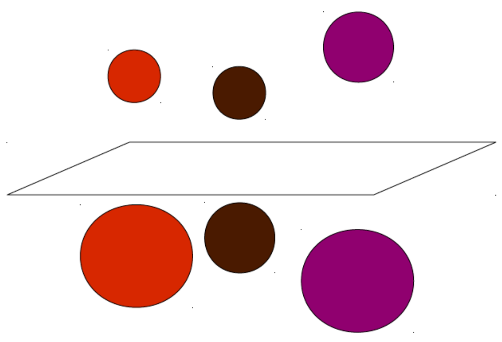
N=2* Yang-Mills on the Lattice

Anosh Joseph
International Centre for Theoretical Sciences
Tata Institute of Fundamental Research (ICTS-TIFR)
Bangalore, INDIA

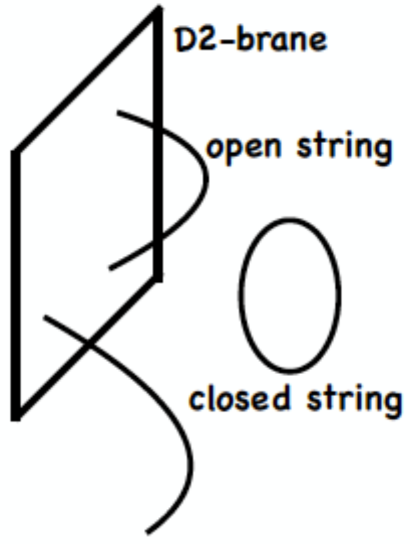
BANGALORE AREA STRINGS MEETING 2017
01 AUG 2017

COLLABORATORS

- Poul Damgaard (DENMARK)
- So Matsuura (JAPAN)

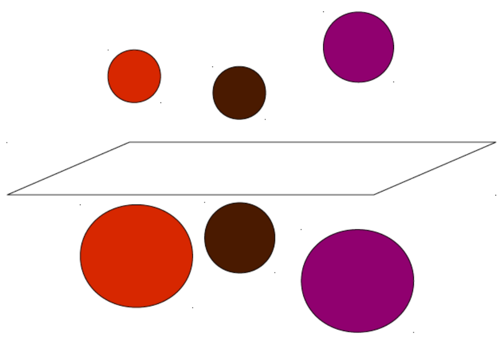


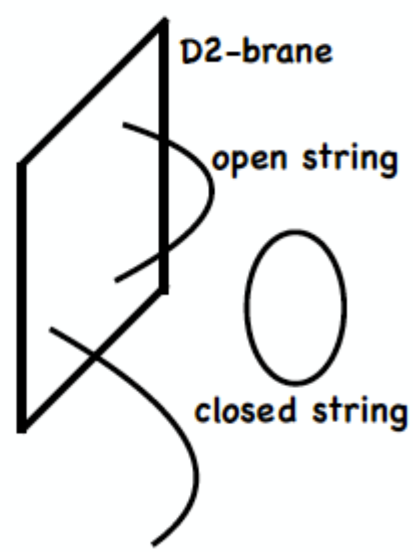
Work in progress!



OUTLINE

- N=4 Supersymmetric Yang-Mills
- Operator Deformation and N=2* SYM
- Gravitational Dual
- Some Results from Literature
- Lattice Formulation
- Conclusions and Future Directions





$\mathcal{N} = 4$ SYM Theory in 4D

A theory with maximal number of supersymmetries

Takes part in the AdS/CFT correspondence

Dual gravitational theory - D3 brane geometry on $AdS_5 \times S^5$

Lower dimensional versions - well explored on the lattice

Catterall, Wiseman (2007-2010), Catterall, Joseph, Wiseman (2010), Hanada, Nishimura et al. (2007-2016), Kadoh, Kamata (2012-2016) …

Excellent validations of gauge-gravity duality on the lattice

Need for testing gauge-gravity duality in higher dimensions

$\mathcal{N} = 4$ SYM Theory in 4D

Field content: A vector multiplet and 3 chiral multiplets

$$V \rightarrow A_\mu,\ \lambda_{4\alpha},\ \overline{\lambda}^{4}{}_{\dot{\alpha}},$$
$$\Phi_s, \Phi^{\dagger s} \rightarrow \phi_s,\ \lambda_{s\alpha},\ \phi^{\dagger s},\ \overline{\lambda}^{s}{}_{\dot{\alpha}}$$

Global symmetry group

$$SU(2)_L \times SU(2)_R \times SU(4)$$

Euclidean Lorentz symmetry

Internal symmetry

$\mathcal{N} = 2^*$ SYM Theory

A mass deformation of $\mathcal{N} = 4$ SYM theory

Introduced by Polchinski and Strassler (hep-th/0003136)

Combine 2 chiral multiplets $\longrightarrow$ $\mathcal{N} = 2$ hypermultiplet

Action

$$S_{\mathcal{N}=2^*} = S_{\mathcal{N}=4} + S_m$$

Mass deformation:

$$\begin{aligned} S_m \; = \; \frac{1}{g^2} \int d^4x \, \mathrm{Tr} \Big(& im\lambda_1{}^{\alpha}\lambda_{2\alpha} - im\overline{\lambda}^1{}_{\dot{\alpha}}\overline{\lambda}^{2\dot{\alpha}} + m^2\phi_1\phi_1^\dagger + m^2\phi_2\phi_2^\dagger \\ & - m\phi_3[\phi_1, \phi_1^\dagger] - m\phi_3[\phi_2, \phi_2^\dagger] - m\phi_3^\dagger[\phi_1, \phi_1^\dagger] - m\phi_3^\dagger[\phi_2, \phi_2^\dagger] \Big) \end{aligned}$$

$\mathcal{N} = 2^*$ SYM Theory

Convenient to express mass deformation using 2 operators

$$\mathcal{O}_2 = \frac{1}{3}\Big(\phi_1\phi_1^\dagger + \phi_2\phi_2^\dagger - 2\phi_3\phi_3^\dagger\Big)$$

dimension 2

$$\begin{aligned}\mathcal{O}_3 \;\; = \;\; & 2\Big(i{\lambda_1}^{\alpha}\lambda_{2\alpha} - i\overline{\lambda}^{1}{}_{\dot\alpha}\overline{\lambda}^{2\dot\alpha} \\ & -\phi_3[\phi_1, \phi_1^\dagger] - \phi_3[\phi_2, \phi_2^\dagger] - \phi_3^\dagger[\phi_1, \phi_1^\dagger] - \phi_3^\dagger[\phi_2, \phi_2^\dagger]\Big) \\ & + \frac{2}{3}m\Big(\phi_1\phi_1^\dagger + \phi_2\phi_2^\dagger + \phi_3\phi_3^\dagger\Big).\end{aligned}$$

dimension 3

Correspond to turning on bosonic and fermionic scalars in dual gravitational theory

$\mathcal{N} = 2^*$ SYM Theory

Action

$$S_{\mathcal{N}=2^*} \quad = \quad S_{\mathcal{N}=4} - \frac{1}{2g^2}\int d^4x \, m^2 \, \mathrm{Tr}\, \mathcal{O}_2 - \frac{1}{2g^2}\int d^4x \, m \, \mathrm{Tr}\, \mathcal{O}_3$$

Soft SUSY breaking

$$m \xrightarrow{m \to 0} \mathcal{N} = 4 \ \mathrm{SYM}$$

$$m \xrightarrow{m \to \infty} \mathcal{N} = 2 \ \mathrm{SYM}$$

Gravitational Dual

Constructed by Pilch and Warner [Nucl. Phys. B594 (2001) 209-228]

A product of deformed AdS_5 and a deformed five-sphere

Supergravity scalars: α and χ

bosonic

$$\alpha \longleftrightarrow \mathcal{O}_2$$

$$\chi \longleftrightarrow \mathcal{O}_3$$

fermionic

Supergravity action

$$I_5 = \frac{1}{4\pi G_5} \int_{\mathcal{M}_5} d\xi^5 \sqrt{-g} \left(\frac{1}{4} R - \mathcal{L}_{\text{matter}} \right)$$

Gravitational Dual

Supergravity action

$$I_5 = \frac{1}{4\pi G_5}\int_{\mathcal{M}_5} d\xi^5 \sqrt{-g}\left(\frac{1}{4}R - \mathcal{L}_{\rm matter}\right)$$

$$\mathcal{L}_{\rm matter} = -3(\partial\alpha)^2 - (\partial\chi)^2 - \mathcal{P}$$

Potential

$$\mathcal{P} = \widehat{g}^2\left(\frac{1}{16}\left[\frac{1}{3}\left(\frac{\partial W}{\partial\alpha}\right)^2 + \left(\frac{\partial W}{\partial\chi}\right)^2\right] - \frac{1}{3}W^2\right)$$

Superpotential

$$W = -e^{2\alpha} - \frac{1}{2}e^{4\alpha}\cosh(2\chi)$$

Some Results from Literature

Finite temperature Pilch-Warner solution

Constructed by Buchel, Peet and Polchinski **[Phys. Rev. D63 (2001) 044009]**

$\mathcal{N} = 2^*$ plasma has been studied

Buchel, Deakin, Kerner and Liu **[Nucl. Phys. B784 (2007) 72]**
Hoyos, Paik and Yaffe **[JHEP 1110 (2011) 062]**
Buchel and Liu **[JHEP 1311 (2013) 031]**

Much easier to implement bosonic deformation

Consistent truncation possible

$$m_f = 0,\ m_b \neq 0 \quad \longleftrightarrow \quad \chi = 0, \alpha \neq 0$$

Some Results from Literature

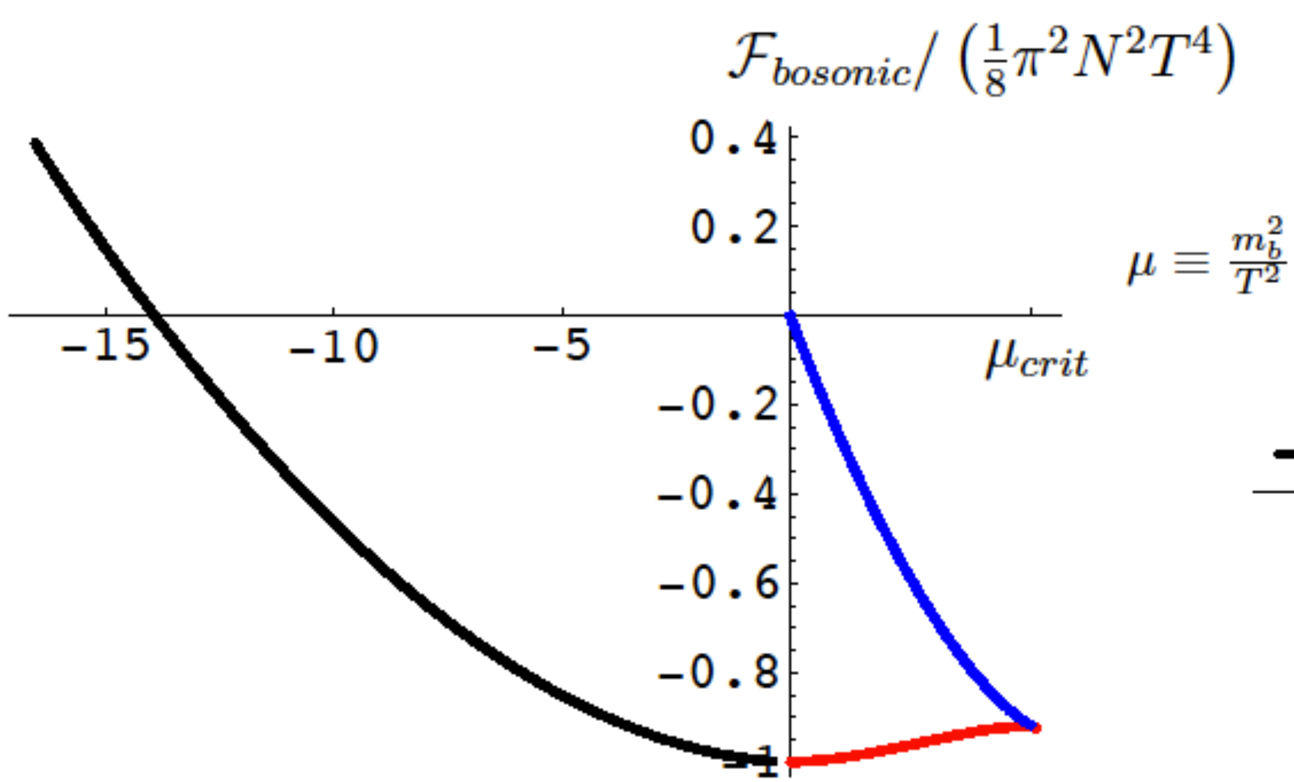
Free energy: $\mathcal{F}_{bosonic}$

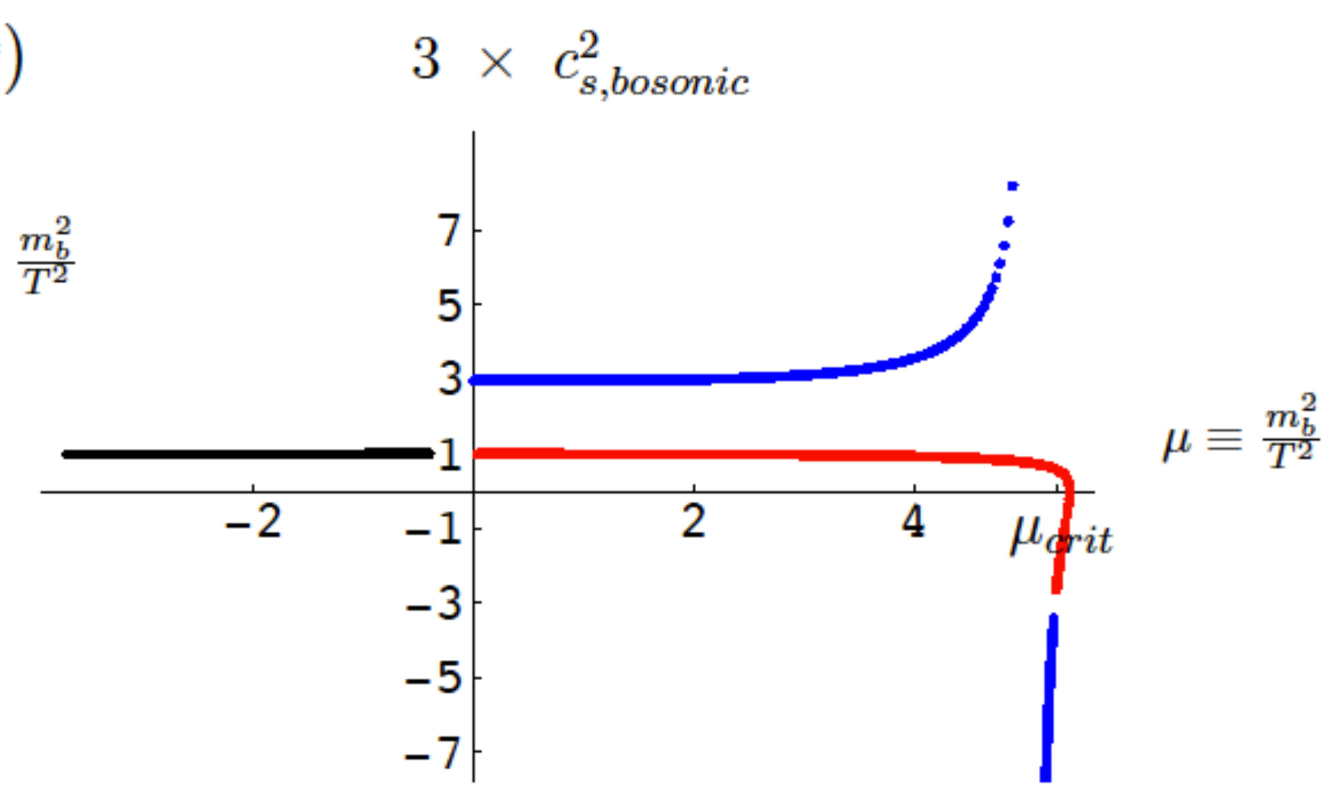
Speed of sound: $c^2_{s,bosonic}$

$$\mathcal{F} = \mathcal{E} - sT$$

$$c_s^2 = -\frac{\partial \mathcal{F}}{\partial \mathcal{E}}$$

$$c^2_{s,\ bosonic,\ red}(\mu = \mu^*) = 0$$





Buchel, Deakin, Kerner and Liu **[Nucl. Phys. B784 (2007) 72]**

Going to the Lattice and Vafa-Witten Twist

Use a twisted version of the theory

$\mathcal{N} = 4$ SYM can be twisted in 3 different ways

1.) Half twist
2.) Vafa-Witten twist
3.) GL twist

Use Vafa-Witten twist [Nucl. Phys. B431 (1994) 3-77]

Global symmetry group

Twisted symmetry group

$$SU(2)_L \times SU(2)_R \times SU(4) \xrightarrow{\text{twist}} SU(2)'_L \times SU(2)_R \times SU(2)_F$$

$$SU(2)'_L = \mathrm{diag}\Big(SU(2)_L \times SU(2)_I\Big)$$

Vafa-Witten Twist

Twisted fields of $\mathcal{N} = 4$ SYM

$$\begin{array}{rcl} A_\mu & \longrightarrow & A_\mu, \\ \lambda_{u\alpha} & \longrightarrow & \eta, \zeta, \chi_{\mu\nu}, \psi_{\mu\nu}, \\ \overline{\lambda}^u{}_{\dot{\alpha}} & \longrightarrow & \psi_\mu, \chi_\mu, \\ \phi_{uv} & \longrightarrow & B_{\mu\nu}, \phi, \overline{\phi}, C \end{array}$$

Twist gives 2 scalar supercharges: Q and $\widetilde{Q}$

$$Q^2 X = 2\sqrt{2}[\phi, X] \qquad \widetilde{Q}^2 X = -2\sqrt{2}[\overline{\phi}, X]$$

Vafa-Witten Twist

Q supersymmetry transformations

$$
\begin{aligned}
QA_\mu &= \psi_\mu, & Q\psi_\mu &= -2\sqrt{2}D_\mu\phi,\\
Q\phi &= 0, & &\\
Q\overline{\phi} &= \sqrt{2}\eta, & Q\eta &= 2[\phi, \overline{\phi}],\\
Q\chi_{\mu\nu} &= 2H_{\mu\nu}, & QH_{\mu\nu} &= \sqrt{2}[\phi, \chi_{\mu\nu}],\\
QC &= \sqrt{2}\zeta, & Q\zeta &= 2[\phi, C],\\
QB_{\mu\nu} &= \sqrt{2}\psi_{\mu\nu}, & Q\psi_{\mu\nu} &= 2[\phi, B_{\mu\nu}],\\
Q\chi_\mu &= 2H_\mu, & QH_\mu &= \sqrt{2}[\phi, \chi_\mu].
\end{aligned}
$$

Similar transformations for $\widetilde{Q}$

Vafa-Witten Twist

Action takes following form

$$S = Q\widetilde{Q}\ \frac{1}{g^2}\int d^4x\ \mathrm{Tr}\ \mathcal{F}.$$

Action potential

$$\mathcal{F} \;=\; \Big(-\frac{1}{2\sqrt{2}}B_{\mu\nu}F_{\mu\nu} - \frac{1}{24\sqrt{2}}B_{\mu\nu}[B_{\mu\rho}, B_{\rho\nu}] - \frac{1}{8}\chi_{\mu\nu}\psi_{\mu\nu} + \frac{1}{8}\psi_\mu\chi_\mu + \frac{1}{8}\eta\zeta\Big)$$

Twisted $\mathcal{N} = 2^*$ SYM Theory

Mass deformation (untwisted theory)

$$S_m = \frac{1}{g^2}\int d^4x \, \mathrm{Tr}\Big(im\lambda_1{}^{\alpha}\lambda_{2\alpha} - im\overline{\lambda}^1{}_{\dot\alpha}\overline{\lambda}^{2\dot\alpha} + m^2\phi_1\phi_1^\dagger + m^2\phi_2\phi_2^\dagger$$
$$-m\phi_3[\phi_1,\phi_1^\dagger] - m\phi_3[\phi_2,\phi_2^\dagger] - m\phi_3^\dagger[\phi_1,\phi_1^\dagger] - m\phi_3^\dagger[\phi_2,\phi_2^\dagger]\Big)$$

Using twisted variables: $S_{\mathcal{N}=2^*} = S_{\mathcal{N}=4} + S_m$

$$\boxed{S_m = -\frac{1}{2g^2}\mathrm{Tr}\int d^4x \left(m^2\,\mathcal{O}_2 + m\,\mathcal{O}_3\right)}$$

$$\mathcal{O}_2 = \frac{1}{3}(B_{\mu\nu}^2 + C^2 + 2\phi\overline{\phi}),$$

bosonic

$$\begin{aligned}\mathcal{O}_3 = {} & -4i(\psi_{12}\psi_{23} + \psi_{13}\zeta + \chi_1\chi_2 + \chi_3\chi_4) \\ & +2i(\phi[B_{12},B_{23}] - \overline{\phi}[B_{12},B_{23}] + \phi[B_{13},C] - \overline{\phi}[B_{13},C]) \\ & -(\phi[B_{\mu\nu},B_{\mu\nu}] + \overline{\phi}[B_{\mu\nu},B_{\mu\nu}] + \phi[C,C] + \overline{\phi}[C,C]) \\ & +\frac{2}{3}m(B_{\mu\nu}^2 + C^2 - \phi\overline{\phi}).\end{aligned}$$

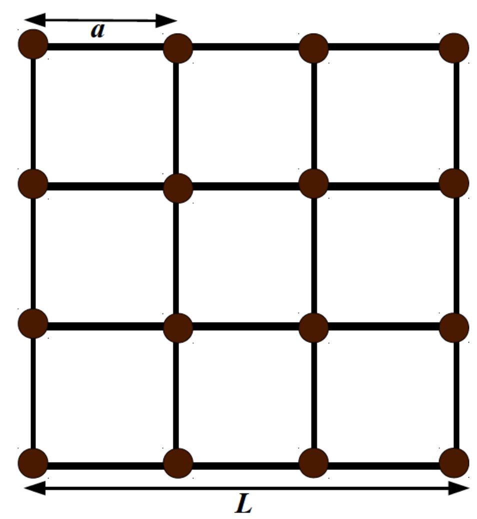
fermionic

Lattice Formulation

Use discretization prescription by Sugino F. Sugino [JHEP 0401 (2004) 015]

Gauge field on links

$$U_\mu(\mathbf{n}) = e^{A_\mu(\mathbf{n})}$$



All other fields on sites

Makes lattice theory local, gauge invariant and doubler free

Appropriate for lattice: Balanced Topological Field Theory Form (BTFT) of action

R. Dijkgraff and G. Moore [Commun.Math.Phys. 185 (1997) 411-440]

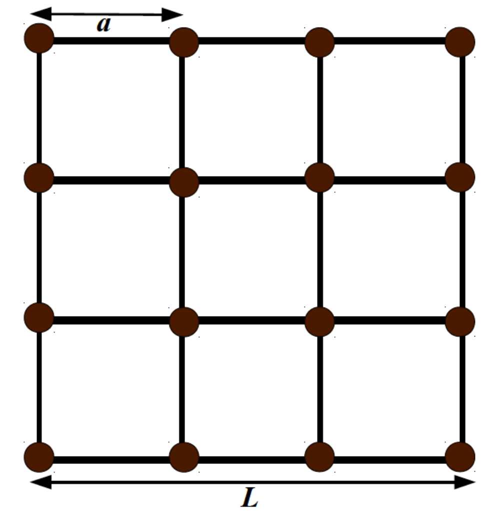
Field strength on lattice - functional of link fields

$$\Phi_A = -\Big(U_{A4}(\mathbf{n}) - U_{4A}(\mathbf{n}) + \frac{1}{2}\sum_{B,C=1}^{3} \epsilon_{ABC}(U_{BC}(\mathbf{n}) - U_{CB}(\mathbf{n}))\Big)$$

Lattice Formulation

Straightforward to write down lattice action F. Sugino [JHEP 0401 (2004) 015]

Issue with vacuum degeneracy - need to be resolved



Twisted theory gauge action:

$$\frac{1}{2g_0^2}\sum_{\mathbf{n}}\sum_{\mu<\nu}\mathrm{Tr}\left[-\left(U_{\mu\nu}(\mathbf{n})-U_{\nu\mu}(\mathbf{n})\right)^2\right]$$

Standard Wilson:

$$\frac{1}{2g_0^2}\sum_{\mathbf{n}}\sum_{\mu<\nu}\mathrm{Tr}\left[2-U_{\mu\nu}(\mathbf{n})-U_{\nu\mu}(\mathbf{n})\right]$$

$$U_{\mu\nu} = \mathrm{diag}(\pm 1, \cdots, \pm 1)$$

many classical vacua

$$U_{\mu\nu} = 1$$

unique minimum

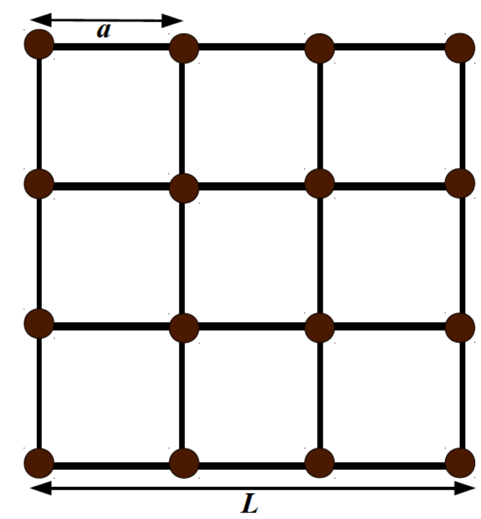
Could resolve by adding standard Wilson term - softly breaks SUSY

Lattice Formulation

Goal - investigate finite temperature theory

With bosonic deformation

Lattice action



$$S^{L}_{\mathcal{N}=2^*\ \text{bosonic}} = \beta_L Q\widetilde{Q} \sum_{\mathbf{n}} \mathrm{Tr}\ \mathcal{F}(\mathbf{n}) + \beta_L m_L^2 \sum_{\mathbf{n}} \mathrm{Tr}\ \mathcal{O}_2(\mathbf{n})$$

$$\mathcal{O}_2(\mathbf{n}) = \frac{1}{3}\Big(B^2_{\mu\nu}(\mathbf{n}) + C^2(\mathbf{n}) + 2\phi(\mathbf{n})\overline{\phi}(\mathbf{n})\Big)$$

Local, doubler free, gauge invariant, twisted Lorentz invariant

Gravitational dual has been investigated

Supergravity action

$$I_5 = \frac{1}{4\pi G_5}\int_{\mathcal{M}_5} d\xi^5 \sqrt{-g}\left(\frac{1}{4}R - \mathcal{L}_{\text{matter}}\right)$$

with α turned on

Conclusions and Future Directions

- Presented lattice formulation of N=2* super Yang-Mills
  - Bosonic deformation much easier to implement
  - Could simulate thermodynamics
  - Could validate gauge-gravity duality in non-conformal setting
- Need to enumerate symmetries of lattice theory
- Study renormalization and fine tuning
- Study finite temperature properties
  - First law of thermodynamics in AdS/CFT setting
  - Does the speed of sound vanish? $c_s^2(\mu = \mu^*) = 0, \quad \mu \equiv \frac{m_b^2}{T^2}$
  - Thermal phases in the theory
- Sign problem?
- Dynamical/thermodynamic stability?

THANK YOU!