Smooth Causal Patches for AdS Black Holes

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References and Collaborators

This talk is based on

S.R, “Smooth Causal Patches for AdS Black Holes”, arXiv:1604.03095.

Also based on previous work with Kyriakos Papadodimas (CERN & Groningen), Souvik Banerjee (Groningen) and Jan-Willem Bryan (Groningen) and work in progress with Sudip Ghosh (ICTS-TIFR).

Context

- The context for this talk is the Information Paradox. In its modern avatar, this turns into the question:

  "Can Holography describe the BH Interior?"

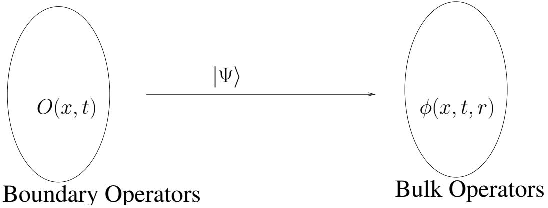
  [Mathur, Almheiri, Marolf, Polchinski, Sully, Stanford, 2009-2015]

- This version is not restricted to evaporating BHs, but also applies to thermodynamically stable large black holes in AdS.

Context

- Resolution: Paradox can be completely resolved using a state-dependent map between interior bulk observables and boundary observables.

[K.P., S.R, 2013–15]



- Is this consistent? Or does it violate the linearity of QM?

Summary

- Marolf and Polchinski suggested that state-dependent constructions of the BH interior violate a general rule of statistical mechanics

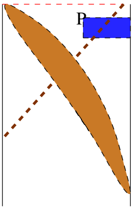
  "Low energy excitations in a large thermal system have small effects on observables"

- I will show that if these violations are unobservable due to causality as manifested in properties of AdS position-space correlators.

Outline

Outline

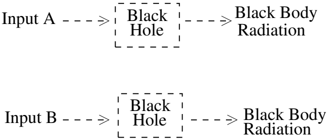
The Old Information Paradox



- In the shaded patch, physics is independent of details of collapse.

$$\langle a^{\dagger}_{\omega} a_{\omega'} \rangle = \frac{e^{-\beta\omega}}{1 - e^{-\beta\omega}} \delta(\omega - \omega')$$

- Suggests that for different inputs, we get the same output.



Resolution to the Old Information Paradox

- Very small corrections of the order of e^{-S} can restore unitarity.

  [Maldacena, 2001]

- Pure density matrix in a very large system can mimic a thermal density matrix to extreme accuracy

$$\mathrm{Tr}\left(\rho_{\text{pure}} A_\alpha\right) = \frac{1}{\mathcal{Z}} \mathrm{Tr}\left(e^{-\beta H} A_\alpha\right) + \mathrm{O}\left(e^{-\frac{S}{2}}\right),$$

  for a large class of observables A_α.

- Another way to state this is

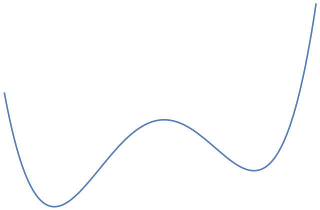
$$\rho_{\text{pure}} = \frac{1}{\mathcal{Z}} e^{-\beta H} + e^{-S} \rho_{\text{corr}}; \quad \rho_{\text{pure}}^2 = \rho_{\text{pure}}$$

Path Integral Perspective

- Effective field theory insufficient to control such corrections.

- A semi-classical spacetime is a saddle point of the QG path-integral.

$$\mathcal{Z} = \int e^{-S} \mathcal{D} g_{\mu\nu}$$



- Perturbative effective field theory (used to derive the Hawking answer) is an asymptotic series expansion of this path-integral.

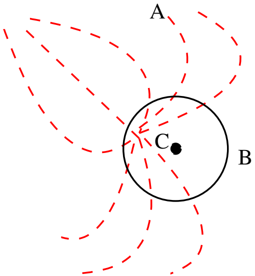
- Non-perturbatively, the notion of local spacetime breaks down.

Complementarity

- For example, in a very high-point correlator

$$\langle \phi(x_1) \ldots [\phi(x_{\text{out}}), \phi(x_{\text{in}})] \ldots \phi(x_S) \rangle \neq 0.$$

- Hilbert space does not factorize into far-away region and near-horizon region.



- Concrete example in empty AdS.

[S. Banerjee, J.W. Bryan, K. Papadodimas, S. R. ,2016]

Old Information Paradox: Slogan

Hawking's calculation is not precise enough to lead to a paradox.

Outline

Modern Information Paradox

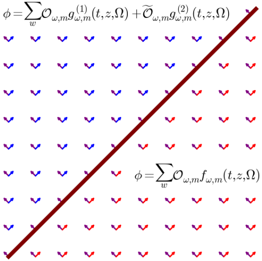
Can the black hole interior be described holographically?

This may look different from the information paradox. But it is still

Unitarity vs Effective Field Theory

Inside a Black Hole

- To describe a local field inside the black hole, we need both left and right movers.



- Can think of $\widetilde{\mathcal{O}}_{\omega,m}$ as modes that have bounced off the origin (Hawking)

- Can also think of them as modes coming from left asymptotic region of the eternal black hole.

Smoothness of the Horizon and Two-Point correlators

Smoothness of the horizon $\leftrightarrow$ KMS condition for $\mathcal{O}_{\omega,m}$ and $\widetilde{\mathcal{O}}_{\omega,m}$

$$\langle\Psi|\widetilde{\mathcal{O}}_{\omega,m}\widetilde{\mathcal{O}}^{\dagger}_{\omega',m'}|\Psi\rangle = \langle\Psi|\mathcal{O}_{\omega,m}\mathcal{O}^{\dagger}_{\omega',m'}|\Psi\rangle = \frac{1}{1-e^{-\beta\omega}}\delta(\omega-\omega')\delta_{mm'}C_{\omega,m}$$

$$\langle\Psi|\widetilde{\mathcal{O}}_{\omega,m}\mathcal{O}_{\omega',m'}|\Psi\rangle = C_{\omega,m}\frac{e^{\frac{-\beta\omega}{2}}}{1-e^{-\beta\omega}}\delta(\omega-\omega')\delta_{mm'}$$

$$\langle[\mathcal{O}_{\omega,m},\mathcal{O}^{\dagger}_{\omega',m'}]\rangle = C_{\omega,m}\delta(\omega-\omega')\delta_{mm'}$$

These must hold in typical states if typical states correspond to smooth-horizons.

How does one describe the $\widetilde{\mathcal{O}}_{\omega,m}$ in the CFT?

Unusual Properties of Mirror Modes

- From analysis of large diffeomorphisms, we find

$$[H, \widetilde{\mathcal{O}}_\omega] = \omega \widetilde{\mathcal{O}}_\omega$$

- Effective field theory requires the KMS condition

$$\langle \Psi | \widetilde{\mathcal{O}}_\omega \widetilde{\mathcal{O}}_\omega^\dagger | \Psi \rangle = e^{\beta \omega} \langle \Psi | \widetilde{\mathcal{O}}_\omega^\dagger \widetilde{\mathcal{O}}_\omega | \Psi \rangle$$

- If the black hole state is approximately thermal

$$\langle \Psi | \widetilde{\mathcal{O}}_\omega \widetilde{\mathcal{O}}_\omega^\dagger | \Psi \rangle \approx \frac{1}{Z(\beta)} \mathrm{Tr}(e^{-\beta H} \widetilde{\mathcal{O}}_\omega \widetilde{\mathcal{O}}_\omega^\dagger) = \frac{1}{Z(\beta)} e^{-\beta \omega} \mathrm{Tr}(e^{-\beta H} \widetilde{\mathcal{O}}_\omega^\dagger \widetilde{\mathcal{O}}_\omega)$$
$$\approx e^{-\beta \omega} \langle \Psi | \widetilde{\mathcal{O}}_\omega^\dagger \widetilde{\mathcal{O}}_\omega | \Psi \rangle ?$$

using equivalence of microcanonical and canonical ensembles, cyclicity of trace and commutator with Hamiltonian.

The Little Hilbert Space

- $|\Psi\rangle \equiv$ Black Hole Microstate

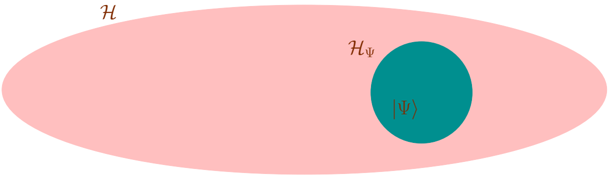
- Little Hilbert Space: all possible effective field theory excitations of $|\Psi\rangle$

$$\mathcal{H}_\Psi = \mathcal{A}|\Psi\rangle,$$

$$\mathcal{A} = \text{span}\{\mathcal{O}_{\omega_1},\ \mathcal{O}_{\omega_1}\mathcal{O}_{\omega_2}, \ldots, \mathcal{O}_{\omega_1}\mathcal{O}_{\omega_2}\ldots\mathcal{O}_{\omega_K}\}.$$

with

$$\omega_m \ll \mathcal{N}, \quad K \ll \mathcal{N}$$



- $H_\Psi = H_{\text{code}}$ in QEC discussions.

Definition of $\widetilde{\mathcal{O}}_\omega$

- Define $\widetilde{\mathcal{O}}_\omega$ precisely within H_Ψ

$$S A_\alpha |\Psi\rangle = A_\alpha^\dagger |\Psi\rangle$$

and

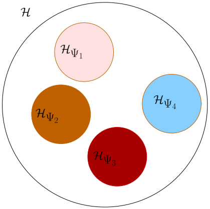
$$\boxed{\widetilde{\mathcal{O}}_\omega = S e^{\frac{\beta H}{2}} \mathcal{O}_\omega e^{-\frac{\beta H}{2}} S}$$

[KP, SR, 2013]

- This is closely related to the isomorphism used in Tomita-Takesaki theory.

- $\phi(t, \Omega, \lambda)$ constructed using this $\widetilde{\mathcal{O}}_\omega$ is a linear operator on H_Ψ and has the correct effective field theory correlators

State-dependence



- no single linear operator $\widetilde{\mathcal{O}}_\omega$ behaves correctly in all H_ψ.
- State-dependence means that we must use different operators $\widetilde{\mathcal{O}}_\omega$ in different H_ψ.

Consistency of State-Dependence

- State-dependence resolves all paradoxes that suggested black-hole firewalls.
- Also leads to a precise description of ER=EPR.
- "Does interior exist in holography?" $\equiv$ "Is state-dependence consistent?"
- **Rest of the talk:** A paradox invented by Marolf and Polchinski (2015) to test consistency of state-dependence and its resolution.

Outline

A Theorem in Statistical Mechanics

- A typical state in a system with many degrees of freedom is thermal for coarse-grained probes:

$$\langle \Psi | A_\alpha | \Psi \rangle = \frac{1}{Z(\beta)} \text{Tr}(e^{-\beta H} A_\alpha) + \text{O}\left(\frac{1}{\sqrt{S}}\right),$$

- Consider a low energy excitation

$$\langle \Psi | U^\dagger H U | \Psi \rangle - \langle \Psi | H | \Psi \rangle = \delta E.$$

- Theorem:

$$\langle \Psi | U^\dagger A_\alpha U | \Psi \rangle - \langle \Psi | A_\alpha | \Psi \rangle \leq 2\sqrt{\beta \delta E} \sigma_\alpha$$

$(\sigma_\alpha^2 = \langle A_\alpha^2 \rangle - \langle A_\alpha \rangle^2)$

Sketch of Proof

$$U : \mathcal{H}_E \to \mathcal{H}_{E+\delta E},$$
$$\dim(\mathcal{H}_E) = e^{S(E)}, \quad \dim(\mathcal{H}_{E+\delta E}) = e^{S(E)+\beta E}$$

So, decompose a typical state $|\Psi_{E+\delta E}\rangle \in \mathcal{H}_{E+\delta E}$,

$$|\Psi_{E+\delta E}\rangle = (1 - \frac{\beta \delta E}{2}) U |\Psi_E\rangle + (\beta \delta E)^{\frac{1}{2}} |\Psi_{\text{orth}}\rangle + \text{O}\left((\beta \delta E)^{\frac{3}{2}}\right).$$

Ensemble at higher energy has same temperature

$$\langle \Psi_{E+\delta E} | A_\alpha | \Psi_{E+\delta E} \rangle = \langle \Psi_E | A_\alpha | \Psi_E \rangle + \text{O}\left(\frac{1}{\sqrt{S}}\right).$$

Therefore,

$$\langle \Psi_{E+\delta E} | A_\alpha | \Psi_{E+\delta E} \rangle - \langle \Psi_E | U^\dagger A_\alpha U | \Psi_E \rangle$$
$$= \delta \langle A_\alpha \rangle = \sqrt{\beta \delta E} \left(\langle \Psi_E | U^\dagger A_\alpha | \Psi_{\text{orth}} \rangle + \langle \Psi_{\text{orth}} | A_\alpha U | \Psi_E \rangle \right) + \text{O}\left(\beta \delta E\right).$$

Low Energy Excitations

$$\delta\langle A_\alpha\rangle \leq 2\sqrt{\beta\delta E}\sigma_\alpha$$

Low energy excitations have small effects: it is impossible to definitively excite a thermal system with energy less than kT.

Marolf-Polchinski Paradox

- With

$$\mathcal{O}_\omega = \int_{-T}^{T} \mathcal{O}(t) e^{i\omega t} dt, \quad A_\alpha = \mathcal{O}_\omega \widetilde{\mathcal{O}}_\omega.$$

we need, for a smooth horizon,

$$\langle \Psi | \mathcal{O}_\omega \widetilde{\mathcal{O}}_\omega | \Psi \rangle = C_\omega \frac{e^{-\frac{\beta\omega}{2}}}{1 - e^{-\beta\omega}}$$

$(C_\omega = \langle [\mathcal{O}_\omega, \mathcal{O}_\omega^\dagger] \rangle.)$

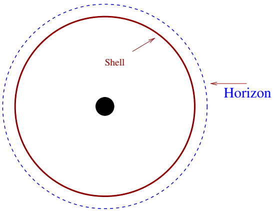
- Take $U_{\text{MP}} = e^{i\theta \frac{\mathcal{O}_\omega \mathcal{O}_\omega^\dagger}{C_\omega}}$

$$\langle \Psi | U_{\text{MP}}^\dagger \mathcal{O}_\omega \widetilde{\mathcal{O}}_\omega^\dagger U_{\text{MP}} | \Psi \rangle = e^{i\theta} C_\omega \frac{e^{-\frac{\beta\omega}{2}}}{1 - e^{-\beta\omega}}$$

- But $\delta E \propto \beta\theta^2 (\delta\omega)^2 \propto \beta\theta^2 / T^2$.

Appears to violate $\delta\langle A_\alpha \rangle \leq 2\sqrt{\beta \delta E} \sigma_\alpha$

Intuition: Firewalls near the Horizon

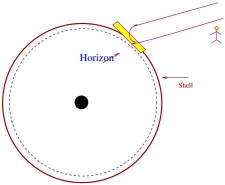


- For every black hole with an empty interior, consider a configuration with a ultra-relativistic shell close to the horizon.
- Binding energy with BH cancels rest+kinetic energy of shell $\Rightarrow$ increase in AdM energy is small.
- Paradox suggests that $\delta \langle A_{\alpha} \rangle \leq 2\sqrt{\beta \delta E}\sigma_{\alpha}$ is violated unless typical states are firewalls.

[Marolf-Polchinski, 2015]

Significance of the Infalling Observer

- Paradox only inside the horizon: outside observer cannot distinguish shell from the Unruh effect.



- Technical version: operators outside the horizon given by HKLL construction; automatically obey $\delta\langle A_\alpha\rangle \leq 2\sqrt{\beta\delta E}\sigma_\alpha$.

- Interior ops state-dependent; inequality not guaranteed.

M-P paradox is a test of whether state-dependence leads to observable violations of standard rules of statistical mechanics.

Outline

Resolution

- Start with an equilibrium state $|\Psi\rangle$.
- Create the excitation actively by modifying the boundary Hamiltonian

$$H_{\text{CFT}} = H_{\text{CFT}} + J(t)\mathcal{O}^J(t)$$

- Then, as a result of a remarkable property of position space AdS correlators

$$\delta\langle A_\alpha\rangle \leq 2\sqrt{\beta\delta E}\sigma_\alpha$$

No observer can start with an equilibrium state; excite it and jump into the horion to observe the violation.

Resolution: Alternate language

We can prove a stronger result

If the unitary U and the observable A_α fit in the same causal patch, we can prove $\delta\langle A_\alpha \rangle \leq 2\sqrt{\beta\delta E}\sigma_\alpha$. Therefore violations are unobservable!

Sources and Causal Patches

Deform

$$H_{\text{CFT}} = H_{\text{CFT}} + J(t)\mathcal{O}^J(t)$$

Then, bulk observables modified to

$$\phi^J(t, r_*, \Omega) = \overline{\mathcal{T}}\{e^{i\int_{\vartheta}^{t+r_*} J(x)\mathcal{O}(x)dx}\}\phi(t, r_*, \Omega)\mathcal{T}\{e^{-i\int_{\vartheta}^{t+r_*} J(x)\mathcal{O}(x)dx}\}.$$

Only the part of the source in the causal past of the bulk point affects the field there.

Therefore the differences

$$\langle\phi^J(t_1, r_{*1}, \Omega_1)\ldots\phi^J(t_n, r_{*n}, \Omega_n)\rangle - \langle\phi(t_1, r_{*1}, \Omega_1)\ldots\phi(t_n, r_{*n}, \Omega_n)\rangle$$

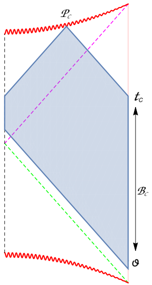
with pts in same causal patch obey constraints of statistical mechanics.

Causal Patches

$$U = e^{i \int_{\vartheta}^{t_{\mathcal{C}}} A(t) dt}.$$

$A(t)$ is a simple boundary operator. With x_i in causal patch of $t_{\mathcal{C}}$

$$\delta = \langle \Psi | U^{\dagger} \phi(x_1) \ldots \phi(x_n) U | \Psi \rangle - \langle \Psi | \phi(x_1) \ldots \phi(x_n) | \Psi \rangle$$



Most general test of inequality possible within constraints of causality.

Summary of Resolution

For this observable

$$\delta \leq 2\sqrt{\beta\delta E}\sigma.$$

as a result of a very non-trivial property of position-space AdS correlators.

So, no observer or army of observers can detect a violation of the inequality by doing experiments that obey the constraints imposed by bulk causality.

Sketch of Proof

From the definition of mirror operators

$$\widetilde{\phi}_r(t, r_*, \Omega) A_\alpha U|\Psi\rangle = A_\alpha U e^{\frac{-\beta H}{2}} \widehat{\phi}(t, r_*, \Omega) e^{\frac{\beta H}{2}} |\Psi\rangle.$$

where $\widehat{\phi}(t, r_*, \Omega)$ is an ordinary operator.

Main technical step is

$$[\widehat{\phi}(t_1, r_{*1}, \Omega_1), \mathcal{O}(t_2, \Omega_2)] = 0.$$

where $\mathcal{O}(t_2, \Omega_2)$ is a boundary operator and point 1 is behind the horizon but in the same causal patch as point 2.

Sketch of Proof

So

$$\langle\Psi|UA_\alpha\widetilde{\phi}_r(t,r_*,\Omega)U^\dagger|\Psi\rangle = \langle\Psi|UA_\alpha e^{-\frac{\beta H}{2}}\widehat{\phi}(t,r_*,\Omega)e^{\frac{\beta H}{2}}U^\dagger|\Psi\rangle + \mathrm{O}\left(\beta\delta E\right).$$

But then

$$\begin{aligned}&\langle\Psi|UA_\alpha\widetilde{\phi}_r(t,r_*,\Omega)U^\dagger|\Psi\rangle - \langle\Psi|A_\alpha\widetilde{\phi}_r(t,r_*,\Omega)|\Psi\rangle \\ &= \langle\Psi|UA_\alpha e^{-\frac{\beta H}{2}}\widehat{\phi}(t,r_*,\Omega)e^{\frac{\beta H}{2}}U^\dagger|\Psi\rangle - \langle\Psi|A_\alpha e^{-\frac{\beta H}{2}}\widehat{\phi}(t,r_*,\Omega)e^{\frac{\beta H}{2}}|\Psi\rangle \\ &+ \mathrm{O}\left(\beta\delta E\right).\end{aligned}$$

Correlator on right is just an ordinary correlator; automatically obeys $\delta \leq 2\sqrt{\beta\delta E}\sigma$.

Therefore, the correlator of mirror operators in the same causal patch as U also obeys the constraints of statistical mechanics.

Resolution of the Marolf-Polchinski Paradox: Summary

- In frequency space, the state-dependence of mirror operators is manifested as an anomalously large change in correlators under low-energy excitations.
- When we consider position space observables in the same causal patch as the excitation, these anomalous transformations cancel.
- So, the anomalous properties of the interior state-dependent operators are not visible in any physical experiment.

Outline

Summary

- The modern information paradox can be rephrased as a question about the existence of CFT operators dual to bulk fields.
- Can be resolved using a state-dependent map between boundary and bulk fields.
- Question is whether state-dependence is consistent.

Summary

- Marolf-Polchinski suggested that state-dependence contradicts a general rule of statistical mechanics.

$$\langle \Psi | U^\dagger A_\alpha U | \Psi \rangle - \langle \Psi | A_\alpha | \Psi \rangle \leq 2\sqrt{\beta \delta E} \sigma_\alpha$$

  "low-energy excitations have small effects."

- Here, we argued that this paradox is unobservable.

- If we consider boundary excitations U and local bulk observables A_α in the same causal patch as U then the inequality is obeyed.

- Non-trivial property of position-space AdS correlators.

Open Question

- This shows that it is possible to describe the BH interior holographically and consistently.
- But, why is this the “correct description”?
- Requires a dynamical understanding of why the bulk-observer measures the fields he does.
- Analogous to the Unruh-de Witt answer for why a particular observer uses a particular definition of “particle number”; here we want to explain why a specific CFT operator is the correct local bulk field ϕ.

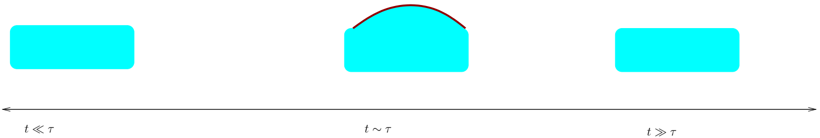
Appendix

Autonomously Excited States

- Consider
$$|\Psi^{\text{ne}}\rangle = U(\tau)|\Psi\rangle,$$
as an autonomously excited state.

- On the boundary, think of $|\Psi^{\text{ne}}\rangle$ as a fluid about to undergo a spontaneous excitation around τ.

- What does an infalling observer in $|\Psi^{\text{ne}}\rangle$ experience?

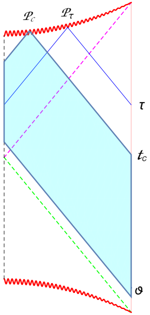


Paradox in Spontaneously Excited States

If an observer can prepare $U(\tau)|\Psi\rangle$ and compare with $|\Psi\rangle$

$$\delta = \langle\Psi|U(\tau)^\dagger\phi(x_1)\ldots\phi(x_n)U(\tau)|\Psi\rangle - \langle\Psi|\phi(x_1)\ldots\phi(x_n)|\Psi\rangle$$

where x_i are in a causal patch but τ is beyond the causal patch, then he would observe a violation of $\delta \leq 2\sqrt{\beta\delta E}\sigma$.



(Would also observe a violation of the second law of thermodynamics!).

Proposal of Causal Patch Complementarity

Write

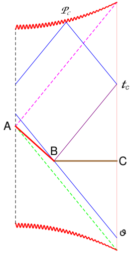
$$U(\tau) = U^{\mathcal{C}} \widehat{V}^{\mathcal{C}},$$

where

$$U^{\mathcal{C}} = e^{i \int_{\vartheta}^{t_{\mathcal{C}}} A_{\gamma}(t) dt}$$

and, $\forall A_{\alpha}(t)$ with $t < t_{\mathcal{C}}$,

$$\langle \Psi | U(\tau)^{\dagger} A_{\alpha}(t) U(\tau) | \Psi \rangle = \langle \Psi | U^{\mathcal{C}} A_{\alpha}(t) U^{\mathcal{C}} | \Psi \rangle$$



Proposal of Causal Patch Complementarity

Fields appropriate for causal patch corresponding to $t_{\mathcal{C}}$ satisfy

$$\langle \Psi | U^{\dagger} \phi_{\mathcal{C}}(x_1) \ldots \phi_{\mathcal{C}}(x_n) U | \Psi \rangle = \langle \Psi | U^{\mathcal{C}^{\dagger}} \phi(x_1) \ldots \phi(x_n) U^{\mathcal{C}} | \Psi \rangle,$$

Infalling observer is only sensitive to "part of the boundary excitation in the same patch as the observer"

Consequences: Causal Patch Complementarity

- Ensures that

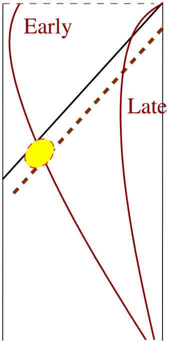
$$\delta = \langle\Psi|U^{\dagger}\phi_{\mathcal{C}}(x_1)\ldots\phi_{\mathcal{C}}(x_n)U|\Psi\rangle - \langle\Psi|\phi_{\mathcal{C}}(x_1)\ldots\phi_{\mathcal{C}}(x_n)|\Psi\rangle$$

  satisfies $\delta \leq 2\sqrt{\beta\delta E}\sigma$ for all U provided $x_1, \ldots x_n$ are in a single causal patch.

- Correlators of $\phi_{\mathcal{C}}$ and the HKLL ϕ agree in all equilibrium states and in all equilibrium states excited with a source.

- For spontaneously excited states, this modifies HKLL outside the horizon.

Example: Stanford-Shenker State



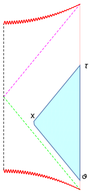
Observer outside infers that the early observer had a trans-Planckian collision. But early observer perceives a smooth geometry.

['t Hooft, Susskind, Thorlacius, Uglum, Kiem, Verlinde^2, 85–95]

Teleological Property of HKLL Construction

- In empty AdS, HKLL construction is causal. But, in the presence of a black hole, HKLL construction is teleological

$$\phi_{\text{HKLL}}(x) = \int O(t)K(t,x)dt$$



- Fields ϕ_{HKLL} and $\phi_{\mathcal{C}}$ differ in their response to future excitations in spontaneously excited states.

- How does observer at x “know” whether he should use ϕ_{HKLL} or ϕ_{caus}?