Bangalore Area String Meeting

Sachdev-Ye-Kitaev Model : Global and Super Symmetry

Junggi Yoon

August 2, 2017

ICTS

ICTS INTERNATIONAL CENTRE *for* THEORETICAL SCIENCES

TATA INSTITUTE OF FUNDAMENTAL RESEARCH

“Your beginnings will seem simple,
so chaotic will your future will be.”

–SYK Model 1:1

Contents

ICTS

Sachdev-Ye-Kitaev Model: Review

Based on bi-local collective field theory

Why “Bi-local Collective Field Theory”?

SYK Model

- Quantum Mechanics of N Majorana Fermions : $\chi^i \quad (i = 1, 2, \cdots, N)$
  【Sachdev, Ye, cond-mat/9212030】【Kitaev (2015)】
- Action : $$S = \int d\tau \left[\frac{1}{2} \sum_{i=1}^{N} \chi^i \partial_\tau \chi^i - \sum_{i,j,k,l=1}^{N} J_{ijkl} \chi^i \chi^j \chi^k \chi^l \right]$$
- Random Coupling Constant : J_{ijkl}
  - ✓ Gaussian Ensemble : $\mathcal{P}[J_{ijkl}] \sim \exp \left[-\frac{2N^3}{J^2} J_{ijkl} J_{ijkl} \right]$

    J : play a role of coupling constant
- Quenched Average : $\langle \log Z \rangle_J$
  - ✓ Replica Trick : $\lim_{n \to 0} \frac{\langle Z^n \rangle_J - 1}{n}$ 【Jevicki, Suzuki, JY, 1603.06246】

    cf. Renyi Entropy

    cf【Nishinaka, Terashima, 1611.10290】

ICTS

Bi-local Collective Theory

- O(N) Invariants: Bi-local Field 【Jevicki, Suzuki, JY, 1603.06246】

$$\Psi(\tau_1,\tau_2)=\frac{1}{N}\sum_{i=1}^{N}\chi^i(\tau_1)\chi^i(\tau_2)$$

  - ✓ cf) Radial Coordinate, 3d Free O(N) vector model
    【Jevicki, Koch, Jin, Rodrigues, 1008.0633】
    【Jevicki, Koch, Rodrigues, JY, 1408.4800】
  - ✓ Can be treated as a matrix in bi-local space

- Jacobian:

$$\log\mathcal{J}=-\frac{1}{2}\left(N-1+\delta(0)\right)\mathrm{Tr}\,\log\Psi$$

  - ✓ Transformation from χ to Ψ in path integral
  - ✓ cf) Jacobian for radial coordinates
  - ✓ -1, $\delta(0)$: Not so important for a while. Will discuss in SUSY SYK model.

Bi-local Collective Theory

✳ Bi-local Collective Action 【Jevicki, Suzuki, JY, 1603.06246】

$$S_{col} = \frac{N}{2}\mathrm{Tr}\left[-D\star\Psi + \log\Psi\right] - \frac{NJ^2}{8}\int d\tau_1 d\tau_2\ \left[\Psi(\tau_1,\tau_2)\right]^4$$

❶ ❷ ❸

✓ ❶ Kinetic term: $\frac{1}{2}\int d\tau\ \chi^i\partial_\tau\chi^i$

- Bi-local Derivative: $D(\tau_1,\tau_2) = \partial_{\tau_1}\delta(\tau_1-\tau_2)$
- Star (matrix) product in bi-local space: $(A\star B)(\tau_1,\tau_2) = \int d\tau_3\ A(\tau_1,\tau_3)B(\tau_3,\tau_2)$

✓ ❷ Jacobian: $\log\mathcal{J} = -\frac{N}{2}\mathrm{Tr}\ \log\Psi$

✓ ❸ Potential: $-\int d\tau \sum_{i,j,k,l=1}^{N} J_{ijkl}\chi^i\chi^j\chi^k\chi^l$

- Disorder average over Gaussian Ensemble: $\exp\left[-\frac{2N^3}{J^2}J_{ijkl}J_{ijkl}\right]$

Emergent Reparametrization

- At strong coupling limit $|J\tau| \gg 1$

$$S_{col} = \frac{N}{2}\mathrm{Tr}\left[\cancel{-D\star}\,\Psi + \log\Psi\right] - \frac{NJ^2}{8}\int d\tau_1 d\tau_2\ \left[\Psi(\tau_1,\tau_2)\right]^4$$

  - Kinetic can be ignored

- Under reparametrization

$$\Psi(\tau_1,\tau_2) \longrightarrow [f'(\tau_1)]^{\frac{1}{4}}\,\Psi\left(f(\tau_1), f(\tau_2)\right)[f'(\tau_2)]^{\frac{1}{4}}$$

✓ Collective action is invariant at strong coupling limit.

✓ Emergent Reparametrization symmetry

Symmetry Breaking

$$S_{col} = \frac{N}{2}\mathrm{Tr}\,\boxed{[-D\star\Psi +} \log\Psi] - \frac{NJ^2}{8}\int d\tau_1 d\tau_2\ [\Psi(\tau_1,\tau_2)]^4$$

- Reparametrization

$$\Psi(\tau_1,\tau_2) \longrightarrow [f'(\tau_1)]^{\frac{1}{4}}\Psi\left(f(\tau_1),f(\tau_2)\right)[f'(\tau_2)]^{\frac{1}{4}}$$

- Spontaneously Broken by Classical Solution : $\Psi_{cl}(\tau_1,\tau_2) = \Lambda\frac{\mathrm{sgn}\,(\tau_1-\tau_2)}{|\tau_1-\tau_2|^{\frac{1}{2}}}$
  - ✓ Still Invariant under SL(2,R) : Global
- Explicitly Broken by Kinetic term
- Leads to Pseudo-Nambu-Goldstone Boson
  - ✓ Crucial for maximal chaos

ICTS

Effective Action

- Transformation of Classical solution by $f(\tau)$

$$[f'(\tau_1)]^{\frac{1}{4}}\,\Psi\left(f(\tau_1), f(\tau_2)\right)[f'(\tau_2)]^{\frac{1}{4}}$$

- Substitute into Kinetic term: $-\frac{N}{2}\mathrm{Tr}\, D \star \Psi$

- Schwarzian Effective Action

$$S_{\mathrm{eff}} = -\frac{N\alpha}{J}\int d\tau\, \mathrm{Sch}(f,\tau)$$

✓ SL(2,R) invariant

$$\mathrm{Sch}(f,\tau) \equiv \frac{f'''(\tau)}{f'(\tau)} - \frac{3}{2}\left(\frac{f''(\tau)}{f'(\tau)}\right)^2$$

ICTS

Maximal Chaos

✴ Infinitesimal fluctuation in Effective Action: $f(\tau) = \tau + \epsilon(\tau)$

$$\langle \epsilon_n \epsilon_{-n} \rangle = \frac{\beta J}{N\alpha} \frac{1}{n^2(n^2-1)}$$

【Maldacena, Stanford, 1604.07818】

✴ Contribution of the effective action

$$\langle \Psi(\tau_1,\tau_2)\Psi(\tau_3,\tau_4)\rangle = \underline{\psi_{cl}(\tau_1,\tau_2)\psi_{cl}(\tau_3,\tau_4)} + \mathcal{O}\left(\frac{1}{N}\right)$$

$$\big\downarrow$$

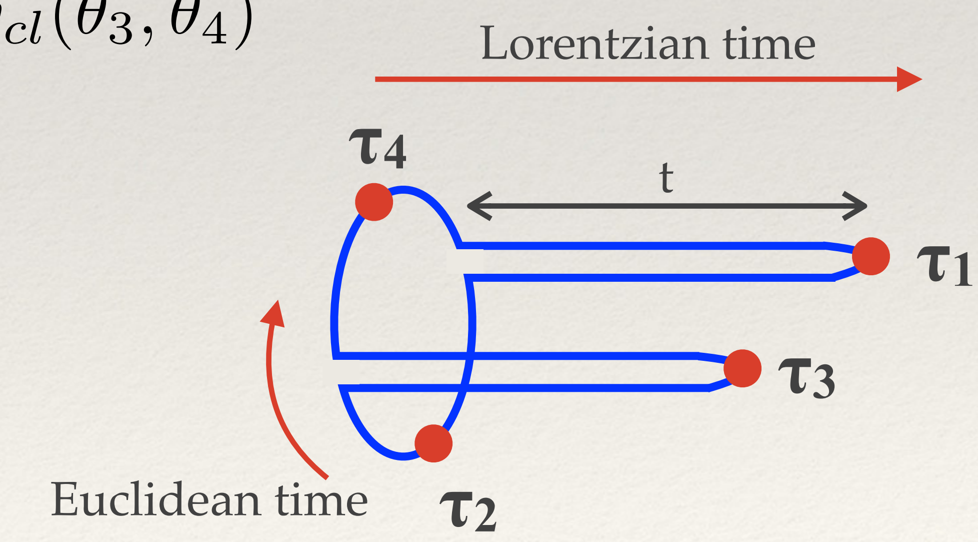
$$\langle \epsilon_n \epsilon_{-n} \rangle\, \delta_{\epsilon_n}\psi_{cl}(\theta_1,\theta_2)\delta_{\epsilon_{-n}}\psi_{cl}(\theta_3,\theta_4)$$

✴ Out-of-time-ordered Correlator:

【Maldacena, Stanford, 1604.07818】

$$\mathcal{F}(t) \sim e^{\frac{2\pi}{\beta}t}$$

✓ Maximally Chaotic: $\lambda_L = \frac{2\pi}{\beta}$



ICTS

Extension of Bi-local Space

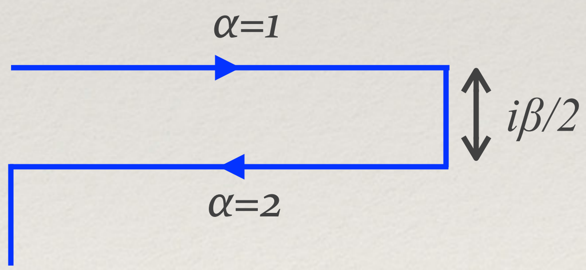
- Add Extra structure to field: Extend bi-local space

$$\Psi(\tau_1,\tau_2) \longrightarrow \Psi(\tau_1,\alpha_1;\tau_2,\alpha_2)$$

- ✓ Matrix in the extended bi-local space:

$$(A\star B)(\tau_1,\alpha_1;\tau_2,\alpha_2)=\int d\tau_3\sum_{\alpha_3}A(\tau_1,\alpha_1;\tau_3,\alpha_3)B(\tau_3,\alpha_3;\tau_2,\alpha_2)$$

- ✓ Replica space: $\alpha = 1,2,\cdots,n$ 【Jevicki, Suzuki, JY, 1603.06246】
- ✓ Doubled space in Thermofield Dynamics: $\alpha = 1,2$ 【Jevicki, JY, 1503.08484】【Jevicki, Suzuki, 1508.07956】



- ✓ Flavor: $\alpha = 1,2,\cdots,M$ 【JY, 1707.01740】
- ✓ Superspace: $\alpha = \theta$ 【JY, 1706.05914】

I have an SYK, I have a flavor

Uh, SYK-flavor!

Global Symmetry: SO(M)

- SYK model with SO(M) global symmetry 【JY, 1707.01740】

$\alpha_1, \alpha_2 = 1, 2, \cdots, M$

$$S = \int d\tau \left[\frac{1}{2} \sum_{i=1}^{N} \sum_{\alpha=1}^{M} \chi^{i\alpha} \partial_\tau \chi^{i\alpha} - \sum_{i,j,k,l=1}^{N} \sum_{\alpha_1,\alpha_2=1}^{M} J_{ijkl} \chi^{i\alpha_1} \chi^{j\alpha_1} \chi^{k\alpha_2} \chi^{l\alpha_2} \right]$$

  - ✓ Manifestly SO(M) Invariant   cf) 【Sachdev, 1506.05111】 【Sachdev et al., 1612.00849】 【Gross, Rosenhaus, 1610.01569】

- Bi-local field

$$\Psi^{\alpha_1\alpha_2}(\tau_1, \tau_2) \equiv \frac{1}{N} \sum_{i=1}^{N} \chi^{i\alpha_1}(\tau_1) \chi^{i\alpha_2}(\tau_2)$$

  - ✓ M×M matrix: $\mathbf{\Psi}(\tau_1, \tau_2)$
  - ✓ Decomposition: $\square \otimes \square = (\text{singlet}) \oplus (\text{adjoint})$

ICTS

Emergent Local Symmetry

- Collective Action 【JY, 1707.01740】

$$S_{col}[\mathbf{\Psi}] = \frac{N}{2}\underbrace{\mathrm{Tr}\,\left[\cancel{-D\circledast\mathbf{\Psi}} + \log\mathbf{\Psi}\right]}_{\text{Matrix in the extended bi-local space}} - \frac{NJ^2}{8M}\int d\tau_1 d\tau_2\ \underbrace{\left[\mathrm{tr}\,\left(-\mathbf{\Psi}(\tau_1,\tau_2)\mathbf{\Psi}(\tau_2,\tau_1)\right)\right]}_{\text{M×M Matrix}}^2$$

- At Strong Coupling, neglect Kinetic term

- Emergent Reparametrization

$$\mathbf{\Psi}(\tau_1,\tau_2) \longrightarrow (f'(\tau_1))^{\frac{1}{4}}\,\mathbf{\Psi}(f(\tau_1),f(\tau_2))\,(f'(\tau_2))^{\frac{1}{4}}$$

- Global SO(M) is enhanced to Local SO(M) symmetry

$$\mathbf{\Psi}(\tau_1,\tau_2) \longrightarrow \boldsymbol{g}(\tau_1)\mathbf{\Psi}(\tau_1,\tau_2)\boldsymbol{g}^{-1}(\tau_2)$$

Symmetry Algebra

- Symmetry: Semi-direct product $Diff \ltimes SO(M)$

$$\boldsymbol{\Psi}(\tau_1,\tau_2) \longrightarrow (f'(\tau_1))^{\frac{1}{q}}\, \boldsymbol{g}(\tau_1)\boldsymbol{\Psi}(f(\tau_1), f(\tau_2))\boldsymbol{g}^{-1}(\tau_2)\,(f'(\tau_2))^{\frac{1}{q}}$$

- Symmetry Algebra:

$$[L_n, L_m] = (n-m)L_{m+n}$$

$$[J_n^a, J_m^b] = i f^{abc} J_{m+n}^c$$

$$[L_n, J_m] = -mJ_{m+n}$$

Thank Prithvi
for the proof of this.

✓ Virasoro and Kac-Moody without central extension: $Diff \ltimes \hat{so}(M)$
【JY, 1707.01740】

ICTS

Symmetry Breaking

- Classical Solution: Invariant under Global SO(M)

$$\boldsymbol{\Psi}_{cl}(\tau_1, \tau_2) = \Lambda \frac{\mathrm{sgn}\,(\tau_1 - \tau_2)}{|\tau_1 - \tau_2|^{\frac{2}{q}}} \boldsymbol{I}$$

- Spontaneously broken by Classical Solution

$$\boldsymbol{\Psi}(\tau_1, \tau_2) \longrightarrow (f'(\tau_1))^{\frac{1}{q}} \boldsymbol{g}(\tau_1) \boldsymbol{\Psi}(f(\tau_1), f(\tau_2)) \boldsymbol{g}^{-1}(\tau_2) \, (f'(\tau_2))^{\frac{1}{q}}$$

- Explicitly Broken by Kinetic term

$$S_{col}[\boldsymbol{\Psi}] = \frac{N}{2} \mathrm{Tr}\left[\boxed{-D \circledast \boldsymbol{\Psi}} + \log \boldsymbol{\Psi}\right] - \frac{NJ^2}{8M} \int d\tau_1 d\tau_2 \, \left[\mathrm{tr}\,(-\boldsymbol{\Psi}(\tau_1, \tau_2) \boldsymbol{\Psi}(\tau_2, \tau_1))\right]^2$$

- Pseudo-Nambu-Goldstone Boson

ICTS

Effective Action

✳ Substitute transformed Classical Solution to Kinetic term

【JY, 1707.01740】

$$\hookrightarrow (f'(\tau_1))^{\frac{1}{q}}\, \boldsymbol{g}(\tau_1)\boldsymbol{\Psi}(f(\tau_1), f(\tau_2))\boldsymbol{g}^{-1}(\tau_2)\, (f'(\tau_2))^{\frac{1}{q}}$$

$$S_{\text{eff}} = \underbrace{-\frac{MN\alpha_{\text{diffeo}}}{J}\int d\tau\ \text{Sch}(f,\tau)}_{❶} \underbrace{-\frac{N\alpha_{\text{SO(M)}}}{J}\int d\tau\ \frac{1}{2k^2}\text{tr}\ [\boldsymbol{J}(\tau)\boldsymbol{J}(\tau)]}_{❷}$$

$$\boldsymbol{J}(\tau) = -k\partial_\tau \boldsymbol{g}(\tau)\cdot \boldsymbol{g}^{-1}(\tau)$$

- ❶ Schwarzian Action: Reparametriztion
  - ✓ Invariant under SL(2,R)
- ❷ A particle on Group Manifold: Local SO(M) symmetry
  - ✓ Invariant under global SO(M)

  cf 【Witten, Stanford, 1703.04612】
- Coadjoint Orbit 【Wadia, Manda, Nayak, 1702.04266】 【Witten, Stanford, 1703.04612】

Contribution of Effective Action

* Infinitesimal Fluctuation: $f(\tau)=\tau+\epsilon(\tau)$ $\quad \boldsymbol{g}=\boldsymbol{I}+i\boldsymbol{\rho}(\tau)$

  Reparametrization $\quad$ Local so(M)

* Correlation functions (from effective action):

$$\langle \epsilon_n \epsilon_{-n}\rangle = \frac{\beta J}{MN\alpha_{\text{diffeo}}}\frac{1}{n^2(n^2-1)} \qquad \langle \rho_n^a \rho_{-n}^b\rangle = \delta^{ab}\frac{\beta J}{N\alpha_{\text{SO(M)}}}\frac{1}{n^2}$$

* Contribution to four point function: $\langle \boldsymbol{\Psi}(\tau_1,\tau_2)\boldsymbol{\Psi}(\tau_3,\tau_4)\rangle = \boldsymbol{\Psi}_{cl}(\tau_1,\tau_2)\boldsymbol{\Psi}_{cl}(\tau_3,\tau_4)+\mathcal{O}(\frac{1}{N})$

  【JY, 1707.01740】

$$\square \otimes \square = (\text{singlet}) \oplus (\text{adjoint})$$

  ✓ Singlet Channel: only $\boldsymbol{\epsilon}$ gives a contribution

$$\frac{1}{M}\langle \epsilon_n \epsilon_{-n}\rangle \,\text{tr}\,(\delta_{\epsilon_n}\boldsymbol{\Psi}_{cl}(\theta_1,\theta_2))\,\text{tr}\,\left(\delta_{\epsilon_{-n}}\boldsymbol{\Psi}_{cl}(\theta_3,\theta_4)\right)$$

$$\frac{1}{M}\langle \rho_n \rho_{-n}\rangle \,\text{tr}\,(\delta_{\rho_n}\boldsymbol{\Psi}_{cl}(\theta_1,\theta_2))\,\text{tr}\,\left(\delta_{\rho_{-n}}\boldsymbol{\Psi}_{cl}(\theta_3,\theta_4)\right)=0 \qquad \delta_{\rho_n}\boldsymbol{\Psi}_{cl}\ :\text{traceless}$$

  ✓ Adjoint Channel: only $\boldsymbol{\rho}$ gives a contribution

$$\frac{1}{2\text{g}}\langle \epsilon_n \epsilon_{-n}\rangle \,\text{tr}\,(\delta_{\epsilon_n}\boldsymbol{\Psi}_{cl}(\theta_1,\theta_2)\boldsymbol{T}^a)\,\text{tr}\,(\delta_{\epsilon_{-n}}\boldsymbol{\Psi}_{cl}(\theta_3,\theta_4)\boldsymbol{T}^b)=0 \qquad \delta_{\epsilon_n}\boldsymbol{\Psi}_{cl}\sim\boldsymbol{I}$$

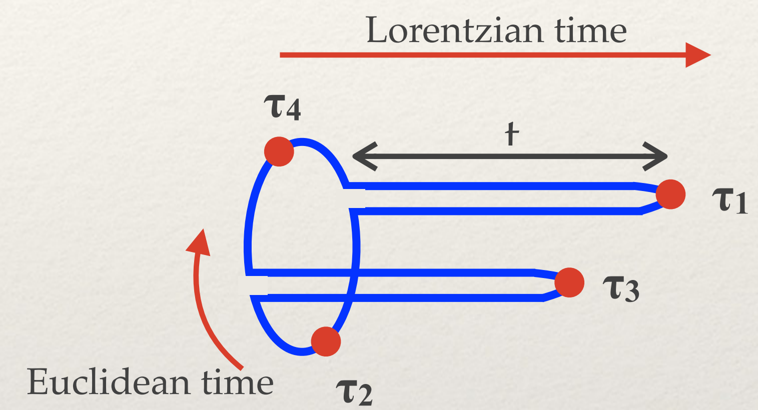
$$\frac{1}{2\text{g}}\langle \rho_n \rho_{-n}\rangle \,\text{tr}\,(\delta_{\rho_n}\boldsymbol{\Psi}_{cl}(\theta_1,\theta_2)\boldsymbol{T}^a)\,\text{tr}\,(\delta_{\rho_{-n}}\boldsymbol{\Psi}_{cl}(\theta_3,\theta_4)\boldsymbol{T}^b)$$

ICTS

Summary of Four point Function

- Out-of-time-ordered Correlators: Analytic Continuation

$$\mathcal{F}(\tau_1, \tau_2, \tau_3, \tau_4)$$



- Singlet: Maximally chaotic

$$\mathcal{F}_{\text{signlet}}(t) \sim e^{\frac{2\pi}{\beta}t}$$

  - ✓ Contribution from reparametrization Goldstone boson

- Adjoint: No Exponential Growth

【JY, 1707.01740】

$$\mathcal{F}_{\text{adjoint}}(t) \sim t$$

  - ✓ Contribution from so(M) Goldstone boson

I have a SUSY, I have an SYK

Uh, SUSY-SYK!

$\mathcal{N}=1$ SUSY SYK Model

【Maldacena, Sachdev, Gaiotto, Fu, 1706.08917】

- Lagrangian for $\mathcal{N}=1$ SUSY SYK Model

$$\mathcal{L} = \sum_i \left[\frac{1}{2}\chi^i \partial \chi^i - \frac{1}{2} b^i b^i + i \sum_{1 \leqq j < k \leqq N} C_{ijk} b^i \chi^j \chi^k \right] \qquad (i = 1, 2, \cdots, N)$$

  - ✓ (Gaussian) Random coupling constant : C_{ijk}
  - ✓ Non-dynamical boson : $b^i(\tau)$
  - ✓ Supercharge : $Q = iC_{ijk}\chi^i\chi^j\chi^k$

- In superspace,

Superderivative

$$\mathcal{L} = \int d\theta \left[\frac{1}{2}\psi^i D_\theta \psi^i - iC_{ijk}\psi^i\psi^j\psi^k \right]$$

  - ✓ Superfield : $\psi^i(\tau, \theta) = \chi^i(\tau) + \theta b^i(\tau)$

Bi-local Superspace

- O(N) Invariant Bi-local superfield:

$$\Psi = \frac{1}{N}\sum_{i=1}^{N}\psi^i(\tau_1,\theta_1)\psi^i(\tau_2,\theta_2)$$

- Extend to Bi-local Superspace: 【JY, 1706.05914】

$$(\tau_1,\tau_2) \longrightarrow (\tau_1,\theta_1;\tau_2,\theta_2)$$

- General Bi-local Superfields:

$$A(\tau_1,\tau_2) \longrightarrow A(\tau_1,\theta_1;\tau_2,\theta_2)$$

  - ✓ Could be Grassmannian odd or Even

ICTS

Bi-local Superfield

- Bi-local superfield in components: 【JY, 1706.05914】

$$A(\tau_1,\theta_1;\tau_2,\theta_2) = A_0(\tau_1,\tau_2) + \theta_1 A_1(\tau_1,\tau_2) - A_2(\tau_1,\tau_2)\theta_2 - \theta_1 A_3(\tau_1,\tau_2)\theta_2$$

  - ✓ A_0, A_1, A_2, A_3 could be Grassmannian Odd/Even
  - ✓ Specific ordering and sign
  - ✓ Our "convention":
    - $A(\tau_1,\theta_1;\tau_2,\theta_2)$: Odd if $A_1(\tau_1,\tau_2)$, $A_2(\tau_1,\tau_2)$ are odd (A_0, A_3: even)
    - $A(\tau_1,\theta_1;\tau_2,\theta_2)$: Even if $A_1(\tau_1,\tau_2)$, $A_2(\tau_1,\tau_2)$ are even (A_0, A_3: odd)

ICTS

Star Product

- Star product in bi-local superspace

$$(A \star B)(\tau_1, \tau_2) = \int d\tau_3 A(\tau_1, \tau_3) B(\tau_3, \tau_2) \quad \text{: non-supersymmetric}$$

$$\longrightarrow \quad (A \circledast B)(\tau_1, \theta_1; \tau_2, \theta_2) = \int A(\tau_1, \theta_1; \tau_3, \theta_3) d\tau_3 d\theta_3 B(\tau_3, \theta_3; \tau_2, \theta_2)$$

  - Measure is Grassmannian odd
  - Position of measure

- Example: $(A^- \circledast B^-)(\tau_1, \theta_1; \tau_2, \theta_2)$

$$= \cdots\cdot$$

$$= (A_0^+ \star B_1^- + A_2^- \star B_0^+) + \theta_1 (A_1^- \star B_1^- + A_3^+ \star B_0^+) - (A_0^+ \star B_3^+ + A_2^- \star B_2^-)\theta_2 \; -\theta_1 (A_1^- \star B_3^+ + A_3^+ \star B_2^-)\theta_2$$

ICTS

Supermatrix Formulation

✴ Supermatrix : $A(\tau_1,\theta_1;\tau_2,\theta_2)=A_0(\tau_1,\tau_2)+\theta_1A_1(\tau_1,\tau_2)-A_2(\tau_1,\tau_2)\theta_2-\theta_1A_3(\tau_1,\tau_2)\theta_2$

$$\longrightarrow \quad A=\begin{pmatrix} A_1 & A_3 \\ A_0 & A_2 \end{pmatrix}$$ 【JY, 1706.05914】

✓ Specific position of components: Drastically simplify star product

✓ Supermatrix is even/odd if A_1 is even/odd.

- e.g. $\underbrace{A(\tau_1,\theta_1;\tau_2,\theta_2)}_{\text{odd}}=\underbrace{A_0(\tau_1,\tau_2)}_{\text{even}}+\theta_1\underbrace{A_1(\tau_1,\tau_2)}_{\text{odd}}-\underbrace{A_2(\tau_1,\tau_2)}_{\text{odd}}\theta_2-\theta_1\underbrace{A_3(\tau_1,\tau_2)}_{\text{even}}\theta_2$

$$\longrightarrow \begin{pmatrix} \text{odd} & \text{even} \\ \text{even} & \text{odd} \end{pmatrix}$$

- Bi-local Superfield $\Psi(\tau_1,\theta_1;\tau_2,\theta_2)$: Grassmannian Odd Supermatrix

$$\Psi=\frac{1}{N}\sum_{i=1}^{N}\psi^i(\tau_1,\theta_1)\psi^i(\tau_2,\theta_2)$$

ICTS

Supermatrix Formulation

- Star(matrix) product for supermatrix 【JY, 1706.05914】

$$A \circledast B = \begin{pmatrix} A_1 & A_3 \\ A_0 & A_2 \end{pmatrix} \circledast \begin{pmatrix} B_1 & B_3 \\ B_0 & B_2 \end{pmatrix} = \begin{pmatrix} A_1 \star B_1 + A_3 \star B_0 & A_1 \star B_3 + A_3 \star B_2 \\ A_0 \star B_1 + A_2 \star B_0 & A_0 \star B_3 + A_2 \star B_2 \end{pmatrix}$$

  - ✓ Usual Matrix multiplication with non-SUSY star product

$$(A \star B)(\tau_1, \tau_2) = \int d\tau_3 \; A(\tau_1, \tau_3) B(\tau_3, \tau_2)$$

- All supermatrix notations appear naturally in SUSY SYK Model
  - ✓ Identity: $(\theta_1 - \theta_2)\delta(\tau_1 - \tau_2) \longrightarrow \begin{pmatrix} \delta(\tau_1 - \tau_2) & 0 \\ 0 & \delta(\tau_1 - \tau_2) \end{pmatrix}$
  - ✓ Supertrace: $\mathrm{str} A = \mathrm{tr}\, A_1 - (-1)^{|A|} \mathrm{tr}\, A_2$ $\qquad$ $|A| = 0$ for even supermatrix, $|A| = 1$ for odd supermatrix
  - ✓ Superderivative, Supertranspose, Berezian, $\cdots\cdots\cdots$

ICTS

Jacobian

- Jacobian: $$\log \mathcal{J} = -\frac{N-1}{2}\,\mathrm{str}\log\Psi$$ 【JY, 1706.05914】

$$\log \mathcal{J} = -\frac{N-1+\delta(0)}{2}\,\mathrm{tr}\;\log\Psi \quad \text{: non-supersymmetric case}$$

- ✓ Appear in transf from superfield $\psi(\tau,\theta)$ to bi-local superfield $\Psi(\tau_1,\theta_1;\tau_2,\theta_2)$
- ✓ supertrace log of supermatrix (or, log of Berezian)
- ✓ No counter term $\delta(0)$: $(\theta-\theta)\delta(\tau-\tau)=0 \longleftarrow \begin{pmatrix} \delta(\tau_{12}) & 0 \\ 0 & \delta(\tau_{12}) \end{pmatrix} = (\theta_1-\theta_2)\delta(\tau_{12})$
  - As usual SUSY model — $\delta(\tau-\tau)$: non-supersymmetric case
- ✓ Shift in N by -1

Digression: Free Theory

- Free SUSY 1d O(N) Vector Model: $S_{\text{free}} = \int d\tau \left[\frac{1}{2}\chi^i \partial \chi^i - \frac{1}{2} b^i b^i\right]$
- Collective Action: $S_{col} = \text{str}\left[\underbrace{-\frac{N}{2}\mathfrak{D} \circledast \Psi}_{\text{kinetic}} + \underbrace{\frac{N-1}{2}\log \Psi}_{\text{Jacobian}}\right]$ 【JY, 1706.05914】
- Large N Classical Solution: Same as Exact Answer

$$\Psi_{cl}(\tau_1,\theta_1;\tau_2,\theta_2) = \frac{1}{2}\left(\text{sgn}\left(\tau_{12}\right) - \theta_1 2\delta(\tau_{12})\theta_2\right) = \begin{pmatrix} 0 & \delta(\tau_{12}) \\ \frac{1}{2}\text{sgn}\left(\tau_{12}\right) & 0 \end{pmatrix}$$

- 1/N Corrections?
  - ✓ tr log Ψ generates infinite vertices (cubic, quartic, $\cdots$)

$$\Psi = \Psi_{cl} + \frac{1}{\sqrt{N}}\Phi \longrightarrow \frac{N-1}{2}\sum_{m=1}^{\infty}\frac{(-1)^{m+1}}{mN^{\frac{m}{2}}}\text{str}\left[\left(\Psi_{cl}\circledast\Phi\right)^{\circledast m}\right]$$

  - ✓ (Completely) cancelled $\frac{1}{2N}\left\langle \Phi(\tau_1,\theta_1;\tau_2,\theta_2)\text{str}\left[-\Psi_{cl}^{-1}\circledast\Phi + \frac{1}{3}\left(\Psi_{cl}^{-1}\circledast\Phi\right)^{\circledast 3}\right]\right\rangle = 0$

ICTS

Repeat the Same Analysis

- Bi-local Collective Action for $\mathcal{N}=1$ SUSY SYK Model

$$S_{col} = \frac{N}{2}\mathrm{str}\left[-\mathfrak{D}\circledast\Psi + \log\Psi - \frac{J}{3}\Psi\circledast[\Psi]^2\right]$$ 【JY, 1706.05914】

- Large N Expansion: Classical Solution, Quadratic action, ⋯
- Emergent Super reparametrization【Maldacena, Sachdev, Gaiotto, Fu, 1706.08917】
  - ✓ Spontaneously and Explicitly broken
  - ✓ Leads to Super Schwarzian Effective Action
- Richer structure in $\mathcal{N}=2$ SUSY SYK Model:
【Maldacena, Sachdev, Gaiotto, Fu, 1706.08917】
  - ✓ Chiral/Anti-chiral bi-local Superfield【JY, 1706.05914】

ICTS

Concluding Remarks

- Essence of SYK(-like) Models
  - ✓ Emergent symmetry
  - ✓ Spontaneous and explicit symmetry breaking
  - ✓ Effective Action
- SYK(-like) Model:
  - ✓ Hope that they play a role of basic building block like harmonic oscillators to understand strongly interacting systems.
- Bi-local Collective field theory with extended bi-local space
  - ✓ can provide a tool to construct generalized models from the basic building block.

We have an SYK.

And, what do you have?