Local quenches and quantum chaos from higher spin perturbations

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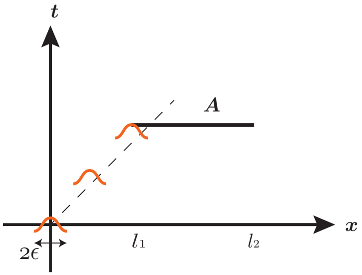
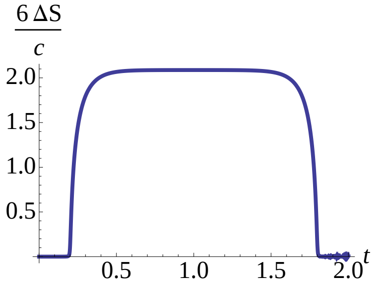
Introduction





- Local quench with temporal width ϵ is inserted in a CFT at origin.

- We show that when $t \sim \frac{l_1+l_2}{2}$ and ϵ is small, the quench correlator can be analytic continued to OTO correlator.

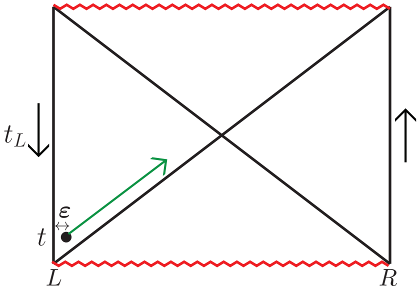
- This enables us to extract the Lyapunov exponent from the quench correlator.

Introduction

- Consider local quench in one CFT of TFD.

- **Mutual information**, $I_{A:B} = S_A + S_B - S_{A\cup B}$.

- The time when due to perturbation, the mutual information vanishes, $I_{A:B}(t^*) = 0$, is the **scrambling time**.



$$t^* = f(L,\beta) + \frac{\beta}{2\pi}\ln\left(\frac{\epsilon S_{\text{density}}}{4\Delta}\right) + O(\epsilon)$$

- We evaluate the correction to scrambling time and Lyapunov exponent, when the infalling massive particle has a spin-3 charge.

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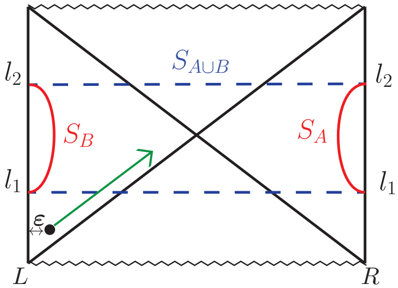

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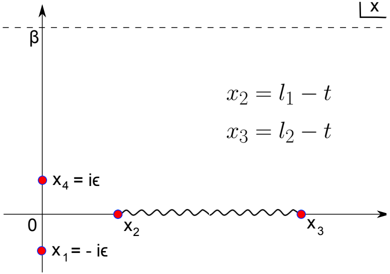
HHLL correlator

- Reduced density matrix:

$$\hat{\rho}_\epsilon = \mathcal{N} e^{-iHt}\left(e^{-\epsilon H}\mathcal{O}(0)e^{\epsilon H}\right)\rho_\beta\left(e^{\epsilon H}\mathcal{O}^\dagger(0)e^{-\epsilon H}\right)e^{iHt}$$
$$= \mathcal{N} e^{-iHt}\mathcal{O}(x_1,\bar{x}_1)\rho_\beta\,\mathcal{O}^\dagger(x_4,\bar{x}_4)\,e^{iHt}$$

- Entanglement entropy,

$$S_A \propto \ln \mathrm{Tr}_{\bar{A}}\,\hat{\rho}_\epsilon = \ln\langle\mathcal{O}^\dagger(x_4)\sigma(x_2)\bar{\sigma}(x_3)\mathcal{O}(x_1)\rangle$$



Holographic dual

- Backreacted geometry is conical defect geometry,

$$ds^2 = -(r^2 + R^2 - m)\, d\tilde{\tau}^2 + \frac{R^2\, dr^2}{r^2 + R^2 - m} + r^2\, d\phi^2\,.$$

$\tanh\lambda = \sqrt{1 - M\epsilon^2}$ and $m = \frac{24\Delta}{c} R^2$. Caputa, et. al. 1410.2287

- The co-ordinates (r, τ, ϕ) are related to BTZ co-ordinates (z, t, x) by following transformation,

$$D_L = \left|\cosh\left(\frac{2\pi}{\beta}\ell\right) - \cosh\left(\frac{2\pi}{\beta}t_L\right)\right|$$

$$e^{\rho_L} = \frac{R\Lambda\beta^2}{4\pi^2\epsilon}\, D_L$$

$$\tan(\tau_L) = \frac{2\pi\epsilon}{\beta}\,\frac{\sinh\left(\frac{2\pi}{\beta}t_L\right)}{D_L}$$

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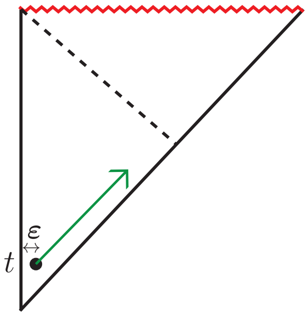


SL(2) connections and WL prescription

The flat $\mathrm{sl}(2,\mathbb{R})$ Chern-Simons connections in the radial gauge,

$$a = \left(L_1 + \frac{\alpha^2}{4} L_{-1}\right)(d\tau + d\phi) \qquad \bar{a} = -\left(L_{-1} + \frac{\alpha^2}{4} L_1\right)(d\tau - d\phi)$$

where $\alpha = \sqrt{1 - \frac{24\Delta_{\mathcal{O}}}{c}}$. Defining matrices,

$$g = \exp\left(a \times (\tau + \phi)\right)\, b(\rho), \qquad \bar{g} = \exp\left(\bar{a} \times (\tau - \phi)\right)\, b^{-1}(\rho), \qquad b(\rho) = e^{\rho L_0}$$

the holomorphic prescription to obtain Wilson line is,

$$W_{\mathrm{fund}}(P, Q) = \mathrm{Tr}_{\mathrm{fund}}\left[\bar{g}^{-1}(P)\, \bar{g}(Q)\, g^{-1}(Q)\, g(P)\right].$$

$$S_A = \frac{c}{6} \ln W_{\mathrm{fund}}(P, Q) = \ln \frac{c}{6}\left(\ln \frac{r_\infty^{(1)} r_\infty^{(2)}}{R^2} + \ln \frac{2\cos\left(\alpha|\Delta\tilde{\tau}_\infty|\right) - 2\cos\left(\alpha|\Delta\phi_\infty|\right)}{\alpha^2}\right)$$

Single interval entanglement entropy

Transformation is substituted to obtain time dependent jump in EE,

$$\frac{6}{c}\,\Delta S_{\mathrm{EE}}\Big|_{\ell_2>t>\ell_1} = \ln\left(1 + 12\left(\frac{\beta E}{\pi\, c}\right)\mathcal{Z}^{-1}_{\ell_1,\ell_2}(t)\right) + O(\epsilon)$$

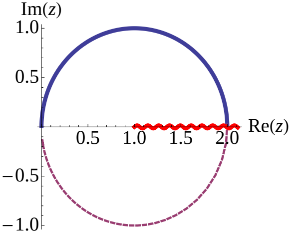
where,

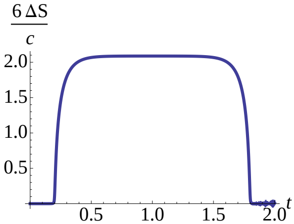
$$\mathcal{Z}_{\ell_1,\ell_2}(t) \equiv \frac{\sinh\frac{\pi}{\beta}(\ell_2-\ell_1)}{\sinh\frac{\pi}{\beta}(t-\ell_1)\,\sinh\frac{\pi}{\beta}(\ell_2-t)}\,.$$

Cross-ratio

Lets consider the cross-ratio,

$$z = \frac{i \sin\left(\frac{2\pi\epsilon}{\beta}\right) \sinh \frac{\pi}{\beta}(\ell_2 - \ell_1)}{\sinh \frac{\pi}{\beta}(\ell_1 - t + i\epsilon) \sinh \frac{\pi}{\beta}(\ell_2 - t - i\epsilon)}$$





When $t \sim (l_1 + l_2)/2$, and ϵ is small, $\Rightarrow |z| << 1$.

To avoid the branch cut when taking the $t >> l_2$ limit,

$$t \to t + i\epsilon_1$$

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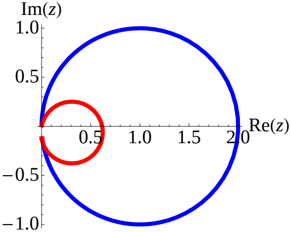

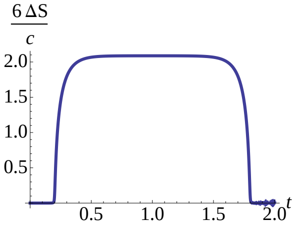

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To avoid the branch cut when taking the $t >> l_2$ limit,

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OTOC

Transformation is substituted to obtain time dependent jump in EE,

$$\frac{6}{c}\,\Delta S_{\rm EE}\Big|_{\ell_2>t>\ell_1} = \ln\left(1\,+\,12\left(\frac{\beta E}{\pi\,c}\right)\,\mathcal{Z}^{-1}_{\ell_1,\ell_2}(t)\right)\,+\,O(\epsilon)$$

where,

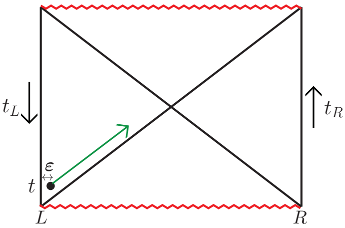
$$\mathcal{Z}_{\ell_1,\ell_2}(t)\;\equiv\;\frac{\sinh\frac{\pi}{\beta}(\ell_2-\ell_1)}{\sinh\frac{\pi}{\beta}(t-\ell_1)\,\sinh\frac{\pi}{\beta}(\ell_2-t)}\,.$$

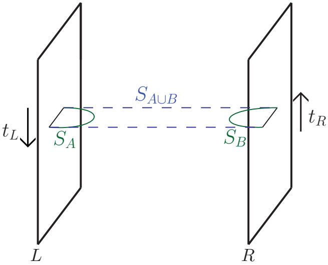
$$W_{\rm fund}\Big|_{\rm OTO}\;\to\;\frac{8(R\Lambda)^2\beta^2}{\pi^2}\,\sinh^2\frac{\pi}{\beta}\,(\ell_2-\ell_1)\,\left(1\,-\,\frac{6\beta E}{\pi\,c}\,e^{\frac{2\pi}{\beta}(t+i\epsilon_1-\ell_2)}\right)\,.$$

Taking $\epsilon_1=\beta/2$ gives the standard OTOC.

Kruskal extension of BTZ

The holographic dual local quench in one CFT of the TFD is the Kruskal extension of BTZ with an infalling particle on one side of the extension.





Evaluate the Mutual information, $I_{A:B} = S_A + S_B - S_{A\cup B}$.

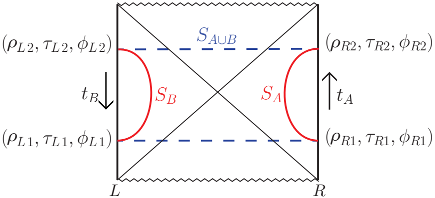
Transformation in two regions

$$D_L = \left|\cosh\left(\tfrac{2\pi}{\beta}\ell\right) - \cosh\left(\tfrac{2\pi}{\beta}t_L\right)\right|$$
$$e^{\rho_L} = \frac{R\Lambda\beta^2}{4\pi^2\epsilon} D_L$$
$$\tan(\tau_L) = \frac{2\pi\epsilon}{\beta}\frac{\sinh\left(\tfrac{2\pi}{\beta}t_L\right)}{D_L}$$
$$\tan(\phi_L) = -\frac{2\pi\epsilon}{\beta}\frac{\sinh\left(\tfrac{2\pi}{\beta}\ell\right)}{D_L}$$

$$D_R = \left|\cosh\left(\tfrac{2\pi}{\beta}\ell\right) + \cosh\left(\tfrac{2\pi}{\beta}t_R\right)\right|$$
$$e^{\rho_R} = \frac{R\Lambda\beta^2}{4\pi^2\epsilon} D_R$$
$$\tan(\tau_R) = -\frac{2\pi\epsilon}{\beta}\frac{\sinh\left(\tfrac{2\pi}{\beta}t_R\right)}{D_R}$$
$$\tan(\phi_R) = -\frac{2\pi\epsilon}{\beta}\frac{\sinh\left(\tfrac{2\pi}{\beta}\ell\right)}{D_R}$$

Related by the replacement, $t_R \to t_L + i\beta/2$.

$$S_{A\cup B} \sim \ln\frac{r_L r_R}{R^2}\,\frac{2\cos(\alpha|\Delta\tilde{\tau}_\infty|) - 2\cos(\alpha|\Delta\phi_\infty|)}{\alpha^2}$$



Scrambling time

Mutual information, $I_{A:B}(t) = S_A + S_B - S_{A\cup B}$, $I_{A:B}(t^*) = 0$
in the limit $t \to \infty$,

$$t^* = \frac{l_1 + l_2}{2} + \frac{\beta}{\pi} \ln\left(2\sinh\frac{\pi(l_2 - l_1)}{\beta}\right) - \frac{\beta}{2\pi}\ln\left(\frac{\beta}{\pi\epsilon}\frac{\sin\pi\sqrt{1-\frac{24\Delta}{c}}}{\sqrt{1-\frac{24\Delta}{c}}}\right)$$

Using $\frac{\Delta}{c} = \frac{E}{\pi c}\epsilon$, where the energy E of the infalling particle is finite,
Caputa, et. al 1503.08161

$$t^* = f(L,\beta) + \frac{\beta}{2\pi}\ln\left(\frac{\pi S_{\text{density}}}{4E}\right)$$

Since, $S_{\rm density} = \frac{\pi c}{3\beta}$ and Lyapunov exponent is read from,

$$t^* \sim \frac{1}{\lambda_L}\log c \quad \Rightarrow \quad \lambda_L = \frac{2\pi}{\beta}.$$

Charged conical defect

- Infalling massive particle has spin-3 charge, q

$$ds^2 = -\,d\tau^2\left(r^2+R^2-m+\frac{q^2}{\left(r+\sqrt{r^2+R^2-m}\right)^4}\right)$$
$$+\,dr^2\frac{R^2}{r^2+R^2-m}+d\phi^2\left(r^2+\frac{q^2}{\left(r+\sqrt{r^2+R^2-m}\right)^4}\right)$$

- The flat $\mathrm{sl}(3,\mathbb{R})$ Chern-Simons connections in the radial gauge,

$$a \;=\; \left(L_1+\frac{\alpha^2}{4}\,L_{-1}\;-\;\frac{\pi\mathcal{W}}{8k_{\mathrm{cs}}}\,W_{-2}\right)d\xi^+\,,$$

$$\bar{a} \;=\; -\left(L_{-1}+\frac{\alpha^2}{4}L_1\;-\;\frac{\pi\overline{\mathcal{W}}}{8k_{\mathrm{cs}}}\,W_2\right)d\xi^-$$
$$\alpha=\sqrt{1-\frac{24\Delta}{c}}$$

Charged CD

$$W_{\mathrm{Ad}}(P,Q) = \frac{2^{14}(R\Lambda)^8\beta^8}{\pi^8}\sinh^8\tfrac{\pi}{\beta}(\ell_2-\ell_1)\times$$
$$(\lambda_1\lambda_2\lambda_3)^{-2}(z\,\bar{z})^{-4}\left[\sum_{j=1}^{3}\frac{1}{\lambda_j}\left(\frac{(1-z)^{i\lambda_j}-1}{(1-z)^{i\lambda_j/2-1}}\right)^2\right]\left[\sum_{j=1}^{3}\frac{1}{\lambda_j}\left(\frac{(1-\bar{z})^{i\lambda_j}-1}{(1-\bar{z})^{i\lambda_j/2-1}}\right)^2\right]$$

- Using the substitution

$$\frac{\Delta_{\mathcal{O}}}{c} = \frac{E}{\pi c}\,\epsilon\,, \qquad \mathcal{W} = \epsilon^2\,\frac{4q}{\pi^2}\,, \qquad \epsilon \ll \beta\,.$$

Single side EE, is

$$\Delta S_{\mathrm{EE}}(PQ) = \frac{c}{24}\left(\ln W_{\mathrm{Ad}} - \ln W_{\mathrm{Ad}}|_{t=0}\right)$$
$$= \frac{c}{24}\,\ln\left[\left(1+\frac{12\beta E}{\pi c}\mathcal{Z}^{-1}_{\ell_1,\ell_2}(t)\right)^4 - \frac{q^2}{c^2}\left(\frac{12\beta}{\pi}\,\mathcal{Z}^{-1}_{\ell_1,\ell_2}(t)\right)^4\right] + O(\epsilon).$$

Bound on charge

Let's consider the following limit of entanglement entropy,

$$\Delta S_{\mathrm{EE}}\Big|_{\ell_2\to\infty,\, t\gg\ell_1} = \frac{c}{24}\,\ln\left[\left(1+\frac{6\beta E}{\pi c}\right)^4 - \frac{q^2}{c^2}\left(\frac{6\beta}{\pi}\right)^4\right]$$

The EE is real and finite when,

$$\sqrt{\frac{|q|}{c}} < \frac{E}{c} + \frac{S_\beta}{2c}\,, \qquad S_\beta = \frac{\pi c}{3\beta}$$

$$\Rightarrow \sqrt{\frac{|\mathcal{W}|}{c}} < \frac{2\Delta_{\mathcal{O}}}{c}$$

For, $l_1 < t < l_2$, using $t \to t + i\epsilon_1$ and then taking $t >> l_2$,

$$W_{\mathrm{Ad}}\Big|_{\mathrm{OTO}} \to \frac{2^{12}(R\Lambda)^8\beta^8}{\pi^8}\,\sinh^8\frac{\pi}{\beta}(\ell_2-\ell_1)$$
$$\left[\left(1 - \frac{2E}{S_\beta}\,e^{\frac{2\pi}{\beta}(t+i\epsilon_1-\ell_2)}\right)^4 - \left(\frac{36q\beta^2}{\pi^2 c}\,e^{\frac{4\pi}{\beta}(t+i\epsilon_1-\ell_2)}\right)^2\right].$$

Scrambling time

We derive scrambling time from Mutual information

$$I_{L:R}(t_L,\, t_R) \,=\, \frac{c}{24}\left(\ln W^{R}_{\rm Ad} \,+\, \ln W^{L}_{\rm Ad} \,-\, \ln W^{LR,1}_{\rm Ad} \,-\, \ln W^{LR,2}_{\rm Ad}\right)$$

where the two sided Wilson line is,

$$\begin{aligned} W^{LR,i}_{\rm Ad} &= \frac{2^{14}(R\Lambda)^8\beta^8}{\pi^8}\, \cosh^8 \tfrac{\pi}{\beta}(t_L - t_R)\times \\ &\left[\left(1 \,-\, \frac{12E\beta}{\pi c}\mathcal{T}_i(t_L,t_R)\right)^4 - \frac{q^2}{c^2}\left(\frac{12\beta}{\pi}\mathcal{T}_i(t_L,t_R)\right)^4\right] \\ \mathcal{T}_i(t_L,t_R) &= \frac{\sinh\tfrac{\pi}{\beta}(\ell_i \,-\, t_L)\,\cosh\tfrac{\pi}{\beta}(\ell_i \,-\, t_R)}{\cosh\tfrac{\pi}{\beta}(t_L \,-\, t_R)}\,. \end{aligned}$$

Scrambling time is,

$$t_* \,=\, \frac{\ell_1 + \ell_2}{2} \,+\, \frac{\beta}{\pi}\ln\left[\sinh\tfrac{\pi}{\beta}(\ell_2 - \ell_1)\right] \,+\, \frac{\beta}{2\pi}\ln\left|\frac{E^4}{S_\beta^4} \,-\, \frac{9q^2\beta^2}{\pi^2 S_\beta^2}\right|^{-\frac{1}{4}}$$

Lyapunov exponent

$$t_* = \frac{\ell_1 + \ell_2}{2} + \frac{\beta}{\pi} \ln\left[\sinh \tfrac{\pi}{\beta}(\ell_2 - \ell_1)\right] + \frac{\beta}{2\pi} \ln\left|\frac{E^4}{S_\beta^4} - \frac{9q^2\beta^2}{\pi^2 S_\beta^2}\right|^{-\frac{1}{4}}$$

The scrambling time increases as q is increased from 0 to critical value $|q| = E^2/c$. It is singular at this value, then decreases.

If energy, E is very small,

$$t_* \simeq \ell_2 + \frac{\beta}{4\pi} \ln\left(\frac{\pi^2 c}{36|q|\beta^2}\right)$$

Define the 'spin-3 scrambling time', $t_3^* = \frac{\beta}{4\pi}\log c$, Perlmutter, 1602.08272

Hence spin-3 Lyapunov exponent is : $\lambda_L^{(3)} = \frac{4\pi}{\beta}$

This is a holographic verification Perlmutter's claim , $\lambda_L^{(N)} = \frac{2\pi}{\beta}(N-1)$,

Conical Defect with HS chemical potential

The flat $\mathrm{sl}(3,\mathbb{R})$ Chern-Simons connections in the radial gauge,

$$a = \left(L_1 - \frac{\pi\mathcal{L}}{2k}L_{-1} - \frac{\pi\mathcal{W}}{8k}W_{-2}\right)d\xi^+$$

$$+ \mu_{\mathcal{O}}\left(W_2 - \frac{\pi\mathcal{L}}{k}W_0 + \frac{\pi^2\mathcal{L}^2}{4k^2}W_{-2} + \frac{\pi\mathcal{W}}{k}L_{-1}\right)d\xi^-$$

where,

$$\mathcal{L} = -\frac{\alpha^2 k}{2\pi}\left(1 - 20\frac{\mu_{\mathcal{O}}^2\,\alpha^2}{3R^2}\right) + \ldots, \qquad \mathcal{W} = \mu_{\mathcal{O}}\frac{4\alpha^4 k}{3\pi R} + \ldots$$

$$\frac{\mu_{\mathcal{O}}}{R} = \epsilon^2\frac{72}{\pi}\frac{q}{c}$$

In CFT, such a state is given by density matrix,

$$\hat{\rho}_\epsilon = \mathcal{N}\,e^{-iHt}\left(\sum_n e^{\mu_{\mathcal{O}}\beta\,\mathcal{W}_{\mathcal{O}_n}}\,\mathcal{O}_n(x_1,\,\bar{x}_1)\,e^{-\beta H}\,\mathcal{O}_n^\dagger(x_4,\,\bar{x}_4)\right)e^{iHt}\,.$$

CFT with Higher Spin symmetry

The bulk background is BTZ with HS chemical potential and infalling massive particle does not have HS charge. Given by following deformation of CFT action,

$$S_{\rm CFT} \,\to\, S_{\rm CFT} \,-\, \int d^2z\, \mu\, W(z) \,+\, {\rm h.c.}$$

In CFT and its thermofield double with large interval length $l >> \beta$,

$$S_{\rm EE}(L \cup R) \,\simeq\, 4 S_\beta\, v\, t\,.$$

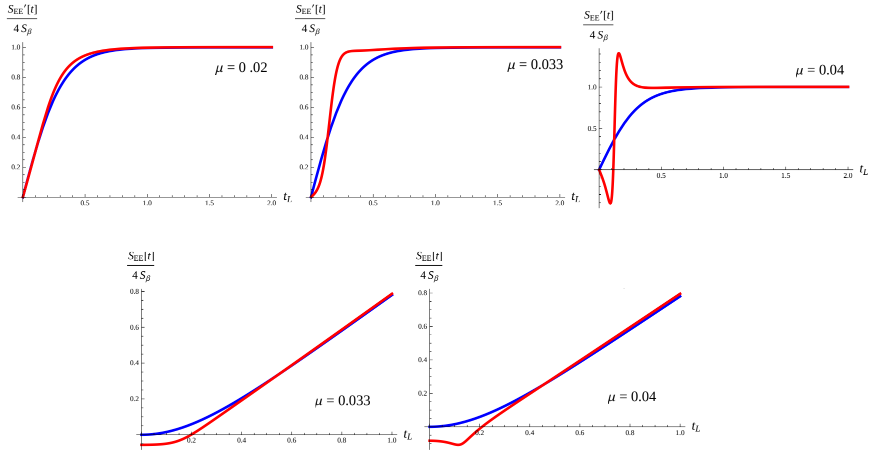
For unperturbed CFT, entanglement speed $v = 1$.

v remains unaffected in presence of HS deformation,

$$S_{\rm EE}(L \cup R) \to 4 \left(\frac{\pi c}{3\beta} + \frac{32 c \pi^3 \mu^2}{9 \beta^3} \right) t \,=\, 4\, S_\beta\, t$$

Bound on μ

$$\frac{\mu_c}{\beta} = \frac{3}{16\pi}\sqrt{2\sqrt{3}-3} \approx 0.04066\,.$$



Scrambling time

CFT evaluation of 6-point correlation function on thermofield double

$$S_{L\cup R} \propto \ln\langle \mathcal{O}^\dagger(x_4)\sigma(x_2)\bar{\sigma}(x_3)\sigma(\hat{x}_2)\bar{\sigma}(\hat{x}_3)\mathcal{O}(x_1)\rangle$$

leads to Scrambling time,

$$t_* = \frac{\ell_2+\ell_1}{2} + \frac{\beta}{2\pi}\frac{6S_{\mathrm{EE}}(\ell)}{c} + \frac{\beta}{2\pi}\left(\ln\left(\frac{S_\beta}{\pi E}\right) + \frac{16\mu^2\pi^2}{3\beta^2} + 2\pi^4\frac{\mu^2}{\beta^2}\right)$$

Lyapunov exponent is same as Einstein gravity.

Summary

- We consider 2d CFT at large-c with local quench by operators carrying higher spin charge.
- We use Wilson line prescription to compute single interval EE following the quench. We find an analytic continuation which takes the quench correlator to the OTO correlator.
- The entanglement entropy is finite and real only if the spin-3 charge satisfies $|q|/c < E^2/c^2$.
- We find correction to scrambling time for two sided mutual information in TFD, when the quench operator has spin-3 charge
- Controlled by spin-3 Lyapunov exponent $4\pi/\beta$.
- We find scrambling time in a CFT with HS charge, where the quench has zero HS charge. Lyapunov exponent is same as Einstein gravity.

Thank You!

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