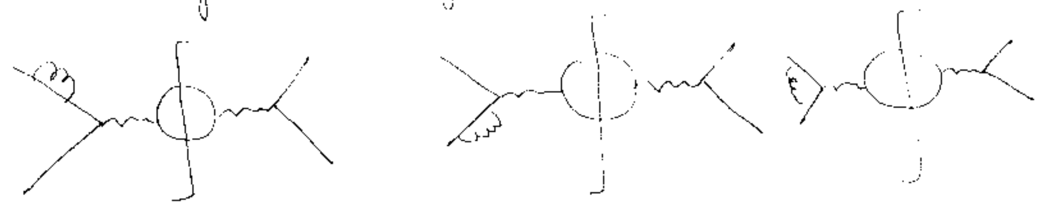
$O(\alpha_s)$ corrections to Born cross section include contributions from virtual corrections also with cut diagrams as follows



matrix elements for virtual corrections are obtained by multiplying the Feynman diagrams for the Born amplitude with graphs containing vertex and quark self energy corrections. Loop integrals contain UV divergences which can be regulated in $\frac{1}{\epsilon}$ poles & removed by adding a counterterm to the QCD Lagrangian. There are also IR divergences from the loop integrals. If all the contributions of virtual corrections are added we get

$$\boxed{\frac{\alpha_s}{\pi}\frac{d\sigma^v}{dQ^2} = \frac{\alpha_s}{\pi}\sigma_B C_F D(\epsilon)\,\delta(1-z)\left[-\frac{2}{\epsilon^2}-\frac{3}{\epsilon}+\frac{2\pi^2}{3}+O(\epsilon)\right]}$$

Adding the real & virtual part, cancels the $\frac{1}{\epsilon^2}$ poles. $\delta(1-z)$ in $\frac{d\sigma^v}{dQ^2}$ reflects the fact that the final state contains only 2 particles.

Adding real & virtual corrections we get

$$\frac{d\sigma^{(1)}}{dQ^2} = \frac{d\sigma^r}{dQ^2} + \frac{d\sigma^v}{dQ^2}$$

$$= \sigma_B \, D(\epsilon) \left[-\frac{2}{\epsilon} P_{qq}(z) + R(z) \right]$$

where

A-P. splitting fn. $\longrightarrow$ $P_{qq}(z) = C_F \left[\frac{(1+z^2)}{(1-z)_+} + \frac{3}{2}\delta(1-z) \right]$

and $R_z = C_F \left[\delta(1-z)\left(\frac{2\pi^2}{3} - 8\right) + 4(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ - 2\left(\frac{1+z^2}{1-z}\right)\ln z \right]$

Note the presence of large logs in $R(z)$

second Term : $\longrightarrow$ $\left(\frac{\ln(1-z)}{1-z}\right)_+$

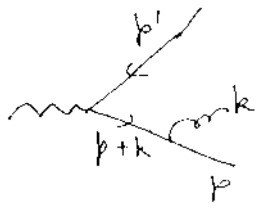
these terms become large in the threshold limit $z \to 1$. It is the exponentiation of these large logs that ~~is~~ ~~the~~ is performed in THRESHOLD RESUMMATION.

So what is the 'physical' origin of these logs?

→ same as that of IR divergences

Recall em vertex fn in massless GT with emission of one real gluon

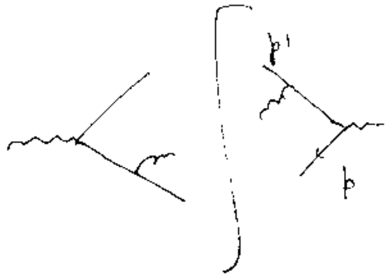
propagator $(p^2 = k^2 = 0)$



$$\frac{1}{(p+k)^2} \longrightarrow \frac{1}{2p\cdot k}$$

$$\frac{1}{2p\cdot k} = \frac{1}{2E_g E_q (1-\cos\theta_{qg})}$$

Phase space integrati:



$$\alpha_s \int \frac{d^4k}{(2\pi)^4} \frac{p\cdot p'}{(p\cdot k)(p'\cdot k)}$$

$$\sim \alpha_s \int \frac{dE_g}{E_g} \int \frac{d\theta_{qg}}{\theta_{qg}} \sim \alpha_s \ln^2(\quad)$$

In dim regularizatn

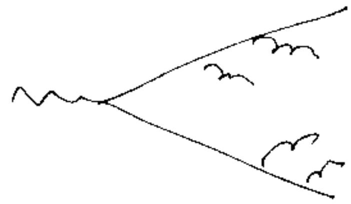
$$\alpha_s \int \frac{d^dk}{(2\pi)^4} \frac{p\cdot p'}{(p\cdot k)(p'\cdot k)} \sim \int \frac{dE_g}{E_g} E_g^{-\epsilon} \int \frac{d\theta_{qg}}{\theta_{qg}} \sin^{\epsilon}\theta_{qg}$$

$$\sim \alpha_s \left[-\frac{1}{\epsilon^2} + \ln^2 K \right]$$

$\Rightarrow$ double logs are directly linked to collinear & IR divergences

In general

n gluon emission



$$\sim (\alpha_s L^2)^n$$

<u>SO</u> : <u>Double</u> logs <u>are associated with soft & collinear emissions & follow from the same integrals as IR & collinear divergences</u>

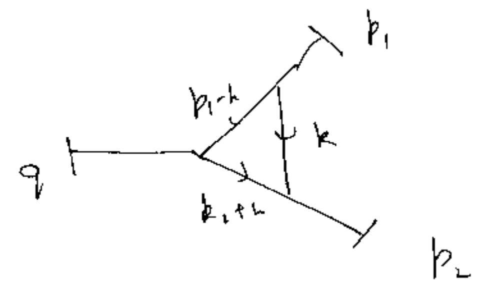
An all order analysis therefore requires an all-order treatment of IR & collinear divergences

→ Collins, Soper & Sterman

Q. How to analyze & classify sources of LD behaviour in PT?

→ Landau eqn. & IR power counting

Ex. Consider fully massless 3-pt function at one loop with two on shell external lines (in scalar theory)



$$I_\Delta = g^3 \mu^{3\epsilon} \int \frac{d^n k}{(2\pi)^n} \frac{1}{[k^2 + i\epsilon][(p_1 - k)^2 + i\epsilon][(p_2 - k)^2 + i\epsilon]}$$

Use Feynman parametrizat^n & n dim integration & calculate

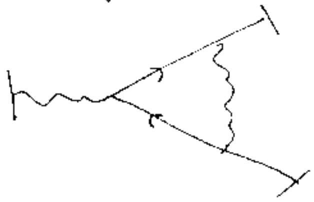
$$I_\Delta = (-i g \mu^\epsilon) \frac{1}{q^2} \frac{g^2}{(4\pi)^2} \left(\frac{4\pi \mu^2}{-q^2 - i\epsilon} \right)^\epsilon \Gamma(1+\epsilon) \times \frac{B(-\epsilon, -1-\epsilon)}{(-\epsilon)}$$

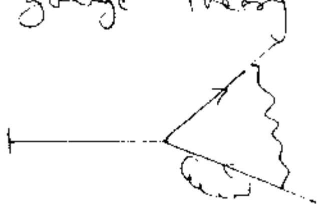
$$q^2 = 2\, p_1 \cdot p_2 \quad , \quad \epsilon = 2 - \frac{n}{2}.$$

integral → UV finite

⟹ Double pole has entirely IR origin

Consider the same result for electromagnetic vertex function in massless gauge theory (16)





For zero mass fermions the em vertex is given in terms of a single form factor

$$\Gamma_\mu(q^2,\epsilon) = -ie\mu^\epsilon\, \bar{u}(p_1)\,\gamma_\mu\, v(p_2)\, F(q^2,\epsilon)$$

At one loop

$$F(q^2) = -\frac{\alpha_s}{2\pi}\, C_F \left(\frac{4\pi\mu^2}{-q^2-i\epsilon}\right)\frac{\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-2\epsilon)}$$

$$\times \left\{\frac{1}{(-\epsilon)^2} - \frac{3}{2}\,\frac{1}{(-\epsilon)} + 4\right\}$$

↳ leading double pole — essentially the same as in scalar case

→ sufficient to consider massless case for the present discussion

Consider I_Δ over a region of momentum space where

$$\boxed{k^2 << p_1 . k ,\ p_2 , k} \quad \text{(Eikonal Approximation)}$$

Choose a frame s.t.

$$p_1 = (p_1^+, 0, \vec{0}_\perp) \quad , \quad p_2^+ = (0, p_2^-, \vec{0}_\perp)$$

$$p^\pm = \frac{1}{\sqrt{2}} (p^0 \pm p^3) \quad , \quad p^2 = 2 p^+ p^- - p_\perp^2$$

$$I_\Delta^{(eik)} \sim \frac{1}{q^2} \int \frac{dk^+ \, dk^- \, d^2k_\perp}{(-k^- + i\epsilon)(k^+ + i\epsilon)(2k^+k^- - k_\perp^2 + i\epsilon)}$$

→ 3 limiting regions that lead to logarithmic divergences

1) Soft region : all four components of k^μ vanish together

2) 3) Collinear .. : component of k^μ parallel to either p_1^μ or p_2^μ remains finite while other components vanish in such a way that $k^+ k^- \sim k_\perp^2$

Momentum components in these regions are of the order of $\sqrt{q^2}$ $\cancel{\times(\lambda)^a}$ time powers of a scaling variable λ which vanishes at the points in momentum space where I_Δ is singular.

1. $k^\mu \sim \lambda \sqrt{q^2}$ soft

2. $k^\pm \sim \sqrt{q^2}$

$k^\mp \sim \lambda \sqrt{q^2}$

$k_\perp \sim \lambda \sqrt{q^2}$

→ change variables in these regions to λ

→ to make log. div. in I_Δ explicit

define scaled momenta

$$\bar{k}^\mu = \frac{k^\mu}{\lambda^a}$$

easy to do in this simple example.

How to do it ~~for~~ at higher orders?

Landau eq^{n}s

Consider an arbitrary Feynman diagram

$$G = \prod_{i=1}^{L} d^d l_i \; N(l_i, p_i) \prod_{j=1}^{I} \frac{1}{k_j^2 - m_j^2 + i\epsilon}$$

L = no. of loops

I = " " internal lines

$\{p_i\}$: external momenta

$\{k_j\}$: internal line momenta (combination of $\{l\}$ & $\{p\}$)

Put $m_j = m$ & use Feynman parametrizat^n

$$G = (I-1)! \prod_{j=1}^{I} \int_0^1 d\alpha_j \, \delta(1-\alpha_1-\dots-\alpha_I)$$

$$\prod_{i=1}^{L} d^d l_i \; N(l_i, p_i) \left[\sum_{j=1}^{I} \alpha_j \left(k_j^2(l,p) - m^2\right) + i\epsilon\right]^{-I}$$

IR type behaviour follow from

$\boxed{D = 0}$

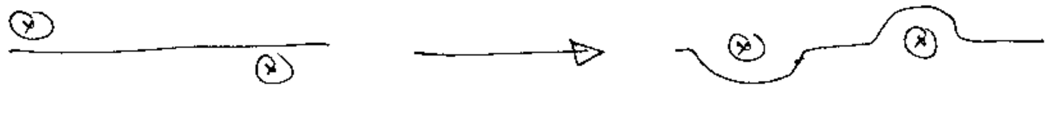
Note that

1) D is quadratic in each of l_i

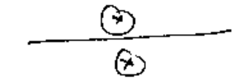
2) " " linear " " " α_j

no l^μ contour passes through the poles due to $i\epsilon$ prescription

→ deform the contour away from pole → get finite result



→ no ~~xx~~ problem

Problem can appear when  i.e the contour is pinched

→ pinch of the contour

→ equivalent to conditions

$$\boxed{D=0,\ \frac{\partial D}{\partial l_i^\mu}=0 \quad \forall i,\mu}$$

↳ a necessary & sufficient conditions for a singularity is to have a pinch (in every loop momentum component)