# Supersolid in the kagome antiferromagnet in a magnetic field

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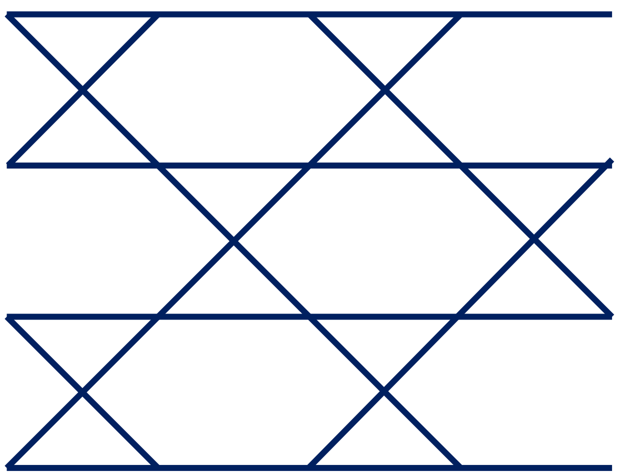
1. Introduction
2. Degenerate perturbation
3. Magnon instability at 5/9 plateau
4. Spin structure below 5/9 plateau
5. Summary

Ref: *X. Plat, T. Momoi, and C. Hotta, Phys. Rev. B* ***98****, 014415 (2018)*

# 1. Introduction

Kagome antiferromagnet

$$H = J\sum_{\langle i,j\rangle}\mathbf{S}_i\cdot\mathbf{S}_j - h\sum_i S_i^z$$

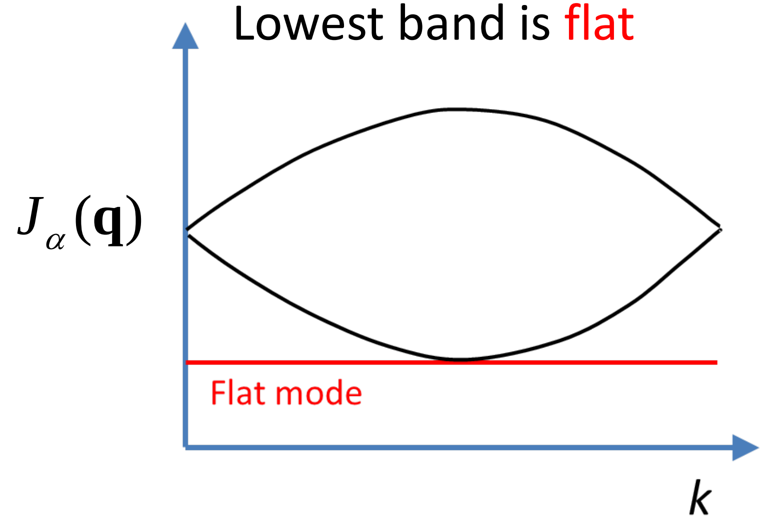


✓ In the classical limit, massively degenerate ground states at both $h$ = 0 and $h$ > 0.

Fourier transform

$$H = \sum_{\alpha=1,2,3}\sum_{\mathbf{q}\in BZ} J_\alpha(\mathbf{q})\left|\mathbf{S}_\alpha(\mathbf{q})\right|^2 - h\sum_i S_i^z$$

Lowest band is flat

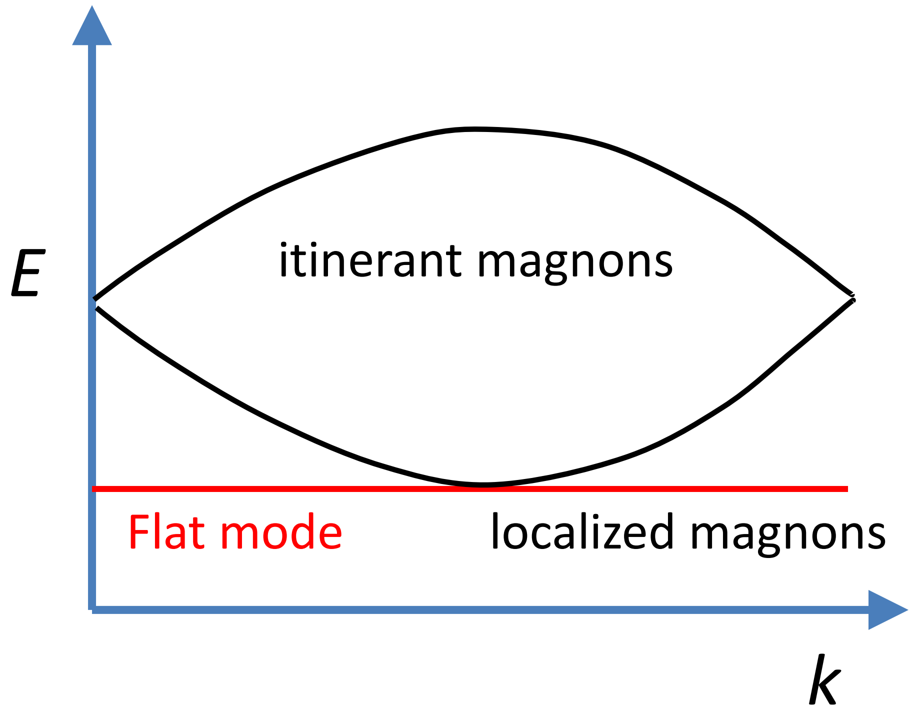


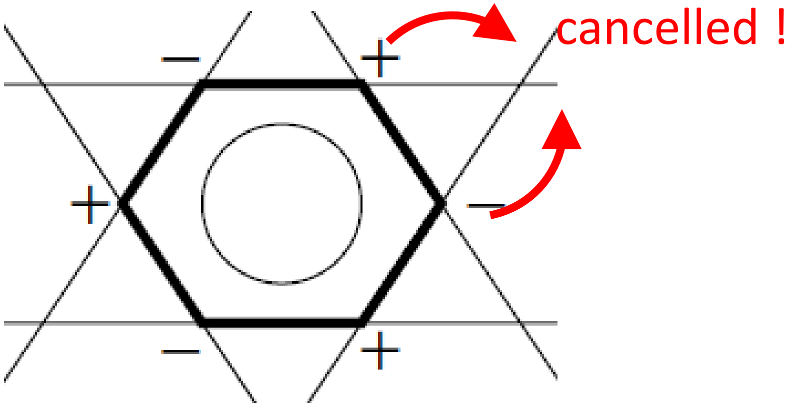
✓ In quantum (S=1/2) case,

magnons are ready to be localized.

At saturation field, one magnon state

$$\sum_r f_r S_r^- \left| \Uparrow\Uparrow\Uparrow \cdots \Uparrow \right\rangle$$




Schulenburg et al., PRL 2003

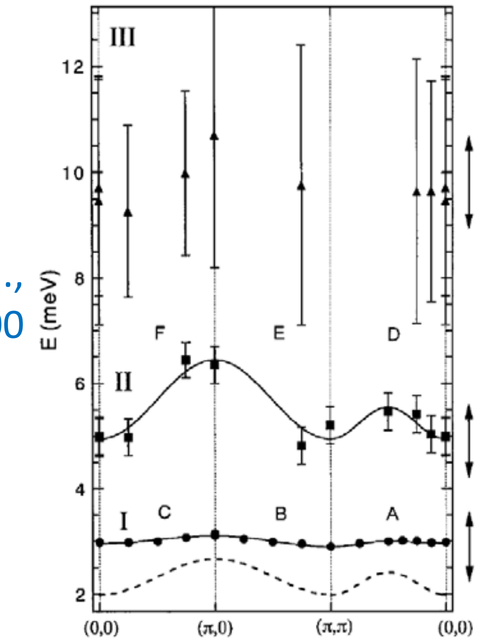
# c.f. Shastry-Sutherland lattice $SrCu_2(BO_3)_2$

Kageyama et al., PRL 1999

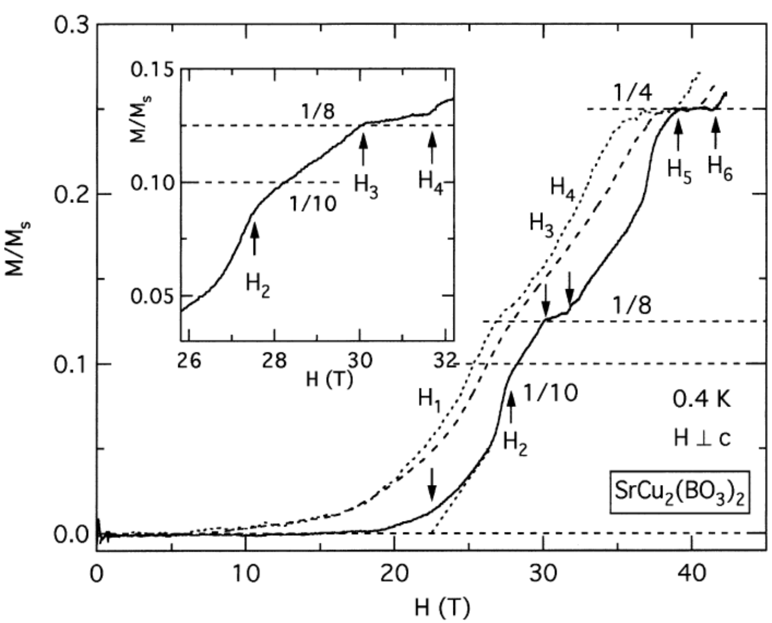
One magnon (triplon) state is nearly localized

Miyahara-Ueda, PRL 1999

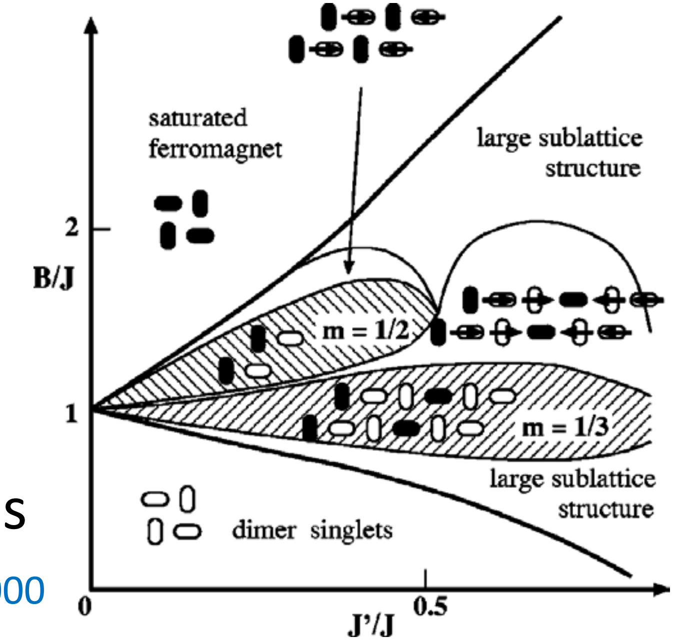
Kageyama et al., PRL 2000




Nearly localized magnons induce many magnetization plateaus




Kageyama et al., JPSJ 1999




Crystals of triplons

Miyahara-Ueda 1999, Momoi-Totsuka PRB 2000

# c.f. Shastry-Sutherland lattice $SrCu_2(BO_3)_2$

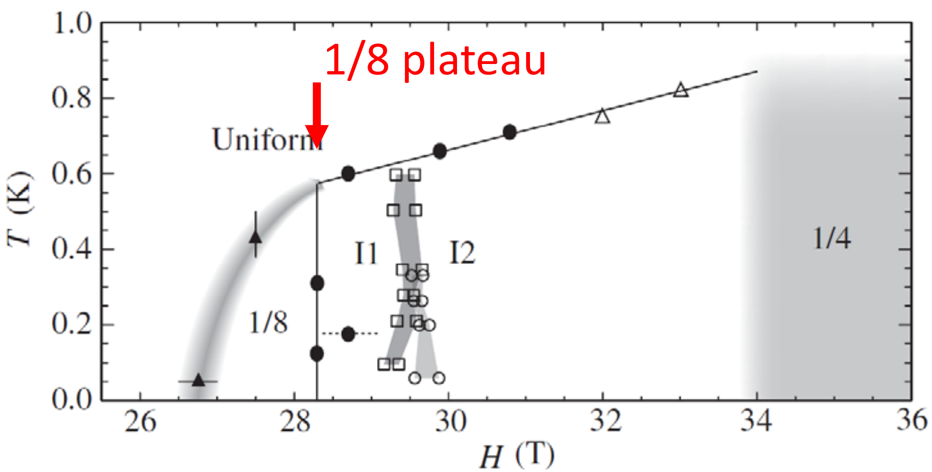
... and nearly localized magnons also induce supersolids
in the vicinity of plateaus.

Coexistence of density wave and superfluid    Momoi-Totsuka PRB 2000

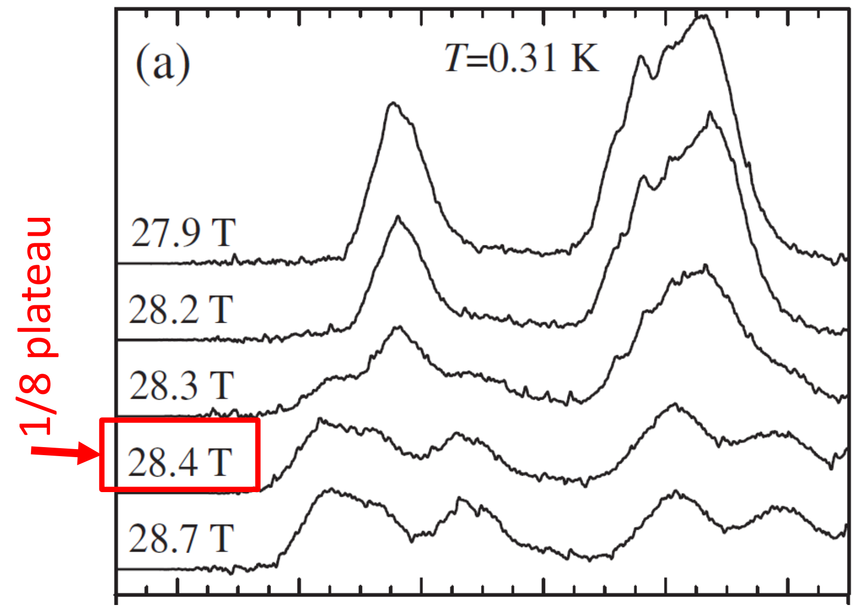
K.P. Schmidt, J. Dorier, A.M. Lauchli, F. Mila PRL 2008

Specific heat    Tsuji et al. JPSJ 2011

Phase diagram

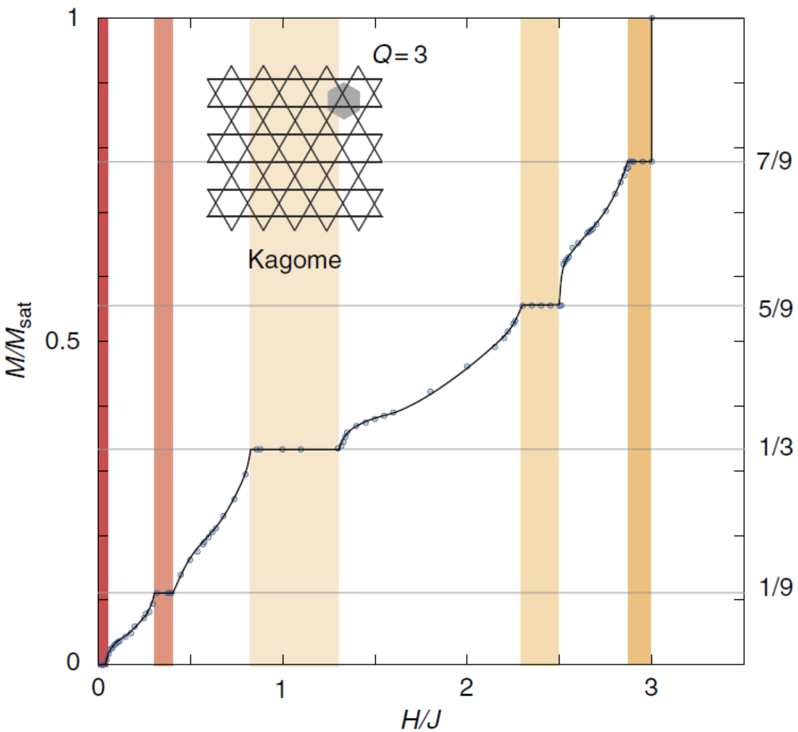



$^{11}$B NMR spectra    Takigawa et al. PRL 2008

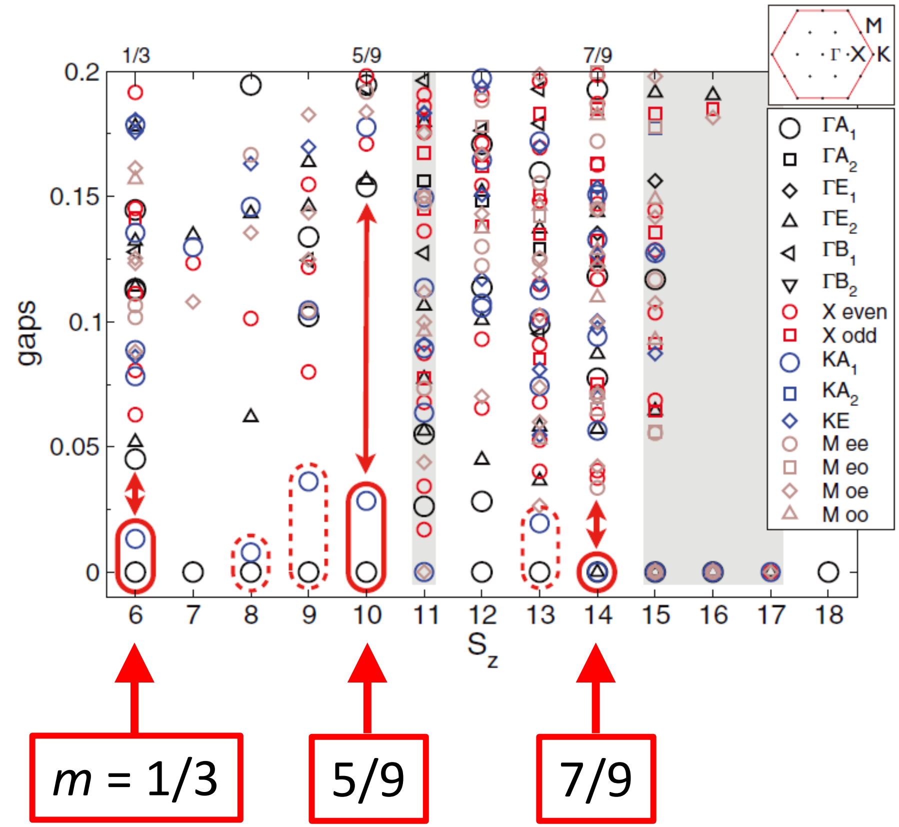


# ✓ Kagome AF indeed shows many magnetization plateaus

Nishimoto-Shibata-Hotta, Nat. Comm. 2013



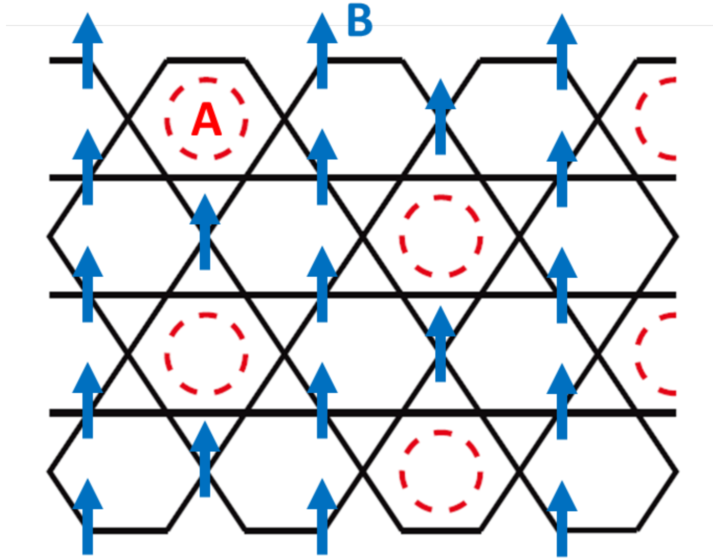

Capponi-Derzhko-Honecker-Lauchli-Richter
PRB 2013




3-fold quasi-degeneracy (*K* and *Γ* points) and a finite energy gap

# SDWs in magnetization plateaus at $m$ = 1/3, 5/9, 7/9

*crystallization*




a good approximation for the plateau state

at $m / m_{sat} = (2n+3)/9$

$$\prod_{\mathbf{A}} \left| \underline{S^z_{hex} = n, \mathrm{gs}} \right\rangle \prod_{i \in \mathbf{B}} \left| \Uparrow \right\rangle_i \qquad (n = 0, 1, 2)$$

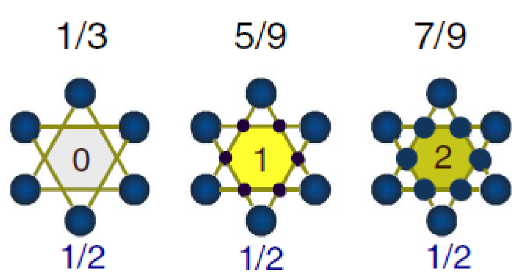
resonating spins on hexagons

Capponi-Derzhko-Honecker-Lauchli-Richter

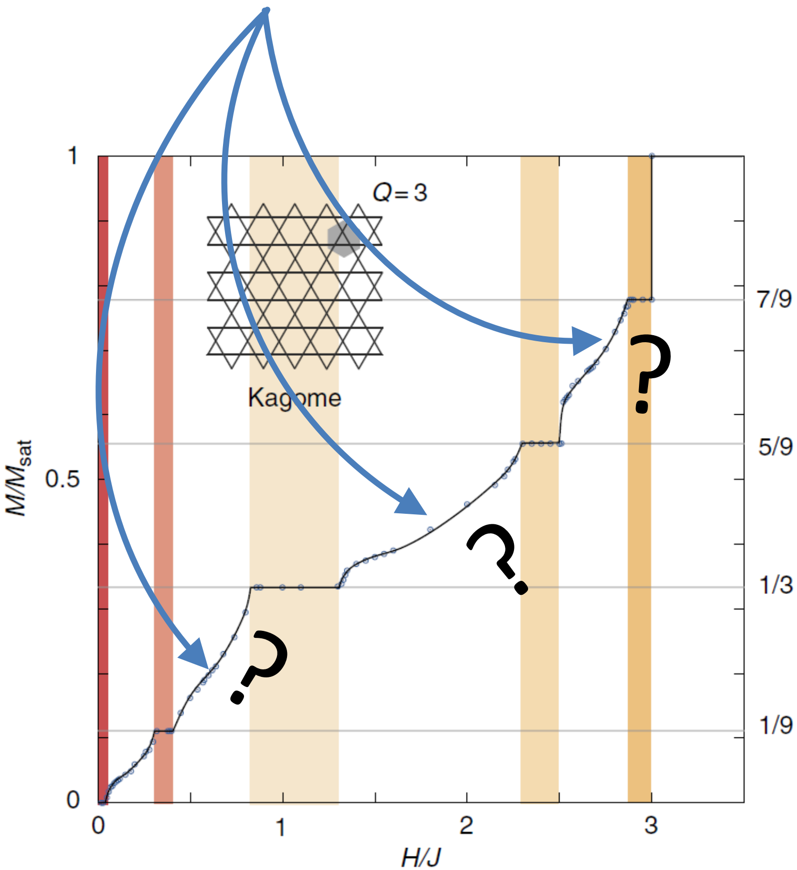
PRB 2013

Nishimoto-Shibata-Hotta

Nat. Comm. 2013



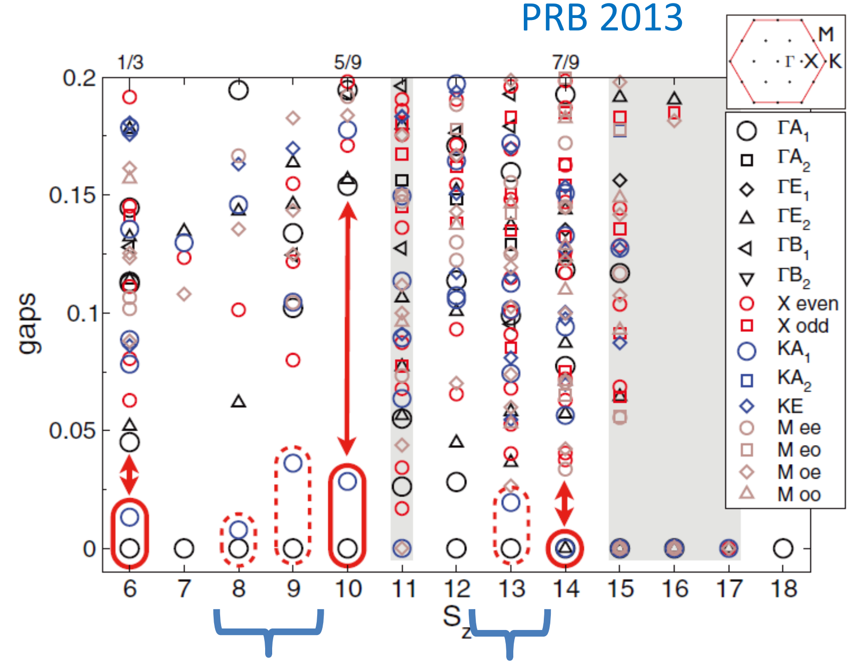
# What is the nature of non-plateau states ?




Nishimoto-Shibata-Hotta, Nat. Comm. 2013

Capponi-Derzhko-Honecker-Lauchli-Richter

PRB 2013




signature of SDW order below

$m/m_{sat}$=5/9 and 7/9 plateaus

*"Supersolid"*

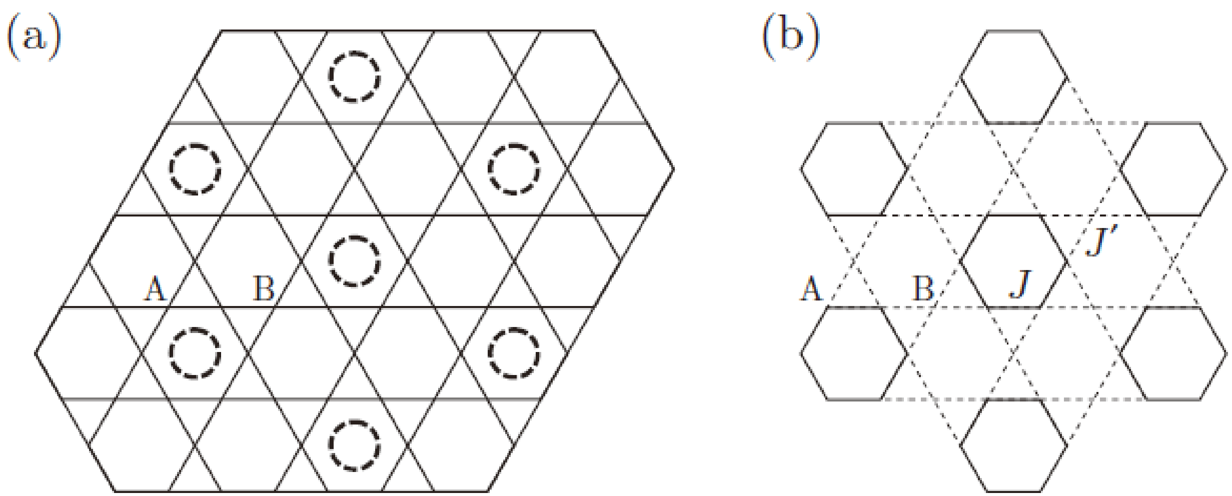
cf. Supershlid phase in Shastry-Sutherland AF Momoi-Totsuka PRB 2000

# 2. Degenerate perturbation theory

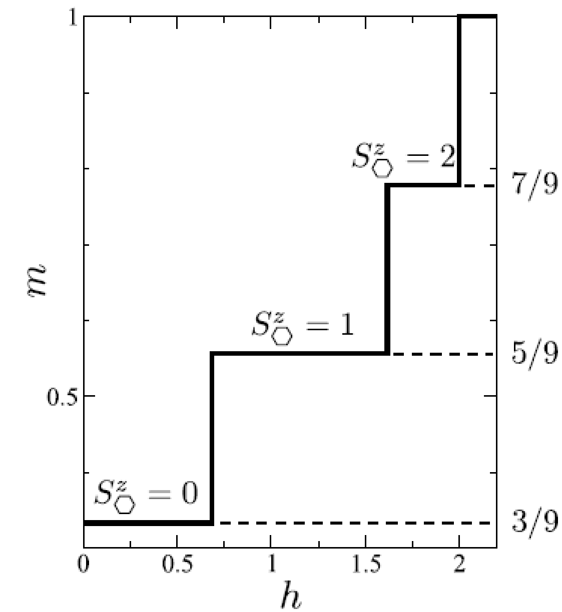
same space symmetry

Plateau states
*m* = 1/3, 5/9, 7/9

Hexamerized kagome lattice




In the $J'$ = 0 limit,




the plateau states are the same as the approximate plateau states in the kagome AF.

- ✓ The decoupled $J'$ = 0 limit is a good starting point for perturbation.
- ✓ Perform the degenerate perturbation by $J'$ up to 3rd order.

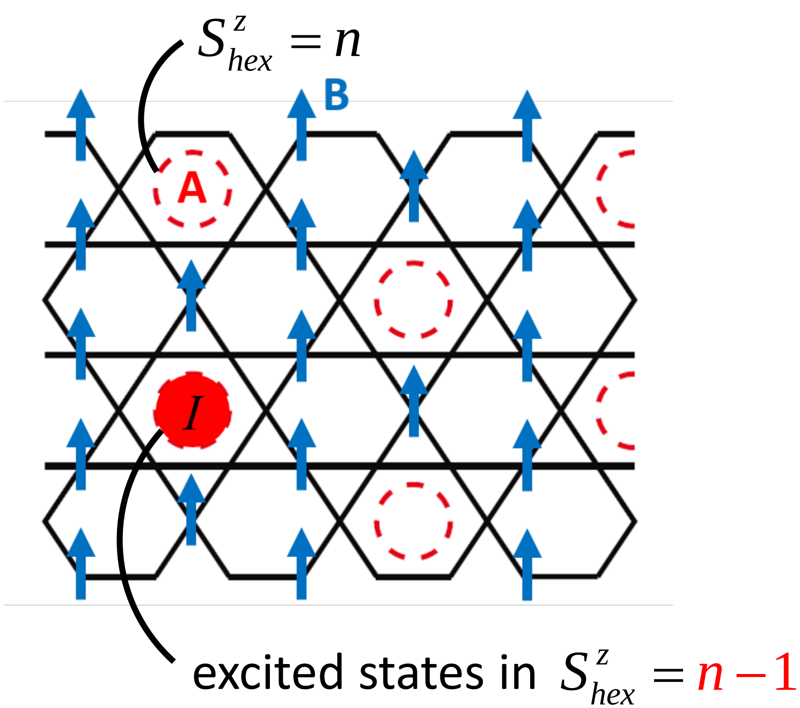
# Degenerate basis in the sector $\Delta S^z = -1$

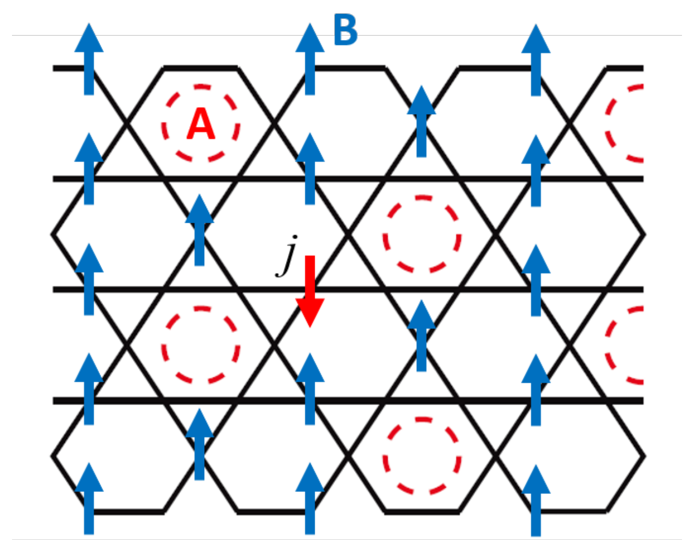
hexagons

$$\underbrace{c_I^{\dagger}|0\rangle,}_{\text{lowest}} \quad \underbrace{a_I^{\dagger}|0\rangle,}_{\text{2nd}} \quad \cdots$$

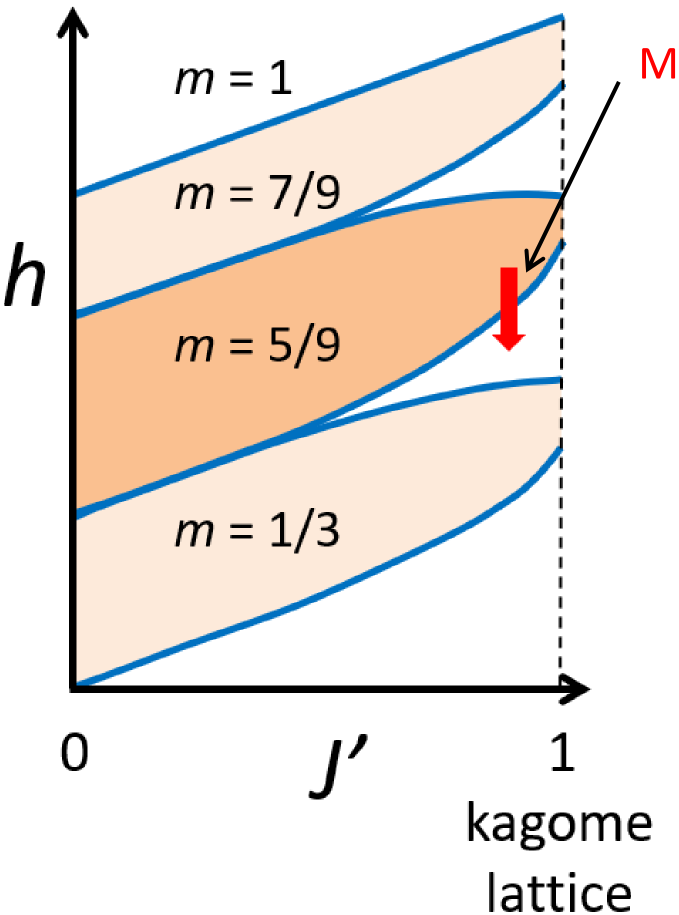
connecting spins

$$b_j^{\dagger}|0\rangle$$



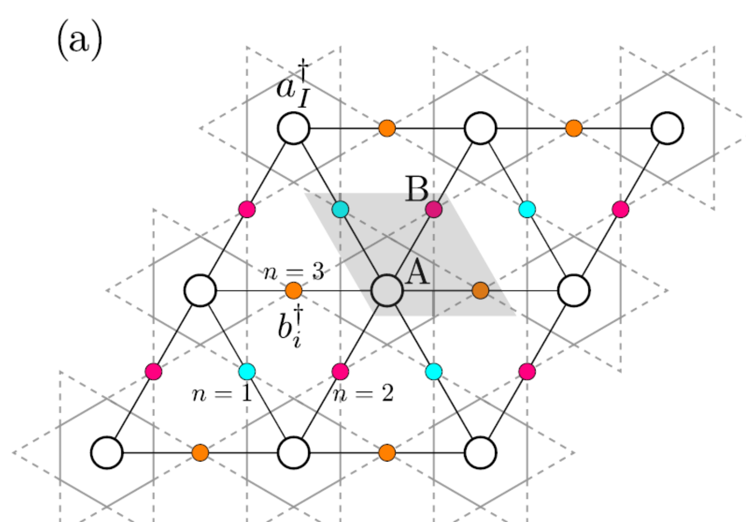



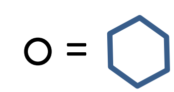
# 3. Magnon instability at 5/9 plateau




Magnetic-field controlled transition (with lowering *h*)

- Lowest energy magnons in hexagons are localized.
  ----- Flat mode
- However, magnons in 2nd energy level can move around. ------ dispersive mode

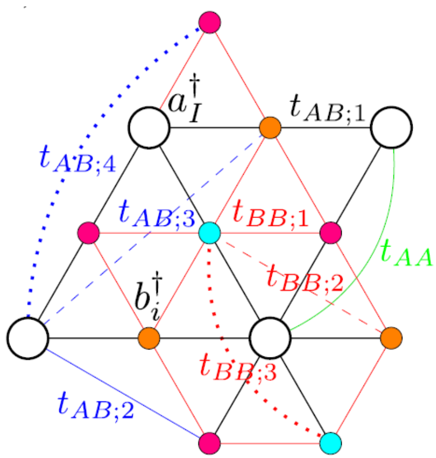


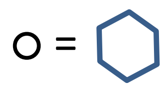



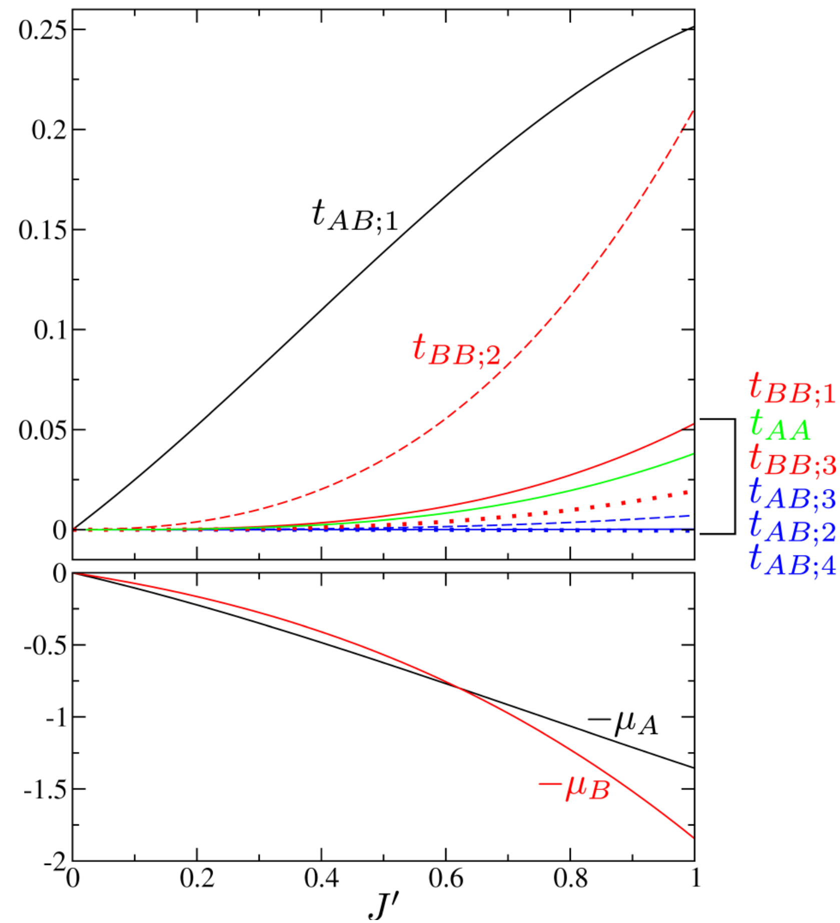
## Itinerant magnons

(2nd energy level of hexagons)

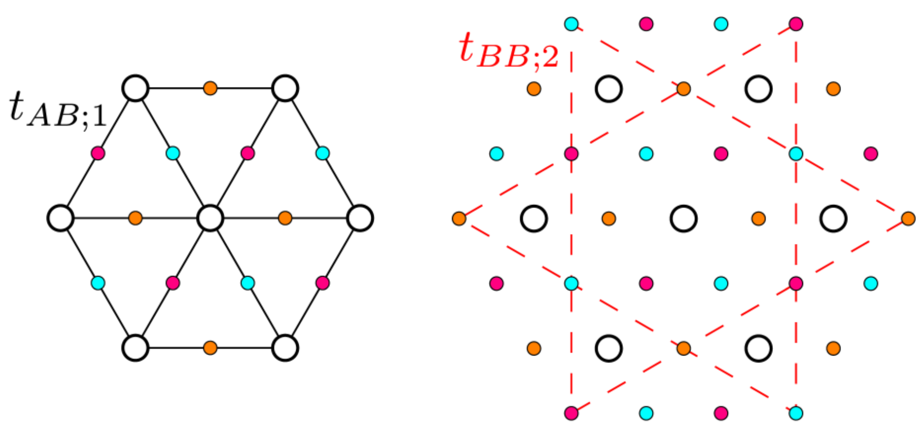
$$\mathcal{H}_{\rm kin} = \sum_{r=1}^{4} \sum_{\langle I,j\rangle_r} t_{AB;r}(a_I b_j^\dagger + \text{h.c.})$$
$$+ \sum_{r=1}^{3} \sum_{\langle i,j\rangle_r} t_{BB;r}(b_i^\dagger b_j + \text{h.c.}) + \sum_{\langle IJ\rangle} t_{AA}(a_I^\dagger a_J + \text{h.c.})$$
$$+ (-\mu_A + h)\sum_I a_I^\dagger a_I + (-\mu_B + h)\sum_i b_i^\dagger b_i,$$



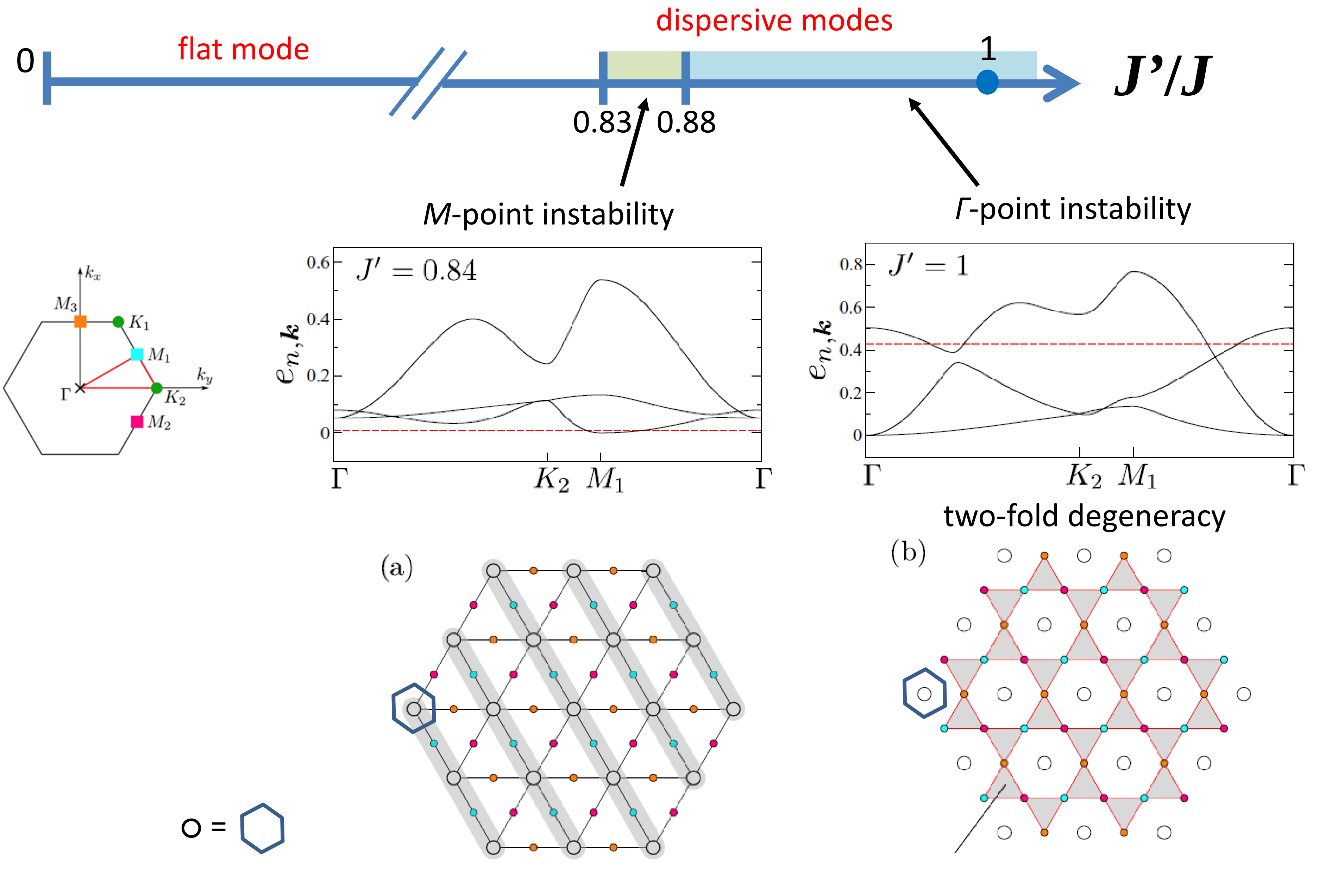




### dominant hoppings

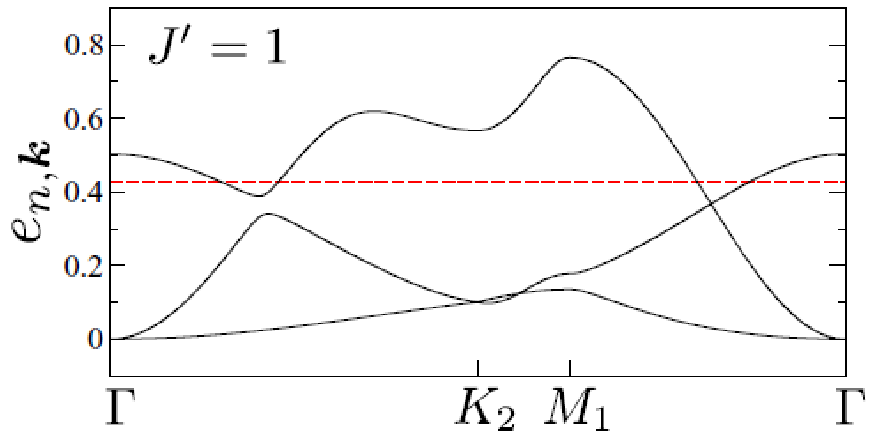


# Magnon instability at $h = h_c$

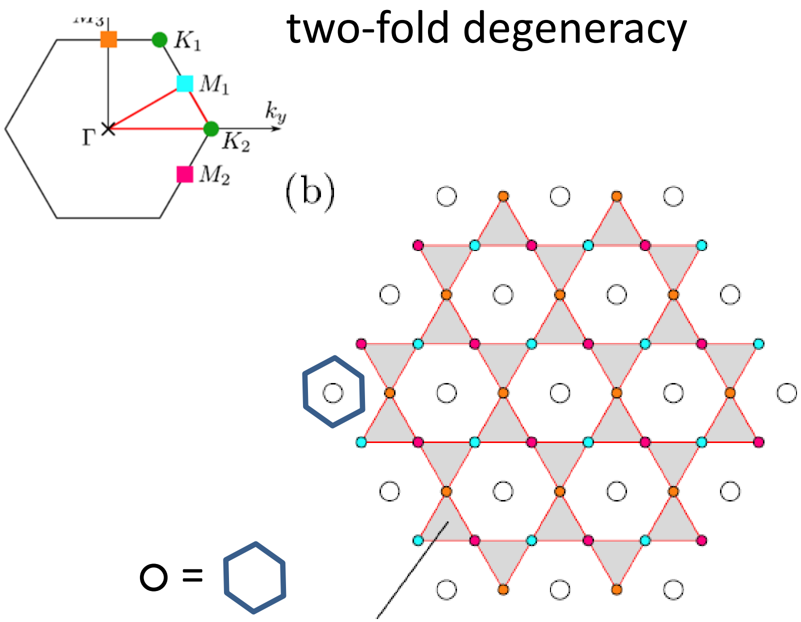


# Magnon instability at $h = h_c$

$\Gamma$-point instability




two-fold degeneracy




Connecting spins with $k = \Gamma$ wave vector have three-sublattice structure.

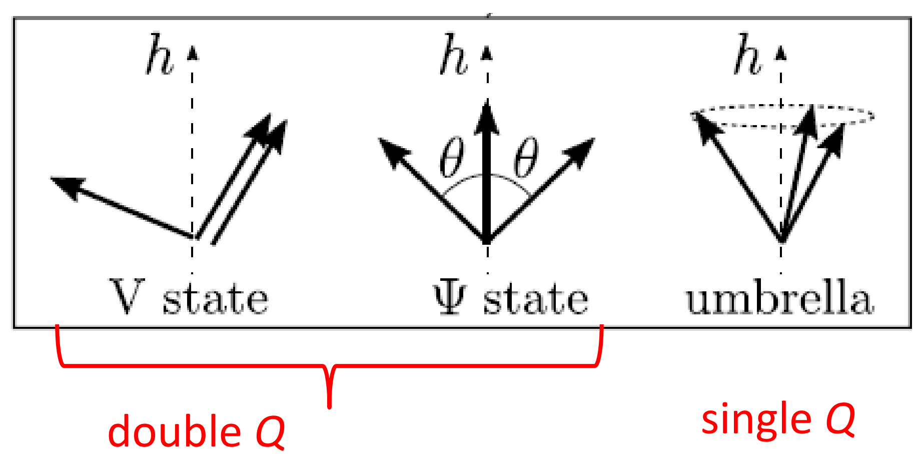
2-fold degenerate magnon instalibity has chiral degrees of freedom

$$b_{\Gamma,+}{}^{\dagger} \sim b_{1,\Gamma}{}^{\dagger} + \omega b_{2,\Gamma}{}^{\dagger} + \omega^2 b_{3,\Gamma}{}^{\dagger}$$

$$b_{\Gamma,-}{}^{\dagger} \sim b_{1,\Gamma}{}^{\dagger} + \omega^2 b_{2,\Gamma}{}^{\dagger} + \omega b_{3,\Gamma}{}^{\dagger}$$

$$\omega = \exp(2\pi i / 3)$$

Possible spin configurations



# 4. Spin structure below 5/9 plateau

Calculate all 2 magnon scattering processes by perturbation up to 3rd order of *J'*

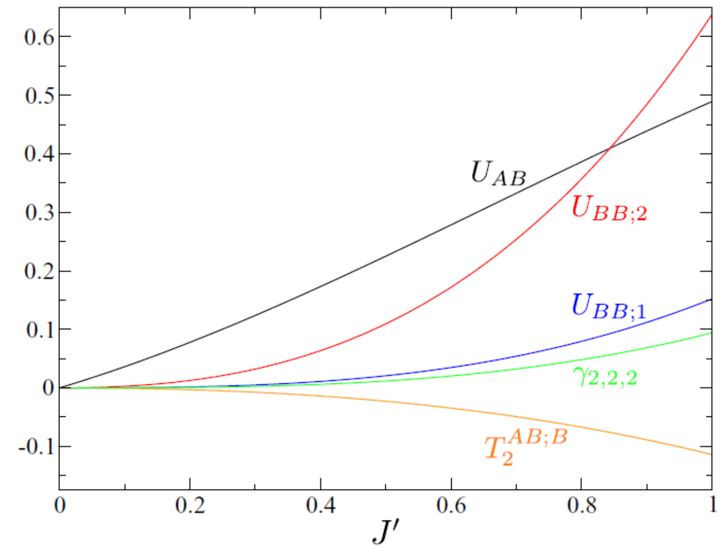
$$\mathcal{V}_{\rm int} = V_{\rm assist} + V_{\rm pair} + V_{\rm repl},$$

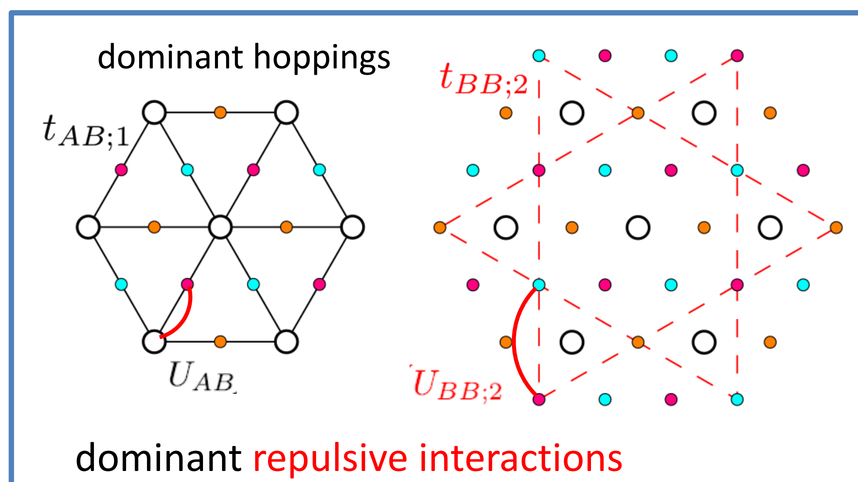
where

$$V_{\rm assist} = \sum_I \Bigg[ \sum_{r=1}^{3} \sum_{\langle jk \rangle_r \in \hexagon_I} \{ T_r^{\rm AB;B} (b_j^\dagger a_I n_k + {\rm H.c.}) + T_r^{\rm BB;A} (b_j^\dagger b_k n_I + {\rm H.c.}) \} + \sum_{jkl \in \hexagon_I} T_{r_1,r_2,r_3}^{\rm BB;B} (b_j^\dagger b_k n_l + {\rm H.c.}) \Bigg],$$

$$V_{\rm pair} = \sum_I \sum_{jkl \in \hexagon_I} \gamma_{r_1,r_2,r_3} (b_j^\dagger b_k^\dagger b_l a_I + {\rm H.c.}),$$

$$V_{\rm repl} = U_{\rm AB} \sum_{\langle iJ \rangle} n_i n_J + \sum_{r=1}^{3} U_{{\rm BB};r} \sum_{\langle ij \rangle_r} n_i n_j + U_{\rm AA} \sum_{\langle IJ \rangle} n_I n_J .$$



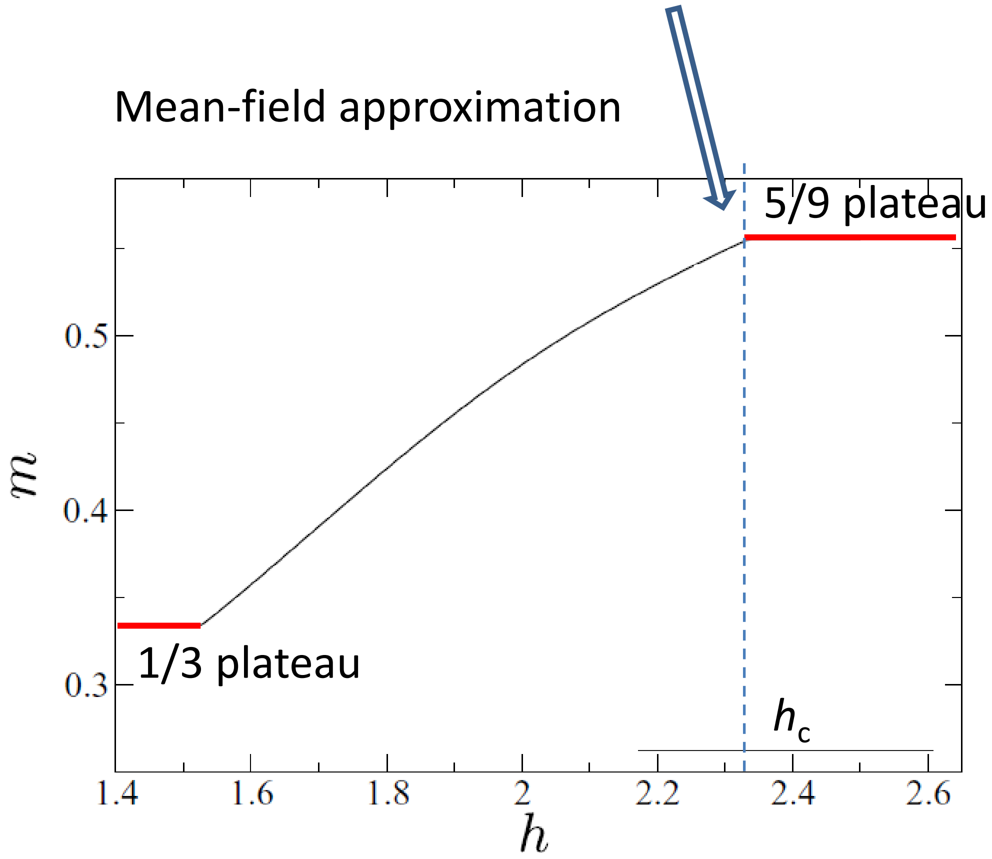
✓ Repulsive interactions stabilize dilute magnon gas.

⇒ continuous transition at $h = h_c$

✓ The lower critical field $h_c$ of 5/9-plateau is evaluated as $h_c = 2.3..$,

……. consistent with numerical results.

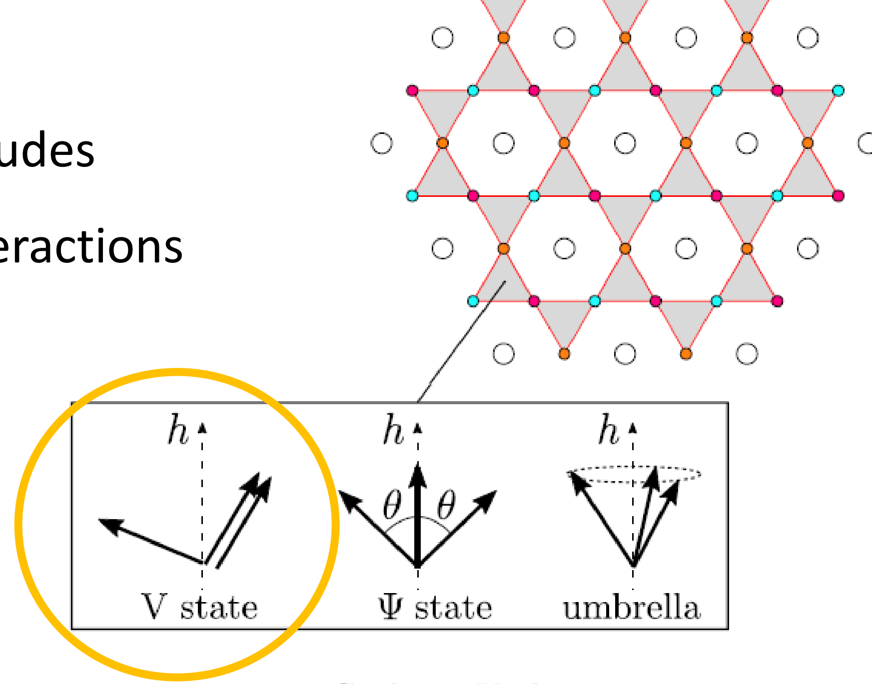
Mean-field approximation



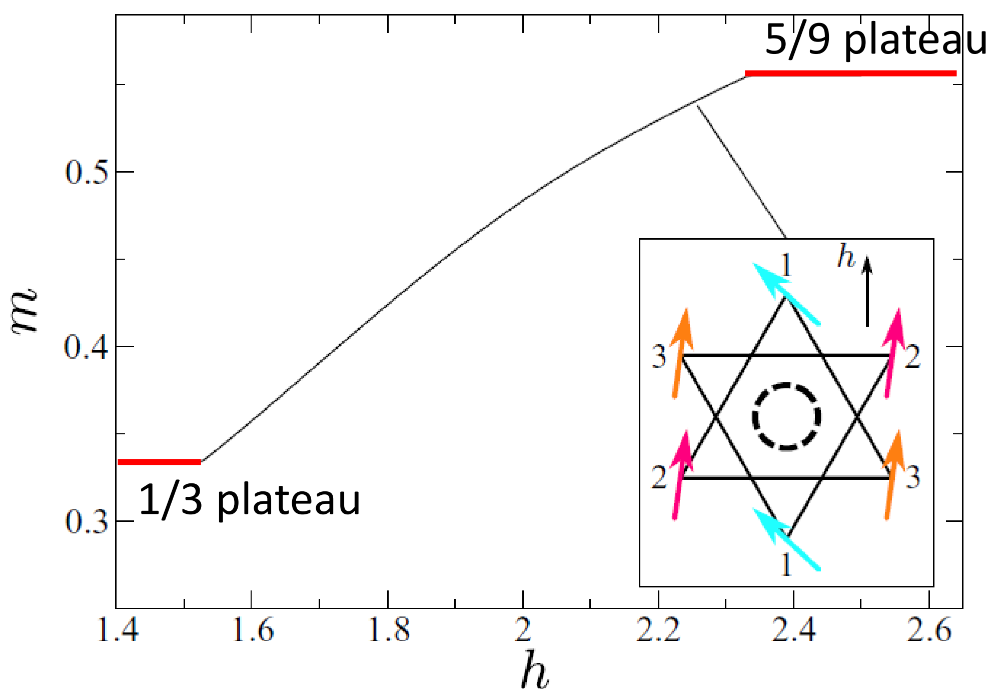
✓ Repulsive interaction >> hopping amplitudes

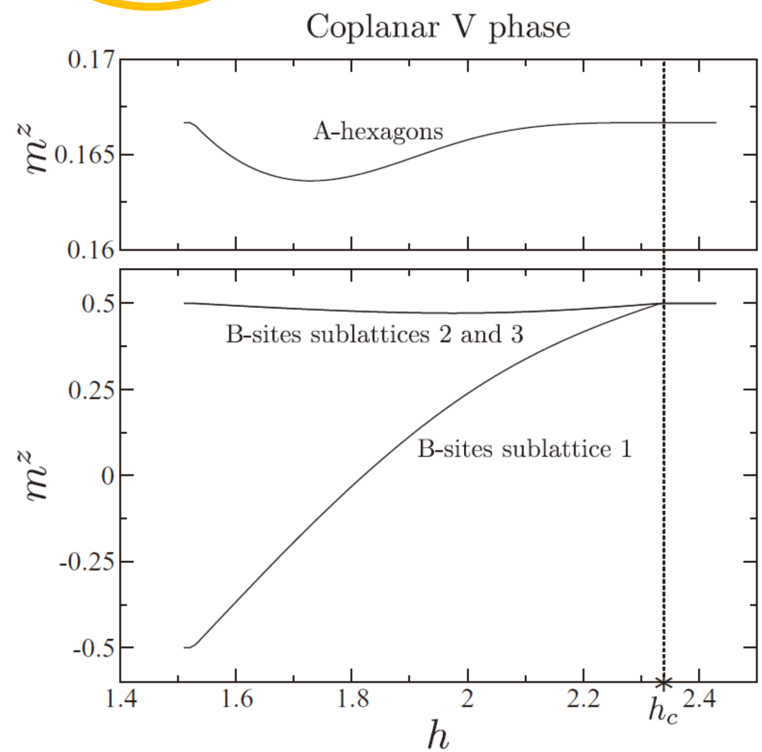
→ Ising anisotropy in effective spin interactions

✓ Ising anisotropy selects *V* state

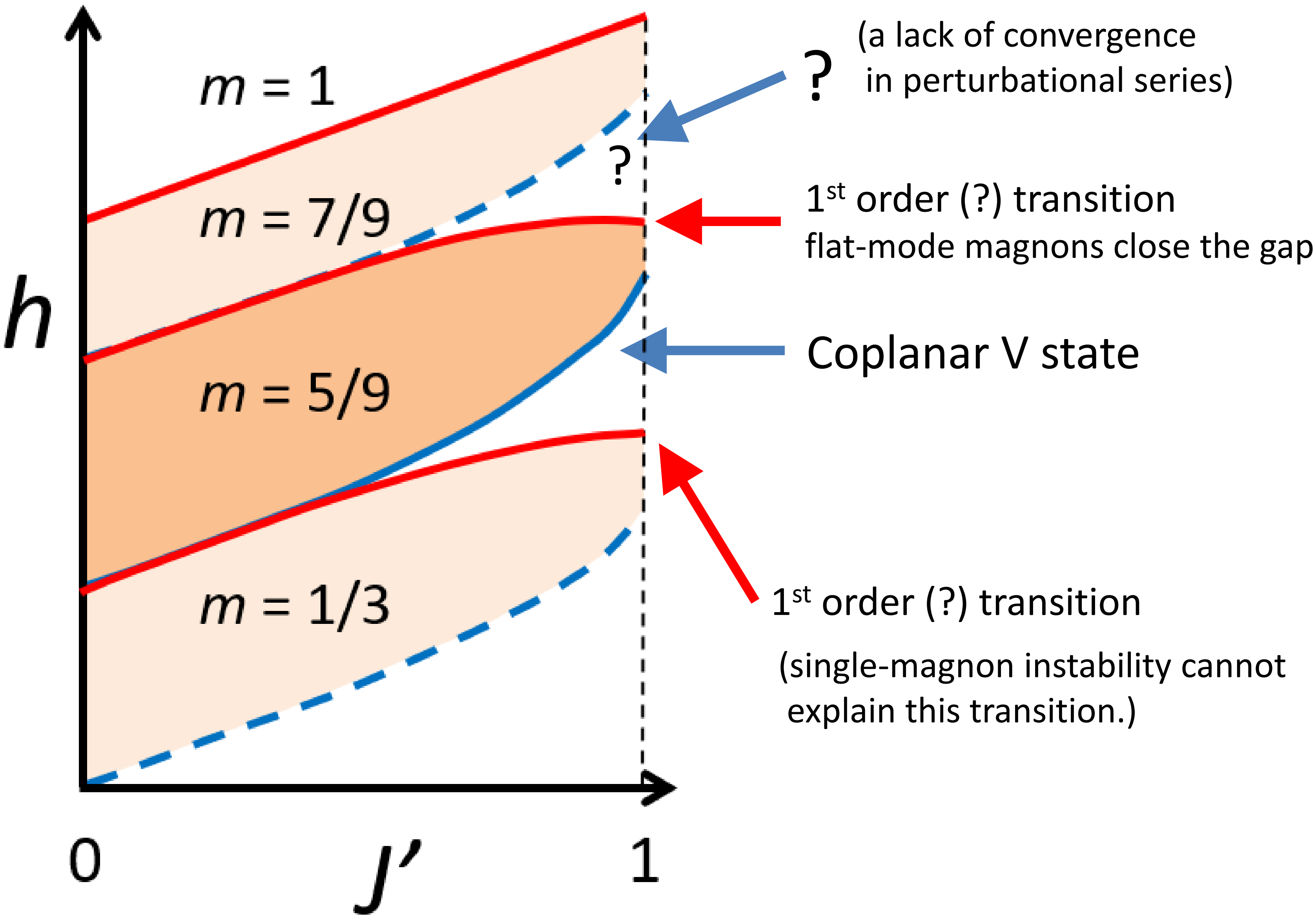



Mean-field approximation



# 5. Other plateaus



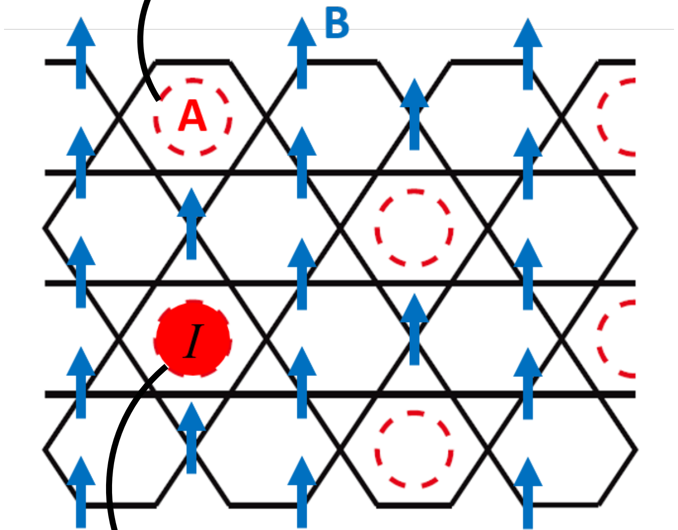
# Degenerate basis in the sector $\Delta S^z = 1$

hexagons $\underbrace{c_I^{\dagger}|0\rangle}_{\text{lowest}},\ \underbrace{a_I^{\dagger}|0\rangle}_{\text{2nd}},\ \cdots$

hopping of these magnons costs a lot of energy

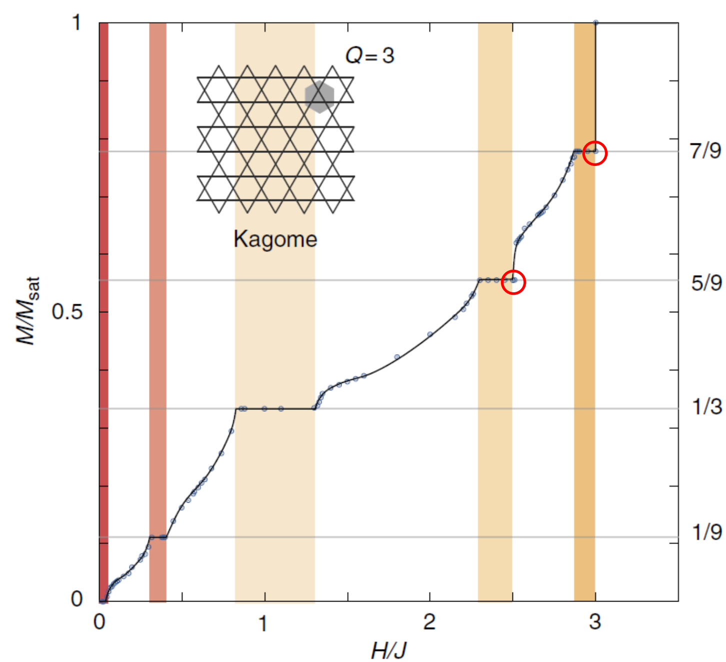
$\Rightarrow$ nearly localized

$S^z_{hex} = n$



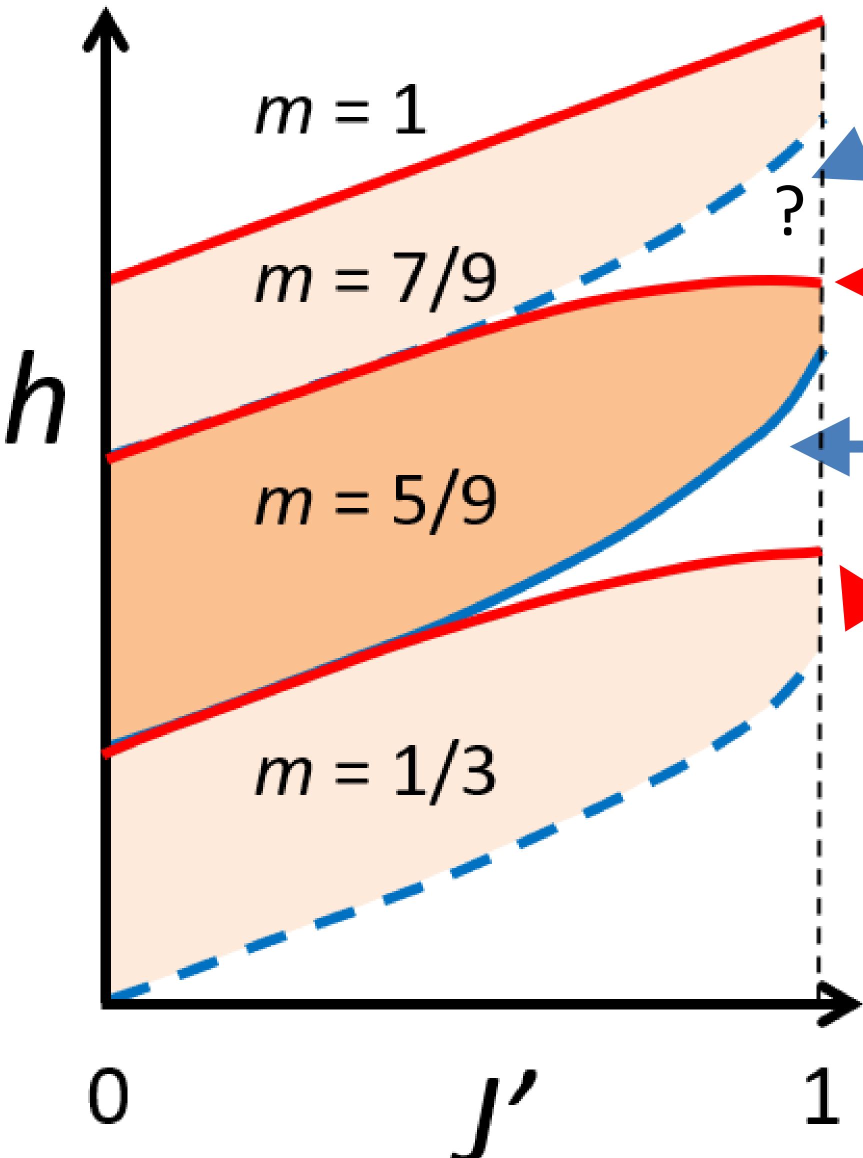

Lowest energy state in $S^z_{hex} = n+1$

Magnetization process is very steep or accompnied with a jump at the upper critical fields of plateaus.




Nishimoto-Shibata-Hotta, Nat. Comm. 2013

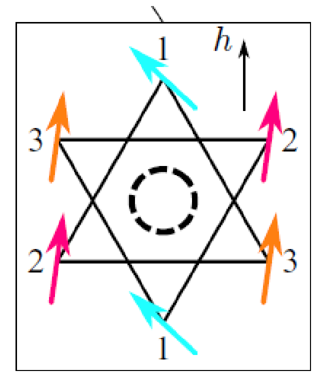
# 6. Summary




? (a lack of convergence in perturbational series)

1st order (?) transition
flat-mode magnons close the gap

Coplanar V state




1st order (?) transition

(single-magnon instability cannot explain this transition.)

Ref: X. Plat, T. Momoi, C. Hotta, Phys. Rev. B **98**, 014415 (2018)