Buoyancy effects in dilute astrophysical plasmas

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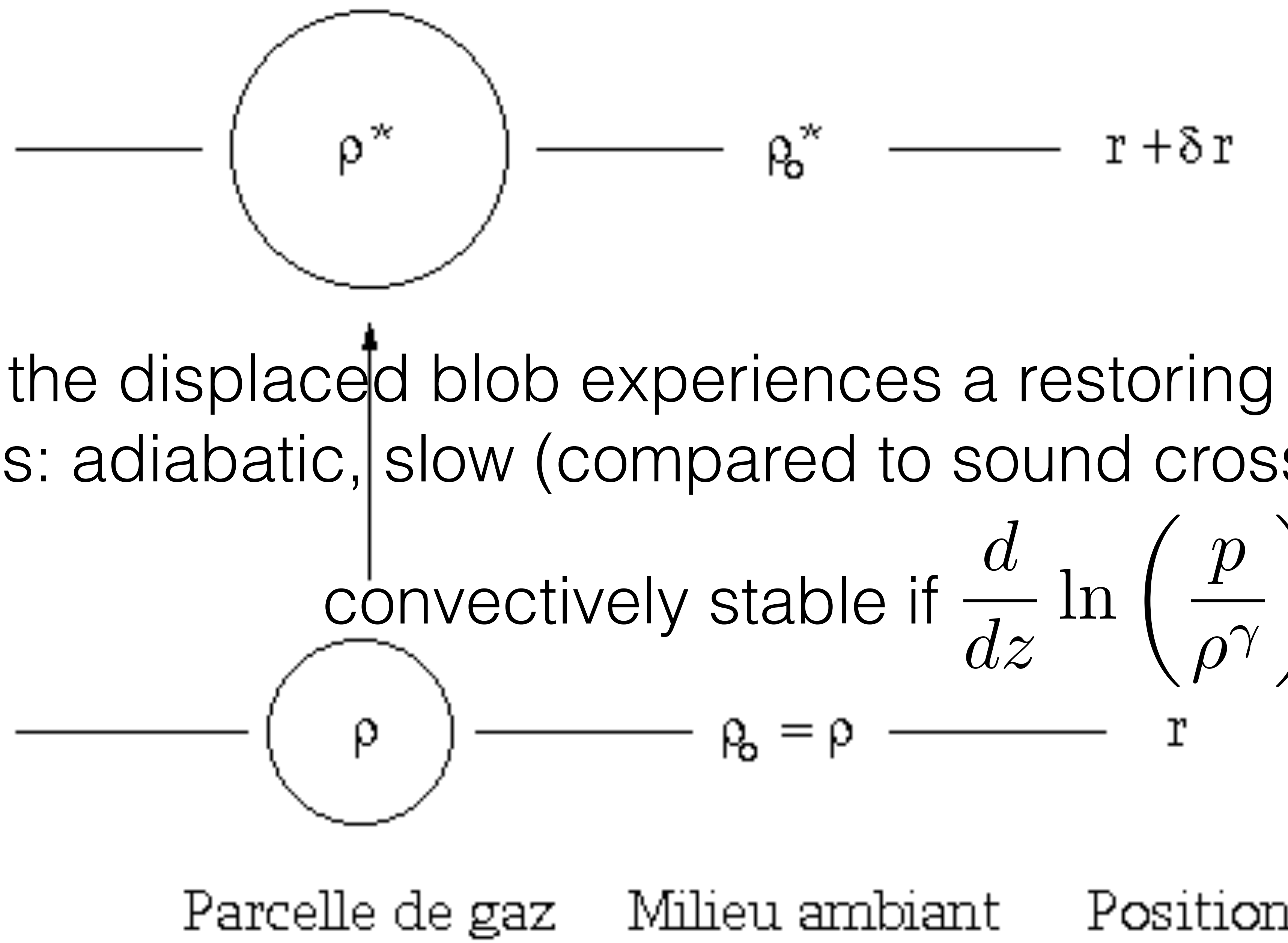
(Buoyancy Driven Flows @ ICTS, June 17, 2017)

Outline

- buoyancy instabilities in dilute astrophysical plasmas: MTI, HBI
- applications: galaxy clusters

Schwarzschild convection

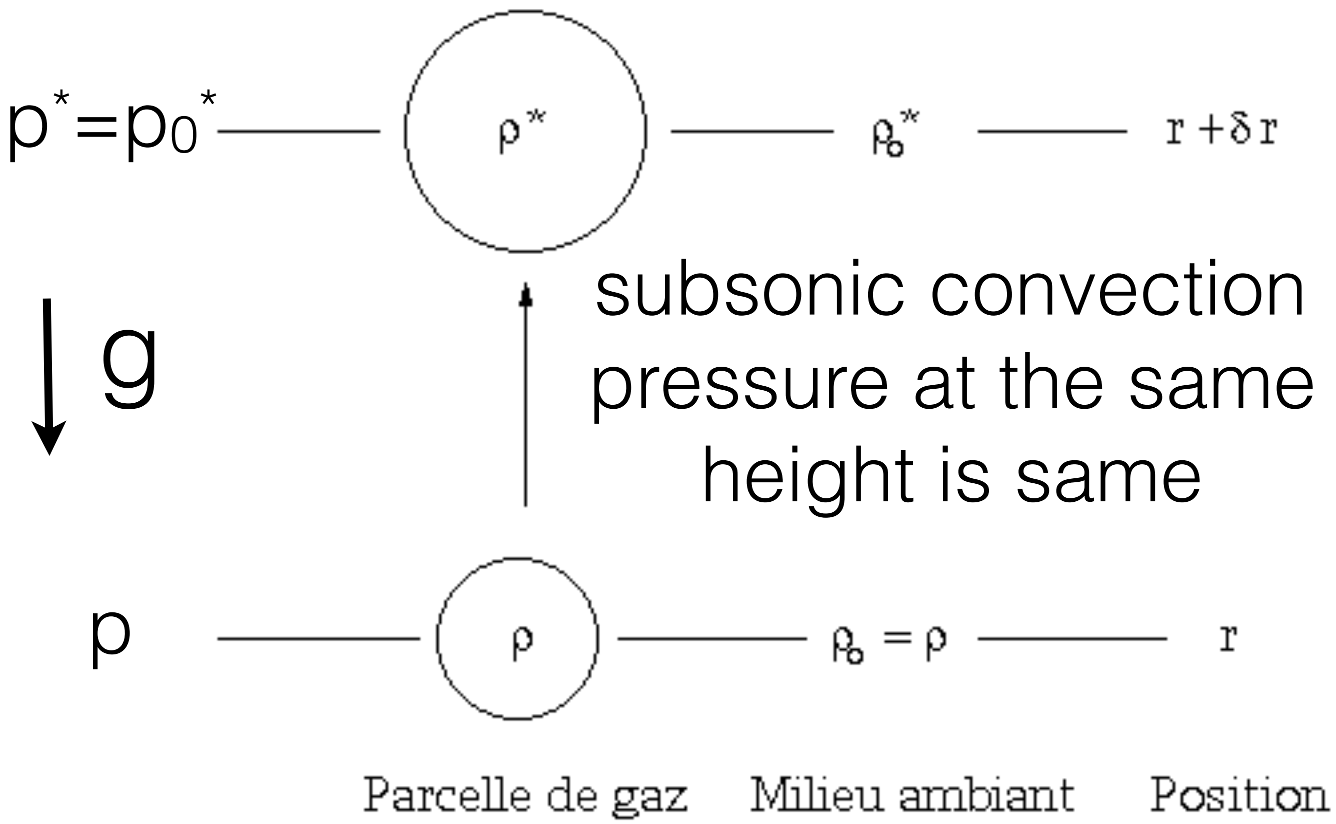


whether the displaced blob experiences a restoring force?

assumptions: adiabatic, slow (compared to sound crossing time)

convectively stable if $\frac{d}{dz} \ln \left(\frac{p}{\rho^\gamma} \right) > 0$

Schwarzschild convection



subsonic convection
pressure at the same
height is same

$$\rho_0^* = \rho_0 + \frac{d\rho}{dr}\delta r$$

$$\frac{p}{\rho^\gamma} = \frac{p^\star}{\rho^{\star\gamma}}$$ adiabatic blob

$$\rho^\star = \rho\left(\frac{p^\star}{p}\right)^{1/\gamma} \approx \rho\left(1 + \frac{1}{\gamma}\frac{d\ln p}{dr}\delta r\right)$$

$$\ddot{\delta r} + N^2\delta r = 0$$

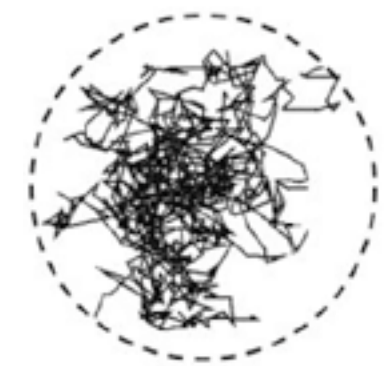
buoyancy force: $(\rho^\star - \rho_0^\star)g = \frac{\rho g}{\gamma}\frac{d}{dr}\ln\left(\frac{p}{\rho^\gamma}\right)\delta r$

$$N^2 = \frac{g}{\gamma}\frac{d}{dr}\ln\left(\frac{p}{\rho^\gamma}\right)$$

stably stratified if entropy higher at top

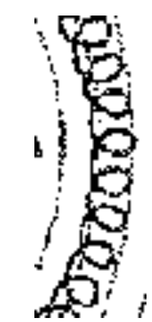
Magnetized, dilute plasmas

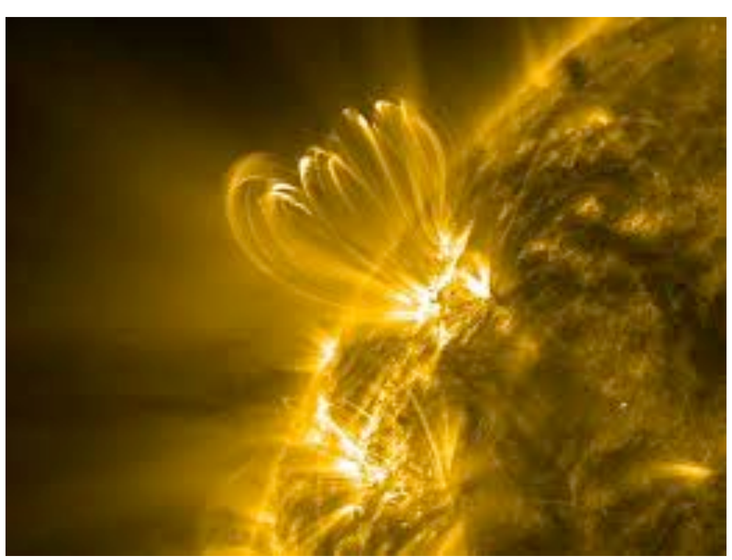
$$\mathbf{Q} = -\kappa \nabla T = -\chi n k_B \nabla T$$ for unmagnetized plasma



$$\mathbf{Q} = -\kappa \hat{b} \nabla_{\parallel} T = -\kappa \hat{b} (\hat{b} \cdot \nabla) T$$ for magnetized plasma

particles move along B w. small Larmor radii but diffuse along B with a path length of mfp; mfp>>ρ_L





solar corona

$$\mathrm{mfp} \sim 0.3\mathrm{kpc}\ T_8^2 n_{0.1}^{-1}$$

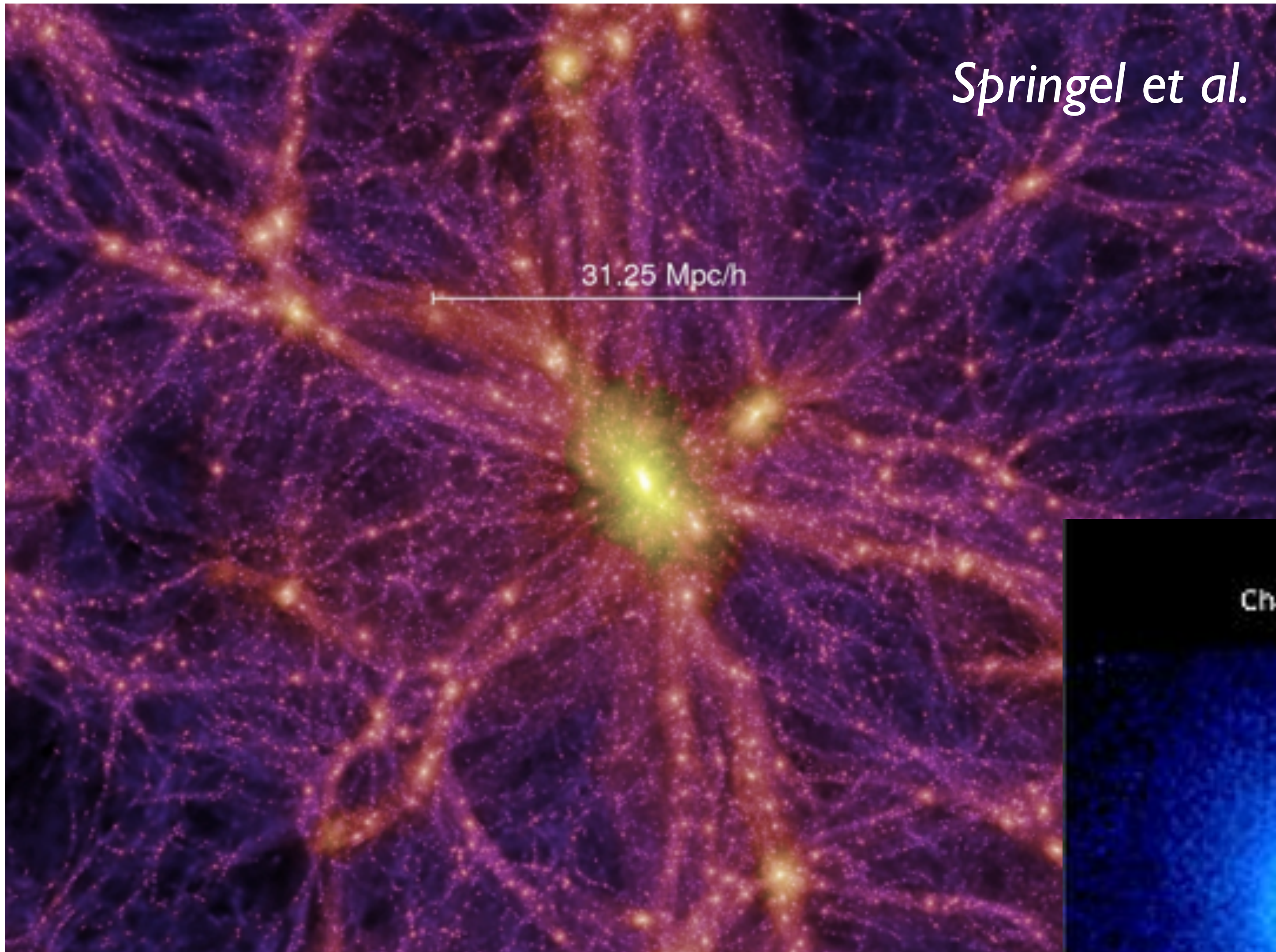
$$\rho_L/\mathrm{mfp} \sim 10^{-11} B_{\mu G}^{-1}$$

for typical clusters

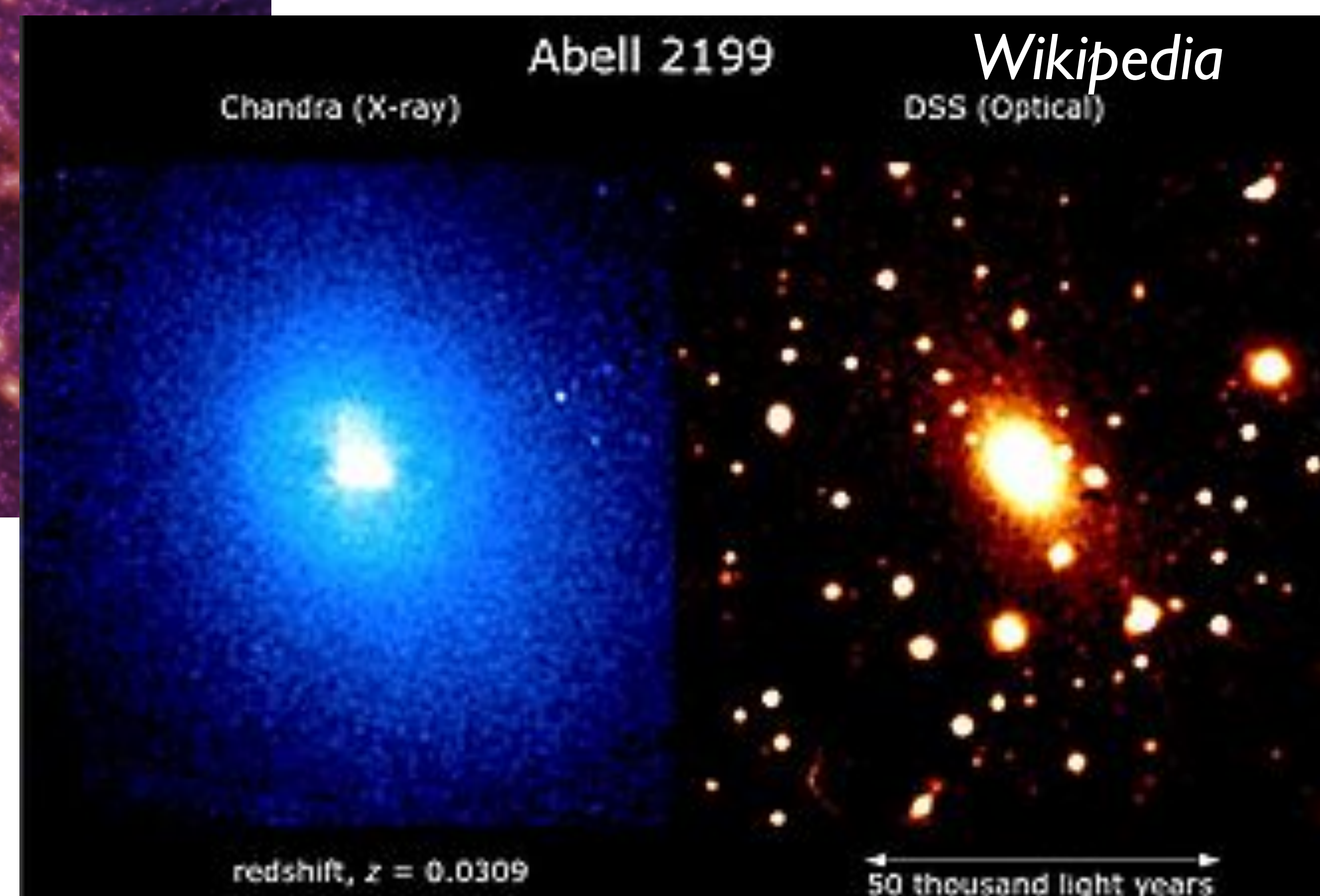
$$D_{\parallel} \sim \mathrm{mfp} \times v_t = v_t^2/\nu \gg D_{\perp} \sim \rho_L^2 \nu$$

true for all transport coeffts.

Galaxy clusters

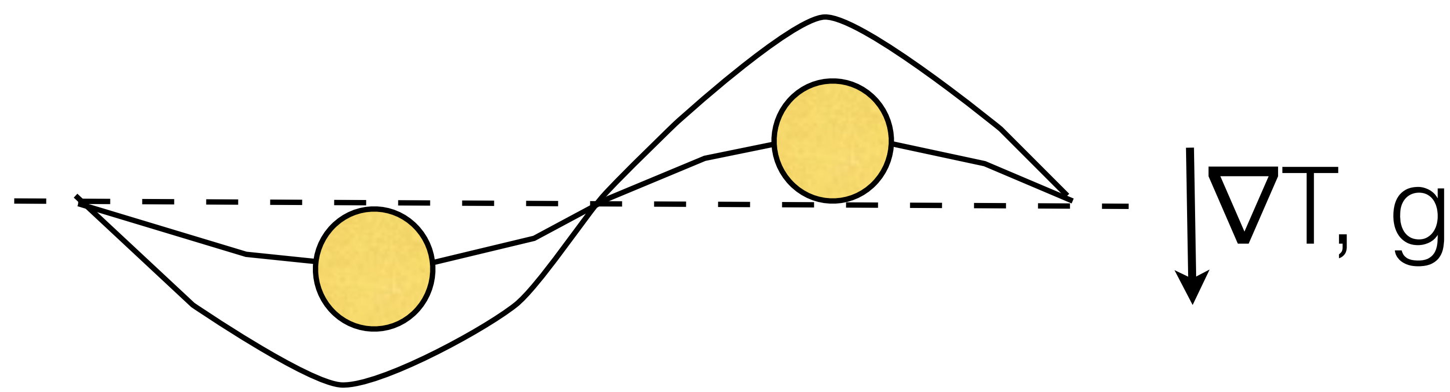


DM provides background gravity



most normal matter is in diffuse X-ray emitting plasma: ICM

Buoyancy instabilities w. anisotropic conduction



ρ_L << mfp < L

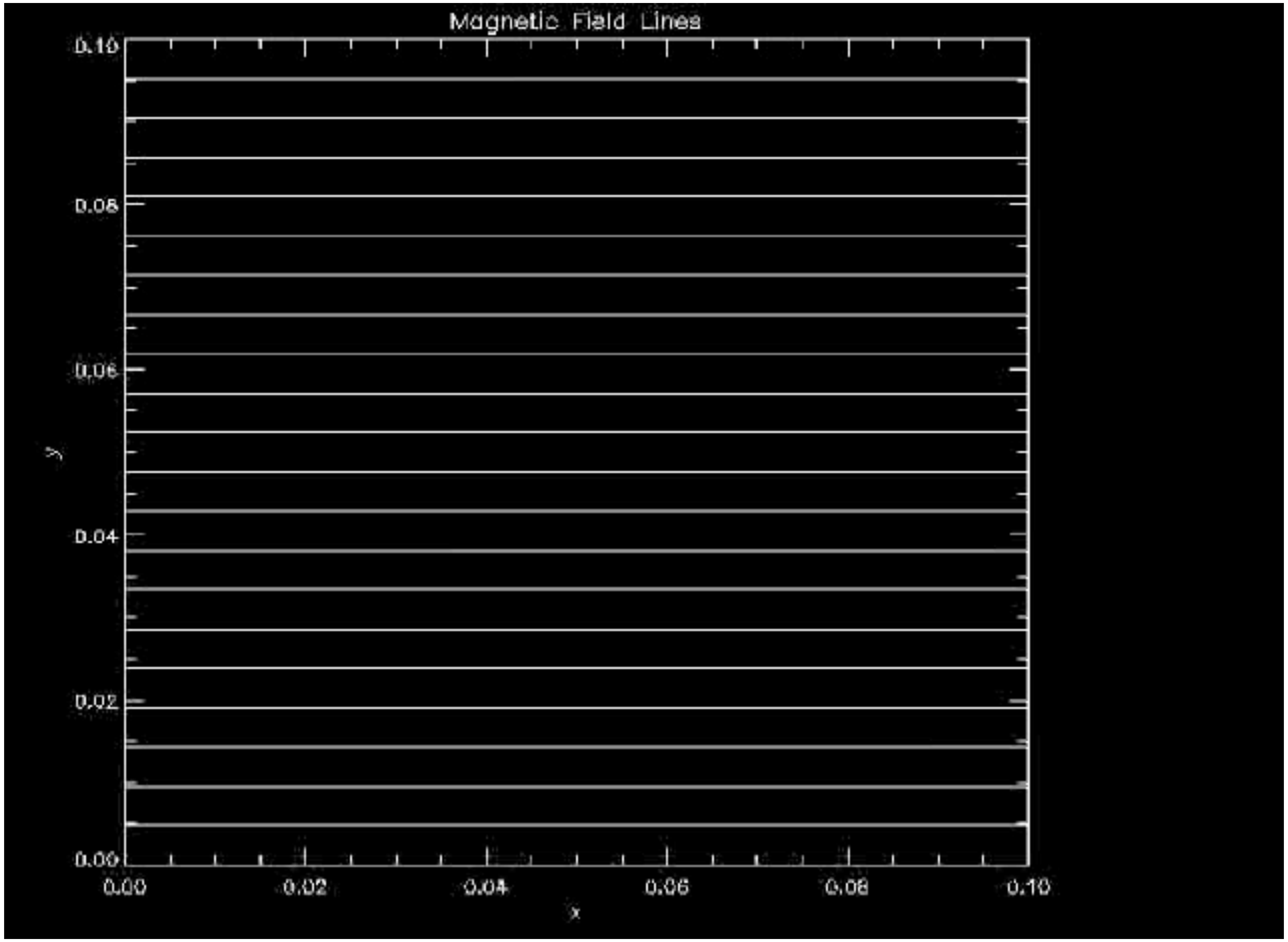
weak B: only role is aniso. cond.
$t_{cond} << t_{buoy} \sim (H/g)^{1/2}$

=> flow is unstable
if dT/dz < 0 even if ds/dz>0!
a similar instability for dT/dz>0!

[Balbus 2000, Quataert 2008]

Magnetothermal Instability (MTI)

[from Ian Parrish's website]



↓∇T, g

temperature maximum at bottom

reflective BCs at top and bottom

outflow more realistic

Equations w. aniso. conduction

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0,$$

$$\rho \frac{\partial \boldsymbol{v}}{\partial t} + \rho(\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = \frac{(\nabla \times \boldsymbol{B}) \times \boldsymbol{B}}{4\pi} - \nabla P + \rho \boldsymbol{g},$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}),$$

$$\rho T \frac{ds}{dt} = -\nabla \cdot \boldsymbol{Q} = \nabla \cdot \left[\kappa \hat{\boldsymbol{b}} (\hat{\boldsymbol{b}} \cdot \nabla) T\right],$$

zero for adiabatic fluids

$$s \equiv \frac{k_B}{\mu m_p} \ln\left(\frac{p}{\rho^\gamma}\right)$$ specific entropy

Linear Analysis

[Quataert 2008]

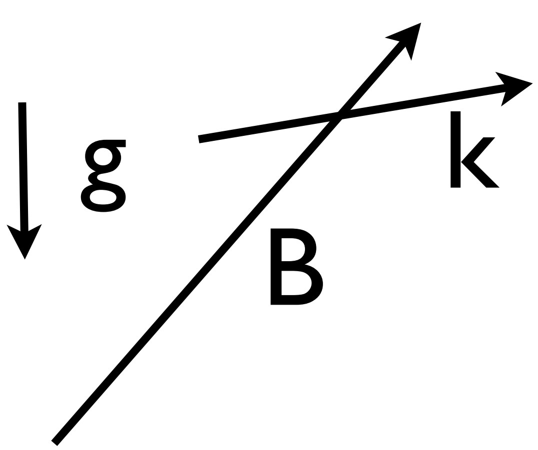
$$\mathbf{g} = -g\hat{z}; \mathbf{B} = B_x\hat{x} + B_z\hat{z}$$

weak B field $$dP/dz = -\rho g.$$

local WKB analysis $\exp(-i\omega t + i\boldsymbol{k}\cdot\boldsymbol{x})$, $\boldsymbol{k} = k_x\hat{\boldsymbol{x}} + k_y\hat{\boldsymbol{y}} + k_z\hat{\boldsymbol{z}}$

$kH \gg 1$ $\quad k_\perp^2 = k_x^2 + k_y^2$

dispersion relation:
$$0 = \omega\tilde{\omega}^2 + i\omega_{\rm cond}\tilde{\omega}^2 - N^2\omega\frac{k_\perp^2}{k^2} - i\omega_{\rm cond}\, g\left(\frac{d\ln T}{dz}\right)\left[\left(1-2b_z^2\right)\frac{k_\perp^2}{k^2} + \frac{2b_xb_zk_xk_z}{k^2}\right],$$



$$\tilde{\omega}^2 = \omega^2 - (\boldsymbol{k}\cdot\boldsymbol{v}_{\rm A})^2, \quad \omega_{\rm cond} = \frac{2}{5}\kappa\left(\hat{\boldsymbol{b}}\cdot\boldsymbol{k}\right)^2$$

Linear Analysis

[Quataert 2008]

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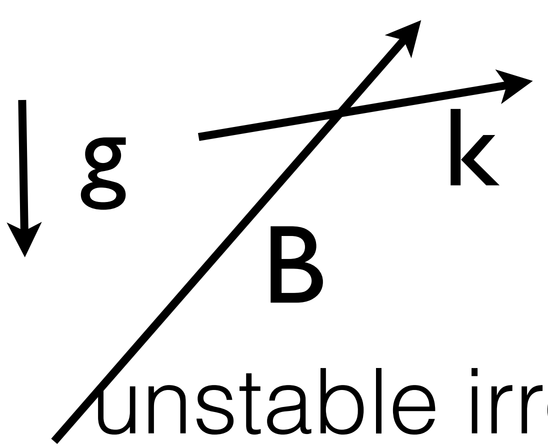
dispersion relation in fast conduction, weak B limit:

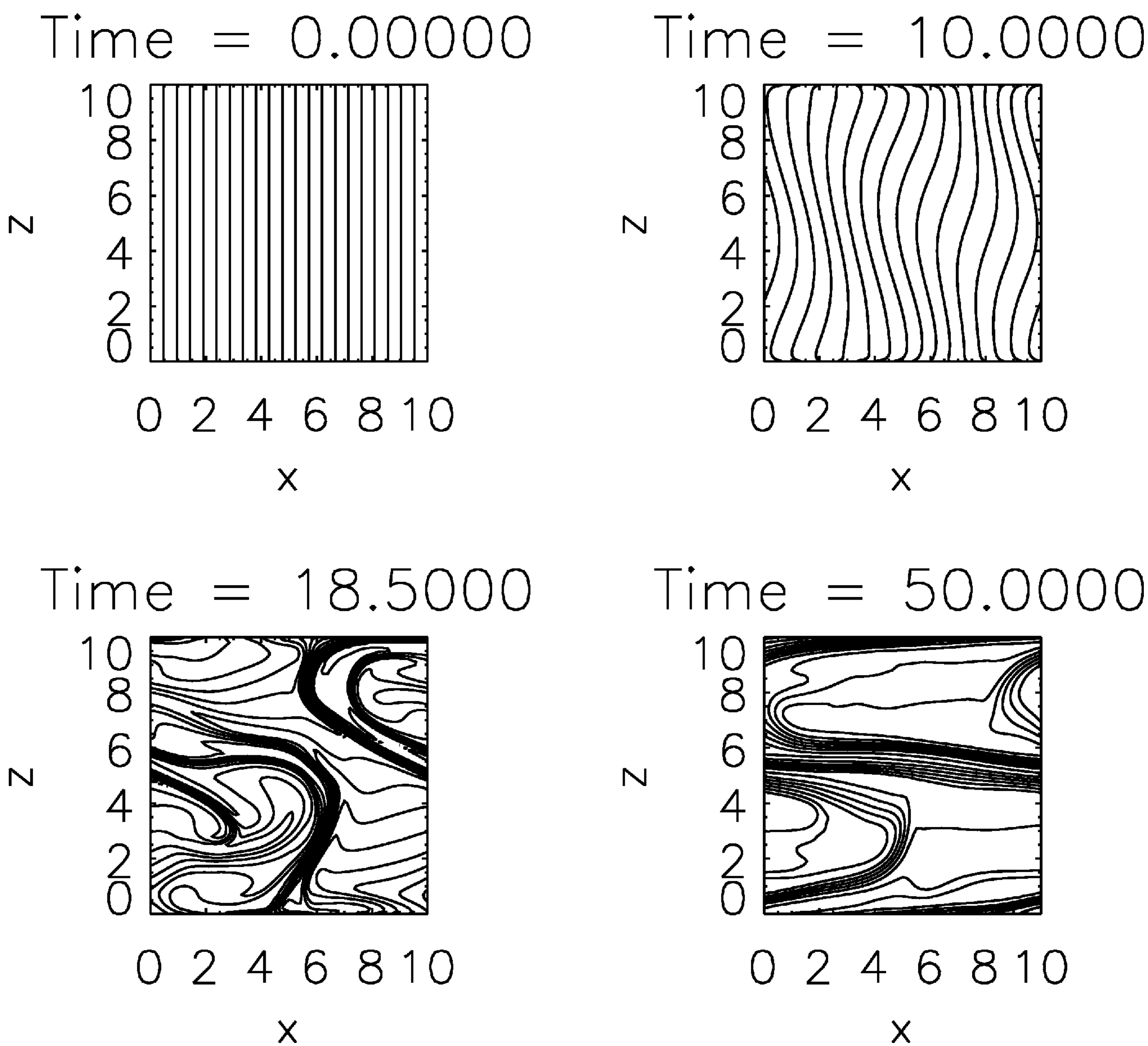


$$\omega^2 \simeq g\left(\frac{d\ln T}{dz}\right)\left[\left(1-2b_z^2\right)\frac{k_\perp^2}{k^2} + \frac{2b_xb_zk_xk_z}{k^2}\right]$$

unstable irrespective of dT/dz if I can choose B geometry!

Heat-flux Buoyancy Instability (HBI)

[Parrish & Quataert 2008]

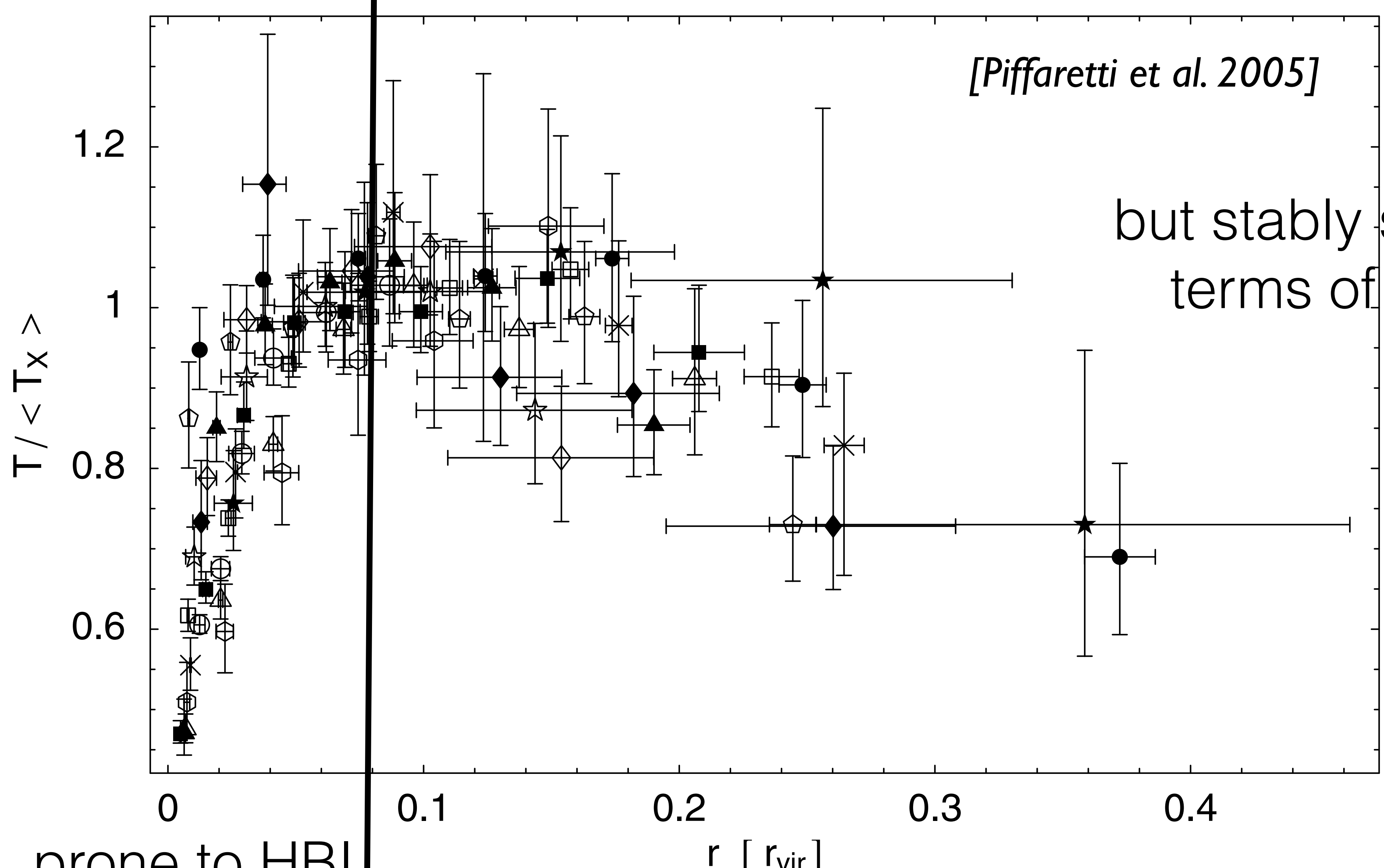


temperature
minimum at bottom

HBI reorients field lines
perpendicular to ∇T

cuts off thermal conduction

Temperature profiles in galaxy clusters



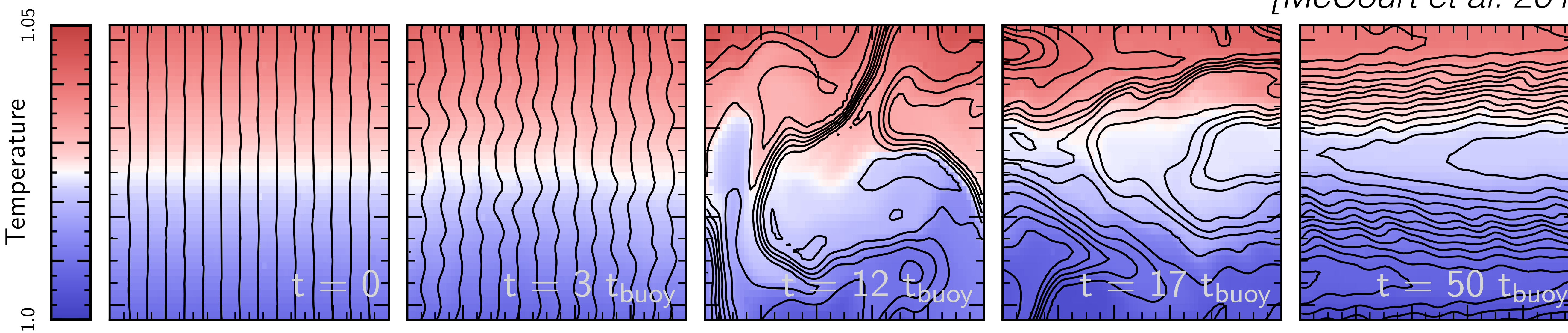
but stably stratified in terms of entropy!

prone to HBI

prone to MTI

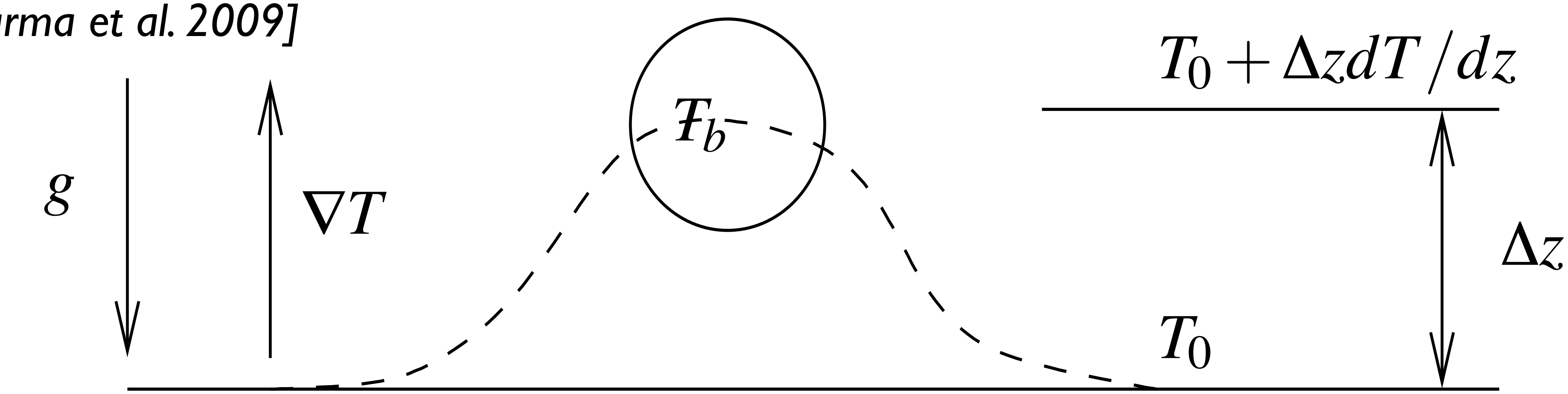
HBI saturated state is stable!

[McCourt et al. 2010]



HBI saturates by reorienting B; negligible fluid motion in saturated state; implication for cooling flow problem

[Sharma et al. 2009]

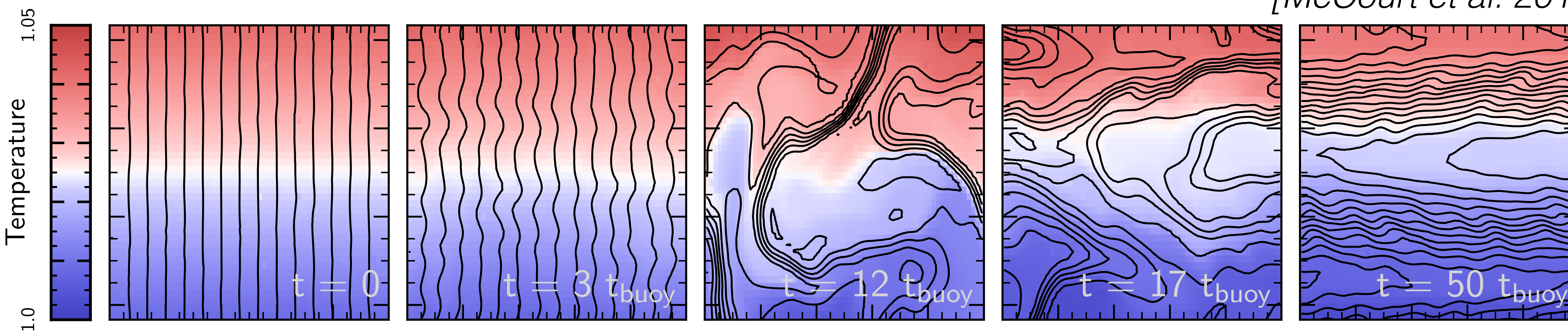


blob experiences a restoring force once B is reoriented; HBI is quenched in absence of vertical B; can define a turbulent Richardson number for mixing!

this stabilization can be overcome by a turbulent velocity of ~ 100 km/s

HBI saturated state is stable!

[McCourt et al. 2010]



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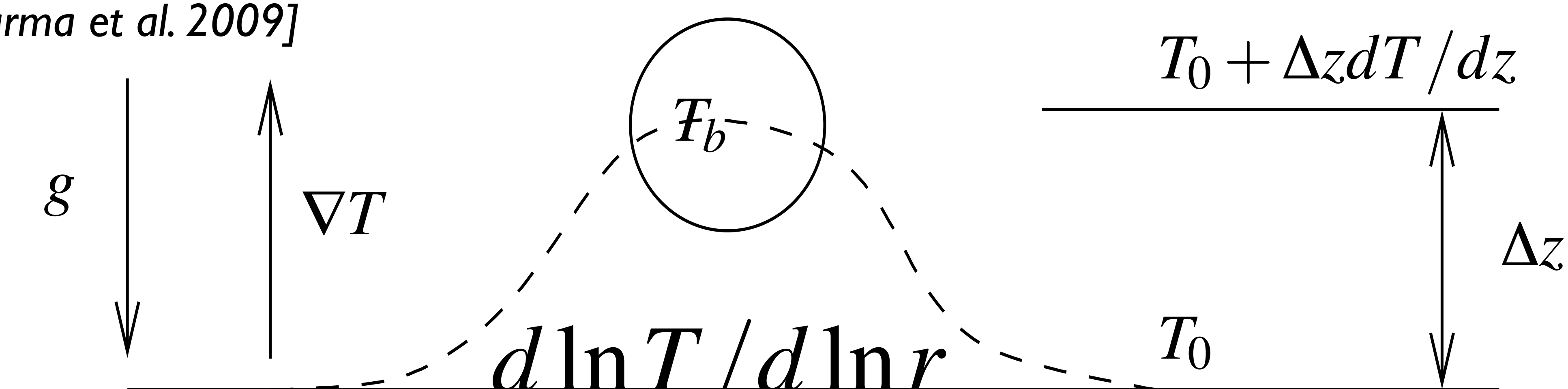


$$Ri \approx 3 g_{-8} r_{10} \frac{d \ln T / d \ln r}{u_{100}^2}$$

Mixing w. conduction

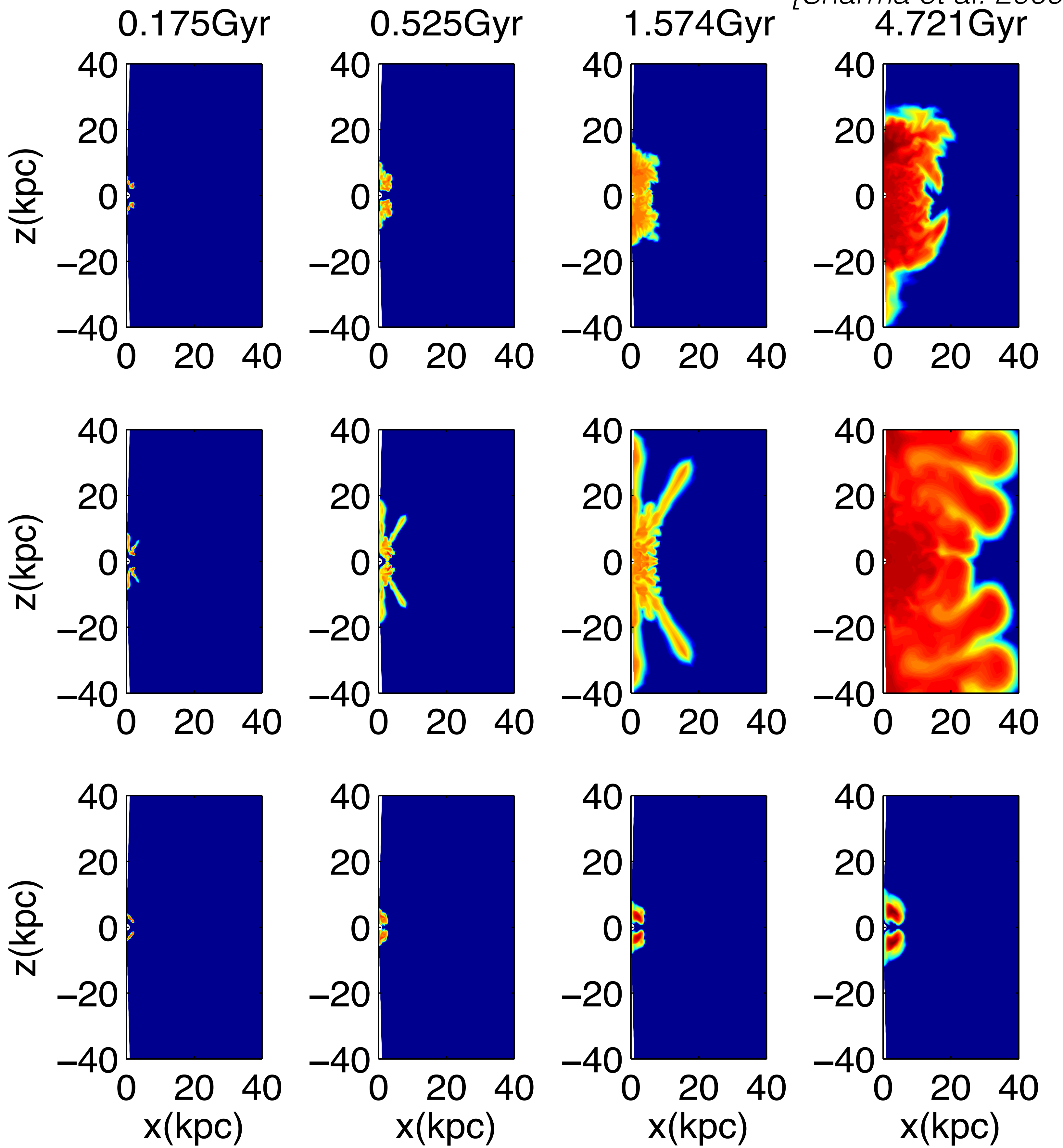
[Sharma et al. 2009]

passive scalar mixing:

anisotropic conduction

isotropic conduction

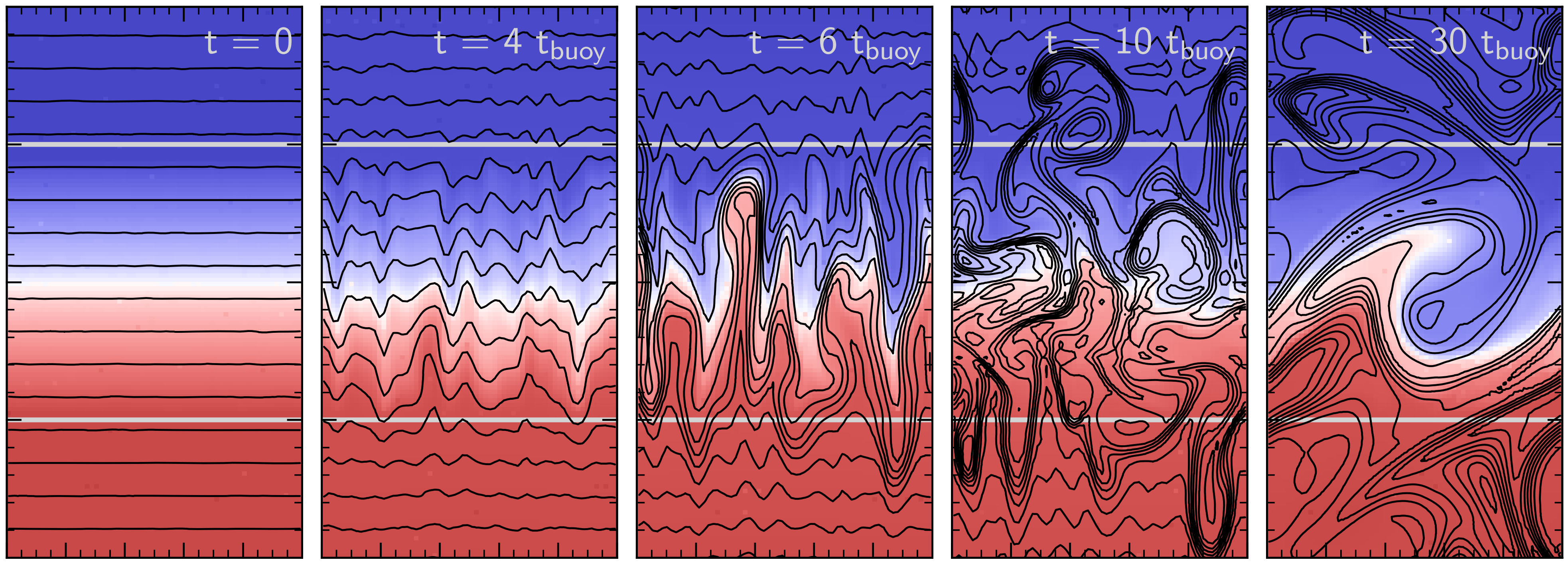
adiabatic



But MTI is a robust instability

[McCourt et al. 2010]

can stir clusters at large radii bias mass measurements of clusters that assume HSE



horizontal motion requires no energy as buoyancy force does not act. Flux-freezing creates horizontal B-field that is again unstable to MTI. Can lead to robust convection!

Magnetic reorientation doesn't shut it off, like normal magneto-convection

Summary

- magnetic tension & Braginskii viscosity further suppress HBI/MTI
- since mfp<~L, kinetic instabilities: mirror, IC, etc. a lot of plasma physics
- turbulence, transport and dynamos in the ICM
- implications for cooling flows and cluster mass estimates; hot accretion flows

Thank You!