Quantification of large-scales turbulence in thermal convection

M. K. Verma , Ambrish Pandey
Abhishek Kumar
A. G. Chatterjee,
Indian Institute of Technology Kanpur, India

mkv@iitk.ac.in
http://turbulencehub.org

Pandey & Verma, Phys. Fluids 2016
Pandey et al., Phys. Rev. E 2016
Verma et al., New J. Phys. 2017

Equations

Velocity field

Pressure

Buoyancy

Ext. Force

$$\partial_t \mathbf{u} + (\mathbf{u}.\nabla)\mathbf{u} = -\nabla\sigma + \alpha g \theta \hat{z} + \nu \nabla^2 \mathbf{u}$$

$$\partial_t \theta + (\mathbf{u}.\nabla)\theta = -\frac{dT}{dz} u_z + \kappa \nabla^2 \theta$$

Kinematic viscosity

Thermal fluctuations

Temperature stratification

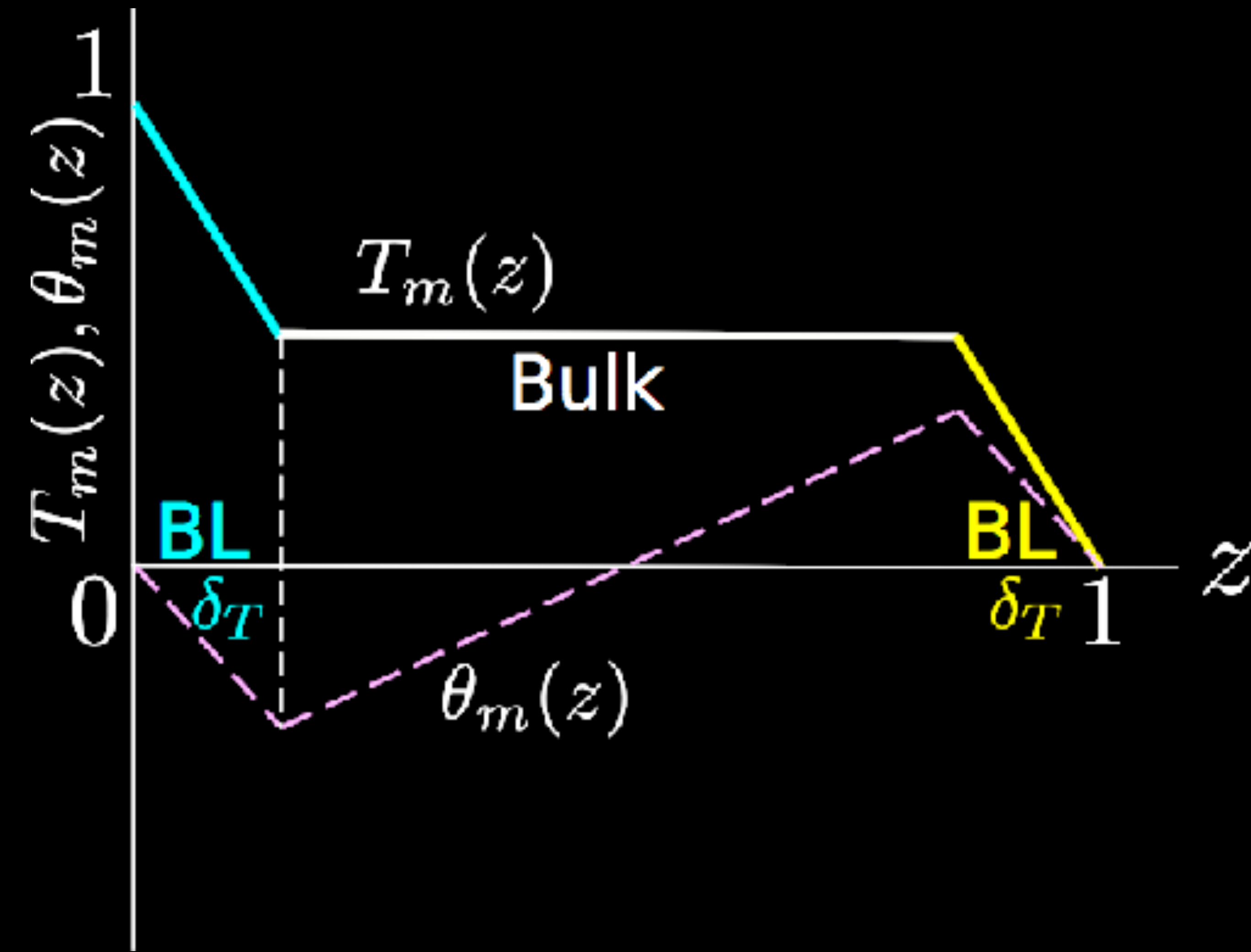
Thermal diffusivity

Bossiness approximation

$T = T_C + \theta$

$$\nabla.\mathbf{u} = 0$$

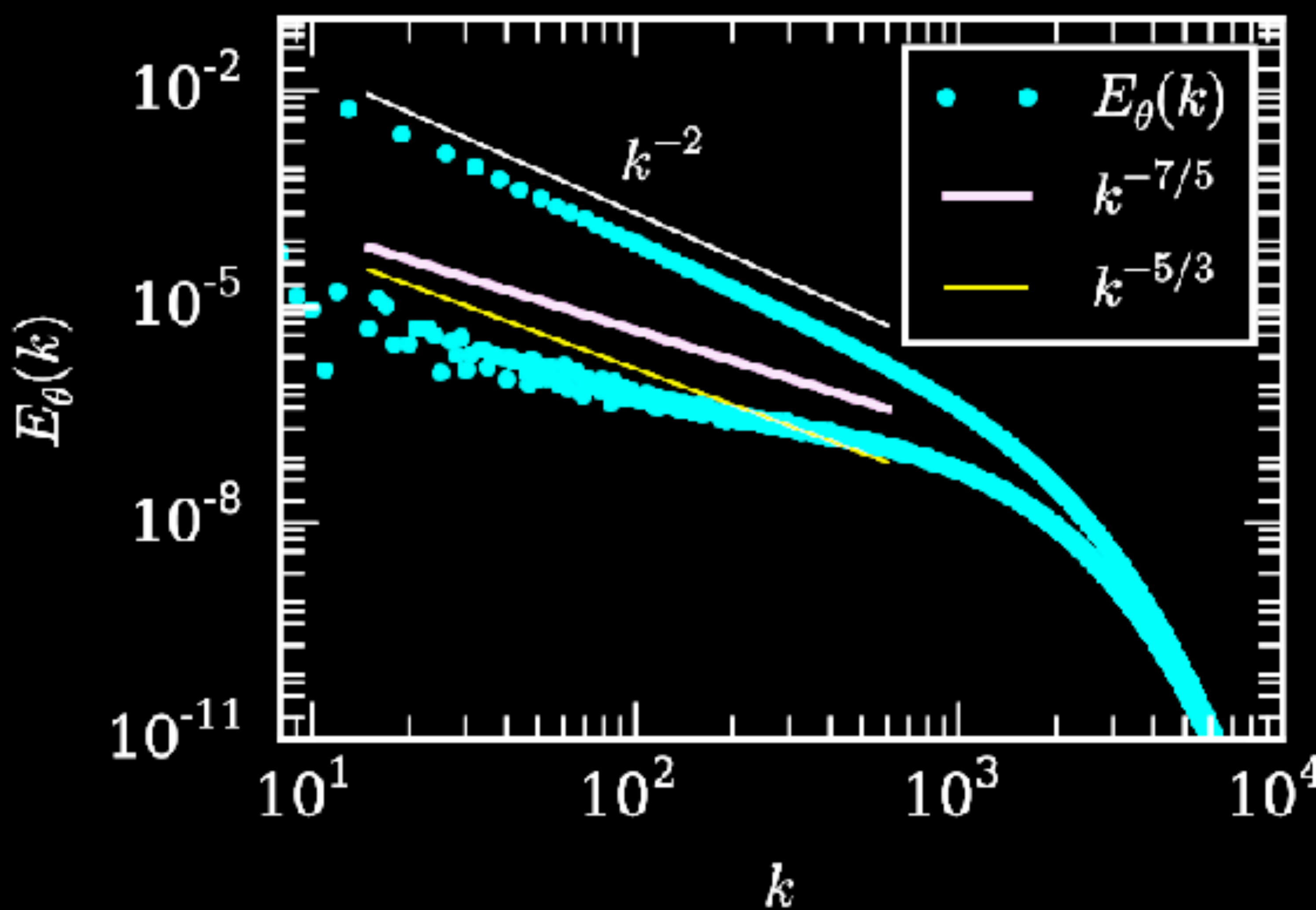
Temperature profile in RBC



$$\theta_m(0,0,k_z) = \int_0^1 \theta_m(z)\sin(k_z \pi z)\,dz$$

$$\approx -\frac{1}{\pi k_z} \qquad k_z \text{ even}$$

$$\approx 0 \qquad k_z \text{ odd}$$



10^{-2}
10^{-5}
10^{-8}
10^{-11}
10^{1}
10^{2}
10^{3}
10^{4}
$E_\theta(k)$
k
k^{-2}
$E_\theta(k)$
$k^{-7/5}$
$k^{-5/3}$

$$u_z(0,0,k_z) = 0$$

$$u_{x,y}(0,0,k_z) = 0 \quad \Rightarrow \quad 0 = -\frac{i\mathbf{k}\sigma_m(\mathbf{k})}{\rho_0} + \alpha g \theta_m(\mathbf{k})\hat{\mathbf{z}}$$

$$\theta = \theta_{\text{res}} + \theta_m; \quad \sigma = \sigma_{\text{res}} + \sigma_m$$

$$\frac{\partial \mathbf{u}(\mathbf{k})}{\partial t} + i \sum_{\mathbf{p}+\mathbf{q}=\mathbf{k}} [\mathbf{k}\cdot\mathbf{u}(\mathbf{q})]\mathbf{u}(\mathbf{p}) = -\frac{i\mathbf{k}\sigma_{\text{res}}(\mathbf{k})}{\rho_0} + \alpha g \theta_{\text{res}}(\mathbf{k})\mathbf{z} - \nu k^2 \hat{\mathbf{u}}(\mathbf{k})$$

$$\partial_t \mathbf{u} + (\mathbf{u}.\nabla)\mathbf{u} = -\nabla\sigma_{res} + \alpha g \theta_{res}\hat{z} + \nu\nabla^2\mathbf{u}$$

Péclet no scaling

$Pe = UL/\kappa$

$$\partial_t \mathbf{u} + (\mathbf{u}.\nabla)\mathbf{u} = -\nabla \sigma_{res} + \alpha g \theta_{res} \hat{z} + \nu \nabla^2 \mathbf{u}$$

$$c_1 \frac{U^2}{d} = c_2 \frac{U^2}{d} + c_3 \alpha g \theta_{\mathrm{res}} - c_4 \nu \frac{U}{d^2}$$

$$c_1 = \frac{|\mathbf{u} \cdot \nabla \mathbf{u}|}{U^2 / d}; \quad c_2 = \frac{|\nabla \sigma|_{\mathrm{res}} / \rho_0}{U^2 / d}; \quad c_3 = |\theta_{\mathrm{res}} / \Delta|; \quad c_4 = \frac{|\nabla^2 \mathbf{u}|}{U / d^2}$$

$$c_1 \mathrm{Pe}^2 = c_2 \mathrm{Pe}^2 + c_3 \mathrm{RaPr} - c_4 \mathrm{PePr}$$

$$\mathrm{Pe} = \frac{-c_4 \mathrm{Pr} + \sqrt{c_4^2 \mathrm{Pr}^2 + 4(c_1 - c_2) c_3 \mathrm{RaPr}}}{2(c_1 - c_2)}$$

Determine C_i's

Finite volume: OpenFOAM

No-slip on a closed cube
conducting top and bottom walls
Insulating side walls

Pr=1,6.8,100,1000

Ra = 10^6 to 5×10^8

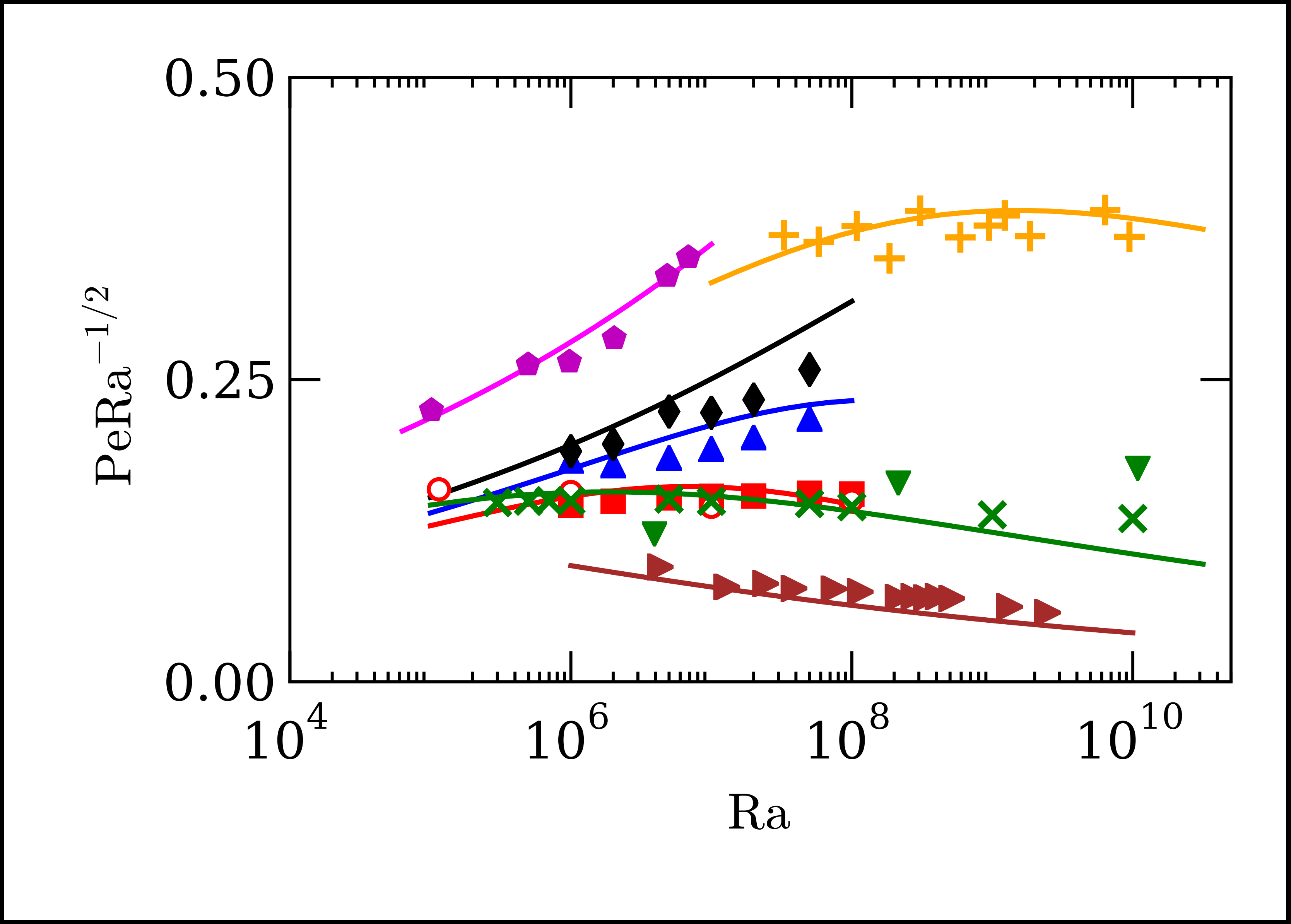
Grid: 256^3

$$c_1 = 1.5\mathrm{Ra}^{0.10}\mathrm{Pr}^{-0.06}$$

$$c_2 = 1.6\mathrm{Ra}^{0.09}\mathrm{Pr}^{-0.08}$$

$$c_3 = 0.75\mathrm{Ra}^{-0.15}\mathrm{Pr}^{-0.05}$$

$$c_4 = 20\mathrm{Ra}^{0.24}\mathrm{Pr}^{-0.08}$$


0.50
0.25
0.00
$PeRa^{-1/2}$
10^4
10^6
10^8
10^{10}
Ra

Viscous regime

Turbulent regime

$$\mathrm{Pe} \approx \frac{c_3}{c_4}\mathrm{Ra} \approx 0.038\mathrm{Ra}^{0.60}$$

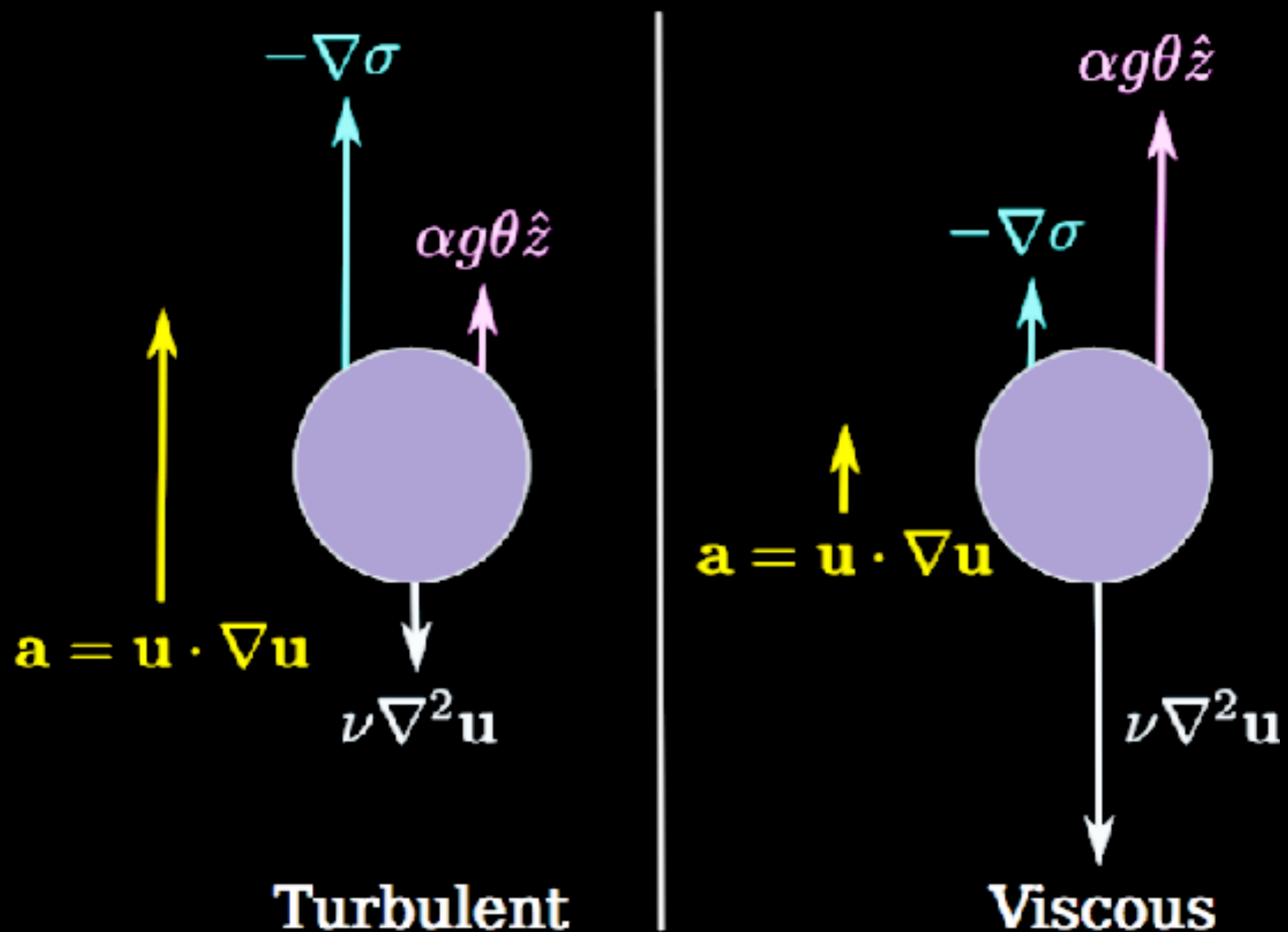
$$\mathrm{Pe} \approx \sqrt{\frac{c_3}{|c_1 - c_2|}\mathrm{RaPr}} \approx \mathrm{Ra}^{0.38}\sqrt{7.5\mathrm{Pr}}$$

crossover

$$\mathrm{Ra} \gg 10^6 \mathrm{Pr}^2$$

For Ra up to 10^8 $\mathrm{Pe} \sim \mathrm{Ra}^{1/2}$

Relative strengths of various forces in RBC



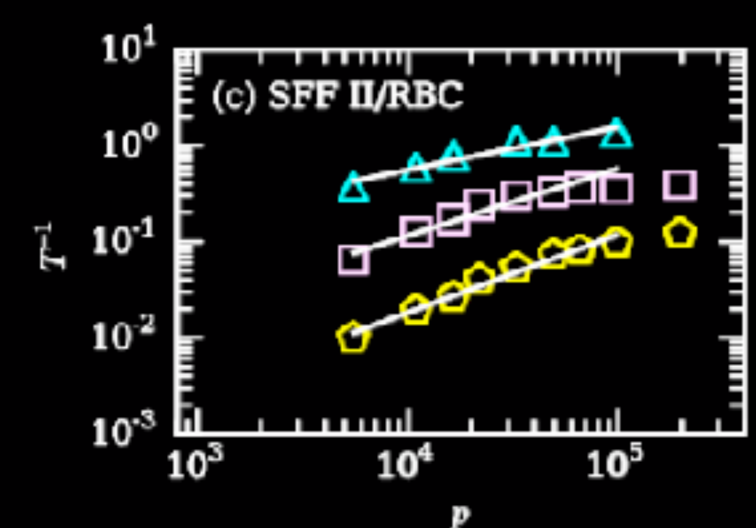
Buoyancy
<< grad(p)

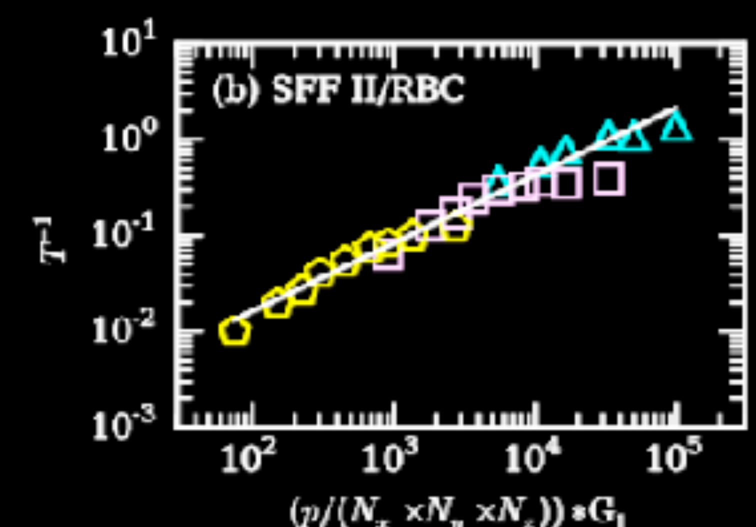
Spectrum studies

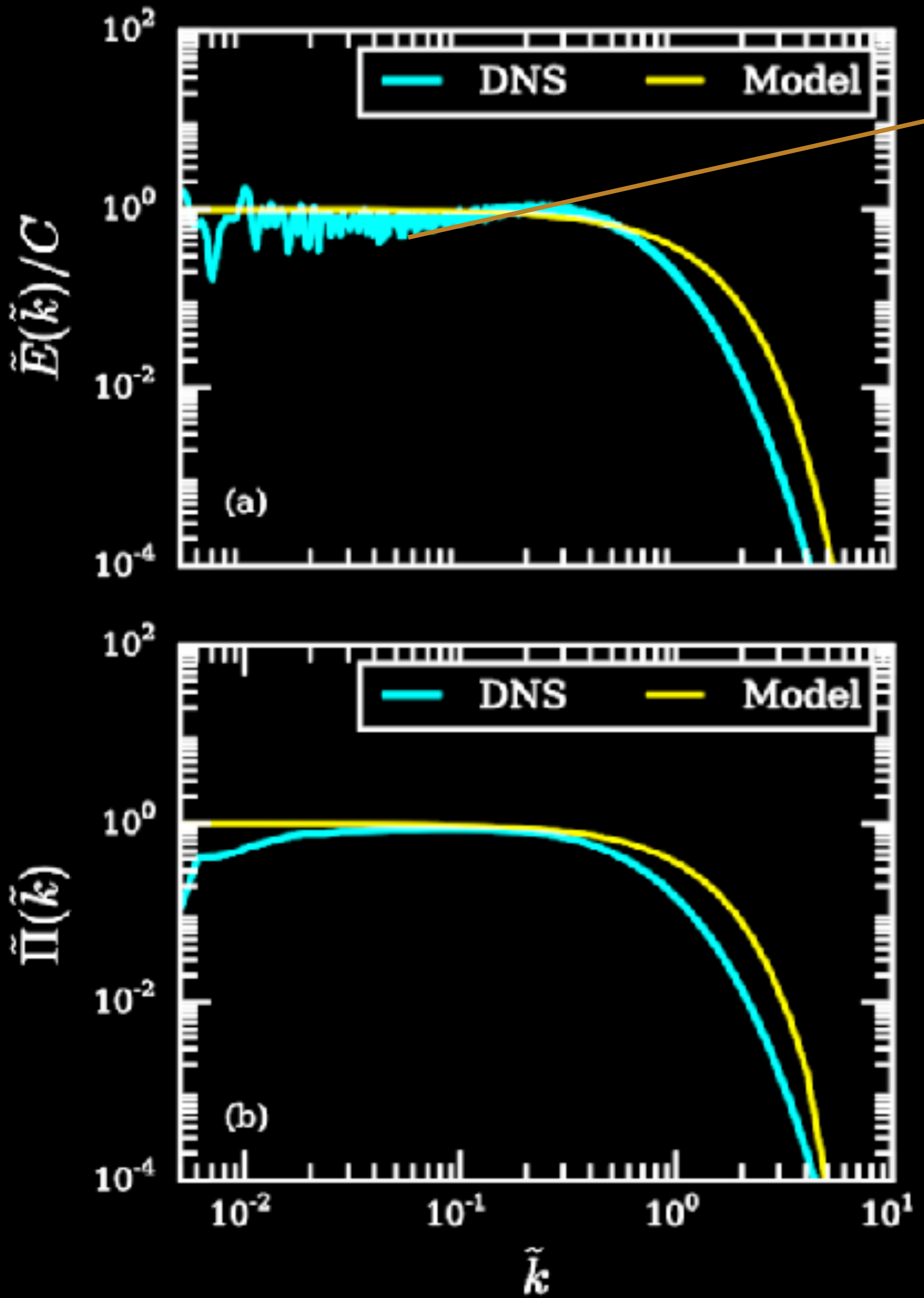
$$\mathrm{Pr} = 1$$
Grid: 4096^3
$$\mathrm{Ra} = 1.1\times10^{11}$$
$$\mathrm{Re} = 4.5\times10^{4}$$

Highest grid achieved so far

on 196608 processors
of Shaheen of KAUST







$E_u(k)k^{5/3}$

Pao's model, 1965

$$E(k) = K_{Ko}\epsilon^{2/3}k^{-5/3}\exp(-\tilde{k}^{4/3})$$

$$\Pi(k) = \epsilon\exp(-\tilde{k}^{4/3})$$

$$\tilde{k} = k / k_d$$

Verma et al., NJP 2016

$\varepsilon = 1.6 \times 10^{-3}$

Nusselt nu scaling

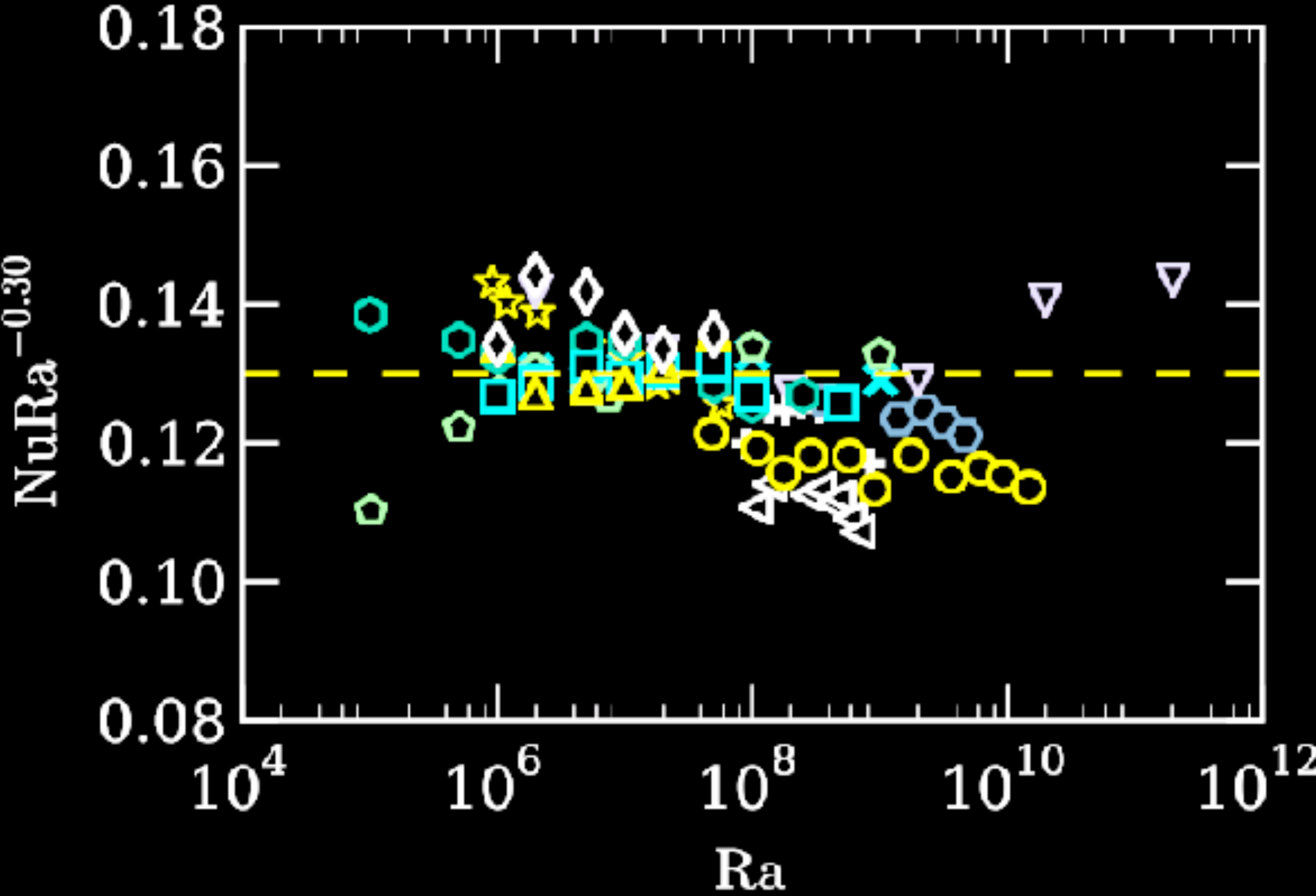
$$\langle u_z \theta \rangle_V = \langle u_z \theta_m \rangle_V + \langle u_z \theta_{\text{res}} \rangle_V$$

$$\mathrm{Nu} = \frac{\kappa\Delta / d + \langle u_z \theta_{\text{res}} \rangle_V}{\kappa\Delta / d} = 1 + \left\langle \frac{u_z d}{\kappa} \frac{\theta_{\text{res}}}{\Delta} \right\rangle_V$$

$$= 1 + C_{u\theta_{\text{res}}} \langle u'_z{}^2 \rangle_V^{1/2} \langle \theta'_{\text{res}}{}^2 \rangle_V^{1/2}$$

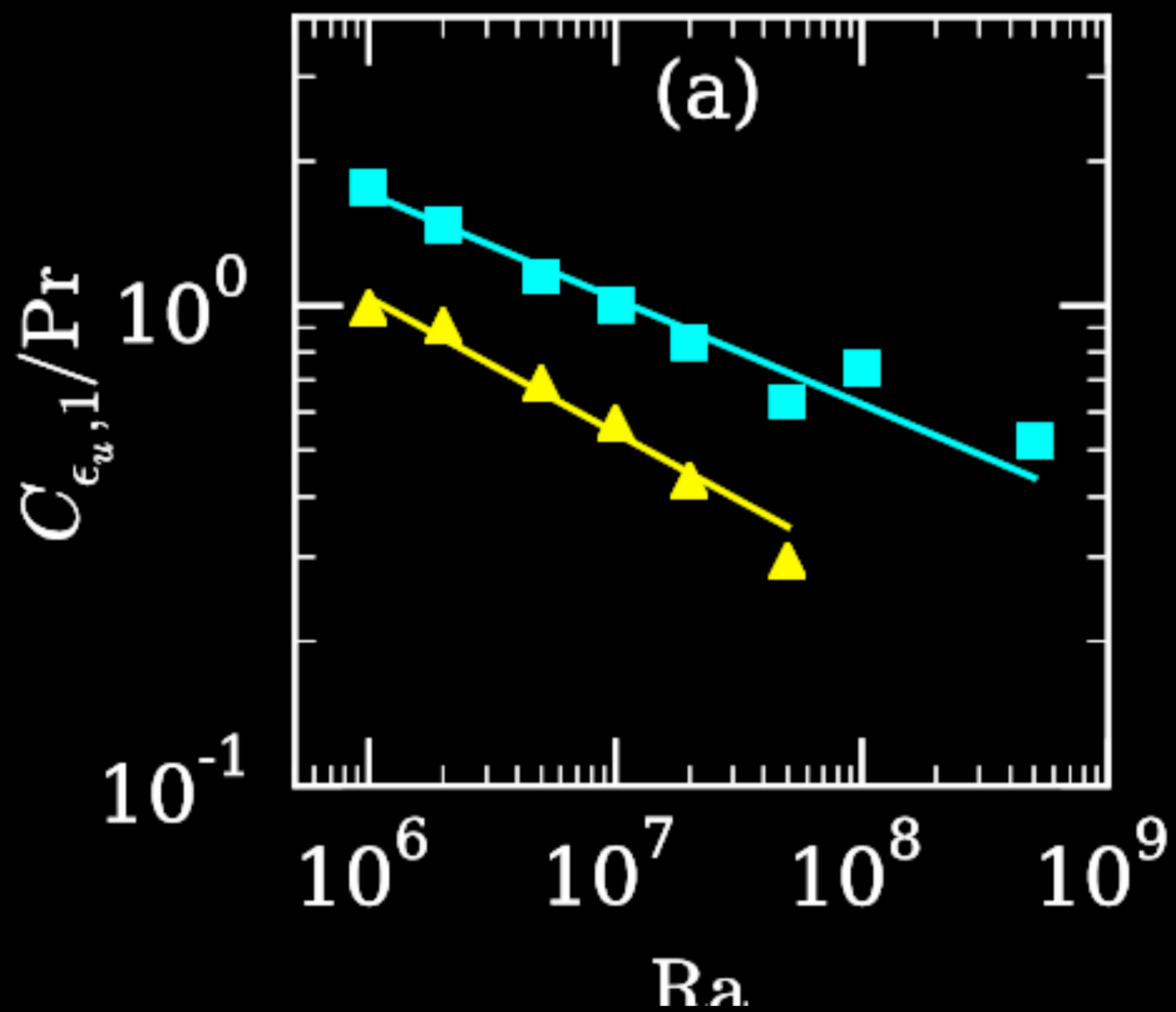
$$C_{u\theta_{\text{res}}} = \frac{\langle u'_z \theta'_{\text{res}} \rangle}{\langle u'_z{}^2 \rangle_V^{1/2} \langle \theta'_{\text{res}}{}^2 \rangle_V^{1/2}}$$

	Turbulent regime	Viscous regime
$C_{u\theta_{\text{res}}}$	$\mathrm{Ra}^{-0.05}$	$\mathrm{Ra}^{-0.07}$
$\langle \theta_{\text{res}}^2 \rangle^{1/2}$	$\mathrm{Ra}^{-0.13}$	$\mathrm{Ra}^{-0.18}$
$\langle u_z^2 \rangle^{1/2}$	$\mathrm{Ra}^{0.51}$	$\mathrm{Ra}^{0.58}$
Nu	$\mathrm{Ra}^{0.32}$	$\mathrm{Ra}^{0.33}$
ϵ_u	$(U^3/d)\mathrm{Ra}^{-0.21}$	$(\nu U^2/d^2)\mathrm{Ra}^{0.17}$


0.18
0.16
0.14
0.12
0.10
0.08
$NuRa^{-0.30}$
10^4
10^6
10^8
10^{10}
10^{12}
Ra

Scaling of viscous dissipation rate

$$\epsilon_u \sim \frac{U^3}{d}\mathrm{Ra}^{-0.18}$$



$$C_{\epsilon_u,1} = \frac{\epsilon_u}{U^3/d} = \frac{(\mathrm{Nu}\text{-}1)\mathrm{RaPr}}{\mathrm{Pe}^3} \sim \mathrm{Ra}^{-0.21}\mathrm{Pr}$$

Grossmann-Lohse model

$$\epsilon_u = \epsilon_{u,BL} + \epsilon_{u,bulk},$$

$$\epsilon_\theta = \epsilon_{\theta,BL} + \epsilon_{\theta,bulk},$$

$$\epsilon_{u,bulk} = \nu \left\langle (\partial_i u_j(\boldsymbol{x} \in bulk, t))^2 \right\rangle_V \sim \frac{U^3}{L} = \frac{\nu^3}{L^4} Re^3. \quad (2$$

$$\epsilon_{u,BL} = \nu \left\langle (\partial_i u_j(\boldsymbol{x} \in BL, t))^2 \right\rangle_V \sim \nu \frac{U^2}{\lambda_u^2} \frac{\lambda_u}{L} \sim \frac{\nu^3}{L^4} Re^{5/2}.$$

Conclusions

θ$_{mean}$(z) is a nontrivial profile

$$\theta_m(0,0,k_z) = -1/(\pi k_z)$$

Péclet scaling

$$c_1\mathrm{Pe}^2 = c_2\mathrm{Pe}^2 + c_3\mathrm{RaPr} - c_4\mathrm{PePr}$$

$$\mathrm{Pe} = \frac{-c_4\mathrm{Pr} + \sqrt{c_4^2\mathrm{Pr}^2 + 4(c_1 - c_2)c_3\mathrm{RaPr}}}{2(c_1 - c_2)}$$

Nusselt scaling: Correlations & scaling of θ$_{res}$ yields Ra$^{0.30}$

$$\epsilon_u \sim \frac{U^3}{d}\mathrm{Ra}^{-0.18}$$

Thank you!