Magnetic confinement of rotating convection: Implications for the Earth's tangent cylinder

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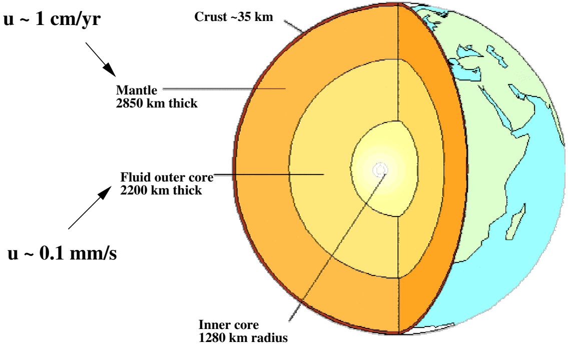
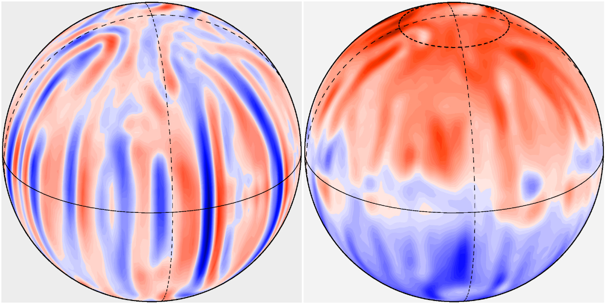
Earth's deep interior



- Thermochemical convection occurs in the liquid iron outer core.
- A highly dynamic, axial dipole-dominated magnetic field is generated.

A computer simulation of Earth's dynamo



$u_r(r = 0.8r_o)$ $B_r(r = r_o)$

Taylor columns; dipolar magnetic fields.

Nonlinear dynamo equations

$$
\begin{aligned}
E Pm^{-1}\left(\frac{\partial \mathbf{u}}{\partial t} + (\nabla \times \mathbf{u}) \times \mathbf{u}\right) + \hat{z} \times \mathbf{u} &= -\nabla p^{\star} + Ra\, Pm Pr^{-1}\; T\, \mathbf{r} \\
&\quad + (\nabla \times \mathbf{B}) \times \mathbf{B} + E \nabla^2 \mathbf{u}, \\
\frac{\partial \mathbf{B}}{\partial t} &= \nabla \times (\mathbf{u} \times \mathbf{B}) + \nabla^2 \mathbf{B}, \\
\frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla) T &= Pm Pr^{-1}\, \nabla^2 T, \\
\nabla \cdot \mathbf{u} &= \nabla \cdot \mathbf{B} = 0.
\end{aligned}
$$

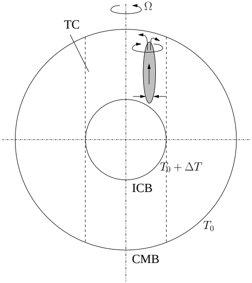
Dimensionless parameters:

$$
E = \frac{\nu}{2\Omega L^2},\; Ra = \frac{g\alpha \Delta T L}{2\Omega\kappa},\; Pr = \frac{\nu}{\kappa},\; Pm = \frac{\nu}{\eta}.
$$

Tangent cylinder convection

- A large-scale anticyclonic zonal flow in the polar regions is suggested, the likely cause of which is a thermal wind (Pedlosky, 1987):

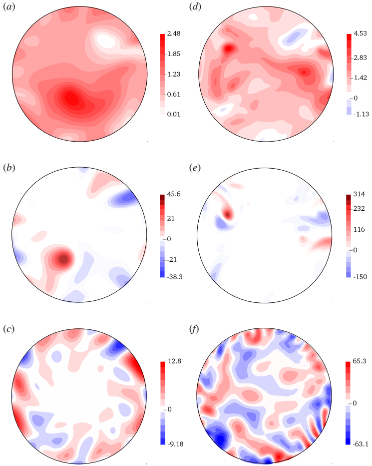
$$2\Omega \frac{\partial u_\phi}{\partial z} = \frac{g\alpha}{r}\frac{\partial T'}{\partial \theta}.$$



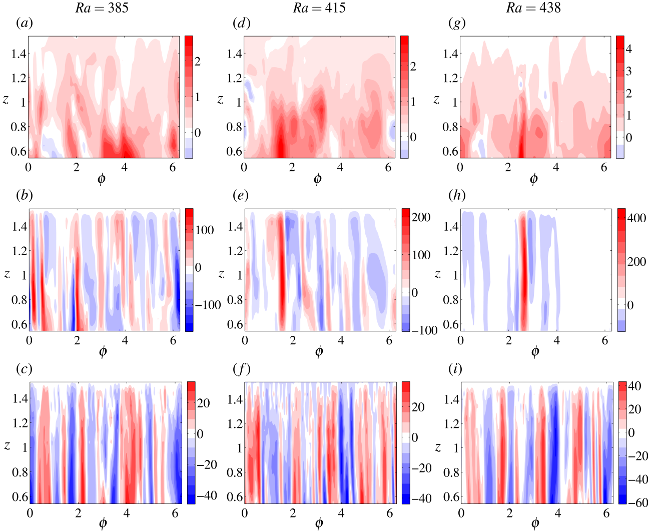
- Dynamos show an isolated off-axis plume within the TC, producing a non-axisymmetric polar vortex.

Dynamo vs non-magnetic convection

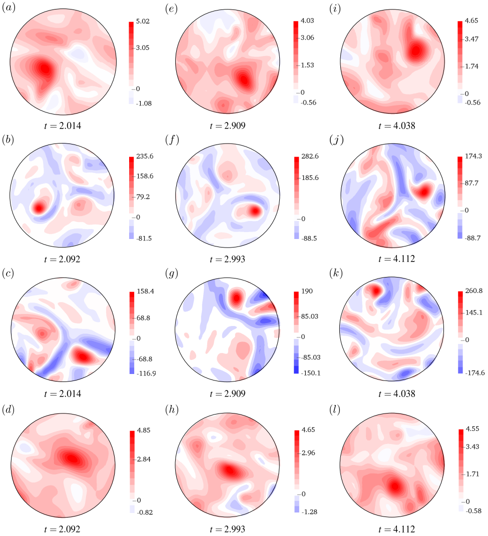
$E = 5 \times 10^{-5}$ $(Ra = 190)$ $E = 5 \times 10^{-6}$ $(Ra = 438)$



Dynamo vs non-magnetic convection



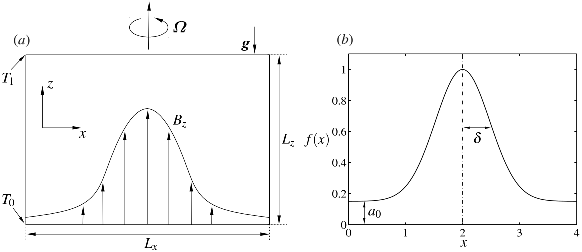
A delayed B_z–u_z correlation ($Ra = 350$)



Summary of results from dynamo simulations

- The magnetic field *upon its formation* appears to strongly localize convection within the tangent cylinder (TC). The onset of an isolated plume is noted for a threshold z-field strength, which is in turn dependent on Ra.
- It is plausible that the laterally varying magnetic field locally reduces the Rayleigh number for onset, which suppresses convection in regions where the field is weak.
- The critical Rayleigh number Ra_c for plume formation increases with decreasing E, suggestive of viscous mode onset.
- In addition, the plume width decreases with decreasing E.

Onset of convection under a laterally varying field



$$\mathbf{B}_0 = \mathcal{B}_0 f(x)\hat{\mathbf{z}};\;\; f(x) = a_0 + a_1 \exp\left[-(x-c)^2/2\delta^2\right].$$

- Background field is small relative to the peak field but not zero.
- The imposed field generates no mean flow as $\nabla \times (\mathbf{j}_0 \times \mathbf{B}_0) = 0$.

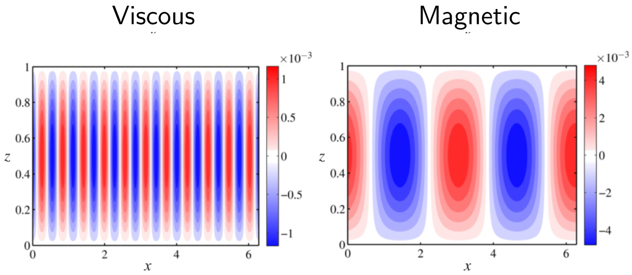
Linearized MHD equations

$$\begin{aligned}
EPm^{-1}\frac{\partial \mathbf{u}}{\partial t} + \hat{\mathbf{z}} \times \mathbf{u} = -\boldsymbol{\nabla} p + \Lambda\big[(\boldsymbol{\nabla} \times \mathbf{B}_0) \times \mathbf{b} + (\boldsymbol{\nabla} \times \mathbf{b}) \times \mathbf{B}_0\big] \\
+ PmPr^{-1}Ra\theta\hat{\mathbf{z}} + E\nabla^2\mathbf{u}, \\
\frac{\partial \mathbf{b}}{\partial t} = \boldsymbol{\nabla} \times (\mathbf{u} \times \mathbf{B}_0) + \nabla^2\mathbf{b}, \\
\frac{\partial \theta}{\partial t} = \mathbf{u}\cdot\hat{\mathbf{z}} + PmPr^{-1}\nabla^2\theta, \\
\boldsymbol{\nabla}\cdot\mathbf{u} = \boldsymbol{\nabla}\cdot\mathbf{b} = 0.
\end{aligned}$$

The operators $(\boldsymbol{\nabla}\times)$ and $(\boldsymbol{\nabla} \times \boldsymbol{\nabla}\times)$ are applied to the momentum equation and $(\boldsymbol{\nabla}\times)$ to the induction equation and their z-components are taken. Time dependence is not considered.

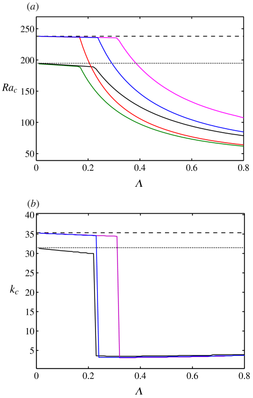
$$\begin{aligned}
\big[u_z', \omega_z', b_z', j_z', \theta'\big](x, y, z) = \big[u_z(x, z), \omega_z(x, z), b_z(x, z), \\
j_z(x, z), \theta(x, z)\big] \exp{(iky)}.
\end{aligned}$$

Solution for a homogeneous magnetic field



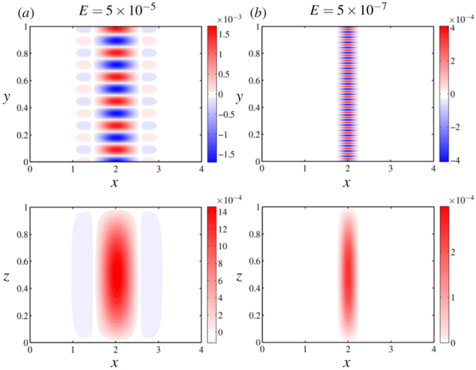
- Two modes of onset showing thin viscous columns and large-scale magnetic rolls. Transition from the viscous to magnetic mode occurs at $\Lambda = O(E^{1/3})$ (e.g. Jones, Mussa & Worland, 2003). ($\Lambda = \mathcal{B}_0^2/2\Omega\rho\mu_0\eta$.)
- Critical (onset) wavenumber decreases sharply at the transition. The onset Rayleigh number progressively decreases in the magnetic mode.

Regime diagram for linear magnetoconvection



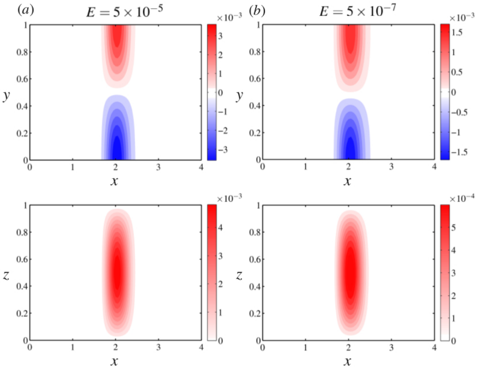
Non-magnetic: horizontal dashed (stress-free) and dotted (no-slip). Uniform field: red (stress-free); green (no-slip). Inhomogeneous field: blue (stress-free), black (no-slip). $E = 5 \times 10^{-5}$, $Pm/Pr = 1$.

Axial velocity (u_z) at viscous mode onset



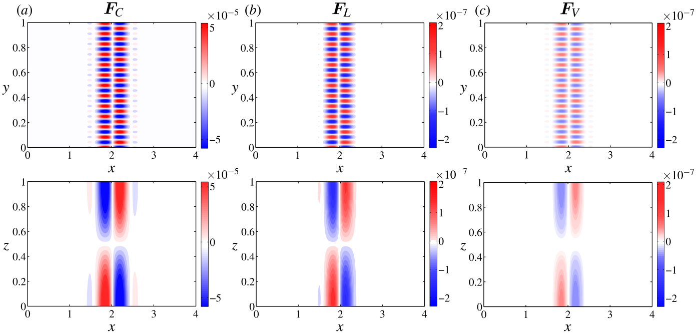
- Elsasser number $\Lambda = 0.04$. (a) $Ra_c = 237.57$; (b) $Ra_c = 1095.7$.
- Convection is strongly localized by the field, but the smallest length scale is viscously controlled!

Axial velocity (u_z) at magnetic mode onset



- Elsasser number $\Lambda = 0.3$. (a) $Ra_c = 189.01$; (b) $Ra_c = 192.2$.
- Convection is strongly localized by the field and magnetically controlled; hence plume width and Ra_c are nearly independent of Ekman number.

Force balance at viscous mode onset



- $E = 5 \times 10^{-6}$, $\Lambda = 0.08$.
- The Lorentz force is influential in setting up convection at the small viscous length scale.

General conclusions

- A laterally varying axial magnetic field localizes convection in a rotating plane layer. An isolated plume follows the path of the peak magnetic field.
- Of particular interest is the localized excitation of viscous-mode convection.
- Earlier studies do not consider this possibility. They predict convection either in the small-scale viscous mode or in the large-scale magnetic mode, with the viscous–magnetic cross-over occurring at $\Lambda = O(E^{1/3})$.
- The critical Rayleigh number for the formation of an isolated plume agrees closely with the onset value for (linear) viscous magnetoconvection in a plane layer.
- Formation of an isolated plume within the TC is therefore approximately linear, even as nonlinear dynamo action exists outside the TC.

Implications for the Earth's core

- The localization of viscous-mode (small-scale) convection provides a mechanism for the formation of isolated plumes within Earth's TC, which can expel flux from high latitudes. This process is inferred from weak flux in the polar region as well as the location of flux lobes just outside the TC (e.g. jackson, Jonkers & Walker, 2000; Gubbins, Willis & Sreenivasan, 2007).
- The polar vortices within the Earth's core can be non-axisymmetric.
- Small-scale convection can exist under an intense, spatially varying field.