Holographic Tensor Models

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Overview

1. Introduction
2. Gurau-Witten Model
3. Klebanov-Tarnopolosky Model
4. Andreev-Altland-Zirnbauer Classification
5. Results for GW and KT Models

SYK Model

- The Hamiltonian is given by

$$H = i^{\frac{q}{2}} \sum_{1 \le i_1 < i_2 < \cdots < i_q \le N} J_{i_1 i_2 \ldots i_q} \, \psi_{i_1} \psi_{i_2} \ldots \psi_{i_q} \tag{1}$$

$$\langle J_{i_1 i_2 \ldots i_q} \rangle = 0, \quad \langle J^2_{i_1 i_2 \ldots i_q} \rangle = \frac{2^{q-1}}{q} \frac{\mathcal{J}^2 (q-1)!}{N^{q-1}}. \tag{2}$$

- The SYK Model has three striking features:
  - Solvable in Large-N
  - Maximally Chaotic
  - Emergent Conformal Symmetry

Gurau-Witten Model

- Bosonic Tensor Models have been discussed in great length in the literature.[Gurau]
- Witten proposed a coloured fermionic tensor model. [Witten '16]
- The Hamiltonian is given by

$$H = \frac{i^{(D+1)/2} J}{n^{D(D-1)/4}} \psi_0 \psi_1 \dots \psi_D \tag{3}$$

- J is a real coupling constant.
- ψ_a's are real fermionic fields, where $a = \{0, 1, \dots, D\}$ indicate the colours.
- Each of the ψ_a transforms under a real irrep of G.
- For any unoredered pair (a, b), $a \neq b$, we can associate a symmetry group $G_{ab} = O(n)$, which gives the full symmetry group (upto an overall discrete group) to be

$$G = \prod_{a<b} \sim O(n)^{D(D+1)/2} \tag{4}$$

Gurau-Witten Model

- Each fermionic field can be thus written as

$$\psi_a^{i_{a0} i_{a1} \ldots \not{i}_{aa} \ldots i_{aD}} \tag{5}$$

where $\not{i}_{aa}$ is an omitted index.

- Each index i_{ab} transforms as a vector under G_{ab}, for $a \neq b$.
- Since each a has D groups G_{ab}, $a \neq b$, ψ_a has n^D components.
- The total number of real fermions in the theory is $N = (D+1)n^D$.
- This is the N that is relevant for large N.
- The simplest case is $D = 1$, where ψ_0 and ψ_1 transforms as a vector under G_{01} and $G_{10} = G_{01}$. The Hamiltonian is

$$H = iJ\,\psi_0^i \psi_1^i. \tag{6}$$

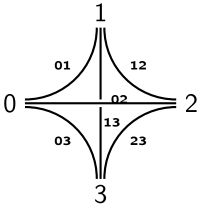
- This is a free theory for any value of n.

Gurau-Witten Model

- Next case to consider is D=3,for which the Hamiltonian is

$$H = -\frac{J}{n^{3/2}}\psi_0^{ijk}\psi_1^{ilm}\psi_2^{pjm}\psi_3^{plk}. \tag{7}$$

- The Feynman vertex is



- The canonical anti-commutation rules are given by

$$\{\psi_a^{ijk}, \psi_b^{plm}\} = \delta_{ab}\delta^{ip}\delta^{jl}\delta^{km}. \tag{8}$$

- The fermion operators can be realized as Euclidean Gamma matrices of $SO(N) = SO((D+1)n^D)$.

Klebanov-Tarnopolosky Model

- This Model is constructed of a smaller symmetry group with same salient features of GW Model.**[Klebanov-Tarnopolosky '16]**
- The theory is constructed of uncoloured fermionic tensors

$$\psi^{i_1 i_2 \ldots i_D}$$

- Each index $i_m = 1, 2, \ldots, n$, giving the total degrees of freedom n^D.
- The symmetry group of the theory is $O(n)^D$.
- The interaction term needs at least $D + 1$ fermions.
- The Hamiltonian is given by

$$H = \frac{J}{\sqrt{8}} \psi^{ijk} \psi^{ilm} \psi^{pjm} \psi^{plk}. \tag{9}$$

Why Tensor Models

- Tensor Models are truly Unitary quantum mechanical system.
- The disorder averaging is avoided, while maintaining the large-N behaviour of SYK.
- SYK and tensor models are all dominated by **melonic** diagrams at large-N.
- All these models saturate the Maldacena-Shenker-Stanford chaos bound.

Andreev-Altland-Zirnbauer Classification

- SYK Model for different values of N belongs to one of the Wigner-Dyson ensembles, namely GUE,GSE,GOE.[Cotler, et. al. '16]
- Andreev-Altland-Zirnbauer ten-fold classification is an extension of the Wigner-Dyson classification.
- The classification is based on the discrete symmetries possessed by the Hamiltonian:
  - S: Spectral Mirror Symmetry operator
    S implies that if the Hamiltonian has an eigenvalue $E_0 + E$, then $E_0 - E$ is also an eigenvalue, E_0 being the mid-level energy value. This means it anti-commutes with the Hamiltonian,

    $$\{H, S\} = 0.$$

  - $\mathcal{T}$: Time Reversal operator
    $\mathcal{T}$ is an anti-unitary operator, which commutes with the Hamiltonian.

Andreev-Altland-Zirnbauer Classification

- For Hamiltonians that have only spectral mirror symmetry, and no time reversal symmetry:
  - S: Unitary; $S^2 = +1$: AIII
  - S: Anti-Unitary; $S^2 = +1$: BD
  - S: Anti-Unitary; $S^2 = -1$: C
- For Hamiltonians that are symmetric under both S and $\mathcal{T}$, with $[\mathcal{T}, S] = 0$, we have four possiblities
  - $S^2 = +1$, $\mathcal{T}^2 = +1$: BDI
  - $S^2 = +1$, $\mathcal{T}^2 = -1$: CII
  - $S^2 = -1$, $\mathcal{T}^2 = +1$: CI
  - $S^2 = -1$, $\mathcal{T}^2 = -1$: DII
- For Hamiltonians that have only time reversal symmetry
  - $\mathcal{T}^2 = +1$: AI (or GOE)
  - $\mathcal{T}^2 = -1$: AII (or GSE)
- When the Hamiltonian is not symmetric under S or $\mathcal{T}$, it belongs to class A (or GUE).

Gamma Matrix assignments

- To recap: The fermions can be realized as Euclidean Gamma matrices of $SO(N) = SO((D+1)n^D)$.
- We will be explicitly diagonalizing the $D = 3, n = 2$ case: which means we need the SO(32) Gamma matrices.
- For $N = 32$, we can find representation that is real and symmetric.
- The mapping is done using the relation

$$\psi_a^{ijk} = \gamma_{8a+4i+2j+k-6}. \tag{10}$$

- Using this, we write down the Hamiltonian, and explicitly diagonalize for the eigenvalues.

Results for GW Model

- The Gamma matrices of $SO(32)$ are 65536×65536 dimensional matrices.
- This already max-es out our computational powers.
- The Hamiltonian is a rather sparse matrix, which helps in GW case (as compared to KT Model).

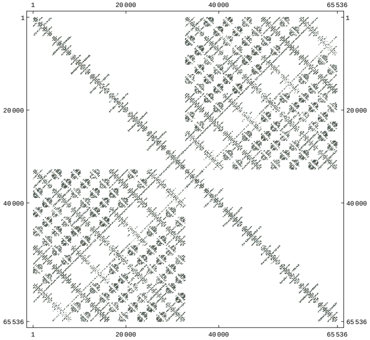
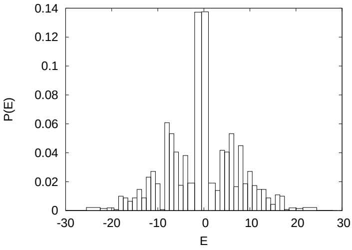


Figure: The MatrixPlot of GW-Hamiltonian

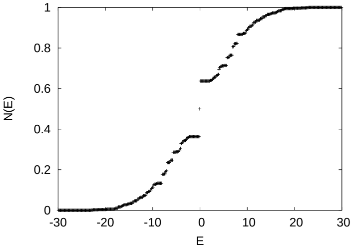
Results for GW Model

Density of States and the Integrated Density of States:

- The eigenvalue spectrum is symmetric about $E = 0$: Spectral Mirror Symmetry.
- The ground state is unique with no degeneracy.
- There is a huge degeneracy around $E = 0$.



(a) The density of states. The d.o.s is symmetric: the slight asymmetry is an artifact of the binning of the eigenvalues.



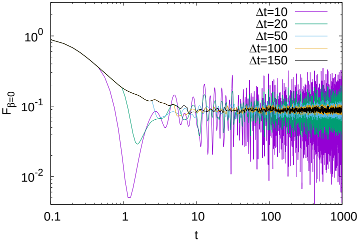
(b) The integrated density of states. The jump around zero is a result of the degeneracy at $E = 0$.

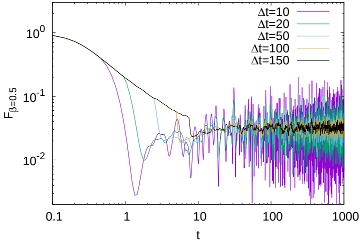
Spectral Form Factor:

- Spectral Form Factor is defined as

$$F_\beta(t) = \frac{|Z(\beta,t)|^2}{|Z(\beta,0)|^2}, \quad \text{where} \quad Z(\beta,t) = \mathrm{Tr}(e^{-(\beta+it)H}) \tag{11}$$

- This was used as a diagnostic of random-matrix behaviour of SYK Model.**[Cotler,et. al. '16]**
- We compute the same quantity in GW case and plot it after a running time average.

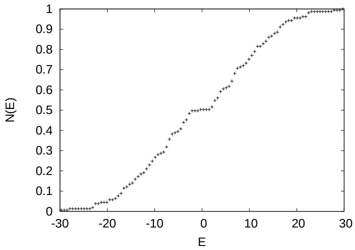




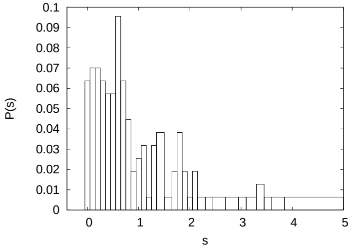
Results for GW Model

Level Repulsion

- After removing the degeneracies, the spectrum is unfolded.
- Integrable systems typically show a Poisson distribution.
- A turnaround in the distibution close to zero spacing is called level repulsion: indicator of chaotic behaviour.



(a) The integrated d.o.s plot after degeneracies have been removed.



(b) Unfolded level spacing distribution showing level repulsion near $s \to 0$.

Choice of Ensembele and Further comments

- The eigenvalue spectrum clearly indicates spectral mirror symmetry, which means we have an operator S such that $\{H, S\} = 0$.
- The operator can be explicitly constructed in terms of γ_i's as

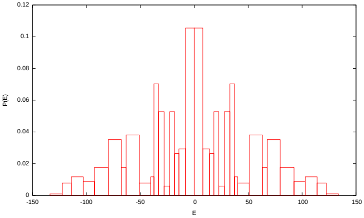
$$S = \gamma_1 \gamma_2 \dots \gamma_8$$

- The operator S is unitary and squares to 1.
- We can also find a Time-Reversal operator $\mathcal{T}$ that squares to 1, thus identifying the ensemble to be **BDI** in the AAZ classification.
- The next simplest GW case is at $D = 3, n = 3$ and $D = 5, n = 2$, which correspond to $N = 108$ and $N = 192$, respectively: computationally inaccessible.

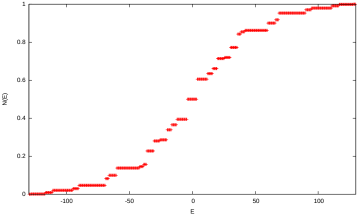
Results for the KT Model

- For the KT model, there are three tractable cases: (i) $D = 3, n = 2$, (ii) $D = 5, n = 2$, (iii) $D = 3, n = 3$.
- $D = 3, n = 2 \Rightarrow N = 8$: does not have enough eigenvalues to indicate chaos.
- $D = 5, n = 2 \Rightarrow N = 32$: matrix too dense to diagonalize.
- $D = 3, n = 3 \Rightarrow N = 27$:

  Eigenvalue spectrum



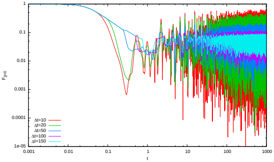
(a) DOS for $D = 3, n = 3$. The spectrum is shifted to look symmetric about $E = 0$.



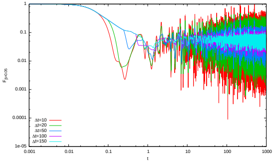
(b) Integrated DOS vs. Energy Levels

Results for the KT Model

Spectral Form Factor

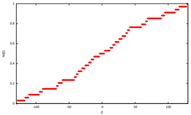


(a) The SFF for $\beta = 0$

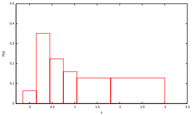


(b) The SFF for $\beta = 0.5$

Level Repulsion



(a) The integrated d.o.s plot after degeneracies have been removed.



(b) Unfolded level spacing distribution showing level repulsion near $s \to 0$.

Comments

- The (un)coloured tensor models have similar features as that of SYK at finite N.
- The Hamiltonians of the tensor models belong to one of the ensembles of AAZ classification.
- Within the numerically tractable cases of the tensor models, one can see indications of chaotic behaviour similar to SYK.