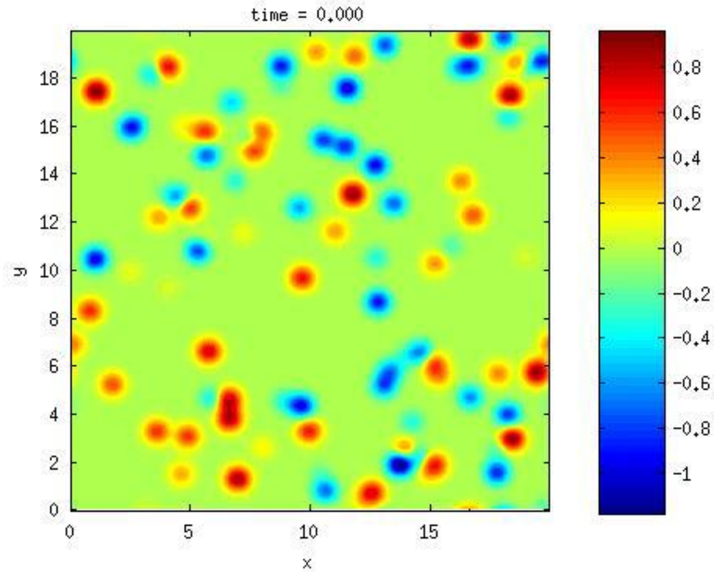
$$St_s = W_0 t_s = W_0 \frac{C r_s^0}{4pan}$$

$$At_{eff} \text{ » } St_p^{1/2} St_s^{1/2}$$

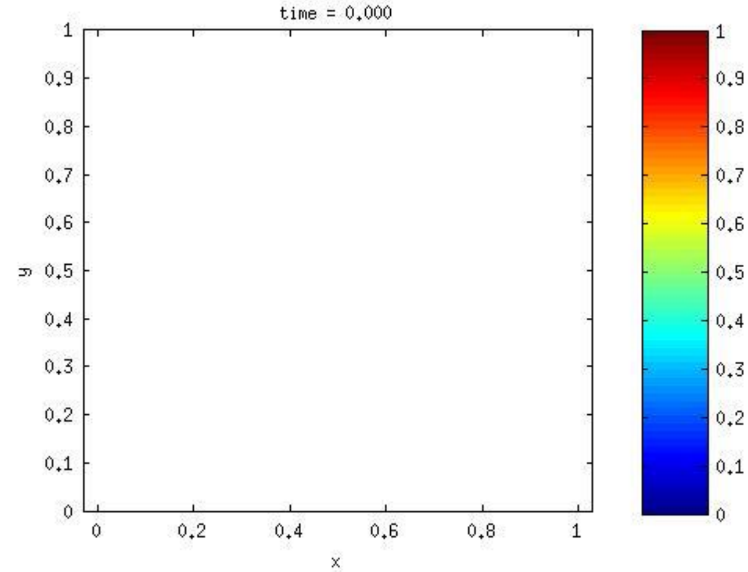


400
300
200
100
25
St_s
0.5
1
2.5
4
5
St_p
300
250
200
150
100
50
$T_{annihilation}$
No annihilation
Annihilation observed
$St_s St_p = 80$

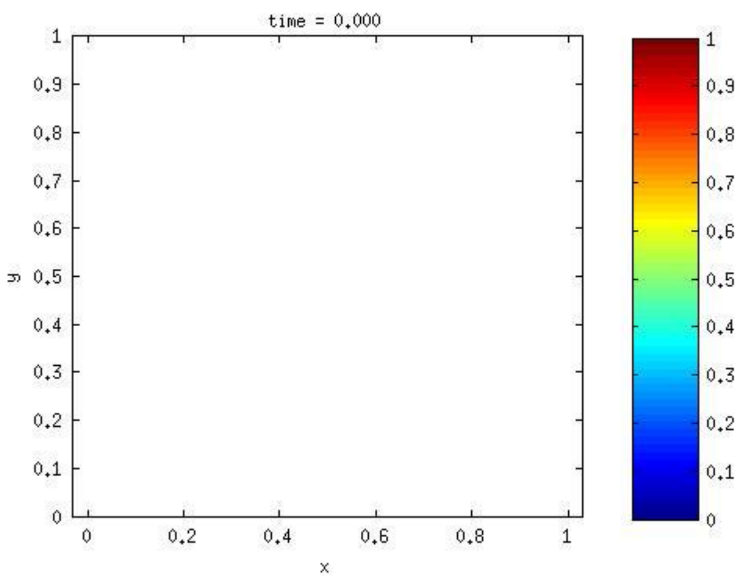
Vorticity



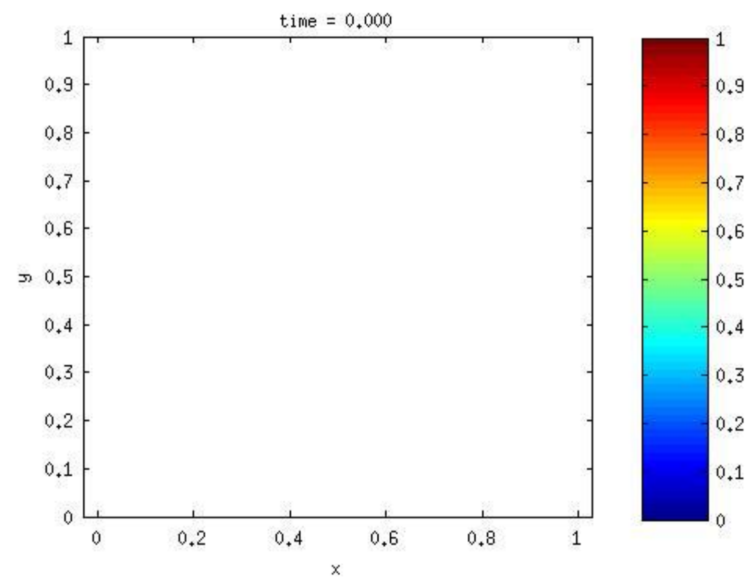
Temperature



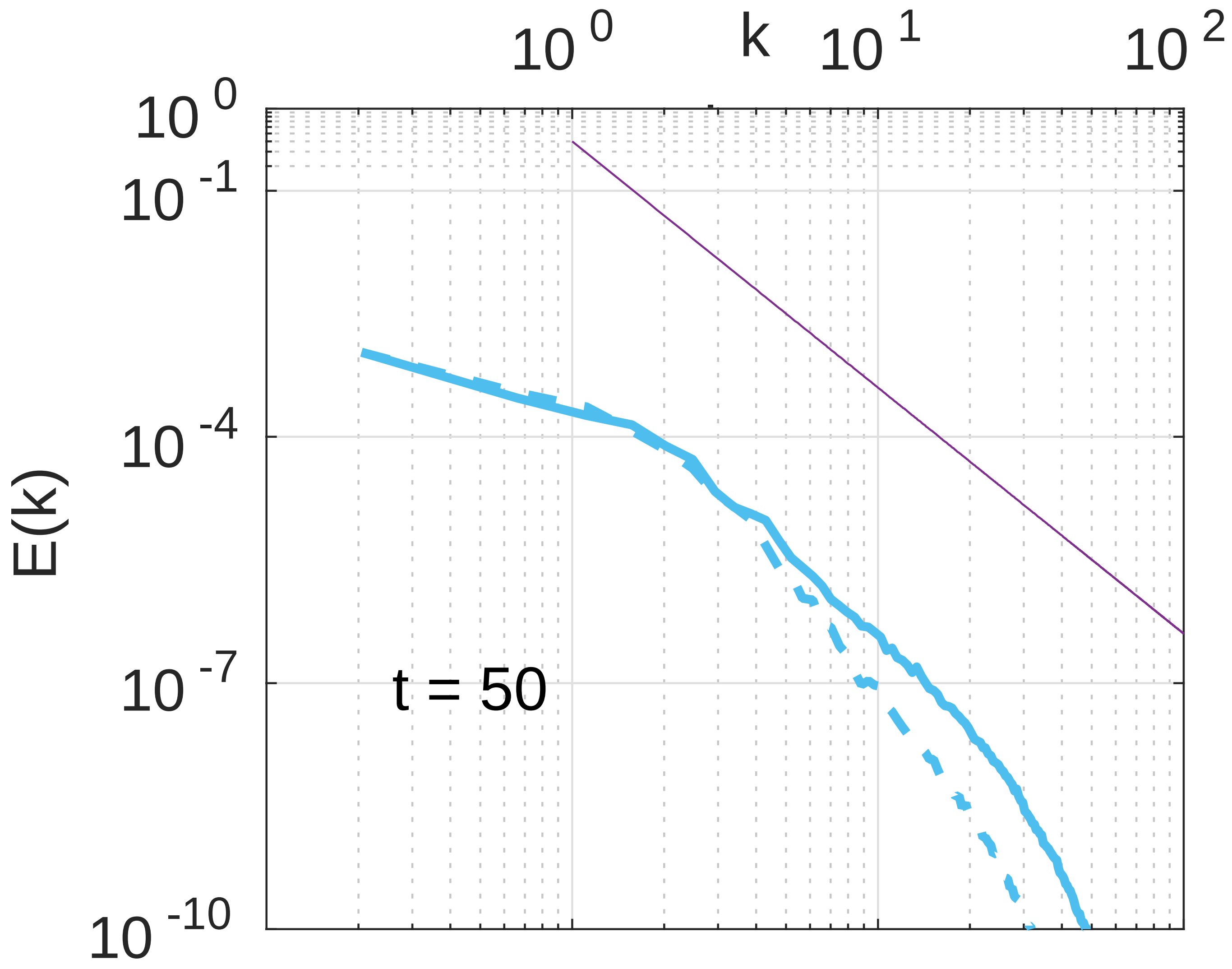
Supersaturation



Droplet number density



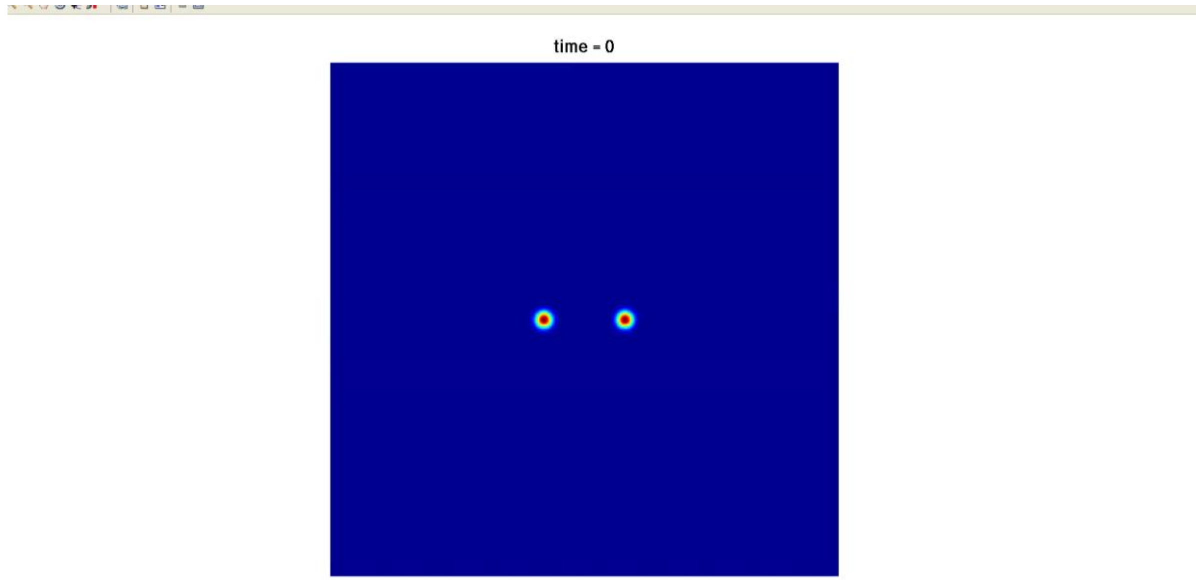
Many vortices with inertia and phase change

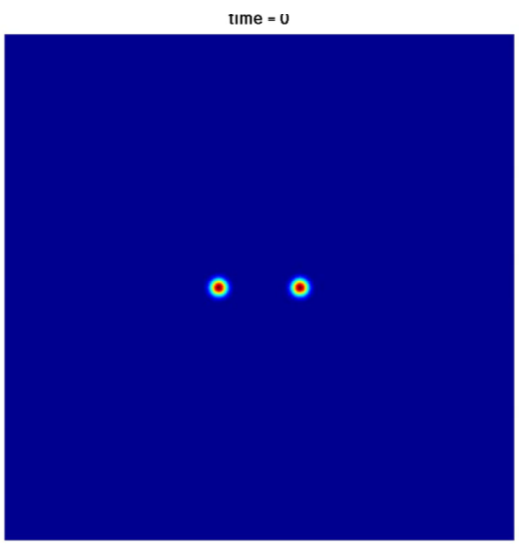


To conclude

Adapted from Roddam Narasimha's concept

Vorticity → Droplets → Phase change → Latent heat release → Buoyancy → Vorticity





Froude=1, Re=5000,
linear stratification, Boussinesq

Dixit & RG Phys. Fluids 2013