Hairy Black Holes in a Box

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Based on JHEP.2016:139 and ongoing work
(With Chethan Krishnan and Pallab Basu)

Motivation

- *No-Hair theorems*: Loosely, Black Holes are completely specified by global charges, like mass, charge, and angular momentum.
- The lack of hair prevents a classical understanding of the Black Hole entropy, which has consequences for the information paradox.
- No-hair theorems can been evaded in some circumstances, for eg. attractor black holes, changing the asymptotics, etc.
- The Hairy Black Hole in AdS was the basis of setting up the holographic superconductor.$_{[Hatnoll, Herzog, Horowtiz'08]}$
- The major difference between asymptotically flat space and AdS has a timelike boundary, like a box, and it only takes finite proper time for signal to get to the boundary.
- A spherical box in flat space has a timelike boundary, just like global AdS.
- **So: Can we evade flat space No-Hair theorem by putting the black hole in a box?**

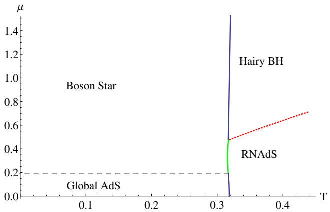
Full phase diagram of global-AdS_4 [Basu,Krishnan,BS:JHEP2016:139]

- To build intuition for the box, we looked at global-AdS_4.
- The Poincaré patch case is analysed in studying the holographic superconductor.[Hartnoll,Herzog,Horowitz(2008), Horowitz,Way(2010)]
- There are three non-trivial solutions that are asymptotically global-AdS:
  1. RNAdS black hole
  2. Boson star: Horizon-less (zero temperature) configuration with a non-trivial scalar profile.
  3. Hairy black hole: RN black hole which supports a non-trivial scalar hair.

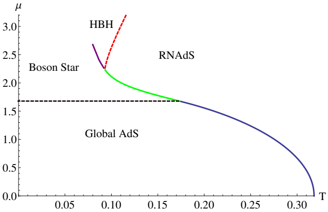
Full phase diagram of global-AdS_4

- Global-AdS is unstable to developing a scalar hair to form boson star above some chemical potential, μ_{bs}, decided by q, the coupling between scalar and gauge field.
- RNAdS develops scalar hair above some μ_{bh} which is a function of q and r_h.
- Boson star and hairy black hole are second order phase transitions from global-AdS and RNAdS respectively.
- The phase diagram is evaluated in the grand canonical ensemble.
- The phase diagrams are different for $q > 1$ and $q \leq 1$

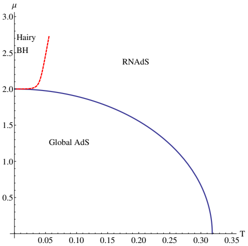
Full phase diagram of global-AdS_4



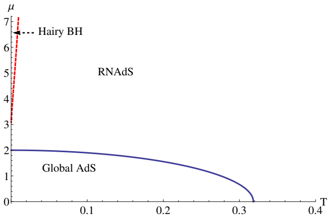
$q = 10$



$q = 1.2$



$q = 1$



$q = \frac{1}{2}$

Flat space: some definitions

- The Einstein-Maxwell-Scalar action with no cosmological constant is given by,

$$S = -\frac{1}{16\pi}\int d^4x\sqrt{-g}\left(R - F^{ab}F_{ab}\right) - \frac{1}{8\pi}\oint d^3x\sqrt{\gamma}\kappa + \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(-|\nabla\psi - iqA\psi|^2\right). \quad (1)$$

- We will be looking at spherically symmetric time-independent solutions, with the ansatz

$$ds^2 = -g(r)h(r)dt^2 + \frac{dr^2}{g(r)} + r^2 d\Omega_2^2,\ A(r) = \phi(r)dt,\ \psi = \psi(r). \quad (2)$$

- Scaling symmetries: There are two scaling symmetries for the equations of motion under this ansatz
  1. $r \to ar$: used to set $r_b = 1$.
  2. $h \to a^2 h, \phi \to a\phi, t \to \frac{t}{a}$: used to set $g_{tt}|_{r_b} = 1$. This sets $a = (g(r_b)h(r_b))^{-\frac{1}{2}}$.

Flat space: some definitions

- The quasilocal energy can be defined by [Brown and York,1992]

$$E = -\frac{1}{8\pi} \lim_{r \to r_b} \int_{S^2} d^2\theta \sqrt{\sigma}\, r^2 \left(k - k_0\right) \tag{3}$$

  where k (k_0) is the extrinsic curvature of the two sphere embedded in the geometry (flat space).
- The temperature of the system is defined by demanding that there are no conical singularities in going to the Euclidean signature, which gives

$$T = \frac{1}{4\pi} \frac{g'(r_h)\sqrt{h(r_h)}}{\sqrt{g(r_b)h(r_b)}}. \tag{4}$$

- The chemical potential is defined as the value of gauge field at $r = r_b$, i.e. $\mu = \phi(r_b)$.
- The charge of the system is defined by

$$Q = \lim_{r \to r_b} \frac{1}{4\pi} \int_{S^2} F_{\mu\nu} t^\mu \eta^\nu \sqrt{\sigma} d^2\theta. \tag{5}$$

Scalar-less solutions

- The simplest possible non-trivial solution in the box, like in the case of asymptotically flat space, is the Schwarzschild black hole, with

$$g(r) = 1 - \frac{r_h}{r}, \;\; h(r) = \frac{1}{g(r_b)}, \;\; \phi(r) = 0, \;\; \psi(r) = 0. \tag{6}$$

  This solution has $\mu = 0 = Q$, and the temperature is given by

$$T = \frac{\left(g'(r_h)^2 \;\, h(r_h)\right)^{\frac{1}{2}}}{4\pi} = \frac{\sqrt{r_b}}{4\pi r_h \sqrt{r_b - r_h}} \tag{7}$$

- This shows that black hole has two solutions for a given temperature above T_{min}, which is the minimum possible temperaure.
- The Brown-York quasilocal energy gives

$$E = \frac{1}{2}\left(r_b - r_b\sqrt{1 - \frac{r_h}{r_b}}\right) \tag{8}$$

Scalar-less solutions

- The free energy for Schwarzschild solution is

$$F = r_b - r_b\sqrt{1 - \frac{r_h}{r_b}} + \frac{r_h\sqrt{r_b}}{4\sqrt{r_b - r_h}}. \tag{9}$$

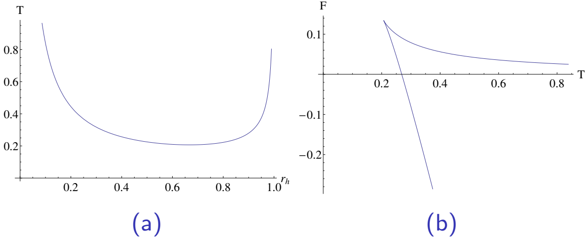


Figure : Schwarzschild plots

- These figures are analogous to the Hawking-Page transition.

Scalar-less solutions

- The RN black hole is studied by Braden, et al., in the pre-AdS/CFT era.
- We do a more systematic study based on free energy which is the language that is more familiar from AdS/CFT.

$$g(r) = 1 - \frac{(1+\epsilon)r_h}{r} + \frac{\epsilon r_h^2}{r^2},\ \phi(r) = \frac{\sqrt{\epsilon} r_h}{\sqrt{g(r_b)}} \left(\frac{1}{r_h} - \frac{1}{r} \right),\ h(r) = \frac{1}{\sqrt{g(r_b)}}.$$

where $r_{in} = \frac{Q^2}{r_h} = \epsilon r_h$ with $0 \le \epsilon \le 1$.

- The temperature,chemical potential and free energy are given by

$$T = \frac{1}{4\pi} \frac{(1-\epsilon)}{r_h \sqrt{g(r_b)}},\ \mu = \frac{\sqrt{\epsilon} r_h}{\sqrt{g(r_b)}} \left(\frac{1}{r_h} - \frac{1}{r_b} \right), \tag{11}$$

$$F = \frac{1}{\sqrt{g(r_b)}} \left(r_b \sqrt{g(r_b)} - r_b + \frac{3r_h}{4} + \frac{\epsilon r_h}{4} \right) \tag{12}$$

Scalar-less solutions

- The $F = 0$ equations gives the following solutions

$$\epsilon = \frac{9r_h - 8r_b}{r_h}, 1. \tag{13}$$

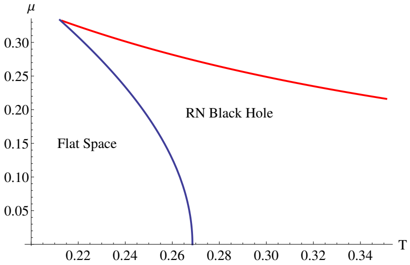


Figure : Phase diagram of Einstein-Maxwell system in a box

- Along the $F = 0$ curve, larger μ black holes in AdS become zero sized, whereas, here it approaches the size of the box.

Scalar-less solutions

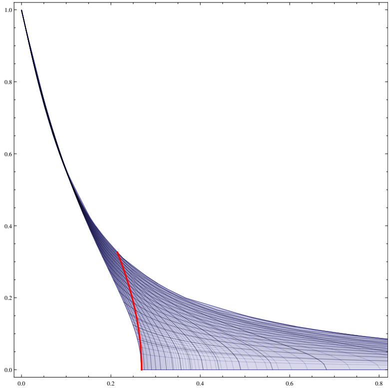


Figure : The parameter space for large black holes. Red line indicates the $F = 0$ curve.

The no-hair theorem, and How the box makes things different

- We can start with a series expansion about infinity in the asymptotically flat space, and impose equations of motion.
- This will lead to setting the expansion coefficient of $\psi(r)$ to zero, order by order, and $h(r) =$ const., which is set to 1, and the general solution becomes the RN Black hole.
- This is our version of the no-hair theorem.
- Asymptotically flat spaces thus do not support a scalar profile.
- However, it turns out to be not the case if the boundary is at some finite r_b.
- The series expansions are determined in terms of six coefficients,

$$\psi_0^b = \psi(r_b),\ \psi_1^b = \psi'(r_b),\ g_0^b = g(r_b),$$
$$\phi_0^b = \phi(r_b),\ \phi_1^b = \phi'(r_b),\ h_0^b = h(r_b) \qquad (14)$$

Some comments on the scalar

- The primary difference is that there is no smooth $r_b \to \infty$ when there is a scalar profile.
- The quasilocal energy definition turns out to be not a good definition of the mass of the scalar.
- When there is no scalar, the information if we truncate the space at finite r_b is the same as that of the asymptotic case.
- We will take the definition of free energy to be

$$F = -T \log \mathcal{Z} = T S_{cl}. \tag{15}$$

- This allows us to find the full set of phase diagrams.

Full set of solutions

- With the scalar turned on, we get the boson star and hairy black hole solutions.
- In all cases we set the boundary condition $\psi_0^b = 0$ (Dirichlet).
- For boson star, the derivatives of all the functions are set to zero at $r = 0$, and also $g(0) = 1$, which leaves three parameters $\psi(0), \phi(0), h(0)$.
- $h(0)$ can be chosen arbitrarily, as we will rescale in the end.
- For a given $\psi(0)$, $\phi(0)$ is chosen such that $\psi_0^b = 0$.
- For the hairy black hole we have to set

$$g(r_h) = 0,\ \phi(r_h) = 0. \tag{16}$$

- The set of solutions are parametrized by $\psi(r_h), \phi'(r_h)$ and $h(r_h)$, which is again arbitrary, and $\phi'(r_h)$ is determined for given values of $\psi(r_h)$.

Boson Star

- The boson star also has zero temperature, and smaller free energy than flat space, and is a second order phase transition from the latter.
- The instability of flat space to become boson star can be computed analytically, as the backreaction for $\psi(r) \ll 1$.
- Keeping terms to linear order in $\psi(r)$, which gives the flat box, and

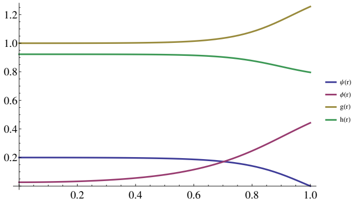
$$\psi(r) = \psi(0)\frac{\sin(\mu q r)}{r}. \tag{17}$$

- From the boundary condition at r_b we will get the instability point

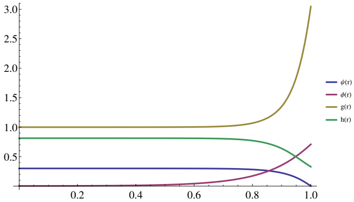
$$\mu\, q\, r_b = n\pi, \;\; n = 1, 2, 3, \dots \tag{18}$$

- We will be looking only at the $n = 1$ modes.
- The fully backreacted solutions are computed numerically.

Plots for the fully backreacted boson star solutions



$q = 40, \psi(0) = 0.2$



$q = 40, \psi(0) = 0.3$

Figure : Profiles of $\psi(r), \phi(r), g(r), h(r)$.

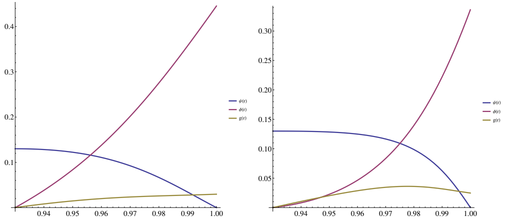
Hairy Black Hole

- The hairy black hole is a finite temperature solution.
- It has a lower free energy than the RN black hole with the same values of (T, μ).
- The hairy black hole is a second order phase transition from the RN black hole.
- The instability of RN black hole can be determined by a probe computation, which is dependent on q, r_h

$$\psi''(r) + \frac{(Q^2 - 2r_h\, r + r_h^2)}{(r - r_h)\,(Q^2 - r_h\, r)}\psi'(r) + \frac{q^2 Q^2 r^2}{(Q^2 - r_h r)^2}\psi(r) = 0. \qquad (19)$$

- The fully backreacted solutions for a given (q, r_h) will have a higher μ and a lower T as we increase $\psi(r_h)$.
- The fully backreacted solutions are computed numerically.

Plots for the fully backreacted hairy black hole solutions



$q = 40$ $q = 100$

Figure : Profiles of $\psi(r), \phi(r), g(r)$ with $r_h = 0.93, \psi(r_h) = 0.13$

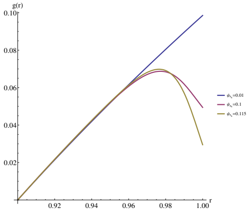


Figure : Profile of $g(r)$ for $\psi(r_h) = 0.01, 0.1, 0.115$

The Phase diagrams

- The phase diagrams are evaluated by comparing the free energies of the different phases.
- There are at most four phases in the diagram.
- The phase diagrams are dependent on q.
- Since the metric is rescaled, the temperatures of the hairy black hole is the same as that of flat space.
- This makes the background subtraction using flat space straightforward.
- There are no divergences, but the subtraction is still done for a common reference.
- There are three types of phase diagrams.

The Phase diagrams: $q_1 < q < \infty$

- In this case all the four phases exist.

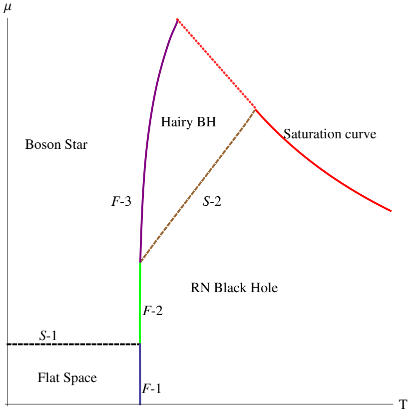
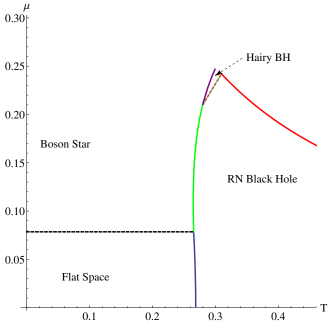
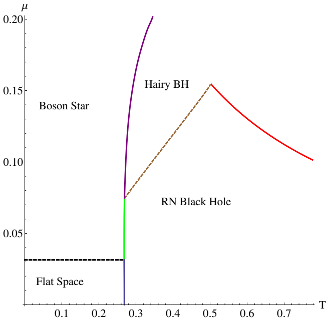


Figure : The sturcture of phase diagram for $q_1 < q < \infty$.

The Phase diagrams: $q_1 < q < \infty$



$q = 40$



$q = 100$

- The region of hairy black hole shrinks as we lower q.
- The hairy black hole phase is entirely absent below $q_1 \approx 36$.

The Phase diagrams: $q_2 < q < q_1$

- The phase diagram has only three of the four phases.
- The hairy black hole solutions happen to be at (T, μ) where the boson star is the favourable phase.

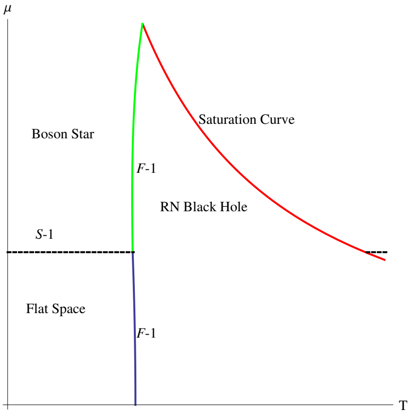
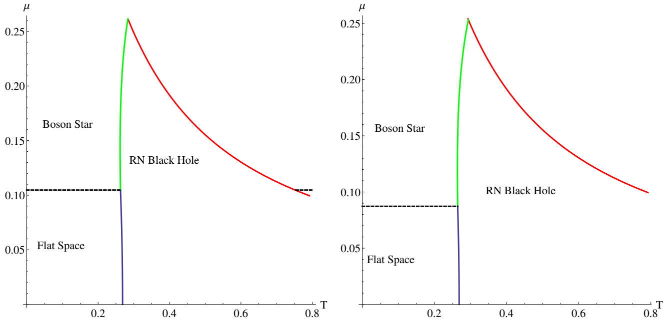


Figure : The sturcture of phase diagram for $q_2 < q < q_1$.

The Phase diagrams: $q_2 < q < q_1$



$q = 30$ $q = 36$

- The boson star to RN black hole transition curve shrinks as we lower q.
- The boson star phase for $q < q_2$ does not share a phase boundary with the RN black hole.

The Phase diagrams: $0 < q < q_2$

- The phase diagram has only three of the four phases.
- The phase diagram for the RN black hole is the same as the scalar-less case.
- The boson star happens only at a larger μ compared to the RN black hole.

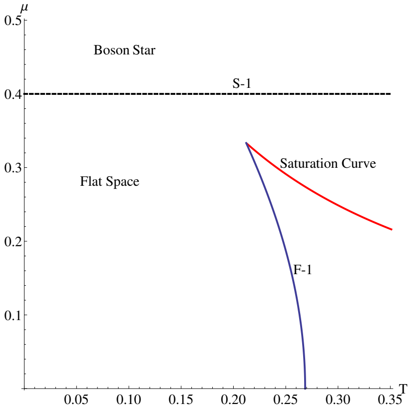


Figure : The sturcture of phase diagram for $0 < q < q_2$.

Key Takeaway Points

- We consider the Einstein-Maxwell system in a box (extending the work of Braden et.al.) and do a detailed study of the phase diagram.
- We solve the fully backreacted Einstein-Maxwell-Scalar system in a box, and find hairy solutions.
- The hairy solutions are the Boson star and the Hairy Black Hole.
- We find that they can be thermodynamically favourable phases.