# SYK like Tensor models on Lattice

with J Yoon [1705.01554]

Bangalore Area Strings Meeting, ICTS

# Overview

# Introduction & Motivation

## SYK Model Sachddev-Ye 1993, Kitaev 2015, Sachdev 2015

- SYK : Quantum Mechanics (with disorder) of $N$ Fermions.
- solvable at large $N$, strong coupling - emergent conformal symmetry for $2, 4$ point functions. Can compute correlators exactly.
- saturates the chaos bound - hint of a gravity dual Refs..

## Tensor Models

- Is disorder fundamental to SYK like physics? No!
- Tensor models - generalizations of Matrix, Vector models. Connection to SYK noticed only recently Witten, Klebanov Tarnopolsky, Gurau 2016 .
- Leading $N$ diagrams are similar to SYK $\rightarrow$ (almost) same physics as SYK at large $N$.

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# Summary of Results

- SYK has been generalized in various directions by now. In particular there are Lattice versions of SYK ( Gu, Qi, Stanford 2016 ).
- We will consider Tensor models on lattice.

## Results

- We study of Tensor models on Lattice - very general formalism.
- Find the saturation of chaos bound. Also find the butterfly velocity. Same as Gu, Qi, Stanford 2016 .
- *Coloured* tensor models (eg Witten 's model) - efficient to think like Klebanov Tarnopolsky models on lattice.
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# Klebanov Tarnopolsky (KT) model

## Hamiltonian

$$H_{\text{KT}} = \frac{1}{4} J N^{-\frac{3}{2}} \; \psi_{i_1 j_1 k_1} \psi_{i_1 j_2 k_2} \psi_{i_2 j_1 k_2} \psi_{i_2 j_2 k_1}$$

$\psi_{ijk}$ : Majorana Fermions

Tetrahedron Interaction



$O(N)^3$ symmetry : $\psi_{ijk} \to \Lambda_i^{\ \tilde{i}} \psi_{\tilde{i}jk}$ and so on.. Will work in large $N$, fixed $J$. Small $N$ studied Krishnan et al

Diagrammatics :




(a) Propagator (b) Vertex

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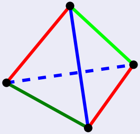

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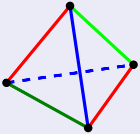

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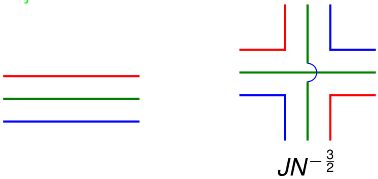


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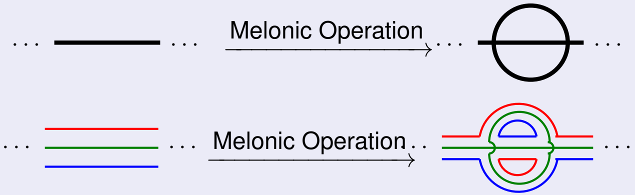
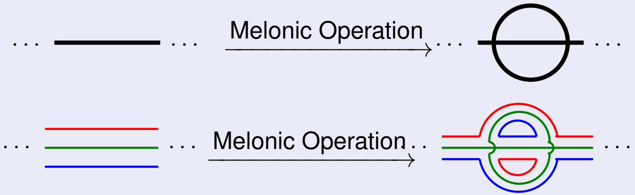



Two Vertices $\sim \frac{1}{N^3}$ , 3 loops $\sim N^3$. Net $N^0$

## Two Point function

Leading diagrams in large $N$: Free propagator + Melonic descendants

# KT Tensor Model : Diagrammatics

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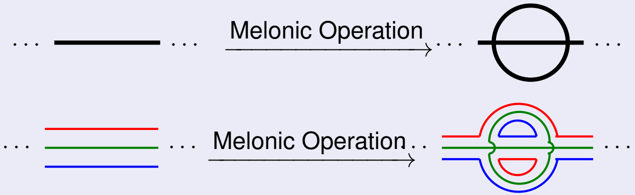


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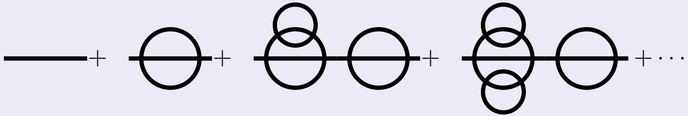

# KT Tensor Model : Two point function

Two Point Function : $G(\tau_1, \tau_2) \equiv \frac{1}{N^3}\langle \psi_{ijk}(\tau_1)\psi_{ijk}(\tau_2)\rangle$

## SD Equation for Two Point Function




$$G(\tau_1, \tau_2) = G_0(\tau_1, \tau_2) + J^2 \int d\tau_3 d\tau_4 G_0(\tau_1, \tau_3)[G(\tau_3, \tau_4)]^3 G(\tau_4, \tau_2)$$

In strong coupling limit $|J\tau_{12}| \gg 1$ : $G(\tau_1, \tau_2) \sim \frac{sgn(\tau_{12})}{\sqrt{|J\tau_{12}|}}$

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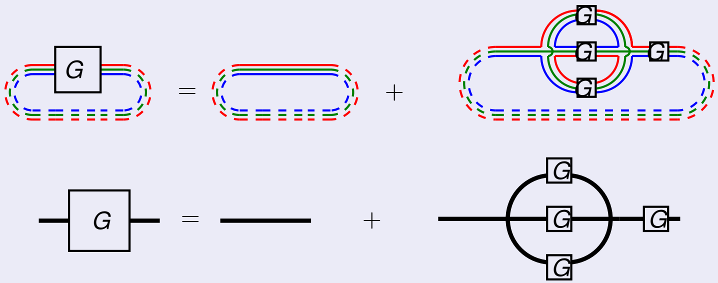


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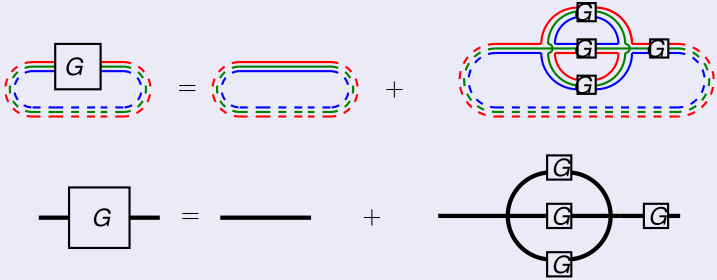


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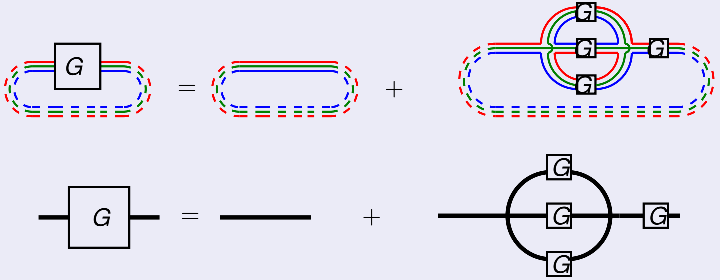


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$$F^C(\tau_1, \tau_2, \tau_3, \tau_4) = \langle \psi_{i_1 j_1 k_1}(\tau_1)\psi_{i_1 j_1 k_1}(\tau_2)\psi_{i_2 j_2 k_2}(\tau_3)\psi_{i_2 j_2 k_2}(\tau_4)\rangle$$

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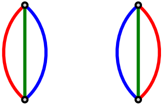

SD

Equation for connected part of 4 point function

$$\mathcal{F}^C(\tau_1, \tau_2, \tau_3, \tau_4) = \int d\tau d\tau' \; \mathcal{K}(\tau_1, \tau_2, \tau, \tau')\mathcal{F}^C(\tau, \tau', \tau_3, \tau_4)$$

where $\mathcal{K}(\tau_1, \tau_2, \tau_3, \tau_4) \equiv -3J^2 G(\tau_{13})G(\tau_{24})[G(\tau_{34})]^2$

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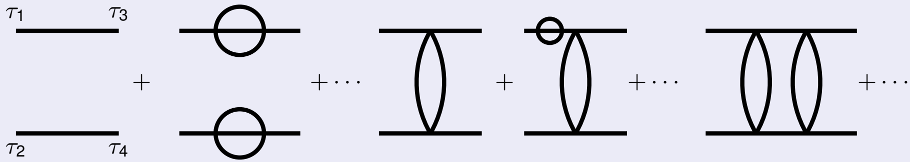
Leading $N$ connected diagrams are Ladder diagrams + melonic descendants :

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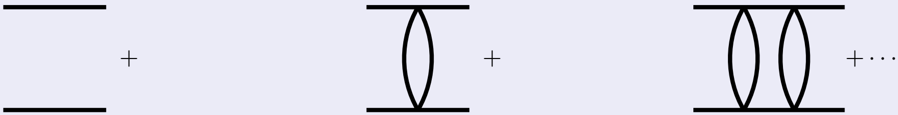
Leading $N$ connected diagrams are Ladder diagrams with full 2pt function :



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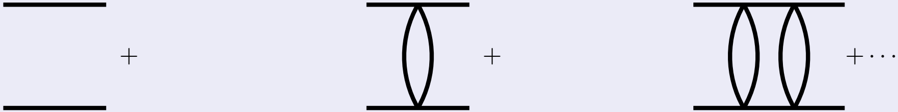

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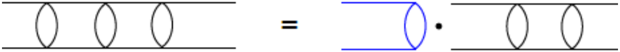

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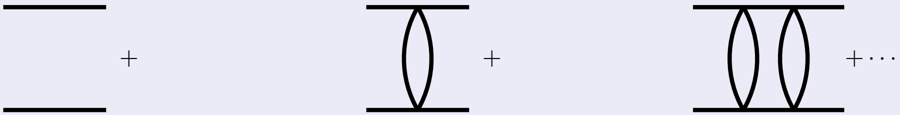

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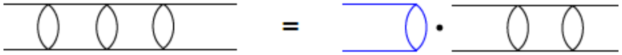

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# KT Tensor Model : Chaos

**Diagnostic of Chaos**

$$F^C(t) \equiv \mathrm{Tr}(e^{\frac{-\beta H}{4}} \psi_{i_1 j_1 k_1}(t) e^{\frac{-\beta H}{4}} \psi_{i_2 j_2 k_2}(0) e^{\frac{-\beta H}{4}} \psi_{i_1 j_1 k_1}(t) e^{\frac{-\beta H}{4}} \psi_{i_2 j_2 k_2}(0))$$

Via an analytic continuation of Euclidean 4 point function

For KT Tensor models,

$$F^C(t) = F^C_d - e^{\lambda_L (t - t_*)}$$

with $t_*/\beta \sim \log \frac{N^3}{\beta J}$.

$$\lambda_L = \frac{2\pi}{\beta}$$

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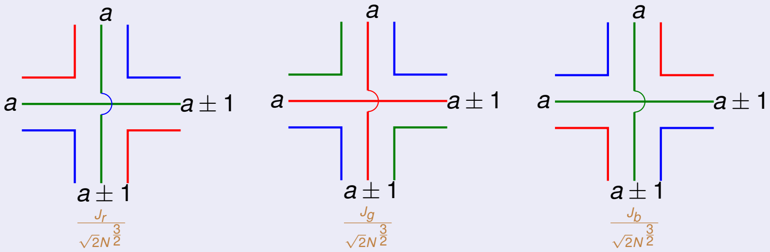
# KT Chain Tensor Model : Setup

$\psi_{ijk} \to \psi^a_{ijk}$.

## KT Model on a Lattice : Hamiltonian

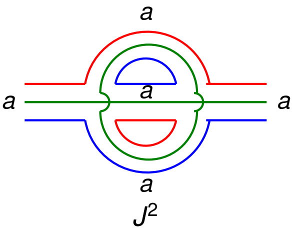
$$H = \frac{JN^{-\frac{3}{2}}}{4}\sum_{a=1}^{L}\psi^a_{i_1j_1k_1}\psi^a_{i_1j_2k_2}\psi^a_{i_2j_1k_2}\psi^a_{i_2j_2k_1} + \frac{J_rN^{-\frac{3}{2}}}{2\sqrt{2}}\sum_{a=1}^{L}\psi^a_{i_1j_1k_1}\psi^a_{i_1j_2k_2}\psi^{a+1}_{i_2j_1k_2}\psi^{a+1}_{i_2j_2k_1}$$
$$+ \frac{J_gN^{-\frac{3}{2}}}{2\sqrt{2}}\sum_{a=1}^{L}\psi^a_{i_1j_1k_1}\psi^{a+1}_{i_1j_2k_2}\psi^a_{i_2j_1k_2}\psi^{a+1}_{i_2j_2k_1} + \frac{J_bN^{-\frac{3}{2}}}{2\sqrt{2}}\sum_{a=1}^{L}\psi^a_{i_1j_1k_1}\psi^{a+1}_{i_1j_2k_2}\psi^{a+1}_{i_2j_1k_2}\psi^a_{i_2j_2k_1}$$

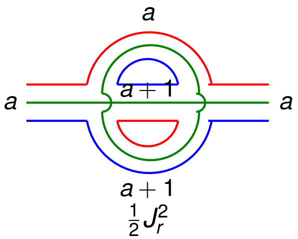



Studied at small $N$ Chaudhuri et al .

# KT Chain Tensor Model Two Point Function

Some contributions to Two Point function :




SD Equation is same as KT model effective coupling $\mathcal{J}$ : $\mathcal{J}^2 \equiv J^2 + J_r^2 + J_g^2 + J_b^2$

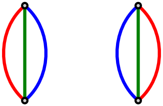
$$G(\tau_1, \tau_2) \sim \frac{sgn(\tau_{12})}{\sqrt{|\mathcal{J}\tau_{12}|}}$$

# KT Chain Tensor Model Four Point Function

## Cooper Contraction

$$F^C_{a_1 a_2} \equiv \langle \psi^{a_1}_{i_1 j_1 k_1}(\tau_1)\psi^{a_1}_{i_1 j_1 k_1}(\tau_2)\psi^{a_2}_{i_2 j_2 k_2}(\tau_3)\psi^{a_2}_{i_2 j_2 k_2}(\tau_4)\rangle$$

Connected four point function : $F^C_{ab}(\tau_1, \tau_2, \tau_3, \tau_4) = N^6 G(\tau_{12})G(\tau_{34}) + N^3 \mathcal{F}^C_{ab}(\tau_1, \tau_2, \tau_3, \tau_4)$



## Tetrahedron Contraction

$$\mathcal{F}^T_{a_1 a_2 a_3 a_4}(\tau_1, \tau_2, \tau_3, \tau_4) \equiv N^{-\frac{9}{2}} \langle \psi^{a_1}_{i_1 j_1 k_1}(\tau_1)\psi^{a_2}_{i_1 j_2 k_2}(\tau_2)\psi^{a_3}_{i_2 j_1 k_2}(\tau_3)\psi^{a_4}_{i_2 j_2 k_1}(\tau_4)\rangle$$

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## Pillow Contraction

$$F^{P,r}_{a_1 a_2} \equiv \langle \psi^{a_1}_{i_1 j_1 k_1}(\tau_1)\psi^{a_1}_{i_2 j_1 k_1}(\tau_2)\psi^{a_2}_{i_1 j_2 k_2}(\tau_3)\psi^{a_2}_{i_2 j_2 k_2}(\tau_4)\rangle$$

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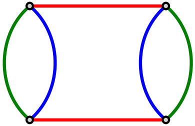

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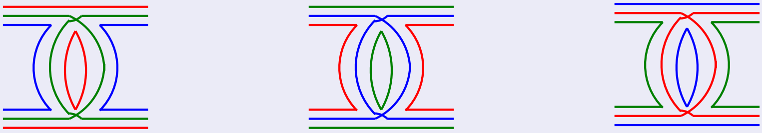
Connected four point function : $F^{P,r}_{ab}(\tau_1,\tau_2,\tau_3,\tau_4) = N^5 G(\tau_{12})G(\tau_{34}) + N^4 \mathcal{F}^{P,r}_{ab}(\tau_1,\tau_2,\tau_3,\tau_4)$

## Tetrahedron Contraction

$$\mathcal{F}^{T}_{a_1 a_2 a_3 a_4}(\tau_1,\tau_2,\tau_3,\tau_4) \equiv N^{-\frac{9}{2}}\langle \psi^{a_1}_{i_1 j_1 k_1}(\tau_1)\psi^{a_2}_{i_1 j_2 k_2}(\tau_2)\psi^{a_3}_{i_2 j_1 k_2}(\tau_3)\psi^{a_4}_{i_2 j_2 k_1}(\tau_4)\rangle$$

## Dipole

The basic object building the ladder



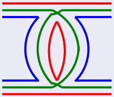
$D^r$ $D^g$ $D^b$

## Contribution to Dipole $D_r$

## Dipole

The basic object building the ladder

1 loop $N$, 2 Vertices $N^{-3}$. Net $N^{-2}$



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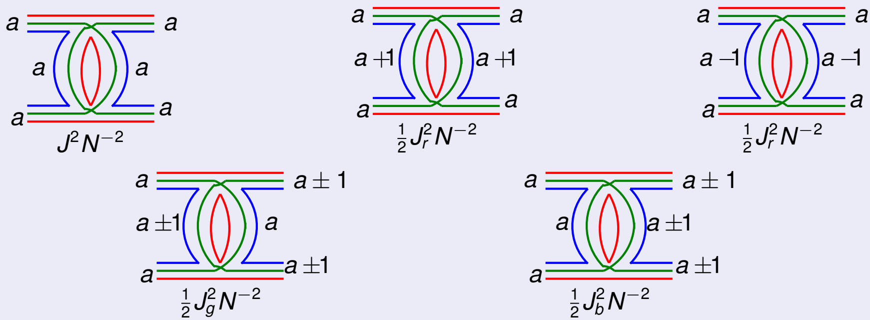
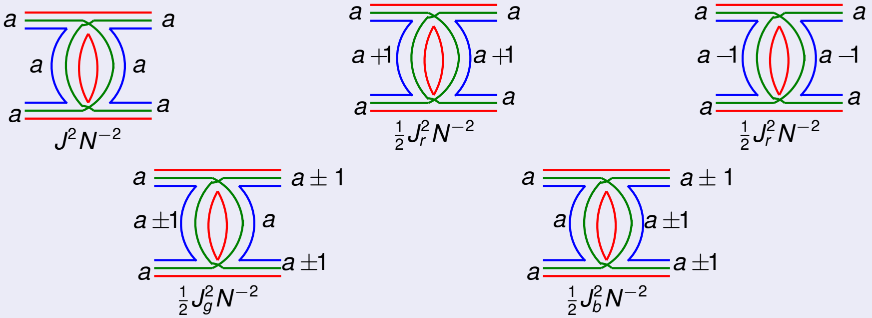


# KT Chain Tensor model Four Point Function, preliminary

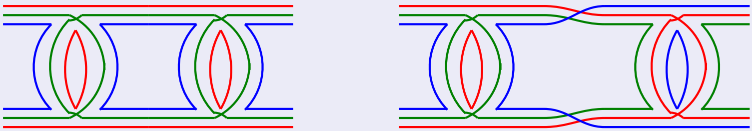
## Dipole

$$N^{-2}D^{r}_{ab}(\tau_1,\tau_2,\tau_3,\tau_4)=\begin{cases}-N^{-2}(J^2+J_r^2)G(\tau_{13})G(\tau_{24})[G(\tau_{34})]^2 & \text{for } a=b\\ -\frac{1}{2}N^{-2}(J_g^2+J_b^2)G(\tau_{13})G(\tau_{24})[G(\tau_{34})]^2 & \text{for } |a-b|=1\end{cases}$$

## Contribution to Dipole $D_r$



Ladder diagrms obtained by stringing Dipoles together. Two types :



(a) Unbroken ladder diagram

(b) Broken ladder diagram

## Cooper Contraction

$$F^{C}_{a_1 a_2} \equiv \langle \psi^{a_1}_{i_1 j_1 k_1}(\tau_1)\psi^{a_1}_{i_1 j_1 k_1}(\tau_2)\psi^{a_2}_{i_2 j_2 k_2}(\tau_3)\psi^{a_2}_{i_2 j_2 k_2}(\tau_4)\rangle$$

Connected four point function : $F^{C}_{ab}(\tau_1,\tau_2,\tau_3,\tau_4) = N^6 G(\tau_{12})G(\tau_{34}) + N^3 \mathcal{F}^{C}_{ab}(\tau_1,\tau_2,\tau_3,\tau_4)$

Both Unbroken and broken diagrams contribute to $\mathcal{F}^C$ at same order.

Ladder diagram :

$$\mathcal{F}^C = \sum_{n=0}^{\infty} (D_r + D_g + D_b)^n \mathcal{F}_0 \qquad \mathcal{F}_0 \equiv -G(\tau_{13})G(\tau_{24}) + G(\tau_{14})G(\tau_{23})$$

Use translational invariance to work in momentum space

$$\mathcal{F}_p(\tau_1, \tau_2, \tau_3, \tau_4) = \frac{1}{1 - s(p)\mathcal{K}} \mathcal{F}_0$$

where $s(p) \equiv 1 - \frac{2\mathcal{J}_{hop}^2}{3\mathcal{J}^2}(1 - \cos p)$ and $\mathcal{K}(\tau_1, \tau_2, \tau_3, \tau_4) \equiv -3\mathcal{J}^2 G(\tau_{13})G(\tau_{24})[G(\tau_{34})]^2$
$\rightarrow$ same as SYK on lattice Gu Qi Stanford 's .

Same spectrum. Same long time behaviour: $e^{-\lambda_L t} \rightarrow e^{-\lambda_L (t - v_B x)}$

**Chaos bound saturated**

$$\lambda_L = \frac{2\pi}{\beta}$$

**Butterfly effect**

$$v_B = \frac{2\pi D}{\beta} \qquad D \sim \frac{\mathcal{J}_{hop}^2}{\mathcal{J}}$$

# KT Chain Tensor model Cooper Channel

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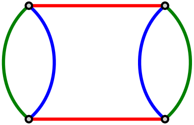

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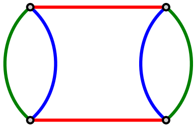

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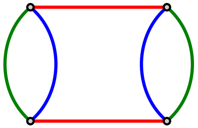

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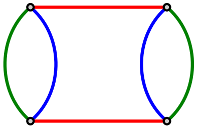

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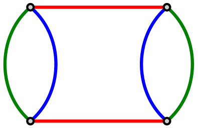

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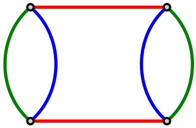

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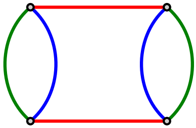

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Finding $1/N$ corrections is very hard. But for rank $D \geq 5$, we can calculate subleading pieces which indeed shows exponential behaviour saturating chaos bound.

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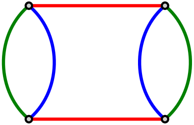

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# Other Tensor Models

Formalism is quite general. Can apply to say Witten 's model.

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$\rightarrow$ KT Model on 4 sites.

Formalism applies to any theory where the quartic interactions have the following property : Given three of the fermions participating in the interaction, the fourth is completely fixed

We also construct a model in which the Lyaponav exponent is non-maximal for some channels.

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# Future Directions

- Large $D$ : Are there simplification in diagrammatics? - More observables..
- Other interesting Tensor models ?
- Generically - index on fermions. Useful for other purposes?