Large Spin asymptotics from conformal bootstrap

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Part 1:
Lightcone
Bootstrap

Operator product expansion (OPE): In a CFT one can expand product of two local operators in a sum:

$$\phi(x)\phi(0) \sim \sum_{O} c_O f_O(x,\partial) O(0)$$

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$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4) \rangle = \frac{1}{\left(x_{12}^2 x_{34}^2\right)^{\Delta_\phi}} \sum_{O} C_O g_O(u,v)$$

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where Δ_ϕ : dimension of ϕ

$$u = \frac{x_{12}^2 x_{34}^2}{x_{24}^2 x_{13}^2} \qquad v = \frac{x_{14}^2 x_{23}^2}{x_{24}^2 x_{13}^2}$$

OPE coefficient

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \frac{1}{\left(x_{12}^2 x_{34}^2\right)^{\Delta_\phi}} \sum_{O} C_O\, g_O(u,v)$$

Conformal block

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Conformal block

s-channel

t-channel

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Must be equal

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \frac{1}{(x_{12}^2 x_{34}^2)^{\Delta_\phi}} \sum_O C_O g_O(u,v)$$

s-channel

Bootstrap eqn (crossing symmetry)

$$u^{-\Delta_\phi} \sum_{\Delta, l} C_{\Delta, l}\, g_{\Delta, l}(u, v) \;=\; v^{-\Delta_\phi} \sum_{\Delta, l} C_{\Delta, l}\, g_{\Delta, l}(v, u)$$

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Here operators are labelled by
spin l,
Dimension Δ.

Take a mean field theory / Free theory in AdS (Infinite N)

$$\frac{1}{(x_{13}^2 x_{24}^2)^{\Delta_\phi}}\left(u^{-\Delta_\phi} + 1 + v^{-\Delta_\phi} \right) = \frac{v^{-\Delta_\phi}}{(x_{13}^2 x_{24}^2)^{\Delta_\phi}}\left(\sum_{\Delta, l} C_{\Delta, l}\, g_{\Delta, l}(v, u)\right)$$

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set limit $u \ll v \ll 1$

↓

Leading term (identity)

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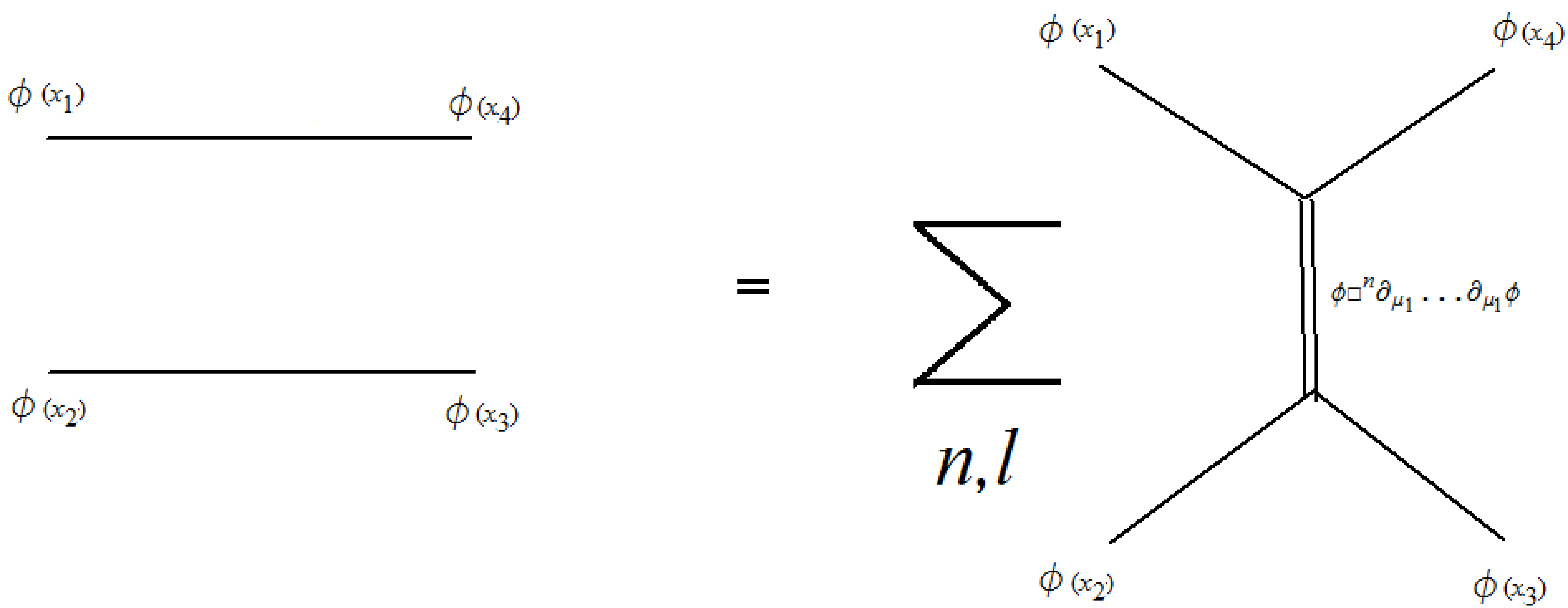
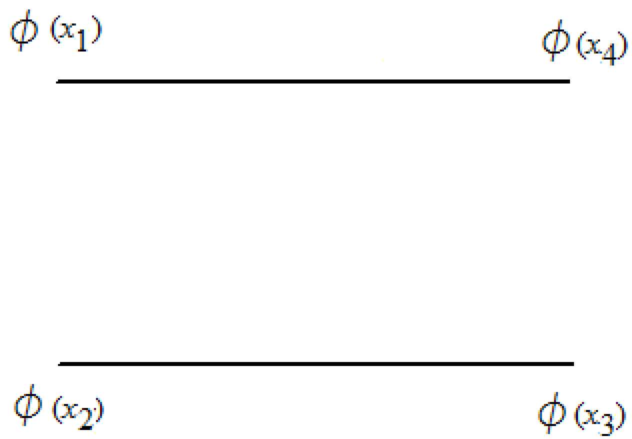
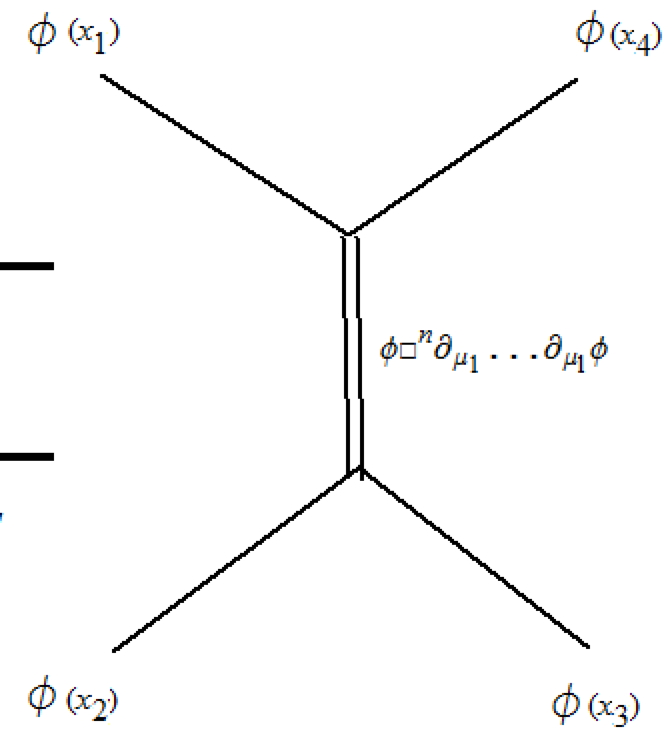
Sum up to give



ϕ (x1)
ϕ (x4)
ϕ (x2)
ϕ (x3)
=
∑ n,l
ϕ (x1)
ϕ (x4)
ϕ□^n ∂μ1 . . . ∂μl ϕ
ϕ (x2)
ϕ (x3)



$$= \sum_{n,l}$$



$$u^{-\Delta_\phi}$$

$$v^{-\Delta_\phi} \sum_{\text{double trace}} C_{\Delta,l}\, g_{\Delta,l}(v,u)$$

Now take a **general CFT**

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Bootstrap eqn

$$u^{-\Delta_\phi} + u^{-\Delta_\phi} \sum_{\Delta,l} C_{\Delta,l}\, g_{\Delta,l}(u,v) \;=\; v^{-\Delta_\phi} + v^{-\Delta_\phi} \sum_{\Delta,l} C_{\Delta,l}\, g_{\Delta,l}(v,u)$$

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↓ $u \ll v \ll 1$

Leading term (from identity operator)

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$\downarrow$ $u \ll v \ll 1$

Leading term (from identity operator)

Claim: Even in a general CFT, the double trace operators exist, and they reproduce the leading term on rhs

[Komargodski, Zhiboedov,(2012); Kaplan et al, (2012)]

Now take a **general CFT**

Bootstrap eqn

$$u^{-\Delta_\phi} + u^{-\Delta_\phi} \sum_{\Delta, l} C_{\Delta, l} g_{\Delta, l}(u, v) = v^{-\Delta_\phi} + v^{-\Delta_\phi} \sum_{\Delta, l} C_{\Delta, l} g_{\Delta, l}(v, u)$$

The double trace operators, at large l, must have

$\Delta = 2\Delta_\phi + 2n + l + O\left(l^{-1}\right)$ or

$\tau = 2\Delta_\phi + 2n + O\left(l^{-1}\right)$

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Summing over large l

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Summing over large l

So at **large spin**, the conformal blocks on rhs are just like the free field theory, and sum up to give the leading term

$$u^{-\Delta_\phi} + O(l^{-1})$$

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There are two corrections (unknowns):

The anomlous dimension $\gamma(n,l) = \Delta - 2\Delta_\phi + 2n + l$

The OPE coeff correction $\delta C(n,l) = C_{\Delta,l} - C_{2\Delta_\phi + 2n + l,\, l}$

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These two **should fall with increasing *l***, and be such that the **subleading terms of lhs are reproduced.**

The subleading term

$$u^{-\Delta_\phi} + u^{-\Delta_\phi + \tau_m} C_m f_{\tau_m, l_m}(0, v) + \ldots \approx v^{-\Delta_\phi} \sum_{\text{double trace ops}} C_{\Delta, l}\, g_{\Delta, l}(v, u)$$

The subleading term

$$u^{-\Delta_\phi} + u^{-\Delta_\phi + \tau_m} C_m f_{\tau_m, l_m}(0, v) + \ldots \approx v^{-\Delta_\phi} \sum_{\text{double trace ops}} C_{\Delta,l}\, g_{\Delta,l}(v, u)$$

Due to an operator of
minimal non-zero twist

twist: $\tau_m = \Delta_m - l_m$
spin: l_m

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$$u^{-\Delta_\phi} + u^{-\Delta_\phi + \tau_m} C_m f_{\tau_m, l_m}(0, v) + \ldots \approx v^{-\Delta_\phi} \sum_{\text{double trace ops}} C_{\Delta,l}\, g_{\Delta,l}(v, u)$$

↓

Due to an operator of
minimal non-zero twist

twist: $\tau_m = \Delta_m - l_m$
spin: l_m

(usually its the stress tensor: $\tau_m = 2,\ l_m = 2$ and $C_m \sim \frac{1}{N^2}$)

The subleading term

$$u^{-\Delta_\phi} + u^{-\Delta_\phi + \tau_m} C_m f_{\tau_m, l_m}(0,v) + \ldots \approx v^{-\Delta_\phi} \sum_{\text{double trace ops}} C_{\Delta,l}\, g_{\Delta,l}(v,u)$$

$\downarrow$

$$\Delta = 2\Delta_\phi + 2n + l \; + \; \gamma(n,l)$$

$$C_{\Delta,l} = C_{2\Delta_\phi + 2n + l,\, l} + \delta C(n,l)$$

Take the log terms

$$u^{-\Delta_\phi} + u^{-\Delta_\phi + \tau_m} C_m f_{\tau_m, l_m}(0,v) + \ldots \approx v^{-\Delta_\phi} \sum_{\text{double trace ops}} C_{\Delta,l}\, g_{\Delta,l}(v,u)$$

$$\downarrow$$

$$C_m u^{-\Delta_\phi + \tau_m} \frac{\Gamma(\tau_m + 2 l_m)}{\Gamma^2(\frac{\tau_m}{2} + l_m)} \log v + O(v \log v)$$

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$$C_m u^{-\Delta_\phi + \tau_m} \frac{\Gamma(\tau_m + 2 l_m)}{\Gamma^2(\frac{\tau_m}{2} + l_m)} \log v \; + O(v \log v)$$

$$\sum_l C_{2\Delta_\phi + l, l}\, u^{-\Delta_\phi + \tau_m} \frac{\sqrt{l}\, 4^l}{\sqrt{\pi}} \gamma(0, l)\, K_0(2 l \sqrt{z}) \log v$$

$$+ O(v \log v)$$

$$\gamma(0,l) = -\frac{C_m}{l^{\tau_m}} \frac{2\,\Gamma^2(\Delta_\phi)\,\Gamma(\tau_m + 2l_m)}{\Gamma^2(\Delta_\phi - \tau_m/2)\,\Gamma^2(\tau_m/2 + l_m)}$$

$$C_m u^{-\Delta_\phi + \tau_m} \frac{\Gamma(\tau_m + 2l_m)}{\Gamma^2(\frac{\tau_m}{2} + l_m)} \log v \;\; + O(v \log v)$$

Converting $\sum_l \to \int dl$

$$\sum_l C_{2\Delta_\phi + l,\, l}\, u^{-\Delta_\phi + \tau_m} \frac{\sqrt{l}\, 4^l}{\sqrt{\pi}}\, \gamma(0,l)\, K_0(2l\sqrt{z}) \log v$$

$$+ O(v \log v)$$

Similar analysis for non-log terms gives OPE coefficients

$$u^{-\Delta_\phi} + u^{-\Delta_\phi + \tau_m} C_m f_{\tau_m, l_m}(0, v) + \ldots \approx v^{-\Delta_\phi} \sum_{\text{double trace ops}} C_{\Delta, l} g_{\Delta, l}(v, u)$$

↓

$$C_m u^{-\Delta_\phi + \tau_m} \frac{\Gamma(\tau_m + 2 l_m)}{\Gamma^2(\frac{\tau_m}{2} + l_m)} (\psi(1) + \psi(\tau_m/2 + l_m)) + \ldots$$

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$$\sum_l C_{2\Delta_\phi + l, l}\, u^{-\Delta_\phi + \tau_m} \frac{\sqrt{l}\, 4^l}{\sqrt{\pi}} (\delta C + \gamma(0, l)) K_0(2 l \sqrt{z})$$

$$+ \ldots$$

Similar analysis for non-log terms gives OPE coefficients

$$\delta C = -\frac{C_m}{l^{\tau_m}} \frac{2\Gamma^2(\Delta_\phi)\Gamma(\tau_m + 2 l_m)}{\Gamma^2(\Delta_\phi - \tau_m/2)\Gamma^2(\tau_m/2 + l_m)} \left(\psi\left(\frac{\tau_m}{2} + l_m\right) + \gamma - \log 2\right)$$

$$C_m u^{-\Delta_\phi + \tau_m} \frac{\Gamma(\tau_m + 2 l_m)}{\Gamma^2\left(\frac{\tau_m}{2} + l_m\right)} \left(\psi(1) + \psi(\tau_m/2 + l_m)\right) + \ldots$$

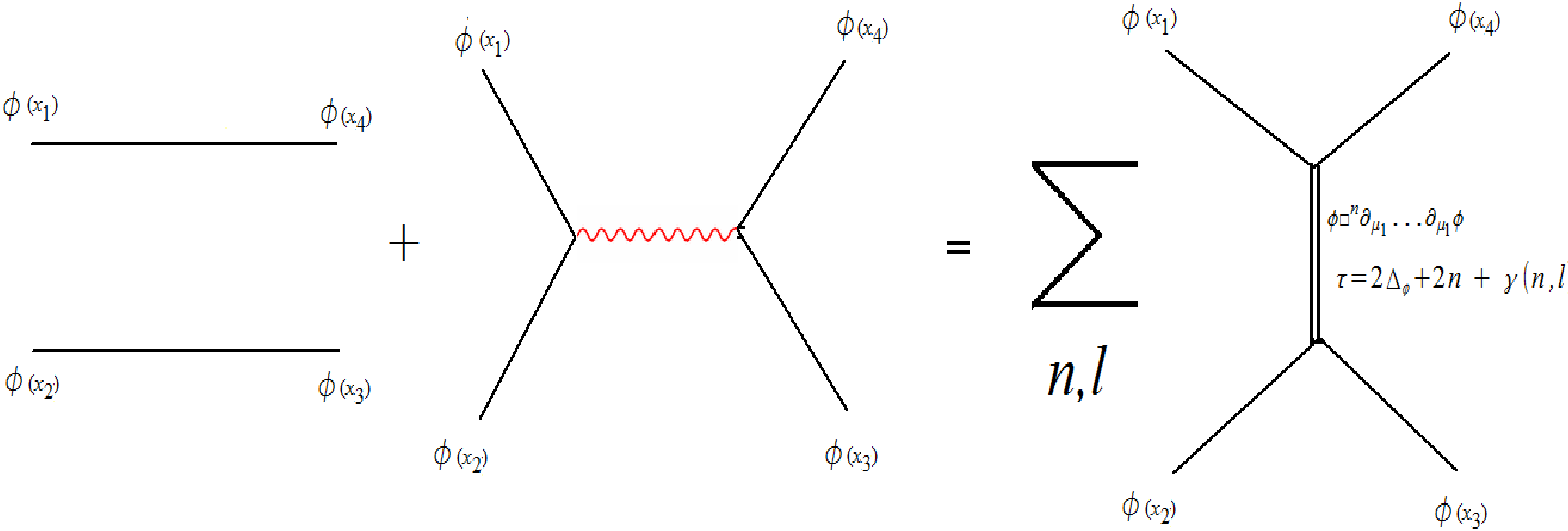
$$\sum_l C_{2\Delta_\phi + l, l} u^{-\Delta_\phi + \tau_m} \frac{\sqrt{l}\, 4^l}{\sqrt{\pi}} \left(\delta C + \gamma(0, l)\right) K_0(2 l \sqrt{z})$$

$$+ \ldots$$

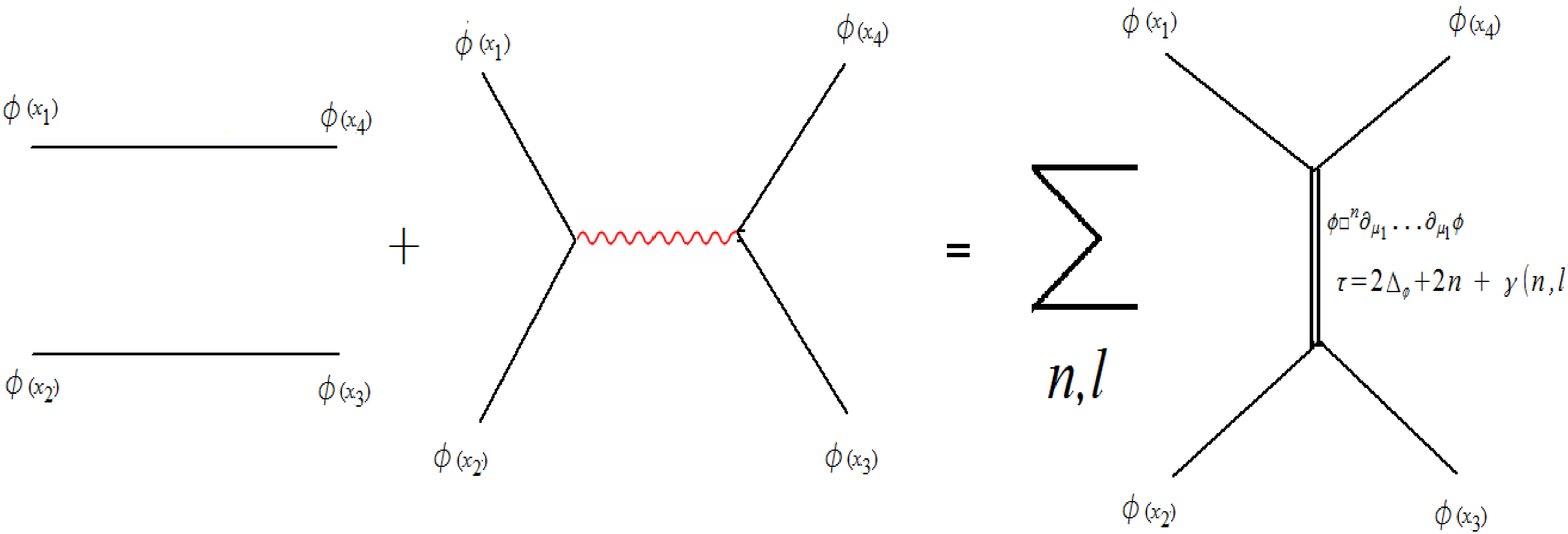
Main results: We have for large spin double trace operators, with n=0

$$\gamma(0,l) = -\frac{C_m}{l^{\tau_m}} \frac{2\Gamma^2(\Delta_\phi)\Gamma(\tau_m+2l_m)}{\Gamma^2(\Delta_\phi-\tau_m/2)\Gamma^2(\tau_m/2+l_m)}$$

$$\delta C(0,l) = -\frac{C_m}{l^{\tau_m}} \frac{2\Gamma^2(\Delta_\phi)\Gamma(\tau_m+2l_m)}{\Gamma^2(\Delta_\phi-\tau_m/2)\Gamma^2(\tau_m/2+l_m)}(\psi(\frac{\tau_m}{2}+l_m)+\gamma-\log 2)$$



$\phi(x_1)$
$\phi(x_4)$
$\phi(x_2)$
$\phi(x_3)$
+
$\phi(x_1)$
$\phi(x_4)$
$\phi(x_2)$
$\phi(x_3)$
=
$\sum_{n,l}$
$\phi(x_1)$
$\phi(x_4)$
$\phi \Box^n \partial_{\mu_1} \ldots \partial_{\mu_l} \phi$
$\tau = 2\Delta_\phi + 2n + \gamma(n,l)$
$\phi(x_2)$
$\phi(x_3)$



$$1 \quad + \quad T_{\mu\nu} \quad = \quad \sum_{\text{double trace with corrections}} C_{\Delta,l}\, g_{\Delta,l}$$

Part 2:
Bootstrap in
Mellin space

Mellin transform of a function:

$$A(u,v) = \int u^s v^t \Gamma^2(-t)\Gamma^2(s+t)\Gamma^2(\Delta_\phi - s)\, M(s,t)\, ds\, dt$$

[Mack (2009); Penedones (2010); Fitzpatrick, Kaplan, Penedones, Raju, van Rees (2001), ...]

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four point fn

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle$$

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four point fn

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle$$

Mellin amplitude

$$M_s(s,t) = \sum_{\Delta,l} \int d\nu \, C^2_{\Delta,l} \;\; m(\nu,s) m(-\nu,s) \; P_\nu(s,t)$$

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$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle$$

Mellin amplitude

$$M_s(s,t) = \sum_{\Delta,l} \int d\nu\, C^2_{\Delta,l}\;\; m(\nu,s)m(-\nu,s)\; P_\nu(s,t)$$

$$\text{where} \quad m(\nu,s) = \frac{\Gamma(\frac{h-\nu-l}{2}-s)\Gamma^2(\Delta_\phi - \frac{h-l-\nu}{2})}{(\Delta-h)-\nu} \times \text{analytic}$$

Mellin transform of a function:

OPE coefficient

Mack Polynomial

$$M_s(s,t) = \sum_{\Delta,l} \int d\nu \, C_{\Delta,l} \; m(\nu,s) m(-\nu,s) \; P_\nu(s,t)$$

where $\quad m(\nu,s) = \dfrac{\Gamma\left(\frac{h-\nu-l}{2}-s\right)\Gamma^2\left(\Delta_\phi-\frac{h-l-\nu}{2}\right)}{(\Delta-h)-\nu} \times \text{analytic}$

The unphysical term:

$$A(u,v) = \int u^s v^t \Gamma^2(-t)\Gamma^2(s+t)\Gamma^2(\Delta_\phi - s)\, M(s,t)\, ds\, dt$$

$$M_s(s,t) = \sum_{\Delta,l} \int d\nu\, C_{\Delta,l}\;\; m(\nu,s) m(-\nu,s)\, P_\nu(s,t)$$

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This s-pole gives unphysical terms
(absent in OPE)

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The unphysical term:

$$A(u,v) = \int u^s v^t \Gamma^2(-t)\Gamma^2(s+t)\Gamma^2(\Delta_\phi - s)\ M(s,t)\ ds\,dt$$

This *s*-pole gives unphysical terms
(absent in OPE)

Residue (shchematically)

$$\sum_l q_l^s(\Delta_\phi)\ Q_l(\Delta_\phi)\ \log u\ +\ q_l^s{}'(\Delta_\phi)\ Q_l(\Delta_\phi)$$

The unphysical term:

$$A(u,v) = \int u^s v^t \Gamma^2(-t)\Gamma^2(s+t)\Gamma^2(\Delta_\phi - s)\, M(s,t)\, ds\, dt$$

$$C_{\Delta,l}\, m(2\Delta_\phi + l - h, \Delta_\phi)\, m(-2\Delta_\phi - l + h, \Delta_\phi)$$

↑

Residue (shchematically)

$$\sum_l q_l^s(\Delta_\phi)\, Q_l(\Delta_\phi) \log u \; + \; q_l^s{}'(\Delta_\phi)\, Q_l(\Delta_\phi)$$

↓

$$P_\nu(\Delta_\phi, t)$$

Strategy:

[Aninda's talk]

$$A(u,v) = \int u^s v^t \Gamma^2(-t)\Gamma^2(s+t)\Gamma^2(\Delta_\phi - s)\, M(s,t)\, ds\, dt$$

Make crossing symm

$$M(s,t) = M_s(s,t) + M_t(s,t) + M_u(s,t)$$

$$\sum_l q_l^s(\Delta_\phi)\, Q_l(\Delta_\phi) \log u \; + \; q_l^s{}'(\Delta_\phi)\, Q_l(\Delta_\phi)$$

s-channel

Strategy:

[Aninda's talk]

$$A(u,v) \;=\; \int u^s v^t \Gamma^2(-t)\Gamma^2(s+t)\Gamma^2(\Delta_\phi - s)\; M(s,t)\; ds\,dt$$

↓ Make crossing symm

$$M(s,t) = M_s(s,t) + M_t(s,t) + M_u(s,t)$$

These polynomials are orthogonal

↑

$$\sum_l q_l^s(\Delta_\phi)\; Q_l(\Delta_\phi)\; \log u \;+\; q_l^s{}'(\Delta_\phi)\; Q_l(\Delta_\phi)$$

s-channel

Strategy:

[Aninda's talk]

$$A(u,v) = \int u^s v^t \Gamma^2(-t)\Gamma^2(s+t)\Gamma^2(\Delta_\phi - s)\, M(s,t)\, ds\, dt$$

$$M(s,t) = M_s(s,t) + M_t(s,t) + M_u(s,t)$$

$$\sum_l q_l^t(\Delta_\phi)\, Q_l(\Delta_\phi) \log u \; + \; q_l^t{}'(\Delta_\phi)\, Q_l(\Delta_\phi)$$

expand in Q_l

$$\sum_l q_l^u(\Delta_\phi)\, Q_l(\Delta_\phi) \log u \; + \; q_l^u{}'(\Delta_\phi)\, Q_l(\Delta_\phi)$$

$$\sum_l q_l^s(\Delta_\phi)\, Q_l(\Delta_\phi) \log u \; + \; q_l^s{}'(\Delta_\phi)\, Q_l(\Delta_\phi)$$

s-channel

Strategy:

[Aninda's talk]

$$A(u,v) \;=\; \int u^s v^t \Gamma^2(-t)\Gamma^2(s+t)\Gamma^2(\Delta_\phi - s)\; M(s,t)\, ds\, dt$$

$$M(s,t) = M_s(s,t) + M_t(s,t) + M_u(s,t)$$

$$\sum_l q_l^t(\Delta_\phi)\, Q_l(\Delta_\phi)\log u \;+\; q_l^t{}'(\Delta_\phi)\, Q_l(\Delta_\phi)$$

$$q_l^t(\Delta_\phi) = \sum_{\Delta',l'} C_{\Delta',l'} \int dt\, d\nu\; m(\nu,\Delta_\phi)\, m(-\nu,\Delta_\phi)\; P(0,t+\Delta_\phi)\; Q_l(\Delta_\phi)$$

$$= q^u(\Delta_\phi)$$

Strategy: **Equate sum of unphysical terms in all channels to 0**

$$q^{s}(\Delta_{\phi})+q^{t}(\Delta_{\phi})+q^{u}(\Delta_{\phi}) = 0 \longrightarrow \text{gives anomalous dimensions}$$

$$q'^{s}(\Delta_{\phi})+q'^{t}(\Delta_{\phi})+q'^{u}(\Delta_{\phi})+q_{disconnected} = 0$$

$$\longrightarrow \text{gives OPE coefficients}$$

$$q^{s}(\Delta_{\phi}) = C_{\Delta,l}\, m(2\Delta_{\phi}+l-h,\Delta_{\phi})\, m(-2\Delta_{\phi}-l+h,\Delta_{\phi})$$

$$q^{t}_{l}(\Delta_{\phi}) = \sum_{\Delta',l'} C_{\Delta',l'} \int dt\, d\nu\; m(\nu,\Delta_{\phi})\, m(-\nu,\Delta_{\phi})\; P(0,t+\Delta_{\phi})\; Q_{l}(\Delta_{\phi})$$

$$= q^{u}(\Delta_{\phi})$$

Large spin approximations

$$Q_l(\Delta_\phi) = \frac{2^l(\Delta_\phi)_l^2}{(2\Delta_\phi + l - 1)_l}\, {}_3F_2(\{-l, 2\Delta_\phi + l - 1, \Delta_\phi + t\}, \{\Delta_\phi, \Delta_\phi\})$$

$$q_l^t(\Delta_\phi) = \sum_{\Delta', l'} C_{\Delta', l'} \int dt\, d\nu\; m(\nu, \Delta_\phi) m(-\nu, \Delta_\phi)\; P(0, t + \Delta_\phi)\; Q_l(\Delta_\phi)$$

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$l \gg s, t$ (large spin)

$$Q_l(\Delta_\phi) \approx \frac{2^l \, l^{-\Delta_\phi - t} \, \Gamma^2(\Delta_\phi + l) \Gamma(\Delta_\phi - t + l - 1)}{\Gamma^2(-t) \Gamma(2\Delta_\phi + 2l - 1)}$$

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Only one pole each of t & ν will contribute at large l

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$$q_l^t(\Delta_\phi) = C_m 2^l e^l l^{-l-\tau_m} f(\tau_m, l_m) + \text{ subleading terms}$$

Only one operator contributes
in *t*-channel

Large spin approximations

In *s*-channel we have

$$q_l^s(\Delta_\phi) = \frac{\Gamma\left(\frac{-\Delta+2\Delta_\phi+l}{2}\right)}{2\Delta_\phi+l-\Delta} \times \text{ other terms}$$

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↓ Assume no minimal twist operator in *t*-channel

$$\text{Solution: } \Delta = 2\Delta_\phi + l + 2n$$

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↓ In presence of minimal twist operator

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Only one operator contributes
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Large spin approximations

For n=0

$$q_l^s(\Delta_\phi) = \frac{2^l e^l l^{-l} \gamma}{\Gamma^2(\Delta_\phi)}$$

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Large spin approximations

For n=0

$$q_l^s(\Delta_\phi) = \frac{2^l e^l l^{-l} \gamma}{\Gamma^2(\Delta_\phi)}$$

$$q_l^s + q_l^t + q_l^u = 0 \quad \rightarrow \quad \gamma = -\frac{C_m}{l^{\tau_m}} \frac{2\Gamma^2(\Delta_\phi)\Gamma(\tau_m + 2l_m)}{\Gamma^2(\Delta_\phi - \tau_m/2)\Gamma^2(\tau_m/2 + l_m)}$$

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When minimal twist operator is present

$$\delta C(0,l) = -\frac{C_m}{l^{\tau_m}} \frac{2\Gamma^2(\Delta_\phi)\Gamma(\tau_m + 2l_m)}{\Gamma^2(\Delta_\phi - \tau_m/2)\Gamma^2(\tau_m/2 + l_m)} \left(\psi\left(\frac{\tau_m}{2} + l_m\right) + \gamma - \log 2\right)$$

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$$\gamma_l - 2\gamma_\phi = \frac{\alpha_0(l)+\alpha_1(g)\log(l)+\alpha_2(g)(\log l)^2+\ldots}{l^{d-2}}$$

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Where g is a small parameter and,

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Where *g* is the small parameter and,

$$\Delta_\phi = \frac{d-2}{2} + g\gamma_\phi^{(1)} + g^2\gamma_\phi^{(2)} + \ldots$$

$$\Delta_{l=0} = d - 2 + g\gamma_0^{(1)} + g^2\gamma_0^{(2)} + \ldots$$

For ϕ^4 in d=4-ε, we have for higher spin operators,

$$\gamma_l = \frac{\epsilon^2}{54}\left(1-\frac{6}{l(l+1)}\right)$$

$$+\,\epsilon^3\left(\frac{-756+218\,l^3+109\,l^4+l\,(-816-432\,H_{l-1})+l^2(373-432\,H_{l-1})}{5832\,l^2(l+1)^2}\right)$$

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$$\gamma_l - 2\gamma_\phi = \frac{\epsilon^2}{9 l^2} + \frac{\epsilon^3}{27 l^2}\log l + \ldots$$

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Behavior agrees with the predictions of Alday-Zhiboedov, and matches with the results of previous slide

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Summary

- Taking the lightcone limit, one can obtain anomalous results and OPE coefficients for large spin operators, in a strongly coupled theory.
- We obtain identical results from the new bootstrap prescription in mellin formalism, more easily.
- We also get new results for large spin operators for a class of weakly coupled theories.
- They match with known predictions