Day two

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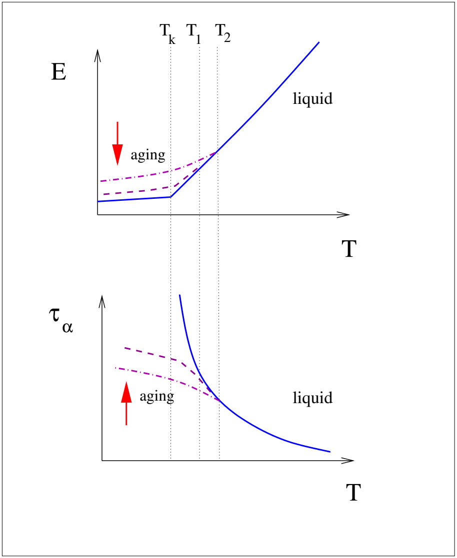
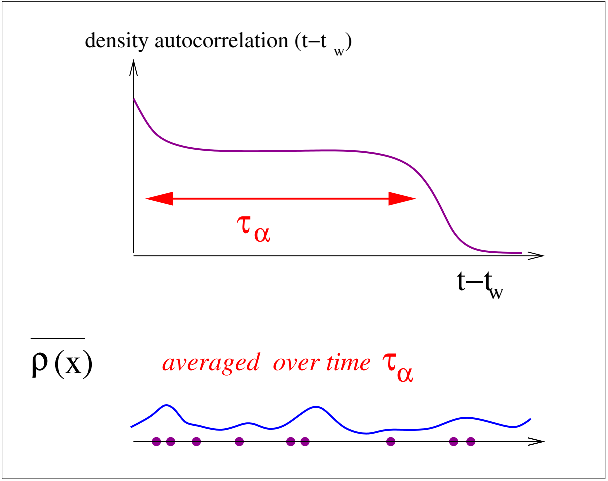
LPS-ENS, Paris

.

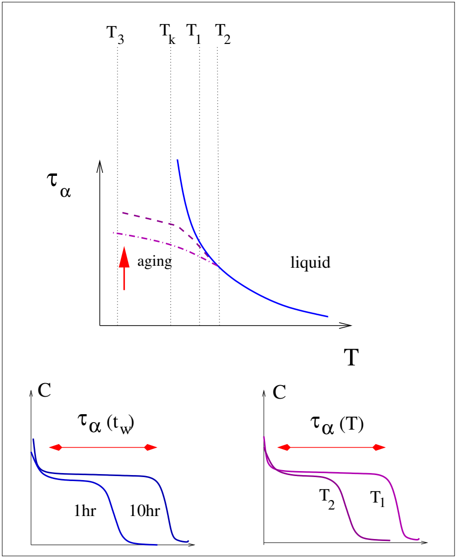
Glassy solid:

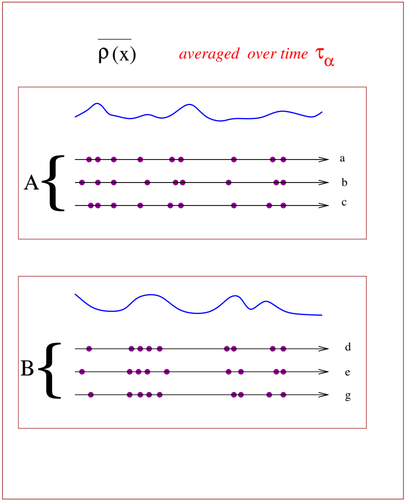


(less and less) transient density profiles

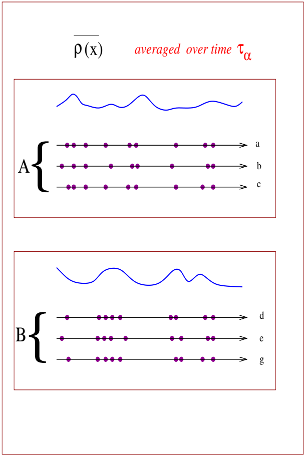


The α scale, in and out of equilibrium

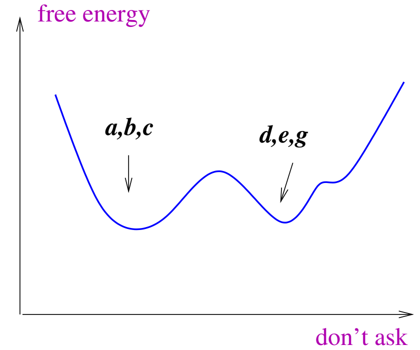




If $\tau_\alpha = \infty$ we have true states


$\overline{\rho(x)}$
averaged over time τ_α
A
a
b
c
B
d
e
g

landscape


free energy
a,b,c
d,e,g
don't ask

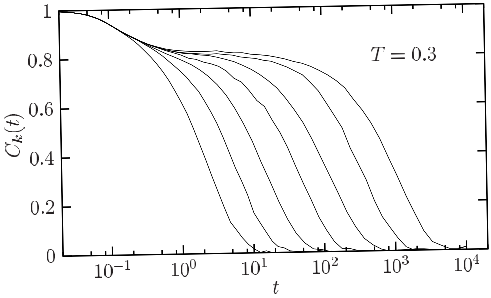
Glasses in this non-ideal world *age*

they are continuously evolving into more and more equilibrated configurations

or, alternatively, they are continuously nucleating better and better equilibrated phases

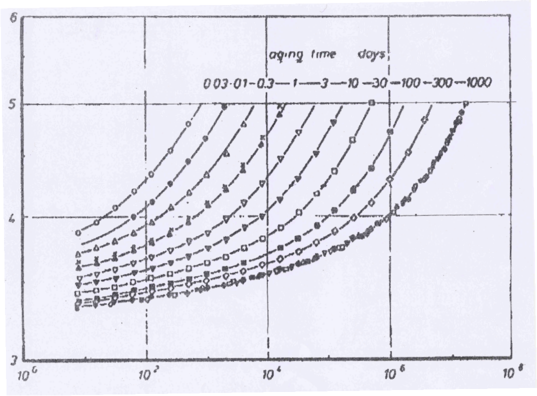
evolution becomes slower as time passes

Aging



Autocorrelation of density fluctuations (Lennard-Jones system, Kob-Barrat)

Aging



the stretching of a plastic bar, from an hour to four years old (Struik)

correlations and responses

A quantity:

$$\rho_k(t) = \int dx\ cos(kx)\rho(x,t)$$

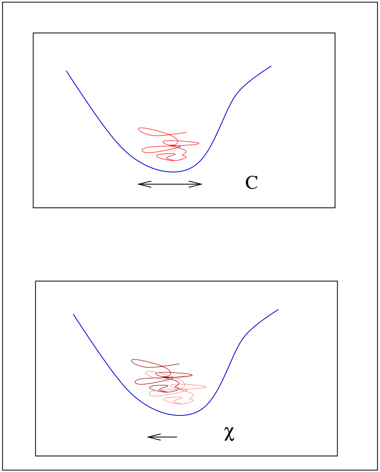
The correlation of its fluctuations:

$$C_k(t,t_w) = \langle \rho_k(t)\rho_k(t_w)\rangle$$

The response at time t to a conjugate field $h\rho_k$ acting from time $-\infty$ to t_w

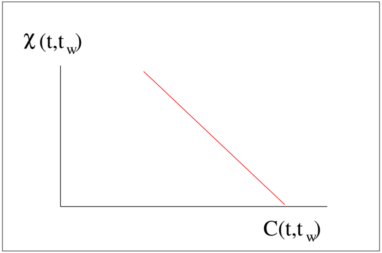
$$\chi(t,t_w) = \frac{\delta\langle\rho_k(t)\rangle}{\delta h}$$

In equilibrium **one should expect χ and C to be related:**



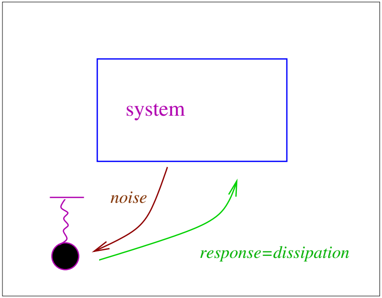
indeed:

$$T\chi(t,t_w) = C(t,t) - C(t,t_w) \qquad \text{or} \qquad T\frac{\partial\chi(t,t')}{\partial t'} = -\frac{\partial C(t,t')}{\partial t'}$$



the fluctuation-dissipation theorem says something important about thermalisation:

$$\mathbf{E}^* = E + x\rho_k + \frac{p_x^2}{2} + \frac{\omega^2 x^2}{2}$$



the fluctuation-dissipation theorem says something important about thermalisation:

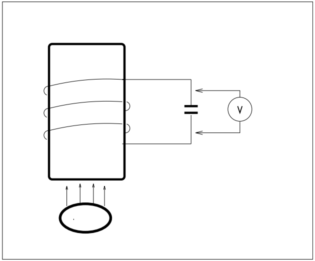
$$\mathbf{E}^* = E + x\rho_k + \frac{p_x^2}{2} + \frac{\omega^2 x^2}{2}$$

$$\ddot{\mathbf{x}} = -\omega \mathbf{x} - \rho_{\mathbf{k}}$$ **but** $$\rho_k = \underbrace{[\rho_k]_o}_{bare} + \underbrace{\int_\infty^t dt' \; \frac{\partial \chi(t,t')}{\partial t'} x(t')}_{back\ reaction}$$

$$\ddot{\mathbf{x}} = -\omega \mathbf{x} + [\rho_{\mathbf{k}}]_{\mathbf{o}} + \int_\infty^{\mathbf{t}} \mathbf{dt'} \; \frac{\partial \chi(\mathbf{t},\mathbf{t'})}{\partial \mathbf{t'}} \mathbf{x}(\mathbf{t'})$$

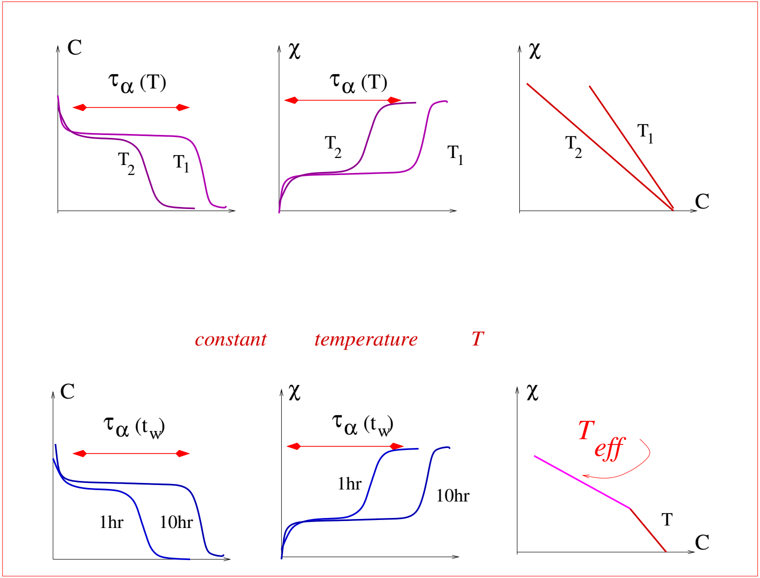
... an oscillator in a good thermal bath of temperature T
provided that $\langle [\rho_k]_o(t)[\rho_k]_o(t') \rangle = -T \frac{\partial \chi(t,t')}{\partial t'}$

A concrete example for a magnetic system:



cf. Grigera Israeloff

Aging in the correlations and the response



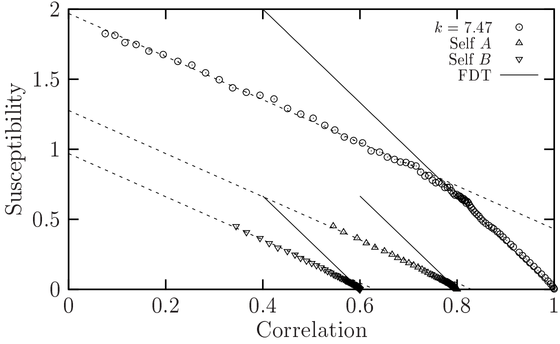
This two-temperature scenario was found analytically to hold for a family of solvable models

the Random First Order (mean-field) theory

... more about which, next lecture

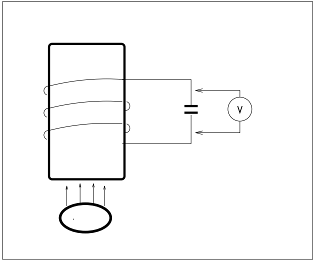
This, in turn, led to the search of effective temperatures in realistic models

all the temperatures in a given timescale should coincide!



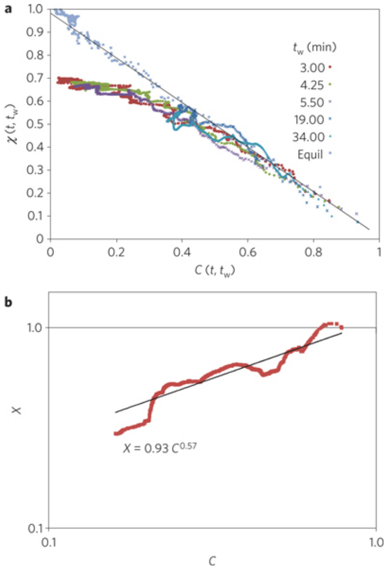
Binary Lennard-Jones glass, simulation L. Berthier and JL Barrat

...and in experimental systems:



cf. Grigera Israeloff

...and in experimental systems:

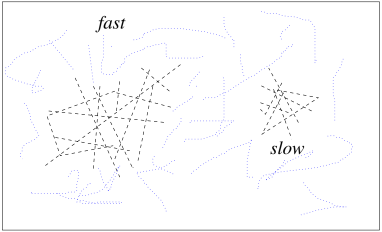


And poses the question of *fictive temperatures*, **introduced long ago** (Tool) **phenomenologically**

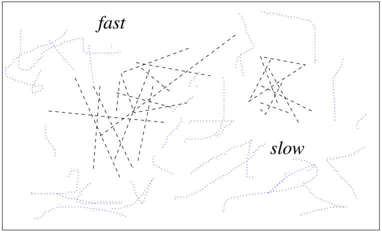
is this the same thing? It seems difficult to avoid identifying them, yet:

it is difficult to compare a definition within a theoretical framework with a phenomenological idea. Certainly many formulas applied to fictive temperatures do not apply to effective temperatures...

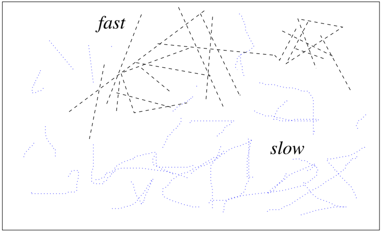
Dynamic heterogeneities and effective temperatures



Dynamic heterogeneities and effective temperatures



Dynamic heterogeneities and effective temperatures



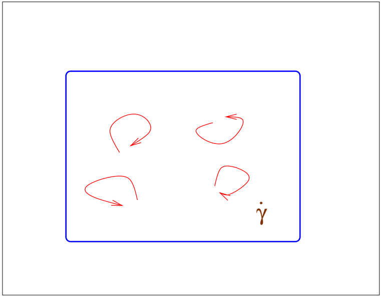
- **Paradoxically, slow regions are responsible for T_{eff}**

- **T_{eff} is not the result of heterogeneities left over from the quench**

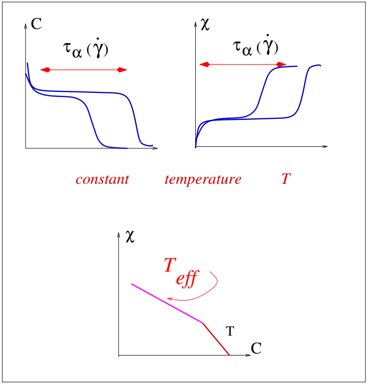
- **The reason why $T_{eff} \sim T_g$ is more subtle!**

Rheology and shear thinning:
the other face of aging

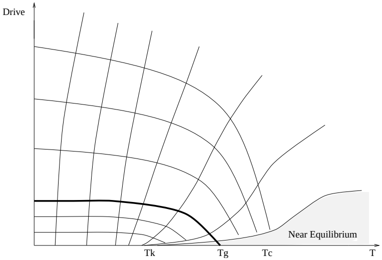
shear-thinning: stirring a liquid makes τ_α smaller, and kills aging



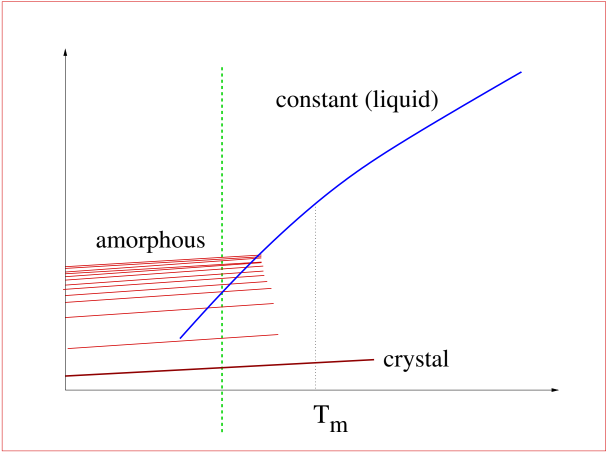
but surprisingly, effective temperatures are still there (for small $\dot{\gamma}$)



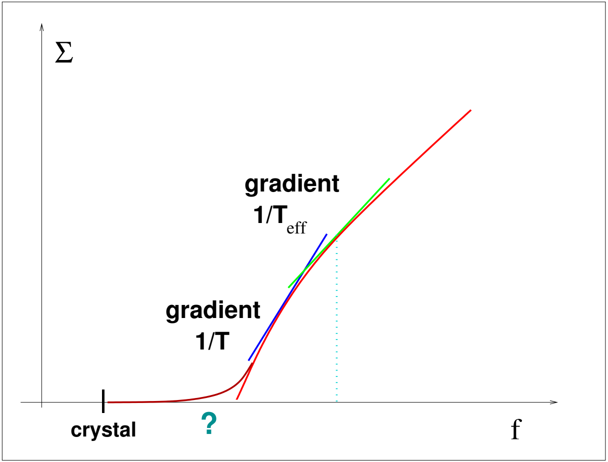
iso-τ_α and iso-T_{eff} lines



many states



with ‘flat’ exploration of states (why?)

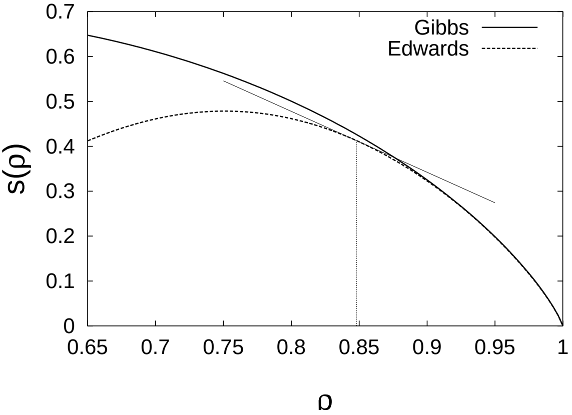


Edwards, (for granular matter)

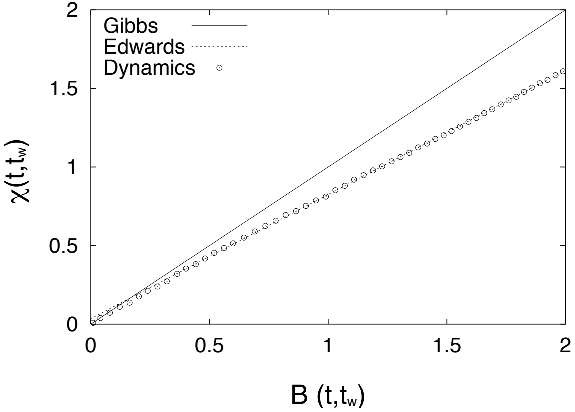
Amazingly, one begins to see laws for the out-of equilibrium regime, quite independent of what the system will do at infinite times

This is a new way of thinking, and it is not well established yet

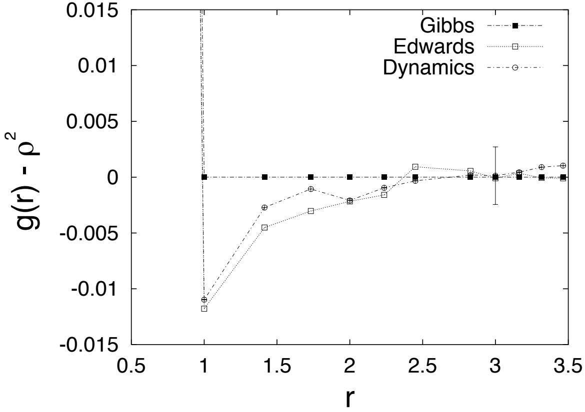
Flat ensemble for the Kob-Andersen model:



Number of blocked states



χ **vs** C



Pair function