Open Quantum Field Theory : ϕ^4 theory

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Outline

What is Open QFT?

- Open QFT is an effective theory which is obtained from a unitary theory by tracing out some degrees of freedom (environment field(s)).
- These theories are not unitary.

Why open QFT?

- To understand dissipation in QFT.
- The field theory living on the exterior of a black hole is also an open QFT and hence studies of open QFT can help us to understand various aspects of black hole physics.

The questions that we want to address

We consider an open QFT which is described by a real single scalar field.

The questions that we want to answer,

- whether the theory is renormalisable.
- whether the Lindblad condition (explained later) at tree level is preserved at one loop.

Schwinger-Keldysh formulation

It is a path integral formulation for the evolution of density matrix.

- Double the degrees of freedom - ket field ϕ_R and bra field ϕ_L
- The generating function,

$$\begin{aligned}\mathcal{Z}_{SK}[J_R, J_L] =& \text{Tr}\Big[U[J_R]\rho_i U^\dagger[J_L]\Big] \\ =& \int_{\rho_i(\phi_R,\phi_L)}^{\phi_R|_{t=\infty}=\phi_L|_{t=\infty}} [\mathcal{D}\phi_R]\,[\mathcal{D}\phi_L]\, e^{iS[\phi_R,J_R]-iS[\phi_L,J_L]}\end{aligned}$$

The Schwinger-Keldysh action of a unitary theory is given by,

$$S_{SK} = S[\phi_R] - S[\phi_L]$$

SK Propagators in scalar theory

There are four propagators and those are following:

$$\begin{aligned}
\mathbf{R:} \quad \langle \phi_R(-k)\phi_R(k)\rangle &= \frac{-i}{k^2+m^2-i\varepsilon} \\
\mathbf{L:} \quad \langle \phi_L(-k)\phi_L(k)\rangle &= \frac{i}{k^2+m^2+i\varepsilon} \\
\mathbf{P:} \quad \langle \phi_R(-k)\phi_L(k)\rangle &= 2\pi\theta(k^0)\delta(k^2+m^2) \\
\mathbf{M:} \quad \langle \phi_L(-k)\phi_R(k)\rangle &= 2\pi\theta(-k^0)\delta_-(k^2+m^2)
\end{aligned}$$

These are related via the following identity,

$$\frac{1}{k^2+m^2-i\varepsilon} - \frac{1}{k^2+m^2+i\varepsilon} = 2\pi i\ \delta(k^2+m^2)$$

Feynman-Vernon

- We start with two fields and construct the SK path integral,

$$\mathcal{Z}_{SK} = \int [\mathcal{D}\phi_R] [\mathcal{D}\chi_R] [\mathcal{D}\phi_L] [\mathcal{D}\chi_L] \, e^{iS[\phi_R,\chi_R]-iS[\phi_L,\chi_L]}$$

- We trace out the χ field and get an effective theory with a reduced SK Lagrangian,

$$S_{SK} = S[\phi_R] - S[\phi_L] + S_{FV}[\phi_R, \phi_L]$$

S_{FV} is called the Feynman-Vernon influence functional.

Lindblad Equation

If we impose that Tr $\rho = 1$ and the eigenvalues of ρ are positive then in the Markovian approximation time evolution of density matrix in open-QM,

$$i\hbar\frac{d\rho}{dt} = [H,\rho] + i\sum_{\alpha\beta}\Gamma_{\alpha\beta}\left(L_\beta\rho L_\alpha^\dagger - \frac{1}{2}L_\alpha^\dagger L_\beta\rho - \frac{1}{2}\rho L_\alpha^\dagger L_\beta\right)$$

In Heisenberg picture the time evolution of any operator,

$$i\hbar\frac{d\mathcal{A}}{dt} = [\mathcal{A},H] + i\sum_{\alpha\beta}\Gamma_{\alpha\beta}\left(L_\alpha^\dagger\mathcal{A}L_\beta - \frac{1}{2}L_\alpha^\dagger L_\beta\mathcal{A} - \frac{1}{2}\mathcal{A}L_\alpha^\dagger L_\beta\right)$$

Feynman-Vernon influence functional takes the form,

$$S_{FV} = i\int\sum_{\alpha\beta}\Gamma_{\alpha\beta}\left(L_\alpha^\dagger(\phi_R)L_\beta(\phi_L) - \frac{1}{2}L_\alpha^\dagger(\phi_R)L_\beta(\phi_R) - \frac{1}{2}L_\alpha^\dagger(\phi_L)L_\beta(\phi_L)\right)$$

Open ϕ^4 theory

The most general SK action with FV influence functional,

$$S = -\int d^4x\Big(\frac{z}{2}(\partial\phi_R)^2 + \frac{1}{2}m^2\phi_R^2 + \frac{\lambda}{4!}\phi_R^4 + \frac{\sigma}{3!}\phi_R^3\phi_L\Big)$$
$$+\int d^4x\Big(\frac{z^\star}{2}(\partial\phi_L)^2 + \frac{1}{2}m^{2\star}\phi_L^2 + \frac{\lambda^\star}{4!}\phi_L^4 + \frac{\sigma^\star}{3!}\phi_L^3\phi_R\Big)$$
$$+ i\int d^4x\Big(z_\Delta\partial\phi_R\partial\phi_L + m_\Delta^2\phi_R\phi_L + \frac{\Delta}{2!2!}\phi_R^2\phi_L^2\Big)$$

This action is CPT invariant. Under CPT,

$$\phi_R \to \phi_L \text{ and } \phi_L \to \phi_R$$

Lindblad conditions

Writing the action in the Lindblad form we obtain the following conditions on the coupling constants.

$$\begin{aligned} \mathrm{Im}\ z - z_{\Delta} &= 0 \\ \mathrm{Im}\ m^2 - m_{\Delta}^2 &= 0 \\ \mathrm{Im}\ \lambda + 4\mathrm{Im}\ \sigma - 3\Delta &= 0 \end{aligned}$$

Energy dependence of coupling constants: Beta functions

The beta functions for the coupling constants are the following:

$$\frac{dm^2}{d\ln\mu} = \frac{m^2}{(4\pi)^2}\Big(\lambda + 2\sigma - i\Delta\Big)$$

$$\frac{dm_\Delta^2}{d\ln\mu} = \frac{2}{(4\pi)^2}\mathrm{Re}\Big[m^2(\Delta + i\sigma)\Big]$$

$$\frac{d\lambda}{d\ln\mu} = \frac{3}{(4\pi)^2}(\lambda + 2\sigma - i\Delta)(\lambda + i\Delta)$$

$$\frac{d\sigma}{d\ln\mu} = \frac{3}{(4\pi)^2}(\lambda + \sigma + \sigma^* + i\Delta)(\sigma - i\Delta)(\sigma - i\Delta)$$

$$\frac{d\Delta}{d\ln\mu} = \frac{1}{(4\pi)^2 i}\Big[(\lambda + 2\sigma^*)(\sigma^* + i\Delta) + 3i\sigma\Delta - c.c.\Big]$$

Universality of Lindblad conditions

The Lindblad conditions are not violated at one loop. Beta functions of Lindblad couplings are,

- No correction to $(\mathrm{Im}\ z - z_\Delta)$ at one loop.
- $$\frac{d}{d\ln\mu}(\mathrm{Im} m^2 - m_\Delta^2) = \frac{2}{(4\pi)^2} m^2 (\mathrm{Im}\lambda + 4\mathrm{Im}\sigma - 3\Delta)$$
- $$\frac{d}{d\ln\mu}(\mathrm{Im}\lambda + 4\mathrm{Im}\sigma - 3\Delta) = \frac{6}{(4\pi)^2}(\mathrm{Im}\lambda + 4\mathrm{Im}\sigma - 3\Delta) \times (\mathrm{Re}\lambda + 2\mathrm{Re}\sigma)$$

We also have a diagrammatic proof that the Lindblad condition is never violated at all order in perturbation theory.

Open Questions

- Open scalar theory with two fields seems to have non-local divergences (work in progress).
- One can extend this idea to fermions (work in progress) and then QED.
- One can also prove systematic breaking of unitarity from a microscopic theory to the effective theory.