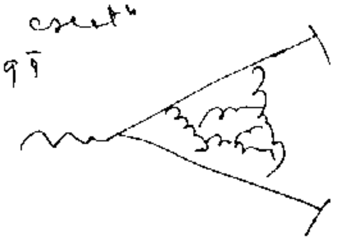
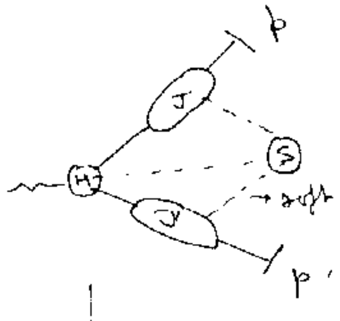
One can identify Landau equations & power counting to identify the sources of IR divergences in any process

Pair creat^n $\gamma^* \to q\bar{q}$

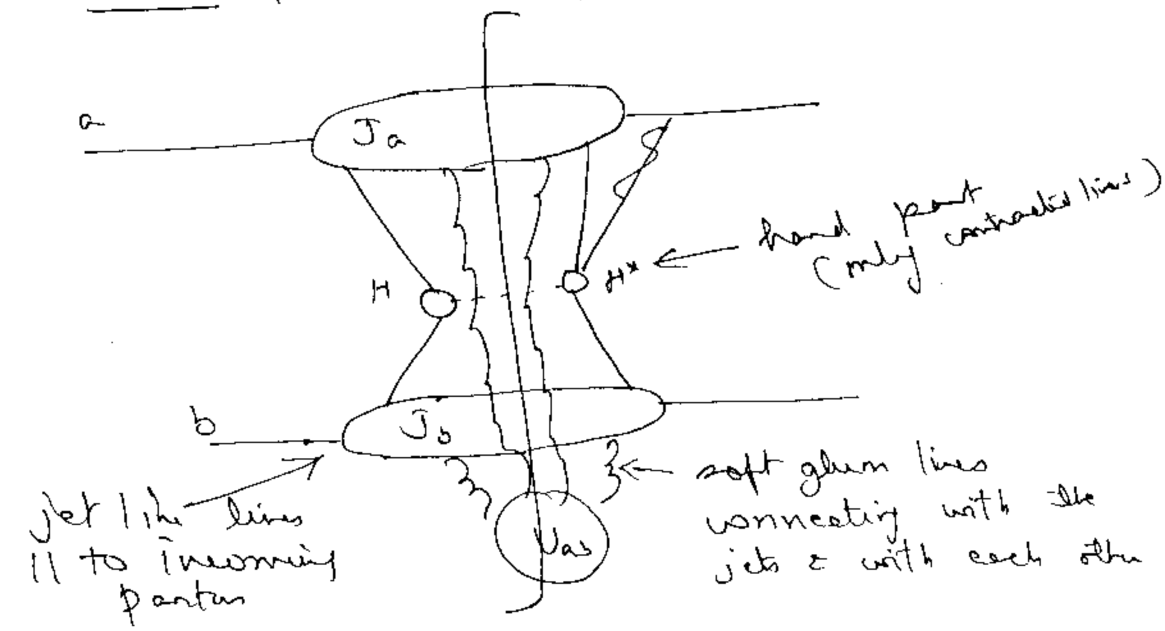


reduced diagrams are all of the form



corresponds to a physical processes in which the decay is followed by two jets (J & J') of virtual particles in the same directions with same momenta p & p' as the two final particles

Ex DY for creating an electroweak boson



Reduced diagram can be expressed as

$$\sum_C \int \frac{dk_a^+}{2\pi} \frac{dk_b^-}{2\pi} H^{(c)}(k_a^+, k_b^-)$$

$$\prod_l \int \frac{d^4 q_l}{(2\pi)^4} \prod_j \frac{d^4 \bar{q}_j}{(2\pi)^4} \quad J_a^{(c)}(k_a^+, q_l^\alpha)^{(\mu_1,\mu_2,\ldots \mu_n)}$$

$$U^{(c)}(q_l^\alpha, \bar{q}_j{}^\beta)_{\{\mu_1 \ldots \mu_n, \nu_1 \ldots\}}$$

$$J_b^{(c)}(k_b^-, \bar{q}_j^\beta)^{\{\nu_1 \ldots \nu_n\}}$$

Collin, Soper, Sterman: Jet fn. may be shown to couple through only one parton to H.

Very complicated. Simplify using

Soft Approximation: Introduce $v^\mu = \delta_{\mu +}$, $u^\mu = \delta_{\mu -}$

$$J^{(c)}(k_a^+, q_a^\alpha)^{\{\mu_1 \ldots \mu_n\}} \Rightarrow J_a^{(c)}(k_a^+, (q_l \cdot v) u^\alpha)^{\{\xi_1 \ldots \xi_n\}}$$

$$u_{\xi_1} \ldots u_{\xi_n} v^{\mu_1} \ldots v^{\mu_n}$$

$$J_b^{(c)}(k_b^-, q_j^{\beta-})^{\{\nu_1 \dots \nu_n\}} \longrightarrow J_b^{(c)}(k_b^-, (q_j \cdot u))^{\{\lambda_1 \dots \lambda_n\}} \times v_{\lambda_1} \dots v_{\lambda_n} u^{\nu_1} \dots u^{\nu_n}$$

means neglect all dependence on soft gluons momenta components in jet functions (both in momentum and polarization) except the opposite moving components.

$\Rightarrow$ ($\Rightarrow J_a$ depends on only minus component of q^μ)

Soft Approximation amounts to eikonal approximation & therefore the sum over all attachments of soft gluons can be replaced by eikonal lines which only remember the direction of jets

$\Rightarrow$ Jet-Soft factorization

$\rightarrow$ Proof non-trivial

Let me try to explain with simple example

Consider coupling of one soft gluon (q^μ) to a fast quark with momentum $p^\mu = (E, 0, 0, E)$ or $(p^+, 0, \bar{0}_T)$ in light-cone notation $\left(p^+ = \frac{E+E}{\sqrt{2}}\right)$

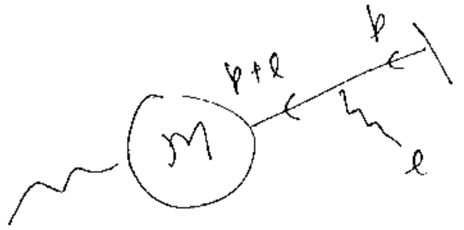
$$g J^\mu(p,q)\, \underbrace{G_{\mu\nu}(q)}_{\text{gluon propagator}} \simeq J^+(p,q)\, G_{+\nu}(q)$$

$$\simeq g J(p) \underbrace{\left(\frac{i p^+}{p^+ q^-} G_{+\nu}(q)\right)}$$

Since $p^\mu >> q^\mu$, the jet sees both for its polarization & the momentum only the q^- component of the soft gluon

$\longrightarrow$ Soft gluon factorizes from fast (jet like) quark.

Eikonal Approximation



$l_\mu \to$ soft

$p_\mu \to$ jet like

$$-ig\, t^a \mathcal{M} \frac{i(\not p + \not l)}{(p+l)^2}\gamma^\mu u(p)\, \epsilon_\mu(l)$$

soft approx. $\quad \not p + \not l \longrightarrow \not p$

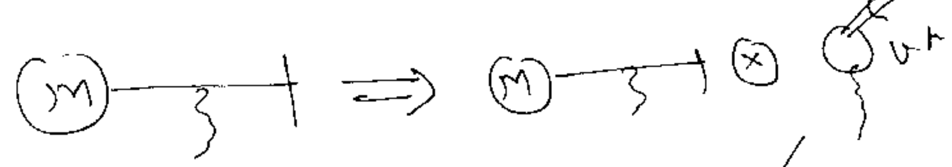
$$(p+l)^2 \longrightarrow 2\, p\cdot l$$

$$\longrightarrow (-g\, t^a)\mathcal{M}\frac{\not p}{2p\cdot l}\not\epsilon\, u(p)$$

$$\xrightarrow{\text{Dirac eq}^n} \left(g\, t^a \mathcal{M} u(p)\right)\frac{p\cdot\epsilon}{p\cdot l}$$

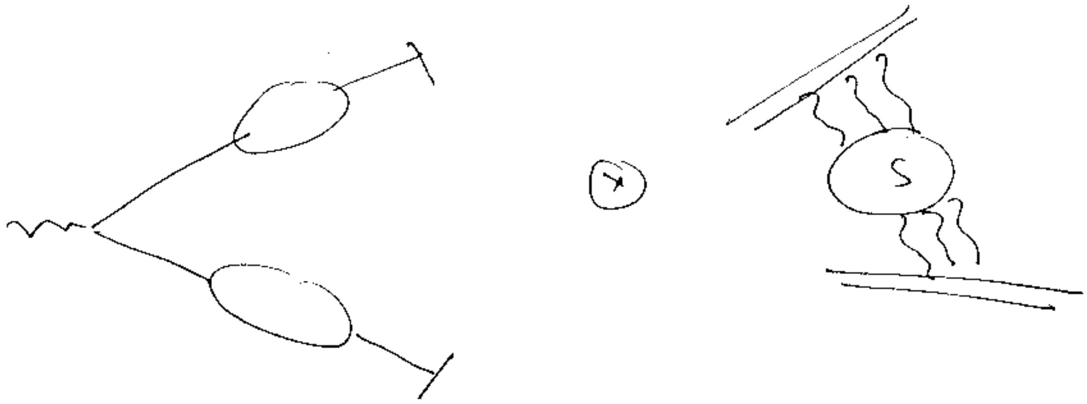
$p^\mu \sim \delta^{\mu +} \implies \epsilon$ couples to only one component of p

Thus



Eikonal Feynman rule

For full graph



Jet degree of freedom factorizes from soft degree of freedom

→ Proof non-trivial.

⇒ Jet Soft factorization

$$d\sigma = \sum_c \int \frac{dk_a^+}{2\pi} \frac{dk_b^-}{2\pi} H(k_a^+, k_b^-) J_a^{(c)}(k_a^+) U^{(c)} J_b^{(c)}(k_b^-).$$

$$\sigma_{DY} = \Phi_A \otimes \Phi_B \otimes H \otimes U$$

When we are well above threshold (plenty of energy) sum over cuts gives

$$\sum_C U_c = 1.$$

and one obtain factorization theorem

$$\sigma_{DY} = \Phi_A \otimes \Phi_B \otimes H_{ab}$$

$$\sim \int dk_a^+ \, dk_b^- \, J_a(k^+) \, H_{ab}(k_a^+, k_b^-) \, J_b(k^-)$$

When we are near threshold ($\tau \simeq 1$, $\tau = \frac{Q^2}{s}$) all radiation is soft

→ jet function is sensitive to changes in the amount of radiation they contribute

⟹ sum ~~of~~ over final state cuts for U becomes effectively weighted

⟹ although singularities cancel, logs remain.

Resummation: Reorganization of these logs

$$\hat{\sigma} = 1 + \alpha_s (L^2 + L + 1) + \alpha_s^2 (L^4 + L^3 + L^2 + 1) + \ldots$$

after resummation

$$\hat{\sigma} \rightarrow \exp\Big[L\, g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s\, g_3(\alpha L) + \dots \Big]\, C(\alpha_s)$$

Why do these logs exponentiate?

Various ways to show

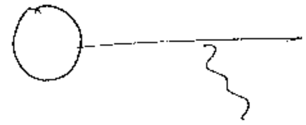
One is using factorization which we will use.

But before that we'll just give a loose argument to show the exponentiation.

Recall: large logs are closely related to IR divergences.

& IR divergences exponentiate

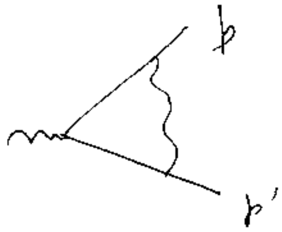
Recall eikonal approximation



$$\sim g\,(\mathcal{M}\,u(p))\,\frac{p^{\mu}}{p\cdot k}$$

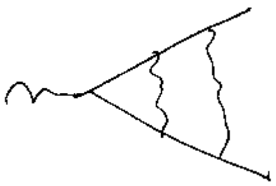
Under eikonal approximation the first virtual correction is

$$\mathcal{M}_0 \int \frac{d^n k}{k^2}\,\frac{p\cdot p'}{(p\cdot k)(p'\cdot k)}$$



as we saw before

Second order correction



$+$

$$\rightarrow \mathcal{M}_0 \int d^n k_1\, d^n k_2\, \frac{(p\cdot p')^2}{k_1^2\, k_2^2}\,\frac{1}{p'\cdot(k_1+k_2)\; p'\cdot k_2}\left[\frac{1}{p\cdot(k_1+k_2)\; p\cdot k_2} + \frac{1}{p\cdot(k_1+k_2)\; p\cdot k_1}\right]$$

Use the identity

$$\frac{1}{p\cdot(k_1+k_2)\;p\cdot k_2} + \frac{1}{p\cdot(k_1+k_2)\;p\cdot k_1}$$

$$= \frac{1}{(p\cdot k_1)(p\cdot k_2)}$$

& symmetrize

$$\longrightarrow \quad A_0 \frac{1}{2}\left(\int d^n k \; \frac{p\cdot p'}{k^2\,(p\cdot k)(p'\cdot k)}\right)^2$$

second term in exponential

In general one uses

$$\sum_{\text{perm}} \frac{1}{p\cdot k_1}\;\frac{1}{p\cdot(k_1+k_2)}\;\cdots\;\frac{1}{p\cdot(k_1+k_2+\cdots k_n)}$$

$$= \frac{1}{p\cdot k_1}\;\frac{1}{p\cdot k_2}\;\cdots\;\frac{1}{p\cdot k_n}$$

To get an all order result

$$A_0 \exp\left[\int \frac{d^n k \quad p\cdot p'}{k^2\,(p\cdot k)(p'\cdot k)}\right]$$