$A_i(\alpha_s)$ & $B_i(\alpha_s)$ are anomalous dimensions & can be calculated by comparison to lower orders.

Note: $A_i(\alpha_s) \rightarrow$ numerator of $\frac{1}{1-x}$ term in the splitting of $P_{ii}(x)$ because its the IR divergent $(x \to 1)$ coefficient of collinear $(b \to \infty)$ singularity

$A^{(1)} \longrightarrow LL$

$B^{(1)}, A^{(2)} \longrightarrow NLL$ and so on.

NLL is good enough as long as

$\alpha_s(Q)\ln(bQ) << 1$

$$A_q(\alpha_s) = \frac{\alpha_s}{\pi} C_q \left(1 + \frac{\alpha_s}{\pi} K + \ldots\right), \quad K = C_A\left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{5n_f}{9}$$

Expanding A_q & B_q in $\left(\frac{\alpha_s}{\pi}\right)$ one can write

$$E_{q\bar{q}}^{PT} \approx 2C_i \int_{1/b^2}^{Q^2} \frac{dk_T^2}{k_T^2}\left[\left\{\frac{\alpha_s(k_T)}{\pi} + K\frac{\alpha_s(k_T)}{\pi}\right\}\ln\left(\frac{Q^2}{k_T^2}\right) + 2\frac{\alpha_s(k_T)}{\pi}\right]$$

$$\sim 2C_F \frac{\alpha_s(Q)}{\pi} \int_{1/b^2}^{Q^2} \frac{dk_T^2}{k_T^2} \left[\left\{ 1 + \left(\frac{\alpha_s(Q)}{\pi} \right) (K - \beta_0) \right\} \ln\left(\frac{Q^2}{k_T^2}\right) + 2 \frac{\alpha_s(Q)}{\pi} \right]$$

$$\sim \alpha_s \ln^2(bQ) \left(1 + \alpha_s \ln(bQ) + \ldots \right)$$
$$+ \alpha_s \ln(bQ) \left(1 + \alpha_s \ln(bQ) + \ldots \right) + \ldots$$

These are LL, NLL ... corrections

After evaluating the resummed exponent, (upto certain order) one performs an inverse Fourier transform

$$\frac{d\sigma(Q_T)}{dQ^2\, d^2Q_T} = \sum_a H_{a\bar{a}}(\alpha_s(Q^2)) \int \frac{d^2b}{(2\pi)^2} e^{i \vec{Q}_T \cdot \vec{b}}$$
$$e^{E^{PT}_{a\bar{a}}(b, Q, \mu)}\; e^{-\mu_I^2 b^2 \left(\ln\left(\frac{Q}{\mu_I}\right) + \ldots\right)}$$
$$\sum_{a = q, \bar{q}} \int_{\xi_1, \xi_2} \frac{d^2\hat{\sigma}_{a\bar{a} \to \mu^+\mu^-}(Q, \mu)}{dQ^2} f_{a/A}\left(\xi_1, \frac{1}{b}\right) f_{\bar{a}/B}\left(\xi_2, \frac{1}{b}\right)$$

$e^{-\mu_I^2 b^2 \ln\left(\frac{Q^2}{\mu_I^2}\right) + \ldots}$ is non-perturbative exponent
→ introduced to deal with an singularity in integrand at $b^2 = Q^2 \exp\left[\frac{-4\pi}{\beta_0 \alpha_s(Q^2)}\right] \sim \frac{1}{\Lambda^2}$

Threshold Resummation

Recall

$$\frac{d^2\hat{\sigma}^{(1)}}{dQ^2 d^2Q_T} = \sigma_0 \frac{\alpha_s C_F}{\pi^2}\left[1 - \frac{Q_T^2}{(1-z)^2 \xi_1 \xi_2 s}\right]^{-1/2}$$

$$\left[\frac{1}{Q_T^2}\cdot\frac{1+z^2}{1-z} - \frac{2z}{(1-z)Q^2}\right].$$

Q_T integrated NLO partonic x-sect^n

$$\frac{d^2\sigma^{(1)}_{q\bar{q}\to\gamma^* X}(z, Q^2, \mu^2)}{dQ^2} = \sigma_0(Q^2)\frac{\alpha_s(\mu)}{\pi}$$

$$\left[2(1+z^2)\left[\frac{\ln(1-z)}{1-z}\right]_+ - \frac{(1+z^2)\ln z}{z}\right.$$

$$\left. + \left(\frac{\pi^2}{3} - 4\right)\delta(1-z)\right] + \sigma_0(Q^2) C_F \frac{\alpha_s}{\pi}\left(\frac{1+z^2}{1-z}\right)_+ \ln\frac{Q^2}{\mu_F^2}$$

I

Recall + distributions are generalized functions defined as

$$\int_0^1 dx \frac{f(x)}{(1-x)_+} = \int_0^1 dx \frac{f(x) - f(1)}{1-x}$$

$$\int_0^1 dx\, f(x)\left(\frac{\ln(1-x)}{1-x}\right)_+ = \int_0^1 dx\,(f(x) - f(1))\frac{\ln(1-x)}{1-x}$$

etc

here $f(x)$ will be pdfs.

ex. Splitting fns $P_{qq}(z) = C_F \frac{\alpha_s}{\pi} \left(\frac{1+z^2}{1-z}\right)_+$

$$\longrightarrow \frac{A(\alpha_s)}{1-z}$$

Threshold resummation is resummation of plus distributions.

For Resummation of plus distribution transform to Mellin space

$$f(N) = \int_0^1 dz \; z^{N-1} f(z)$$

Use $\quad I_n(N) = \int_0^1 dz \; z^{N-1} \left(\frac{\ln^{(n)}(1-z)}{1-z}\right)_+$

with

$$I_0(N) = -\ln\tilde{N} + O\left(\frac{1}{N}\right), \quad \tilde{N} = N e^{\gamma_E}$$

$$I_1(N) = -\frac{1}{2}\ln^2\tilde{N} + \frac{1}{2}\zeta_2 + \dots$$

$$I_2(N) = -\frac{1}{3}\ln^3\tilde{N} - \zeta_2 \ln\tilde{N} - \frac{2}{3}\zeta_3 + \dots$$

$$I_3(N) = \frac{1}{4}\ln^4\tilde{N} + \frac{1}{2}\zeta_2\ln^2\tilde{N} + 2\zeta_3\ln\tilde{N} + \frac{3}{2}\zeta_4 + \dots$$

Also
convolution in z space

$\Longleftrightarrow$ products in N space

$z \to 1$ limit $\Longleftrightarrow$ $N \to$ limit

Using the same method as for k_T resummation, we get

$$\Rightarrow \sigma(N) = \exp\left[-\int_0^1 dx \, \frac{x^{N-1}-1}{1-x} \left\{ \int_{Q^2}^{Q^2(1-x)^2} \frac{d\mu}{\mu} A(\alpha_s(\mu^2)) + D\left(\alpha_s((1-x)^2 Q^2)\right) \right\} \right] \times \left(1 + \alpha_s(Q^2)\frac{C_F}{\pi} + \ldots \right)$$

$$\sim \alpha_s \ln^2 N \left(1 + \alpha_s \ln N + \ldots\right) + \alpha_s \ln N \left(1 + \alpha_s \ln N + \ldots\right)$$

$A(\alpha_s(\mu))$ is coeff of $\frac{1}{1-z}$ term in the splitting function (Note this from eq^n I on previous page)

$A(\alpha_s)$ & $B(\alpha_s)$ are ~~fixed~~ determined by matching to fixed order result

(NNNLL, Moch, Vermaseren, Vogt, 05)

~~A~~ One can expand ~~α_s~~ & A and B in powers of α_s and work upto Leading log (LL) order $\to A_q^{(1)}$, next to leading log NLL " $\to A_q^{(2)}, B_q^{(1)}$ and so on.

Note $\sigma(N)$ → exponential form, two log integr-ry integrands are only fns of α_s

One can carry out the integrals & show that

$$\hat{\sigma}_{DY}(N, Q^2) = g_0^{H(Q)} \cancel{g_0(Q^2)} \exp\left[G^N_{DY}(Q^2) \right]$$

$$G^N_{DY} = \ln N \, g_1(\lambda) + g_2(\lambda) + \alpha_s g_3(\lambda) + \cdots$$

where $\lambda = \beta_0 \alpha_s \ln N$.

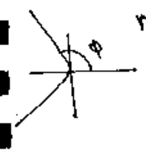
$$g_1(\lambda) = \frac{C_F}{\beta_0 \lambda}\left[\lambda + (1-\lambda)\ln(1-\lambda) \right]$$

Computations ~~does~~ are done upto NNLL &

good convergence observed as logarithmic accuracy of resummation is increased

Result Enhancement observed for Higgs production at LHS (Catani etal 2003) (NNLL vs NNLO)

After summation one performs an inverse Mellin transform via



$$\int_C dN\, e^{zN} f(N) \quad \text{or} \quad \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dN\, x^{-N} f(N)$$

poles on Re N axis

↳ can only be done numerically

Joint Resummation (Laenen, Sterman & Vogelsang 2003)

Combine threshold & recoil resummations into one formalism

→ another kind of refactorization

⟹ logs in $\ln^2 N$ and $\ln^2 b$

~~Expt~~ Phenomenological Studies (some of them!)

1) Higgs Transverse mom. distn at LHC, de Florian, Kulesza & Vogelsang hep-ph/0511205

$\frac{d\sigma}{dp_T}$ $K^{NLL/NLO}$ ≈ 1.1 $m_H = 125$ GeV $0 < p_T < 300$

2) hep-ph/0311194 G. Bozzi

LHC results → NNLL + NLO vs NLO

resummatn effects start showing at low GeV & increase NLO result by 40%.

(58)

3) Drell Yan → Catani → 2003

NNLL + NLO vs D0 data at (1.96 TeV) Tevatron ~~vs NLO~~ $0 < q_T < 50$

$\sqrt{s} = 1.96$ at $\mu_R = \mu_F = m_t$

describes data well

~~Bonvini hep~~

4) Bonvini, hep-ph/1006.5918

NNLO + NNLL for Drell Yan x-sectn

→ inclusion of resummed terms gives a small improvement

at $\sqrt{s} = 7$ TeV (mass of inv pair = 1 TeV)

5) Marzani, arXiv:1006.2314

LHC : DY → Resummation effects correct the NNLO results by a few % for $M^2_{\mu^+\mu^-} < 100$ GeV

6) Bozzi et al, 0705.3887

Higgs $\frac{d^2\sigma}{dq_T\,dy}$ at LHC with $M_H = 125$ GeV

NNLL + NLO vs NLO → 30% higher at $q_T = 50$ GeV.

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