Crossover of Correlation Functions near a Quantum Impurity in a Tomonaga-Luttinger Liquid

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Collaborators

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- Ref.: arXiv:1805.05006

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Ministry of Science and Technology
NCTS

Tomonaga-Luttinger Liquid

- Theory
  - Characterized by the Luttinger parameter g.
  - Critical, with divergent correlation length.
  - Translational/Scale invariance.
  - Power law decay of the correlation functions.

- Numerical study (DMRG)
  - Finite size calculation (OBC, PBC)
    - Finite size effects, finite size scaling.
    - Boundary effects.
  - Infinite size calculation
    - Take advantage of the translational/scale invariance.
    - iDMRG/sMERA.

Tomonaga-Luttinger Liquid + Impurity

- Finite size calculation (OBC, PBC)
  - Very large size is needed.
- Infinite size calculation
  - Broken translational invariance (~~iMPS~~).

- Boundary/Impurity calculation
  - bMERA
  - iMPS with ``infinite boundary conditions (IBC).''
    - H. Phien, G. Vidal, I. P. McCulloch, PRB 86, 245107 (2012).

Transport in a One-Channel Luttinger Liquid

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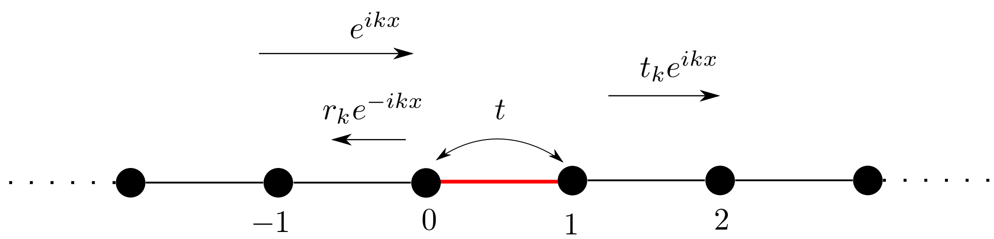
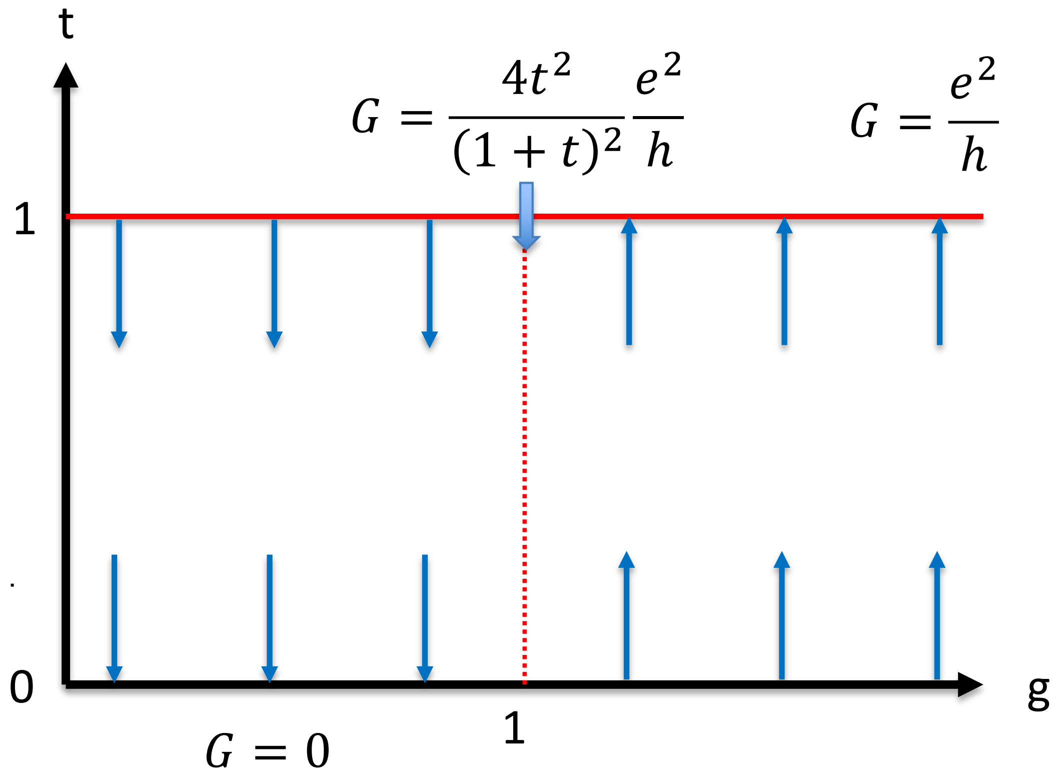
$$H_{wire} = \sum_j -(c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j) + V\left(n_j - \frac{1}{2}\right)\left(n_{j+1} - \frac{1}{2}\right)$$

$$H_{junc} = -t(c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j)$$

$$g = \frac{\pi}{2\arccos(-V/2)}$$





PHYSICAL REVIEW B VOLUME 58, NUMBER 9 1 SEPTEMBER 1998-I

Kondo effect in *XXZ* spin chains

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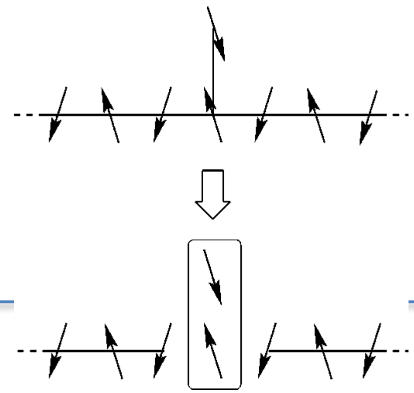
$$H_0 = J\sum_j \left(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y\right) + \Delta S_j^z S_{j+1}^z$$

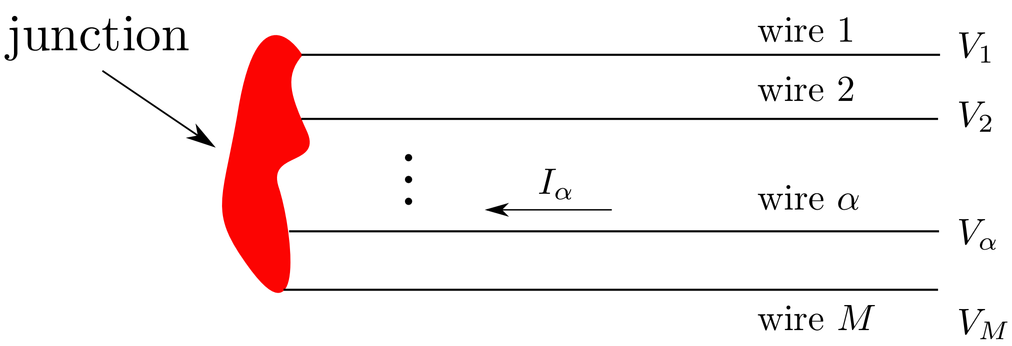
$$H_K = J_K\left(S_0^x S_{imp}^x + S_0^y S_{imp}^y + \Delta S_0^z S_{imp}^z\right)$$



PHYSICAL REVIEW B **85**, 045120 (2012)

General method for calculating the universal conductance of strongly correlated junctions of multiple quantum wires

Armin Rahmani,[1] Chang-Yu Hou,[2] Adrian Feiguin,[3] Masaki Oshikawa,[4] Claudio Chamon,[1] and Ian Affleck[5]



$$\lim_{r\to\infty}\left\langle J_r^\alpha J_r^\beta\right\rangle = \frac{h}{e^2} G_{\alpha\beta}\frac{1}{4\pi}(2r)^{-2}$$

TLL with Impurity
The Conformal Mapping Way

PHYSICAL REVIEW B **85**, 045120 (2012)

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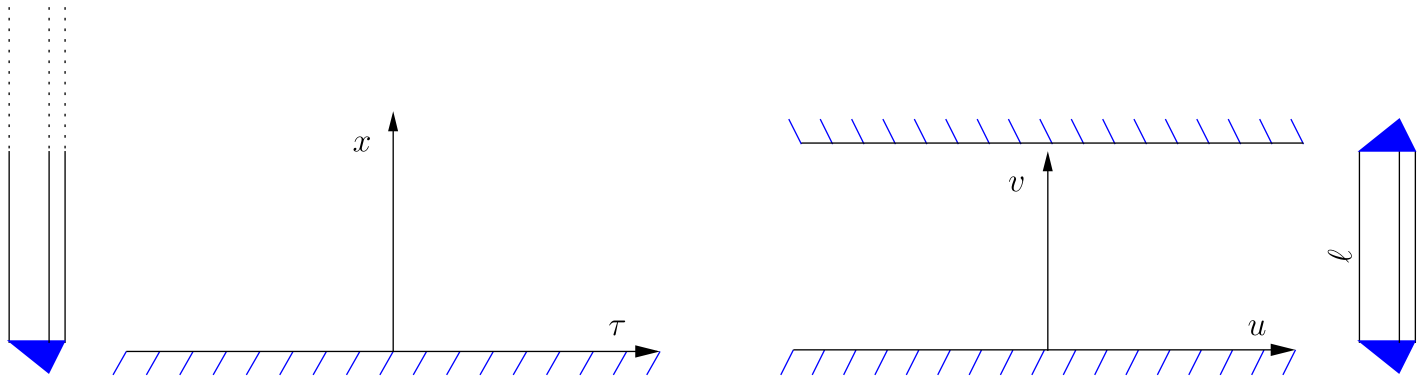
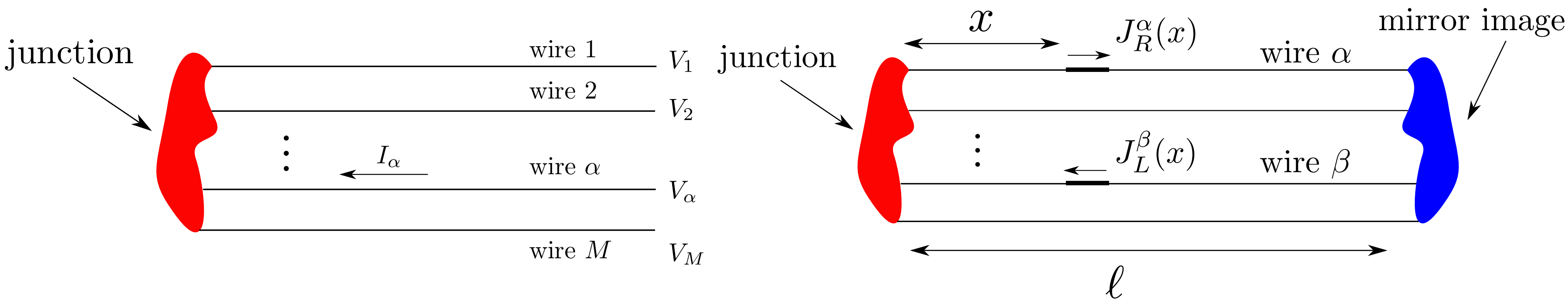


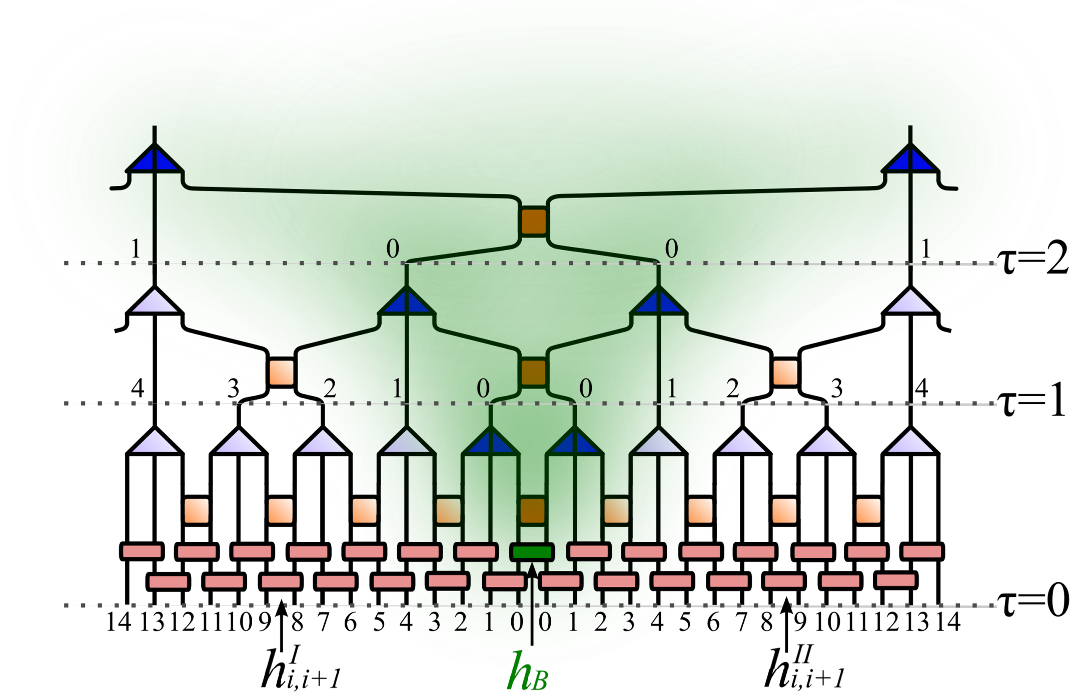


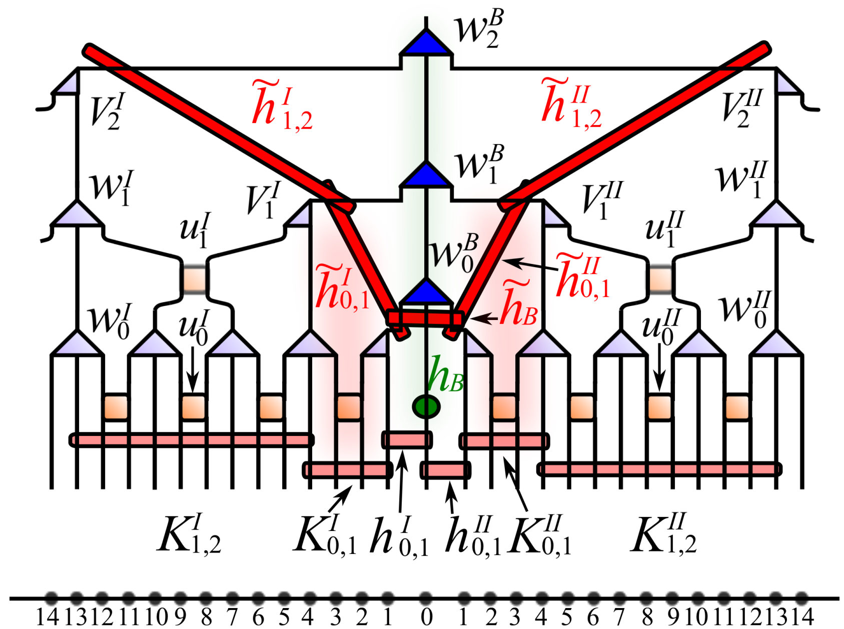
TLL with Impurity
The bMERA Way

PHYSICAL REVIEW B **90**, 235124 (2014)

Quantum impurity in a Luttinger liquid: Universal conductance with entanglement renormalization

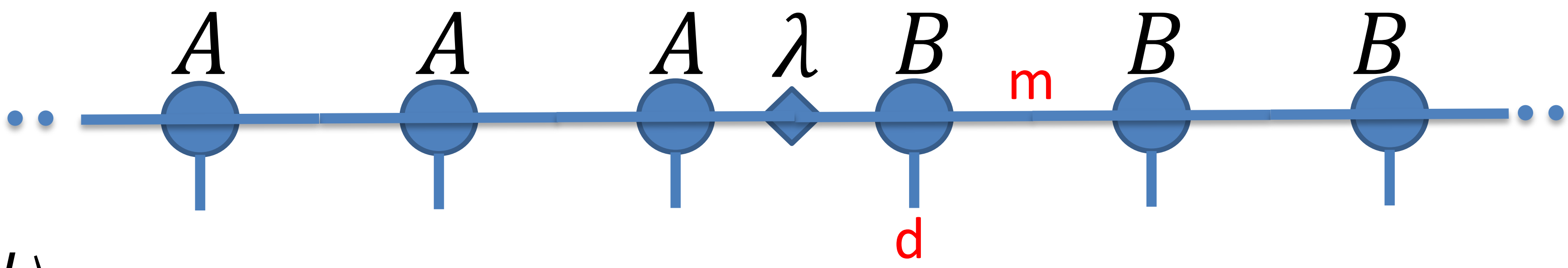
Ya-Lin Lo (羅雅琳),[1,2] Yun-Da Hsieh (謝昀達),[1,2] Chang-Yu Hou,[3]
Pochung Chen (陳柏中),[4,5,*] and Ying-Jer Kao (高英哲)[1,5,6,†]





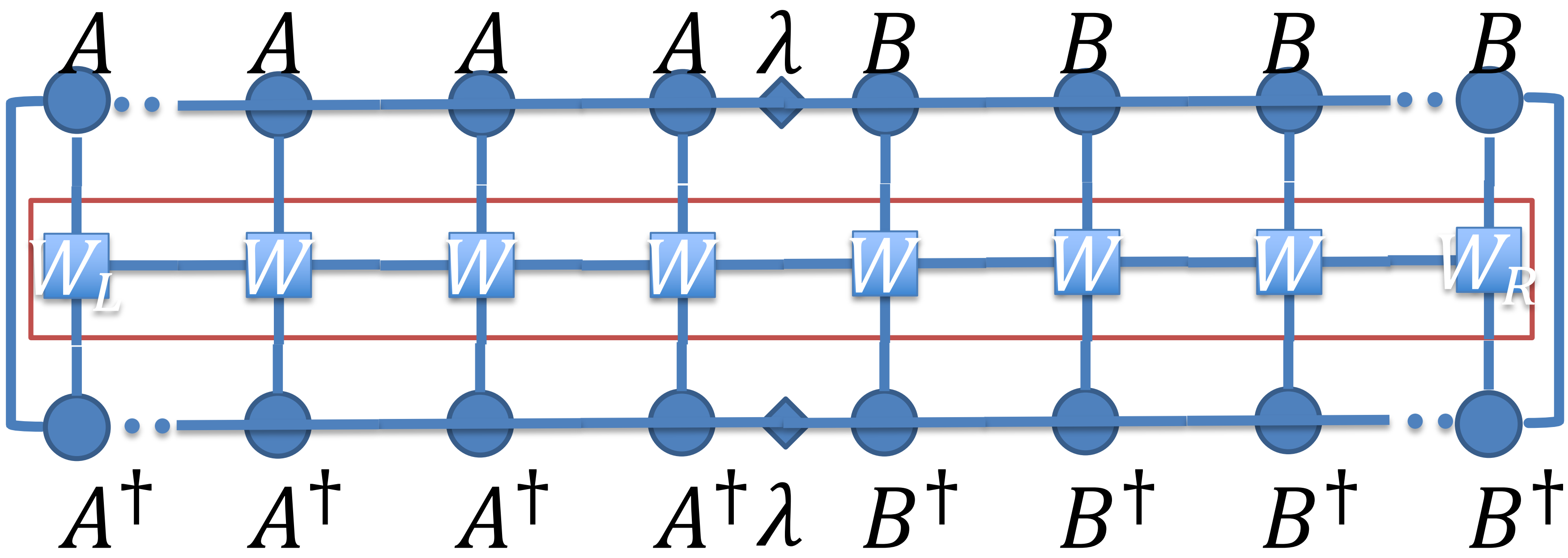
Infinite Matrix Product State (iMPS)

$|\psi\rangle =$

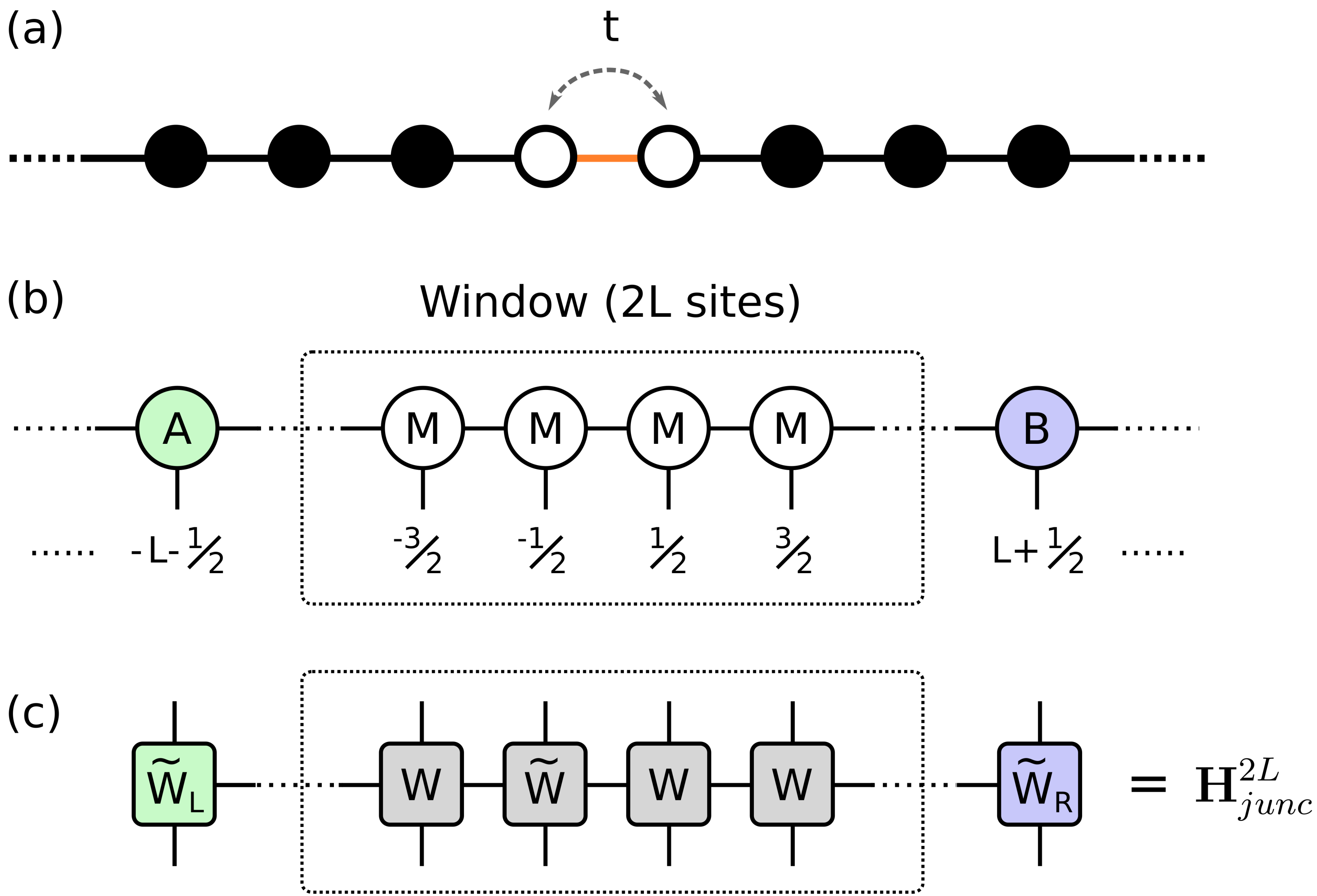


$$\frac{\langle\psi|H|\psi\rangle}{\langle\psi|\psi\rangle} =$$

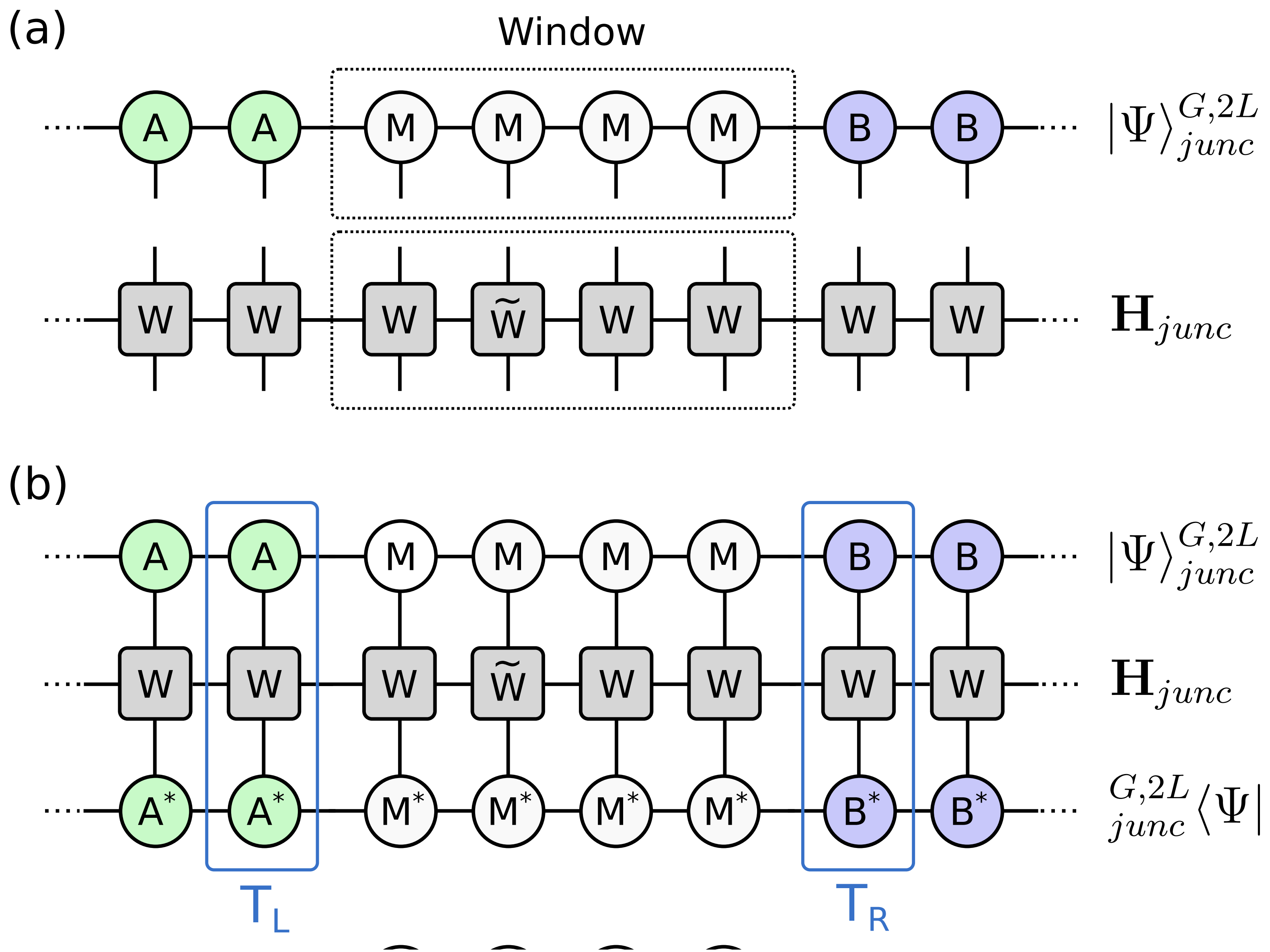
$H =$



Finite Window in an Infinite System

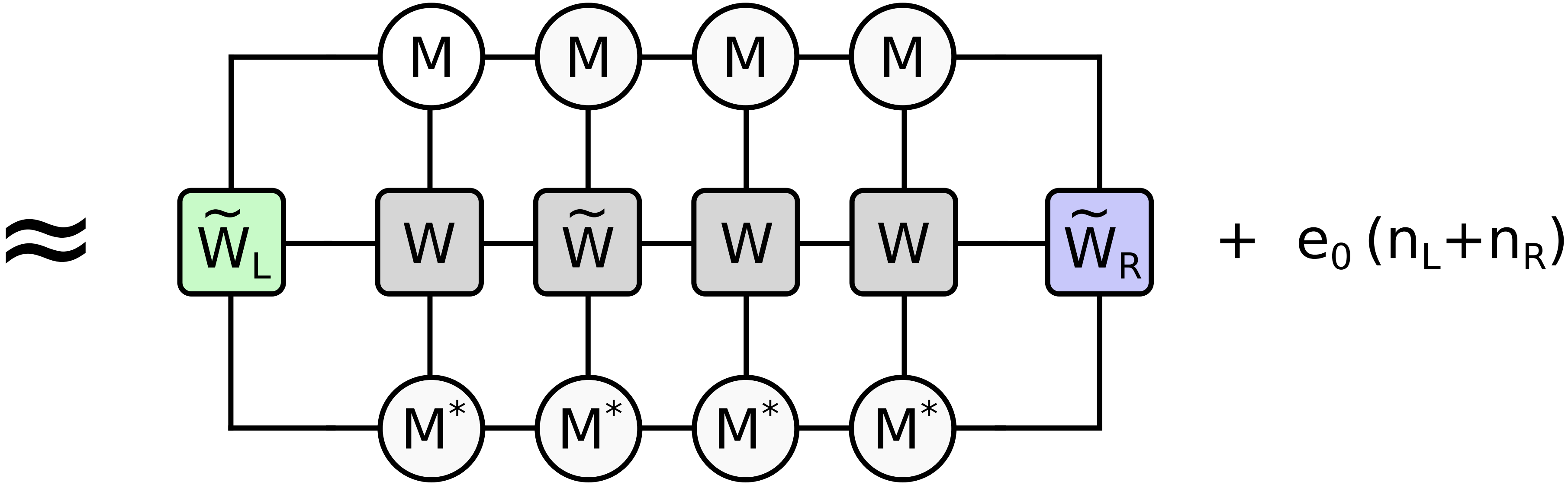


Effective Hamiltonian

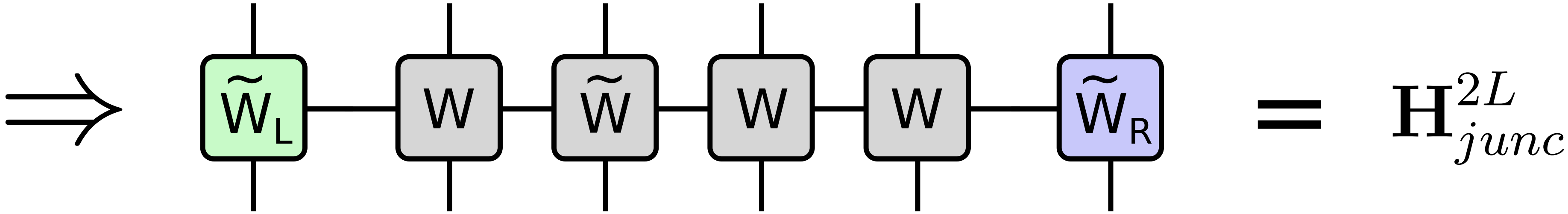


Effective Hamiltonian

(c)



(d)



Bethe Ansatz, Field Theory, iMPS

- Bethe ansatz
  - Exact analytical expression for short range correlation functions.
- Field theory
  - Asymptotically correct analytical expression.
- iMPS
  - Finite correlation length.
  - (More) accurate at short distance.
  - (Less) accurate at long distance.

Bulk Benchmark (Short Distance)

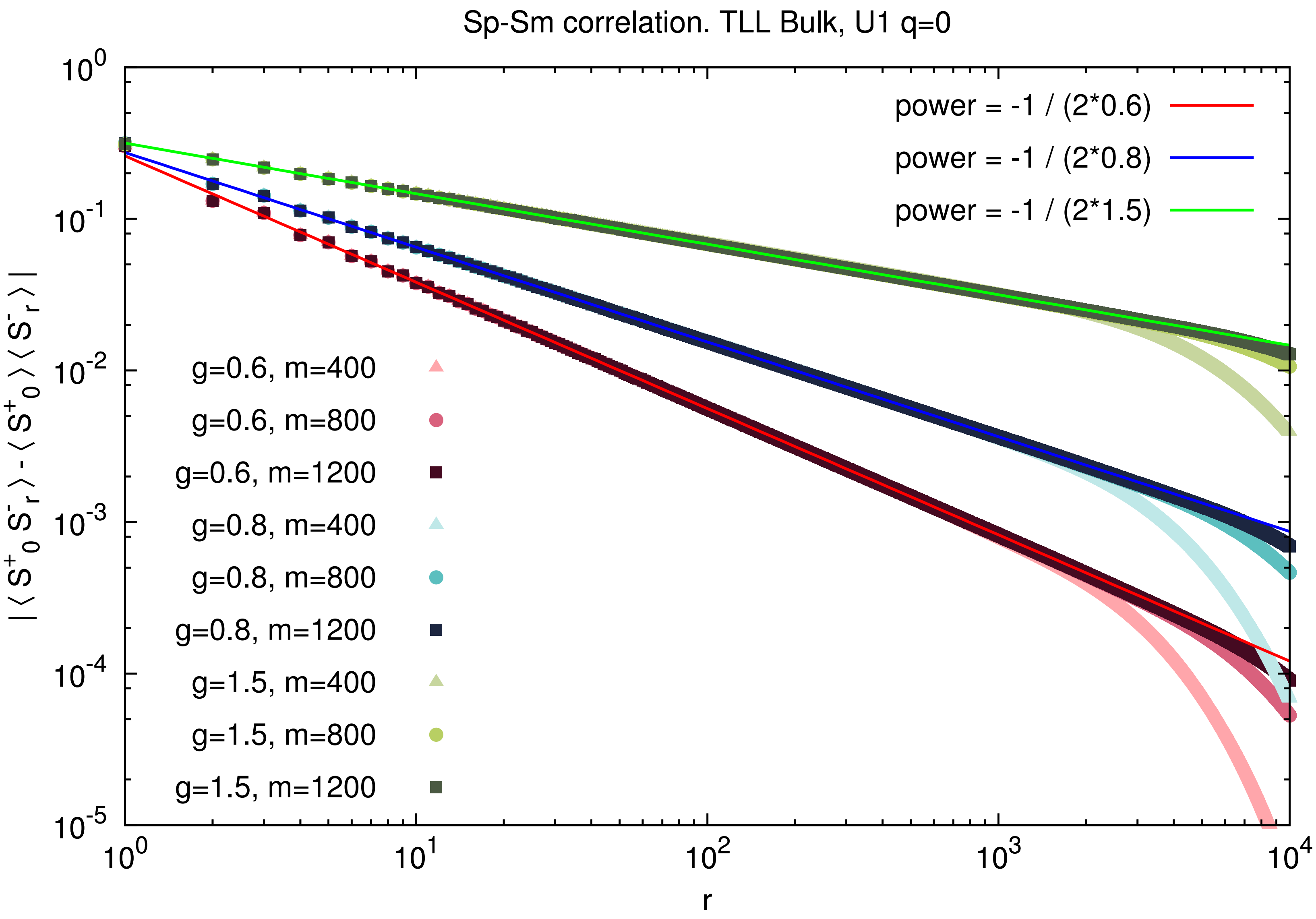
$V = 0.5$

r	$\langle S^z_{-1/2} S^z_{-1/2+r} \rangle$	$\langle S^z_{1/2} S^z_{1/2+r} \rangle$	Average	Bethe Ansatz[7]	Field Theory[9]	Absolute Error
1	−0.5000010645	−0.4999989325	−0.4999999985	−0.5000000000	−0.5805187860	0.0000000015
2	0.1093749813	0.1093749813	0.1093749813	0.1093750000	0.1135152692	0.0000000187
3	−0.0979007322	−0.0978998444	−0.0979002883	−0.0979003906	−0.0993588501	0.0000001023
4	0.0439766803	0.0439766803	0.0439766803	0.0439770222	0.0440682654	0.0000003419
5	−0.0443373471	−0.0443369169	−0.0443371320	−0.0443379157	−0.0444087865	0.0000007837
6	0.0249922056	0.0249922056	0.0249922056	0.0249933420	0.0249365346	0.0000011364
7	−0.0262659910	−0.0262656732	−0.0262658321	−0.0262668452	−0.0262404925	0.0000010131
8	0.0166097867	0.0166097867	0.0166097867	0.0166105110	0.0165641239	0.0000007243

- Bethe ansztz: N. A. Slavnov, Theor Math Phys **150**, 259 (2007).
- Field theory: S. L. Lukyanov and V. Terras, Nucl. Phys. B **654**, 323 (2003).

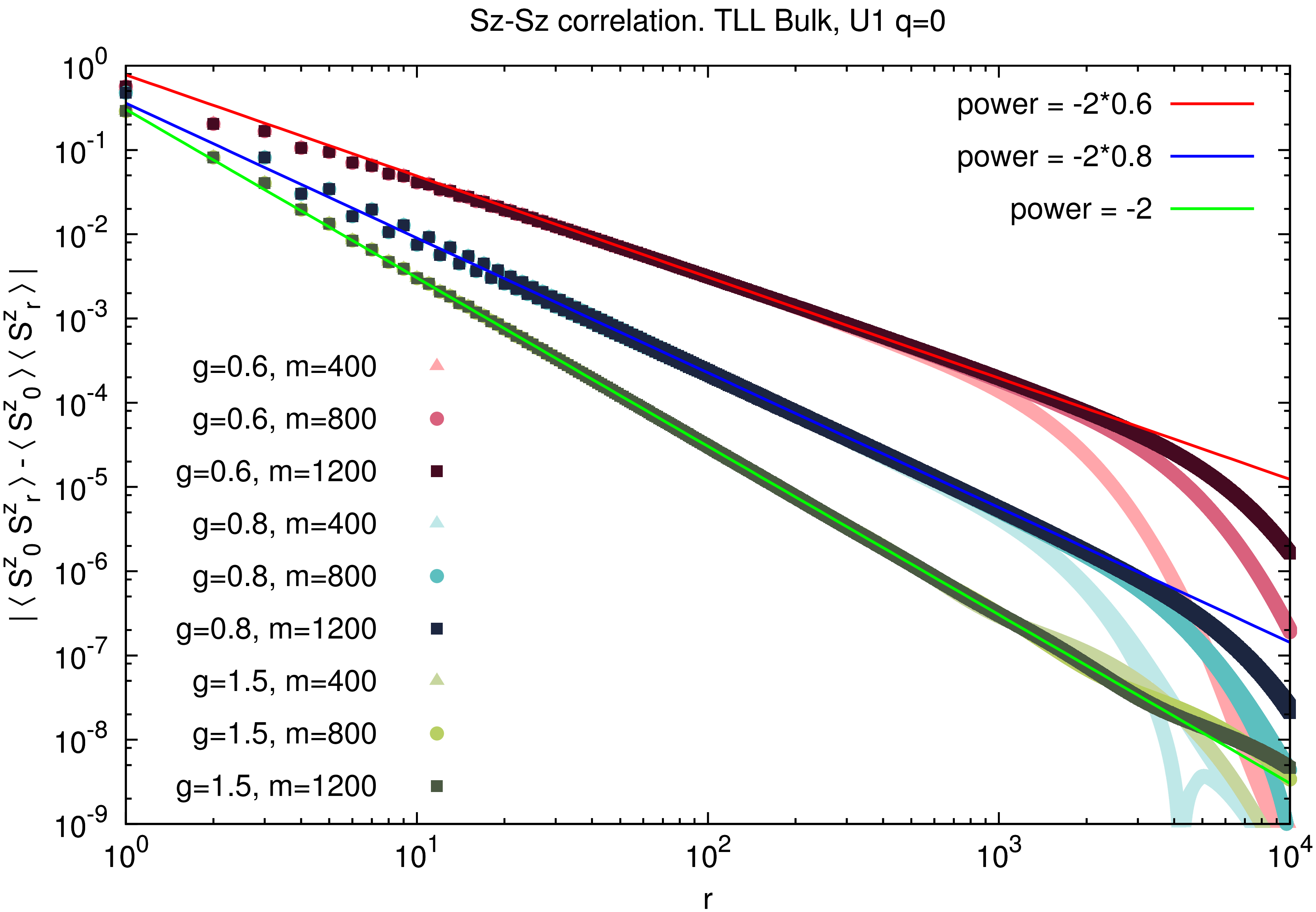
Bulk Benchmark (Long Distance)

$$\langle S^{+}_{-r} S^{-}_{+r} \rangle \propto r^{-1/(2g)}$$



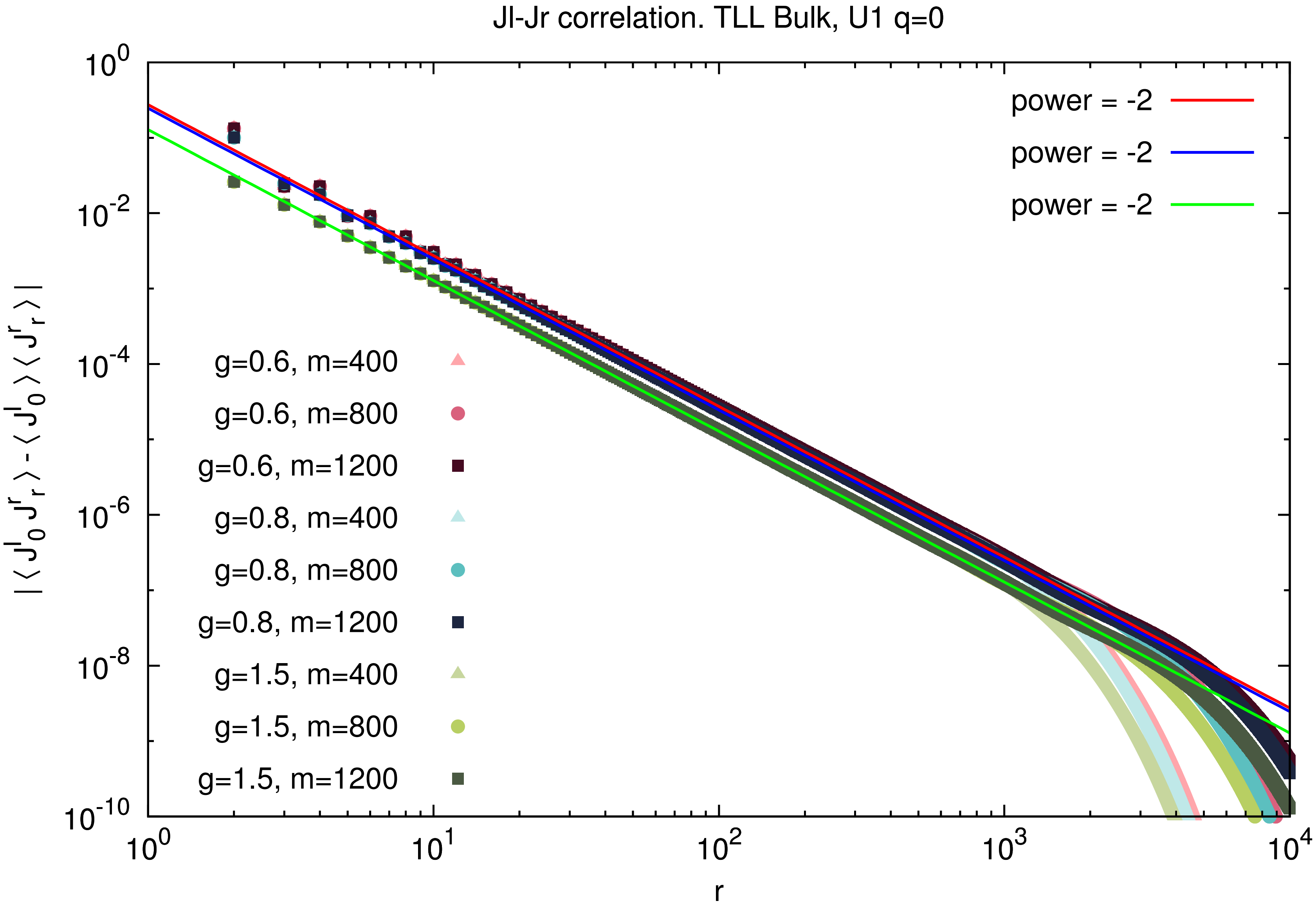
Bulk Benchmark (Long Distance)

$$\langle S^z_{-r} S^z_{+r} \rangle \propto r^{-2g}$$



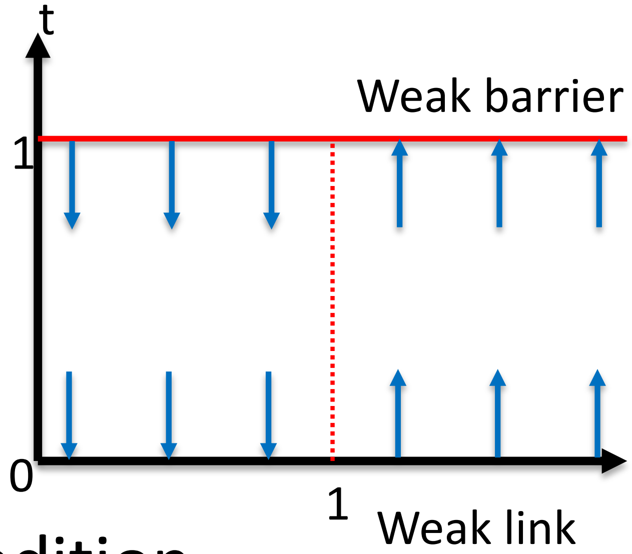
Bulk Benchmark (Long Distance)

$$\langle J_{-r} J_{+r} \rangle \propto r^{-2}$$



Field Theory: Boundary Perturabtion

- Weak barrier (small 1-t): free boundary condition.
  - $\langle S^{+}_{-i} S^{-}_{+i} \rangle = C_0 r^{-\left(\frac{1}{2g}\right)} + D_0 (-1)^r r^{-\left(\frac{1}{2g}+2g\right)}.$
  - $\langle S^{z}_{-i} S^{z}_{+i} \rangle = C_0 r^{-(2)} + D_0 (-1)^r r^{-(2g)}.$
  - $\left\langle J^{+}_{-i-\frac{1}{2}} J^{-}_{+i+\frac{1}{2}} \right\rangle = C_0 r^{-2}.$



- Weak link (small t): Dirichlet boundary condition.
  - $\langle S^{+}_{-i} S^{-}_{+i} \rangle = t \left[C_0 r^{-\left(\frac{3}{2g}-1\right)} + D_0 (-1)^r r^{-\left(\frac{3}{2g}+2g-1\right)} \right].$
  - $\langle S^{z}_{-i} S^{z}_{+i} \rangle = t^2 \left[C_0 r^{-\left(\frac{2}{g}\right)} + D_0 (-1)^r r^{-\left(\frac{2}{g}+2g-2\right)} \right].$
  - $\left\langle J^{+}_{-i-\frac{1}{2}} J^{-}_{+i+\frac{1}{2}} \right\rangle = t^2 C_0 r^{-\frac{2}{g}}.$

Dominant Exponents

	$g > 1$			$g < 1$		
	Bulk IR	IR	UV	Buik IR	IR	UV
$\langle S^{+}_{-i} S^{-}_{+i} \rangle$	$\frac{1}{2g}$	$\frac{1}{2g}$	$\frac{3}{2g} - 1$	$\frac{1}{2g}$	$\frac{3}{2g} - 1$	$\frac{1}{2g}$
$\langle S^{z}_{-i} S^{z}_{+i} \rangle$	2	2	$\frac{2}{g}$	$2g$	$2g + \frac{2}{g} - 2$	$2g$
$\left\langle J^{+}_{-i-\frac{1}{2}} J^{-}_{+i+\frac{1}{2}} \right\rangle$	2	2	$\frac{2}{g}$	2	$\frac{2}{g}$	2

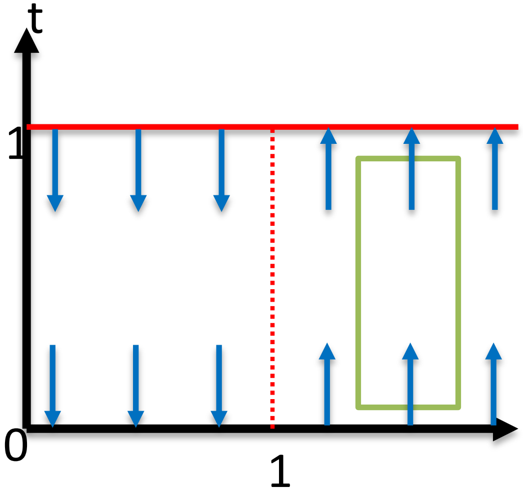
Weak barrier

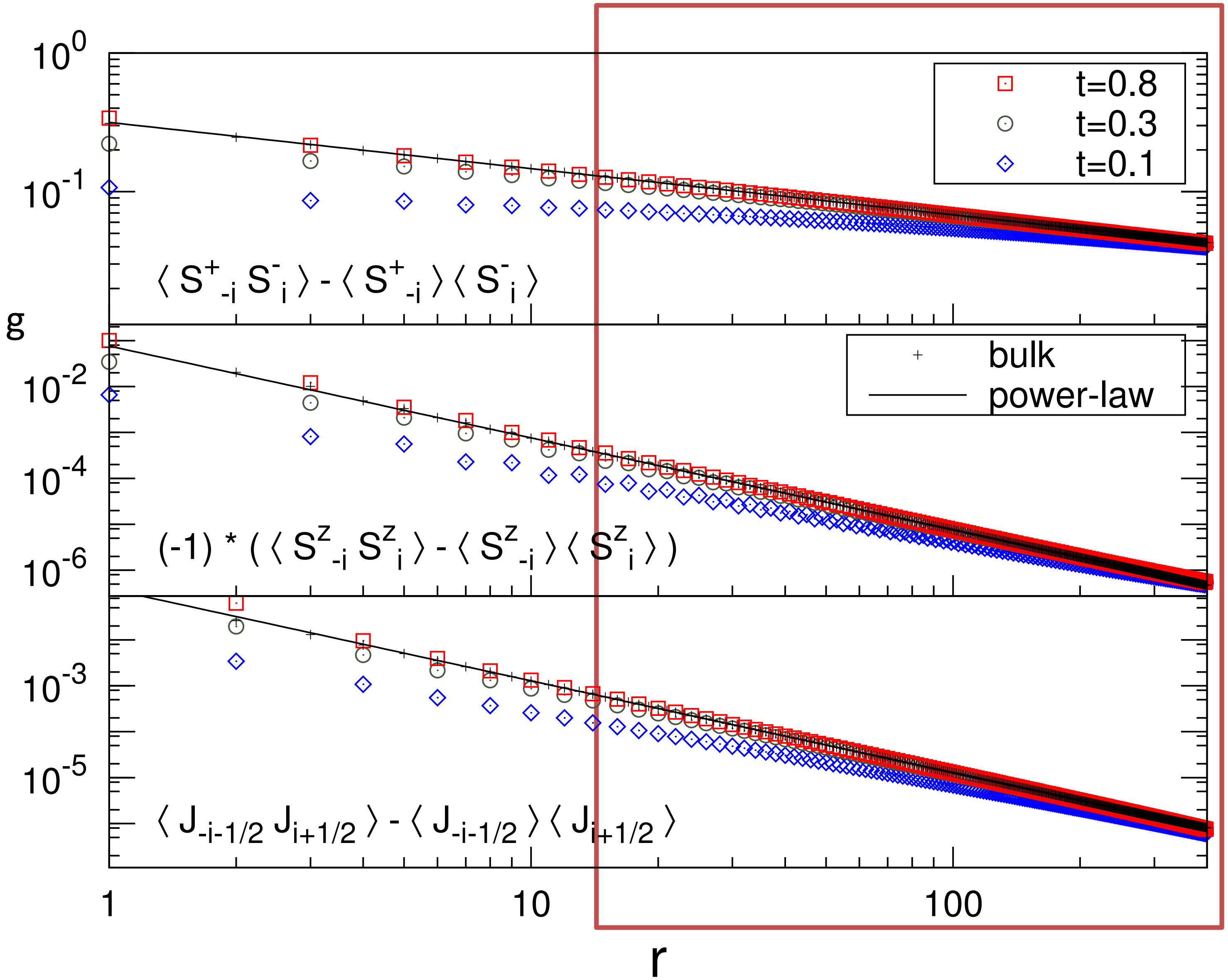
Weak link

Weak link

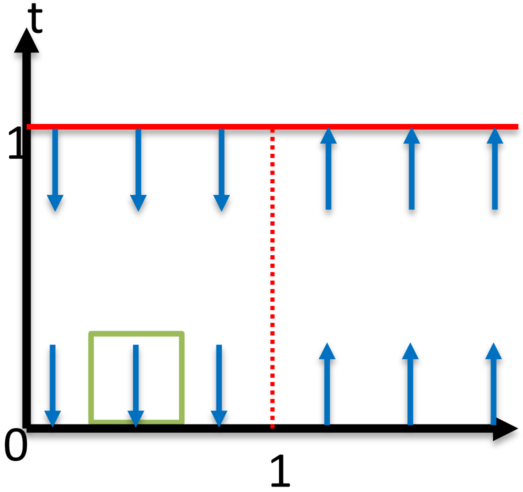
Weak barrier

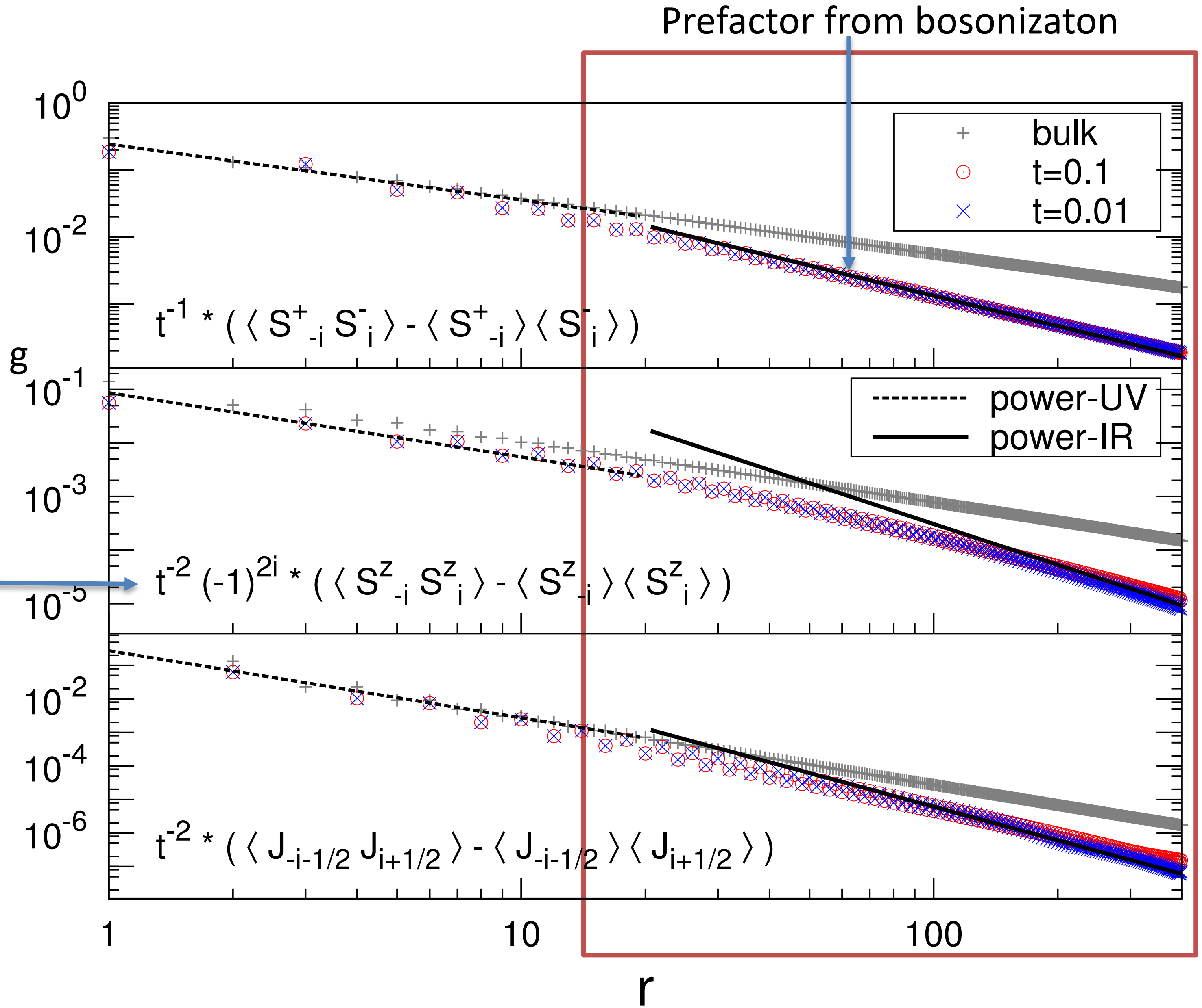
$g = 1.5 > 1$, IR: “Healed” Fixed Point



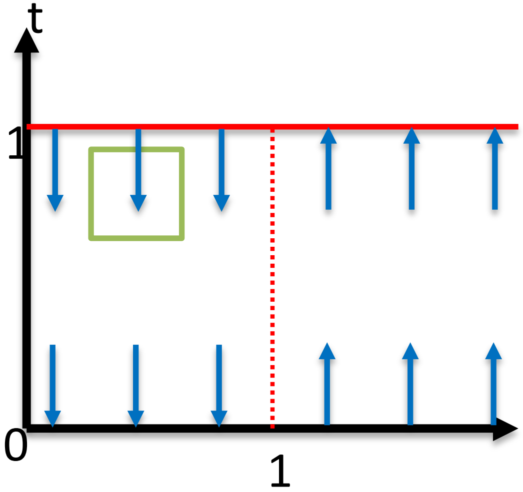


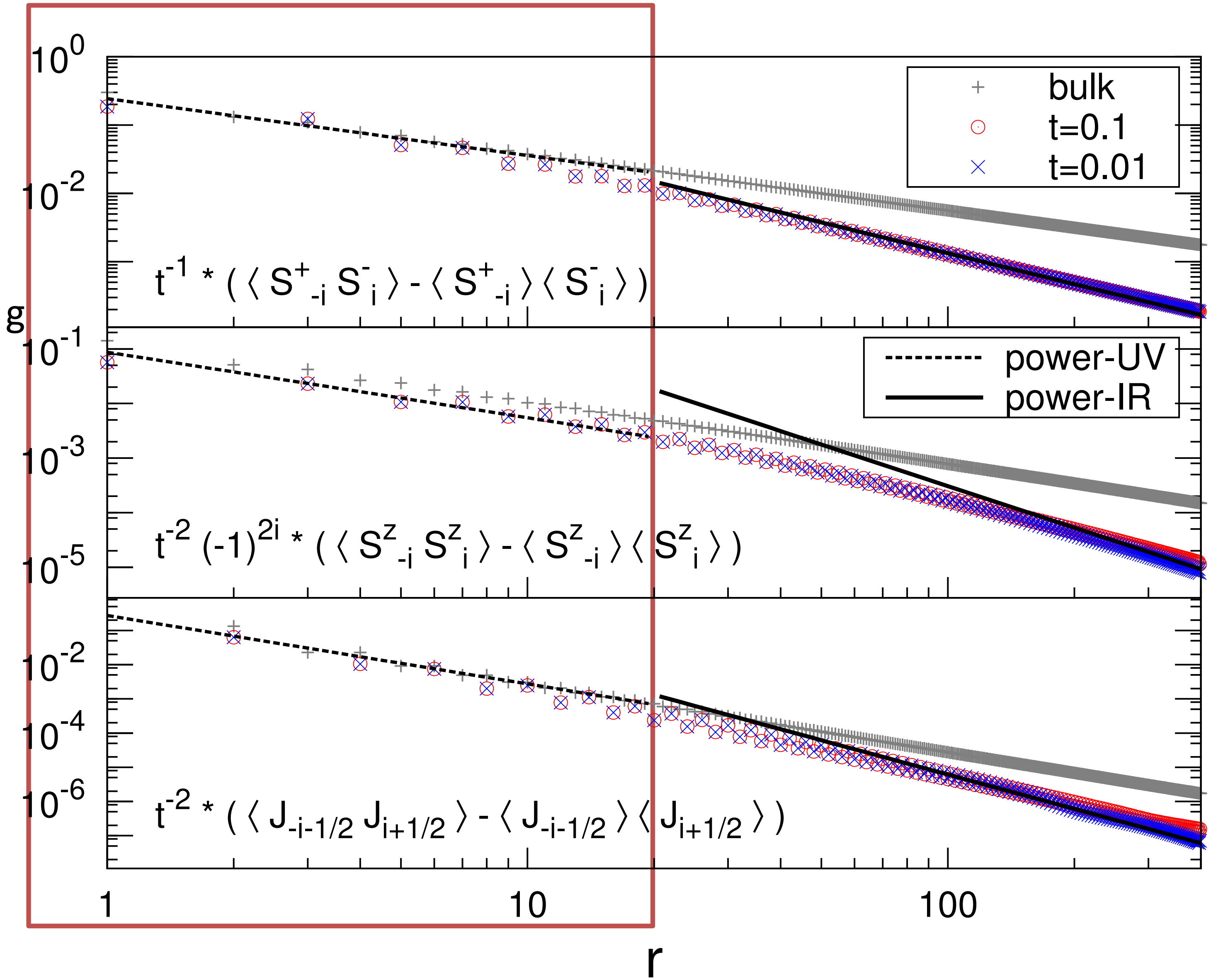
$g = 0.6 < 1$, IR: “Broken” Fixed Point



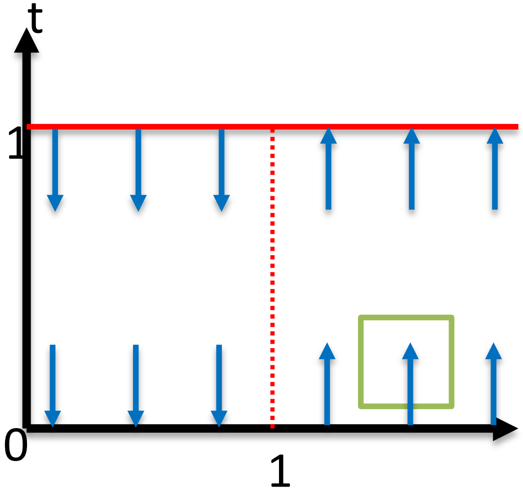


$g = 0.6 < 1$, UV Fixed Point



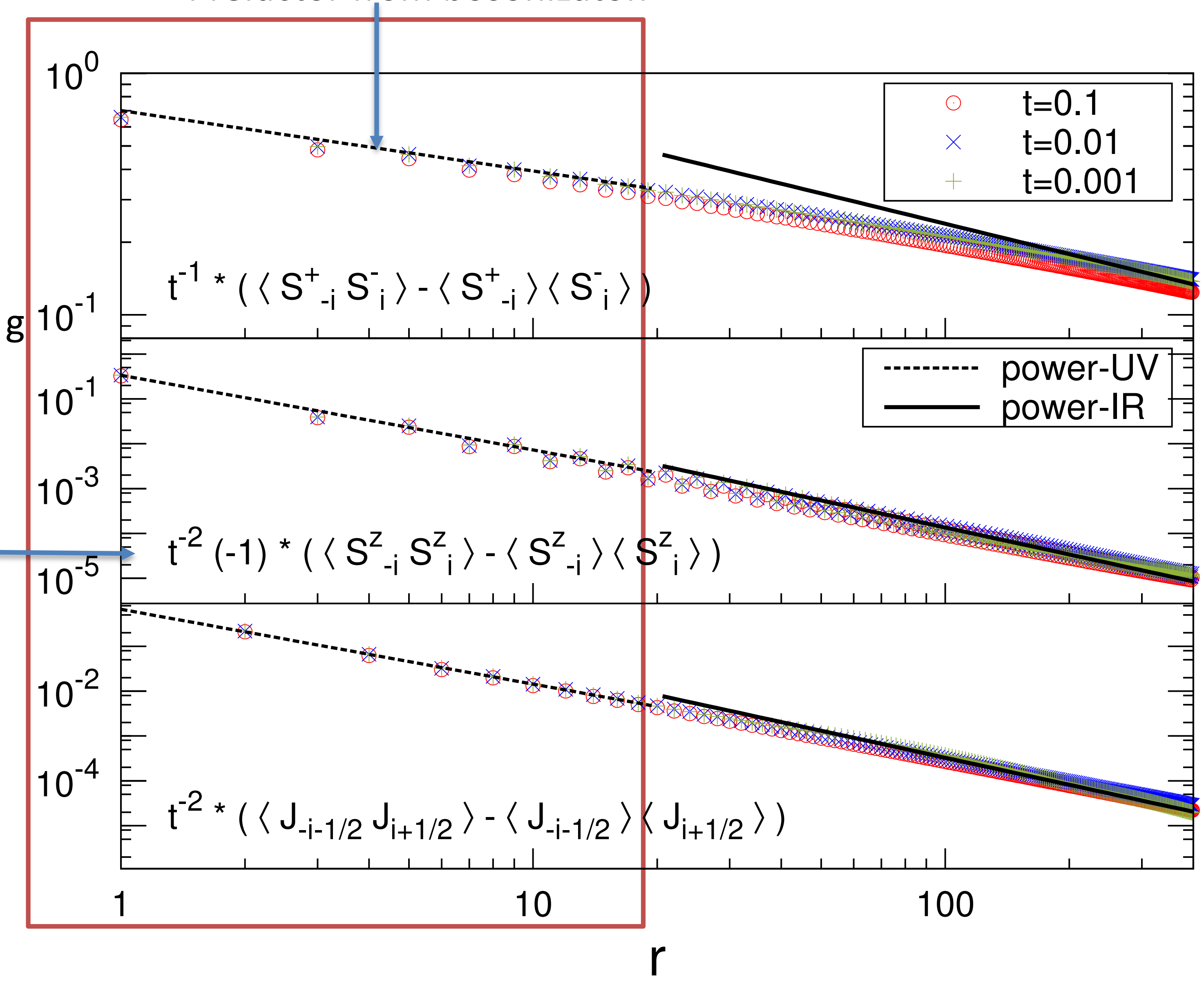


$g = 1.2 > 1$, UV Fixed Point



Prefactor from bosonizaton

Rescaled



Summary & Outlook

- Impurity in a Tomonaga-Luttinger Liquid
  - Finite window iMPS with infinite boundary conditions.
- Spin-1/2 XXZ (Spinless fermion)
  - $\langle S^{+}_{-r} S^{-}_{+r} \rangle, \langle S^{z}_{-r} S^{z}_{+r} \rangle, \langle J_{-r} J_{+r} \rangle$ correlation functions
  - Confirm fixed points : g>1 (heal) and g<1 (broken).
  - Confirm exponents: UV and IR.
  - Crossover of correlation functions from UV to IR.
- Applicable interesting problems:
  - Y-junction, spin-1/2 fermion leads, etc.
- Further generalization:
  - Finite bias, finite temperature.