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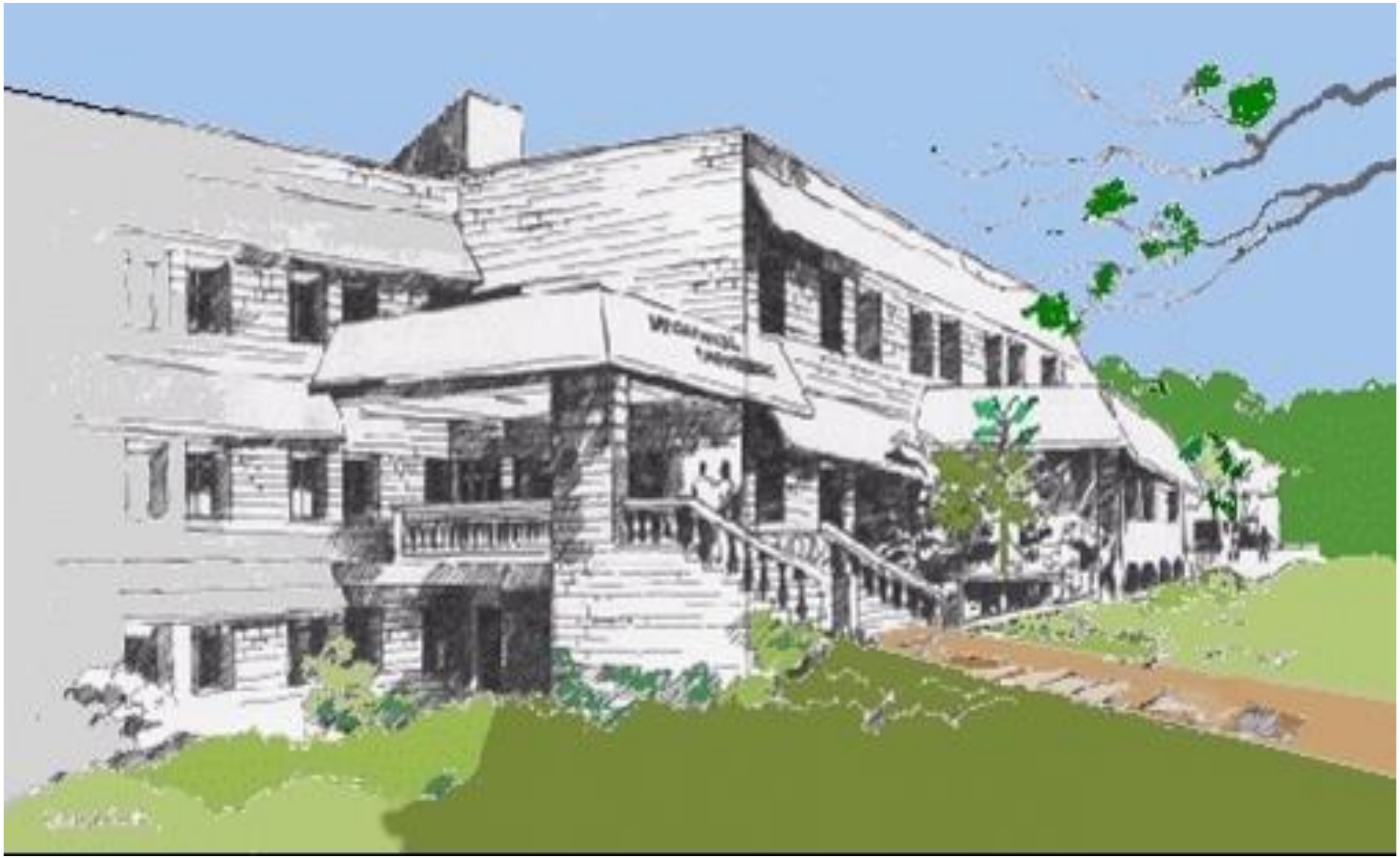

Rayleigh-Benard convection and axially homogeneous convection in vertical tube: a comparison

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   - Nu ~ Ra^1/3; flux independent of length scale
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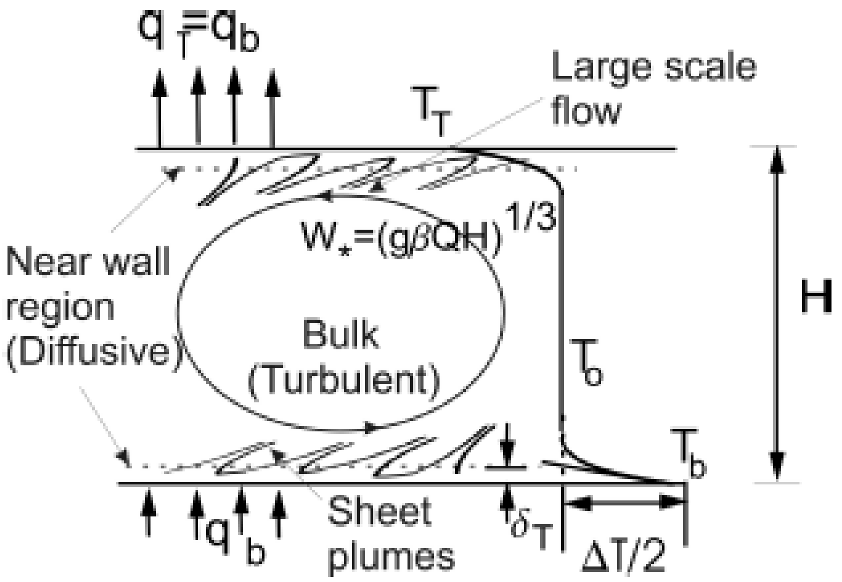
   - free convection analog of fully developed pressure driven pipe flow
   - driven by linear density grad, instead of linear pressure grad.
   - can achieve 'ultimate' regime; Nu ~ (RaPr)^1/2; flux independent of viscosity
   - 2 regimes
3) Moist TC
   - droplet formation and growth in presence of turbulence.
4) Issues of spectra
   - velocity and scalar
   - how does buoyancy affect scaling
5) Light propagation through turbulence

Rayleigh Benard Convection

- Natural convection: Buoyancy is the sole source of motion.



Steady, turbulent Rayleigh Benard Convection (RBC)

- Dimensionless parameters:

$$Nu = f(Ra, Pr, AR) \text{ where,}$$

$$Nu = \frac{Q}{k\Delta T/H}, \quad Ra = \frac{g\beta\Delta TH^3}{\nu\alpha},$$

$$Pr = \frac{\nu}{\alpha} \text{ and } AR = \frac{L}{H}$$

- Near-wall length scale

$$Z_w = \frac{\nu\alpha}{g\beta\Delta T_w} \sim \frac{H}{Ra^{1/3}}$$

Line plumes transport heat from the near-wall diffusive region to the turbulent bulk.

Heat flux in turbulent flow

$$\dot{q}'' = -k\frac{\partial \bar{T}}{\partial z} + \rho C_p \overline{w'T'}$$

Molecular
Diffusion

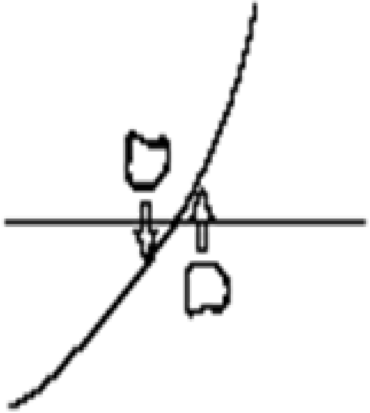
Turbulent
flux

Turbulent kinetic energy

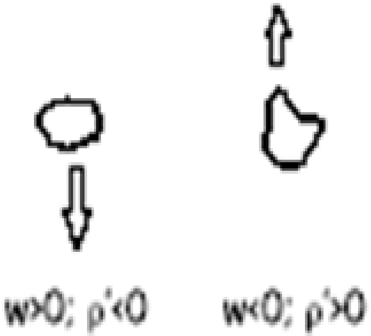
- $k = u'^2 + v'^2 + w'^2$
- $\frac{Dk}{Dt} = \partial_j T_j + P - \varepsilon$ Advection = Transport +Source(*P*) –Sink(ε)
- Production (*P*):

  shear $= \partial_i U_j \, \overline{u'_i u'_j}$

  buoyancy $= -\frac{g}{\rho_0} \overline{w\rho'}$



Shear Production



Buoyancy production

1^{st} type of Nusselt number scaling:
Nu ~ Ra^ 1/3

$$Nu = \frac{q}{q_{conduction}} = \frac{-K\left(\frac{\partial T}{\partial z}\right)}{-K\left(\frac{2T_w}{D}\right)} = \frac{qD}{k(2T_w)}$$

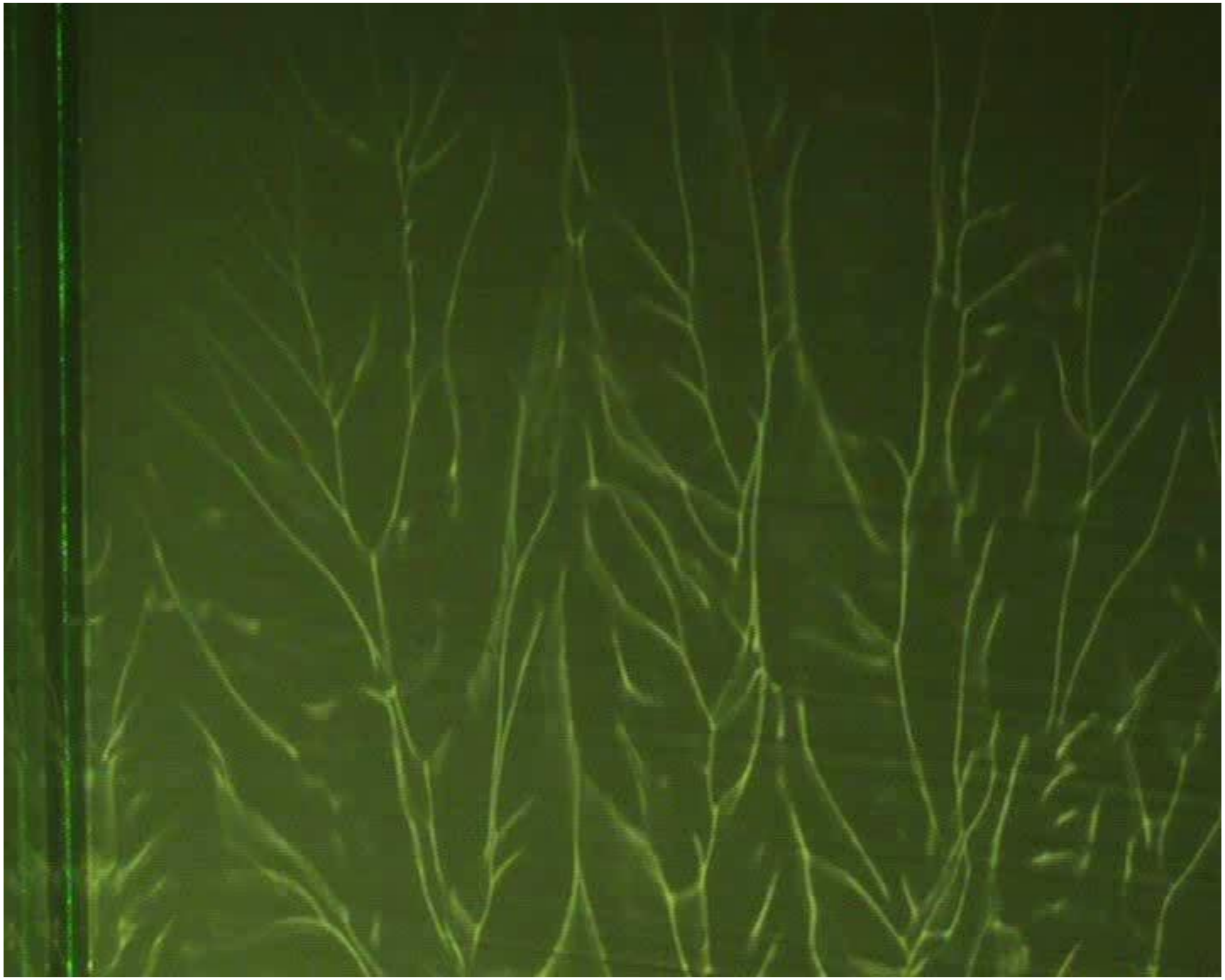
$$Nu = \frac{hd}{k} = c\,Ra^{1/3} = c\,c_1(\Delta T)^{1/3}d$$

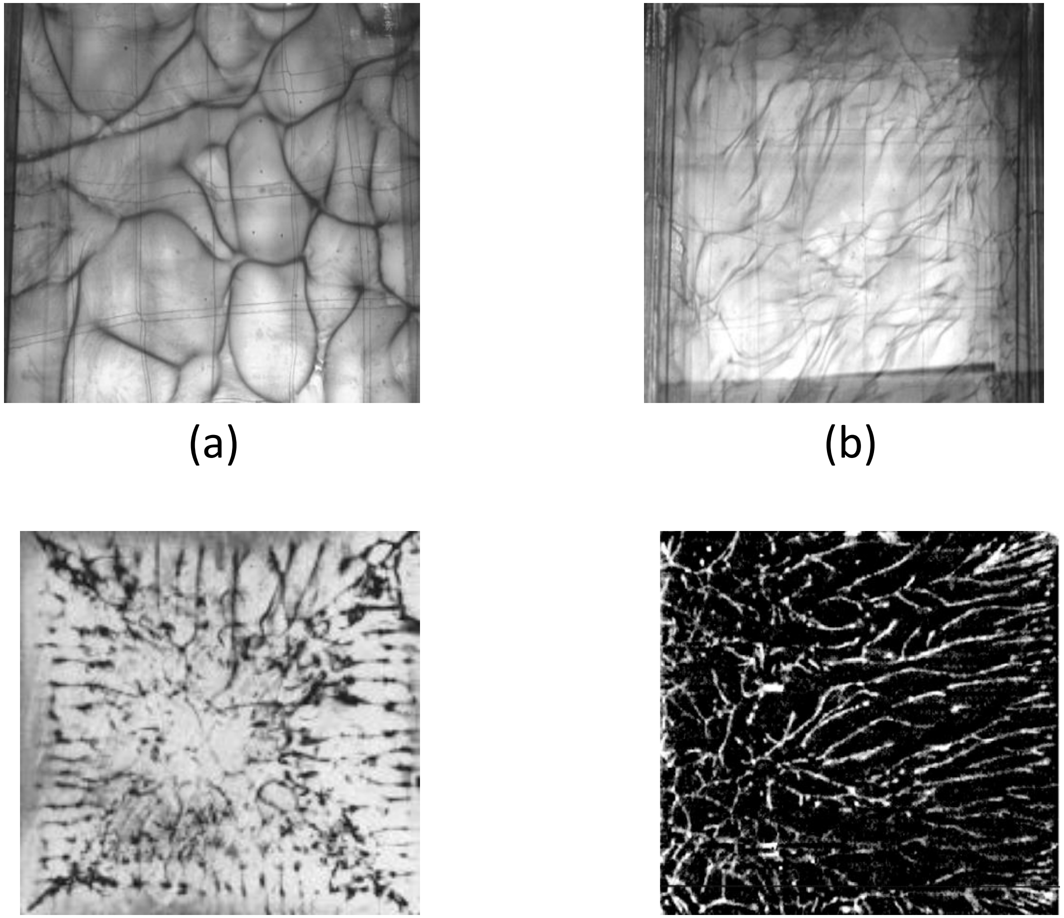
$$h = \ c\,c_1 k(\Delta T)^{1/3}$$

If Nu ~ Ra ^1/3,

heat flux independent of ***length scale***

Near line/sheet plumes in turbulent R-B convection. Equivalent of sublayer vortices in TBL. Not really laminar, but viscous sublayer. (Model for these plumes and near wall scaling: Theerthan & Arakeri, (JFM, 1998), Putheenvital & A (JFM, 2005), Puthneenvital et al (JFM,2011)).



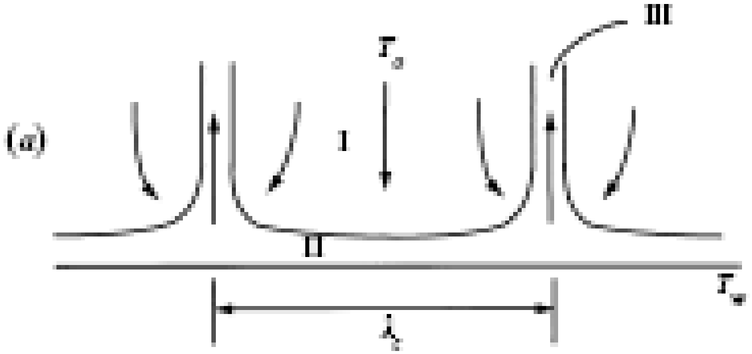


(a) (b)

(c) (d)

(a) R-B convection, AR=6, Ra=4.63×10^5, q=65 W/m^2, ΔT=0.13K. (b) R-B convection, AR=1.5, Ra=1.1×10^9 q=4150 W/m^2, ΔT=4.87K. (c) Convection over 8.9 cm x 8.9 cm, heated horizontal plate, ΔT=27.5K, Ra=5.5×10^8 (From Husar & Sparrow, 1968). (d) High Pr convection, Ra=2.03×10^{11}, AR=0.435 (From Puthenveettil & Arakeri, 2005).

Plumes are result of instability:
Spatial equivalent of Howard’s* model



1) Assume periodic array of line plumes
2) Boundary layer grows (in space)
3) Get plume at distance at which Ra exceeds critical value.

Plume spacing λ given by $Ra_{\lambda} = 50$ for $Pr \cong 1$
$= 90$ for large Pr

* Howard’s model: Intermittent release of thermals; conduction layer grows, erupts into thermal at critical Ra based on conduction layer thickness.

Theerthan & Arakeri, (1998) A model for near-wall dynamics in turbulent Rayleigh Benard convection. JFM 373, 221-254.

Near – wall scales

Townsend- based on flux

$$W_o = (g\beta Q_o \alpha)^{(1/4)}, \qquad \theta_o = \frac{Q_o}{W_o}, \qquad Z_o = \frac{\alpha}{W_o}$$

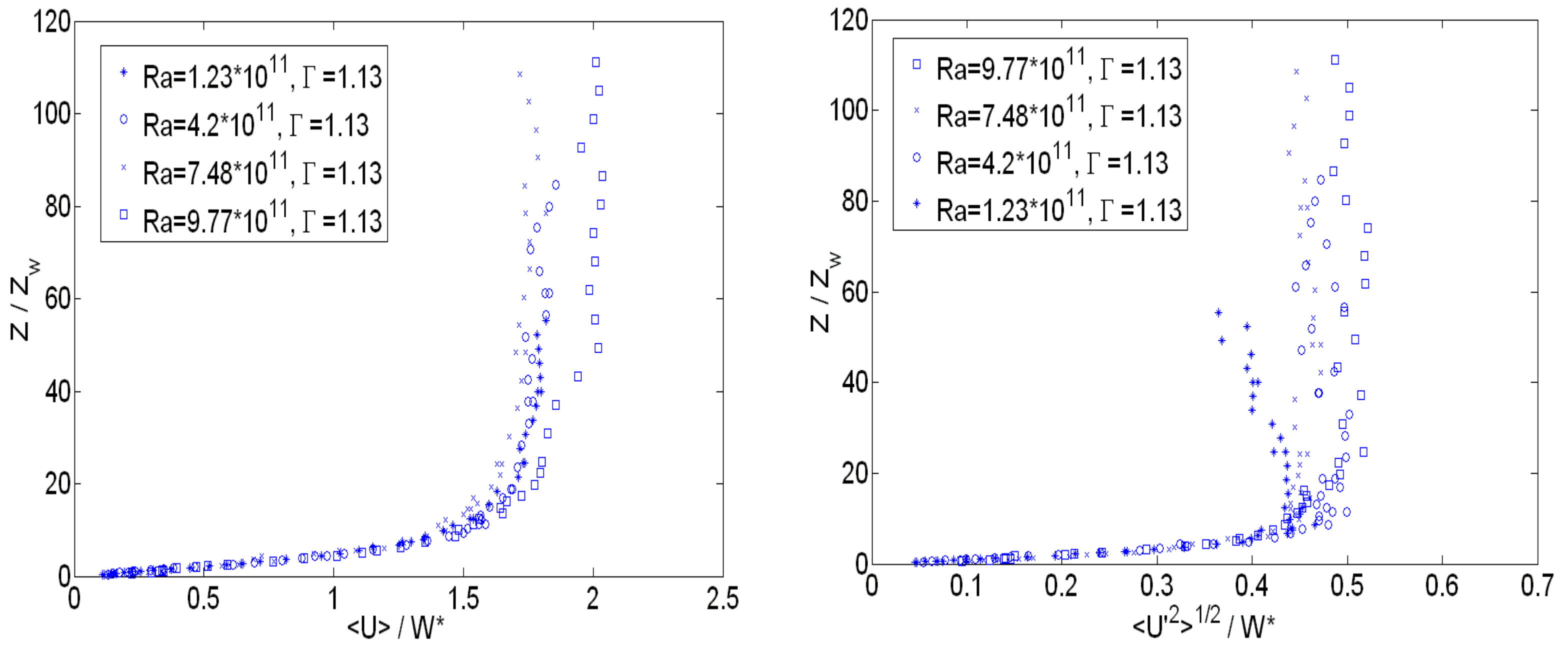
Theerthan&Arakeri – based on temp diff.

$$U_w = (g\beta T_w)^{(1/3)}(\vartheta\alpha)^{(1/6)}, \qquad T_w = \Delta T, \qquad Z_w = \frac{(\alpha v)^{1/2}}{U_w}$$

Outer scales

Deardorf

$$W^* = (\beta g Q Z^*)^{1/3}, \qquad \theta^* = \frac{Q}{W^*}$$



(a) (b)

Profiles at different Ra and AR (Γ) =1.13, for R-B convection in 6.3 m high cell. (a)-the mean horizontal velocity and (b)-the r.m.s. velocity fluctuation. Here < > indicates mean (Data from Puits et al, 2009, rescaled).

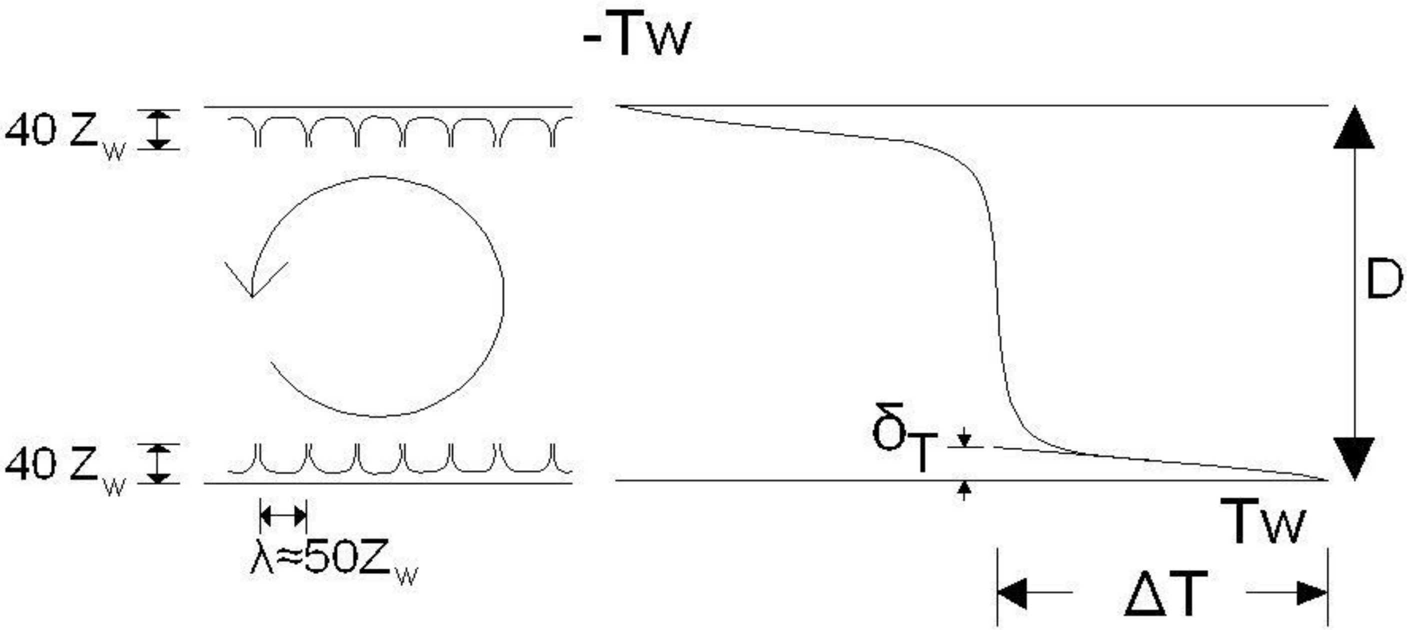
Some comments on turbulent Rayleigh Benard convection

- Flux essentially determined by near wall dynamics,
- Near wall consists of line or sheet plumes

- $Nu = C\,Ra^n \qquad n \sim 1/3$

- Plume spacing λ given by (Ra_λ) ^ (1/3) = 50 for $Pr \cong 1$
  = 90 for large Pr

Plumes come closer as temp. diff. increases $\lambda \sim \frac{1}{(\Delta T)^{1/3}}$ *or* $\lambda \sim \frac{1}{(q)^{1/4}}$



What happens as Ra goes to infinity ?
eg. as D becomes large ~ 1km
Outer flow becomes stronger in relation to the plumes

Kraichnan (1962) predicts ‘**Ultimate Regime**’ $Nu = Ra^{1/2}$

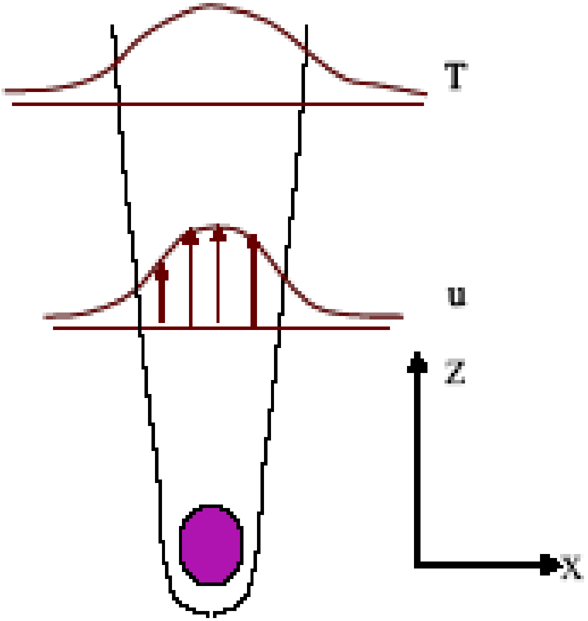
Still controversial if it has been achieved so far in
experiments upto Ra = 10^17

Removal of the top and bottom walls in R-B Convection?

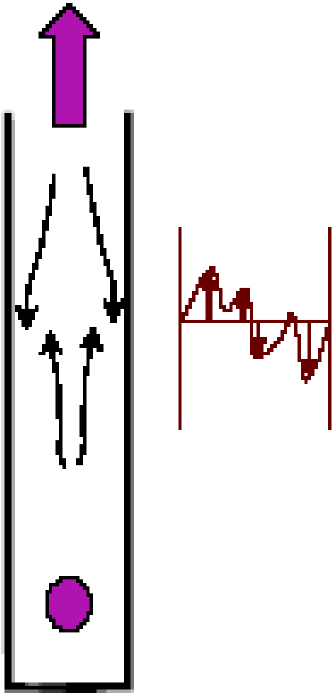
Convection in a tall open-ended vertical tube

Arakeri, J. H., Avila, F. E., Dada, J. M. Tovar, R. O. (2000)
Convection in a long vertical tube due to unstable
stratification-a new type of turbulent flow? Curr. Sci. 79 (6), 859–866.

The flow



Heater in a still ambient



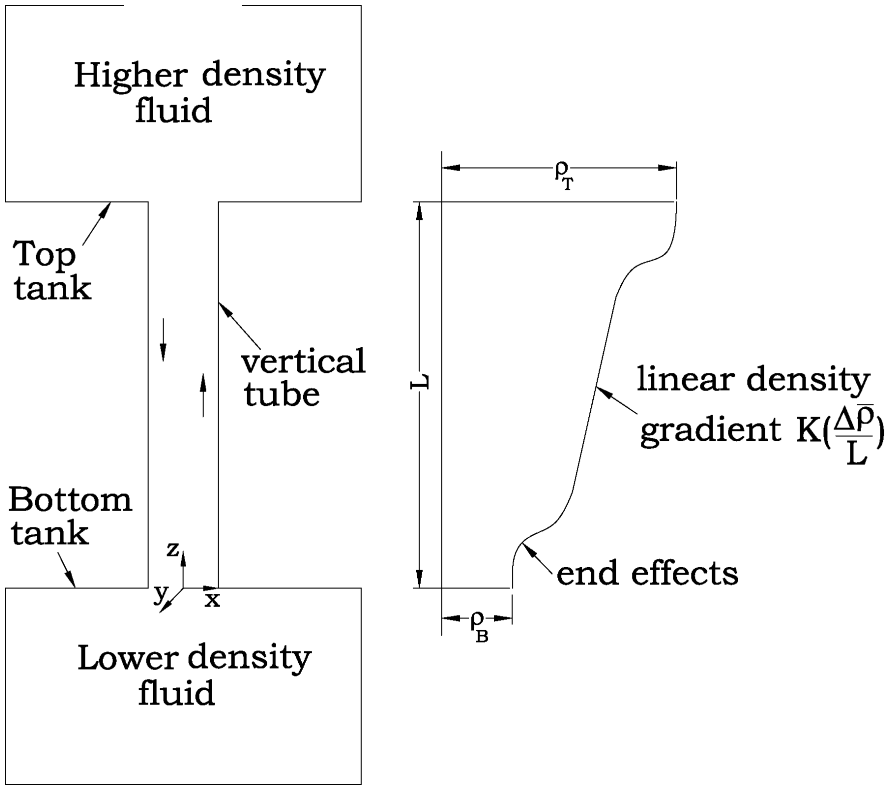
Heater enclosed in a tube

- Cold fluid from the ambient is heated and rises.

- Simultaneous fall of cooler fluid from outside and rise of heated fluid in the tube.
- At any cross section, the net flow is zero.

Tube Convection

- Flow created by an unstable density difference across a long vertical tube (L/d≈10)
- Zero-mean flow:
  $$\int_A w\,dA = 0, \quad \frac{1}{T}\int_0^T u_i\,dt = 0$$
- Zero Reynolds shear stresses $\left(\langle u_i' u_j' \rangle = 0\right)$



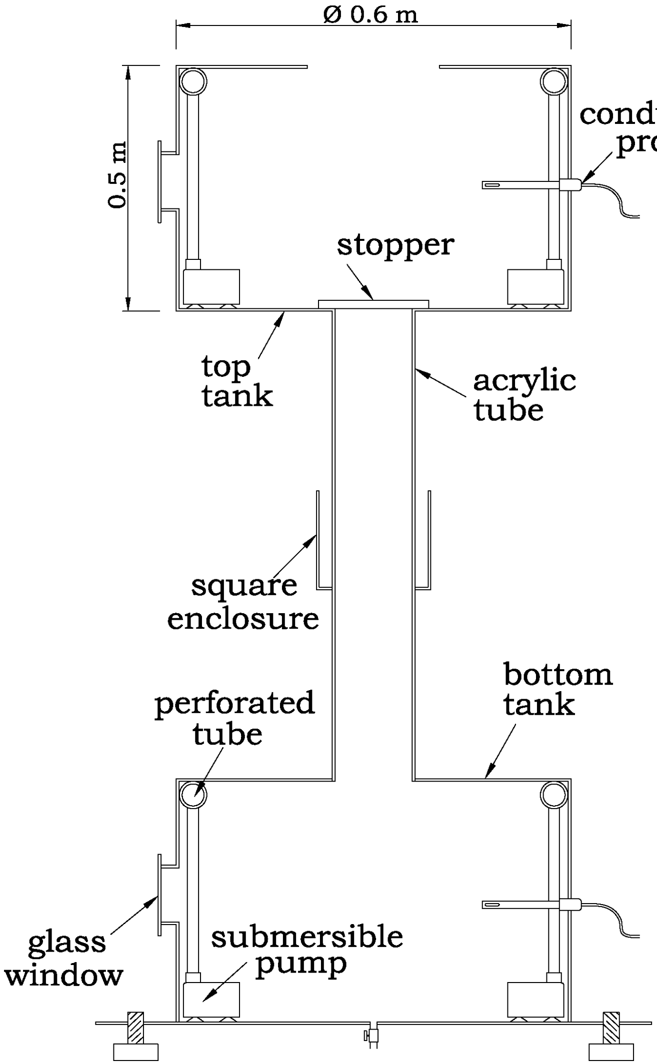
- Axially homogeneous, axisymmetric, linear density gradient along the tube length
- Turbulent energy production solely by buoyancy
- Very high Nusselt and Reynolds numbers

Recent work (S S Pawar)

Pawar & A (POF, 2016; PRFlds (2016); Appld Optics (2016))

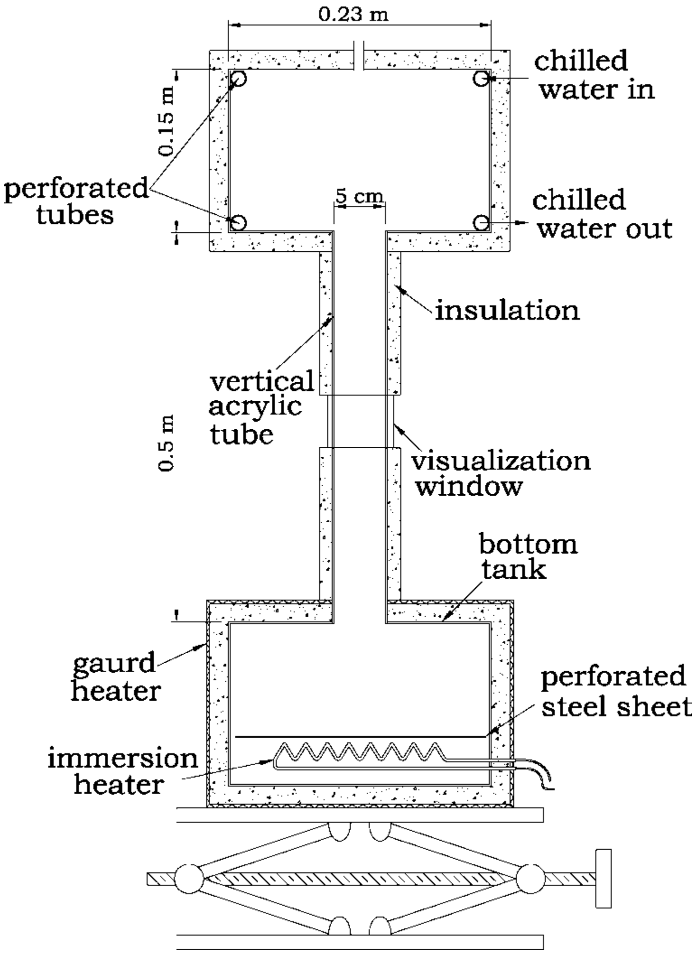
- Experiments using heat (Ra_g = 4×10^4-5×10^6, $Pr\approx6$)
- Experiments using salt (Ra_g = 3×10^8-8×10^9, $Sc\approx600$). Extended Ra range from Cholemari's work, Ra_g~10^8
- Scaling laws for non dimensional flux, Reynolds number, spectra (kinetic energy and scalar)
- Scaling laws for frequency spectra of light intensity and angle of arrival fluctuations

Experimental setup. Using salt. Sc = 600



tube diameter = 140 mm, Length = 1.5 m

Experimental setup: Heat and water, Pr ~ 6



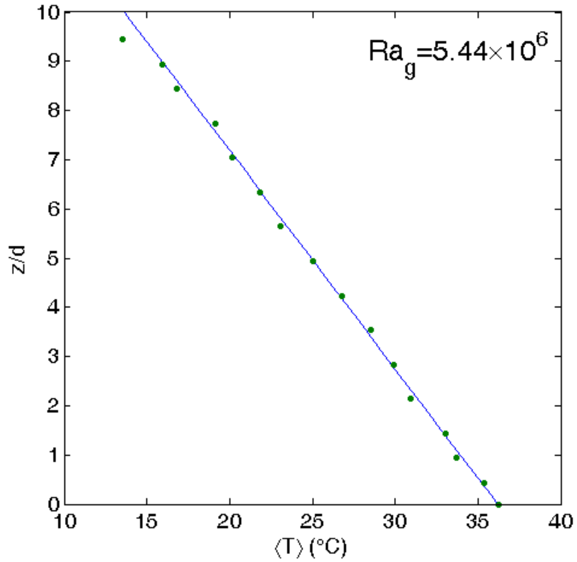
Setup A

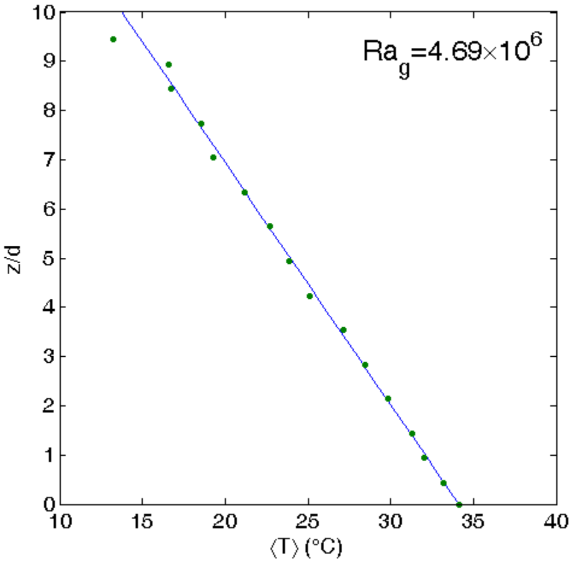
Setup B

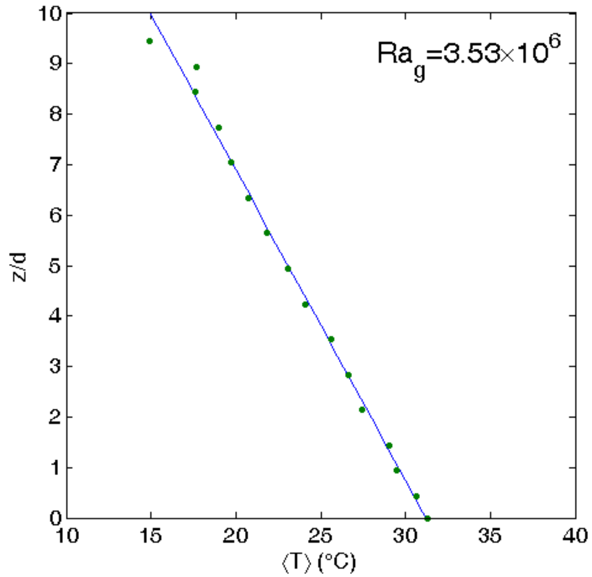
Tube diameters = 15.5, 31.5, 50 mm, L/d = 10,

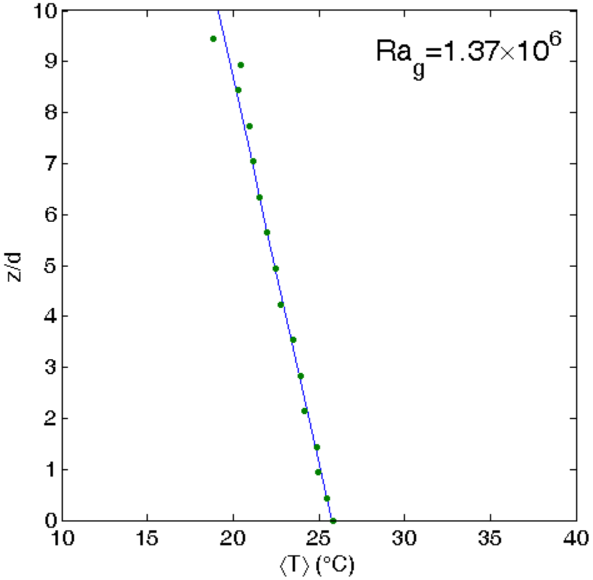
Ra_g range = 4×10^4-2×10^5, 1.6×10^5-1.4×10^6, 9×10^5-5×10^6,

Temperature gradient along the tube length

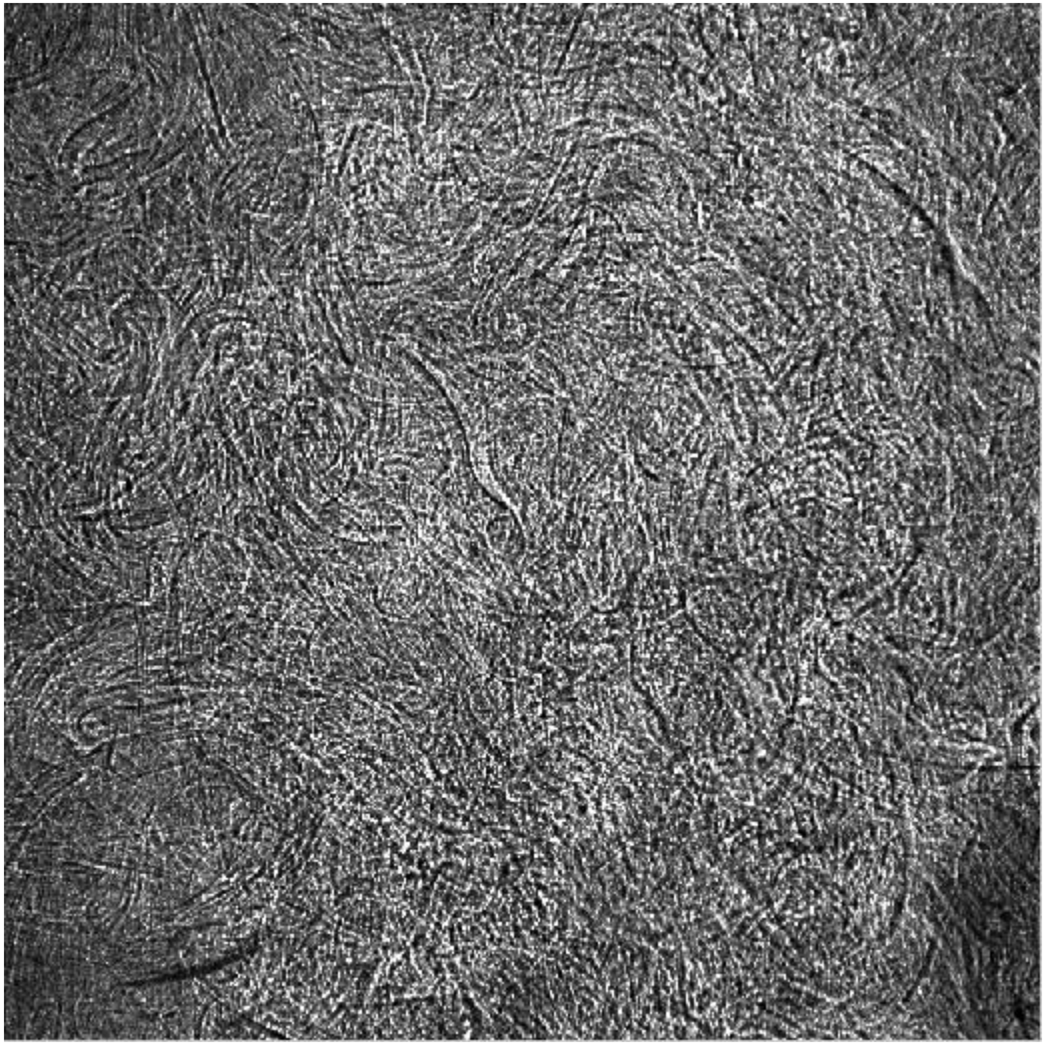




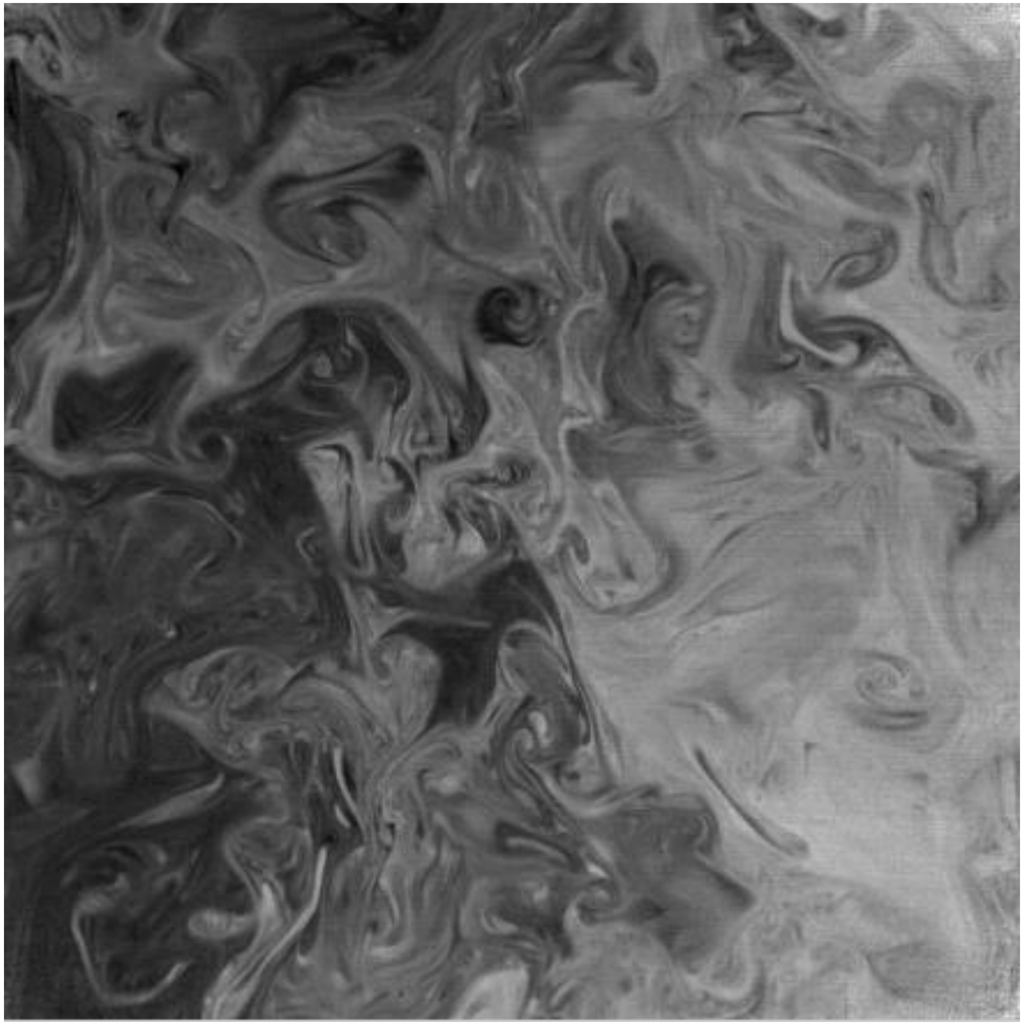




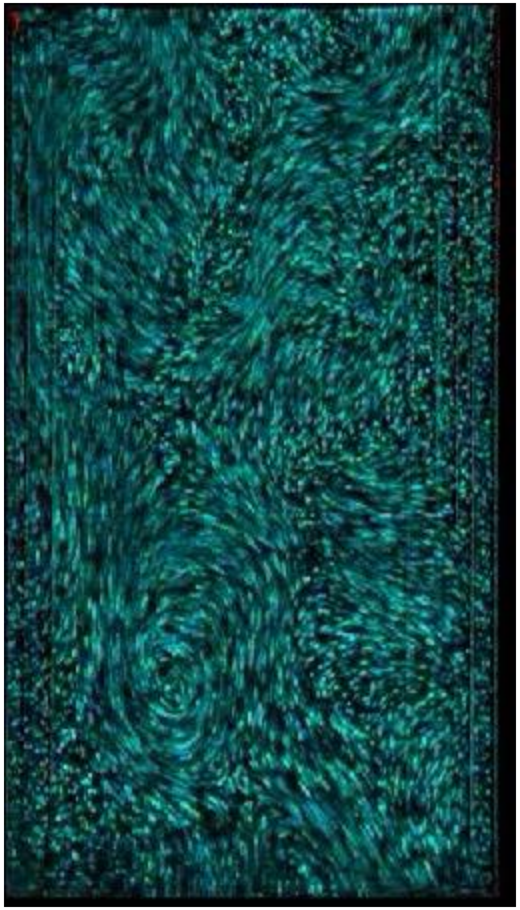
Shadowgraph. Salt expt. high Pr: fine structure; axially homogeneus



Laser induced fluorescence. Salt expt. Range of scales, largest ~ d

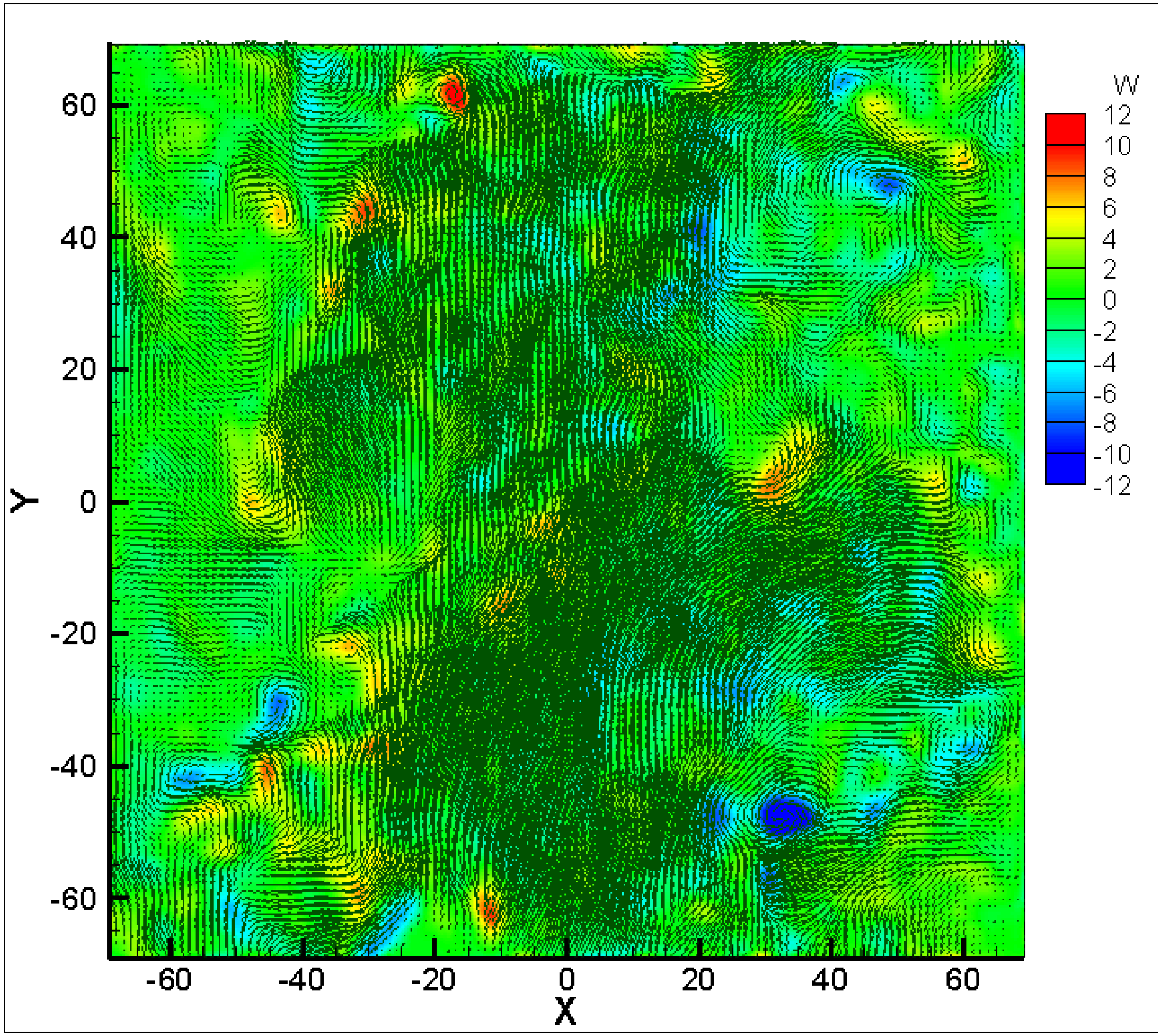


Visualization-particles, velocity-vorticity field (PIV)



Heat experiment

$Ra_g = 1.35\times10^6$, $Pr=6$



Salt experiment, $Ra_g = 7.58\times10^9$, $Sc=605$

Non-dimensional parameters

- Gradient Rayleigh Number,

$\mathrm{Ra_g} = \frac{g}{\rho_0}\left(\frac{d\overline{\rho}}{dz}\right)\frac{d^4}{\nu\alpha}$ ⇒ Buoyancy force / Diffusive effects

- defined based on $\frac{d\overline{\rho}}{dz}$ as the flow is axially homogenous.

- *d* is tube diameter, α is thermal/mass diffusivity (m^2/s) and ν is kinematic viscosity (m^2/s)

- Prandtl or Schmidt Number, Pr (or Sc) = ν/α ⇒ momentum diffusion / thermal (or mass) diffusion

Mixing length model

(Arakeri et al., Current Science, 79, 2000)

- $\frac{d\bar{\rho}}{dz}$, *g* and *d* decide the flow dynamics; ν and α are not important.
- Dimensional arguments give the mixing length scales.

$$\rho' = \left(\frac{d\bar{\rho}}{dz}\right) d \qquad \text{and} \qquad w' = \sqrt{\frac{g}{\rho_0}\left(\frac{d\bar{\rho}}{dz}\right)} d = w_m$$

- Flux $= -\langle \rho' w' \rangle = F_m$
- $Nu = Ra_g^{\frac{1}{2}} Sc^{\frac{1}{2}}$ and $\mathrm{Re} = Ra_g^{\frac{1}{2}} Sc^{-\frac{1}{2}}$ scalings similar to those expected in the 'ultimate regime'; confirmed by the measurements (Cholemari & Arakeri, JFM, 621, 2009) and DNS results (Schmidt et al., JFM, 691, 2012)

2nd type of Nusselt number scaling

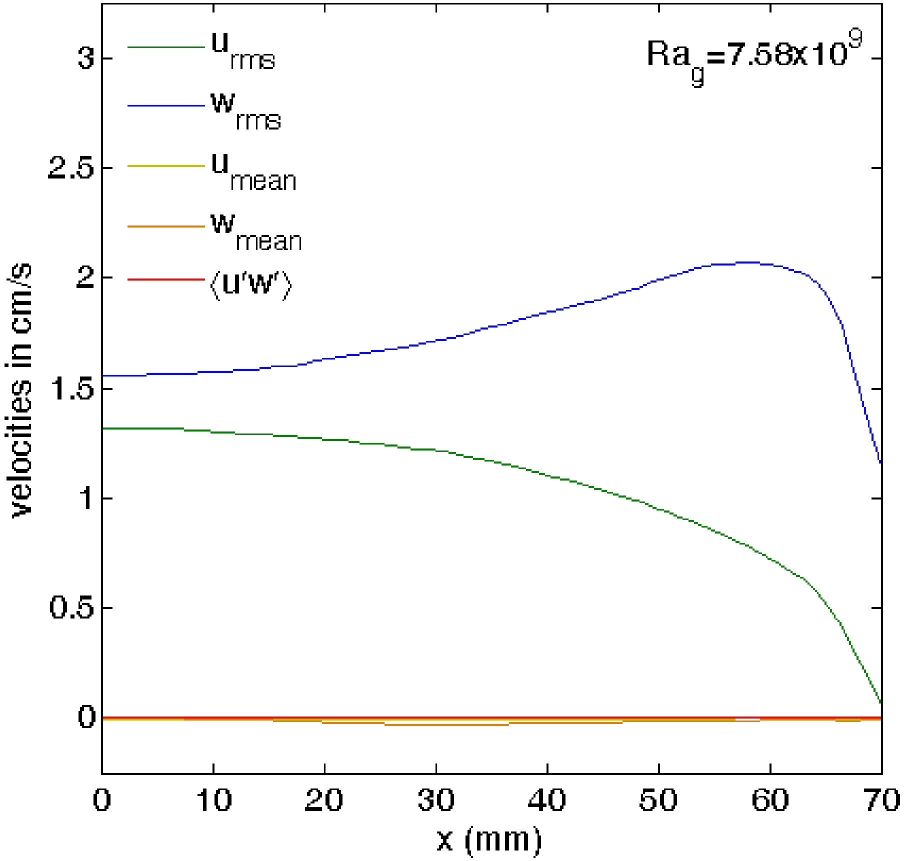
$$\rho' = \left(\frac{d\overline{\rho}}{dz}\right)d \qquad w' = \sqrt{\frac{g}{\rho_0}\left(\frac{d\overline{\rho}}{dz}\right)}d = w_m$$

$$Nu = Ra_g^{\frac{1}{2}} Sc^{\frac{1}{2}} \qquad \mathrm{Re} = Ra_g^{\frac{1}{2}} Sc^{-\frac{1}{2}}$$

Observation of this scaling implies flux independent of viscosity and thermal diffusivity

Velocity measurements. Using PIV. Experiments using salt to create the density difference

Velocity Profiles

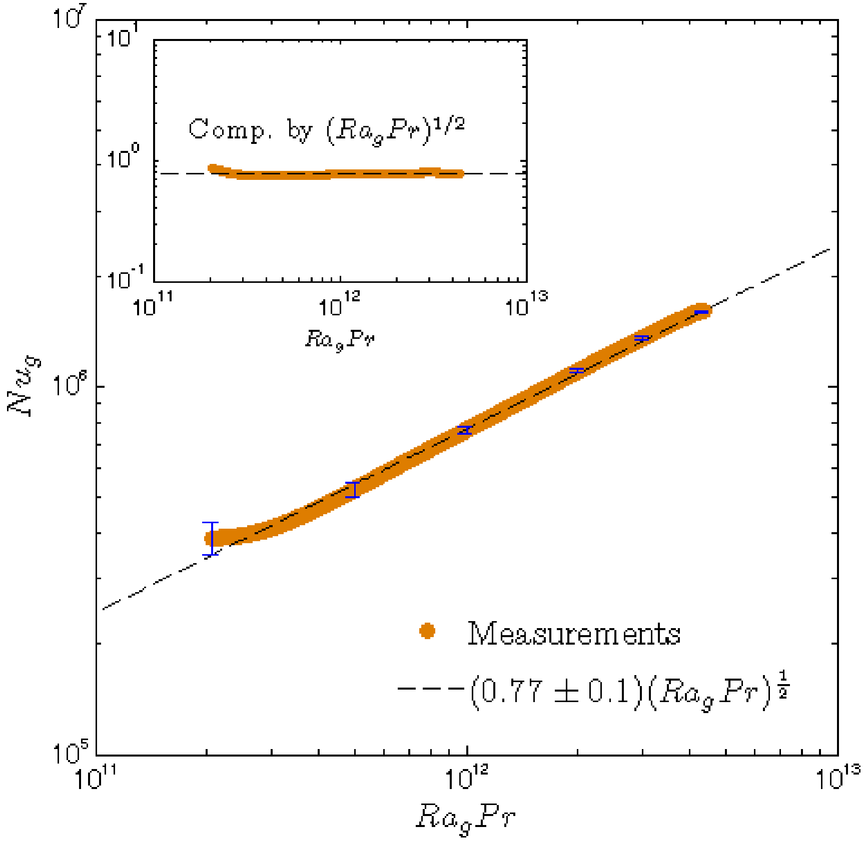


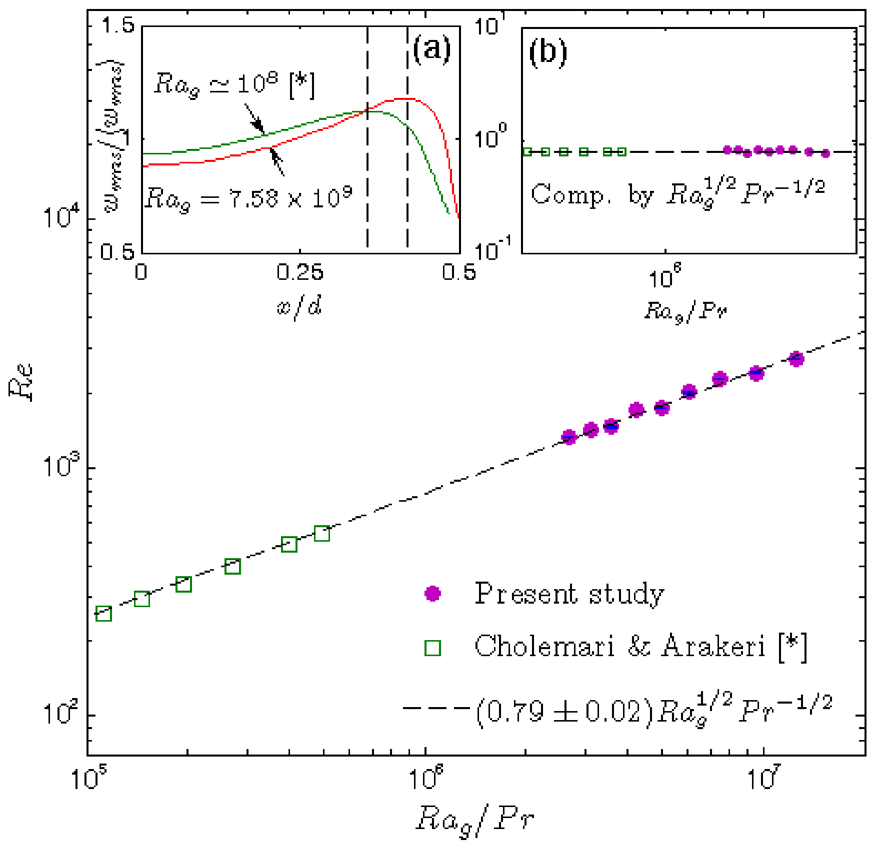
Mean vels, Mean Re shear stress = 0;

rms of vertical velocity has peak near wall;

rms of lateral velocity goes to 0 due to kinematic blocking.

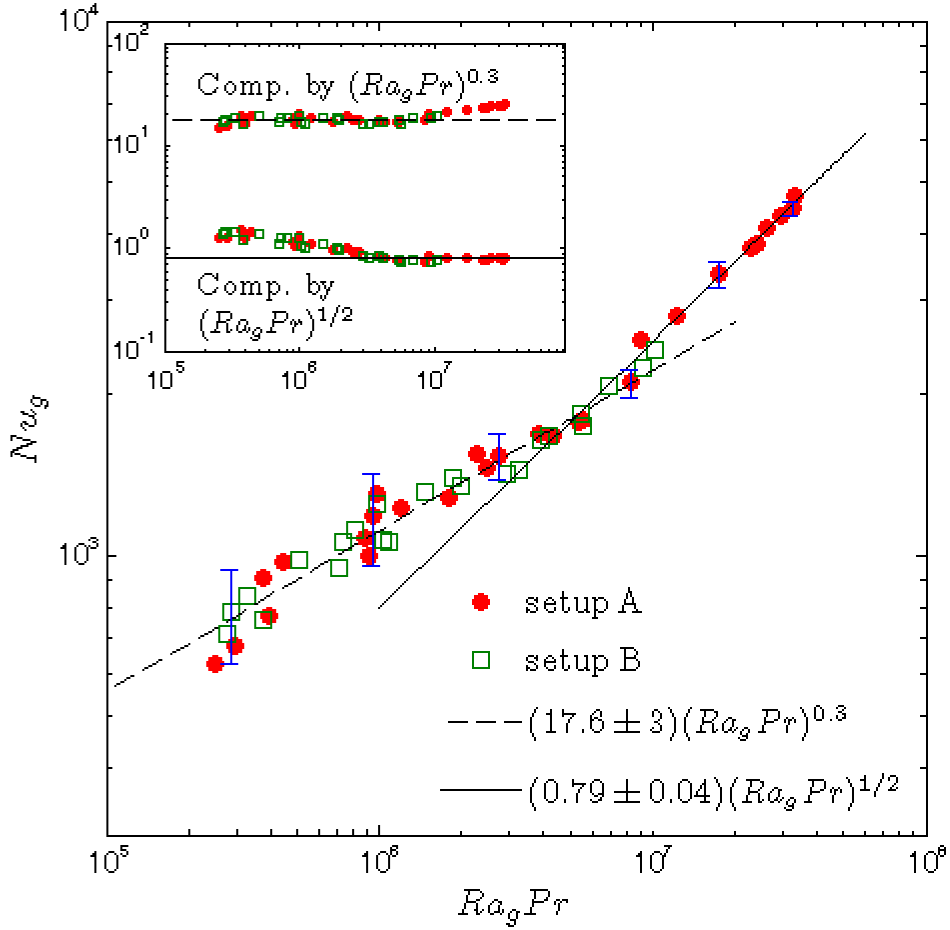
Nu_g and Re scalings - experiments using salt



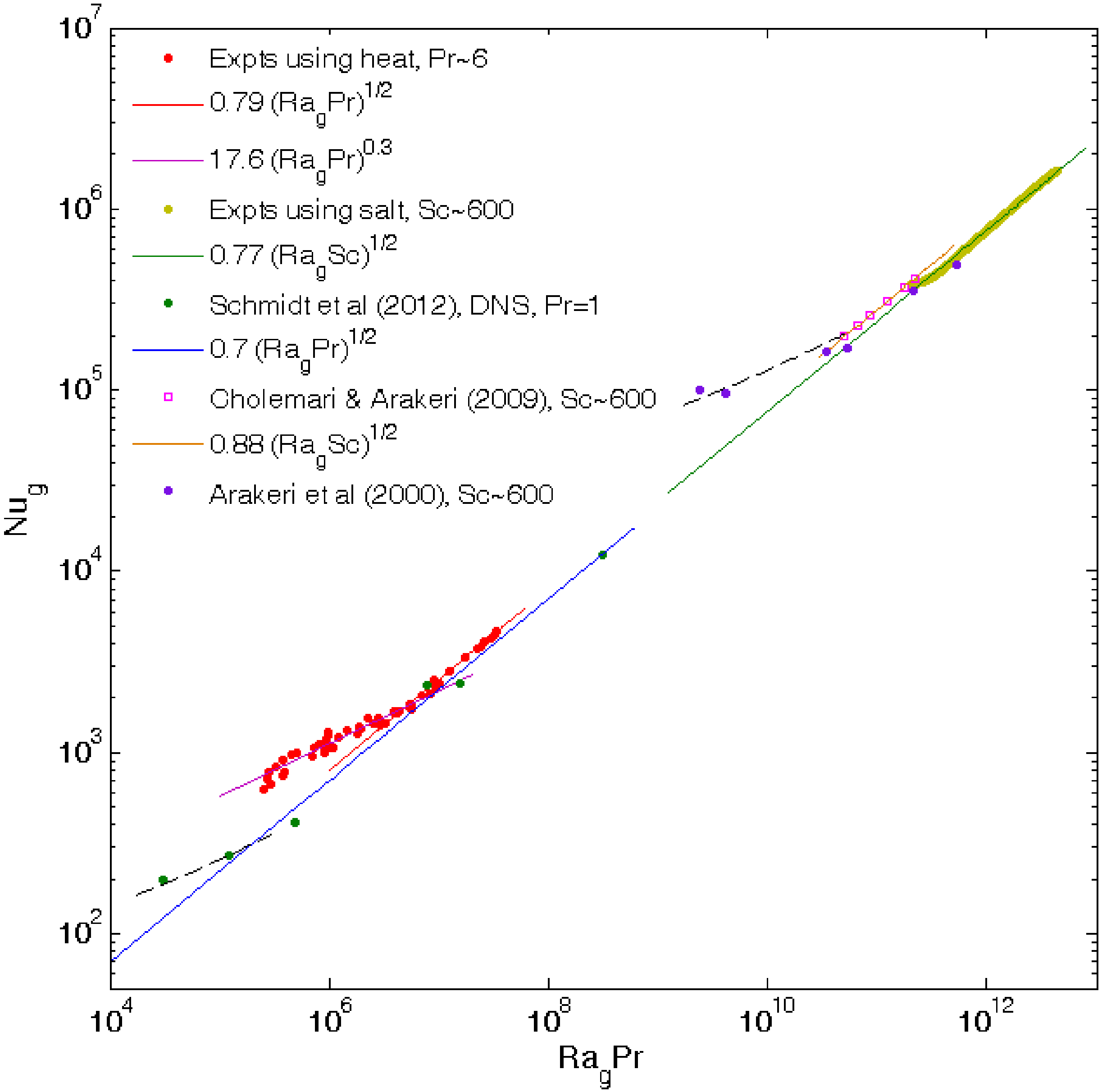


- Nu and Re as per mixing length theory; suggesting independent of visc. and diff.
- For Ra_g=7.5×10^9, Pr≈600, flux four orders of magnitude and Re two orders of magnitude larger than in RBC (Xia et al.(2002) and Lam et al.(2002)

Nu_g scalings - experiments using heat



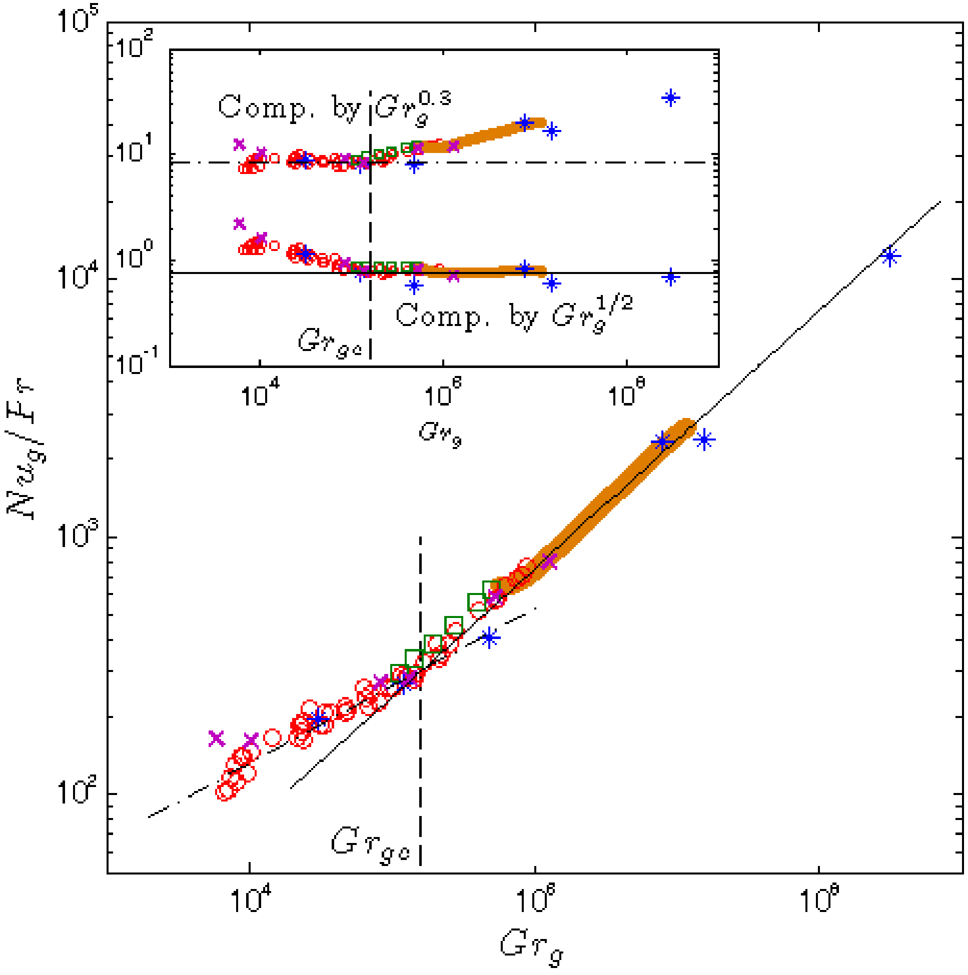
- Two different scalings for Nu; transition at certain Ra, below which exponent = 0.3, similar to RB convection; but Pr dependence different.
- For Ra_g=3×10^5 and Pr=6, flux three orders of magnitude larger than in RBC (Chu & Goldstein (1973))



Data (Nu_g vs Ra_g) from 4 studies; Pr = 1,6, 600. All seem to show two regimes, but transition at different Ra_g values.

Two regimes in tube convection

- Collapse of data by plotting Nu_g/Pr vs $Gr_{g.}$. Gr is Grasshof number.
- Transition between two regimes at single Gr_g Independent of Pr, atleast for Pr > 1
- Two 'universal' correlations for the 2 regimes for Pr > 1.



Analysis of transition at particular Gr

- Scaling $Nu_g \sim (Ra_g \Pr)^{1/2}$ implies flux independent of viscosity and diffusivity. Thus viscous effects become important below critical Gr.

- Define a near wall viscous length scale, $l_v \sim \sqrt{\nu \tau_m}$,

where, $\tau_m = d / w_m$ is mixing length time scale

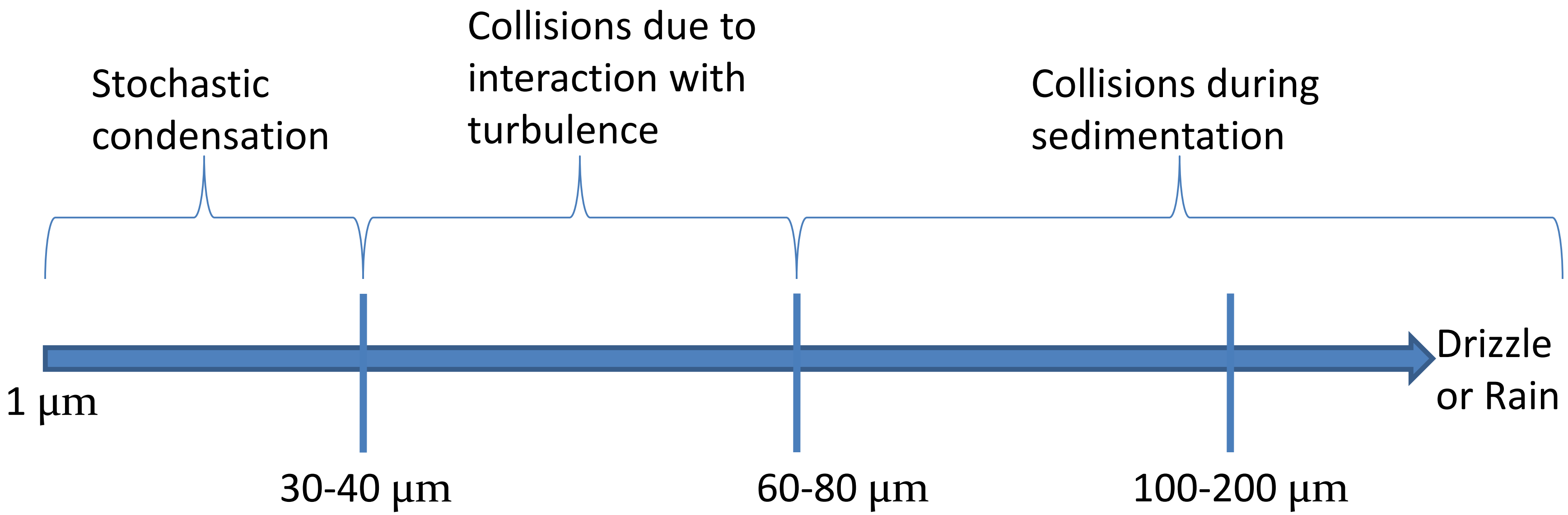
- For $Gr_{gc}=1.6\times 10^5$, get $l_v /d \sim 0.05$.
- $l_v /d > 0.05$ viscous effects become important, and we go into non-ultimate regime.

Droplet growth in moist turbulent natural convection in a tube

- effort to mimic effect of turbulence on condensation processes and droplet growth in clouds

Deepak Madival

Stages of droplet growth in clouds



Stochastic condensation

Collisions due to interaction with turbulence

Collisions during sedimentation

Drizzle or Rain

1 μm

30-40 μm

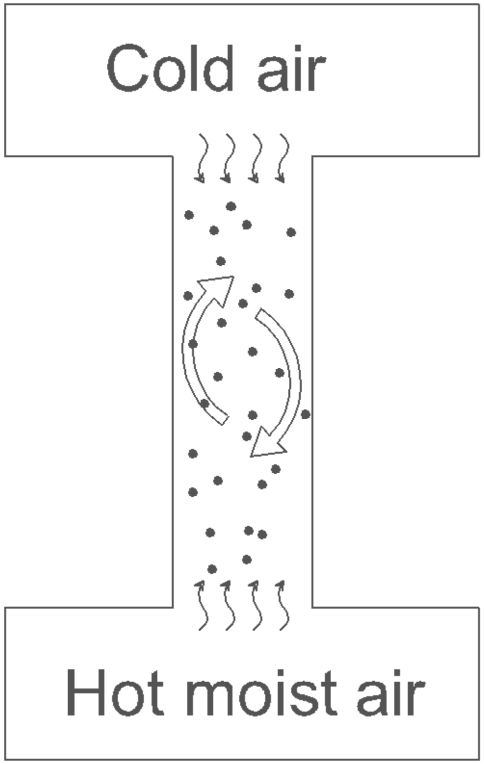
60-80 μm

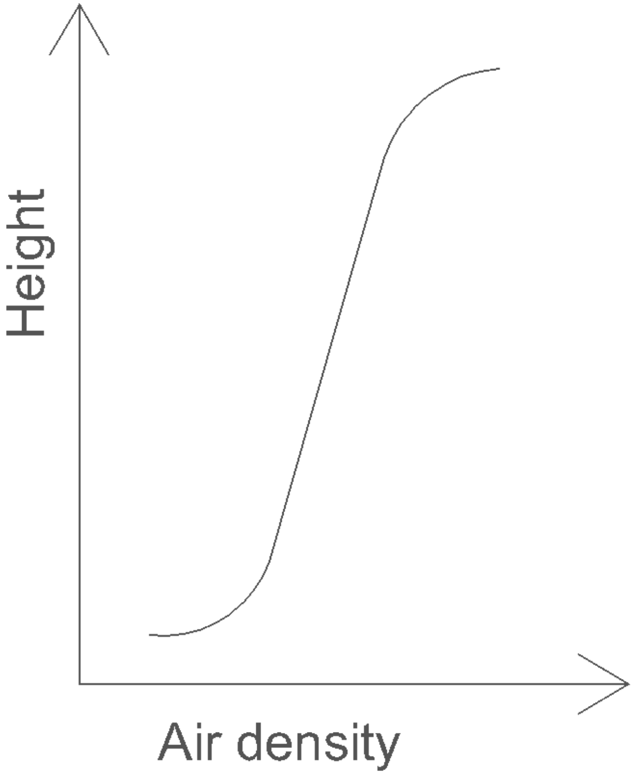
100-200 μm

Cloud droplet diameter

R.A. Shaw (2003) Particle turbulence interactions in atmospheric clouds. *Annu. Rev. Fluid Mech.* 35:183-227.
Grabowski & Wang (2013) Growth of cloud droplets in a turbulent environment. *Annu. Rev. Fluid Mech.* 45:293-324.

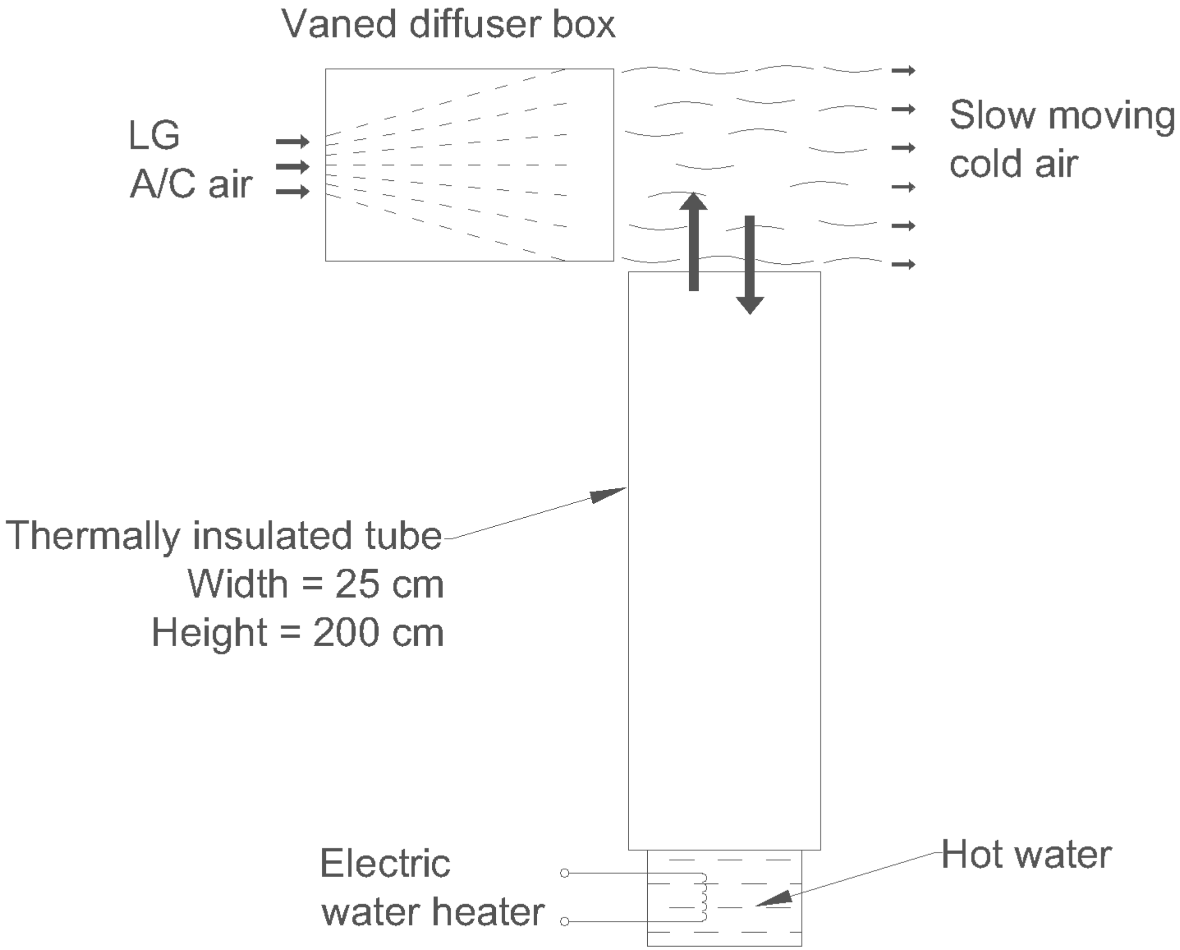
Concept of setup



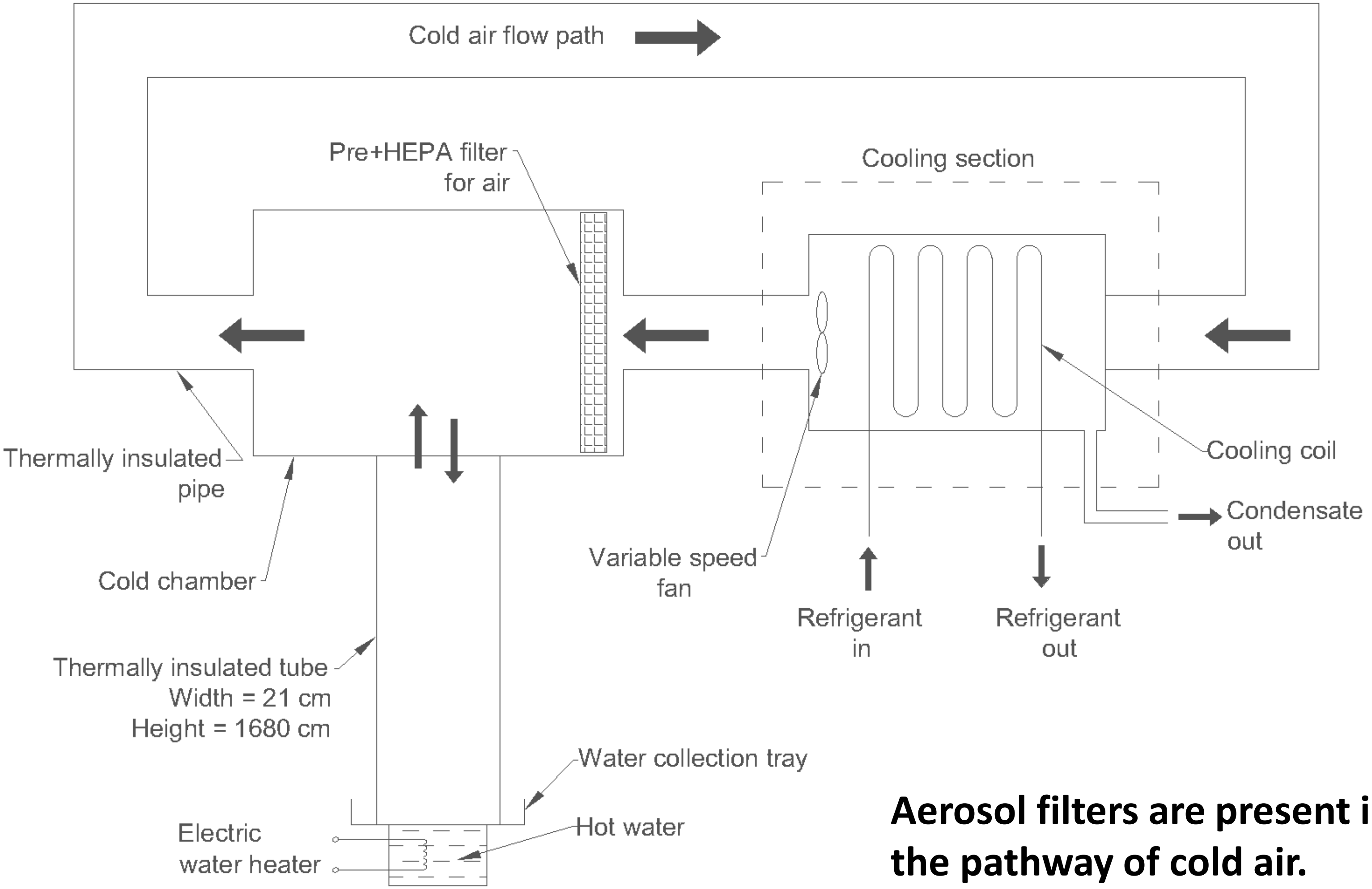


Droplets in turbulent flow

AC setup. Natural aerosols present in atmosphere.



RINAC setup. Aerosols filtered out.



Aerosol filters are present in the pathway of cold air.

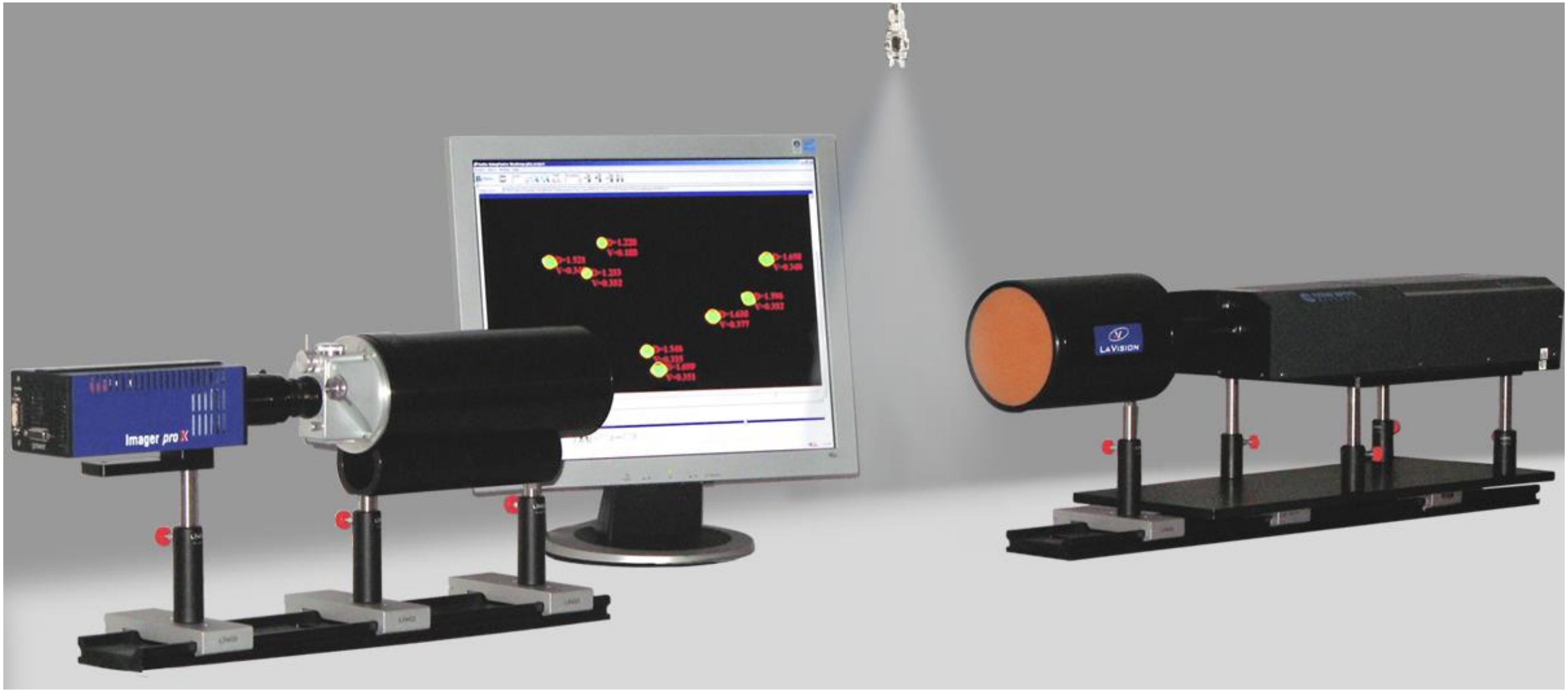
Experimental parameters

- *L/d* ratio of tower (square) = 8
- Temperature of cold air (top) = -3 to +12°C
- Surface temperature of water (bottom) = $60^{\pm 1}$°C
- Velocity of cold air (top)

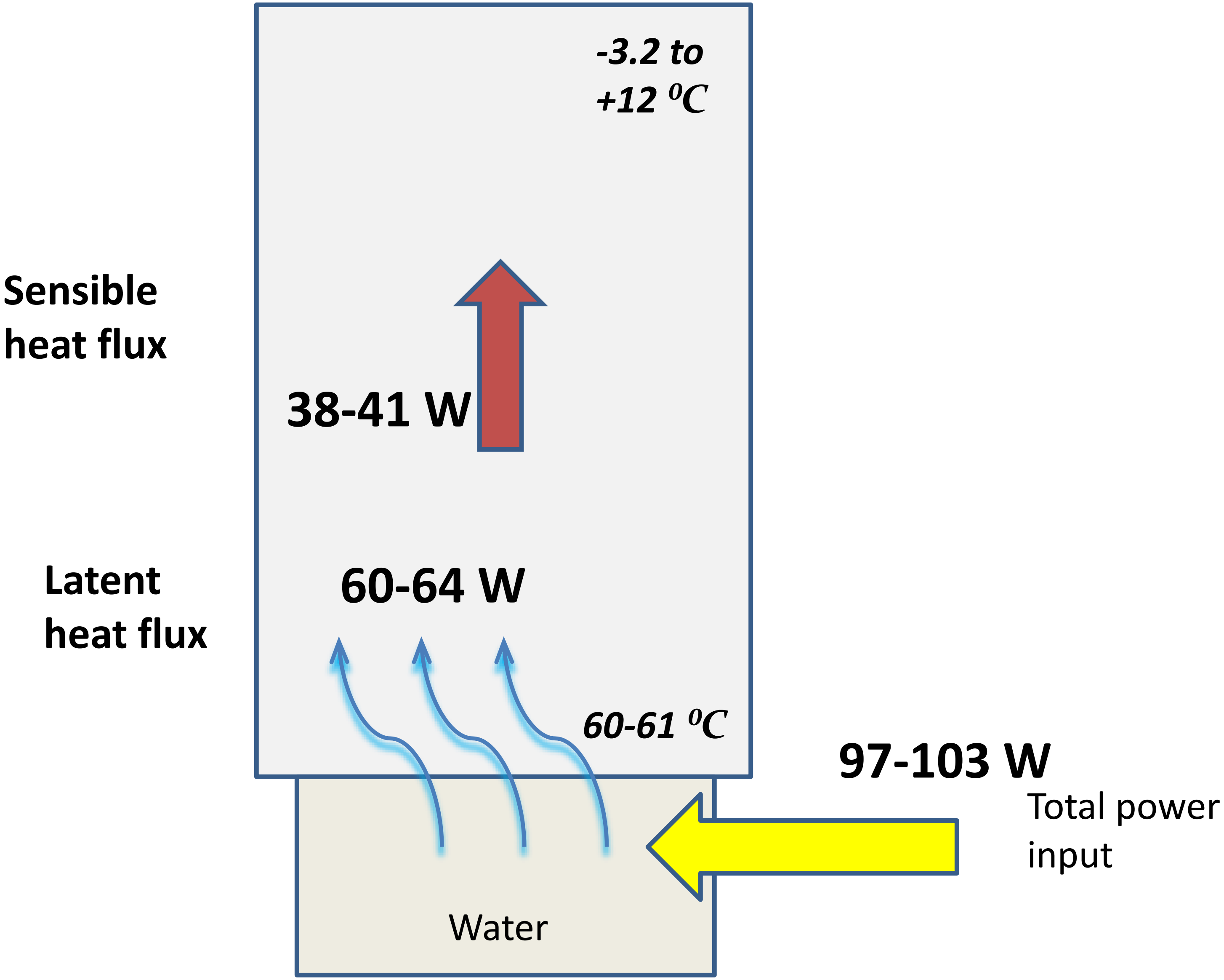
  ~ 0.44-0.48 m/s (AC), ~0.1 m/s (RINAC)

- Ra_g = $g\frac{1}{\rho_0}\frac{\Delta\rho}{L}d^4/(\nu\alpha)$ ~ 10^8 (prefactor 0.6 – 1.6). Density of saturated moist air has been considered.
- Sc =ν/D=Momentum diffusivity/Species diffusivity= 0.67
- Pr (air)=ν/α=Momentum diffusivity/Thermal diffusivity= 0.71

Shadowgraphy for droplet size measurement



Energy budget



-3.2 to +12 ^{0}C

Sensible heat flux

38-41 W

Latent heat flux

60-64 W

60-61 ^{0}C

97-103 W

Total power input

Water

Mean droplet number concentration

- AC setup (#droplets/cc): 431, 445, and 468

  Average = 448 droplets/cc

- RINAC setup (#droplets/cc): 157, 191, and 148

  Average = 165 droplets/cc

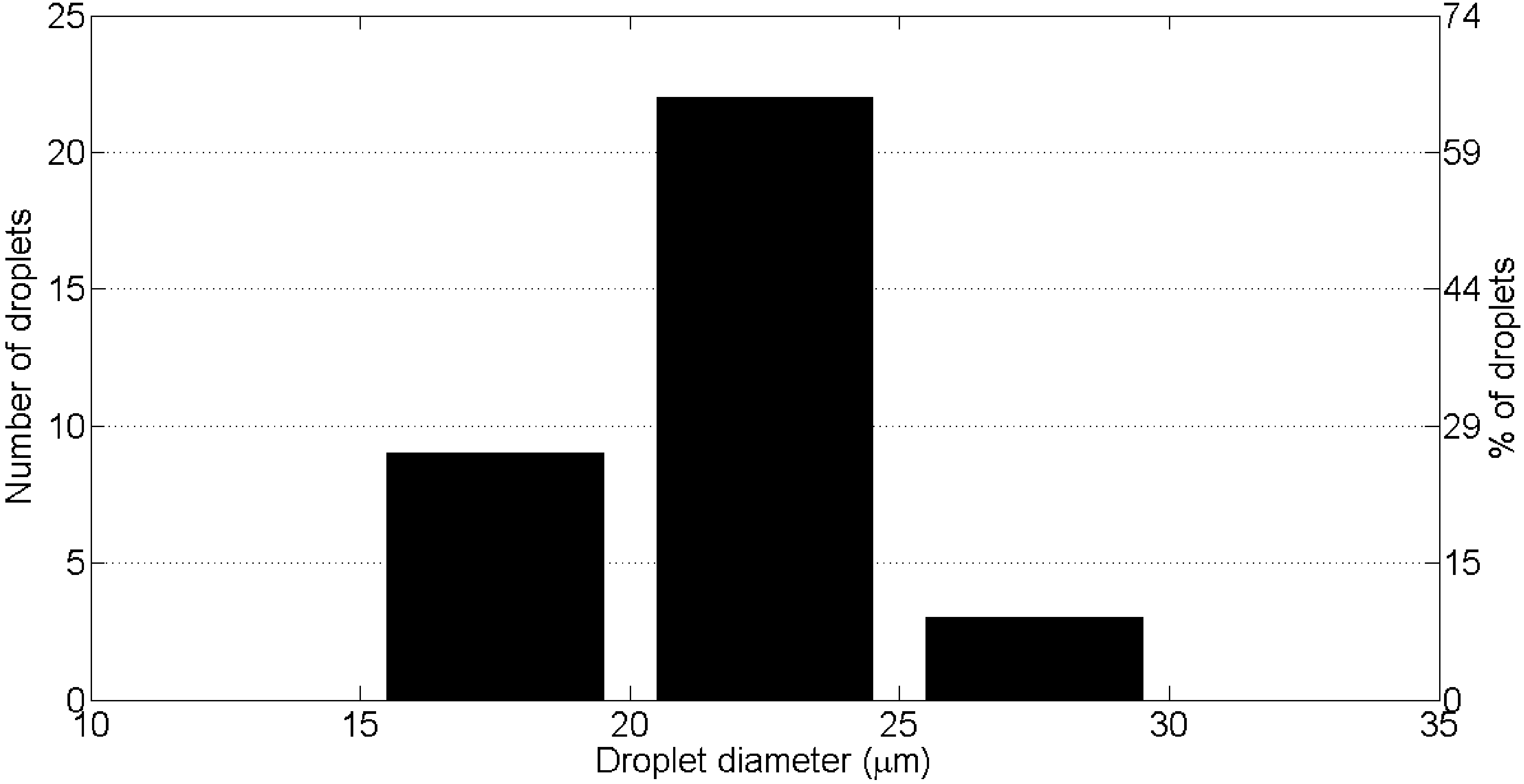
Droplet number concentration in RINAC setup is only 37% of that in AC setup. This is because of presence of aerosol filters in RINAC setup.

Droplet size distribution

- In AC set up all droplets are less than 10 μm.
- This is because of the high aerosol concentration, and low temperature gradients inside the tube.

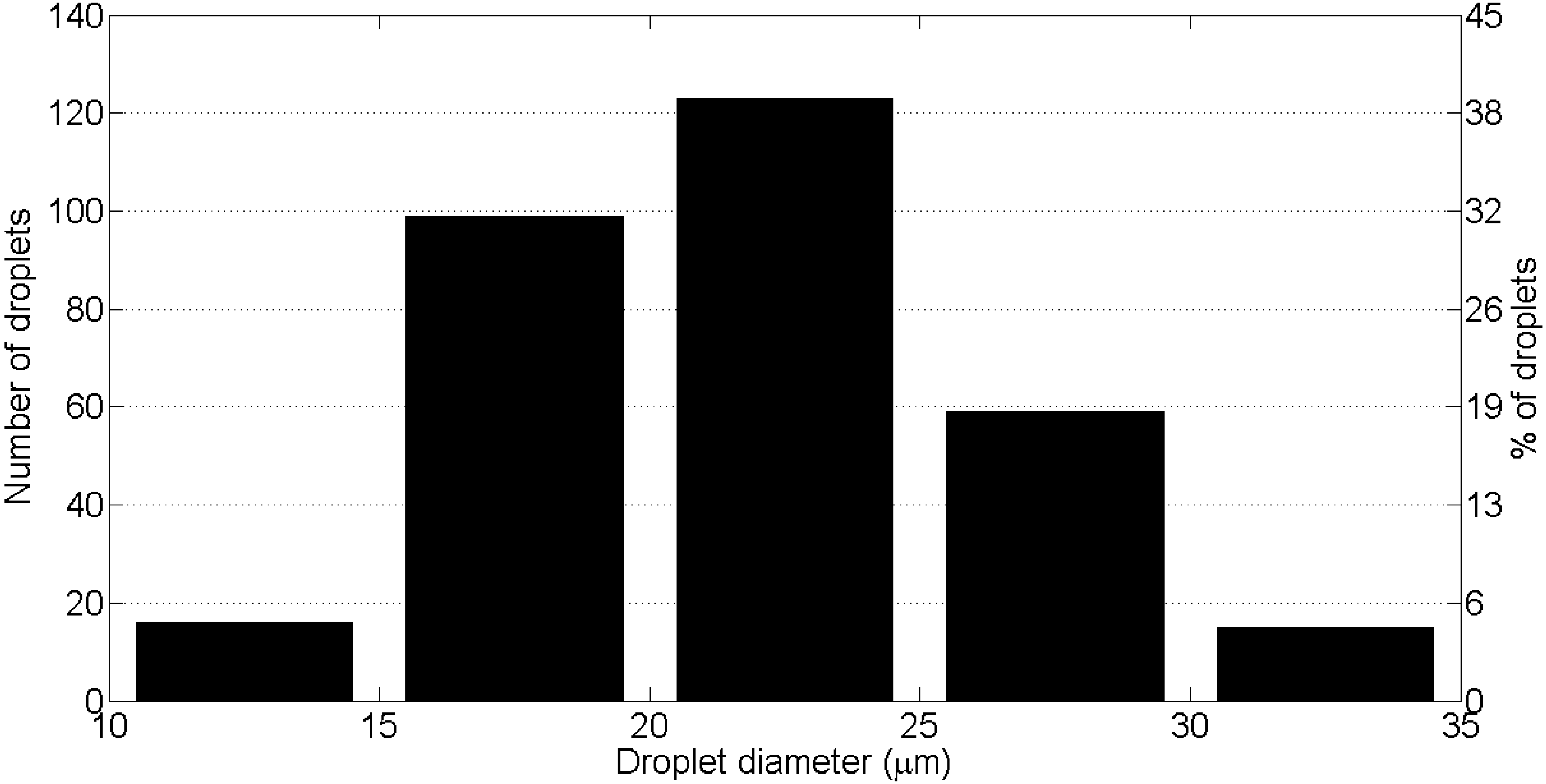
Droplet size distribution
RINAC setup

- T_{hot}=61.5 deg C, T_{cold}=1.5 deg C, 32 droplets detected over ~1 hr measurement



Droplet size distribution RINAC setup

- T_{hot}=61.0 deg C, T_{cold}=-3.2 deg C, 312 droplets detected over ~1 hr measurement



Conclusions

- All droplets in AC setup are less than 10 µm size.
- Largest diameter droplet in RINAC setup is ~36 µm for T_{cold}=-3.2 deg C.

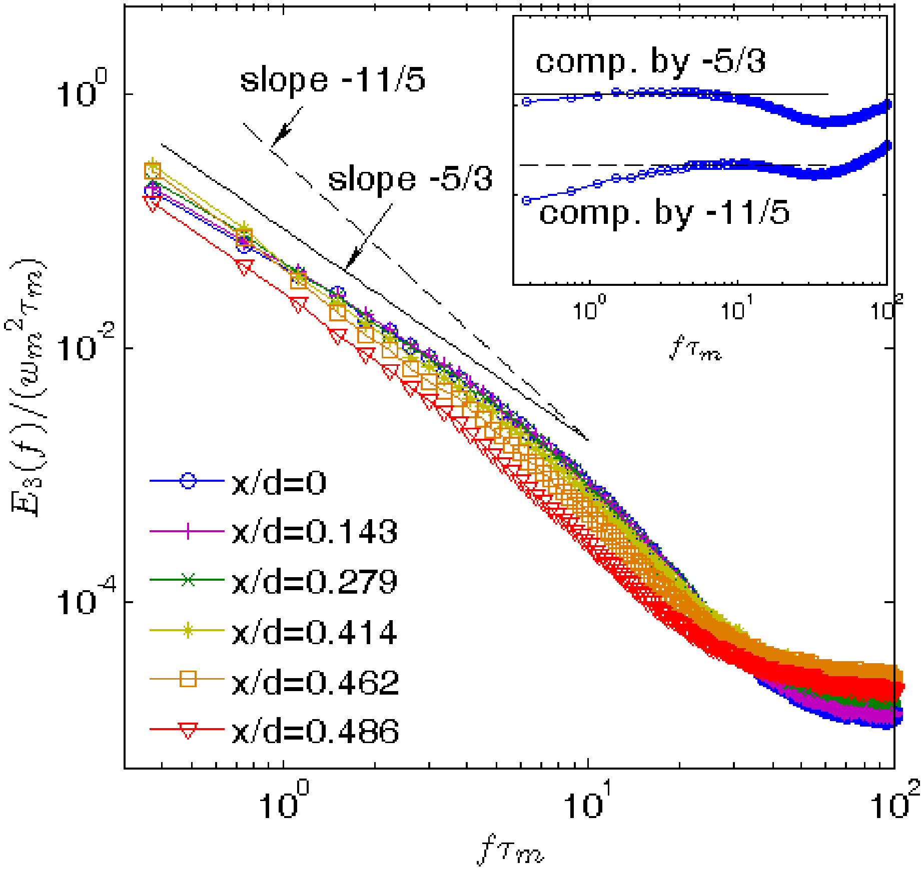
Larger droplets in RINAC setup are formed because

1. Aerosol concentration in RINAC setup is less.
2. Temperature gradient in RINAC setup is larger. Therefore in RINAC setup, mixing of air parcels of more disparate thermodynamic conditions take place, for similar eddy sizes compared to that in AC setup.
3. Absolute temperatures are lower than in AC setup.

The kinetic energy and scalar spectra in tube convection

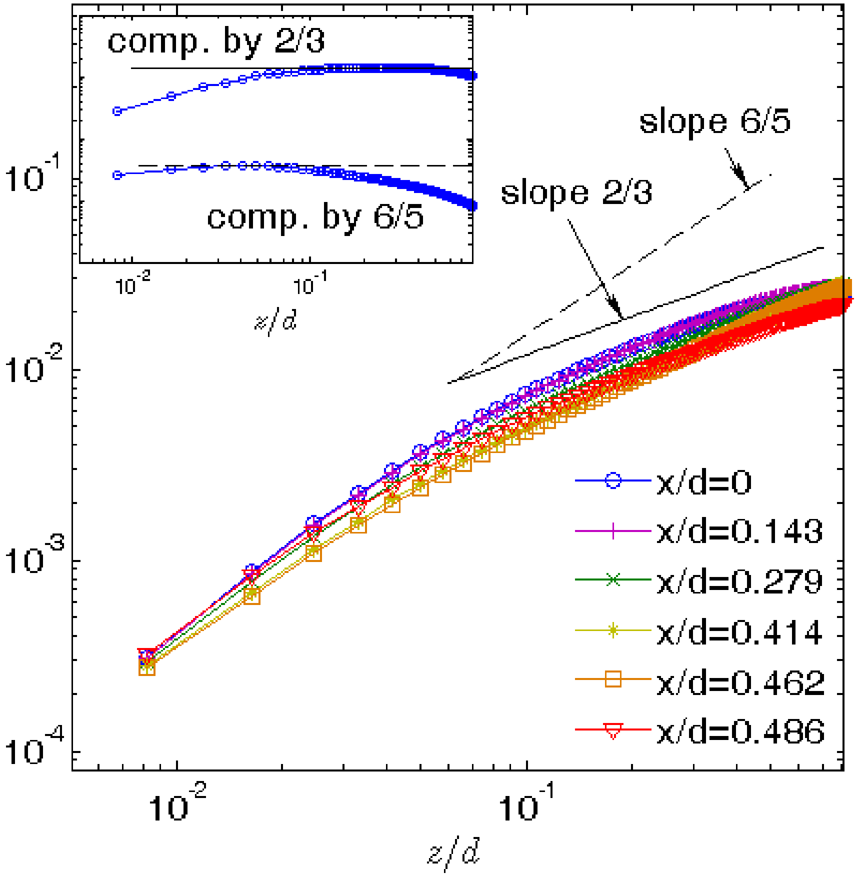
- High Re, axial homogeneity makes TC suitable to study buoyancy effects on turbulence – Kolmogorov or Bolgiano ?
- can have spatial or frequency spectra, for different velocity components
- since mean flow is absent, Taylors hypothesis cannot be used to relate the two (?)

Longitudinal Velocity ($Ra_g = 7.58 \times 10^9$)



Kinetic energy spectra

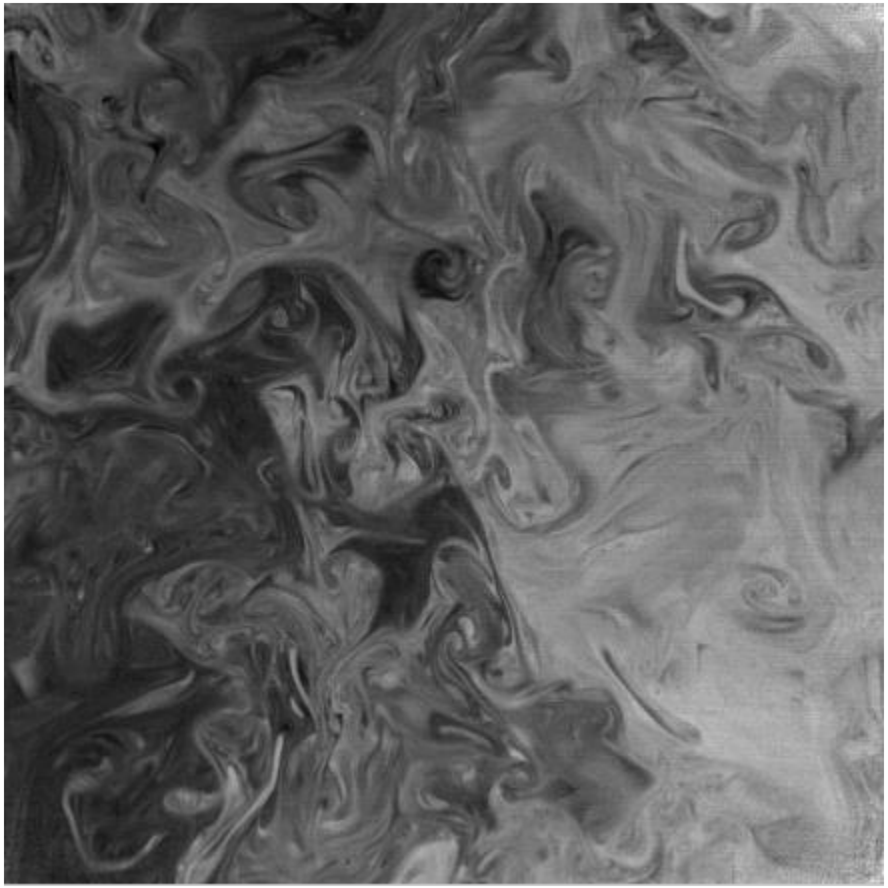
(-5/3 Kolmogorov-Obukhov (KO) scaling,
-11/5 Bolgiano-Obukhov (BO) scaling)



Second-order structure functions

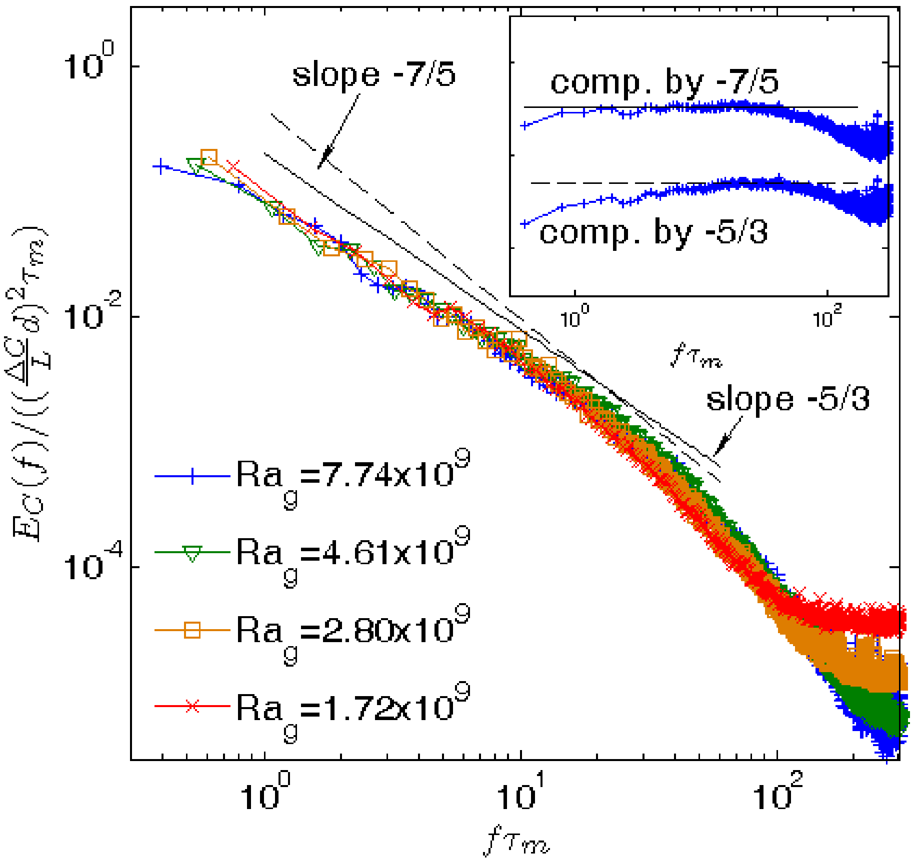
(2/3 KO scaling, 6/5 BO scaling)

PLIF ($Ra_g=6.74\times10^9$)

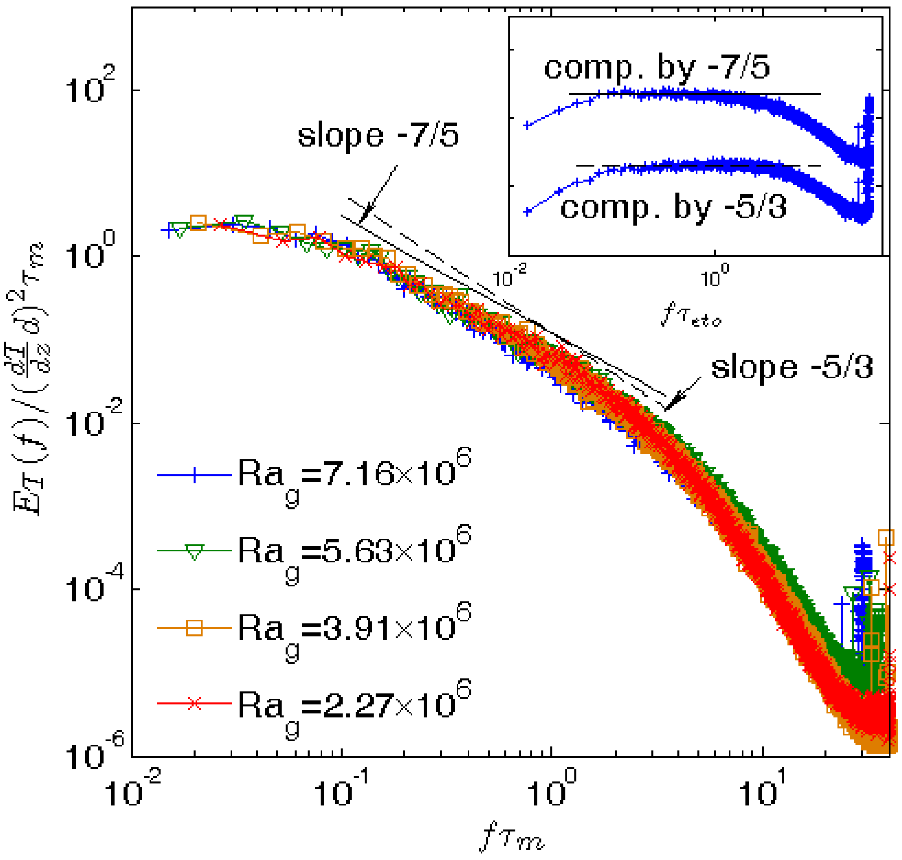


Refractive index matching (Solute pair: NaCl - Ethyl alcohol)

Scalar frequency spectra (salt, heat expts)



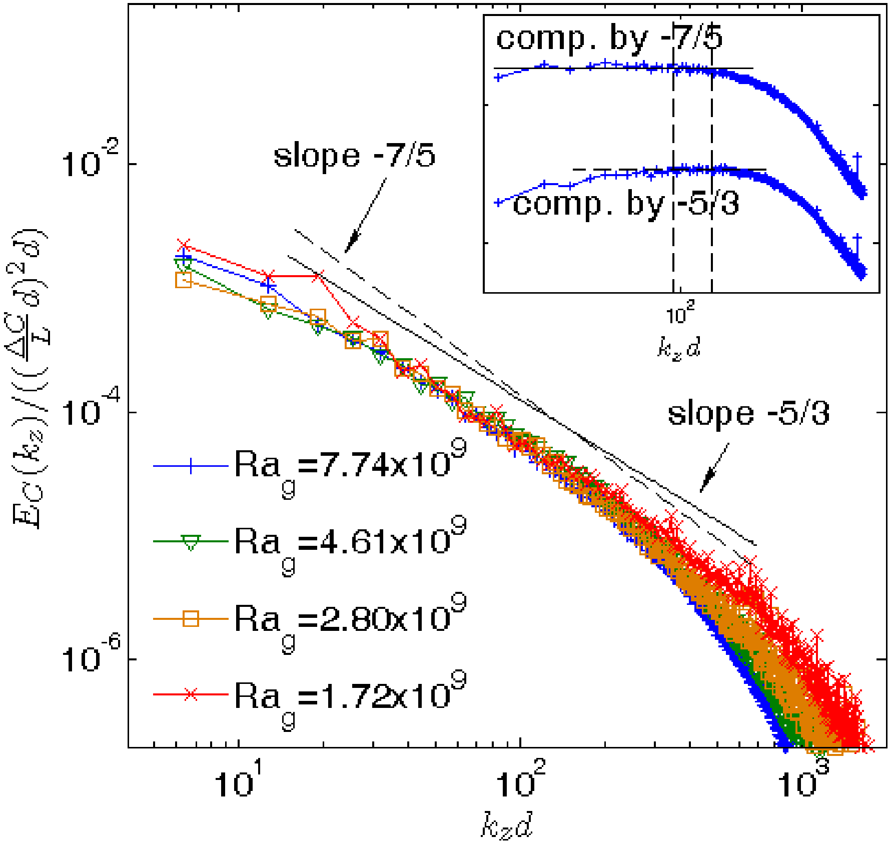
Concentration spectra
(Salt Experiments)



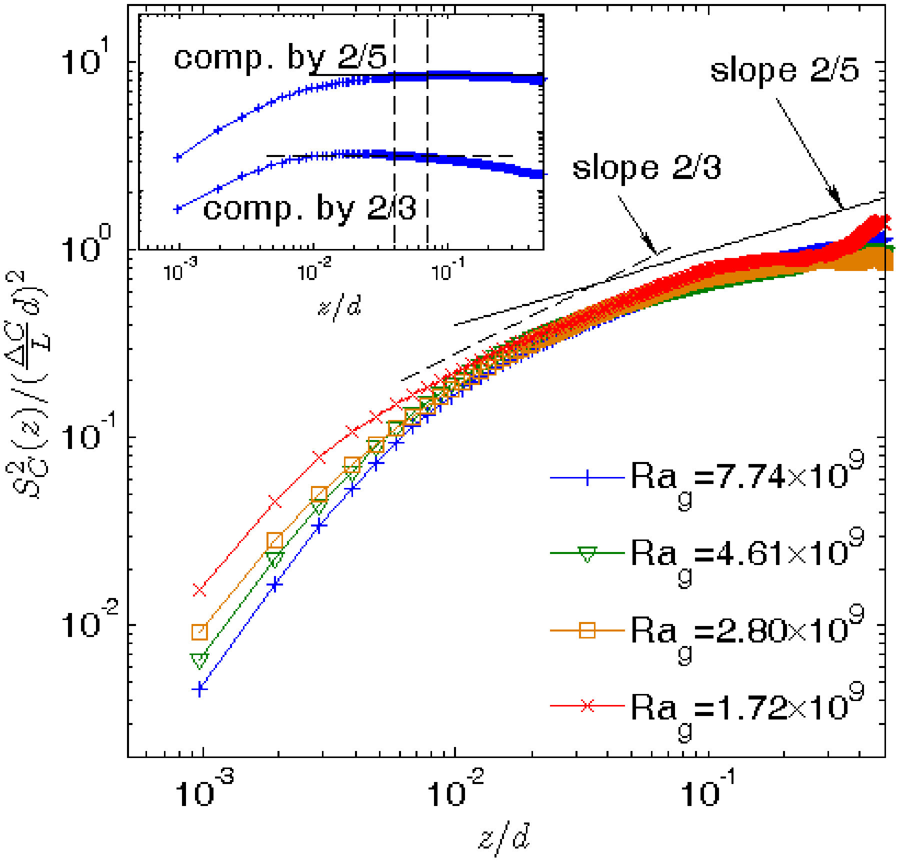
Temperature spectra
(Heat Experiments)

(-5/3 Obukhov-Corrsin (OC) scaling, -7/5 Bolgiano-Obukhov (BO) scaling)

Scalar spatial spectra, structure fn (salt expts)



Spatial spectra



Second-order structure functions

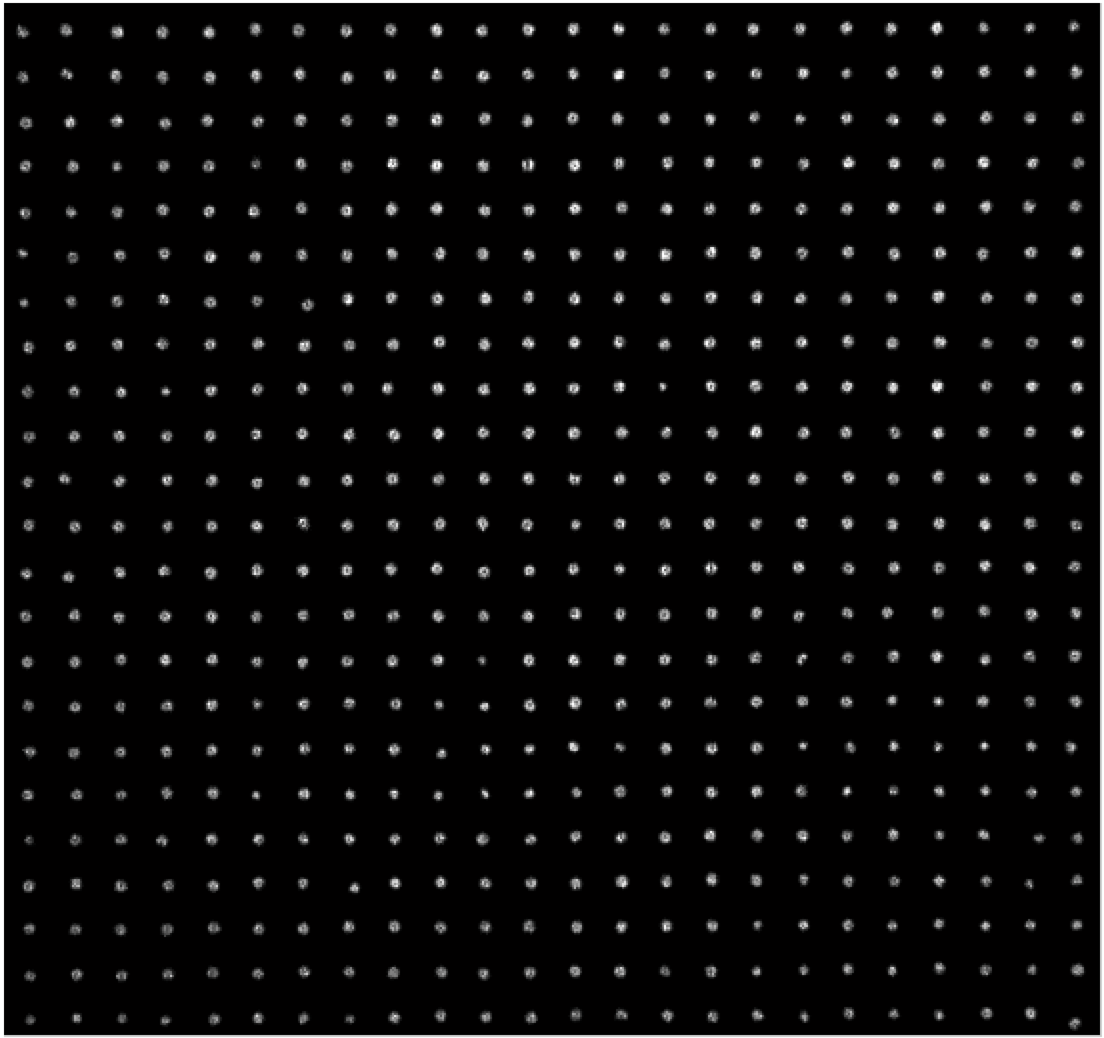
(-5/3 Obukhov-Corrsin (OC) scaling,
-7/5 Bolgiano-Obukhov (BO) scaling)

(2/3 OC scaling, 2/5 BO scaling)

Light propagation through convective turbulence

- Fluctuations of refractive index (n’) in the turbulent atmosphere modify optical wave propagation
- Wave propogation in turbulence has been of interest to communication engineers, astrophysicists etc
- Most experiments done in atmosphere; very few in the lab
- Properties of TC make it suitable for study of light propagation through turbulence - axial homogeneity, high Reynolds numbers
- Methods - Laser shadowgraphy (intensity), deflections of narrow laser beams (AOA),)

Narrow light beams passing through convective turbulence



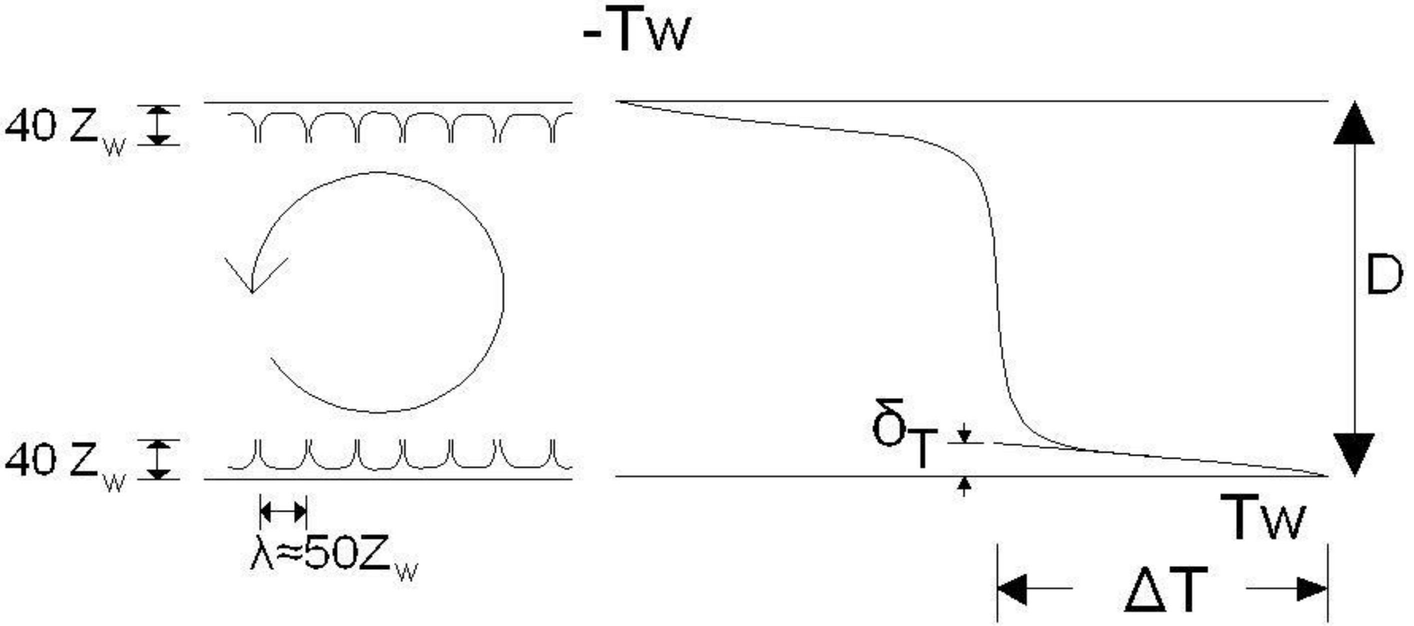
Summary of main results

- Flux and Reynolds number scalings
  - Experiments using salt ($Sc \approx 600$):
    - $Nu_g \sim {Ra_g}^{1/2} Sc^{1/2}$ and $\mathrm{Re} \sim {Ra_g}^{1/2} Sc^{-1/2}$

      ……$3 \times 10^8 < Ra_g < 8 \times 10^9$
  - Experiments using heat ($Pr \approx 6$):
    - $Nu_g \sim (Ra_g \, \mathrm{Pr})^{1/2}$ ……$8 \times 10^5 < Ra_g < 5 \times 10^6$
    - $Nu_g \sim (Ra_g \, \mathrm{Pr})^{0.3}$ ……$5 \times 10^4 < Ra_g < 8 \times 10^5$
  - Unified scalings ($Pr, Sc \geq 1$):
    - $Nu_g / \mathrm{Pr} \sim {Gr_g}^{1/2}$ ……$\mathrm{Gr}_g > 1.6 \times 10^5$
    - $Nu_g / \mathrm{Pr} \sim {Gr_g}^{0.3}$ …… $\mathrm{Gr}_{g_1} < \mathrm{Gr}_g < 1.6 \times 10^5$

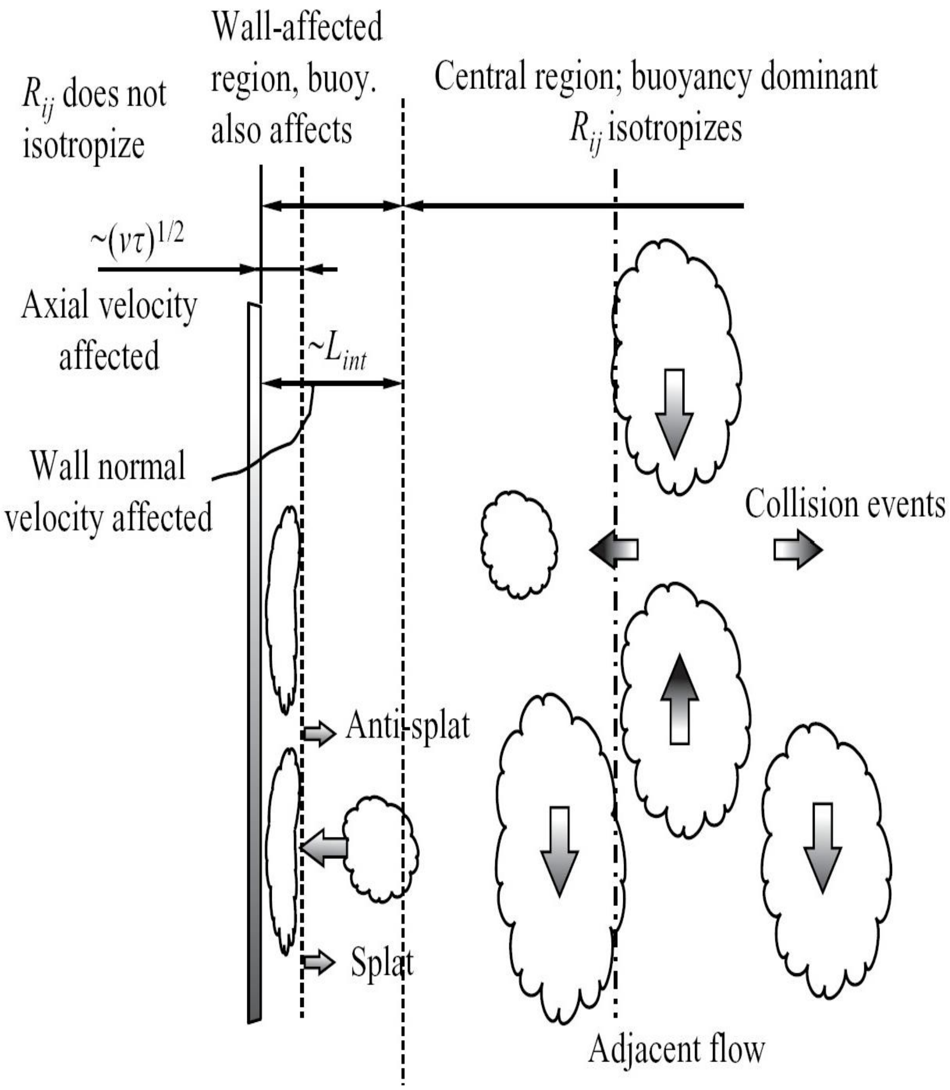
Concluding remarks: R-B convection

- R-B convection dominated by plumes. Plume model to describe near wall dynamics. Wall scales (Uw, Zw) may be used for non-dimensionalising near wall variations; like wall variables in turbulent boundary layers.
- Nu ~ Ra^ (2/7) ; High Ra limit - Ultimate regime (Nu ~ Ra^ (1/2) still elusive



Concluding remarks: Tube convection

- Tube convection very different from R-B convection- axially homogenous, much higher fluxes and Reynolds numbers. Ultimate regime easily realised.
- purely buoyancy driven (all KE production by buoyancy); sustained shear free turbulence near wall
- Zero mean flow; implications for relation between spatial and frequency spectra; Taylor’s hypothesis invalid
- Free convection analog of turbulent fully developed pipe flow – just as useful for fundemental studies of buoyancy driven turbulence



Concluding remarks (contd)

- Preliminary experiments show that moist tube convection may be used to study droplet formation and growth in a convective turbulent environment. Alternate to R-B moist convection.
- Temperature gradients ~ 8 degC/m (AC), 14 degC/m (Rinac).
- Zero mean flow; rms velocities ~ 30 cm/s ; droplet concentration ~ 450 /cc (AC), 165 /cc (Rinac).
- Droplets less than 10 micron in AC set-up, in presence of nuclei. Larger droplets in RINAC system, in absence of aerosols: mean droplet diam ~ 20 micron, largest about 35 micron. Probably due to turbulent fluctuations of temperature and water vapour concentration.

Some open issues

- RB convection: effect of wind/mean shear; roughness
- Tube convection: essentially unexplored;

  Nu = f(Ra,Pr) mapping; regimes; stability; DNS/RANS modellin; exploit axial homgeinity, high Re, fluxes; effect of buoyancy on spectra; spatial-frequency spectra; light propagation.
- explore parameter space for cloud like convection.