

# Stratified Geophysical Flows.

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## The Setup...

- Example from **geophysical fluids**.
- The baroclinic term.
- Boussinesq and other approximations.
- A measure of the strength of stratification.
- Summary of basic equations & conservations laws.

# Basic Features

- Plane waves in stratified flows  $\Rightarrow$  Vortical mode and Internal gravity waves.
- Properties of vortical and wave modes.
- Laboratory examples.

## If time permits...

- Nondimensionalization and asymptotic analysis in the limit of strong stratification.
- Stratified turbulence as viewed through vortical and wave mode decomposition.
- A sketch of the evolution of a stratified flow.



FIGURE 1.1 A surrealistic representation of atmospheric gravity waves. Taken from [Hines \(1974\)](#).

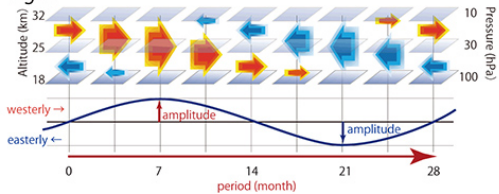
## An Example

- The **Quasi-biennial oscillation** (QBO).
- Easterly and westerly wind regimes in the **equatorial stratosphere**, 24-30 months.
- Earlier hints at changing stratospheric equatorial winds, but an oscillation (later named QBO) was identified around 1960 (independently by Reed and Ebdon).
- *Baldwin et al., The QBO, Reviews of Geophysics, 30, 2001.*
- A remarkably robust feature of the atmosphere.

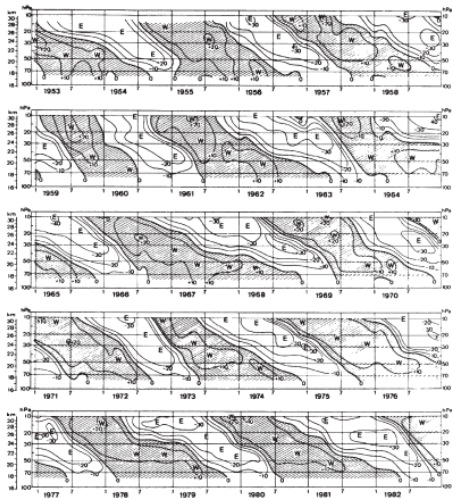
Fig2a



Fig2b

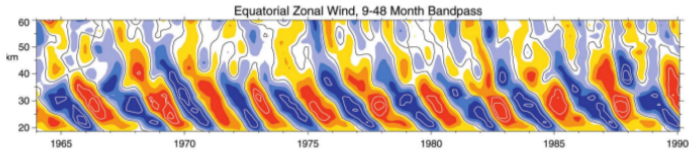


The QBO.



The QBO.





The QBO. From point data (radiosonde below 31 km & rocketsonde above). Longitudinal symmetry is strong, so no need for a zonal average.

## QBO: features

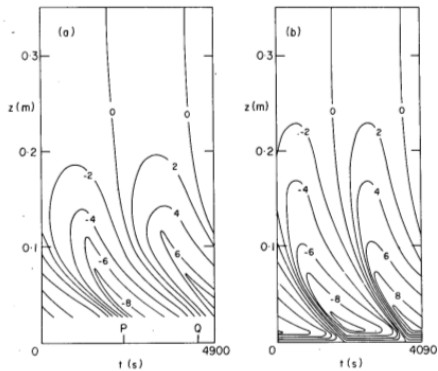
- Zonal winds appear at  $\approx 30$  km, downward propagation at about 1 km/month till approx. 20 km.
- Amplitudes of about 20 m/s and half-width of  $12^\circ$ , symmetric about the equator.
- Key features:
  - Quasi-biennial periodicity.
  - Downward propagation with not much loss of amplitude.
  - Superrotation  $\Rightarrow$  eddy momentum fluxes.

## QBO: theory

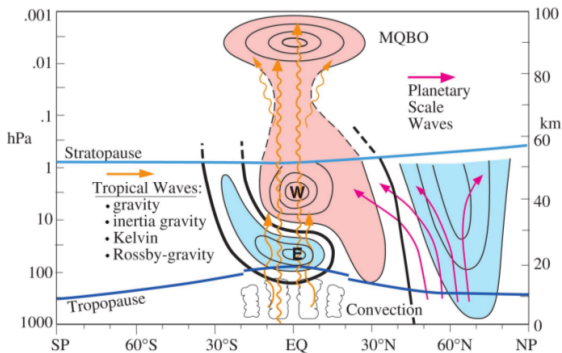
- Longitudinally symmetric but **requires zonally asymmetric waves**.
- Wallace & Holton 1968 — mechanism denial for extratropical planetary waves.
- Vertically propagating internal waves are a key ingredient (Lindzen & Holton 1968, 1972) — **mixed Rossby-gravity, Kelvin** and inertia-gravity waves.
- Essentially, **a wave-mean flow** interaction problem.
- The dynamics of two internal waves interacting with a mean flow brought out clearly in Plumb 1977.

## QBO: lab analog

- Remarkable laboratory experiment by Plumb & McEwan 1978.
- Salt stratified fluid in an annulus.
- Bottom membrane oscillated to generate **clockwise and anti-clockwise vertically propagating gravity waves**.
- Recorded a wave induced mean flow, downward progressing with periodic reversals!



From Plumb & McEwan 1978. Lab measurement (left panel) and theoretical prediction (right panel) of the wave induced mean flow.



Modern schematic of QBO dynamics (Baldwin et al. 2001). Actual spectrum of tropical waves for QBO? In GCMs? Parameterization? **Why is it so regular?**

- Begin basic equations...

# Bjerknes 1898

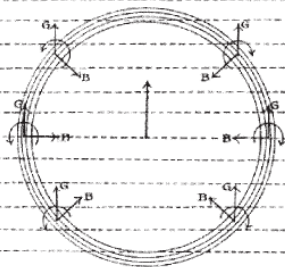


FIG. 4. Reproduction of Fig. 6 from Bjerknes (1898), shows isobaric surfaces with dashed lines (with the pressure gradient vector  $G$ ) and isosteric surfaces as solid lines. This illustrates Archimedes' principle with a mass of less dense air and the consequent development of circulation leading to the ascent of the mass.



## Bjerkenes' examples

- Rising blob of low density fluid.
- Sea breeze — quantitative estimate!
- Extratropical storms.
- Review in *Thorpe et al., The Bjerkenes' Circulation Theorem, BAMS, 2003.*
- Another excellent resource is, *Norbury & Roulstone, Invisible in the Storm, Princeton Univ. Press, 2013.*

## Boussinesq Equations

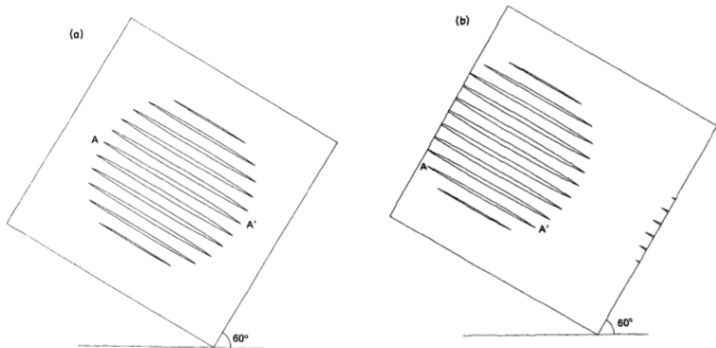
- In terms of  $(u, v, w, T', \phi')$ ,

$$\begin{aligned}\frac{D\mathbf{u}_h}{Dt} &= -\nabla\phi', \\ \frac{Dw}{Dt} &= \frac{\partial\phi'}{\partial z} + (\alpha g)T', \\ \frac{DT'}{Dt} &= \lambda w, \\ \nabla \cdot \mathbf{u} &= 0.\end{aligned}$$

- Note,  $\phi' = \frac{\rho'}{\rho_0}$  and  $\lambda = \frac{\partial\bar{T}}{\partial z}$ .
- Simplest case,  $\lambda$  is a constant, i.e., linear  $\bar{\rho}(z)$ ,  $\bar{T}(z)$  profiles.

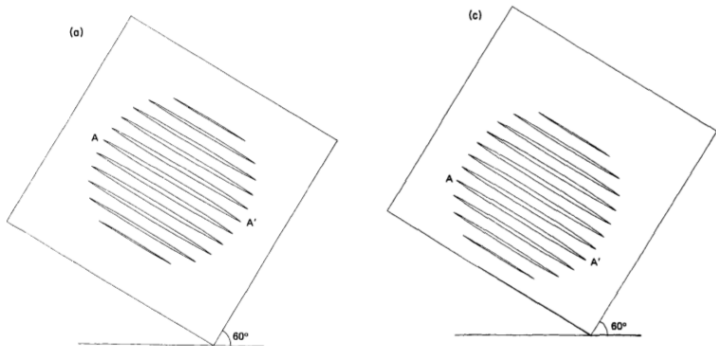
- Second Lecture...

# Internal Gravity Waves



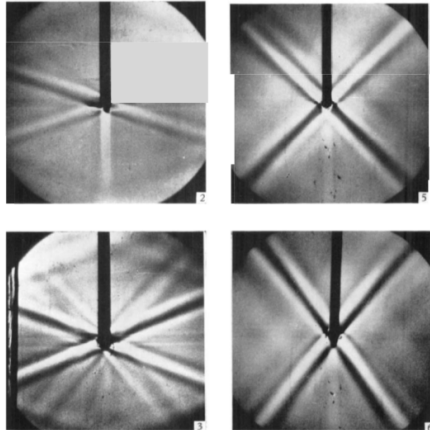
From Gill, Atmosphere Ocean Dynamics.

## External (surface) Gravity Waves



From Gill, Atmosphere Ocean Dynamics.

## Lab Internal Gravity Waves



The image is a series of four photographs showing the development of internal gravity waves in a stratified fluid. The waves are generated by a vertically oscillating source, which is represented by the vertical black line in the center of each frame. The waves propagate outwards from the source, creating a series of bright and dark radial bands. The patterns evolve from a simple vertical line to a complex, multi-lobed structure.

From Mowbray & Rarity, JFM, 28, 1967. Stratified fluid with a vertically oscillating source.