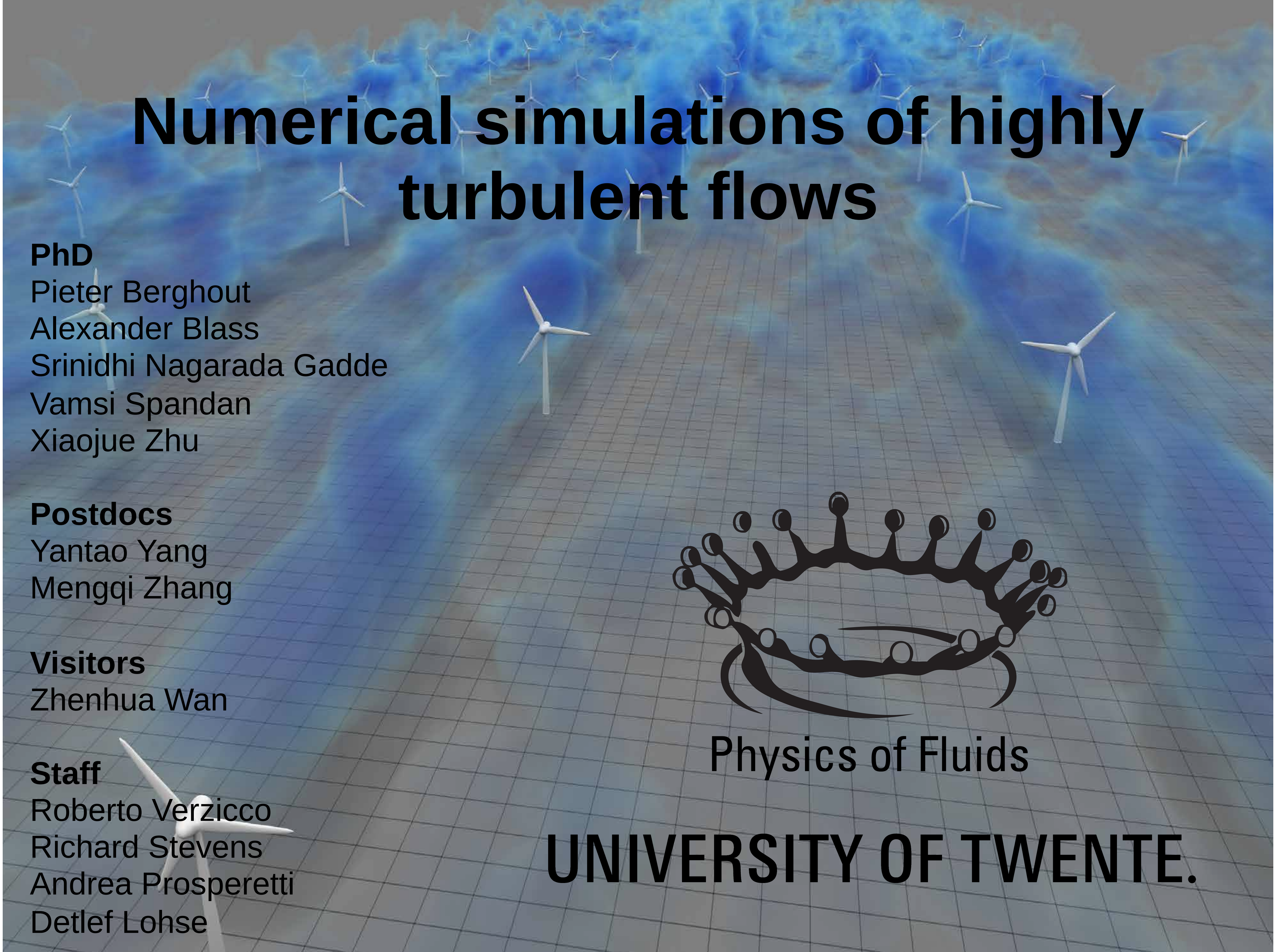


Numerical simulations of highly turbulent flows

PhD
Pieter Berghout
Alexander Blass
Srinidhi Nagarada Gadde
Vamsi Spandan
Xiaojue Zhu

Postdocs
Yantao Yang
Mengqi Zhang

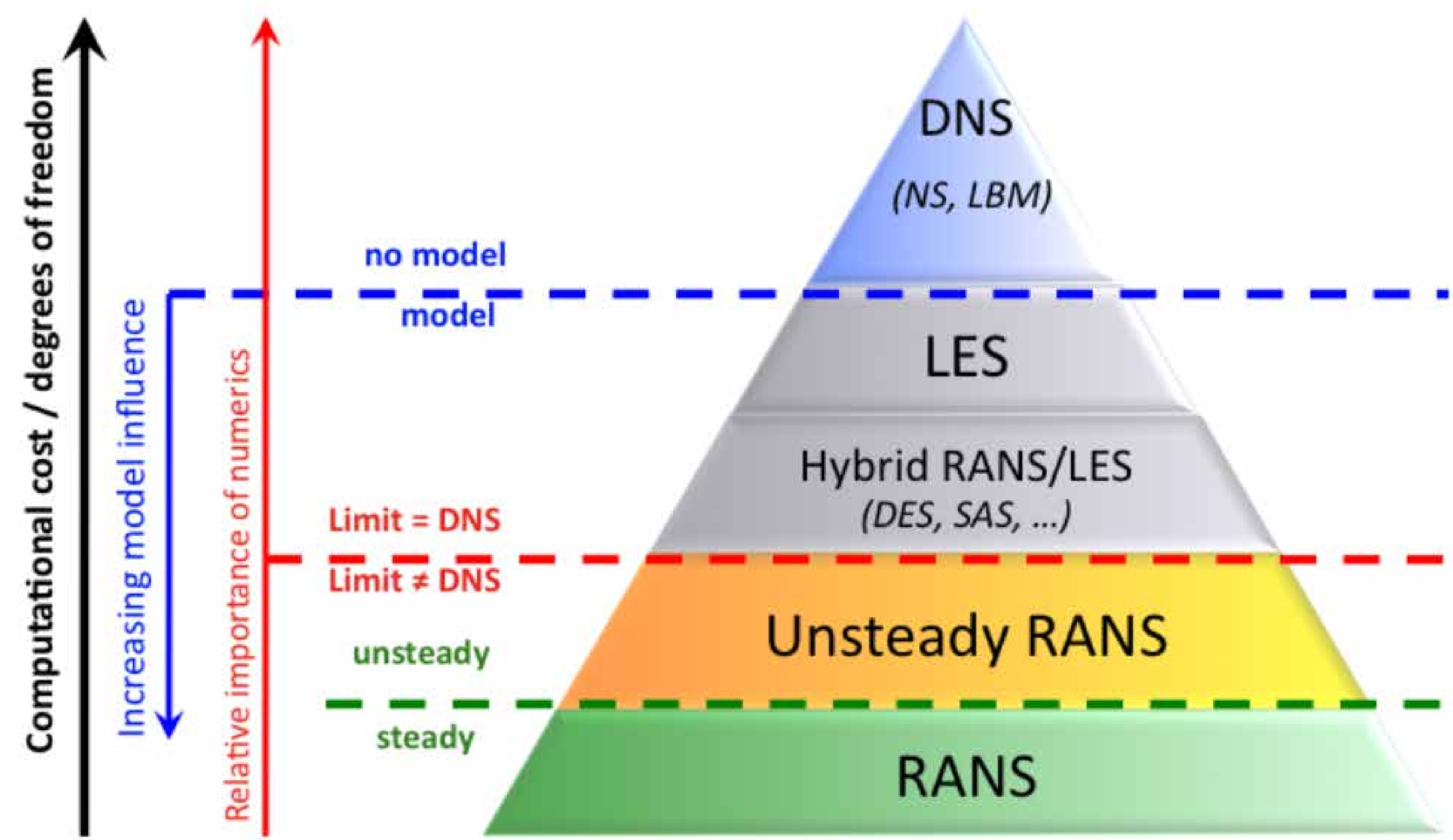
Visitors
Zhenhua Wan

Staff
Roberto Verzicco
Richard Stevens
Andrea Prosperetti
Detlef Lohse

Physics of Fluids

UNIVERSITY OF TWENTE.

Modeling approaches



Multiscale & Multiresolution approaches for turbulence
Sagaut, Deck & Terracol, Imperial College Press, 2006

Industrial framework

- Need to simulate complex flows
  - Multi-components problems (airplane, car)
  - Non-homogeneous flows
  - High Reynolds numbers
  - Multi phase or reacting flows
- Need a result in 1 hour to maximum 1 day
  - Research and development tool
  - Large number of parameters to investigate
- Limited computational resources
- The cost of the numerical simulations is important

- Need fast and robust methodologies
- Does not have the time to understand the physics.

Research framework

- Restricted to simple (or simplified) geometries
  - Isotropic turbulence (period boundary conditions)
  - Turbulent boundary layer flows
  - Rayleigh-Benard flow
  - Pipe, channel flow, etc.
- Not limited in time: can afford a one year simulation
- Access to largest computers (>10 Peta Flops)
- The cost is not so important
  - The cost is focused in some large simulations
  - The data are accessible to the research community
  - Cost needs to be related to the scientific outcome
- Numerical simulation is a tool to understand physics
- Need optimized algorithm and accurate results

Industrial and research practices

- Industrial practices
  - Most of simulations in steady states (RANS models)
  - Unsteady simulations restricted to specific parts of the flow (URANS,DES, LES)
  - Use of robust numerical techniques (usually not accurate)

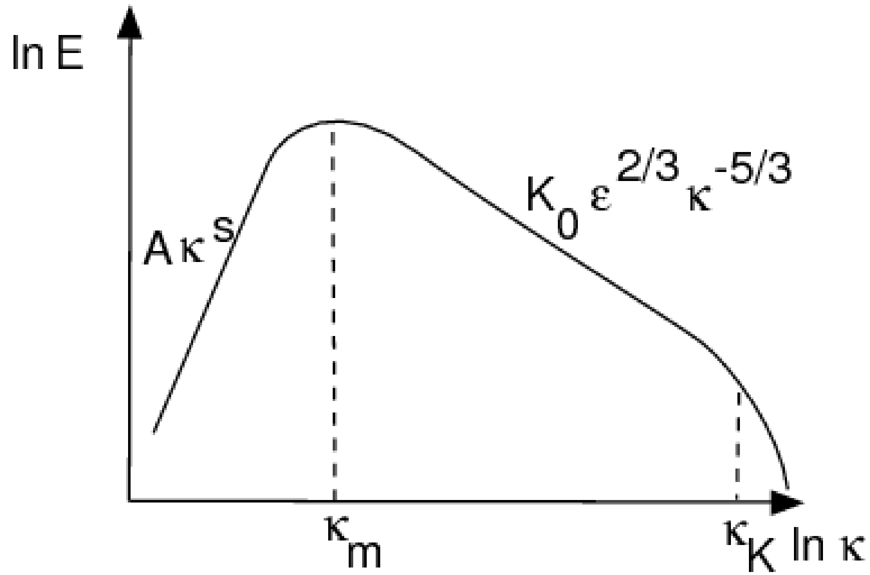
- Research practice
  - Interest on unsteady flows (turbulence) to understand the physics
  - Increasing popularity and use of DNS and LES
  - Intensive research on turbulence models
    - Increasing number of LES models since 1980’s
    - New development in RANS models (more than 100 models !)

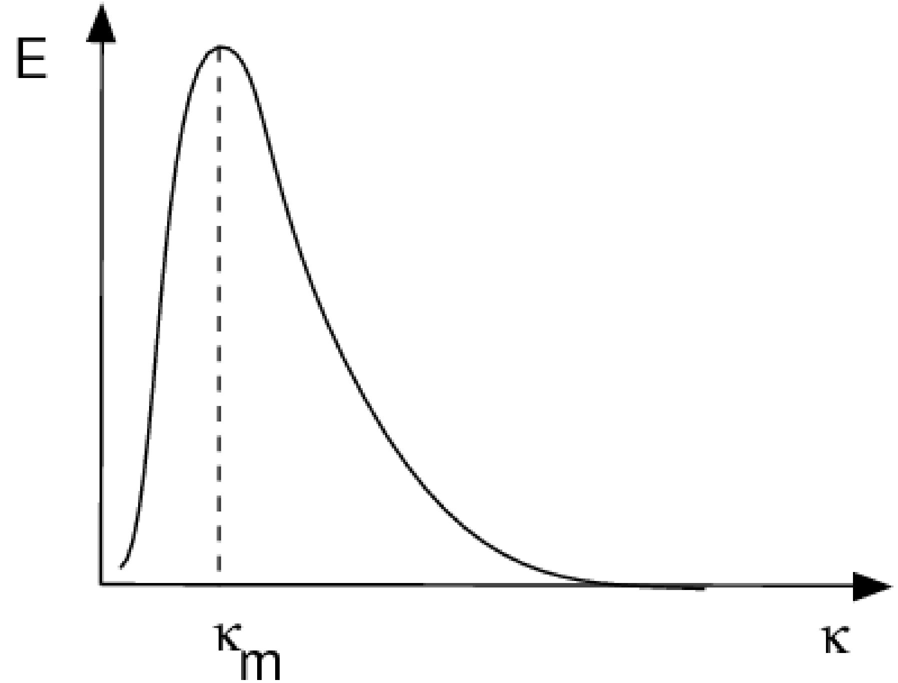
How to select your model?

- What is the Reynolds number?
- Is the flow turbulent?
- Can I perform a DNS of this flow at reasonable cost?
- What is the maximum cost (money and time) I can afford?
- Is it important to do unsteady simulations (noise emission, ...) ?
- Do I need statistics at small scales (chemical reactions, ...)?

Then, you must do some **compromises** between the accuracy and the cost of your simulation.

Energy spectrum of turbulent flows

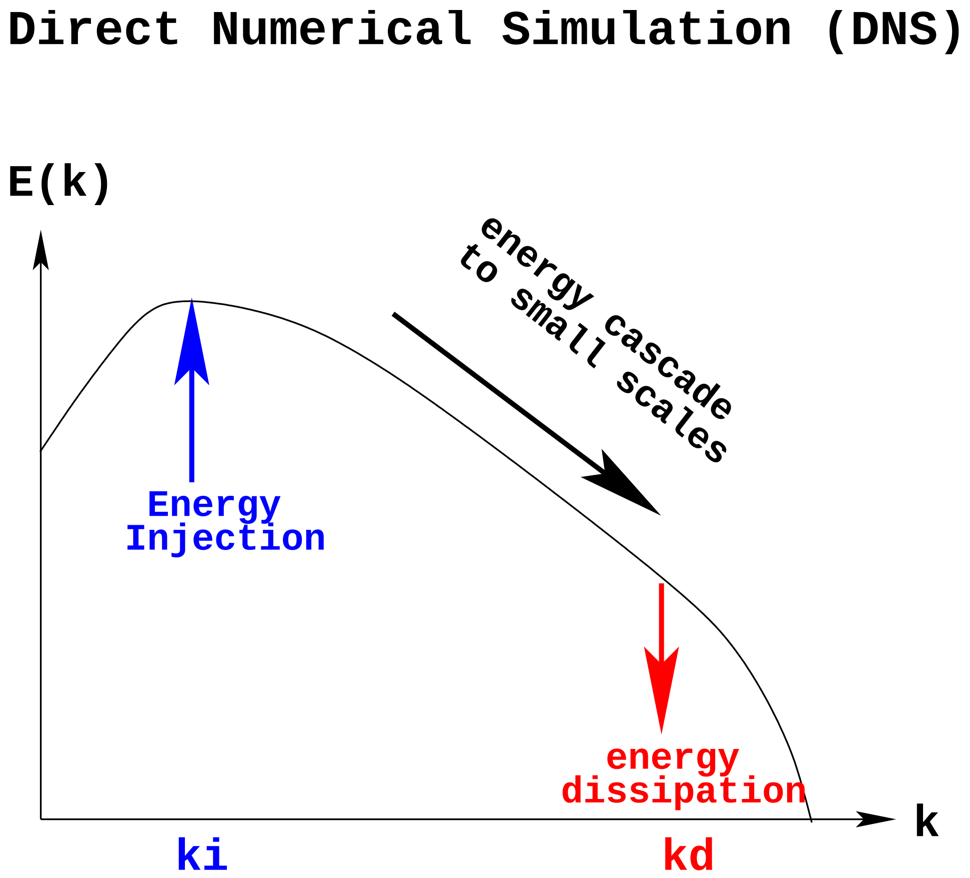


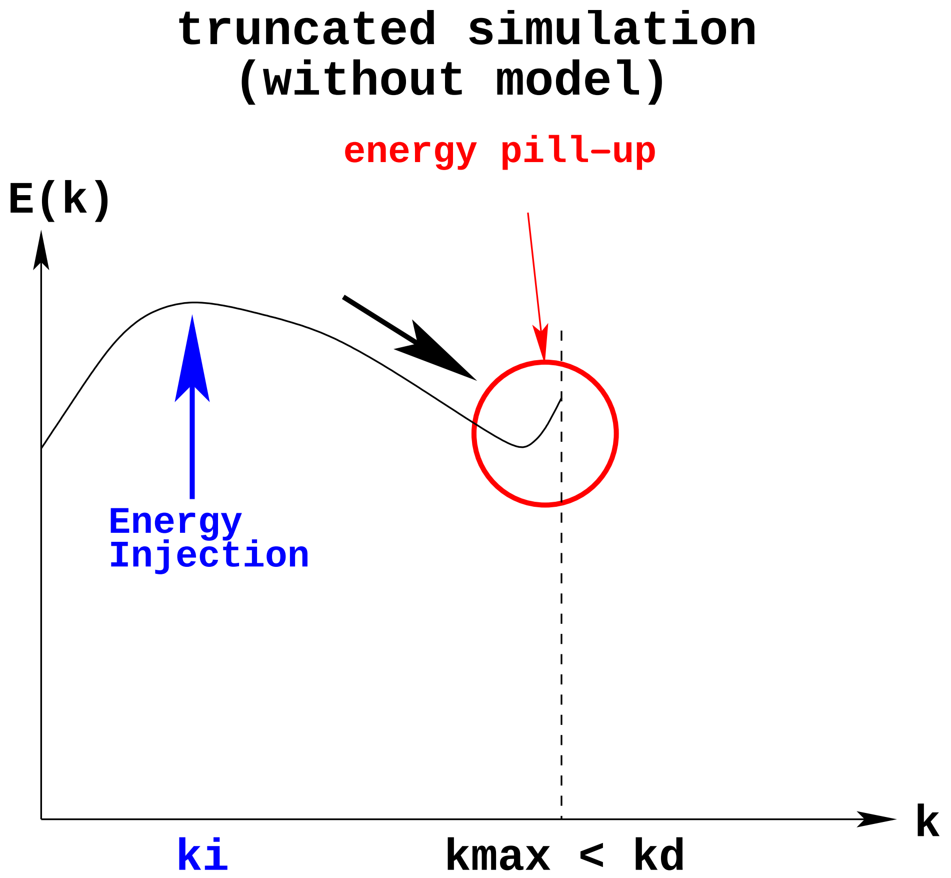


$$E(\kappa) = \begin{cases} A\kappa^s & \text{when } \kappa \leq \kappa_m \\ K_0 \varepsilon^{2/3} \kappa^{-5/3} & \text{when } \kappa \geq \kappa_m \end{cases}$$

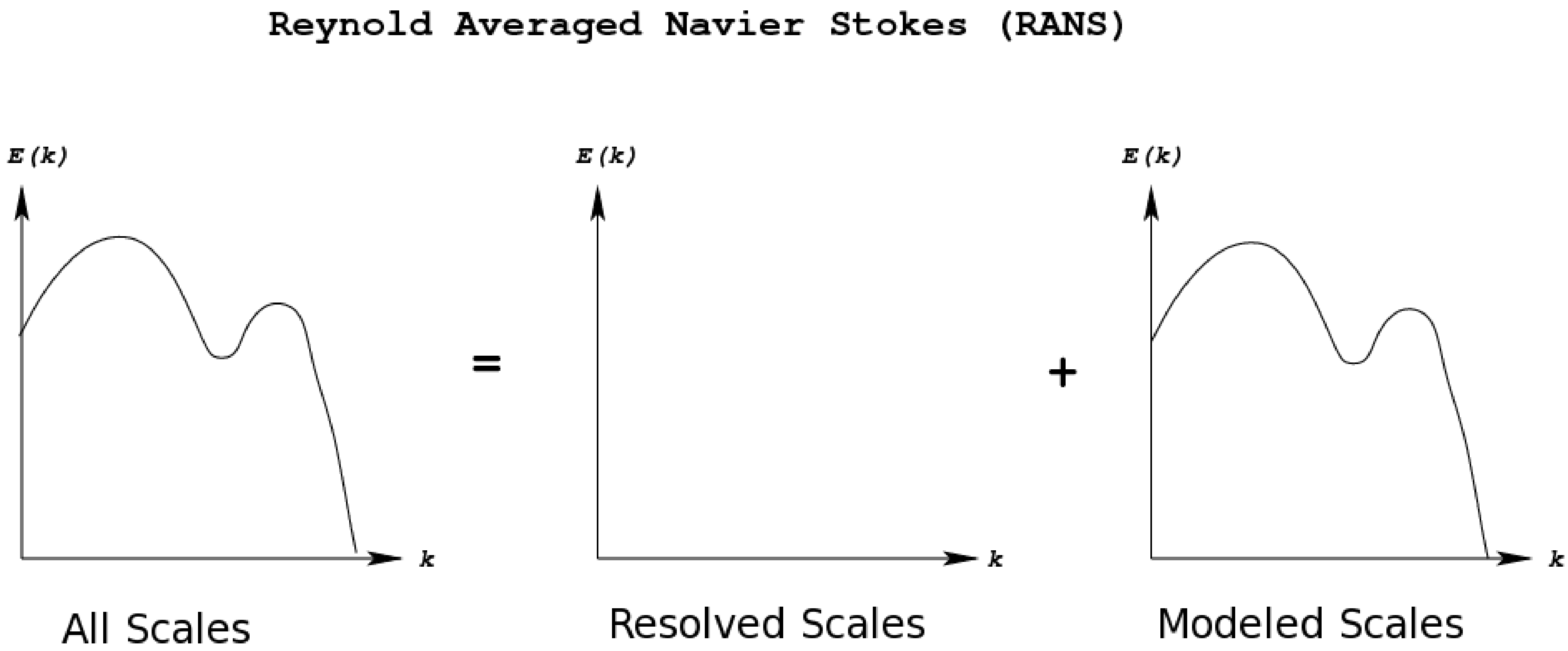
$\Rightarrow k_m$ is the inverse of the energy injection scale

Why do we need models





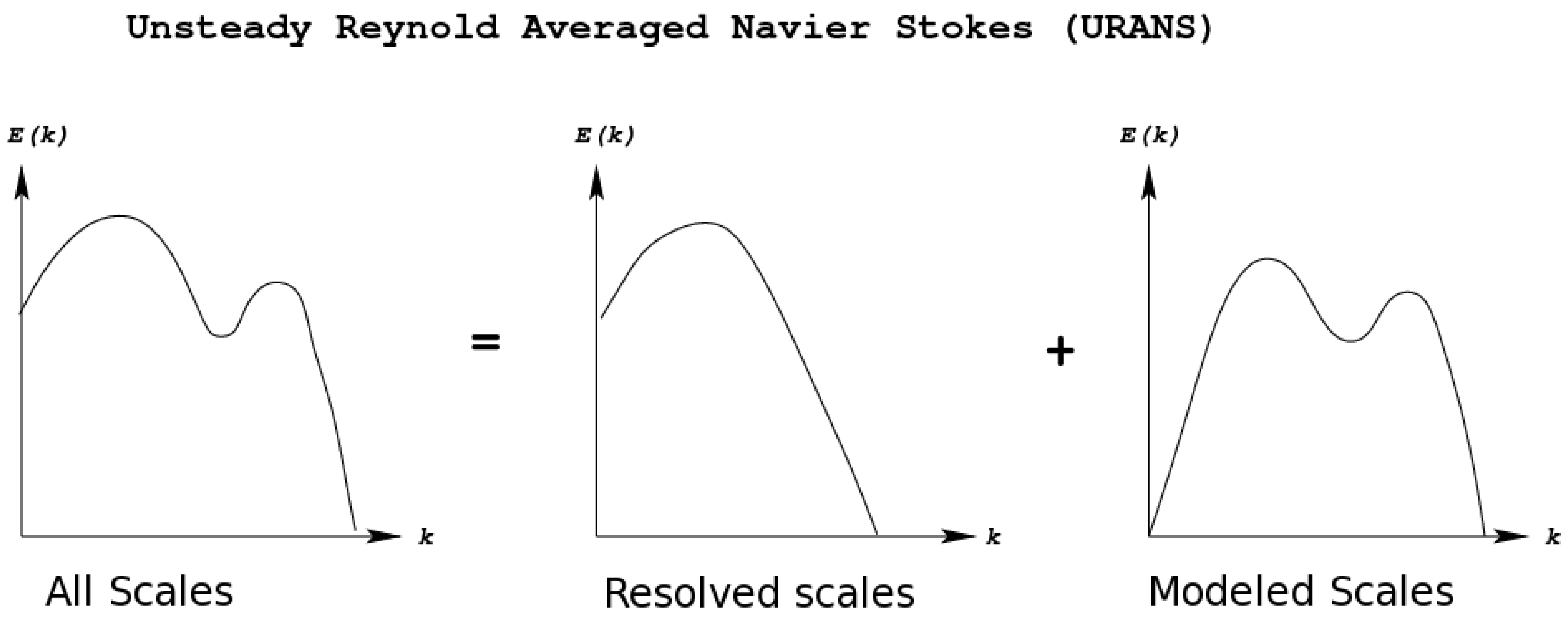
RANS modeling



⇒ Simulation of the statistical average

⇒ All physical scales are modeled

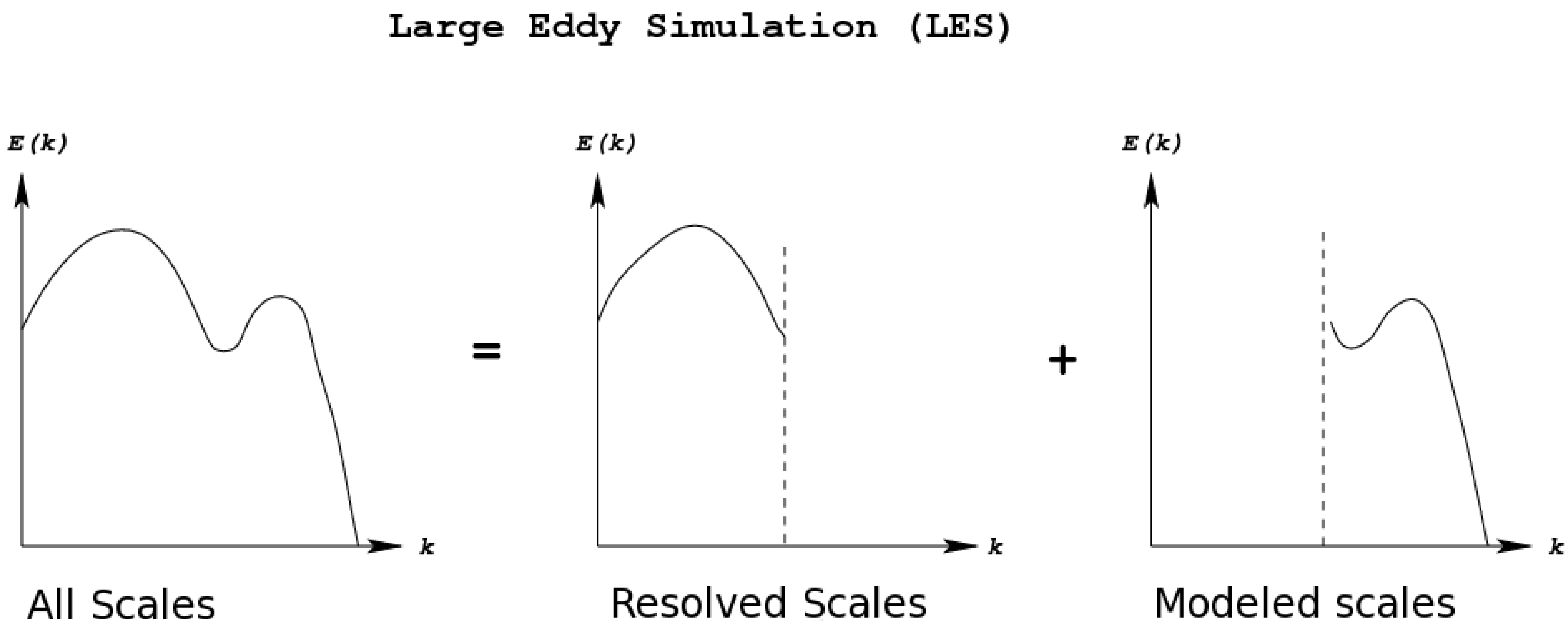
URANS modeling



$\Rightarrow$ Unsteady simulation of the largest scales

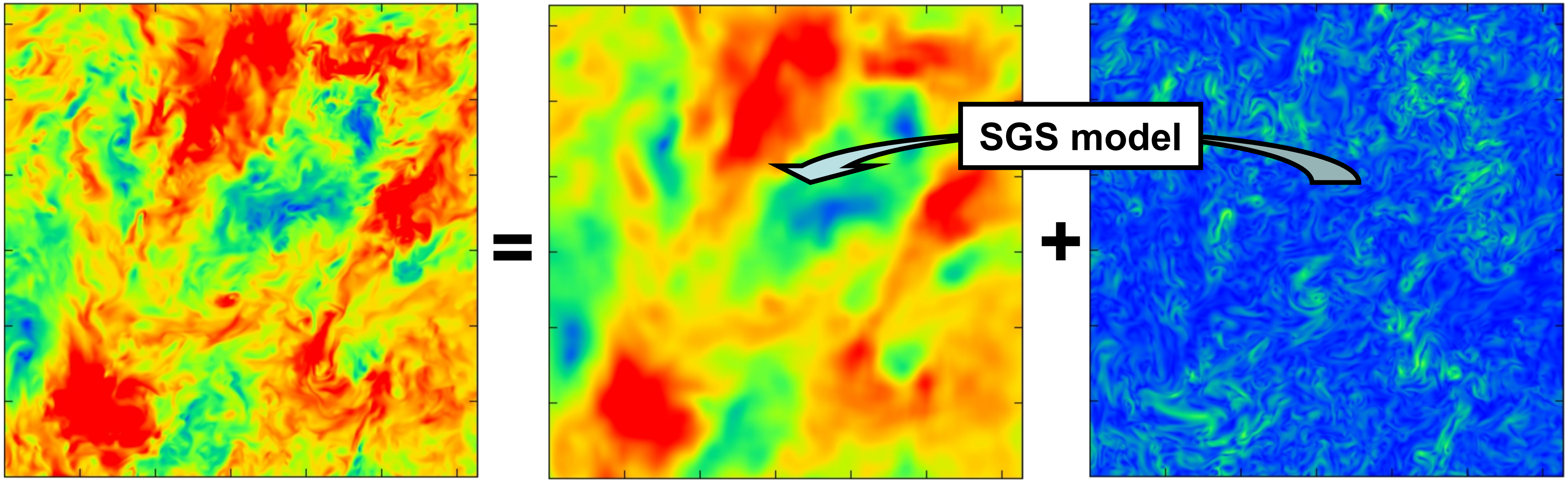
$\Rightarrow$ All physical scales are affected by the model

LES modeling



- ⇒ Unsteady simulation of large scales
- ⇒ Modeling of sub-grid scales only

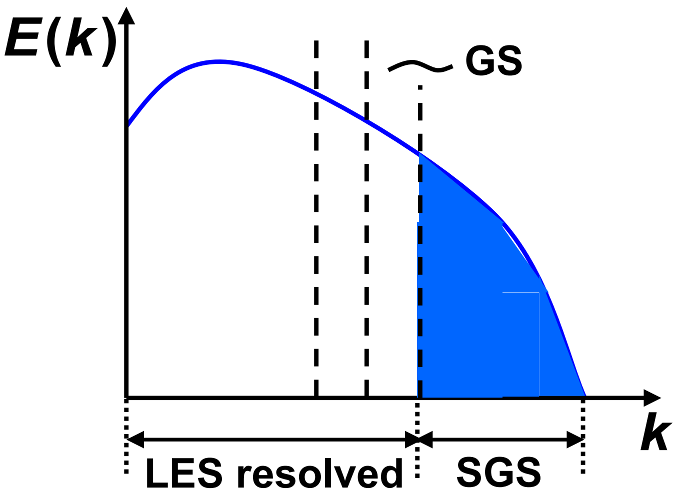
Large Eddy Simulations (LES)



$$\frac{\partial u_i}{\partial t}+\frac{\partial u_i u_j}{\partial x_j}=-\frac{1}{\rho}\frac{\partial p}{\partial x_i}+\frac{\partial}{\partial x_j}\left(\nu\frac{\partial u_i}{\partial x_j}\right)$$

$$\frac{\partial \tilde{u}_i}{\partial t}+\frac{\partial \tilde{u}_i \tilde{u}_j}{\partial x_j}=-\frac{1}{\rho}\frac{\partial \tilde{p}}{\partial x_i}+\frac{\partial}{\partial x_j}\left(\nu\frac{\partial \tilde{u}_i}{\partial x_j}\right)+\frac{\partial \tau_{ij}^{SGS}}{\partial x_j}$$

$$\tau_{ij}^{SGS}=\widetilde{u_i u_j}-\tilde{u}_i\tilde{u}_j$$



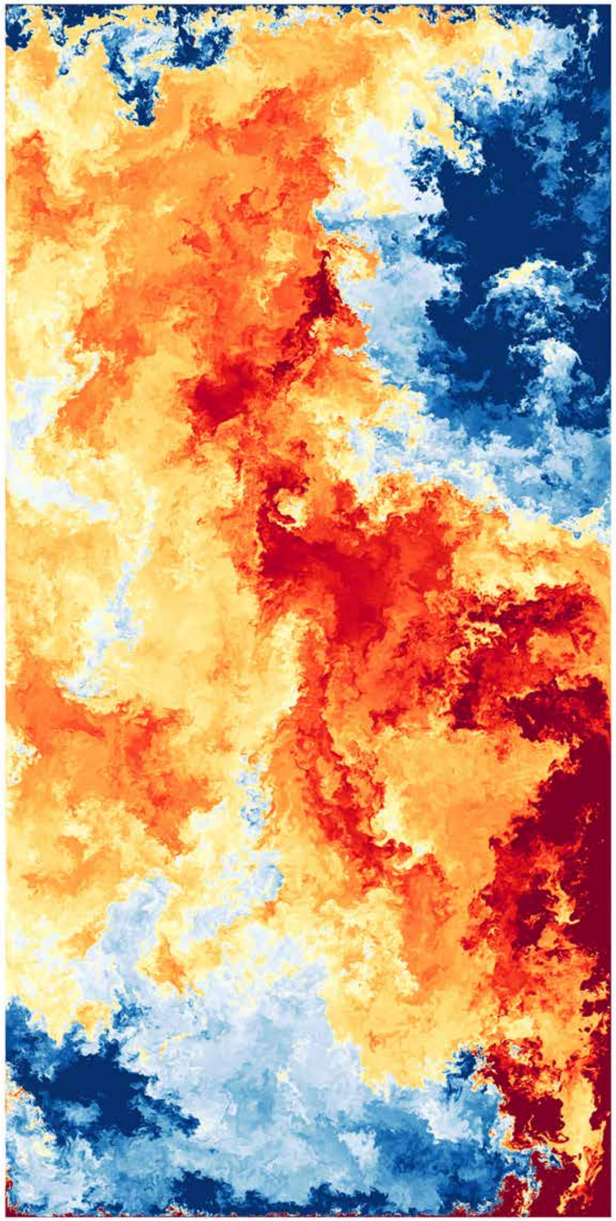
➢ **Eddy viscosity type subgrid-scale model:**

$$\tau_{ij}^{SGS}=-2\nu_t\tilde{S}_{ij} \qquad \nu_t=c_s^2\Delta^2\left|\tilde{S}\right|$$

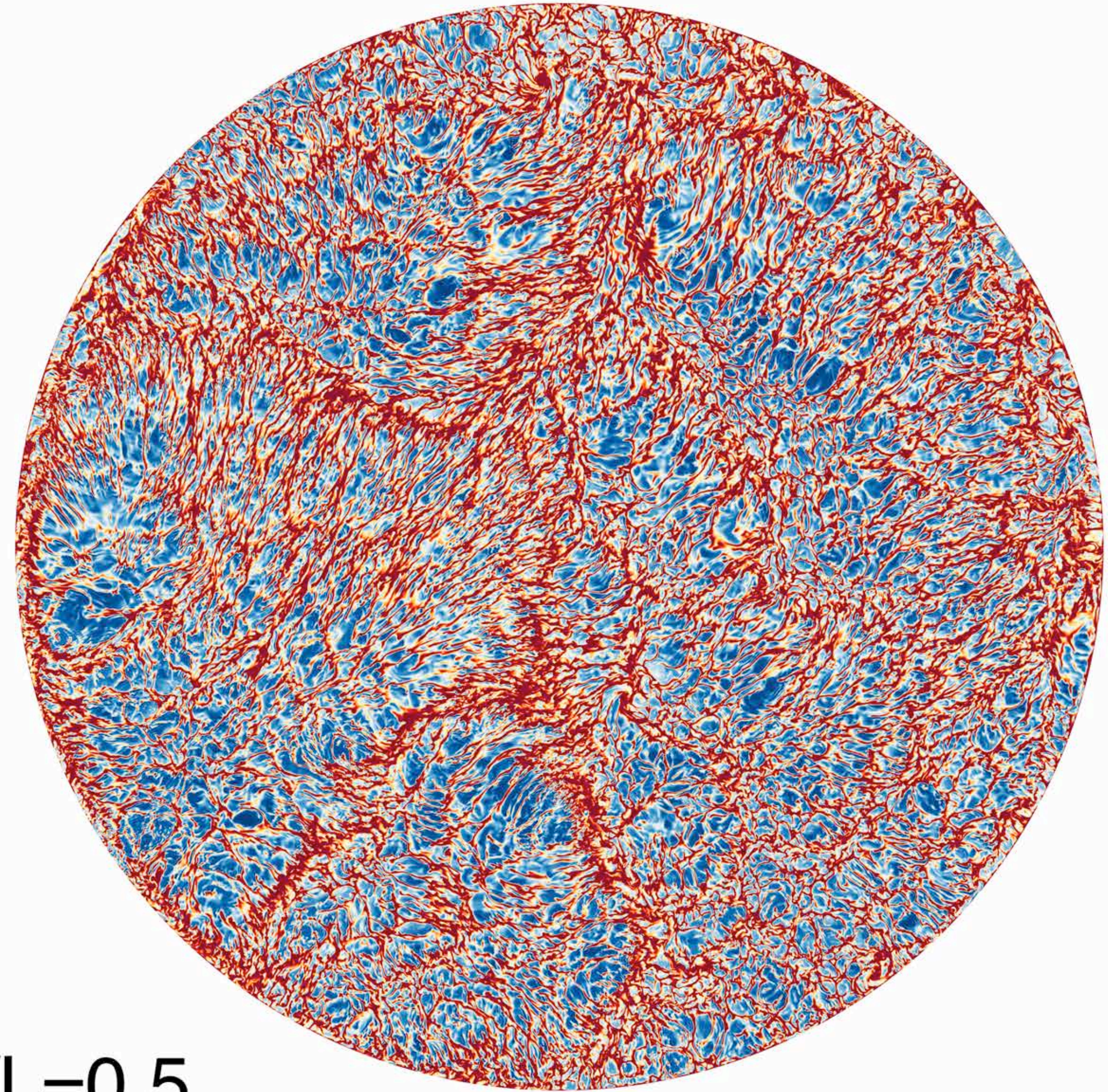
Lagrangian-averaged scale-dependent dynamic model

Bou-Zeid, Meneveau, Parlange, Phys. Fluids 17, 025105 (2005)

Direct Numerical Simulations (DNS)



D/L=0.5



Navier-Stokes equations for incompressible flow

Conservation of momentum

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j^2}$$

Conservation of mass

$$\frac{\partial u_i}{\partial x_i} = 0$$

4 equations for 4 unknowns (3 velocity components and pressure)

Closed system of equations for given initial and boundary conditions

Scaling of the smallest eddies

Dynamics of the smallest scales in turbulence are dominated by viscous dissipation $-->$ parameters ν and ϵ

Smallest turbulence scales ate the Kolmogorov scales

$$\eta = (\nu^3/\epsilon)^{1/4}$$
$$u_\eta = (\epsilon\nu)^{1/4}$$
$$\tau_\eta = (\nu/\epsilon)^{1/2}$$

Low Re consistent with dominance of viscous dissipations

$$Re = \frac{\eta u_\eta}{\nu} = 1$$

Ratio of Kolmogorov scales to macroscales

$$\eta/l_0 = Re_0^{-3/4}$$
$$u_\eta/u_0 = Re_0^{-1/4}$$
$$\tau_\eta/\tau_0 = Re_0^{-1/2}$$

Example

$$Re_0 = \frac{u_0 l_0}{\nu} = 10^5$$
$$\eta/l_0 \sim \frac{1}{6 \cdot 10^3}$$

Increase in range of scales with increasing Reynolds number

Required spatial resolution

$$N = \frac{L}{\eta}$$

$$\eta = \frac{\nu^{3/4}}{\epsilon^{1/4}}$$

$$\epsilon = \frac{U^3}{L}$$

$$\eta = \frac{\nu^{3/4} L^{1/4}}{U^{3/4}}$$

Required spatial resolution

$$N = \frac{L}{\eta} \qquad \eta = \frac{\nu^{3/4}}{\epsilon^{1/4}} \qquad \epsilon = \frac{U^3}{L}$$

$$\frac{L}{\eta} \sim L\frac{U^{3/4}}{\nu^{3/4}L^{1/4}} \qquad \eta = \frac{\nu^{3/4}L^{1/4}}{U^{3/4}}$$

Required spatial resolution

$$N = \frac{L}{\eta} \qquad \eta = \frac{\nu^{3/4}}{\epsilon^{1/4}} \qquad \epsilon = \frac{U^3}{L}$$

$$\frac{L}{\eta} \sim L\frac{U^{3/4}}{\nu^{3/4}L^{1/4}}$$

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$$\eta = \frac{\nu^{3/4}L^{1/4}}{U^{3/4}}$$

Required spatial resolution

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$$\eta = \frac{\nu^{3/4}L^{1/4}}{U^{3/4}}$$

$$\frac{L}{\eta} \sim \frac{L^{3/4}U^{3/4}}{\nu^{3/4}}$$

$$\frac{L}{\eta} \sim Re_L^{3/4}$$

$$Re_L^{1/2} \sim Re_\lambda$$

Required spatial resolution

$$N = \frac{L}{\eta} \qquad \eta = \frac{\nu^{3/4}}{\epsilon^{1/4}} \qquad \epsilon = \frac{U^3}{L}$$

$$\frac{L}{\eta} \sim L\frac{U^{3/4}}{\nu^{3/4}L^{1/4}}$$

$$\eta = \frac{\nu^{3/4}L^{1/4}}{U^{3/4}}$$

$$\frac{L}{\eta} \sim \frac{L^{3/4}U^{3/4}}{\nu^{3/4}}$$

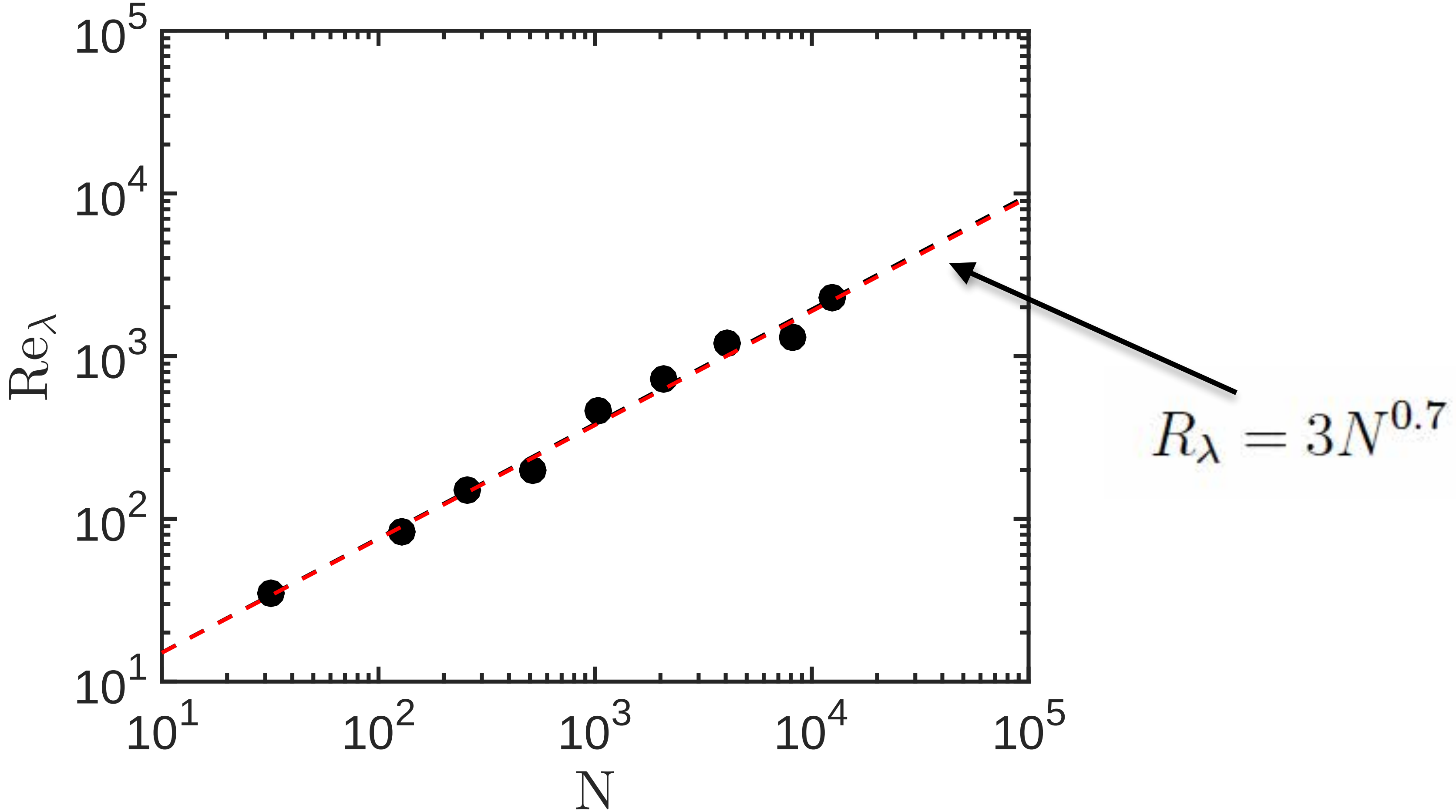
$$\frac{L}{\eta} \sim Re_L^{3/4}$$

$$Re_L^{1/2} \sim Re_\lambda$$

$$\frac{L}{\eta} \sim Re_\lambda^{3/2}$$

$$\boxed{Re_\lambda \sim N^{2/3}}$$

Taylor-based Reynolds number



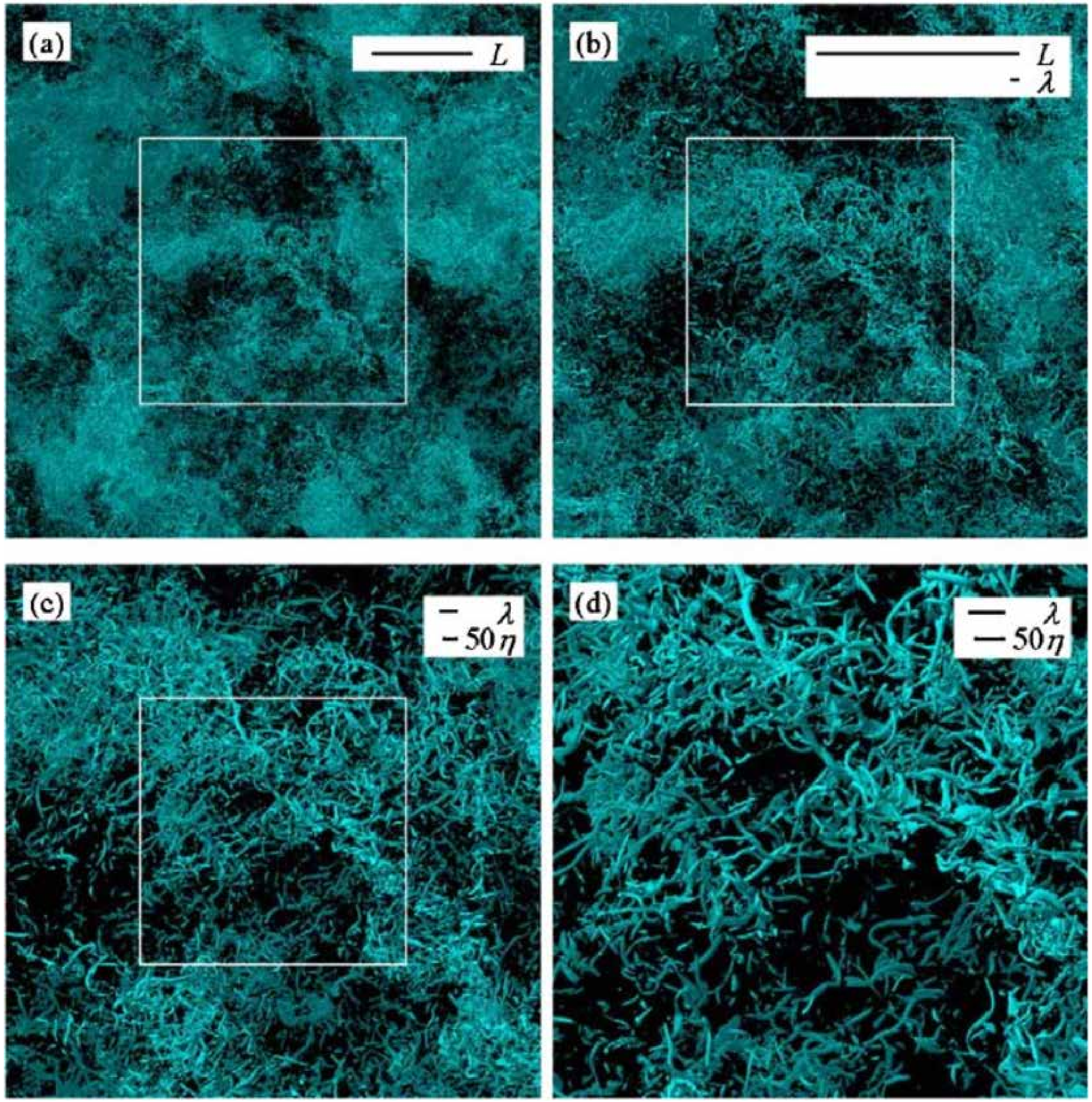
Data points $R_\lambda = 2.3N^{3/4}$

Dimensional analysis $R_\lambda \sim N^{2/3}$

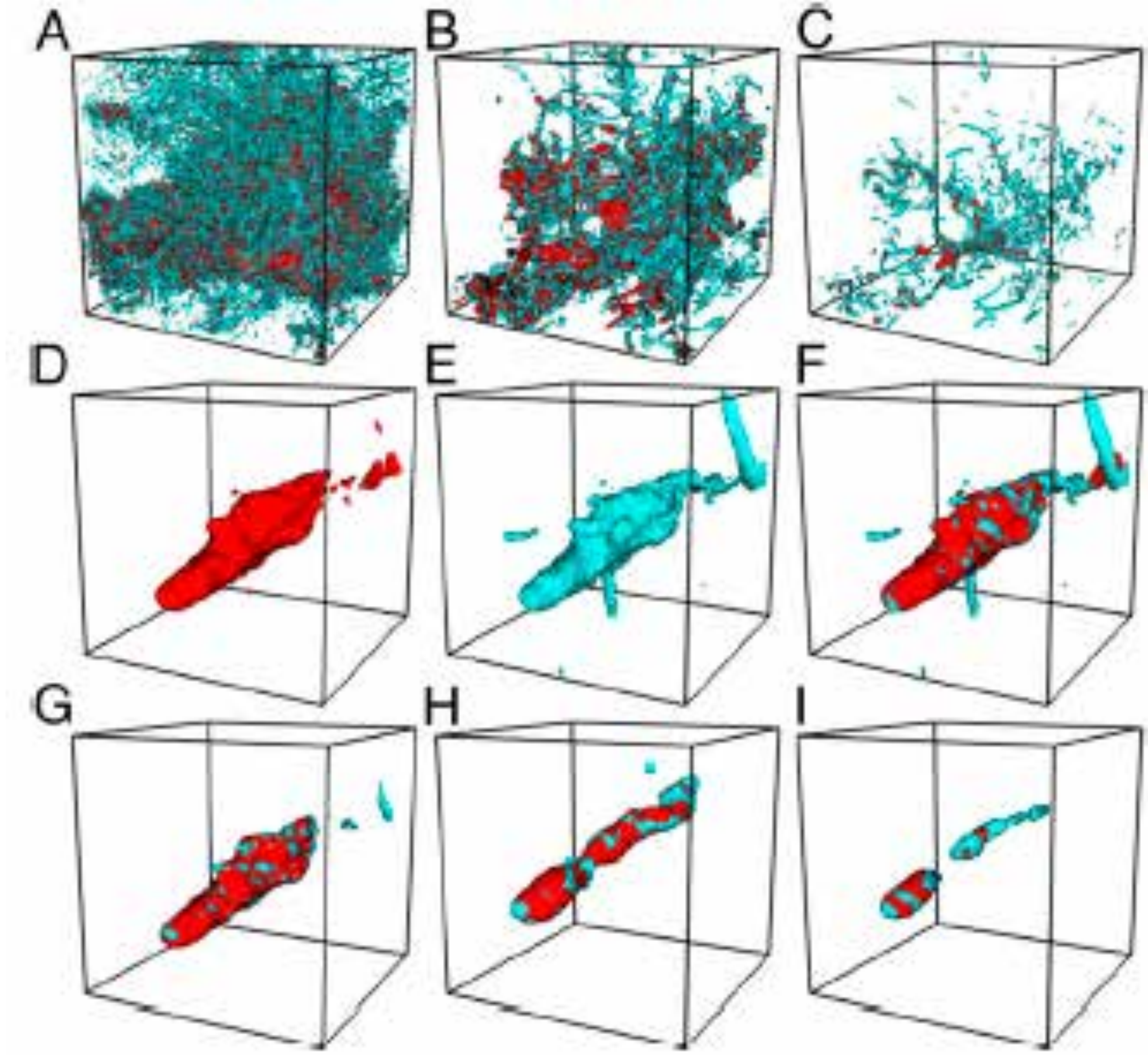
Recent results might be marginally resolved.

A. Celani, Journal of Turbulence, The frontiers of computing in turbulence: challenges and perspectives 8, 2007

Kaneda et al. 2003 DNS 4096^3



Yeung et al. 2015 DNS 8192^3



Ishihara, Morishita, Yokokawa, Uno, Kaneda 2016 DNS 12288^3

How much CPU time is required?

$$N = \frac{L}{\eta} \qquad \frac{L}{\eta} \sim Re_{\lambda}^{3/2}$$

Calculate the total number of modes

$$N^3 = \left(\frac{L}{\eta}\right)^3 \sim Re_L^{9/4} \sim Re_{\lambda}^{9/2}$$

How much CPU time is required?

$$N = \frac{L}{\eta} \qquad \frac{L}{\eta} \sim Re_\lambda^{3/2}$$

Calculate the total number of modes

$$N^3 = \left(\frac{L}{\eta}\right)^3 \sim Re_L^{9/4} \sim Re_\lambda^{9/2}$$

Time advancement also depends on Re_L

T = Large eddy turnover time $\sim \frac{U^2}{\epsilon}$

$$\tau_\eta = \left(\frac{\nu}{\epsilon}\right)^{1/2}$$

How much CPU time is required?

$$M = \frac{T}{\tau_\eta} = \frac{U^2}{\epsilon}\frac{\epsilon^{1/2}}{\nu^{1/2}}$$

How much CPU time is required?

$$M = \frac{T}{\tau_\eta} = \frac{U^2}{\epsilon}\frac{\epsilon^{1/2}}{\nu^{1/2}}$$

$$M = \frac{U^2}{\nu^{1/2}\epsilon^{1/2}}$$

How much CPU time is required?

$$M = \frac{T}{\tau_\eta} = \frac{U^2}{\epsilon}\frac{\epsilon^{1/2}}{\nu^{1/2}}$$

$$M = \frac{U^2}{\nu^{1/2}\epsilon^{1/2}}$$

$$M = \frac{U^2 L^{1/2}}{\nu^{1/2} U^{3/2}}$$

Remember $\epsilon = \frac{U^3}{L}$

How much CPU time is required?

$$
\begin{aligned}
M = \frac{T}{\tau_\eta} &= \frac{U^2}{\epsilon}\frac{\epsilon^{1/2}}{\nu^{1/2}} \\
M &= \frac{U^2}{\nu^{1/2}\epsilon^{1/2}} \\
M &= \frac{U^2 L^{1/2}}{\nu^{1/2}U^{3/2}} \\
M &= \frac{U^{1/2}L^{1/2}}{\nu^{1/2}}
\end{aligned}
$$

How much CPU time is required?

$$
\begin{aligned}
M = \frac{T}{\tau_\eta} &= \frac{U^2}{\epsilon}\frac{\epsilon^{1/2}}{\nu^{1/2}} \\
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M &= \frac{U^{1/2} L^{1/2}}{\nu^{1/2}} \\
M &= Re_L^{1/2} = Re_\lambda
\end{aligned}
$$

How much CPU time is required?

$$
\begin{aligned}
M = \frac{T}{\tau_\eta} &= \frac{U^2}{\epsilon}\frac{\epsilon^{1/2}}{\nu^{1/2}} \\
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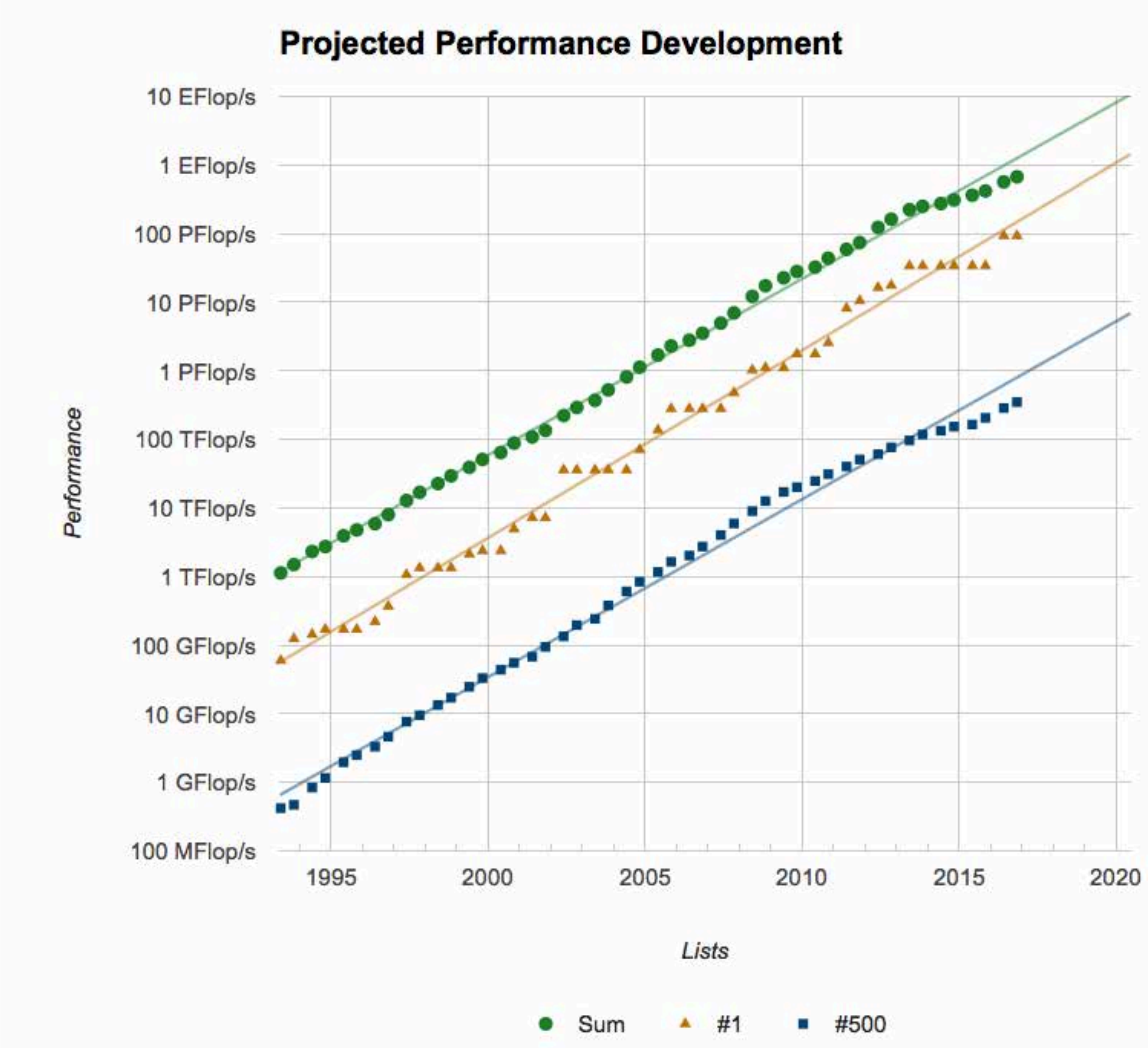
M &= \frac{U^{1/2}L^{1/2}}{\nu^{1/2}} \\
M &= Re_L^{1/2} = Re_\lambda
\end{aligned}
$$

$$
\text{CPU-time} \sim N^3 M \sim Re_L^{11/2} \sim Re_\lambda^{11/4}
$$

Remember $N^3 = \left(\frac{L}{\eta}\right)^3 \sim Re_L^{9/4} \sim Re_\lambda^{9/2}$

Development supercomputers

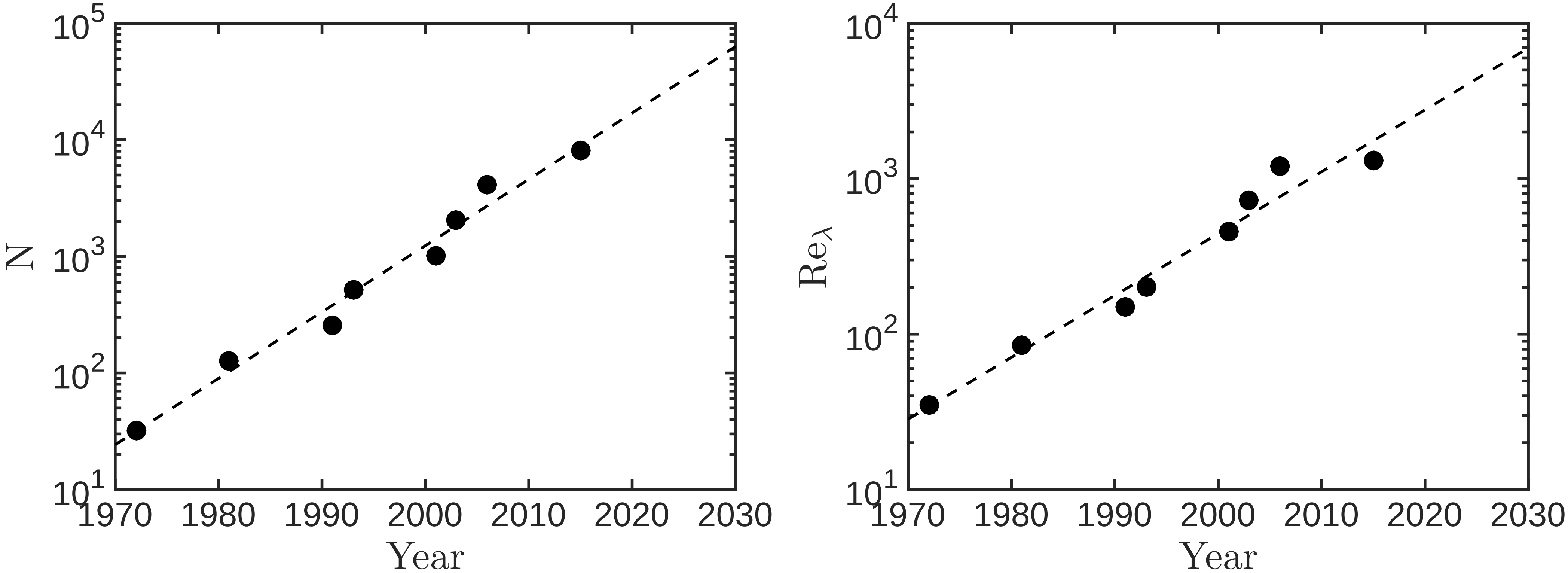


Top supercomputers

Rank	Site	System	Cores	(TFlop/s)	(TFlop/s)	(kW)
1	National Supercomputing Center in Wuxi China	**Sunway TaihuLight** - Sunway MPP, Sunway SW26010 260C 1.45GHz, Sunway NRCPC	10,649,600	93,014.6	125,435.9	15,371
2	National Super Computer Center in Guangzhou China	**Tianhe-2 (MilkyWay-2)** - TH-IVB-FEP Cluster, Intel Xeon E5-2692 12C 2.200GHz, TH Express-2, Intel Xeon Phi 31S1P NUDT	3,120,000	33,862.7	54,902.4	17,808
3	DOE/SC/Oak Ridge National Laboratory United States	**Titan** - Cray XK7 , Opteron 6274 16C 2.200GHz, Cray Gemini interconnect, NVIDIA K20x Cray Inc.	560,640	17,590.0	27,112.5	8,209
4	DOE/NNSA/LLNL United States	**Sequoia** - BlueGene/Q, Power BQC 16C 1.60 GHz, Custom IBM	1,572,864	17,173.2	20,132.7	7,890
5	DOE/SC/LBNL/NERSC United States	**Cori** - Cray XC40, Intel Xeon Phi 7250 68C 1.4GHz, Aries interconnect Cray Inc.	622,336	14,014.7	27,880.7	3,939

www.top500.org, November 2016

Taylor-based Reynolds number



Resolution doubles every 5.5 years
Taylor Reynolds doubles every 8 years

Numerical methods

- Second order central finite difference
  - Energy conserving
  - Better for sharp shocks
  - Easier, more complex physics can be incorporated
  - Efficient, suitable for massively parallel machines.

- Pseudo spectral
  - Higher accuracy for given number degrees of freedom
  - Requires periodic boundary conditions

AFiD: An universal Navier-Stokes solver for wall-bounded flow

R. Verzicco & P. Orlandi, A finite-difference scheme for three-dimensional incompressible flow in cylindrical coordinates, J. Comput. Phys. 123, 402–413 (1996)

E. P. van der Poel, R. Ostilla-Monico, J. Donners, & R. Verzicco, A pencil distributed finite difference code for strongly turbulent wall-bounded flows, Computers and Fluids 166, 10-16 (2015).

R. Ostilla-Monico, Y. Yang, E. P. van der Poel, D. Lohse, R. Verzicco, A multiple–resolution strategy for Direct Numerical Simulation of scalar turbulence, J. Computational Physics 301, 308-321 (2015).

X. Zhu, E. Phillips, V. Spandan, J. Donners, G. Ruetsch, J. Romero, R. Ostilla-Mónico, Y. Yang, D. Lohse, R. Verzicco, M. Fatica, R.J.A.M. Stevens, AFiD-GPU: a versatile Navier-Stokes Solver for Wall-Bounded Turbulent Flows on GPU Clusters, Submitted to Computer Physics Communications (2017).

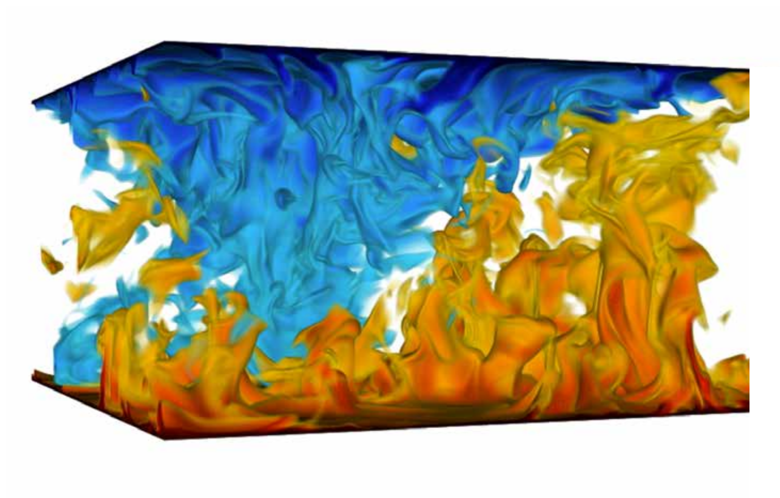
AFiD code for wall bounded turbulence

- Direct numerical simulation of Navier-Stokes equations, no turbulence modeling

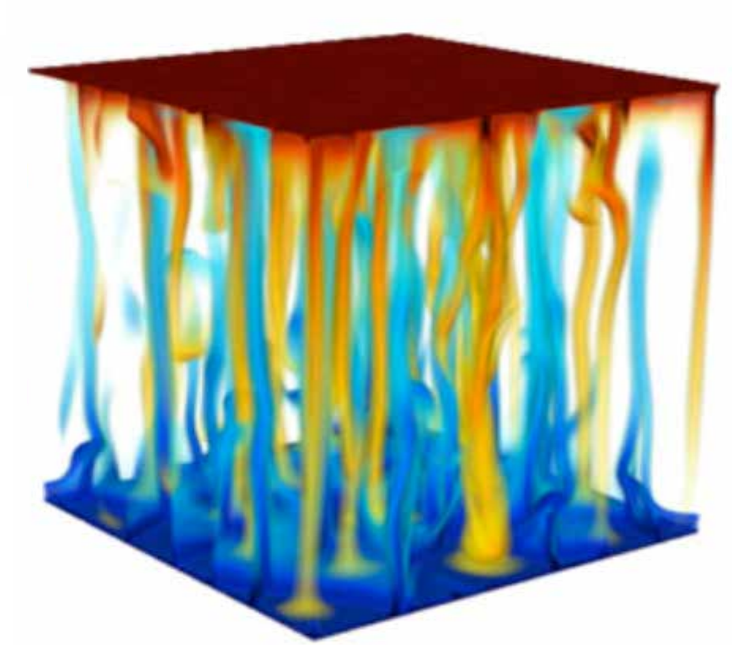
- Spatial discretization: 2nd order finite differences

- Temporal discretization: mixed explicit & implicit treatment of some terms

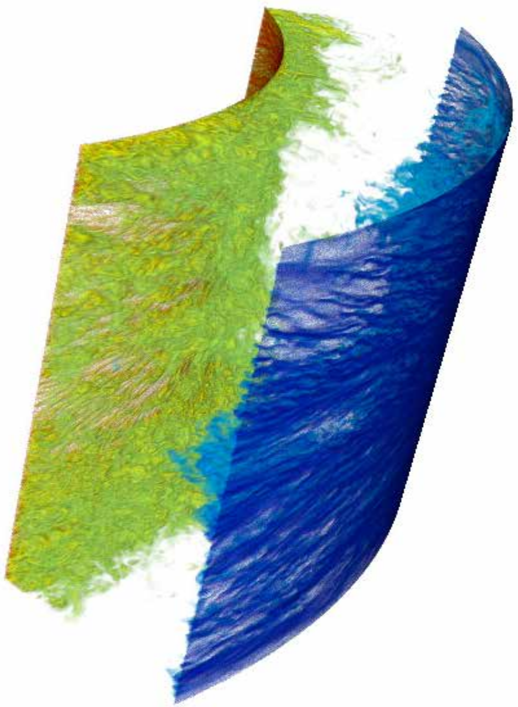
- Pressure correction method: must solve Poisson equation exactly (most expensive!)



Thermal convection

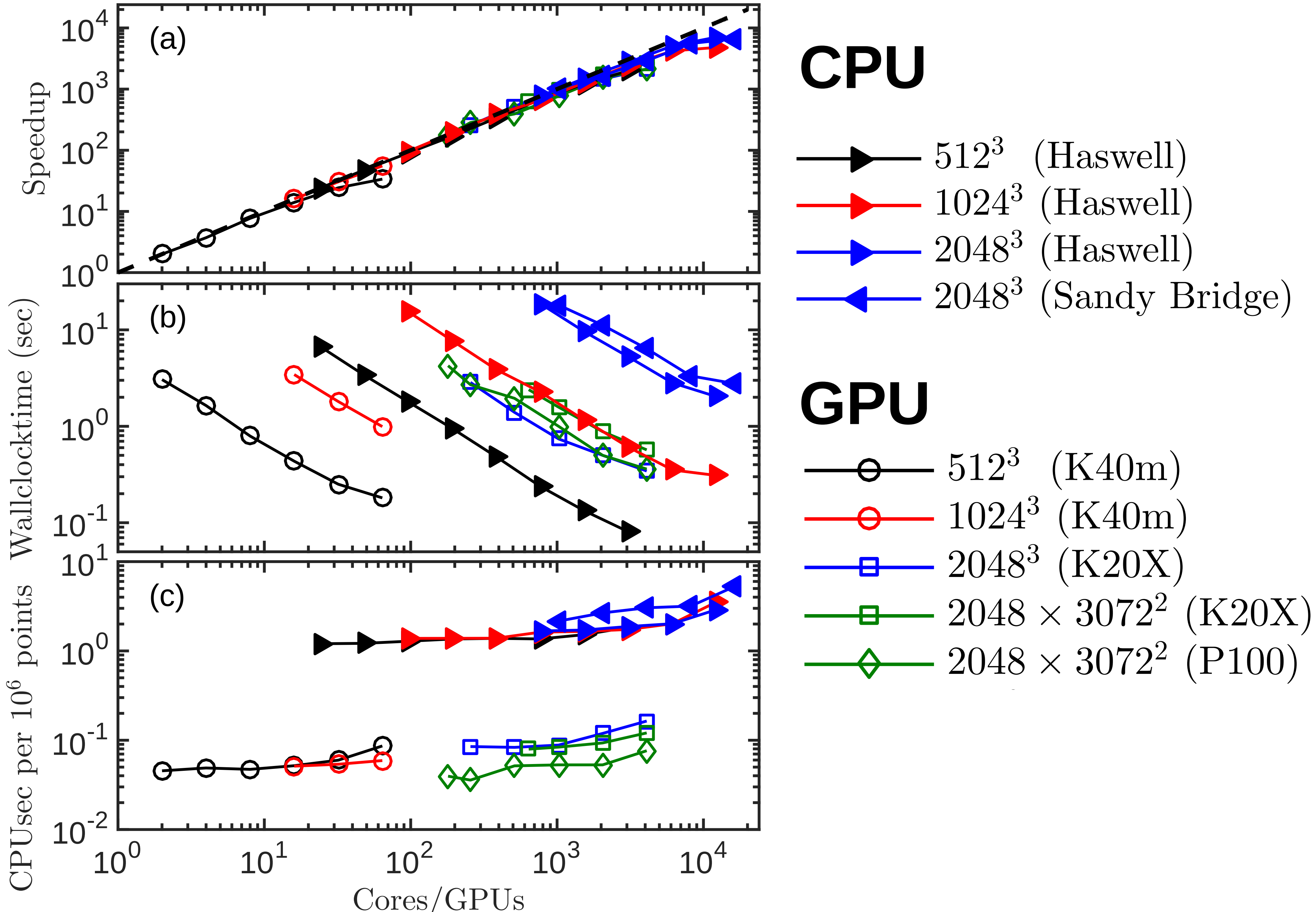


Double diffusive convection

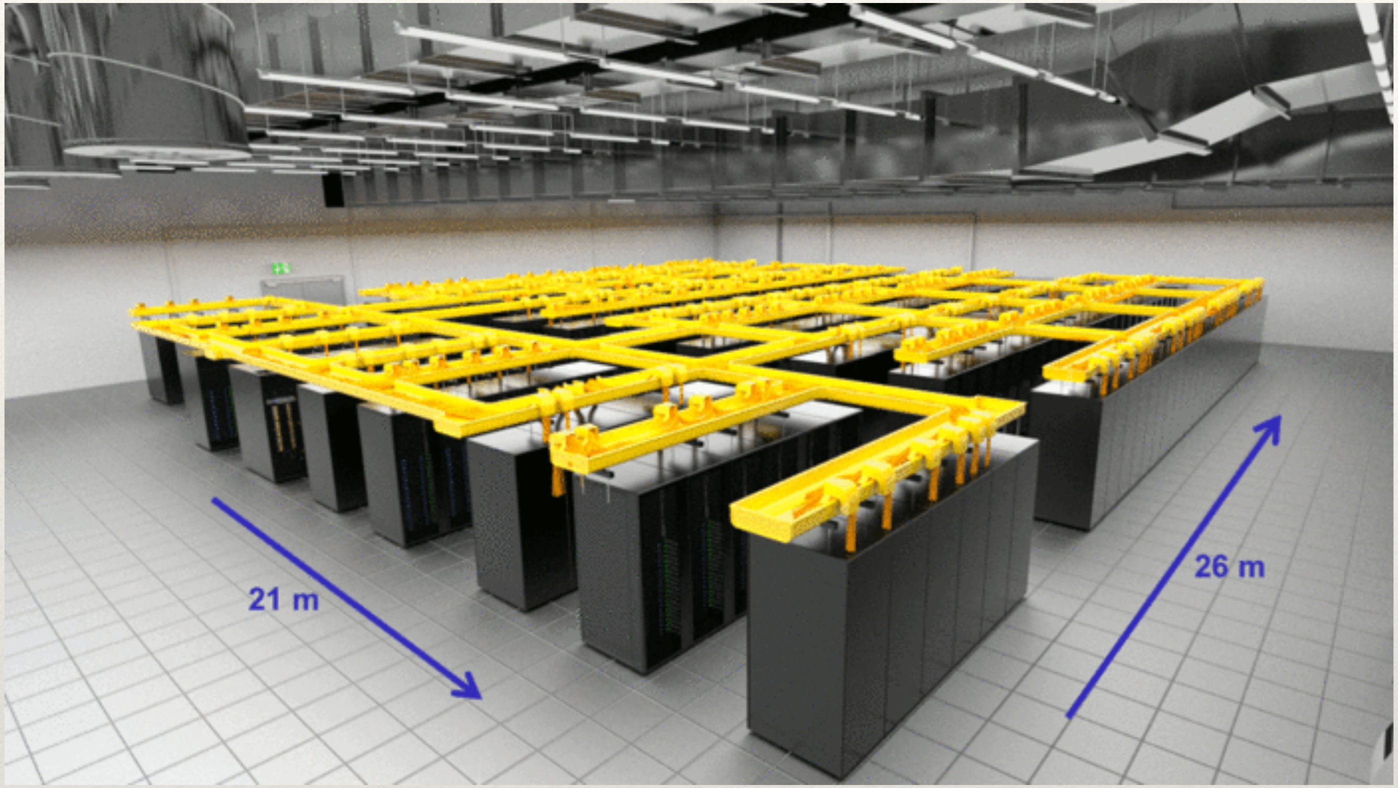


Taylor-Couette flow

Scaling of AFiD code



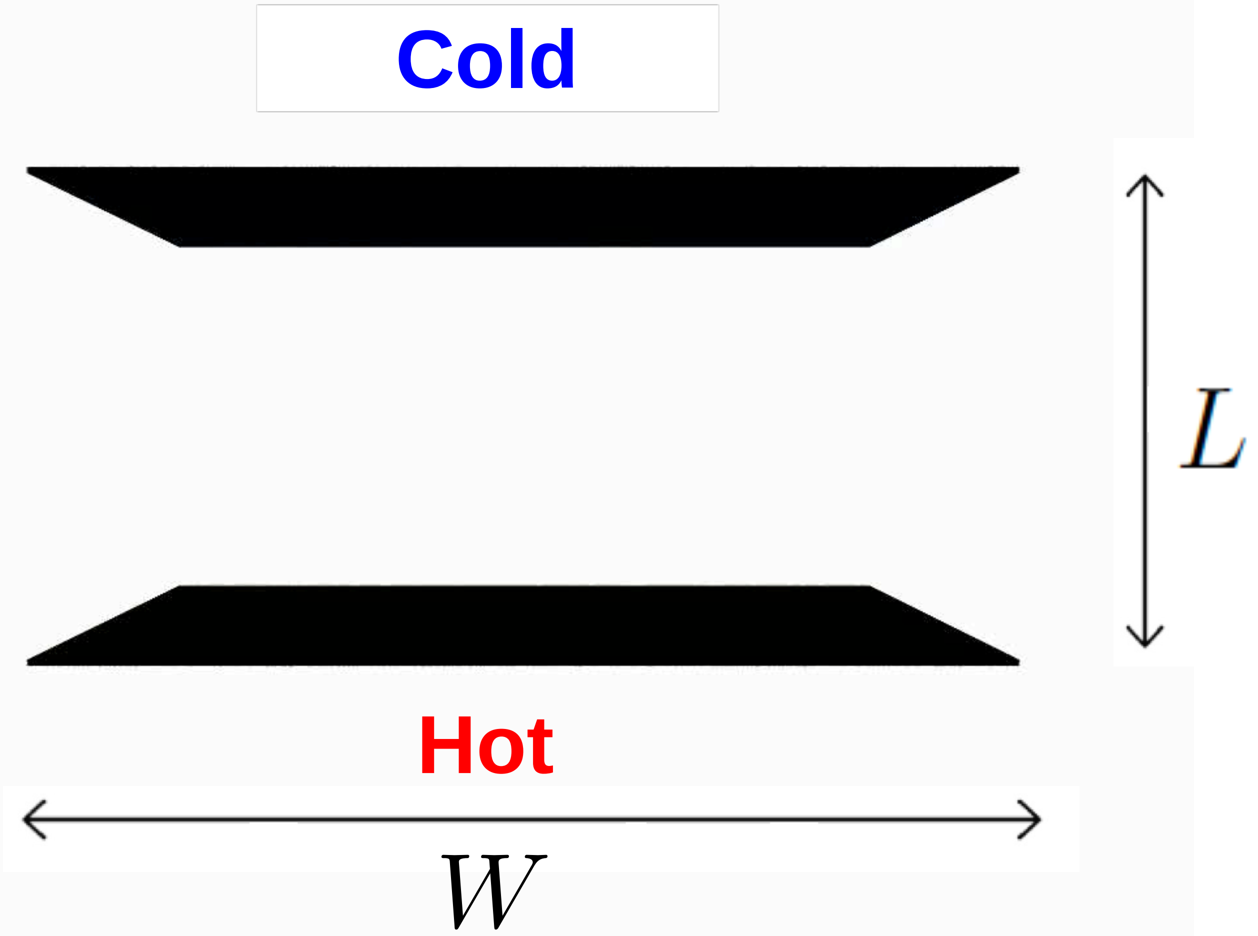
Simulations performed on state of the art supercomputers



SuperMuc (Germany)
Marconi (Italy)
Piz Daint (Switzerland)
Cartesius (Netherlands)
Archer (Great Britain)

3 out of 5 fastest supercomputers in Europe

Rayleigh-Bénard convection



Control parameters

Aspect ratio

$$\Gamma = \frac{W}{L}$$

Rayleigh number

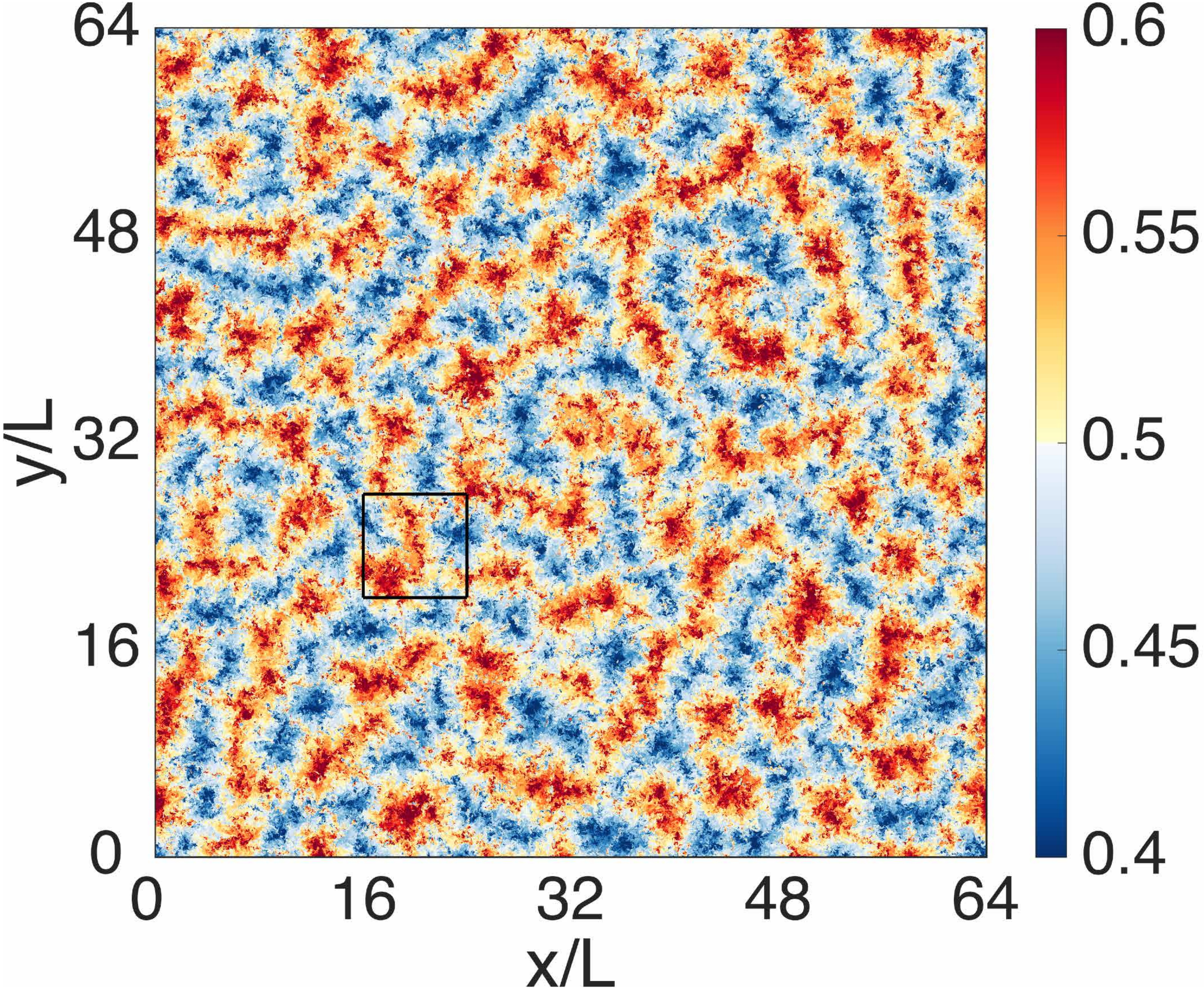
$$Ra = \frac{\beta g L^3 \Delta}{\nu\kappa}$$

Prandtl number

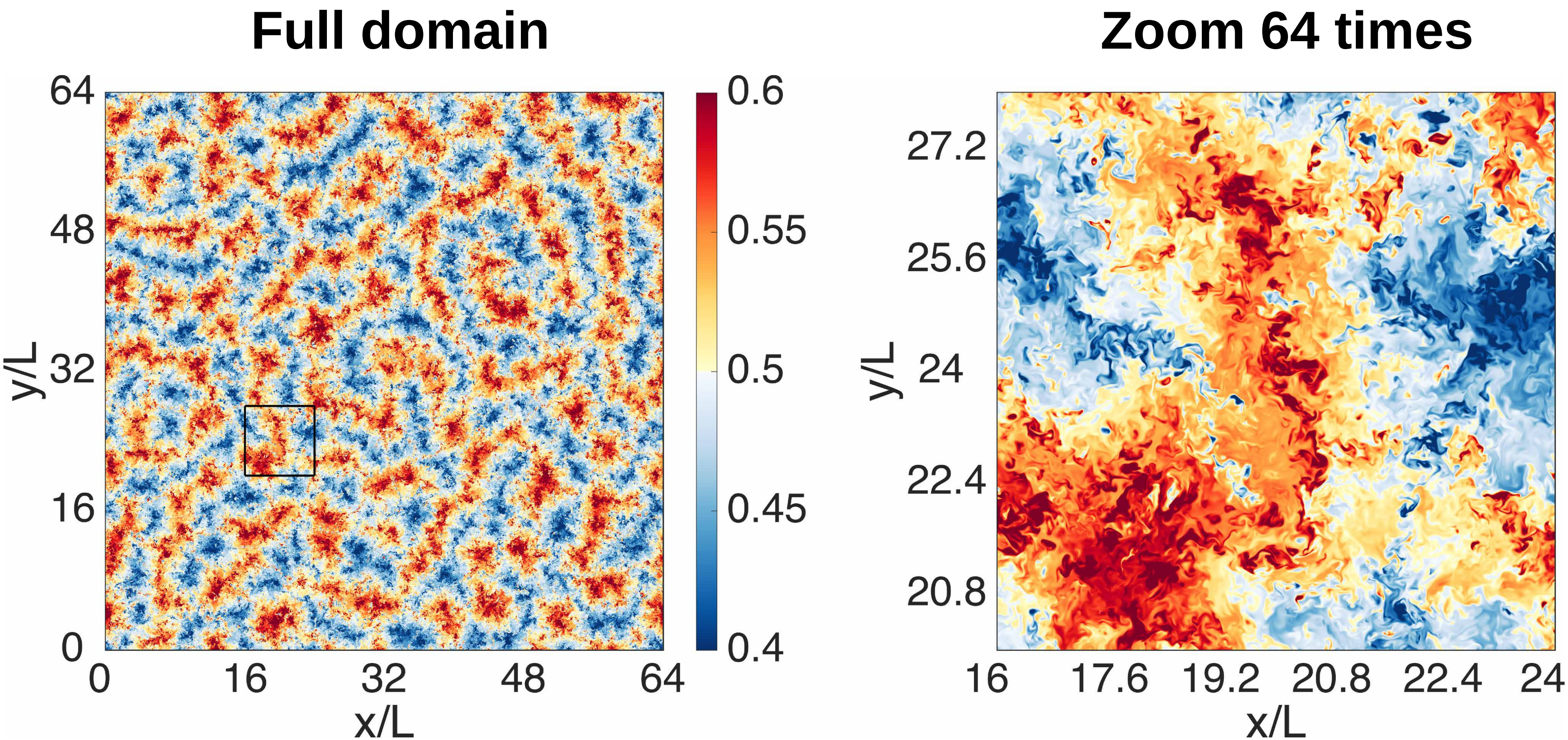
$$Pr = \frac{\nu}{\kappa}$$

van der Poel et al. Computers & Fluids 116 (2015) 10–16

Convection patterns in very large domains

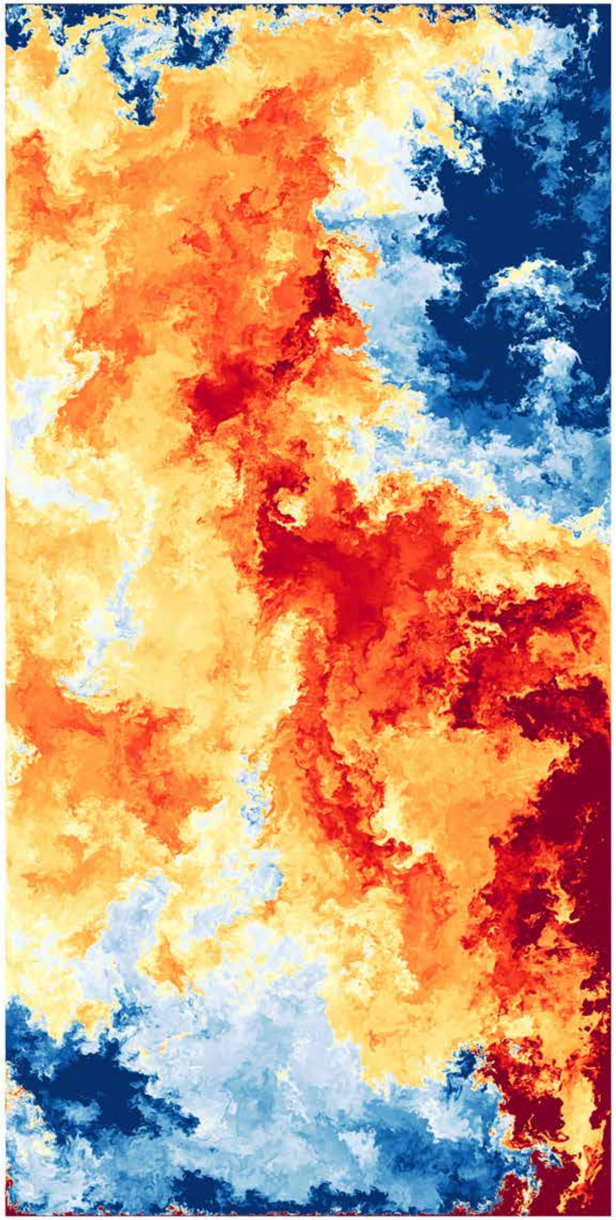


Convection patterns in very large domains



Rayleigh-Bénard convection

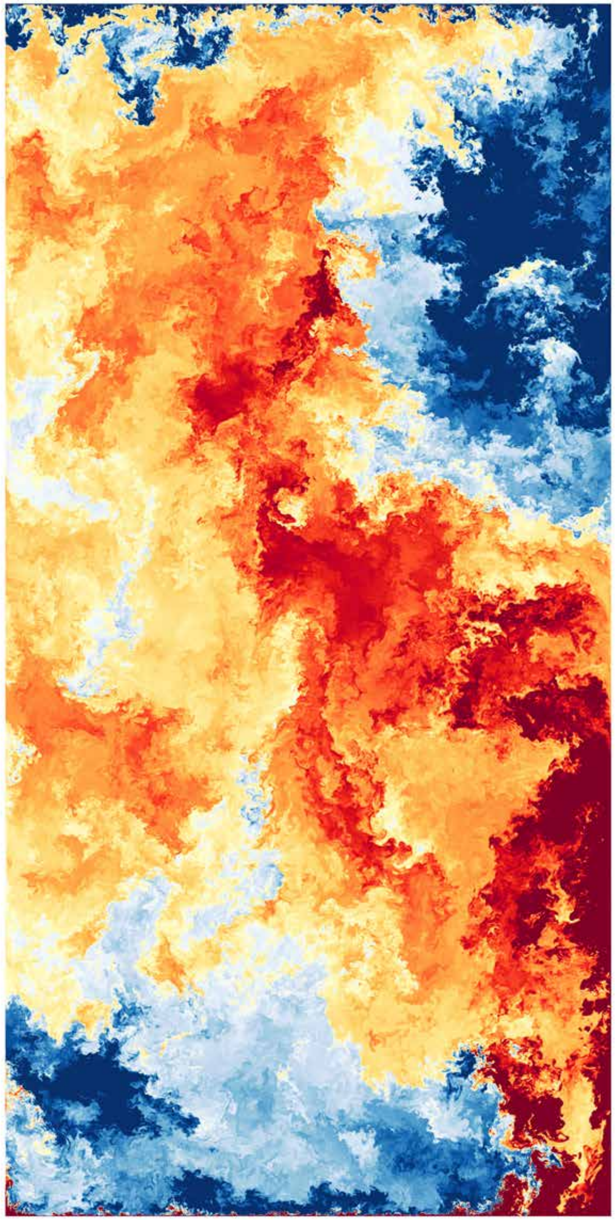
Cold



Hot

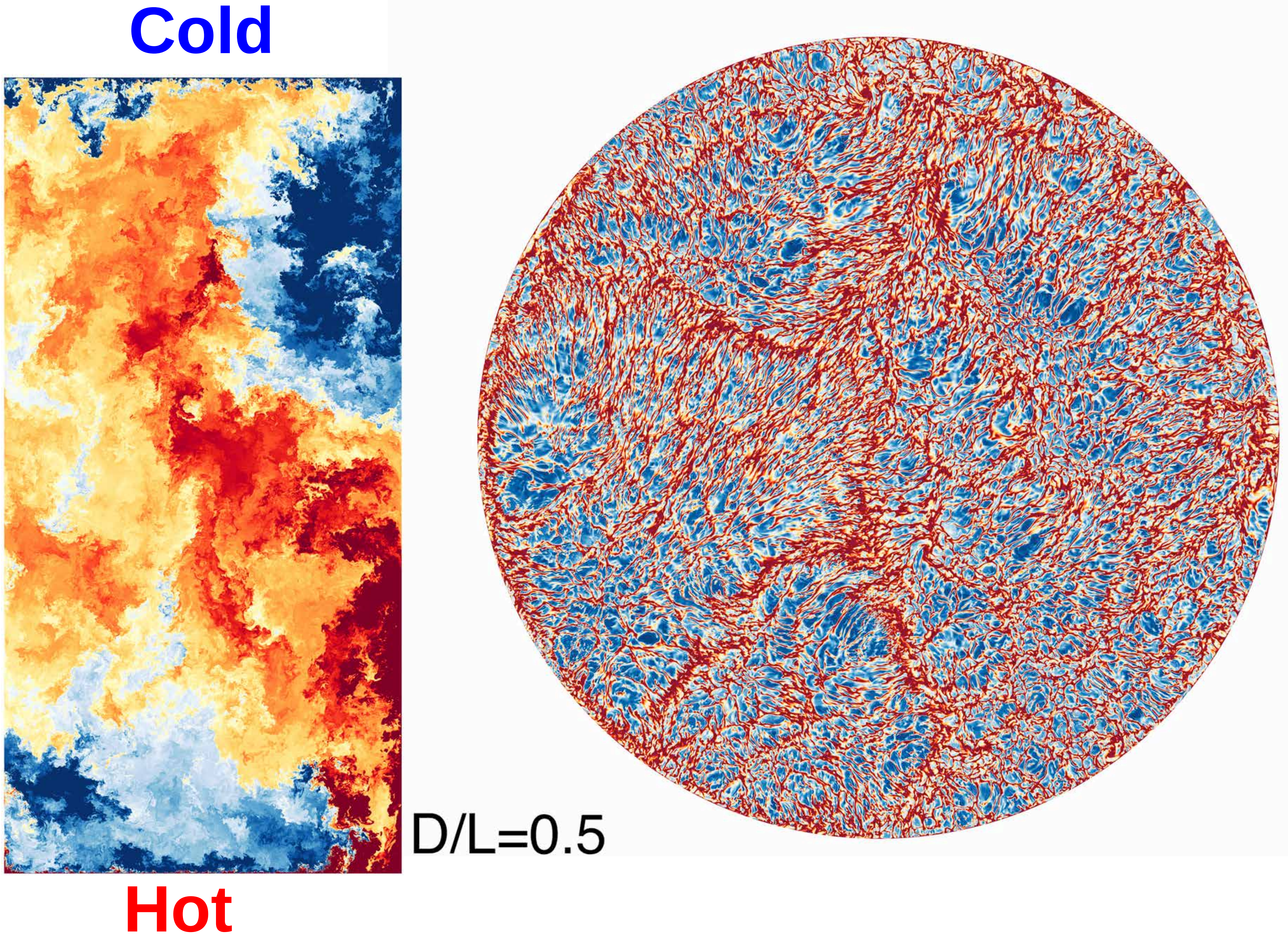
Rayleigh-Bénard convection

Cold

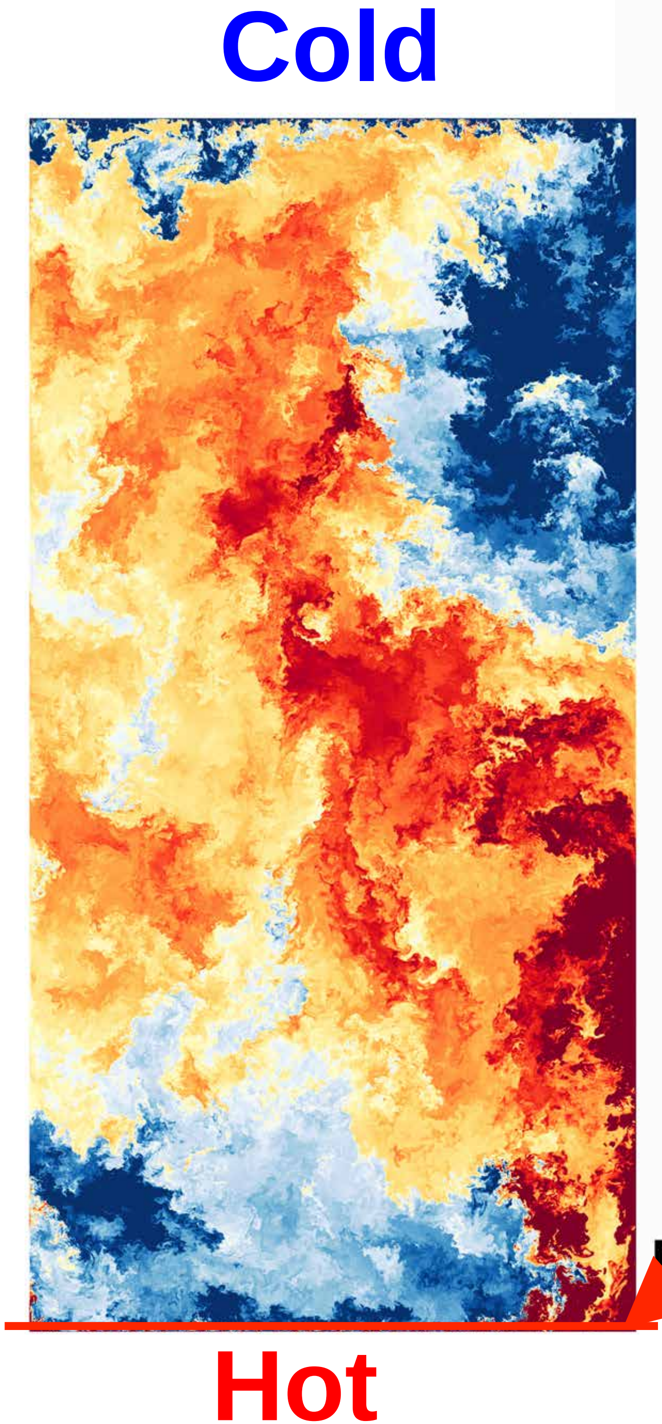


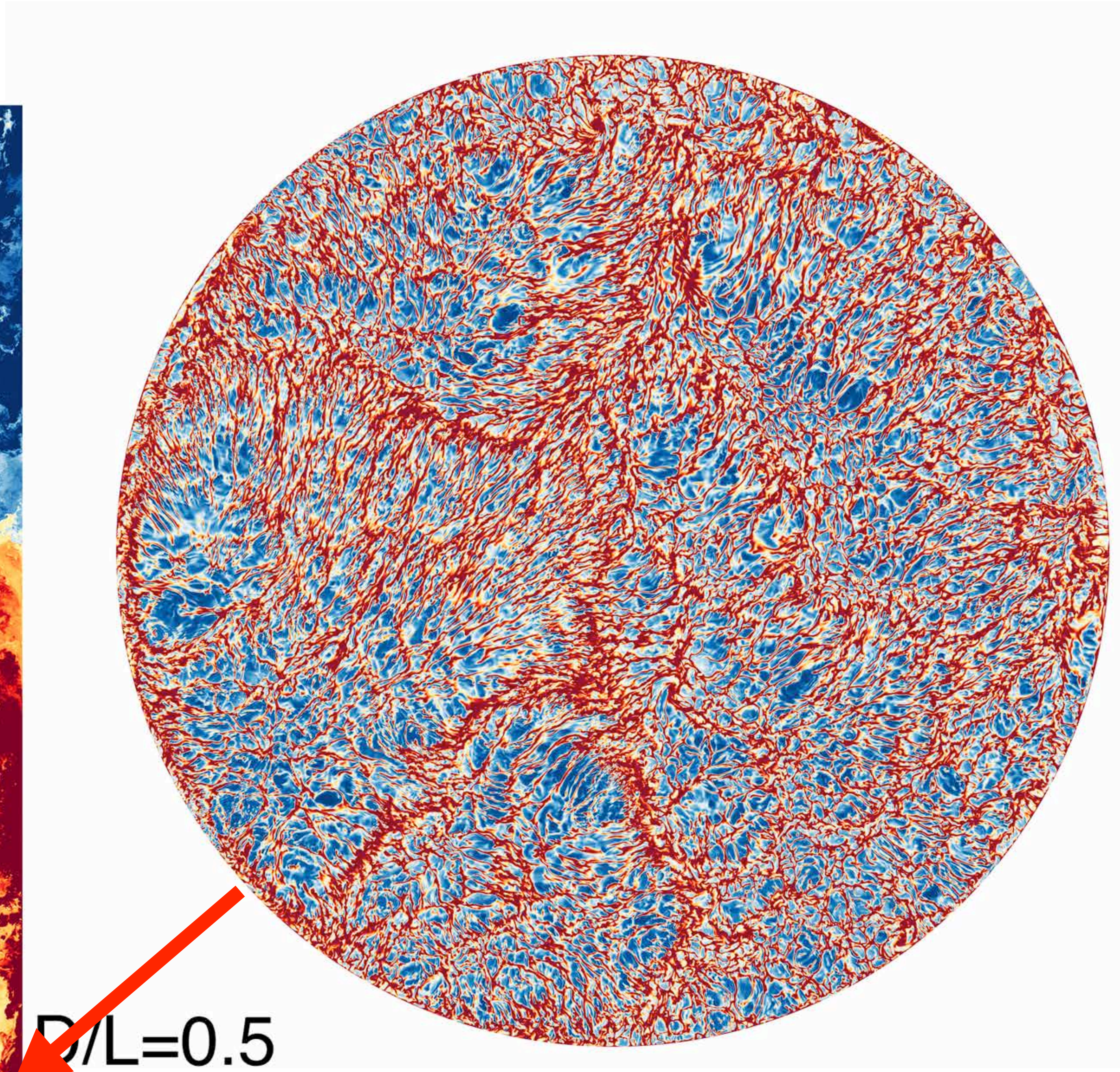
Hot

Rayleigh-Bénard convection

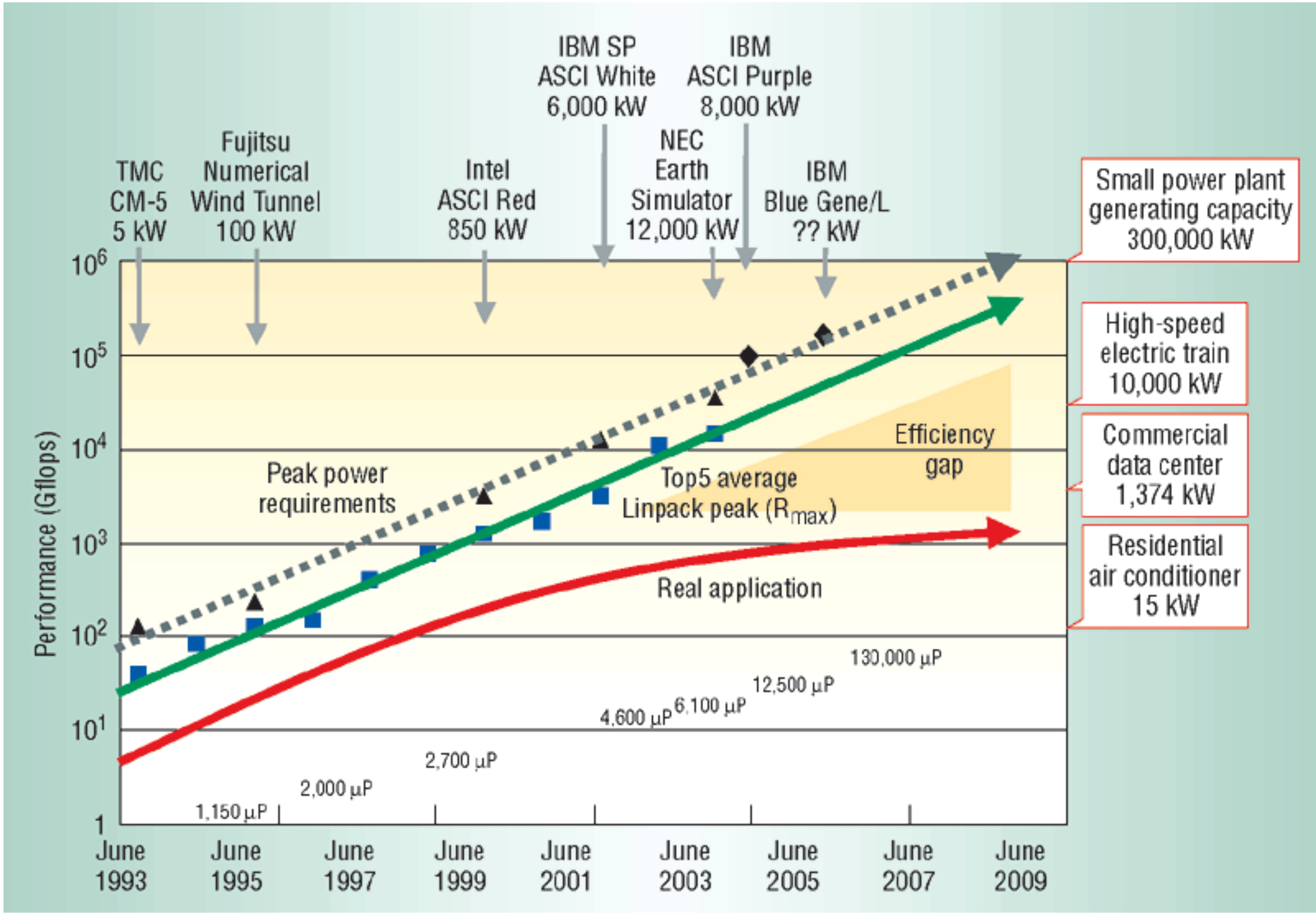


Rayleigh-Bénard convection





Massively parallel supercomputer



Kirk W. Cameron, Rong Ge, Xizhou Feng, High-Performance, Power-Aware Distributed Computing for Scientific Application, *Computer*, vol. 38, no. , pp. 40-47, November 2005

OpenMP versus MPI

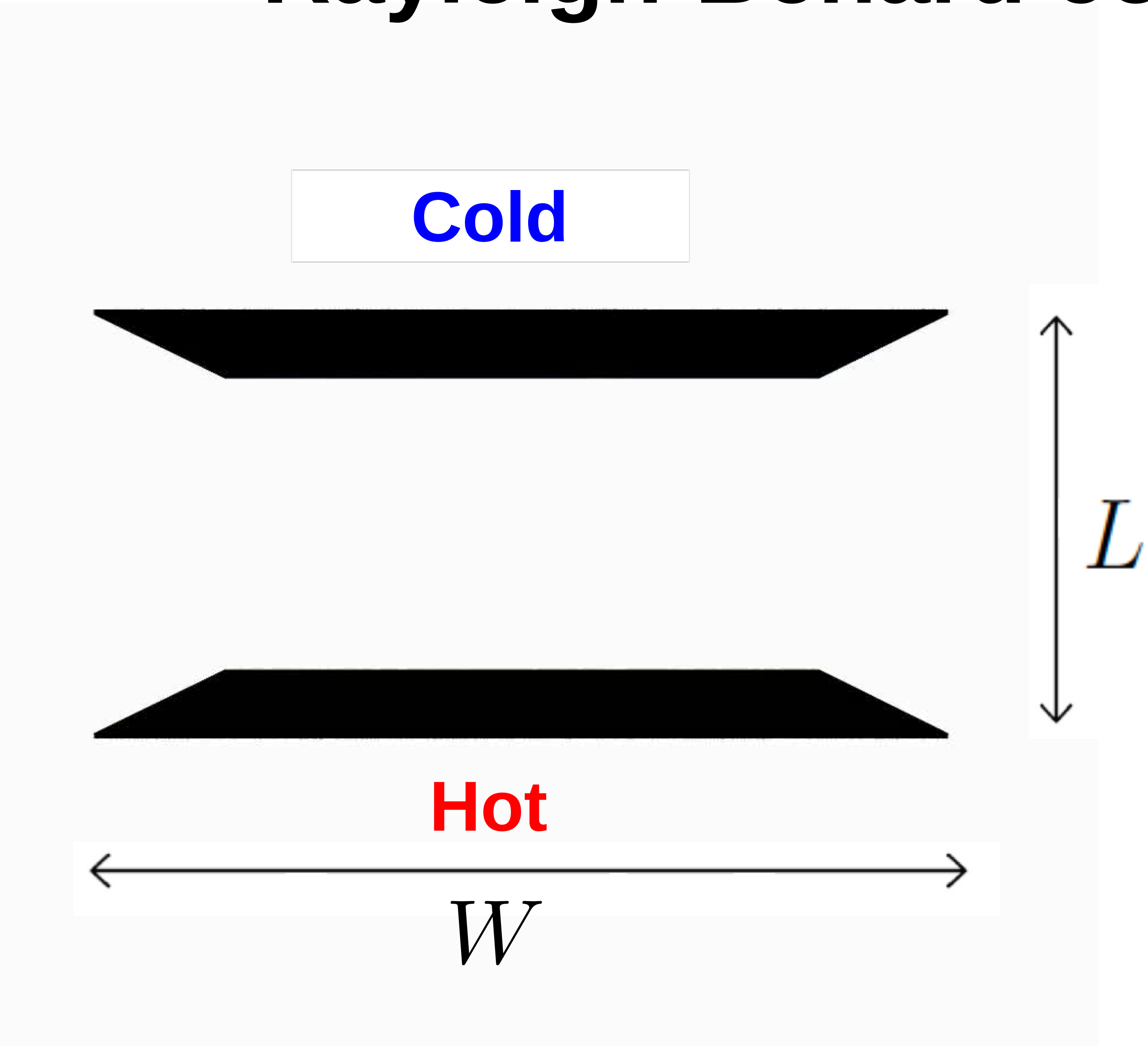
OpenMP

- Pro’s
  - Relatively easy to implement
  - Incremental on a loop per loop basis
- Con’s
  - Works only on shared memory architecture (typically max 32 cores)

MPI

- Pro’s
  - Works for all systems
  - up to an arbitrary number of cores
- Con’s
  - More work to implement

Rayleigh-Bénard convection



Control parameters

Aspect ratio

$$\Gamma = \frac{W}{L}$$

Rayleigh number

$$Ra = \frac{\beta g L^3 \Delta}{\nu \kappa}$$

Prandtl number

$$Pr = \frac{\nu}{\kappa}$$

van der Poel et al. Computers & Fluids 116 (2015) 10–16

Rayleigh-Bénard convection

$$\begin{aligned}
\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} &= -\nabla p + \nu \nabla^2 \mathbf{u} + g\beta T \widehat{z} \\
\frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla) T &= \kappa \nabla^2 T \\
\nabla \cdot \mathbf{u} &= 0
\end{aligned}$$

Rayleigh-Bénard convection

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Introduce the following dimensionless parameters

$$\begin{aligned}\widetilde{x} &= x/H \\ \widetilde{\mathbf{u}} &= \mathbf{u}/U\end{aligned}$$

Rayleigh-Bénard convection

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Introduce the following dimensionless parameters

$$\begin{aligned}
\widetilde{x} &= x/H \\
\widetilde{\mathbf{u}} &= \mathbf{u}/U \\
\widetilde{t} &= tU/H
\end{aligned}$$

Rayleigh-Bénard convection

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\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} &= -\nabla p + \nu\nabla^2\mathbf{u} + g\beta T\widehat{z} \\
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\widetilde{\theta} &= T/\Delta
\end{aligned}$$

Rayleigh-Bénard convection

$$
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$$

Introduce the following dimensionless parameters

$$
\begin{aligned}
\widetilde{x} &= x/H \\
\widetilde{\mathbf{u}} &= \mathbf{u}/U \\
\widetilde{t} &= tU/H \\
\widetilde{\theta} &= T/\Delta \\
\widetilde{p} &= p/U^2
\end{aligned}
$$

Rayleigh-Bénard convection

$$\begin{aligned}
\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} &= -\nabla p + \nu\nabla^2\mathbf{u} + g\beta T\widehat{z} \\
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\end{aligned}$$

$$\begin{aligned}
\frac{\partial \widetilde{\mathbf{u}}}{\partial \widetilde{t}} + (\widetilde{\mathbf{u}}\cdot\widetilde{\nabla})\widetilde{\mathbf{u}} &= -\widetilde{\nabla}\widetilde{p} + \frac{\nu}{\boxed{UH}}\widetilde{\nabla}^2\widetilde{\mathbf{u}} + \frac{g\beta\boxed{\Delta H}}{\boxed{U^2}}\widetilde{T}\widehat{z} \\
\frac{\partial \widetilde{\theta}}{\partial \widetilde{t}} + (\widetilde{\mathbf{u}}\cdot\widetilde{\nabla})\widetilde{\theta} &= \frac{\kappa}{\boxed{UH}}\widetilde{\nabla}^2\widetilde{\theta} \\
\widetilde{\nabla}\cdot\widetilde{\mathbf{u}} &= 0
\end{aligned}$$

Rayleigh-Bénard convection

$$\frac{\partial \tilde{\mathbf{u}}}{\partial \tilde{t}} + (\tilde{\mathbf{u}} \cdot \tilde{\nabla})\tilde{\mathbf{u}} = -\tilde{\nabla}\tilde{p} + \frac{\nu}{UH}\tilde{\nabla}^2\tilde{\mathbf{u}} + \frac{g\beta\Delta H}{U^2}\tilde{T}\hat{z}$$

$$\frac{\partial \tilde{\theta}}{\partial \tilde{t}} + (\tilde{\mathbf{u}} \cdot \tilde{\nabla})\tilde{\theta} = \frac{\kappa}{UH}\tilde{\nabla}^2\tilde{\theta}$$

$$\tilde{\nabla} \cdot \tilde{\mathbf{u}} = 0$$

Setting buoyancy scale order 1 a convenient velocity scale is found: the so called free-fall velocity $U = \sqrt{g\beta\Delta H}$ as typical velocity.

$$\frac{\nu}{UH} = \sqrt{\frac{Pr}{Ra}}$$

$$\frac{\kappa}{UH} = \frac{1}{\sqrt{PrRa}}$$

With

$$Pr = \frac{\nu}{\kappa}$$

$$Ra = \frac{\beta g L^3 \Delta}{\nu\kappa}$$

Rayleigh-Bénard convection

$$
\begin{aligned}
\frac{D\mathbf{u}}{Dt} &= -\nabla P + \left(\frac{Pr}{Ra}\right)^{1/2} \nabla^2 \mathbf{u} + \theta \widehat{z} \\
\frac{D\theta}{Dt} &= \frac{1}{(Pr\,Ra)^{1/2}} \nabla^2 \theta \\
\nabla \cdot \mathbf{u} &= 0
\end{aligned}
$$

With

$$Pr = \frac{\nu}{\kappa} \qquad Ra = \frac{\beta g L^3 \Delta}{\nu \kappa}$$

AFiD code for wall bounded turbulence

Navier-Stokes equations with Boussinesq approximation
and additional equation for temperature

$$
\begin{aligned}
\frac{D\mathbf{u}}{Dt} &= -\nabla P + \left(\frac{Pr}{Ra}\right)^{1/2} \nabla^2 \mathbf{u} + \theta \widehat{z} \\
\frac{D\theta}{Dt} &= \frac{1}{(Pr\,Ra)^{1/2}} \nabla^2 \theta \\
\nabla \cdot \mathbf{u} &= 0
\end{aligned}
$$

With

$$Pr = \frac{\nu}{\kappa} \qquad\qquad Ra = \frac{\beta g L^3 \Delta}{\nu \kappa}$$

Two horizontal periodic directions (*y*-*z*), vertical direction (*x*) is wall-bounded
Mesh is equally spaced in the horizontal directions, stretched in the vertical direction

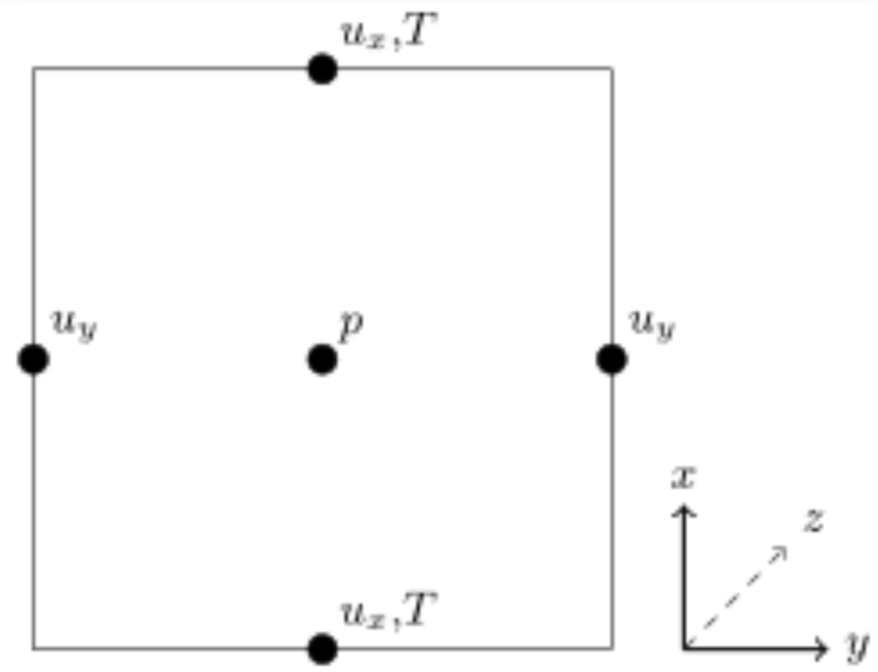
AFiD code: Numerical scheme

Conservative centered finite difference

- Staggered grid
- Fractional step
- Time marching: low-storage RK3

(Verzicco and Orlandi, JCP 1996)

(Orlandi, Fluid Flow Phenomena)



AFiD code: Numerical scheme

At each sub-step:

1) Intermediate non-solenoidal velocity field is calculated using non-linear, viscous, buoyancy and pressure at the current time sub-step

$$\frac{\mathbf{u}^* - \mathbf{u}^j}{\Delta t} = \left[\gamma_l H^j + \rho_l H^{j-1} - \alpha_l \mathcal{G} p^j + \alpha_l (\mathcal{A}_x^j + \mathcal{A}_y^j + \mathcal{A}_z^j) \frac{(\mathbf{u}^* + \mathbf{u}^j)}{2} \right]$$

AFiD code: Numerical scheme

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H - Explicit terms

AFiD code: Numerical scheme

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H - Explicit terms

Ai - Viscous terms in different computational direction

AFiD code: Numerical scheme

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H - Explicit terms

Gradient operator

Ai - Viscous terms in different computational direction

AFiD code: Numerical scheme

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Gradient operator

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u* intermediate, non-solenoidal velocity field u

AFiD code: Numerical scheme

At each sub-step:

1) Intermediate non-solenoidal velocity field is calculated using non-linear, viscous, buoyancy and pressure at the current time sub-step

Time integration coefficients: depend on the used scheme

$$\frac{\mathbf{u}^* - \mathbf{u}^j}{\Delta t} = \left[\gamma_l H^j + \rho_l H^{j-1} - \alpha_l \mathcal{G} p^j + \alpha_l \left(\mathcal{A}_x^j + \mathcal{A}_y^j + \mathcal{A}_z^j\right) \frac{(\mathbf{u}^* + \mathbf{u}^j)}{2}\right]$$

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u* intermediate, non-solenoidal velocity field u

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2) Pressure correction is calculated solving the following Poisson equation

$$\nabla^2 \phi = \frac{1}{\alpha_l \Delta t} (\nabla \cdot \mathbf{u}^*),$$

AFiD code: Numerical scheme

At each sub-step:

1) Intermediate non-solenoidal velocity field is calculated using non-linear, viscous, buoyancy and pressure at the current time sub-step

$$\frac{\mathbf{u}^* - \mathbf{u}^j}{\Delta t} = \left[\gamma_l H^j + \rho_l H^{j-1} - \alpha_l \mathcal{G} p^j + \alpha_l (\mathcal{A}_x^j + \mathcal{A}_y^j + \mathcal{A}_z^j) \frac{(\mathbf{u}^* + \mathbf{u}^j)}{2}\right]$$

2) Pressure correction is calculated solving the following Poisson equation

$$\nabla^2 \phi = \frac{1}{\alpha_l \Delta t} (\nabla \cdot \mathbf{u}^*),$$

3) The velocity and pressure are then updated using:

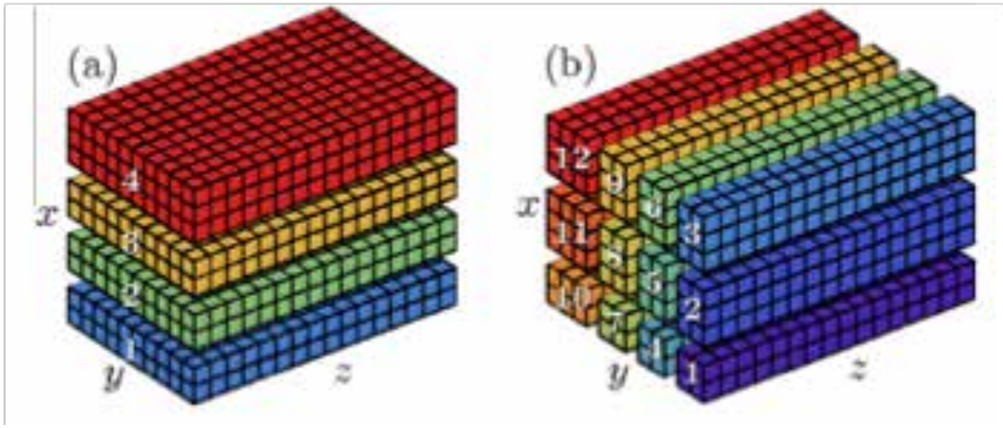
$$\mathbf{u}^{j+1} = \mathbf{u}^* - \alpha_l \Delta t (\mathcal{G} \phi),$$

$$p^{j+1} = p^j + \phi - \frac{\alpha_l \Delta t}{2Re} (\mathcal{L} \phi),$$

Making uj+1 divergence free

AFiD code: Parallel implementation

- For large Ra numbers (large temperature difference), the implicit integration of the viscous terms in the horizontal directions becomes unnecessary (prevents solving tridiagonal matrices in the horizontal directions)
- This simplifies the parallel implementation:
  - Only the Poisson solver requires global communication
- The code uses a pencil-type decomposition, more general than a slab-type one



- The pencil decomposition is based on the Decomp2D library www.2decomp.org)

AFiD code: Poisson solver

- The solution of the Poisson equation is always the critical part in incompressible solvers

- Direct solver:
  - Fourier decomposition in the horizontal plane
  - Tridiagonal solver in the normal direction

$$\left(\frac{\partial^2}{\partial x^2} - \omega_{y,j}^2 - \omega_{z,k}^2\right)\mathcal{F}(\phi) = \mathcal{F}\left[\frac{1}{\alpha_l \Delta t}(\mathcal{D}\mathbf{u}^*)\right]$$

$$\omega_{y,j} = \begin{cases} \left(1 - \cos\left[\frac{2\pi(j-1)}{N_y}\right]\right)\Delta_y^{-2} & : \text{for } j \leqslant \frac{1}{2}N_y + 1 \\ \left(1 - \cos\left[\frac{2\pi(N_y-j+1))}{N_y}\right]\right)\Delta_y^{-2} & : \text{otherwise} \end{cases}$$

AFiD code: Poisson solver

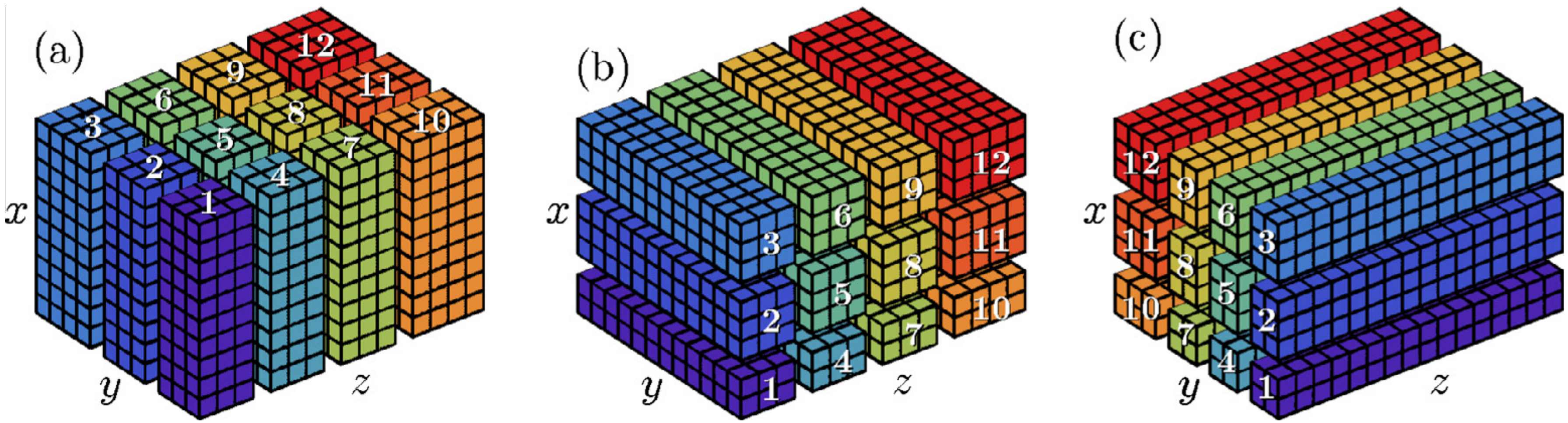
1) FFT the r.h.s along y – *(b)* (from real NX x NY x NZ to complex NX x (NY+1)/2 x NZ)

2) FFT the r.h.s. along z – *(c)* (from complex NX x (NY+1)/2 x NZ to complex NX x (NY+1)/2 x NZ)

3) Solve tridiagonal system in x for each y and z wavenumber - *(a)*

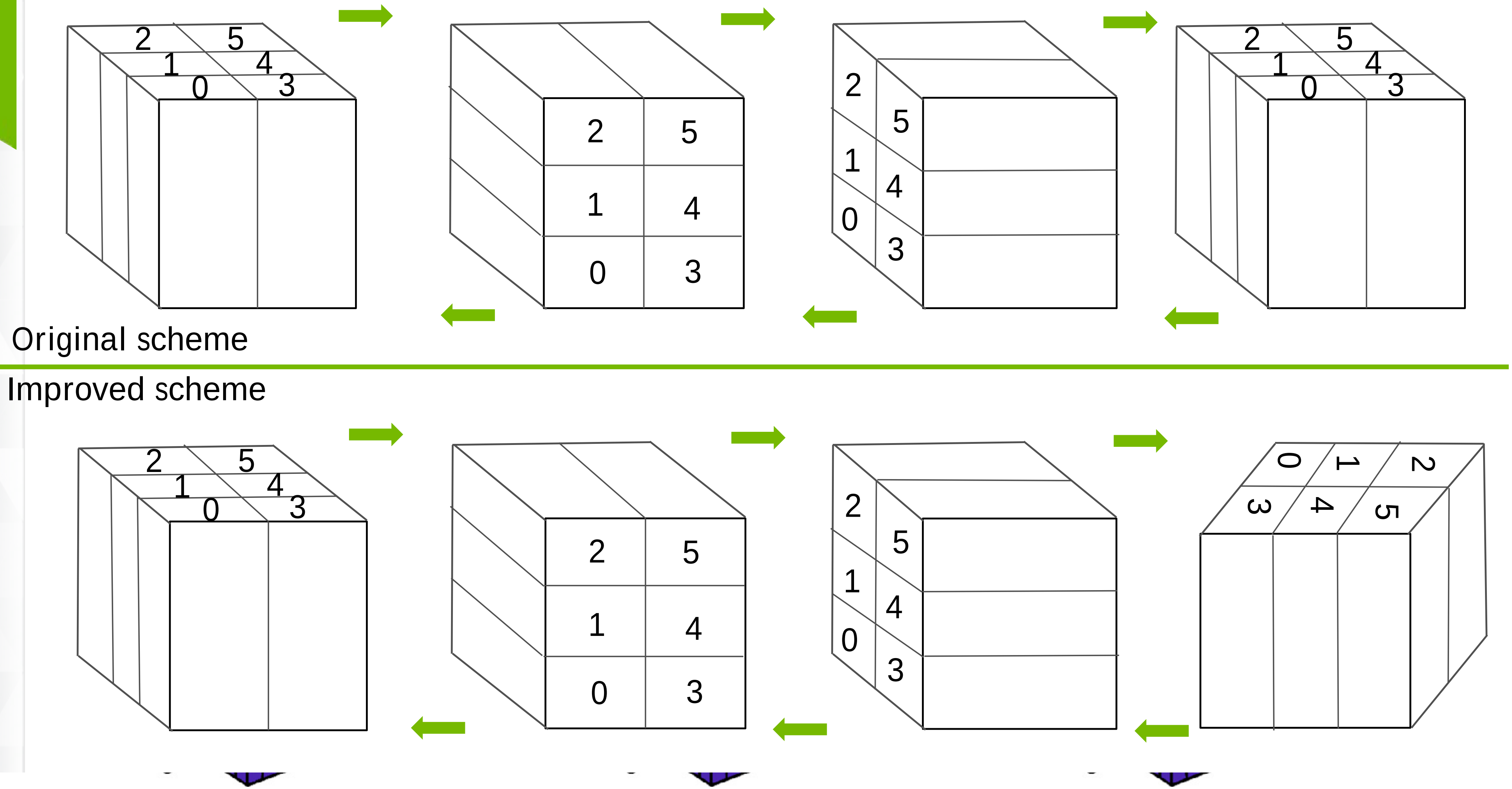
4) Inverse FFT the solution along z – *(c)* (from complex NX x (NY+1)/2 x NZ to complex NX x (NY+1)/2 x

5) Inverse FFT the solution along y – *(b)* (from complex NX x (NY+1)/2 x NZ to real NX x NY x NZ)



AFiD code: Poisson solver

Transpose



AFiD Code — Libraries

CPU

I/O: HDF5

FFT: FFTW (guru plan)

Linear algebra: BLAS+LAPACK

Distributed memory: MPI, 2DDecomp with additional x-z and z-x transpose

GPU

I/O: HDF5

FFT: CUFFT

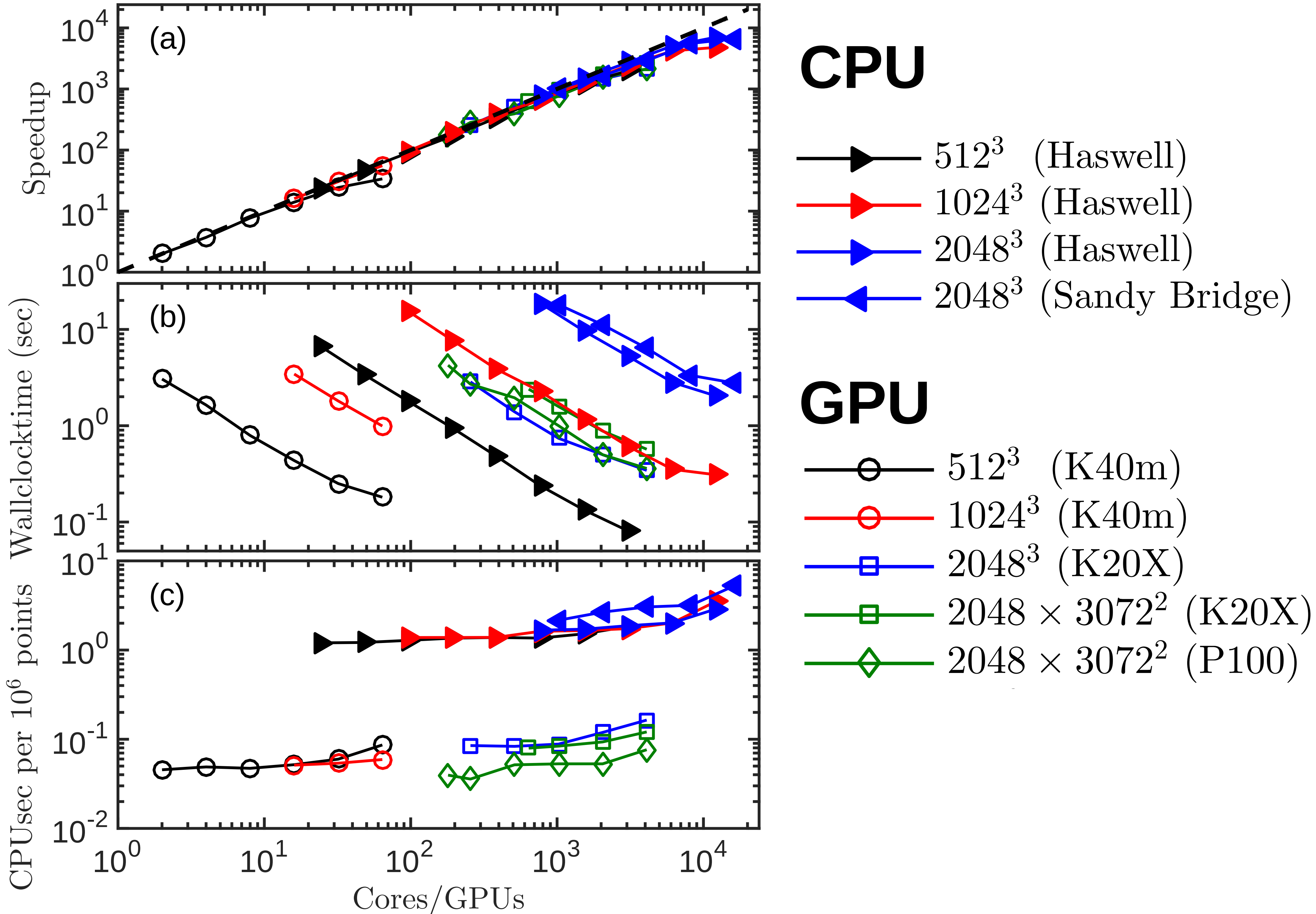
Linear algebra: custom kernels

Distributed memory: MPI, 2DDecomp with improved x-z and z-x transpose

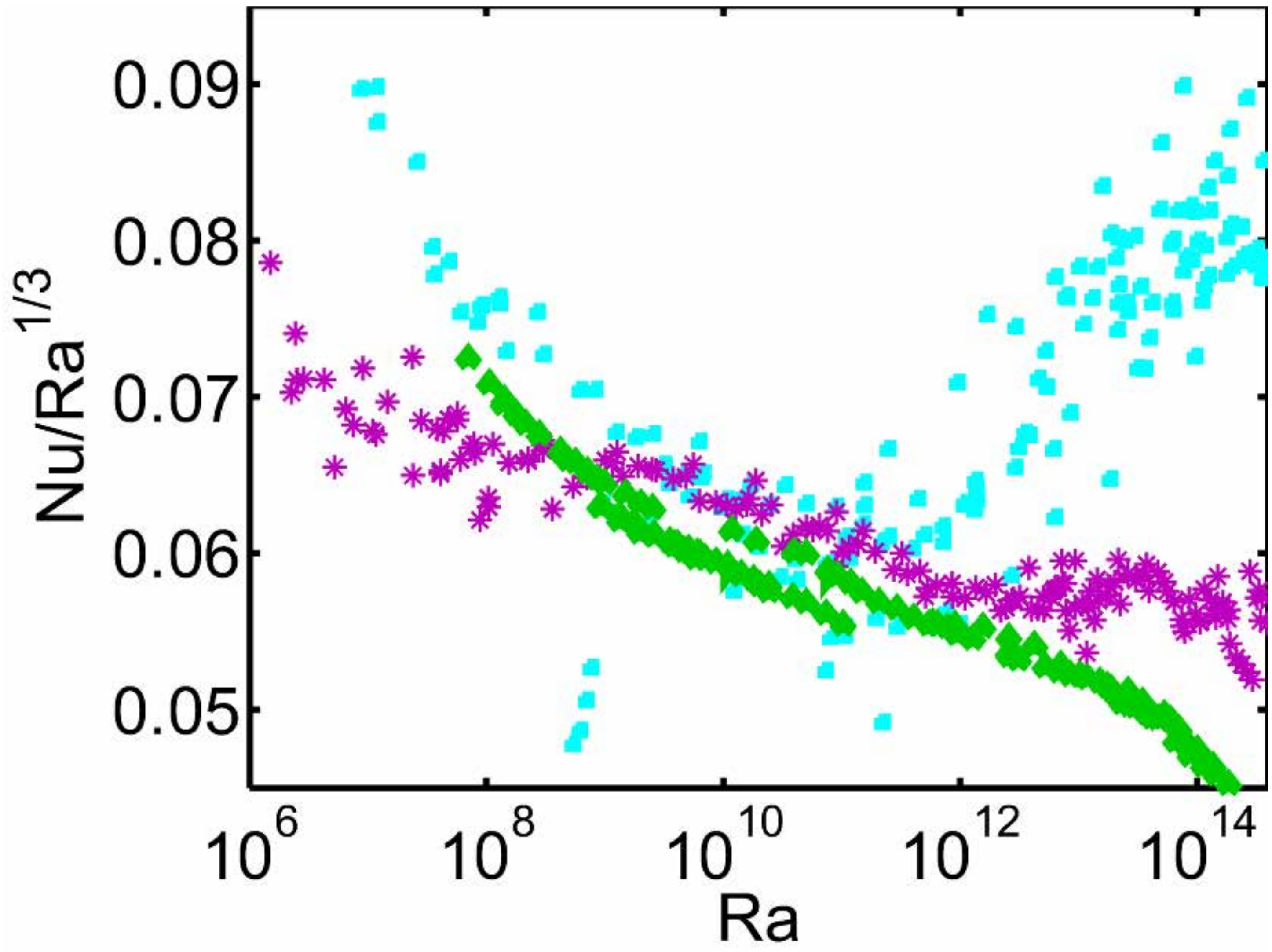
Manycore: CUDA Fortran

Available through Github on **www.afid.eu**

Scaling of AFiD code



Compensated Nusselt Number



Experiments

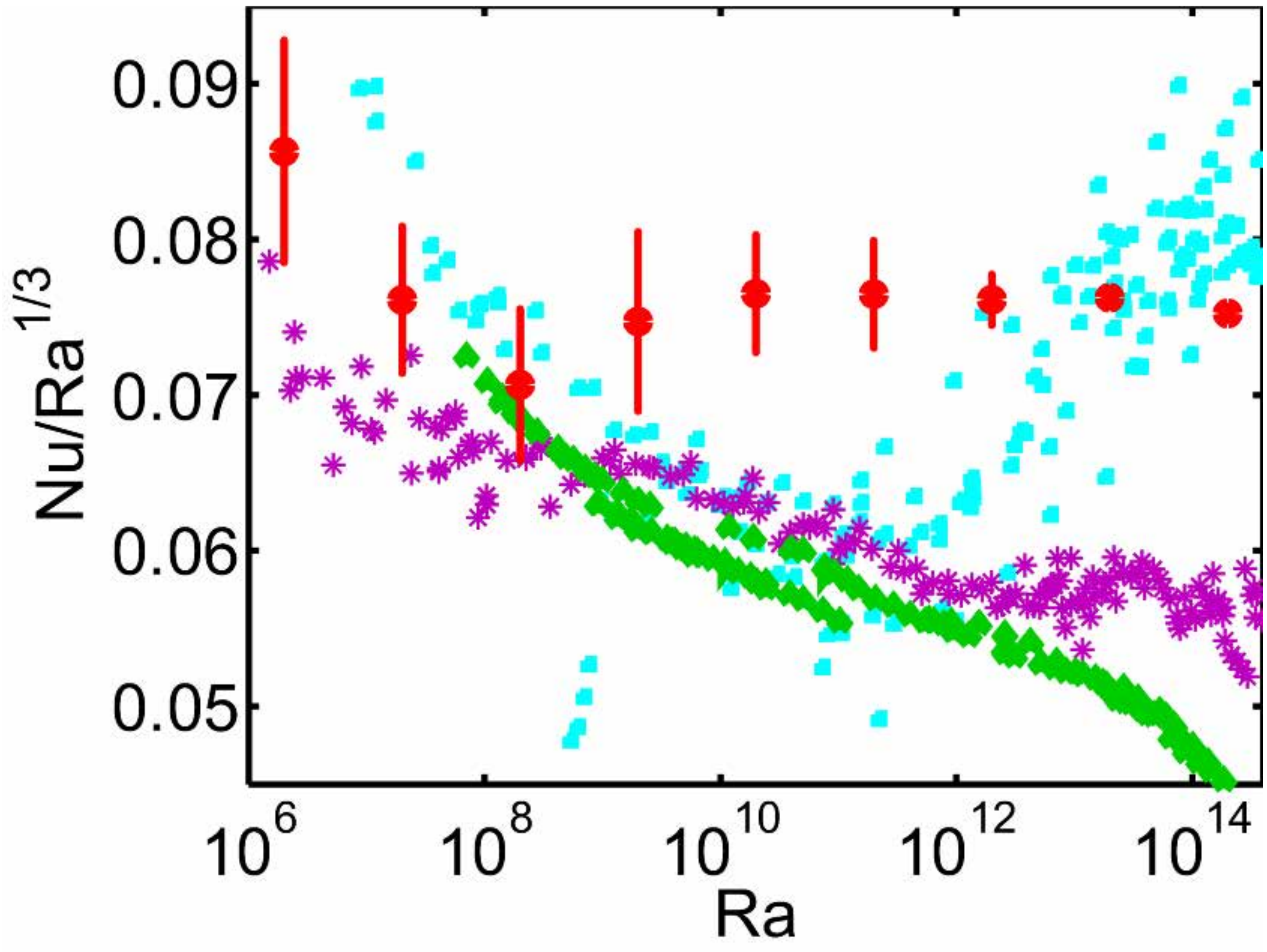
Chavanne *et al.* (2001)

Niemela et al. (2000)

Funfschilling et al. (2009)

$\Gamma = 0.5$, $Pr = 0.7$

Compensated Nusselt Number



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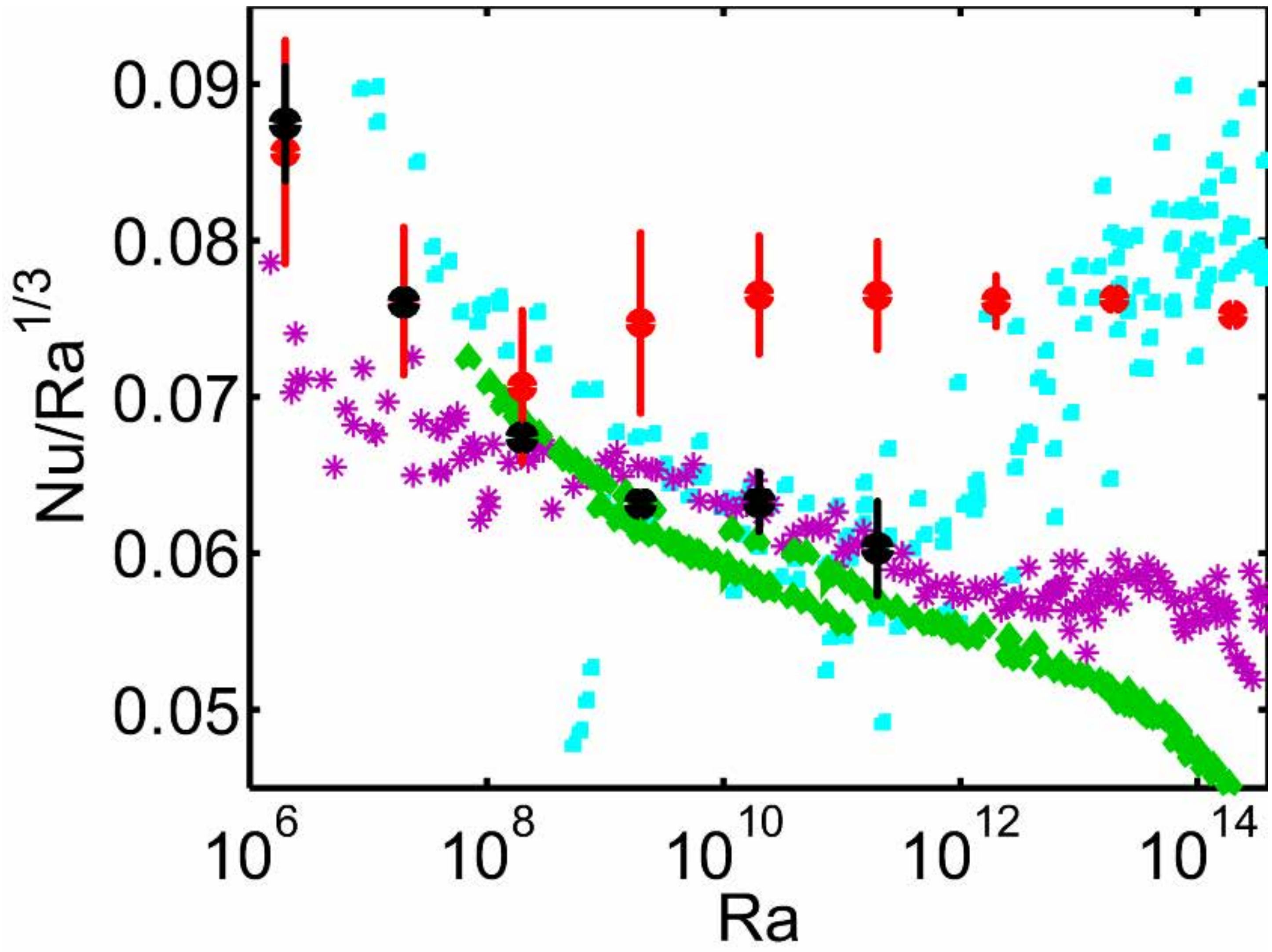
Simulations

Amati et al. (2005)

$\Gamma = 0.5$, $Pr = 0.7$

Compensated Nusselt Number



Γ =0.5, Pr=0.7

Experiments

Chavanne *et al.* (2001)

Niemela et al. (2000)

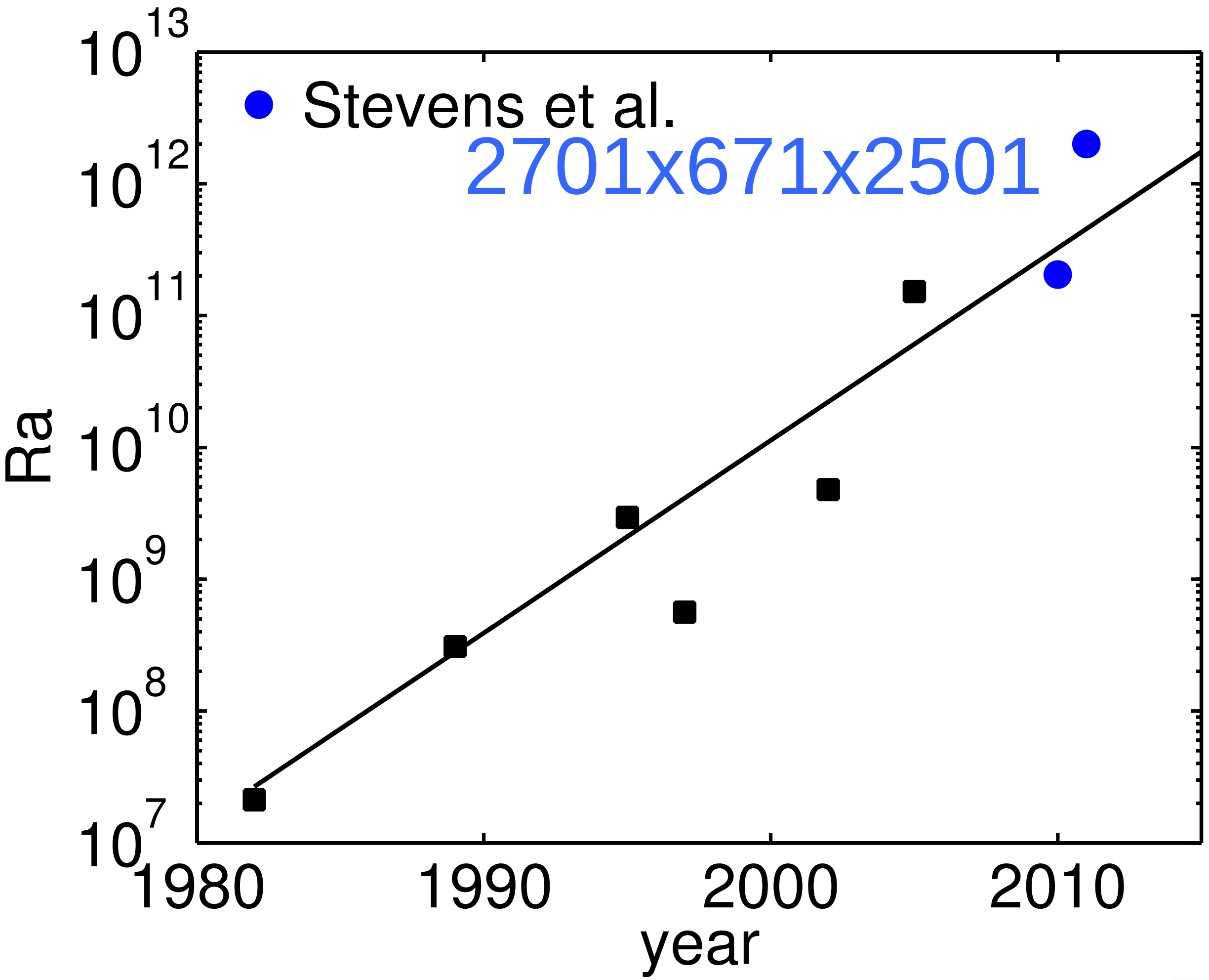
Funfschilling et al. (2009)

Simulations

Amati et al. (2005)

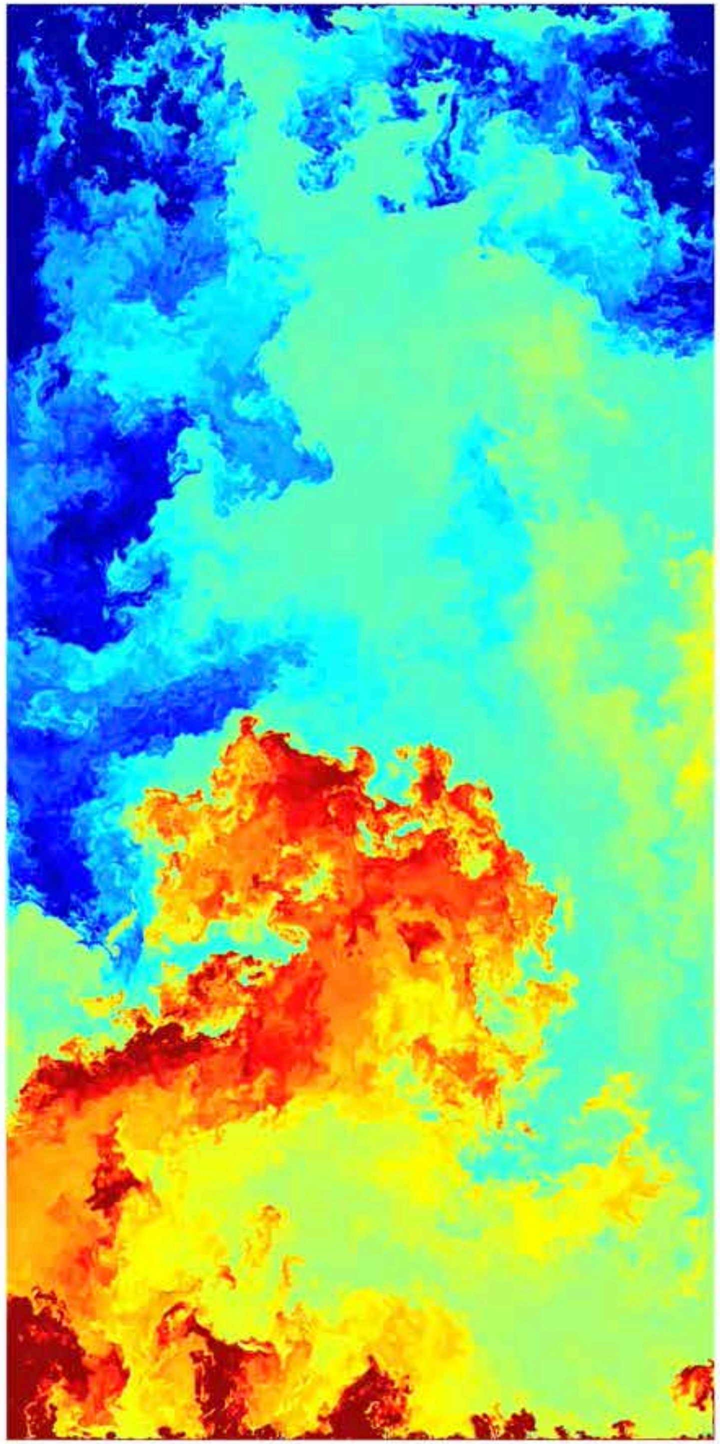
Stevens et al. (2009)

Rayleigh Bénard convection: Direct numerical Simulations

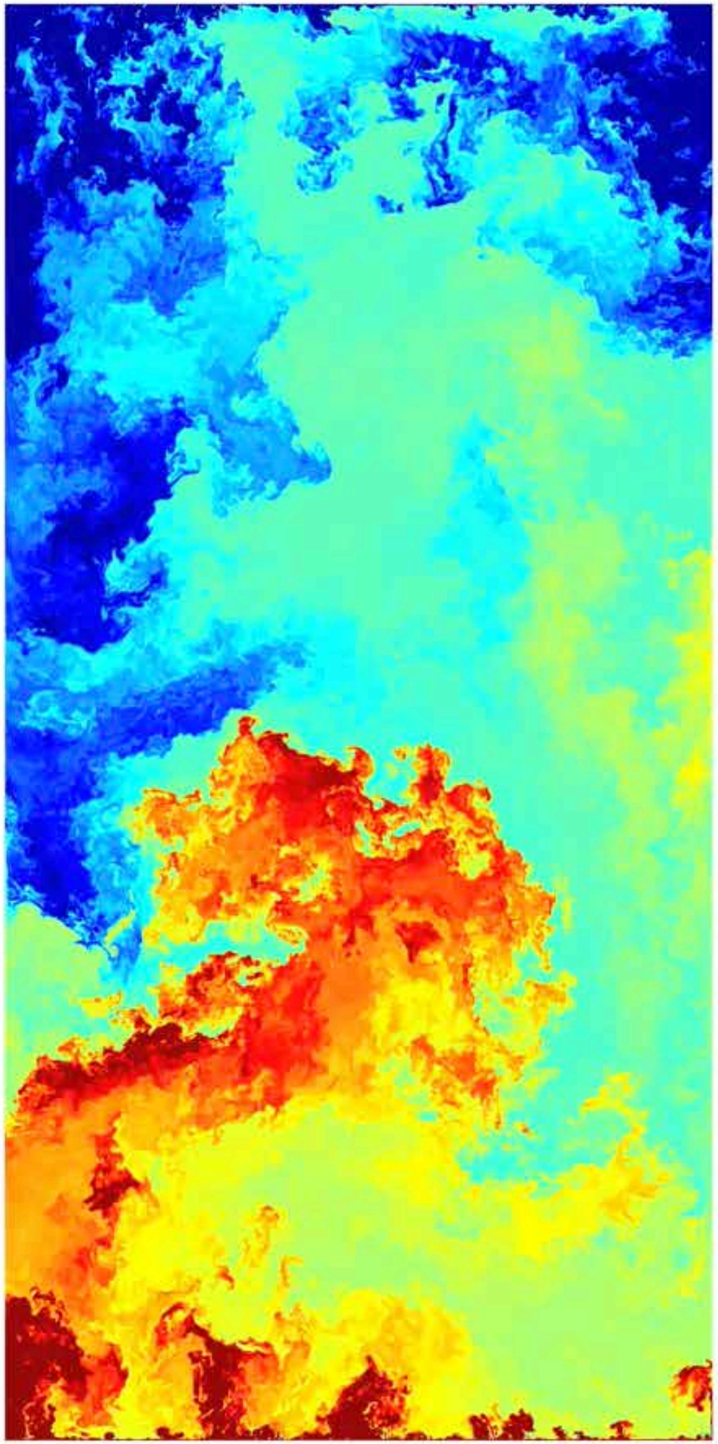


Stevens, Verzicco, Lohse, JFM 643, 495–507 (2010)
Stevens, Lohse, Verzicco, JFM 688, 31-43 (2011)

3D flow



3D flow



Largest DNS

$Ra = 2 \bullet 10^{12}$

$Pr = 0.7$

$\Gamma = 0.5$

3D

2012

Simulation details for Ra = 2×10^{12}

- grid = $N_\phi \times N_r$ X N_z = 2701 x 671 x 2501
- 10^7 DEISA CPU hours = 1000 years!
  (devoted machine in Stuttgart)
- corresponds to 750 Huygens cores
- Upscaling presently limited by ratio #cores/ N_z

Grid resolution

Smallest scales must be resolved, i.e. the Kolmogorov (velocity scale) and the Batchelor scale (temperature scale) have to be fully resolved.

$$h \leq \pi\eta = \pi L \left(\frac{Pr^2}{RaNu}\right)^{1/4} \quad \text{for } Pr \leq 1 \ ,$$

$$h \leq \pi\eta_T = \pi L \left(\frac{1}{RaPrNu}\right)^{1/4} \quad \text{for } Pr \geq 1 \ ,$$

Note that the flow has to be solved properly in all directions, i.e. h=max$(\Delta x, \Delta y, \Delta z)$

Stevens, Verzicco, Lohse, JFM 643, 495–507 (2010).

How to verify resolution?

- Check whether the relevant length scales are properly resolved by calculating them from the kinetic and thermal dissipation rates.
- Kinetic energy dissipation rate

$$\epsilon_u(\overrightarrow{x}) \quad = \quad \nu|\nabla \mathbf{u}|^2$$

- Thermal energy dissipation rate

$$\epsilon_\theta(\overrightarrow{x}) \quad = \quad \kappa|\nabla \theta|^2$$

- These can be checked from exact analytic relationships

Derivation of exact relations: ϵ_u

Multiply Boussinesq equation for u_i

$$\partial_t u_i + u_j \partial_j u_i = -\partial_i p + \nu \partial_j^2 u_i + \beta g \partial_{i3} \theta \tag{1}$$

with u_i:

$$\partial_t \frac{u^2}{2} = -\partial_j \left(u_j \frac{u^2}{2} \right) - u_i \partial_i p + \nu u_i \partial_j^2 u_i + \beta g \theta u_3 \tag{2}$$

Derivation of exact relations: ϵ_u

Multiply Boussinesq equation for u_i

$$\partial_t u_i + u_j \partial_j u_i = -\partial_i p + \nu \partial_j^2 u_i + \beta g \partial_{i3} \theta \tag{1}$$

with u_i:

$$\partial_t \frac{u^2}{2} = -\partial_j \left(u_j \frac{u^2}{2} \right) - u_i \partial_i p + \boxed{\nu u_i \partial_j^2 u_i} + \beta g \theta u_3 \tag{2}$$

Apply product rule for the second but last term:

$$\boxed{\nu u_i \partial_j^2 u_i} = \nu \partial_j (u_i \partial_j u_i) - \nu \partial_j u_i \partial_j u_i \tag{3}$$

Derivation of exact relations: ϵ_u

Multiply Boussinesq equation for u_i

$$\partial_t u_i + u_j \partial_j u_i = -\partial_i p + \nu \partial_j^2 u_i + \beta g \partial_{i3} \theta \tag{1}$$

with u_i:

$$\partial_t \frac{u^2}{2} = -\partial_j \left(u_j \frac{u^2}{2} \right) - u_i \partial_i p + \boxed{\nu u_i \partial_j^2 u_i} + \beta g \theta u_3 \tag{2}$$

Apply product rule for the second but last term:

$$\nu u_i \partial_j^2 u_i = \nu \partial_j (u_i \partial_j u_i) - \nu \partial_j u_i \partial_j u_i \tag{3}$$

Summarize terms which can be written as divergence:

$$\partial_t \frac{u^2}{2} = -\partial_i \left(u_i \frac{u^2}{2} + u_i p \boxed{- \nu \left[\partial_i \frac{u^2}{2} \right]} \right) \boxed{- \nu \partial_j u_i \partial_j u_i} + \beta g \theta u_3 \tag{4}$$

Derivation of exact relations: ϵ_u

Multiply Boussinesq equation for u_i

$$\partial_t u_i + u_j \partial_j u_i = -\partial_i p + \nu \partial_j^2 u_i + \beta g \partial_{i3} \theta \tag{1}$$

with u_i:

$$\partial_t \frac{u^2}{2} = -\partial_j \left(u_j \frac{u^2}{2} \right) - u_i \partial_i p + \nu u_i \partial_j^2 u_i + \beta g \theta u_3 \tag{2}$$

Apply product rule for the second but last term:

$$\nu u_i \partial_j^2 u_i = \nu \partial_j (u_i \partial_j u_i) - \nu \partial_j u_i \partial_j u_i \tag{3}$$

Summarize terms which can be written as divergence:

$$\partial_t \frac{u^2}{2} = -\partial_i \left(u_i \frac{u^2}{2} + u_i p + \nu \left[\partial_i \frac{u^2}{2} \right] \right) - \nu \partial_j u_i \partial_j u_i + \beta g \theta u_3 \tag{4}$$

Apply stationarity, i.e. $lhs = 0$, and volume average: With Gauss's theorem the first term on the rhs vanishes as $u = 0$ at all boundaries. Thus

$$\epsilon_u = \nu \langle \partial_j u_i \partial_j u_i \rangle_V = \beta g \langle \theta u_3 \rangle_V = \beta g \frac{1}{L} \int_0^L \langle \theta u_3 \rangle_A dz \tag{5}$$

Derivation of exact relations: ϵ_u

Use the definition of Nu to write

$$\langle \theta u_3 \rangle_A = k\Delta L^{-1} Nu + k\partial_3 \langle \theta \rangle_A \qquad (6)$$

and perform the integration over z

$$\epsilon_u \quad = \quad \beta g \left(\kappa \Delta L^{-1} Nu + \frac{\kappa}{L} \left[\langle \theta \rangle_A \right]_0^L \right) \qquad (7)$$

Derivation of exact relations: ϵ_u

Use the definition of Nu to write

$$\langle \theta u_3 \rangle_A = k\Delta L^{-1} Nu + k\partial_3 \langle \theta \rangle_A \tag{6}$$

and perform the integration over z

$$\epsilon_u = \beta g \left(\kappa \Delta L^{-1} Nu + \frac{\kappa}{L} [\langle \theta \rangle_A]_0^L \right) \tag{7}$$

$$\epsilon_u = \beta g \left(\kappa \Delta L^{-1} Nu + \frac{\kappa}{L} \left[-\frac{\Delta}{2} - \frac{\Delta}{2} \right]_0^L \right) \tag{8}$$

$$\epsilon_u = \frac{\beta g \kappa \Delta}{L} (Nu - 1) \tag{9}$$

Derivation of exact relations: ϵ_u

Use the definition of Nu to write

$$\langle \theta u_3 \rangle_A = k\Delta L^{-1} Nu + k\partial_3 \langle \theta \rangle_A \quad (6)$$

and perform the integration over z

$$\epsilon_u = \beta g \left(\kappa \Delta L^{-1} Nu + \frac{\kappa}{L} \left[\langle \theta \rangle_A \right]_0^L \right) \quad (7)$$

$$\epsilon_u = \beta g \left(\kappa \Delta L^{-1} Nu + \frac{\kappa}{L} \left[-\frac{\Delta}{2} - \frac{\Delta}{2} \right]_0^L \right) \quad (8)$$

$$\epsilon_u = \frac{\beta g \kappa \Delta}{L} (Nu - 1) \quad (9)$$

$$\epsilon_u = \frac{\nu^3 Pr^{-2} Ra}{L^4} (Nu - 1) \quad (10)$$

Derivation of exact relations: ϵ_θ

Multiply Boussinesq equation for θ

$$\partial_t \theta + u_i \partial_i \theta = \kappa \partial_i \partial_i \theta \tag{11}$$

with θ and obtain

$$\partial_t \frac{\theta^2}{2} + \partial_i \left(\frac{\theta^2}{2} u_i \right) = \kappa \theta \partial_i \partial_i \theta = \kappa \partial_i (\theta \partial_i \theta) - \kappa (\partial_i \theta)^2 \tag{12}$$

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Apply stationarity and volume average, apply Gauss's theorem: Both terms on lhs vanish, the second because $u = 0$ at the boundary. It remains

$$\epsilon_\theta = \kappa \langle (\partial_i \theta)^2 \rangle = \frac{\kappa}{V} \int_V \partial_i (\theta \partial_i \theta) dV = \frac{\kappa}{V} \int_S \langle \theta \partial_i \theta \rangle_A dA_i \tag{13}$$

The surface integral can only contribute at the top and at the bottom, because there is no heat flux through the side walls.

Derivation of exact relations: ϵ_θ

$$\epsilon_\theta = \kappa\langle(\partial_i\theta)^2\rangle = \frac{\kappa}{V}\int_V \partial_i(\theta\partial_i\theta)dV = \frac{\kappa}{V}\int_S \langle\theta\partial_i\theta\rangle_A dA_i \qquad (13)$$

Area/Volume = A/V = 1/L. Thus

$$\epsilon_\theta = \frac{\kappa}{L}\left[\langle\theta\partial_3\theta\rangle_A\right]_{bottom}^{top} = \frac{\kappa}{L}\left(-\frac{\Delta}{2}\partial_3\langle\theta\rangle_{A,top} - \frac{\Delta}{2}\partial_3\langle\theta\rangle_{A,bottom}\right) \qquad (14)$$

Derivation of exact relations: ϵ_θ

$$\epsilon_\theta = \kappa\langle(\partial_i\theta)^2\rangle = \frac{\kappa}{V}\int_V \partial_i(\theta\partial_i\theta)dV = \frac{\kappa}{V}\int_S \langle\theta\partial_i\theta\rangle_A dA_i \tag{13}$$

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Now realize that the Nusselt number and also the dimensional heat flux

$$H = \kappa\Delta L^{-1}Nu = \langle u_3\theta\rangle_A - \kappa\partial_3\langle\theta\rangle_A \tag{15}$$

is independent of the height z.

Derivation of exact relations: ϵ_θ

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$$H = \kappa\Delta L^{-1}Nu = \langle u_3\theta\rangle_A - \kappa\partial_3\langle\theta\rangle_A \tag{15}$$

is independent of the height z.

For $z = 0$ or $z = L$ $u_3 = 0$ and therefore

$$H = -\kappa\partial_3\langle\theta\rangle_{A,top} = -\kappa\partial_3\langle\theta\rangle_{A,bottom} \tag{16}$$

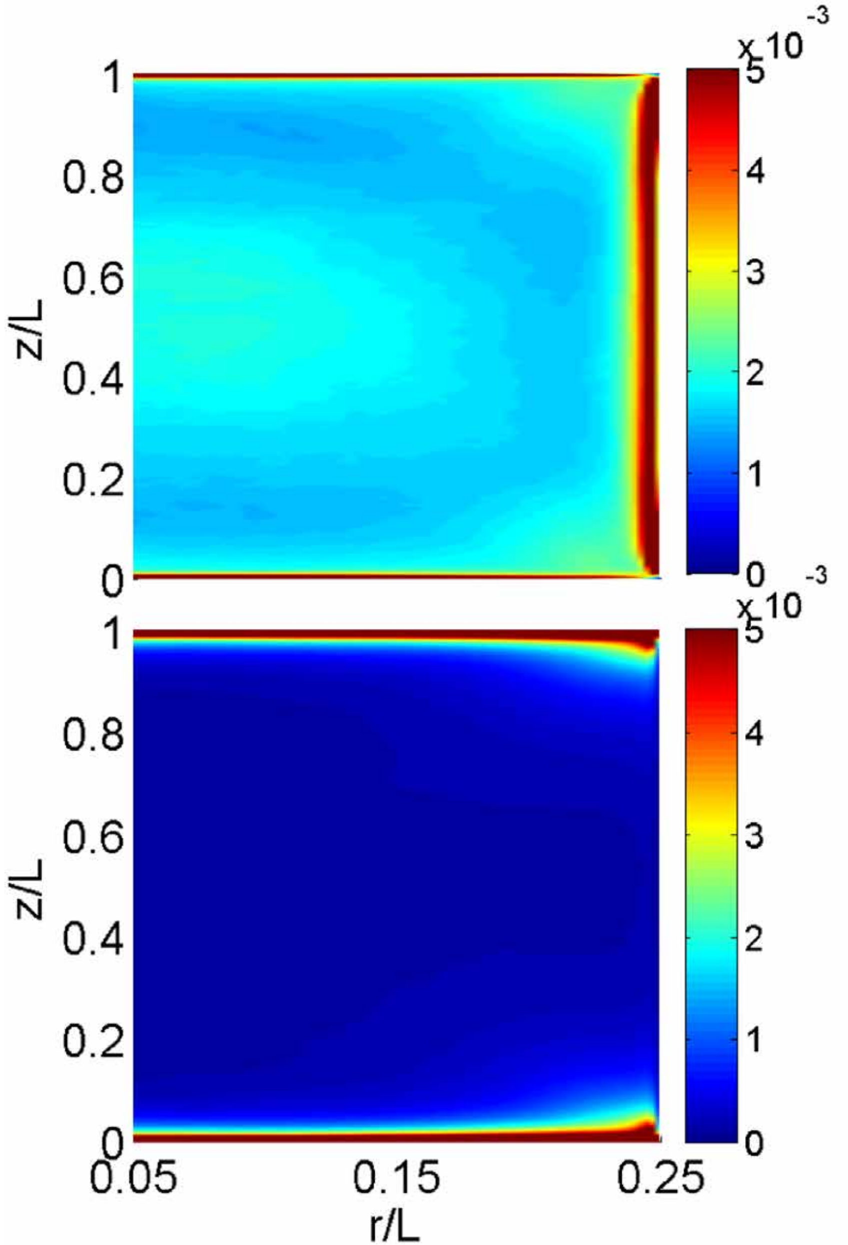
It follows the desired relation

$$\epsilon_\theta = \frac{\Delta H}{L} = \kappa\frac{\Delta^2}{L^2}Nu \tag{17}$$

Check relations in simulations

Ra	$N_\theta \times N_r \times N_z$	Nu	Nu_h	max-diff	N_{BL}	$\frac{\ell_{max,g}}{\eta}$	$\frac{\ell_{max,p}}{\eta}$	$\frac{\frac{\langle\epsilon_u\rangle}{\nu^3 Ra Pr^{-2}/L^4}+1}{Nu}$	$\frac{\frac{\langle\epsilon_\theta\rangle}{\kappa\Delta^2/L^2}}{Nu}$
2×10^6	$97\times49\times129$	10.85	10.92	0.32 %	18	0.42	–	–	–
2×10^6	$97\times49\times129$	10.68	10.32	0.35 %	18	0.42	0.51	0.973	0.978
2×10^6	$129\times65\times193$	10.56	10.86	0.15 %	27	0.31	0.39	0.972	0.986
2×10^6	$193\times97\times257$	11.02	11.03	0.44 %	35	0.21	0.26	0.974	0.991
2×10^7	$129\times49\times193$	20.52	20.56	0.36 %	17	0.66	–	–	–
2×10^7	$193\times97\times257$	20.54	20.69	0.70 %	31	0.46	0.64	0.989	0.987
2×10^7	$289\times129\times353$	20.64	20.53	0.36 %	42	0.34	0.43	0.984	0.991
2×10^8	$97\times49\times193$	40.57	40.71	0.02 %	10	1.84	2.82	1.007	0.926
2×10^8	$193\times65\times257$	39.42	39.52	0.02 %	13	0.92	1.41	0.992	0.950
2×10^8	$257\times97\times385$	39.41	39.10	0.79 %	19	0.70	1.11	0.995	0.973
2×10^9	$129\times65\times257$	89.07	88.25	0.02 %	6	3.01	4.57	1.001	0.858
2×10^9	$193\times65\times257$	84.49	84.46	0.45 %	7	1.99	3.10	1.002	0.879
2×10^9	$193\times65\times257$	84.10	83.66	0.51 %	7	1.98	3.06	1.000	0.877
2×10^9	$385\times97\times385$	79.75	78.70	0.70 %	10	1.15	1.47	0.999	0.935
2×10^9	$513\times129\times513$	79.60	78.89	0.45 %	17	0.93	1.22	1.006	0.962
2×10^{10}	$129\times97\times385$	201.08	201.21	1.01 %	12	6.56	10.88	1.006	0.878
2×10^{10}	$513\times129\times513$	171.79	169.58	2.09 %	19	1.59	2.83	0.994	0.927
2×10^{10}	$385\times257\times1025$	173.13	173.30	0.98 %	29	2.12	–	–	–
2×10^{11}	$769\times193\times769$	387.07	387.53	2.18 %	16	2.31	–	–	–
2×10^{11}	$769\times257\times1025$	373.64	368.88	2.03 %	18	2.28	6.34	0.9883	0.9058
2×10^{11}	$1081\times351\times1301$	352.67	364.75	4.15 %	26	1.60	3.96	1.0244	0.9318

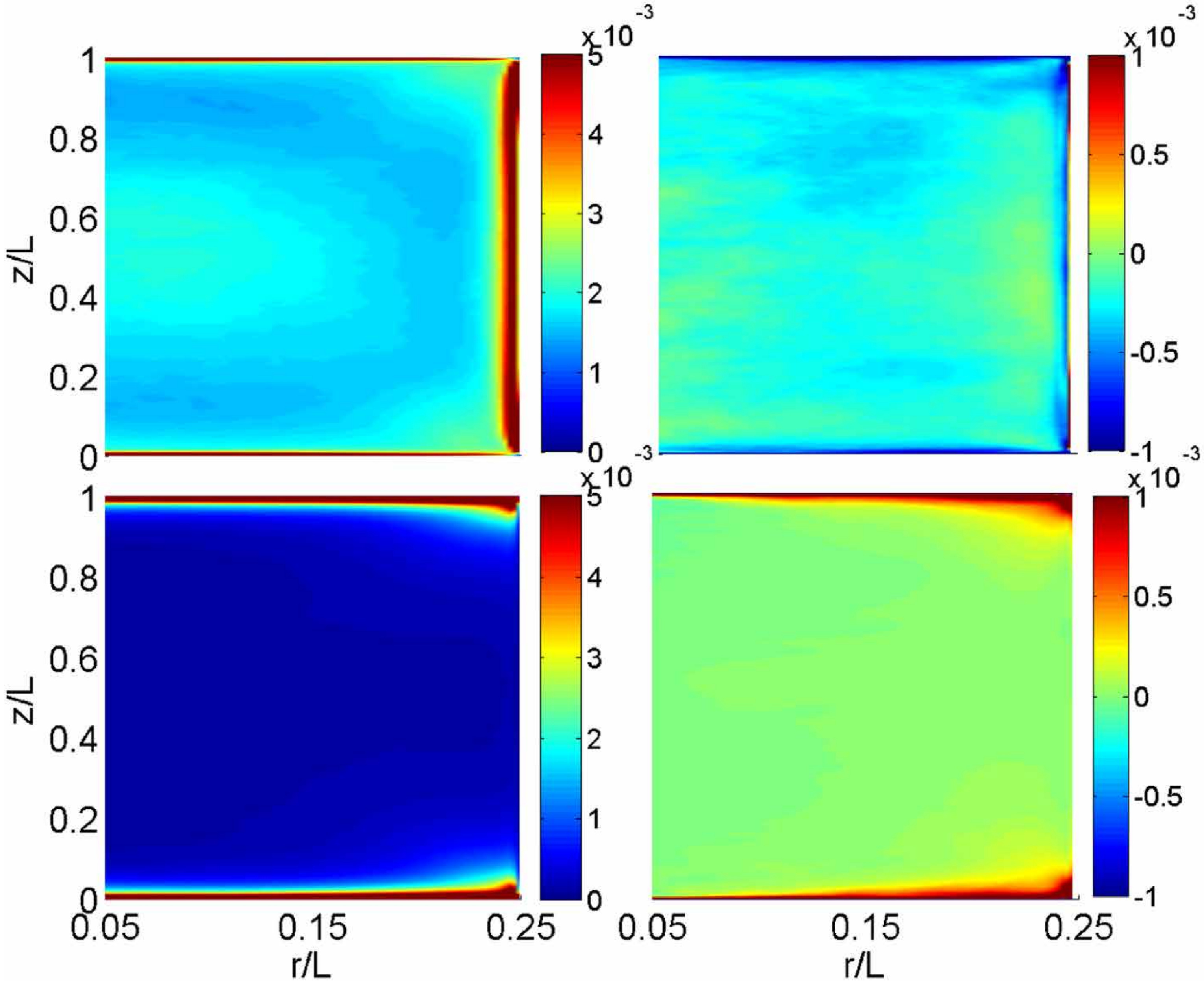
Dissipation rates ($Ra = 2 \cdot 10^9$)



$$\widetilde{\epsilon}_u = \epsilon_u L^3 / U^2$$

$$\widetilde{\epsilon}_\theta = \epsilon_\theta U / (\Delta^2 L)$$

Dissipation rates (Ra = $2\cdot10^9$)



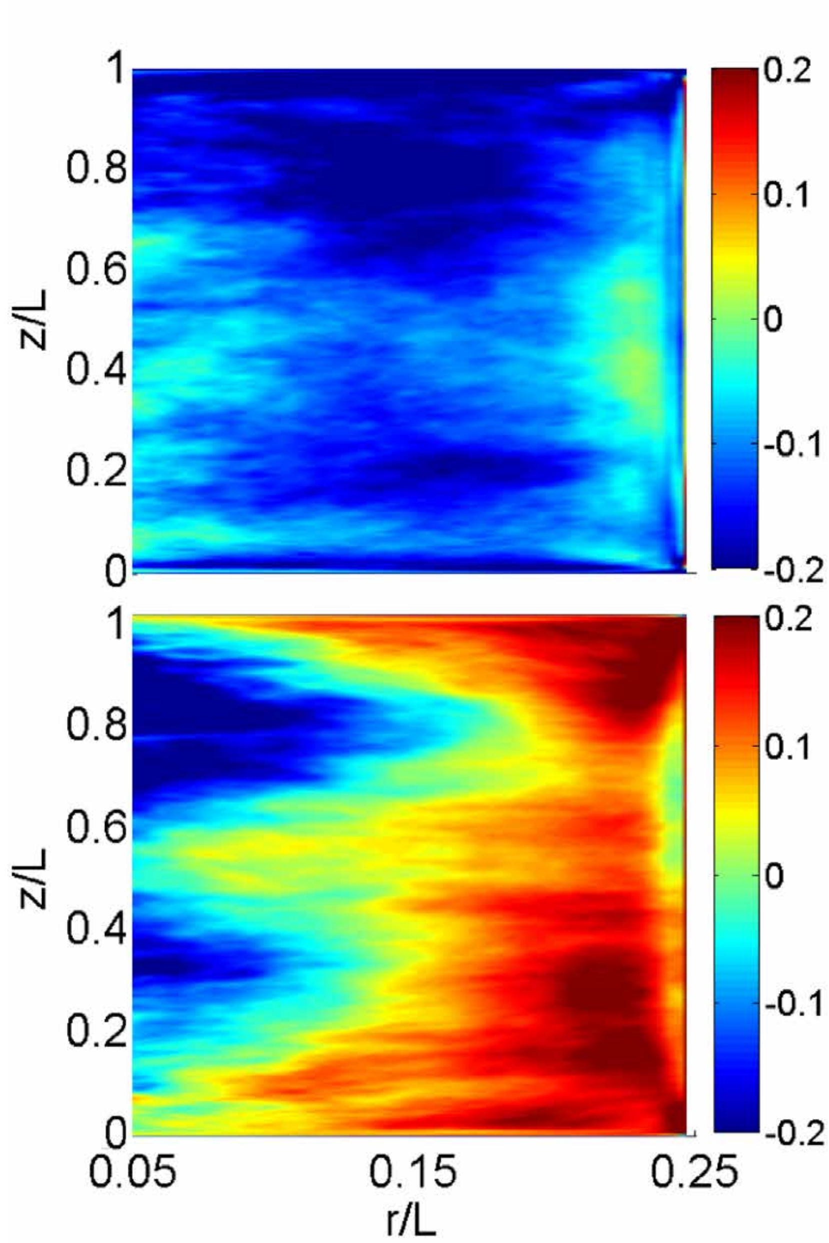
$$\widetilde{\epsilon}_u^H - \widetilde{\epsilon}_u^L$$

$$\widetilde{\epsilon}_\theta^H - \widetilde{\epsilon}_\theta^L$$

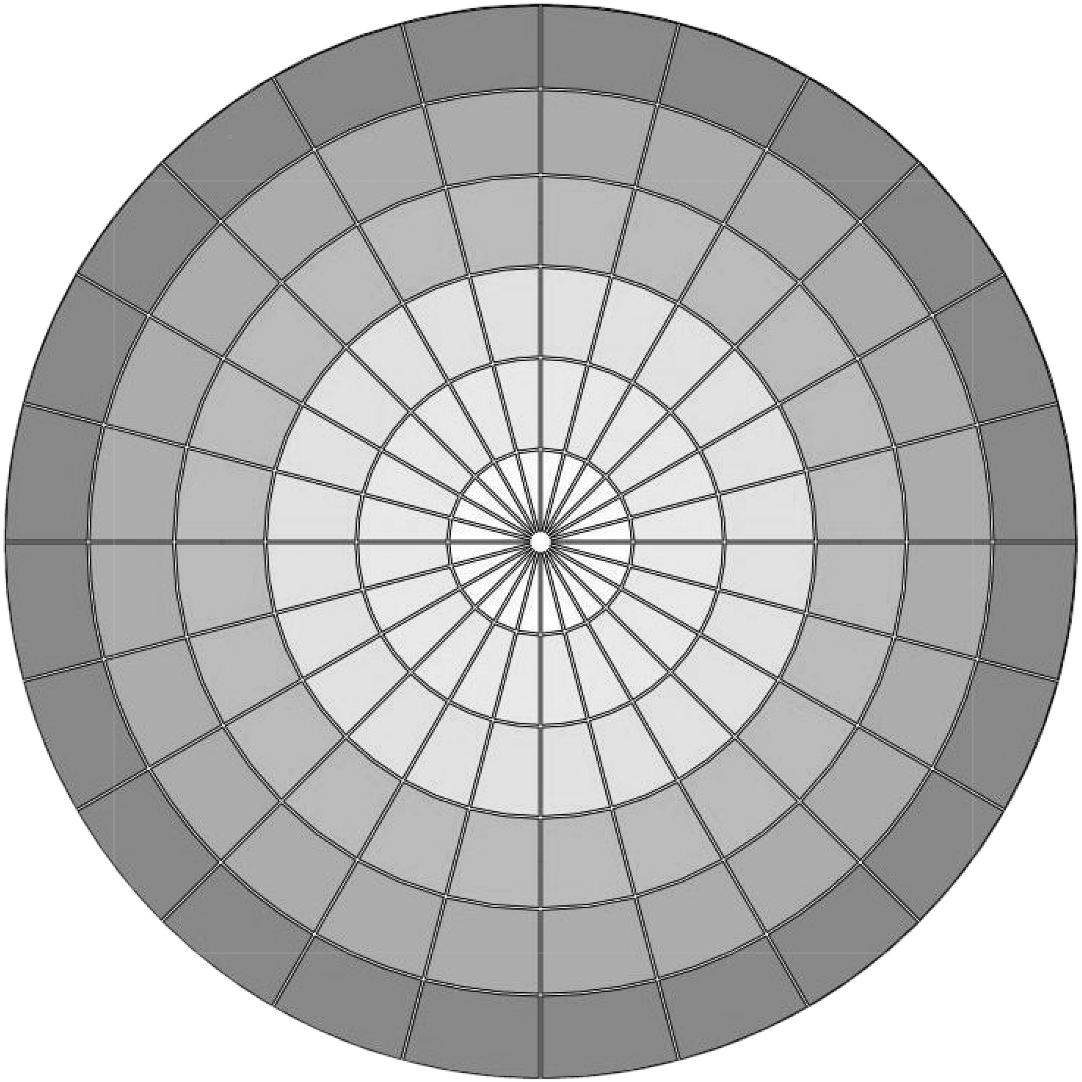
Dissipation rates (Ra = $2 \cdot 10^9$)

$$\frac{\widetilde{\epsilon}_u^H - \widetilde{\epsilon}_u^L}{\widetilde{\epsilon}_u^H}$$

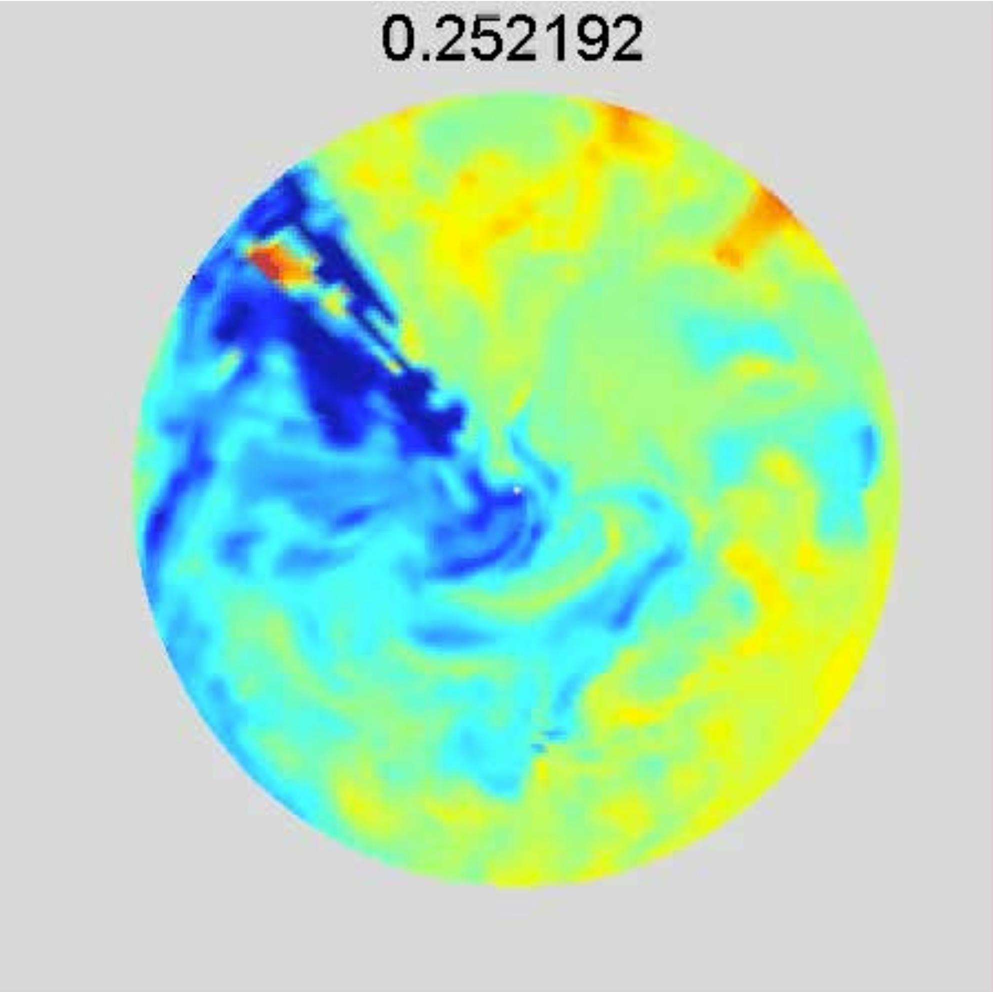
$$\frac{\widetilde{\epsilon}_\theta^H - \widetilde{\epsilon}_\theta^L}{\widetilde{\epsilon}_\theta^H}$$



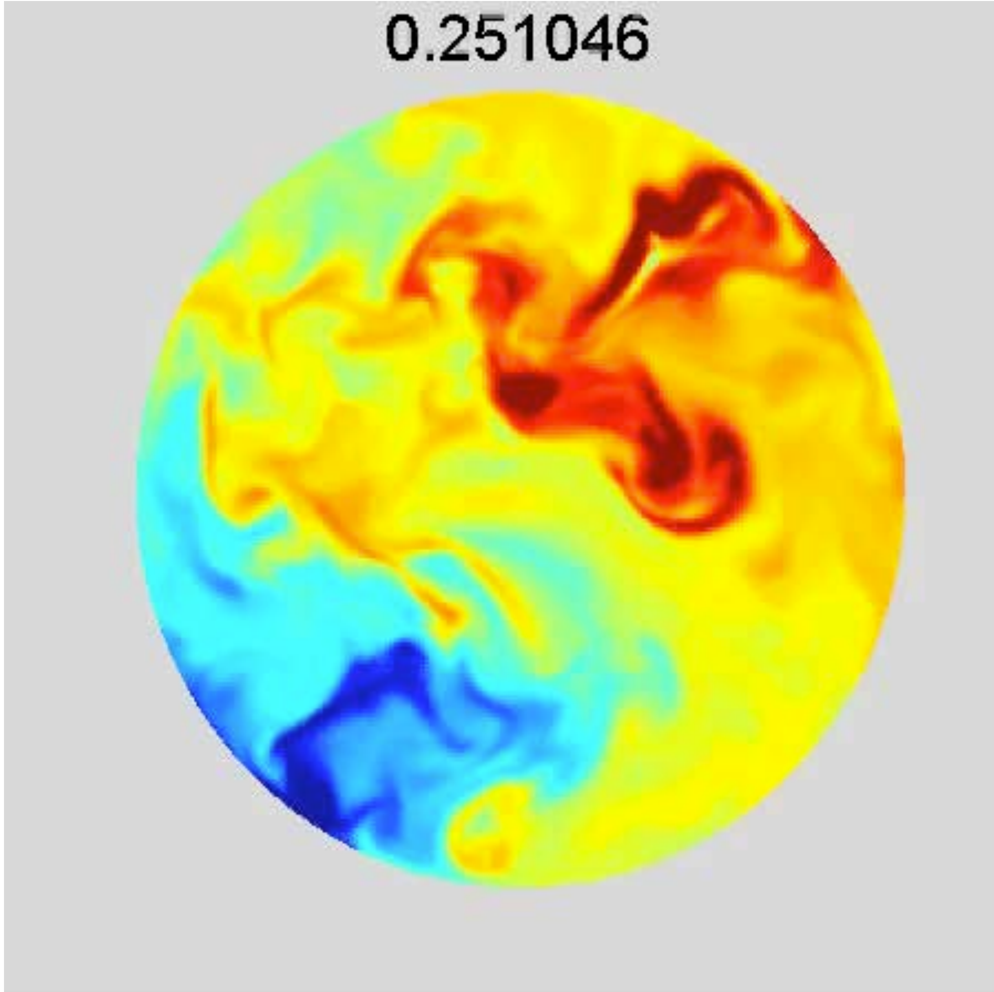
Sketch of grid



Movies at Ra = $2 \cdot 10^9$ (midheight)

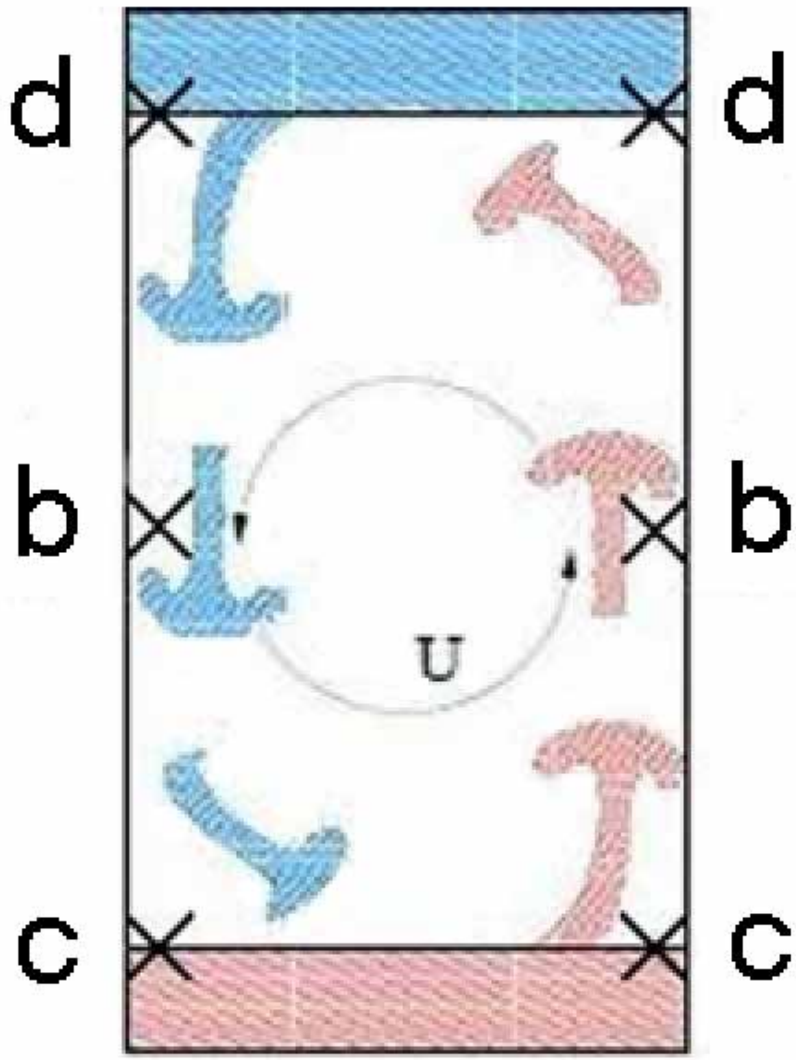


Low resolution

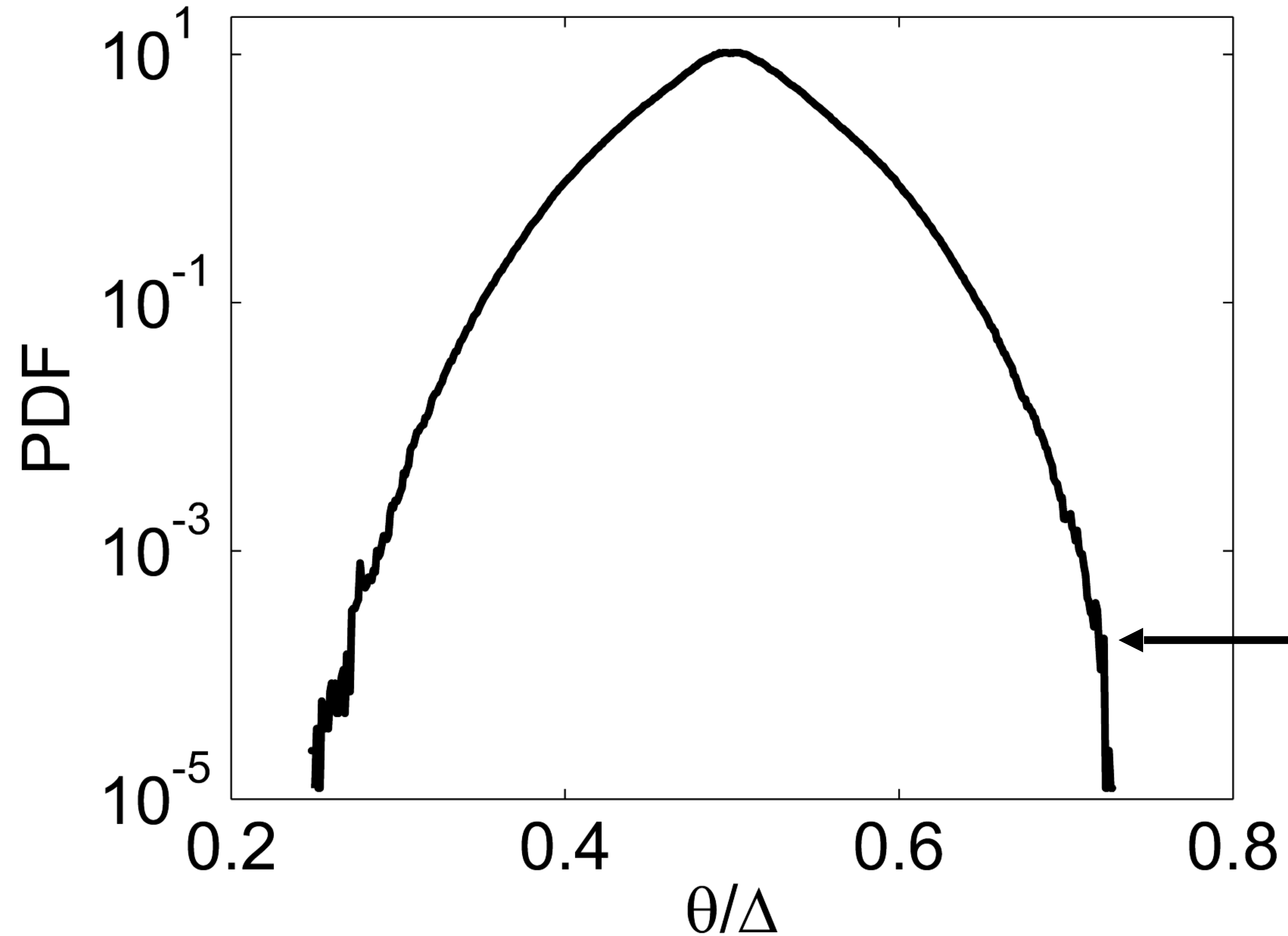


High resolution

PDF locations



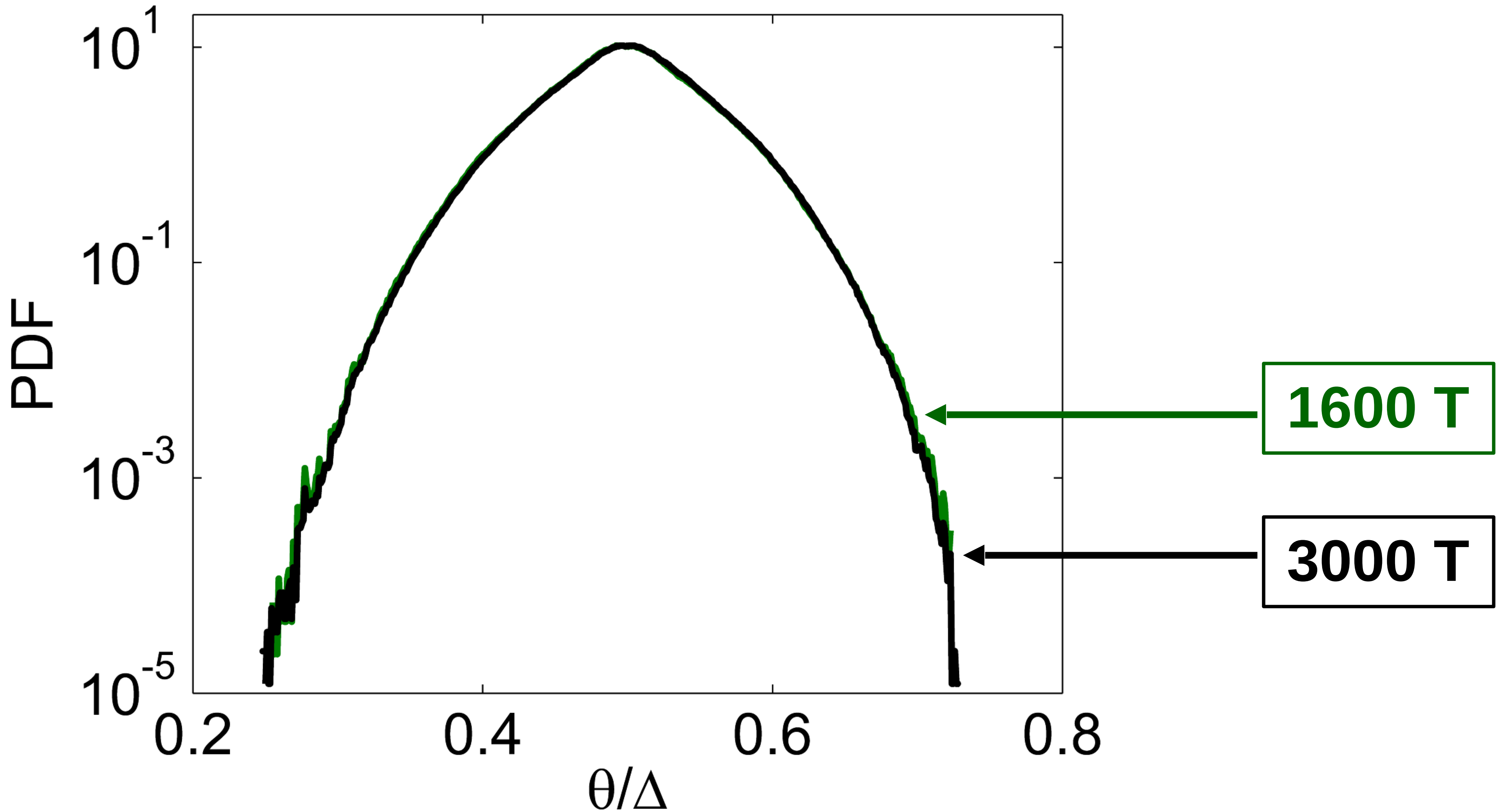
Mid height ($Ra = 2 \cdot 10^8$)



3000 T

M.S. Emran, J. Schumacher, Fine-scale statistics of temperature and its derivatives in convective turbulence. *J. Fluid Mech.* 611, 13-34 (2008)

Mid height ($Ra = 2 \cdot 10^8$)



M.S. Emran, J. Schumacher, Fine-scale statistics of temperature and its derivatives in convective turbulence. *J. Fluid Mech.* 611, 13-34 (2008)

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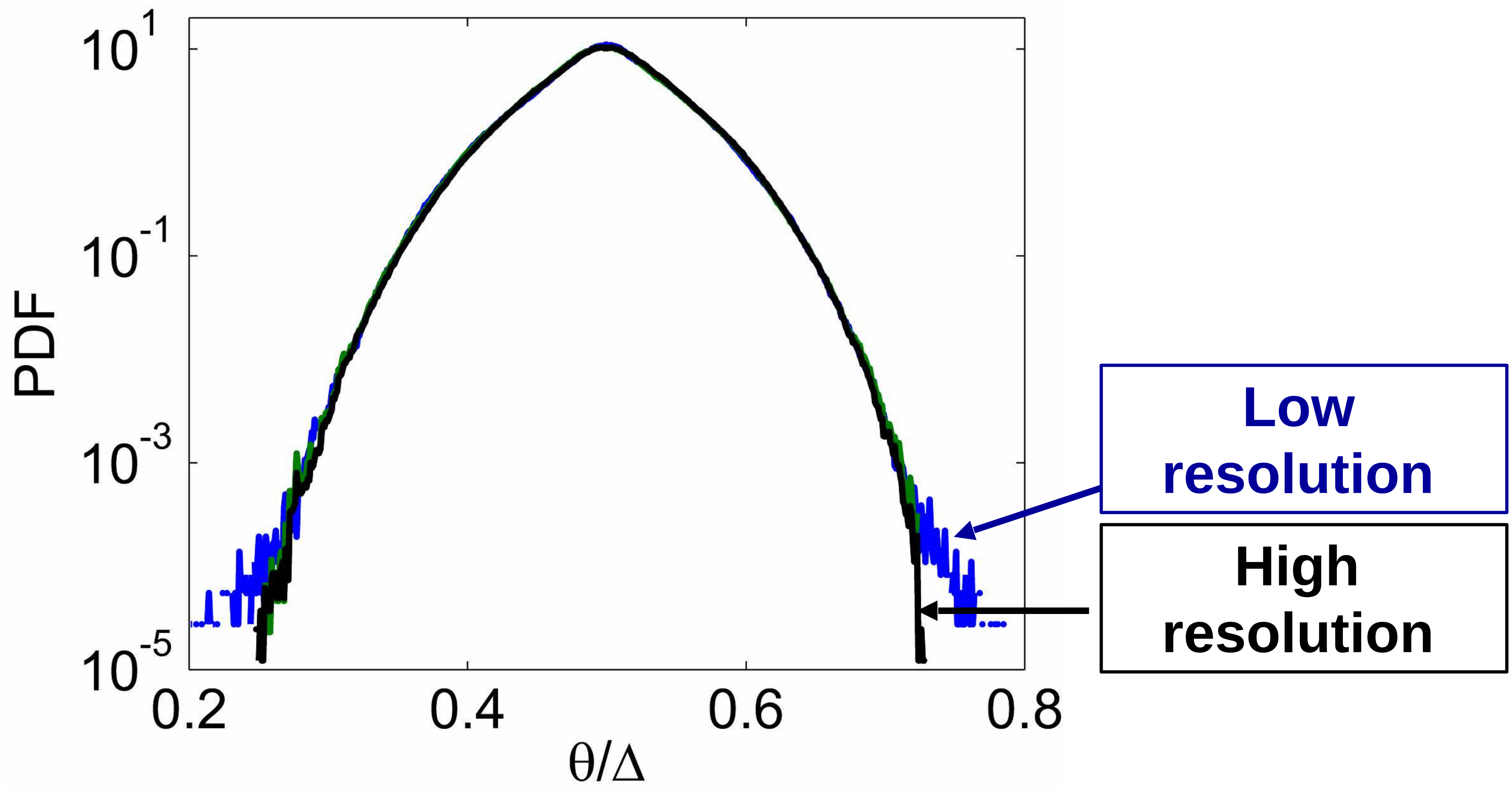


M.S. Emran, J. Schumacher, Fine-scale statistics of temperature and its derivatives in convective turbulence. *J. Fluid Mech.* 611, 13-34 (2008)

New resolution criteria

Following Grötzbach (1983) we get

$$h \leqslant \pi\eta = \pi L \left(\frac{Pr^2}{RaNu}\right)^{1/4} \quad \text{for } Pr \leqslant 1,$$

$$h \leqslant \pi\eta_T = \pi L \left(\frac{1}{RaPrNu}\right)^{1/4} \quad \text{for } Pr \geqslant 1,$$

But not on the average grid size $d=(D_1D_2D_3)^{1/3}$, but by each grid dimension D_1, D_2, D_3 simultaneously!

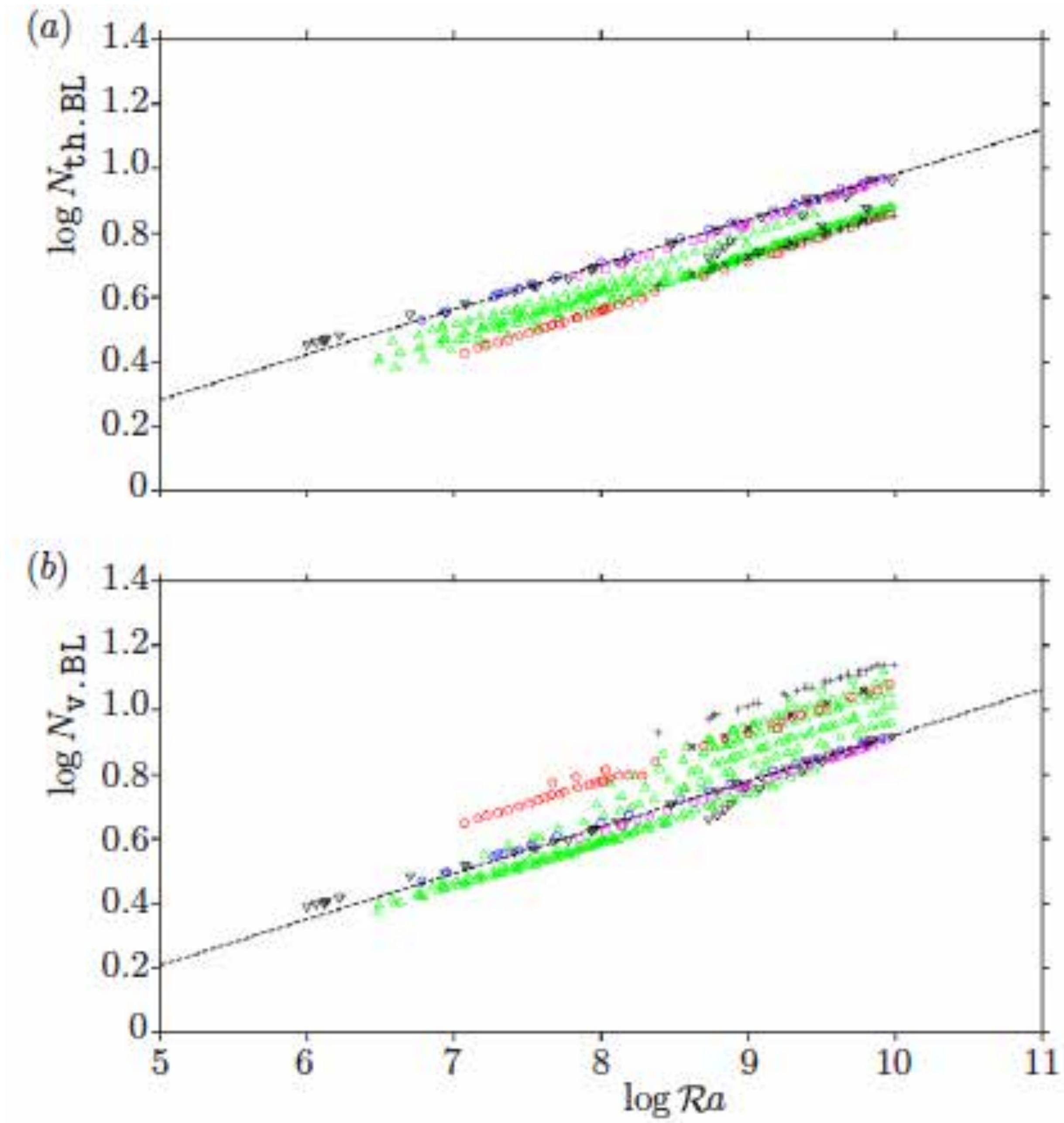
Number of grid points in the boundary layer should be

$$N_{\mathrm{th.BL}} \approx 0.35\mathcal{R}a^{0.15}, \quad 10^6 \leqslant \mathcal{R}a \leqslant 10^{10},$$

$$N_{\mathrm{v.BL}} \approx 0.31\mathcal{R}a^{0.15}, \quad 10^6 \leqslant \mathcal{R}a \leqslant 10^{10}.$$

Stevens, Verzicco, Lohse, **J. Fluid Mech.** 643, 495-507 (2010).
Shishkina, **Stevens**, Grossmann, Lohse, **New J. Phys.** 12, 075022 (2010).

Boundary layer resolution



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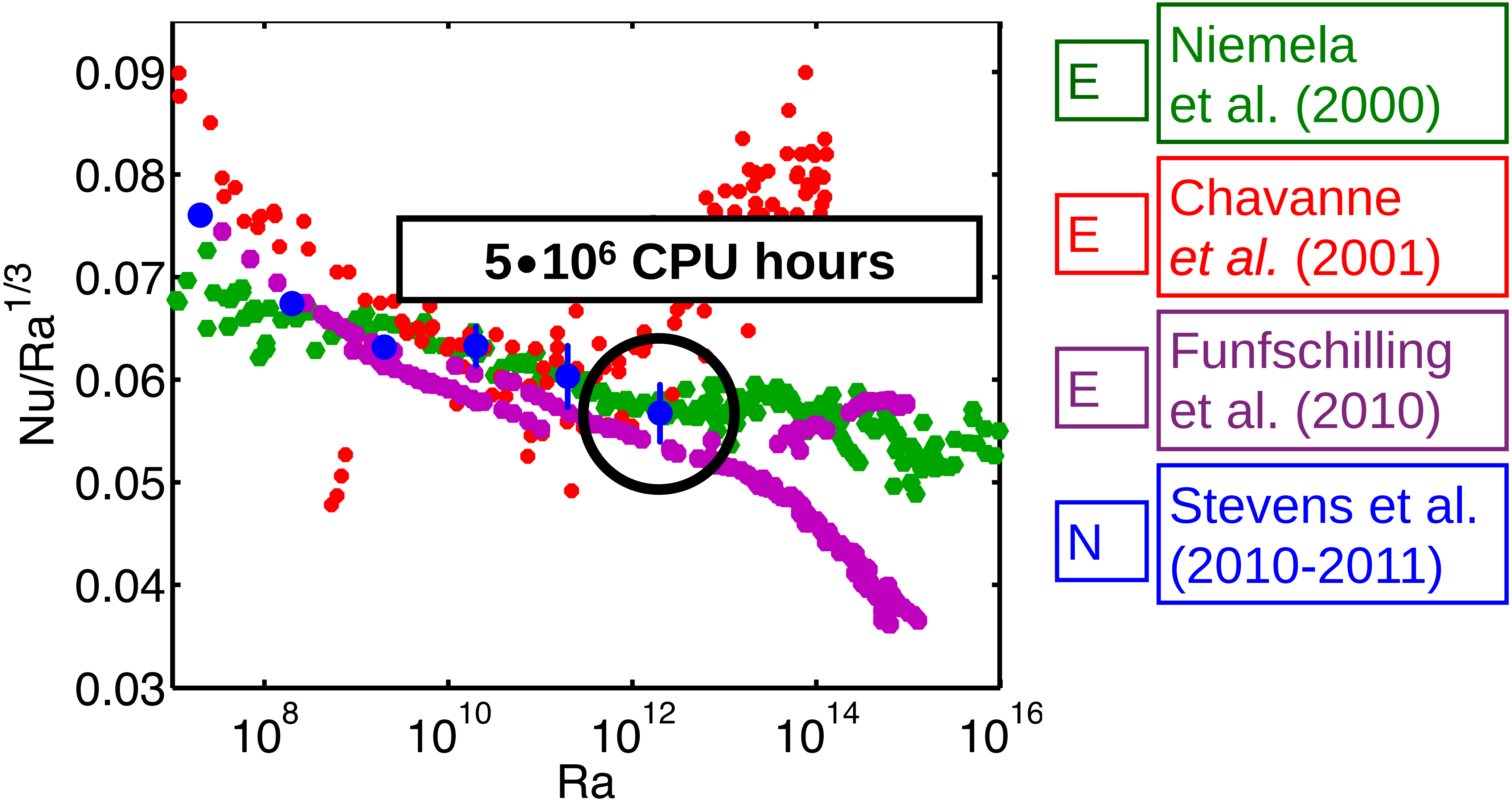
$$N_{\mathrm{v.BL}} \approx 0.31\mathcal{R}a^{0.15}, \quad 10^6 \leqslant \mathcal{R}a \leqslant 10^{10}.$$

For higher Ra this can easily be 20 or more, instead of the fixed value of 3-5 recommended by Grotzbach

Stevens, Verzicco, Lohse, **J. Fluid Mech.** 643, 495-507 (2010).
Shishkina, **Stevens**, Grossmann, Lohse, **New J. Phys.** 12, 075022 (2010).

RB convection (Γ=0.5)

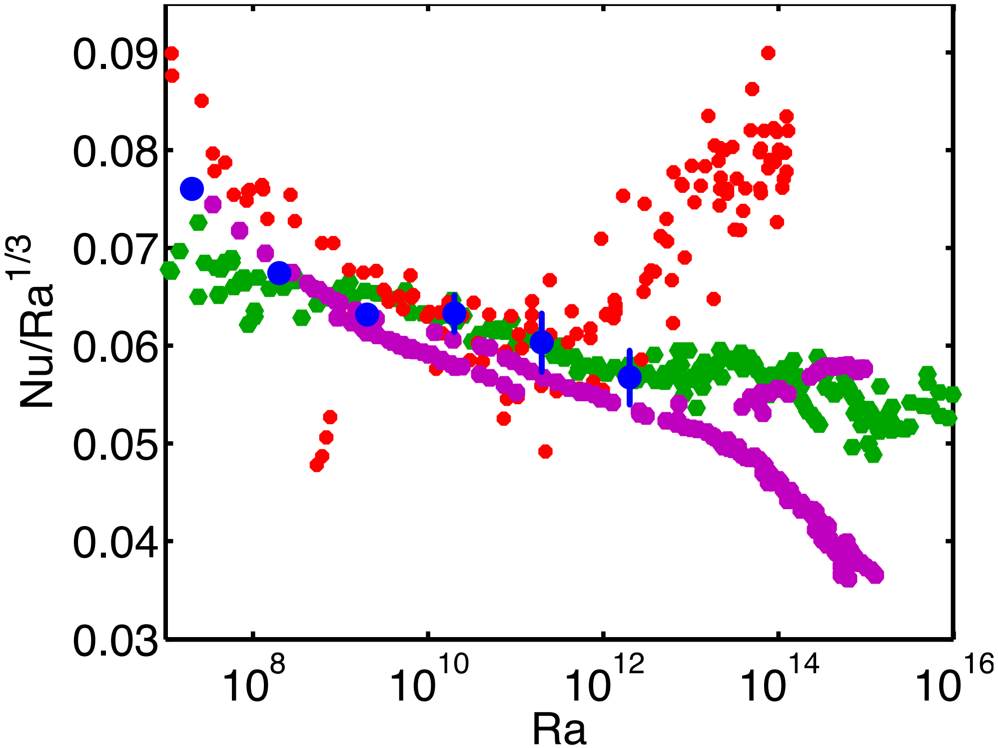


E=Experiments, N=Numerics

Stevens, Verzicco, Lohse, **J. Fluid Mech.** 643, 495-507 (2010).

Why is there a discrepancy between experiments?

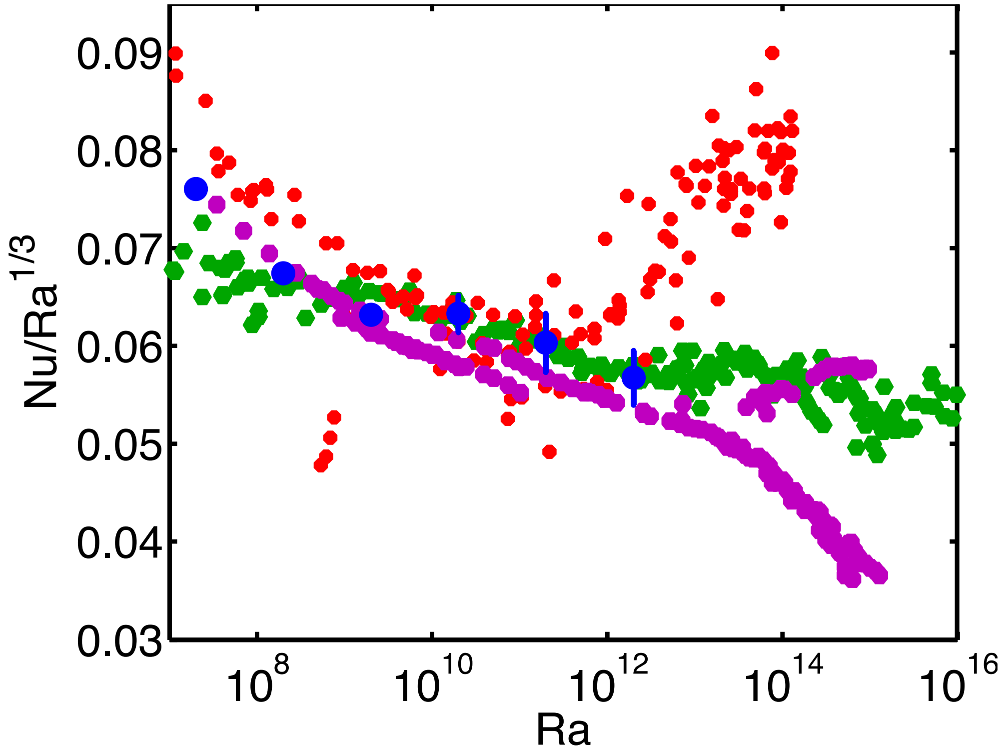
- Prandtl number effect, i.e. different fluid properties

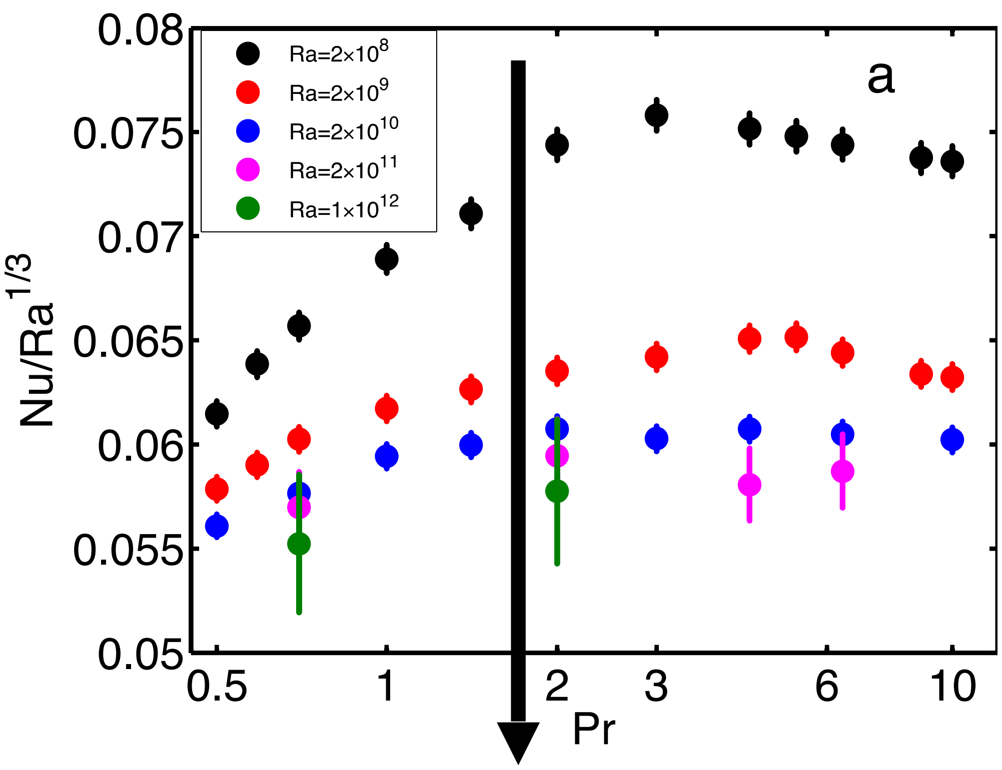


Stevens, Lohse, Verzicco, **J. Fluid Mech.** 688, 31-43 (2011).

Why is there a discrepancy between experiments?

- Prandtl number effect, i.e. different fluid properties
  – Simulations confirm theoretical prediction that Nu becomes independent of Pr for high Ra



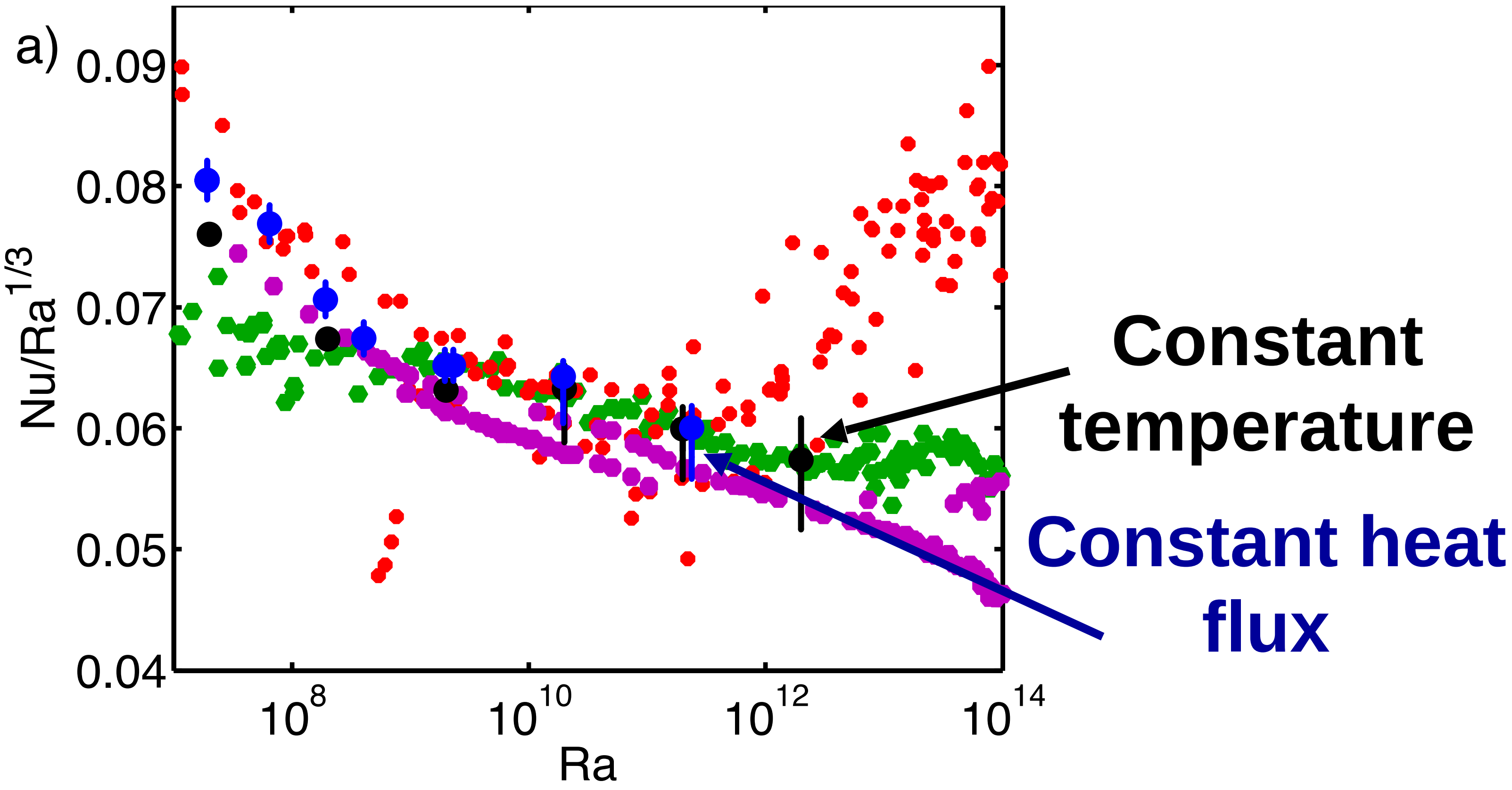


Increasing Ra

Stevens, Lohse, Verzicco, **J. Fluid Mech.** 688, 31-43 (2011).

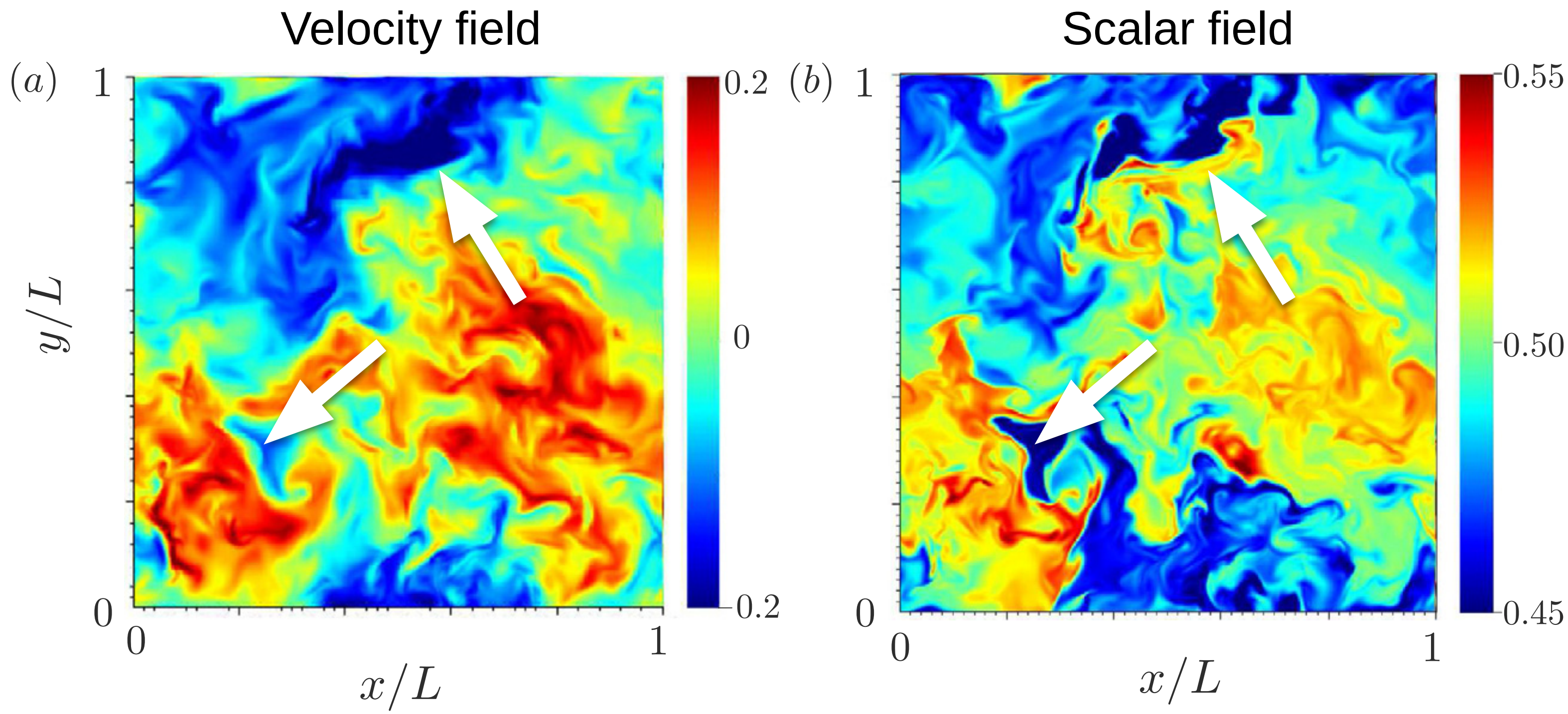
Why is there a discrepancy between experiments?

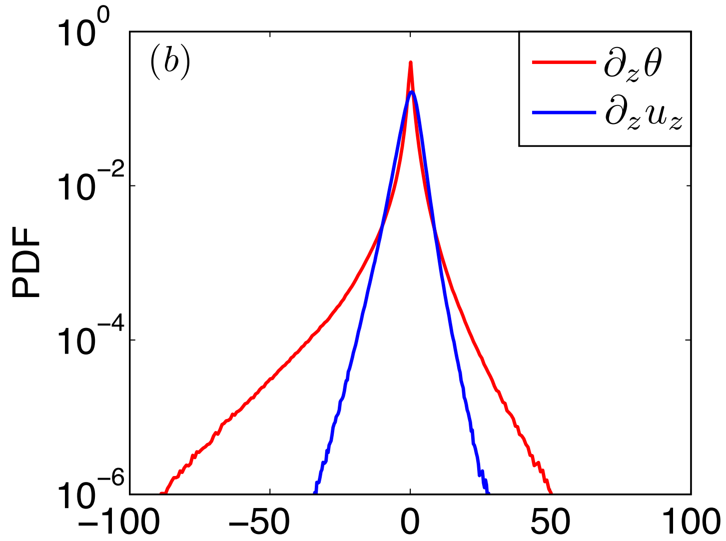
- ~~Prandtl number effect, i.e. different fluid properties~~
- Constant temperature versus constant heat flux boundary condition at the bottom plate



Stevens, Lohse, Verzicco, **J. Fluid Mech.** 688, 31-43 (2011).

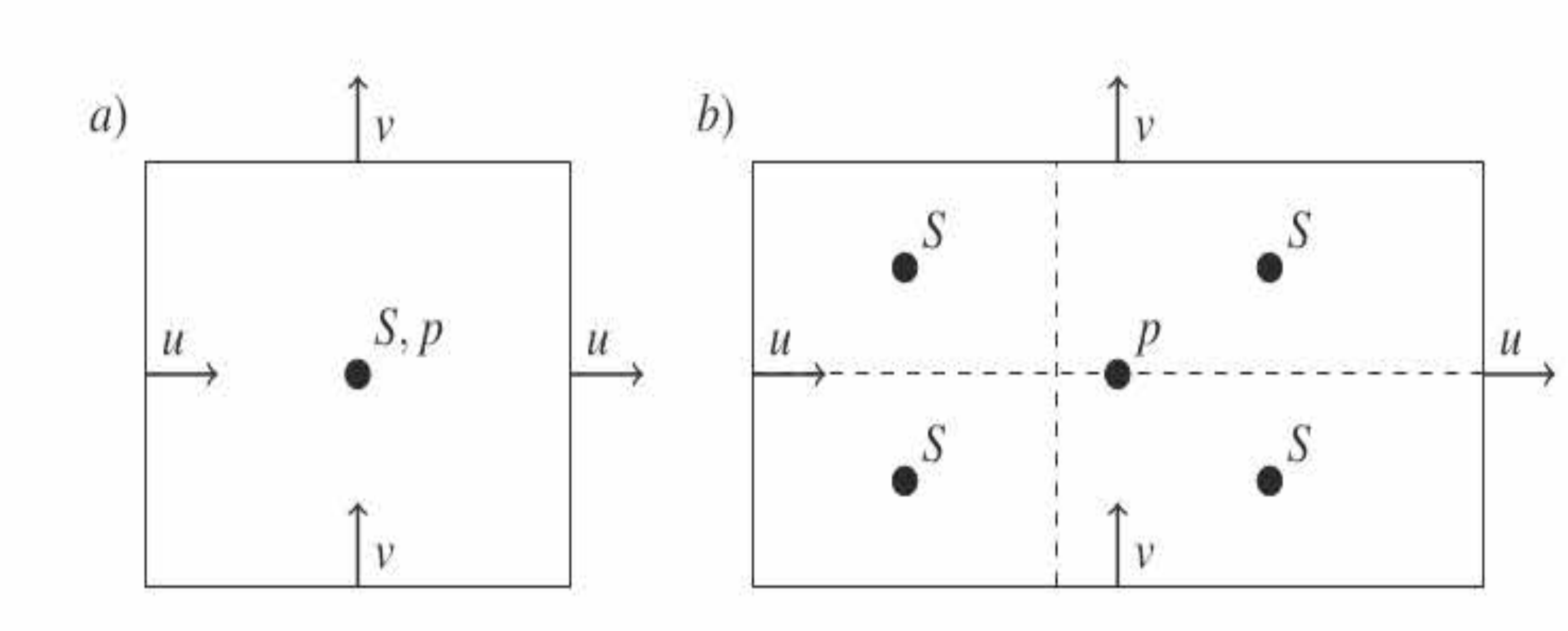
DNS for scalar turbulence requires care: sharp gradients!





Even if the diffusivities are equal (Pr = 1), temperature shows much sharper fronts

Different grids for scalar and momentum

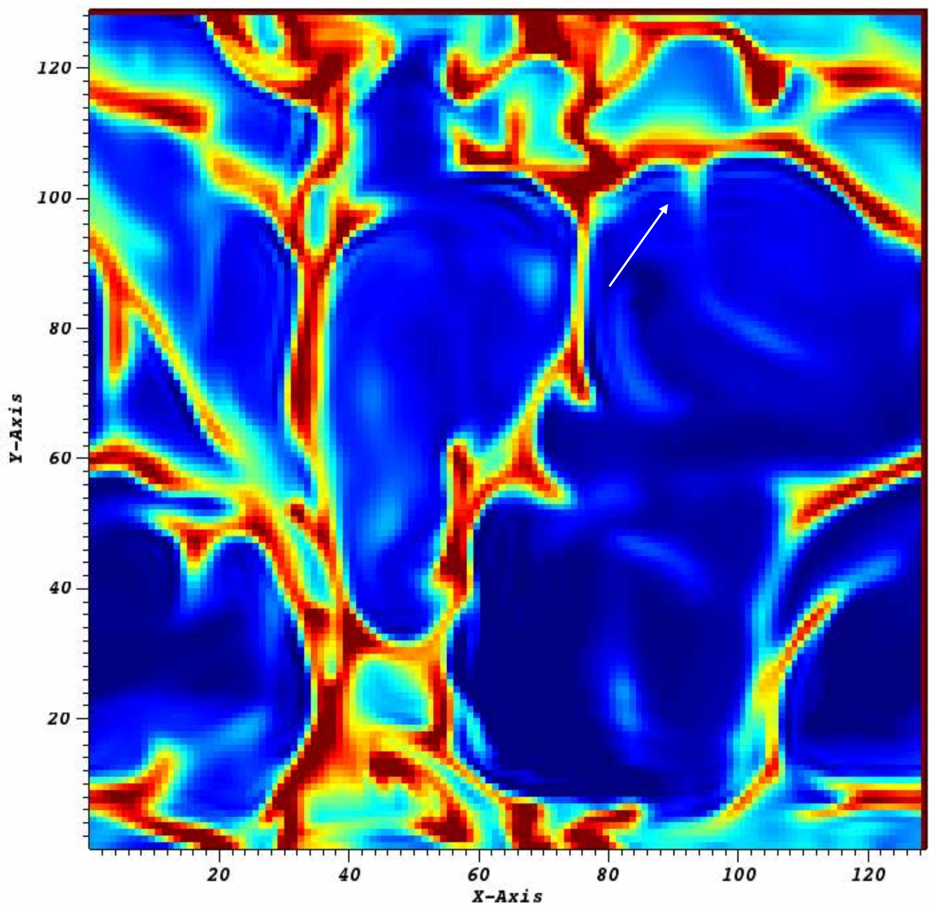


- Interpolation techniques for velocity in space

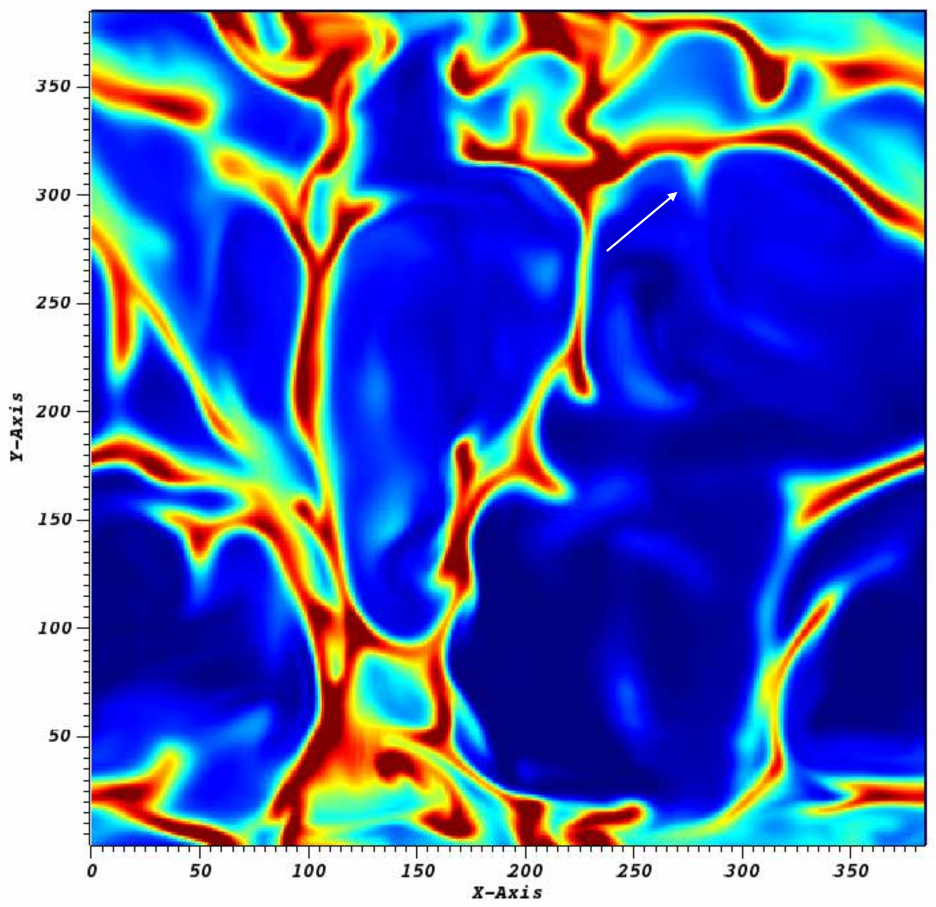
- Needs also sub-time stepping integration due to stability constraints

Refined scalar field resolution

Normal grid



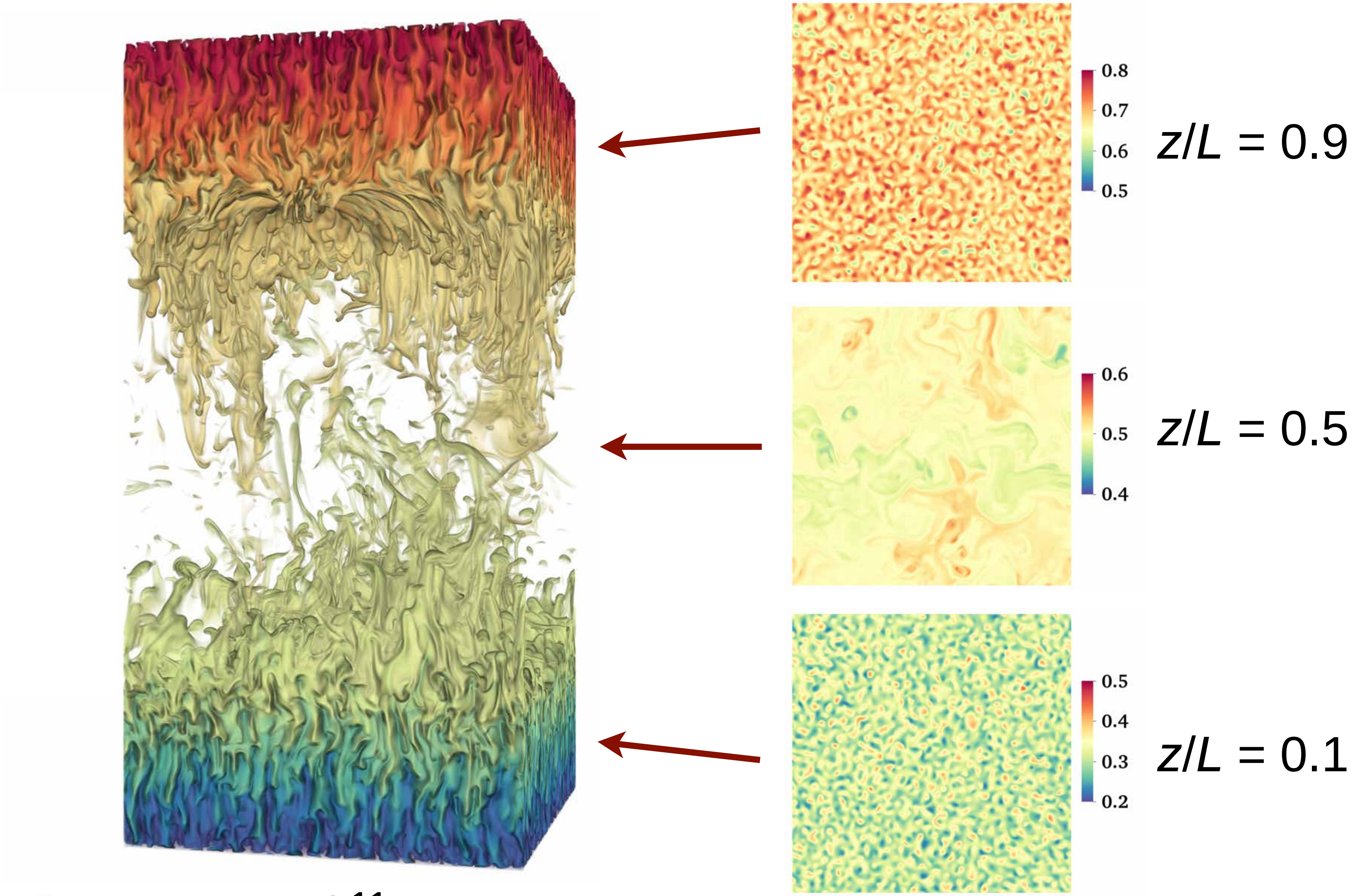
Refined grid



Savings from 2x CPU time for Pr~1 to 4x CPU time for high Pr

Reduces memory use by 3x for medium-size problems

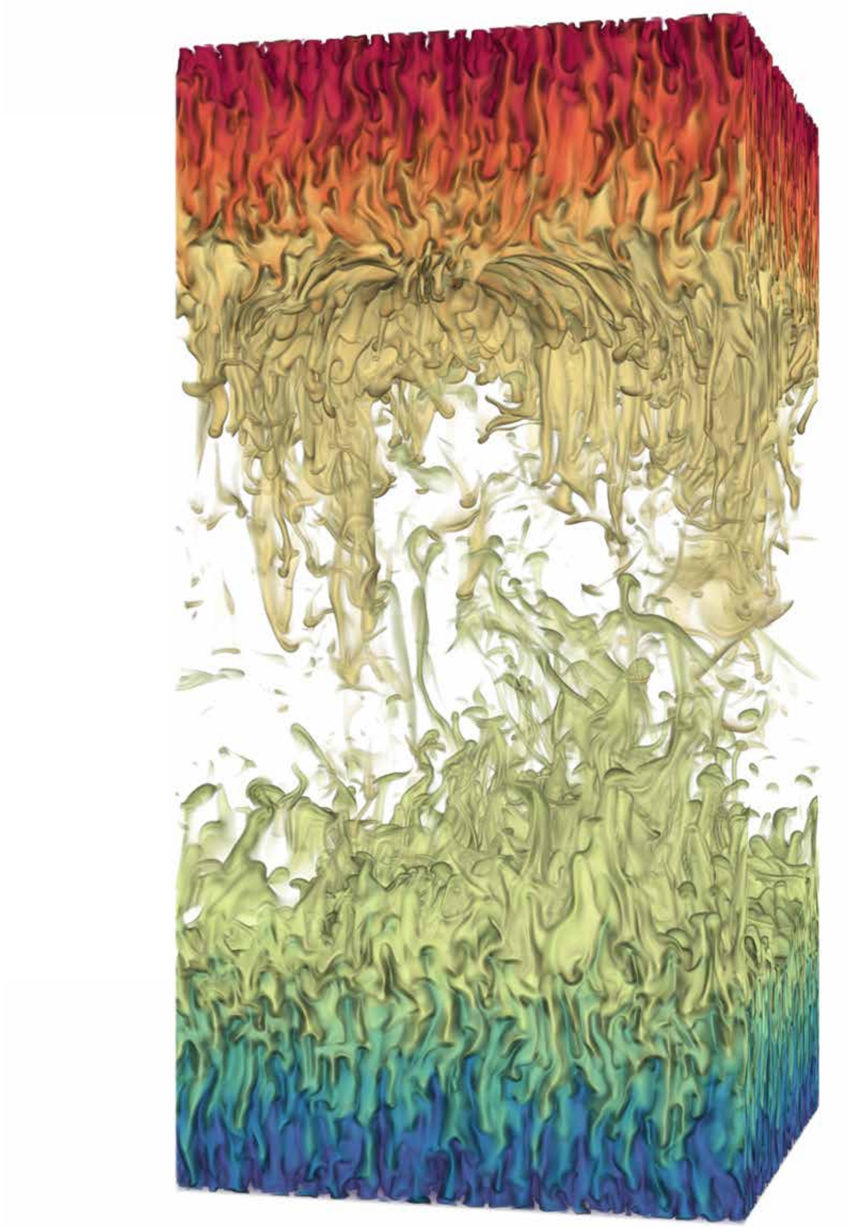
Double diffusive convection



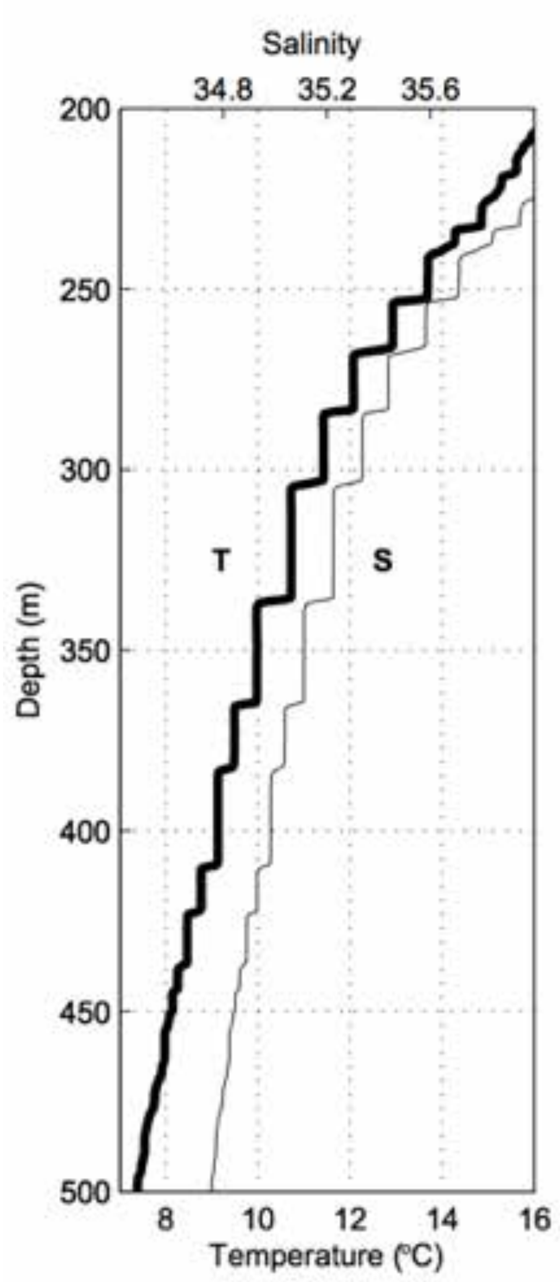
$Ra_S = 1 \times 10^{11}$

by Yantao Yang

Double diffusive convection

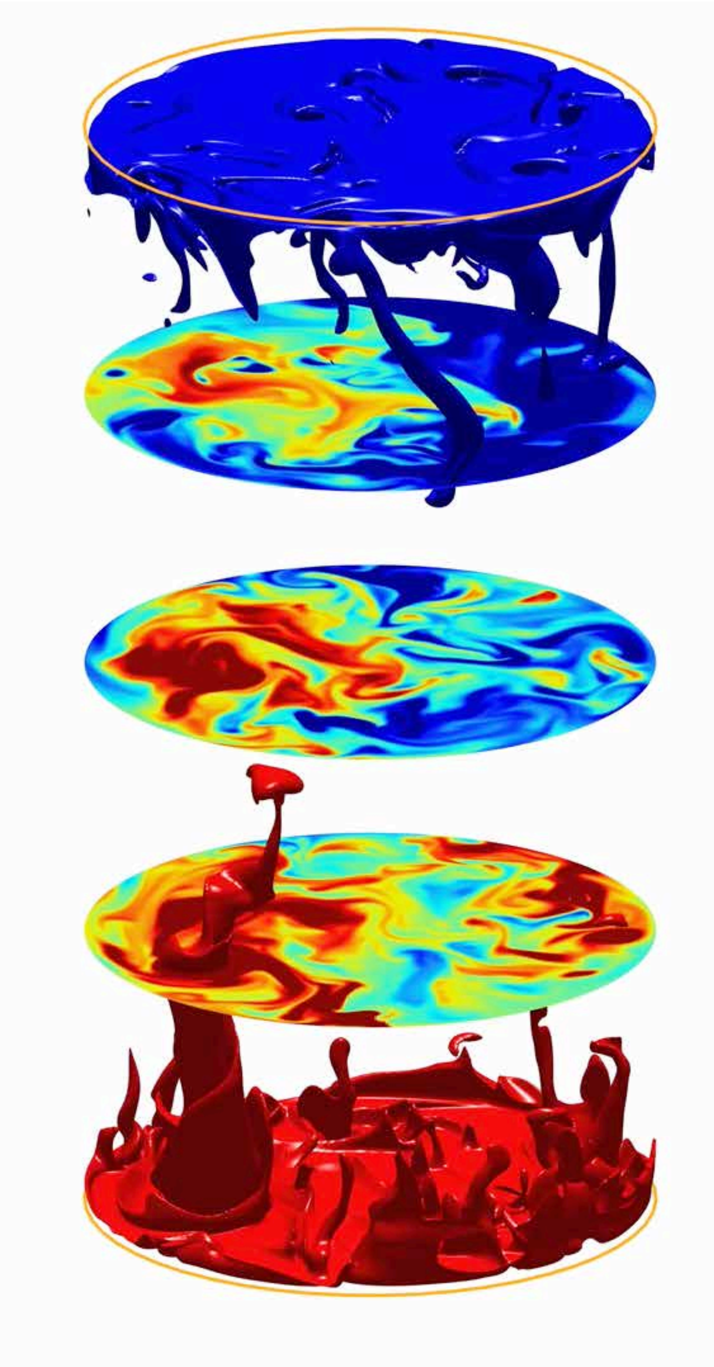


$Ra_S = 1 \times 10^{11}$



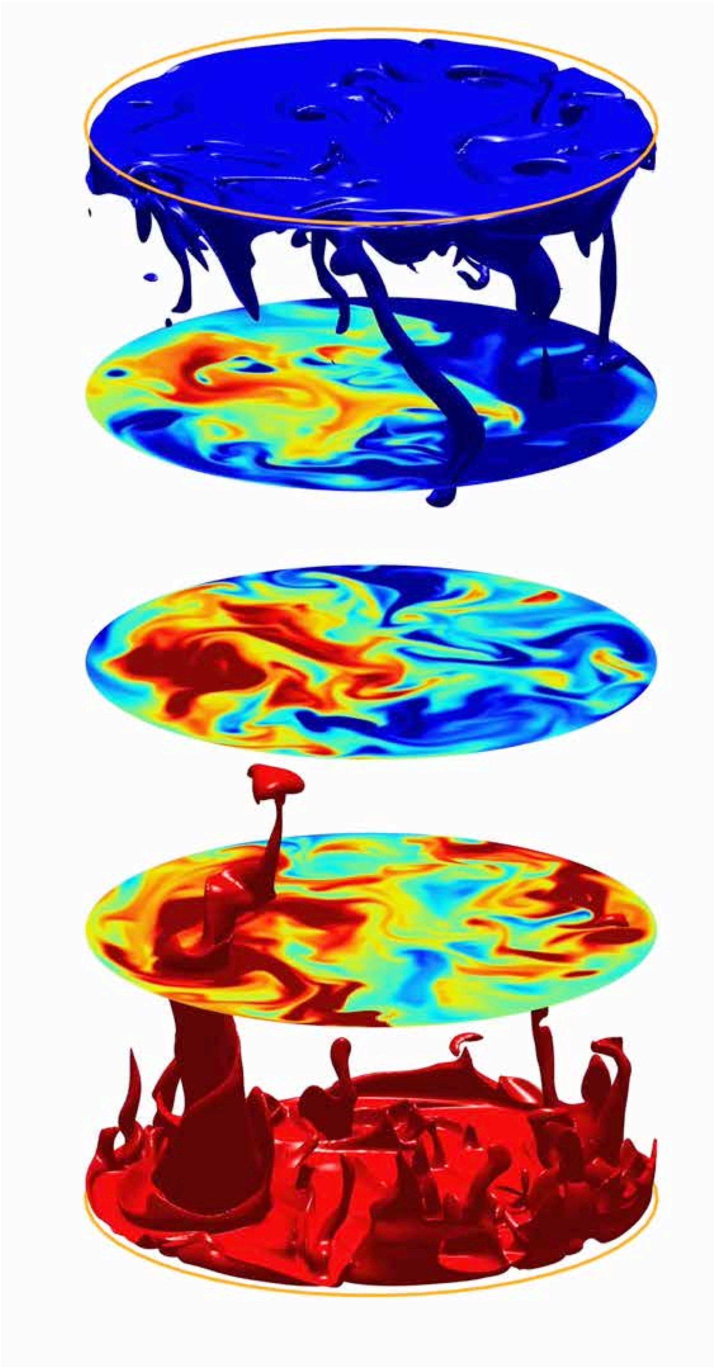
Schmitt et al., *Science* 308, 685 (2005)

Rotating convection



Bubbly convection

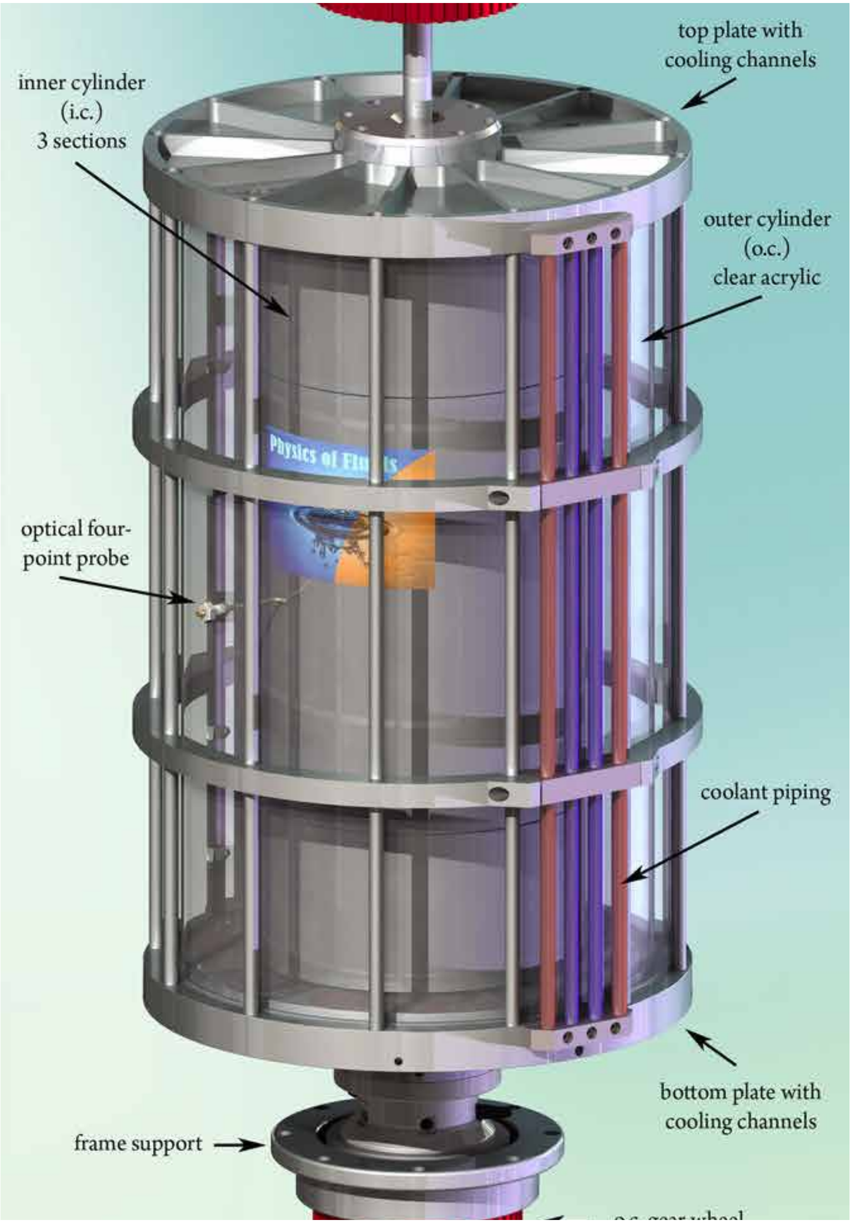
Rotating convection



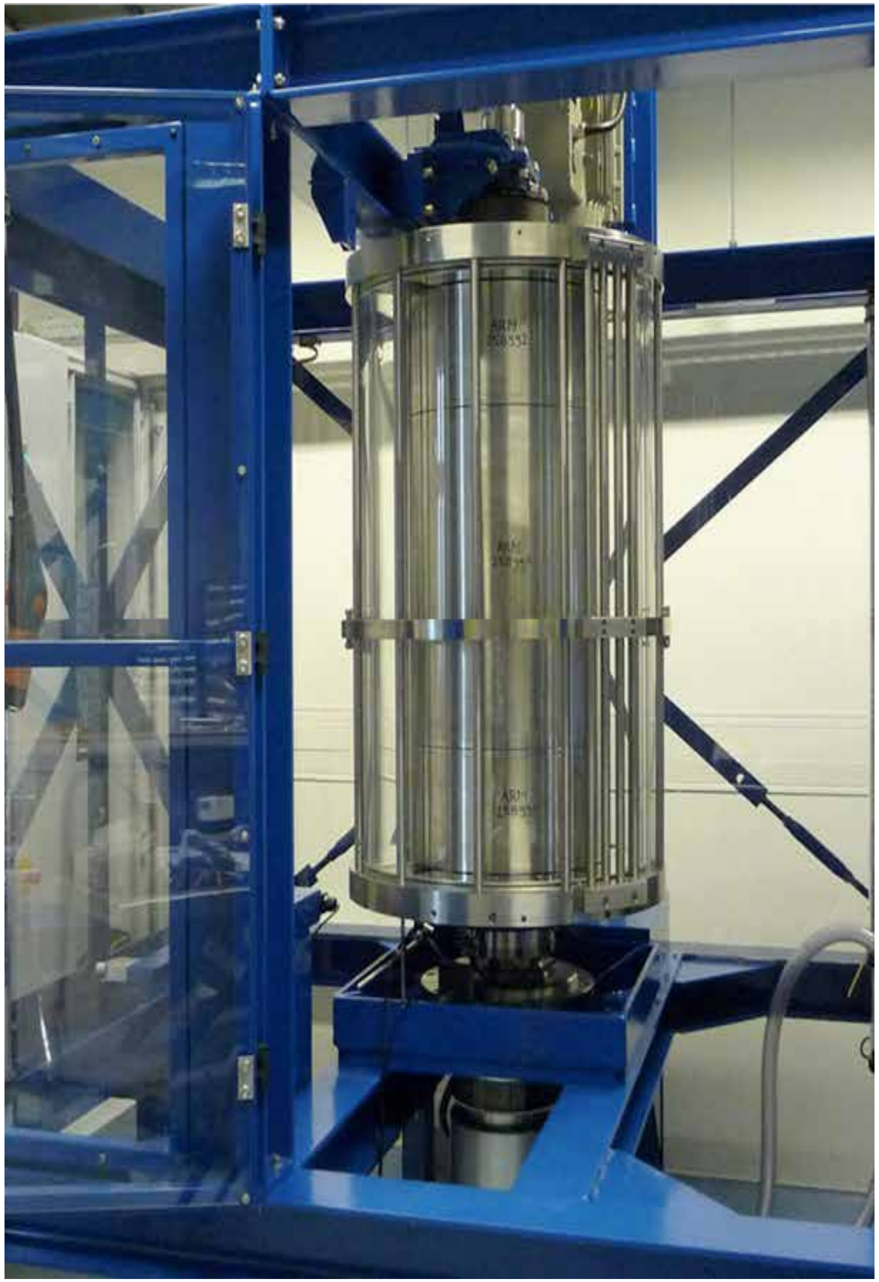
Bubbly convection

by Rajaram Lakkaraju

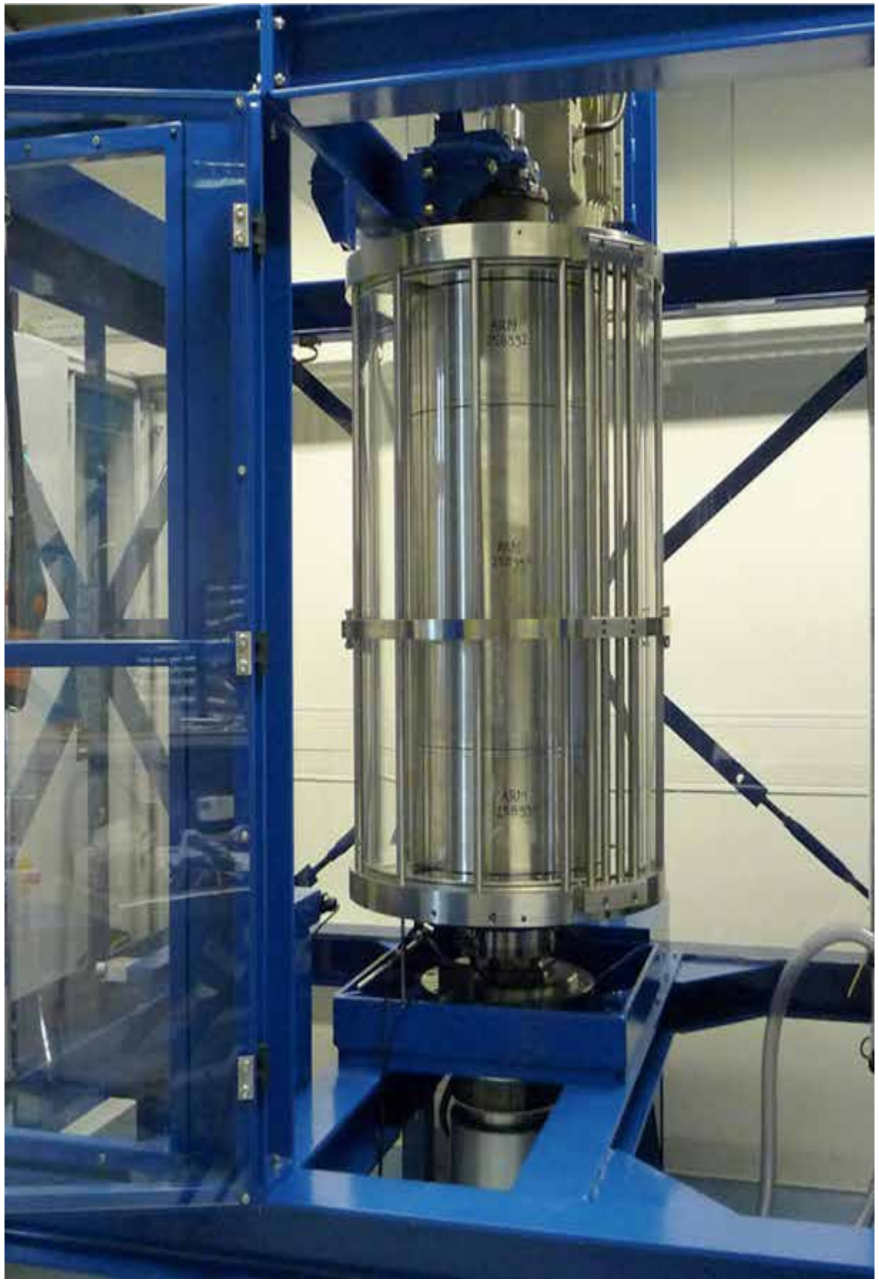
Taylor-Couette flow

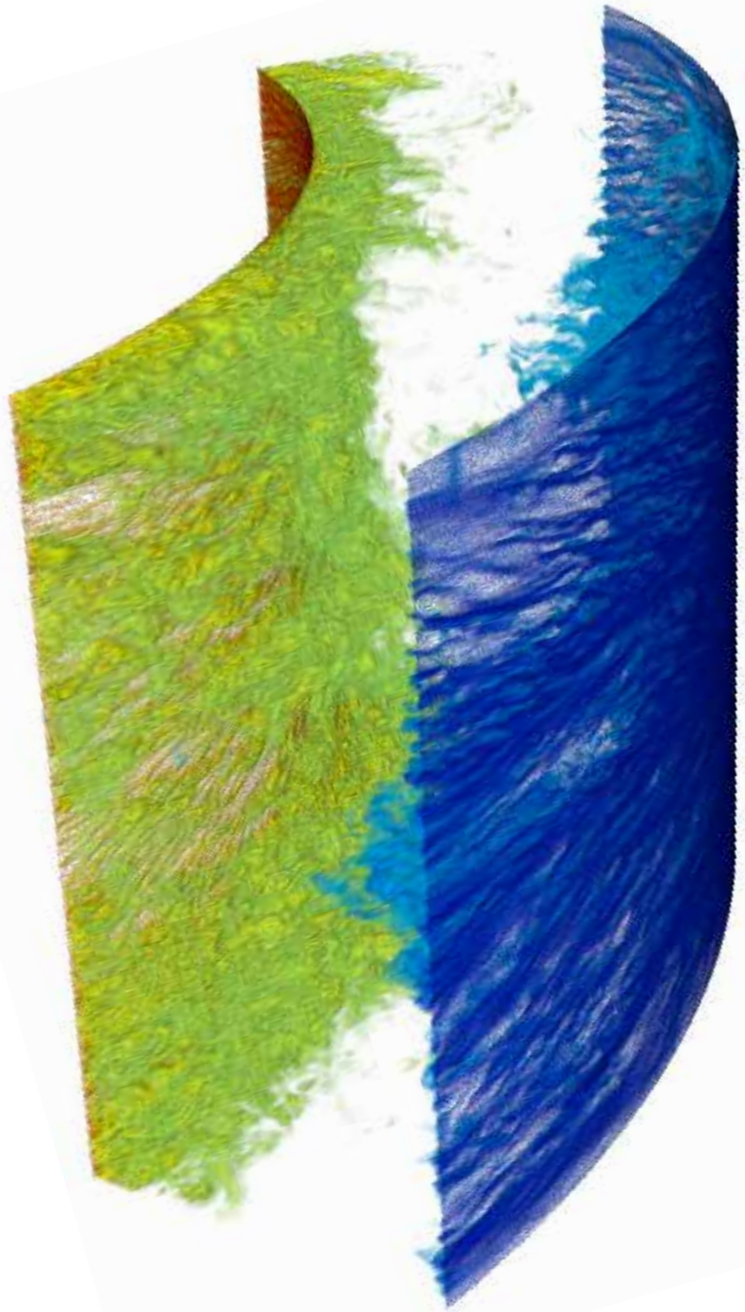


Taylor-Couette flow



Taylor-Couette flow

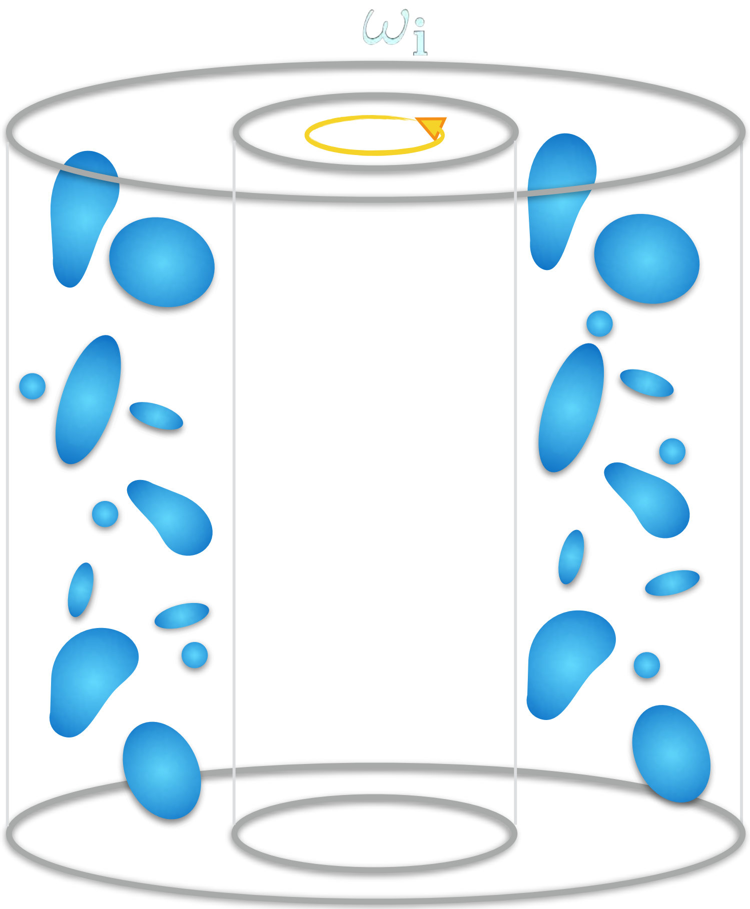


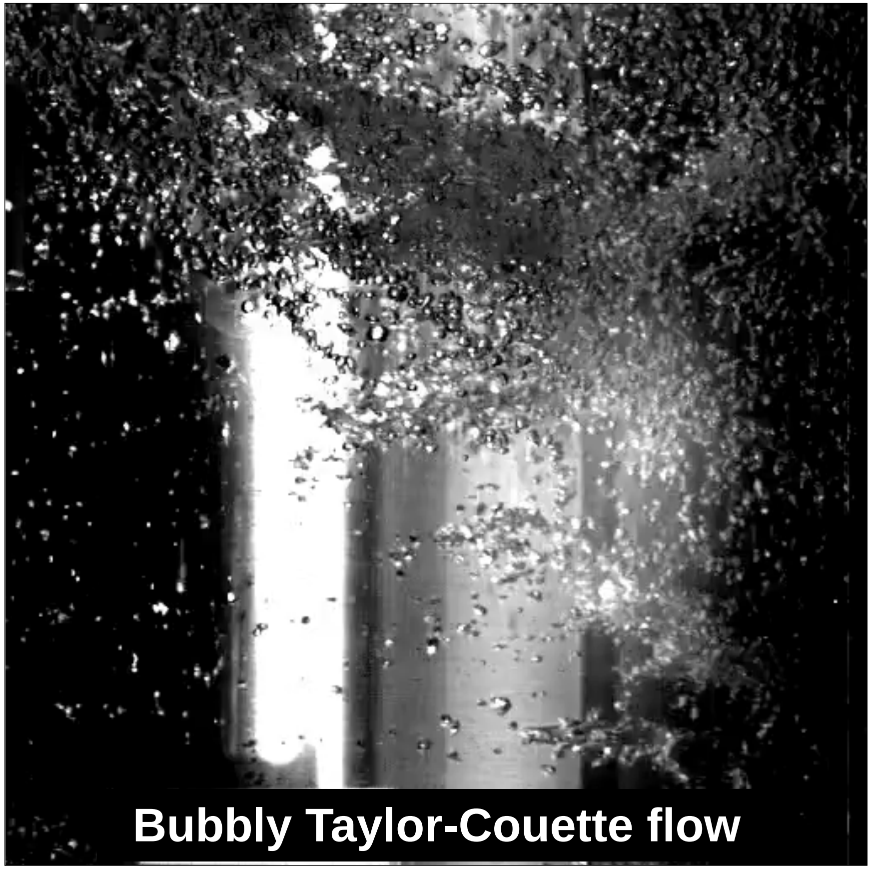


by Rodolfo Ostilla and Xiaojue Zhu

Understanding multi-component/phase flows

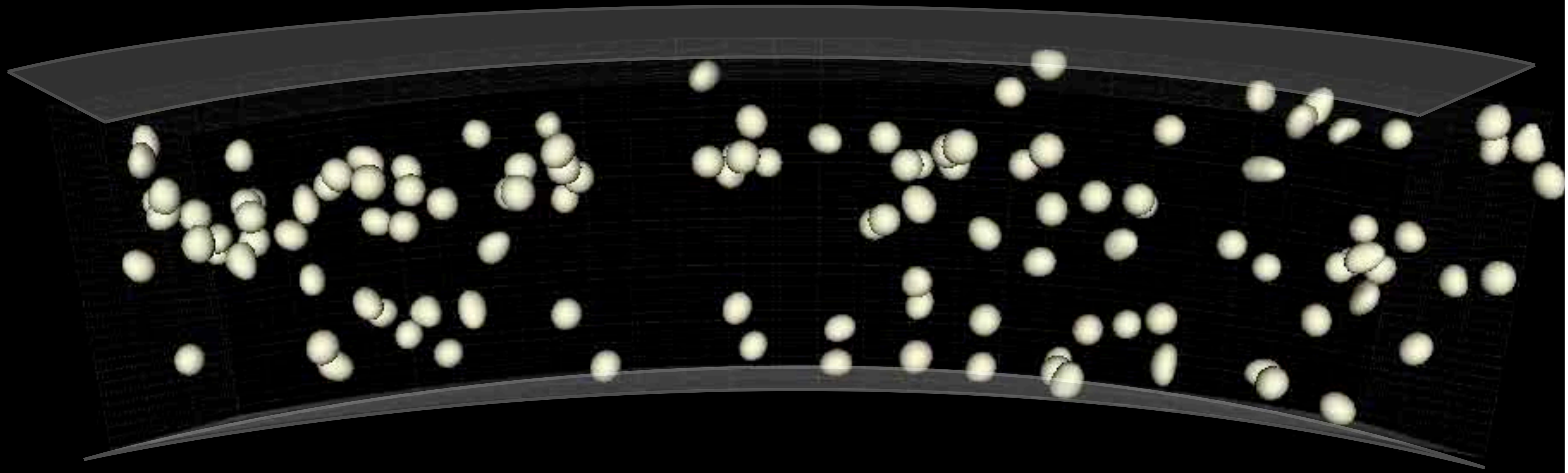
Two-phase Taylor-Couette flow




Bubbly Taylor-Couette flow

van Gils et al., JFM (2013)

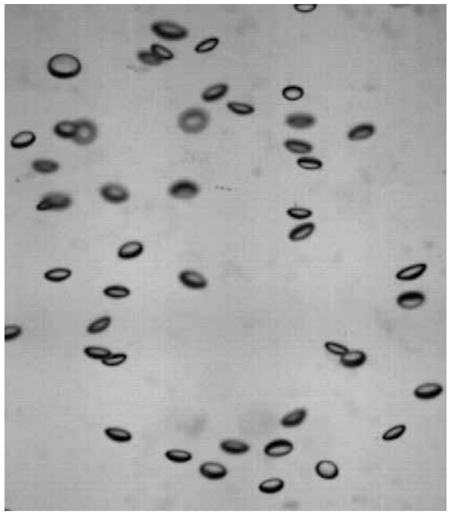
Two-phase Taylor-Couette simulation



by Vamsi Spandan

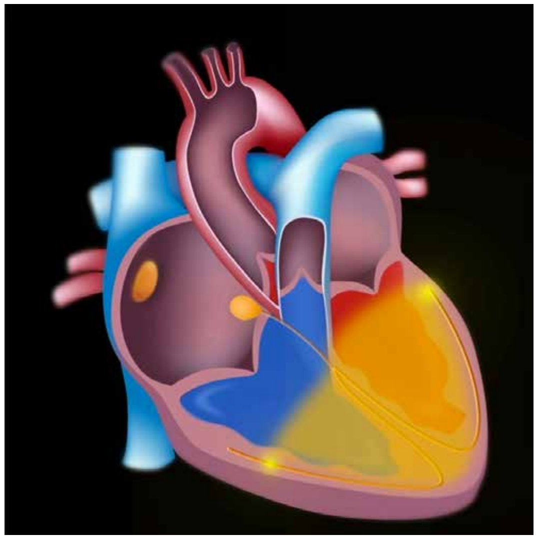
Deformability of bodies immersed in a flow

- Bubble/drop laden turbulent flows - atmosphere, oceans
- Flapping, bending bodies - flags, boat sails
- Biological flows - blood flow, heart-valves

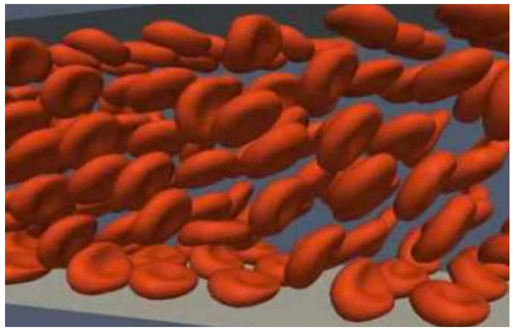


Bubbles

Sails



Heart



Red blood cells