Implications of Gravitational Radiation from Rapidly Spinning Neutron Stars

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Figure: The period and its derivative for various kind of neutron stars.
Spin frequency distribution of AMXPs
sharp cutoff near 730 Hz

Figure: The spin frequency distribution of accreting millisecond X-ray pulsars. **There is a sharp cutoff in the population for spins above 730 Hz.** RXTE has no significant selection biases against detecting oscillations as fast as 2 kHz, making the absence of fast rotators extremely statistically significant (Ref: D. Chakrabarty astro-ph:0809.4031v1).
Neutron star break-up and equation of state (EoS)
Constraining the M-R space for several EoSs

Figure : The solid curves are theoretical mass-radius relations for a variety of models for the equation of state for ultradense matter from. The dashed curves show the limits arising from breakup spin rates of 730 Hz and 2 kHz; the allowed phase space is to the right of the appropriate dashed curve. While a 2 kHz breakup rate is consistent with most equations of state, a 730 Hz breakup rate is inconsistent with most models and excludes the 8-12 km radius range usually inferred for a 1.4 M⊙ neutron star (Ref: D. Chakrabarty astro-ph:0809.4031v1).
Gravitational radiation as a natural reason
Spin frequency distribution of AMXPs

To describe the sharp cutoff in frequency distribution of AMXPs, Gravitational radiation turns out to be a natural reason. There are at least three well-known phenomena which can potentially be sources of GW from a rotating (*isolated*) neutron star:

- r-mode oscillations
- magnetically confined mountain
- bulk and/or crustal deformation (non-axis-symmetric)

Here, we will consider the effect of bulk deformation on GW radiation and its implication in LMXBs.
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GW from an isolated neutron star
Rapidly spinning neutron star

The gravitational wave strain:

\[ h_0 = \frac{4\pi^2 G I_3 f_{gw}^2}{c^4 r} \varepsilon \]  

with \( f_{gw} = 2\pi \omega_{rot} \) and ellipticity \( \varepsilon = \frac{l_1 - l_2}{l_3} \)

\[ h_0 = 1.06 \times 10^{-25} \left( \frac{\varepsilon}{10^{-6}} \right) \left( \frac{I_3}{10^{38} \text{km}^2} \right) \left( \frac{10 \text{ kpc}}{r} \right) \left( \frac{f_{gw}}{1 \text{ kHz}} \right)^2 \]  

NS typically have \( M \sim 1.4M_\odot \), \( R \sim 10 \text{ km} \) and \( I_3 \sim 10^{38} \text{ kg-m}^2 \), \( r = 10 \text{ kpc} \).
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Energy loss of a rotating NS due to GW wave emission:

\[
\frac{dE_{\text{rot}}}{dt} = \frac{-32G}{5c^5} \varepsilon^2 I_3^2 \omega_{\text{rot}}^6
\]  

In the limit of small asymmetry for homogeneous ellipsoid, i.e.,

\[R_1 \approx R_2 \Rightarrow \varepsilon = \frac{R_1 - R_2}{R_3} + O(\varepsilon^2)\]

Each of \(R_1\), \(R_2\) and \(R_3\) depends on the specific EoS, which can potentially be used to find any implication of the cutoff spin frequency for neutron star.
Spin down rate for a rotating NS due to GW wave emission and corresponding torque is:

\[
\dot{\omega}_{\text{rot}} = \frac{-32G}{5c^5} \varepsilon^2 I_3 \omega_{\text{rot}}^5 \tag{4}
\]

\[
\tau_{GW} = I_3 \dot{\omega}_{\text{rot}} = \frac{-32G}{5c^5} \varepsilon^2 I_3^2 \omega_{\text{rot}}^5 \tag{5}
\]

notice that: \( \tau_{GW} \propto I_3^2 \varepsilon^2 \omega_{\text{rot}}^5 \).

Therefore, I have tried to estimate the effect of different EoSs on \( I_3 \).
Using the “Rapidly Rotating Neutron Star” (an open source code), I have computed the stellar structure to find out the $l_3$ along the spin-axis for a stellar spin-frequency $\sim 700$ Hz.

<table>
<thead>
<tr>
<th>Equation of state (EoS)</th>
<th>$l_3$ (in $10^{38}$ kg m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>eosA</td>
<td>0.49</td>
</tr>
<tr>
<td>eosB</td>
<td>0.25</td>
</tr>
<tr>
<td>eosC</td>
<td>1.28</td>
</tr>
<tr>
<td>eosFPS</td>
<td>0.92</td>
</tr>
</tbody>
</table>

eosA -> PANDHARIPANDE NEUTRON: A&B EOS A  
eosB -> PANDHARIPANDE HYPERON: A&B EOS B  
eosC -> BETHE-JOHNSON MODEL 1: A&B EOS C  
eosFPS -> Lorenz, Ravenhall and Pethick, 1993, PRL 70,379

In this results, I DID NOT consider any magnetic field.
Using the “Lorene” (another open source code), I have computed the stellar structure to find out the $R_1$ (along) and $R_2$ (perpendicular to) the magnetic dipole-axis for a non-rotating neutron star.

<table>
<thead>
<tr>
<th>Central B-field</th>
<th>Magnetic/fluid pressure</th>
<th>$(R_1 - R_2)/R_3$</th>
</tr>
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<tbody>
<tr>
<td>$2.15 \times 10^{17}$ (G)</td>
<td>$1.0 \times 10^{-02}$</td>
<td>$1.378 \times 10^{-02}$</td>
</tr>
<tr>
<td>$2.18 \times 10^{16}$ (G)</td>
<td>$1.0 \times 10^{-04}$</td>
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</tr>
<tr>
<td>$2.18 \times 10^{14}$ (G)</td>
<td>$1.0 \times 10^{-08}$</td>
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<tr>
<td>$2.18 \times 10^{12}$ (G)</td>
<td>$1.0 \times 10^{-12}$</td>
<td>numerical error</td>
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In this result, I DID NOT consider any spin of neutron star.
Spin down rate for a rotating NS due to GW wave emission and corresponding torque is:

\[
\dot{\omega}_{\text{rot}} = \frac{-2M_B^2\omega_{\text{rot}}^3}{3c^3 I_3} \sin^2 \theta
\]  

(6)

\[
\tau_{EM} = I_3 \dot{\omega}_{\text{rot}} = \frac{-2M_B^2\omega_{\text{rot}}^3}{3c^3} \sin^2 \theta
\]  

(7)

notice that: \(\tau_{EM} \propto M_B^2\omega_{\text{rot}}^3\).

Therefore, smaller the magnetic-field smaller the \(\tau_{EM}\).
Magnetars Vs LMXBs
Comparison between $\tau_{EM}$ and $\tau_{GW}$

- Note that, the $M_B$ of a magnetar is generally $\gtrsim 10^5$ times larger than that of a NS in LMXBs.

- But, the spin frequency of magnetar is generally $\lesssim 10^{-3}$ times smaller than that of a NS in LMXBs.

- $\tau_{EM} (\text{magnetar})/\tau_{EM} (\text{LMXB}) \sim 10^{10} \times 10^{-9} \simeq 10$

- $\tau_{GW} (\text{magnetar})/\tau_{GW} (\text{LMXB}) \sim 10^{10} \times 10^{-15} \simeq 10^{-5}$

The result is quite conservative; since inside the bulk of a NS in LMXB, magnetic field can be still higher although on surface it is quite small due to field burial.
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Summary
GW may be a dominant mechanism

- As opposed to magnetars, GW may be a dominant mechanism to limit the large spin of the NS in a LMXB.

- In such case, EoSs play a dominant role in determining the cutoff frequency for a distribution of rapidly spinning NSs, iff other effects of GW is negligible.

- If the discussed scenario holds good, we can expect to potentially rule out some of the EoSs by matching the predicted cutoff frequency for each EoS with the observed value.
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THANK YOU for your kind attention.