EXPLORING THE FUZZBALL RESOLUTION TO INFORMATION PARADOX

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THE INFORMATION PARADOX

- Collapsing shell forms a black hole.
- Black hole radiate. The radiation is thermal.

Hawking 1975

- Radiation depends on few parameters, for example, mass, charge and angular momentum.
- Black holes would eventually evaporate.
- Left with thermal radiation, which depends on only few parameters. Hence stores very little information.
- The process is irreversible. Black hole evaporation is non-unitary.

Resolution

- Hawking's calculations are not precise enough to create a paradox.
- In a system with large degrees of freedom, it is difficult to differentiate between a pure state and a thermal state.
- Pure states behave as a thermal state.
- Small corrections in Hawking's computation can lead to "purification" of, apparently, thermal final state.
- In principle, unitarity can be restored.

Cloning Paradox

- Information of infalling matter is present at 2 space-like separated points in a "nice slice".
- We have cloned information.
- The problem can be resolved by realising that degrees of freedom inside the black holes are scrambled version of degrees of freedom outside.

Banrejee, Bryan, Papadodimas and Raju 2016

THE FUZZBALL PROPOSAL

- The fuzzball proposal aims to give a description of black holes devoid of paradoxes, as well as understand the entropy of black holes.
- The fuzzball resolution to information paradox claims that there is no interior to a black hole.
- Black hole geometries are effective description of Fuzzballs, which are obtained by compactifying string theory.
- Fuzzball geometries are smooth and horizon-less.

Two charge solution

For concreteness, let's consider 2 charge fuzzball solutions. Fuzzball metric parametrized by closed curves F is as follows.

Lunin and Mathur 2001

$$
\begin{aligned}
ds^2_{string} &= \frac{1}{\sqrt{f_1 f_5}}\left[-(dt+A)^2+(dy+B)^2\right]+\sqrt{f_1 f_5}\,d\mathbf{x}^2+\sqrt{\frac{f_1}{f_5}}\,d\mathbf{z}^2 \\
f_5 &= 1+\frac{Q_5}{L}\int_0^L \frac{ds}{|\mathbf{x}-\mathbf{F}(s)|^2} \\
f_1 &= 1+\frac{Q_5}{L}\int_0^L \frac{|\mathbf{F}'(s)|^2\,ds}{|\mathbf{x}-\mathbf{F}(s)|\,2} \\
L &= \frac{2\pi Q_5}{R} \\
A &= \frac{Q_5}{L}\int_0^L \frac{F_i\,ds}{|\mathbf{x}-\mathbf{F}(s)|^2}dx^i \\
dB &= *_4 dA
\end{aligned}
$$

Two charge solution

- This solution is asymptotically $M^5 \times S^1 \times T^4$
- y denotes S^1 direction, with radius R. **z** denotes T^4 direction.
- $L = \dfrac{2\pi Q_5}{R}$
- Moduli space is parametrized by closed curves, $x_i = F_i(s)$; $0 < s < L$.
- $Q_5 = g_s N_5,\ Q_1 = \dfrac{g_s}{V_4} N_1$. N_1 and N_5 are number of D-1 and D-5 branes.
- g_s is the string coupling.
- Volume of T^4 is $(2\pi)^4 V_4$.

Haar measure and typical states

- Consider a Hilbert space of dimension e^S.
- $|\psi\rangle = \sum a_i |i\rangle$; where $\sum |a_i|^2 = 1$
- $\ll A \gg \; = \; \langle\psi_{typ}| A |\psi_{typ}\rangle \; = \; \int \langle\psi| A |\psi\rangle \, d\mu$
- $d\mu \; = \; \frac{1}{V}\delta\left(\sum |a_i|^2 - 1\right)\prod_1^{e^S} da_i$
- $\int d\mu \; = \; 1$

Haar measure and typical states

- We can show that

$$\begin{aligned} \ll A \gg &= \frac{1}{e^S} tr(A) \\ \int \left(\langle\psi| A |\psi\rangle - \frac{1}{e^S} Tr(A) \right)^2 &= \frac{1}{e^S+1} \left(\frac{Tr(A^2)}{e^S} - \frac{Tr(A)^2}{e^{2S}} \right) \end{aligned}$$

- Expectation value of any operator A in a typical state is exactly same as in the identity density matrix, $\rho = \mathbb{I}/e^S$.

- Moreover, fluctuations from typical behaviour are suppressed by $e^{-S/2}$.

- We can't differentiate between typical states with low energy experiments.

EXPLORING FUZZBALL PROPOSAL

- As seen earlier, macroscopically, typical states are indistinguishable.
- Fuzzballs can't be typical states. Different fuzzball states correspond to different metric.
- Fuzzball states could form an atypical basis.
- Presumably, typical state should correspond to black hole.
- We would like to compare the properties of fuzzballs with that of black holes.
- We would also be interested in determining the average fuzzball geometry.

Behavior of correlation functions for large space like momenta

- Consider a BTZ black hole

$$ds^2 = -(r^2 - r_h^2)dt^2 + (r^2 - r_h^2)^{-1}dr^2 + r^2dx^2$$

- Solve wave equation with ansatz $e^{-i\omega t}e^{ikx}\psi(r)$
- Obtain boundary 2 point function with respect to Hawking-Hartle state in momentum space, $G_\beta(\omega, k)$.

Papadodimas and Raju 2012

$$\lim_{|k|\to\infty} G_\beta(\omega, k) \sim e^{-\frac{\beta|k|}{2}}$$

Behavior of correlation functions for large space like momenta

- We want to check whether Fuzzballs saturate this bound.
- Fuzzball states should also saturate the above bound for them to be a good description of black holes.
- If fuzzballs don't saturate this bound then it is likely that fuzzball states span only a small subset of whole Hilbert space.

Asymptotically AdS fuzzball solution

- We consider asymptotically $AdS_3 \times S^3 \times T^4$ fuzzball geometry.

Bena et al. 2016

$$
\begin{aligned}
ds_6^2 &= -\frac{2}{\sqrt{P}}(dv+\beta)\left(du+\omega+\frac{1}{2}F(dv+\beta)\right)+\sqrt{P}ds_4^2 \\
ds_4^2 &= \frac{\Sigma}{r^2+a^2}dr^2+\Sigma d\theta^2+\left(r^2+a^2\right)\sin^2(\theta)d\phi^2+r^2\cos^2(\theta)d\psi^2
\end{aligned}
$$

$$
\begin{aligned}
0 &\leq \theta \leq \pi/2 \; ; \quad 0 \leq \phi, \psi \leq 2\pi \\
\Sigma &= r^2 + a^2\cos(\theta) \\
P &= Z_1 Z_2 - Z_4^2 \\
v &= \frac{t+y}{\sqrt{2}} \; ; \quad u = \frac{t-y}{\sqrt{2}} \\
y &\equiv y + 2\pi R_y
\end{aligned}
$$

Asymptotically AdS fuzzball solution

$$\beta = \frac{1}{\sqrt{2}}\frac{a^2 R_y}{\Sigma}\left(\sin^2(\theta)\mathrm{d}\phi - \cos^2(\theta)\mathrm{d}\psi\right)$$
$$F = b_{k,m,n}^2 F_{k,m,n}$$
$$\omega = \omega_0 + b_{k,m,n}^2 \omega_{k,m,n}$$
$$\omega_0 = \frac{1}{\sqrt{2}}\frac{a^2 R_y}{\Sigma}\left(\sin^2(\theta)\mathrm{d}\phi + \cos^2(\theta)\mathrm{d}\psi\right)$$
$$Z_1 = \frac{Q_1}{\Sigma} + \frac{R_y^2}{2Q_5\Sigma} b_{k,m,n}^2 \Delta_{2k,2m,2n}\cos(\hat{v}_{2k,2m,2n})$$
$$Z_2 = \frac{Q_5}{\Sigma}\ ;\ Z_4 = \frac{R_y}{\Sigma} b_{k,m,n}\Delta_{k,m,n}\cos(\hat{v}_{k,m,n})$$
$$\hat{v}_{k,m,n} = \frac{\sqrt{2}}{R_y}(m+n)v + (k-m)\phi - m\psi$$
$$\Delta_{k,m,n} = a^k r^n \left(r^2 + a^2\right)^{-(k+n)/2}\cos^m(\theta)\sin^{k-m}(\theta)$$

Asymptotically AdS fuzzball solutions

- The above metric depend explicitly only on r and θ.
- Ansatz for solving wave equation:

$$\Phi\left(t, y, r, \theta, \phi, \psi\right) = e^{i(-\nu t + ky + p\psi + q\phi)} R(r)\Theta(\theta)$$

- We get a PDE only in r and θ. For $p = q = 0$, the wave equation completely separates.
- We wish to compute correlation functions and determine the large k behaviour.

Mean fuzzball state

The 2 charge solution mentioned earlier has been quantized.

Rychkov 2005.

$$
\begin{aligned}
F^i(s) &= \mu \sum_{k=1}^{\infty} \frac{1}{\sqrt{2k}} \left(a_k^i e^{i\frac{2\pi k}{L}s} + a_k^{i\dagger} e^{-i\frac{2\pi k}{L}s} \right) \\
\mu &= \frac{g_s}{R\sqrt{V_4}} \\
\left[a_k^i, a_{k'}^{j\dagger} \right] &= \delta^{ij}\delta_{kk'}
\end{aligned}
$$

i runs from 1 to 4. We want to compute the average fuzzball metric and also determine the deviation. For simplicity we work with f_5.

Mean fuzzball state

- Let $<< . >>$ denote thermal average.
- We want to compute $<< f_5 >> - << f_5^{naive} >>$.

$$<< f_5^{naive} >>= 1 + \frac{Q_5}{r^2}$$

- This would give us an idea of how different is mean fuzzball state from that of a black hole.

Computing thermal average of f_5

We compute thermal average of $:f_5:$ (normal ordered). We use the following property of thermal average of normal ordered operators.

$$\ll: \mathrm{F}^{2n}(s) :\gg \;=\; \frac{(2n)!}{2^n n!} \ll: \mathrm{F}^2(s) :\gg^n$$

This is just the special case of

$$\ll F_1 F_2 \dots F_{2n} \gg \;=\; G_{12} G_{34} \dots + G_{13} G_{24} \dots + G_{14} G_{23} \dots$$

where

$$G_{ij} \;=\; \ll F_i F_j \gg$$

Mean fuzzball state

Given an operator G[F]

$$G[F] \;=\; g_n F^n$$

We can compute thermal average of :G: as an infinite series.
When computing thermal average of f_5, we found that it is easier to do the following.

$$\frac{1}{|\mathbf{x}-\mathbf{F}(s)|^2} = \int_0^\infty dt\ e^{-t|\mathbf{x}-\mathbf{F}(s)|^2}$$

As the integrand is block diagonal, we only need to compute $\ll e^{-t\left(x^1-F^1(s)\right)^2} \gg$.

Mean fuzzball state

$$\gamma = \ll F^2(s) \gg \; = \; \mu^2 \sum_{k=1}^{\infty} \frac{1}{e^{2\pi k\beta/L} - 1}$$

$$\ll \frac{1}{|\mathbf{x} - \mathbf{F}(s)|^2} \gg \; = \; \int_0^{\infty} dt \, \frac{e^{-\frac{r^2 t}{2\gamma t+1}}}{(2\gamma t + 1)^2} = \frac{1 - e^{-r^2/2\gamma}}{r^2}$$

$$\ll f_5 \gg \; = \; 1 + Q_5 \left(\frac{1 - e^{-r^2/2\gamma}}{r^2} \right)$$

$$\ll f_5 \gg - \ll f_5^{naive} \gg \; = \; -Q_5 \left(\frac{e^{-r^2/2\gamma}}{r^2} \right)$$

All operators are assumed to be normal ordered.

Mean fuzzball state

- $<< f_5 >>$ is not sufficient to conclude that the mean fuzzball geometry is different from black hole geometry.

- We also need to compute the deviation

$$\sigma_5^2 = \ll f_5^2 \gg - \ll f_5 \gg^2$$

- We would need to compute the following term

$$\frac{Q_5}{L^2} \int_0^L ds \int_0^L ds' \ll \frac{1}{|x - F(s)|^2} \frac{1}{|x - F(s')|^2} \gg$$

- We haven't been able to compute the fluctuation analytically.

Mean fuzzball state

- If $\sigma_5 \gtrsim \ll f_5 \gg - \ll f_5^{naive} \gg$, then the mean fuzzball geometry is essentially indistinguishable from black hole.
- If $\left(\ll f_5 \gg - \ll f_5^{naive} \gg\right) \gg \sigma_5$, then the mean fuzzball geometry is indeed different from that of a black hole.

THE END