Introduction to Resummation

One of the major goals of LHC is discovery of Higgs. The main production mechanism would be gluon gluon fusion. An important issue here in determining the total production cross section is calculation of large perturbative corrections in the threshold region defined by $z \to 1$ where $z = \frac{M_H^2}{\hat{s}}$. The leading corrections are enhanced by factors of $\frac{\ln(1-z)}{1-z}$ and invalidate the fixed order perturbation theory.

In the threshold region, the inclusive scattering cross section $\sigma(pp \to HX)$ can be factorized (at leading twist) into a hard part, a soft part and PDF. Renormalization group equations for these parts can be used to resum the large logarithms

for the Higgs production, the calculations up to NNLL accuracy have been done and give a total cross section about 3 times bigger than predicted at LO. [See Kramer, etal NPB 511, 523 (1998)
Catani etal, JHEP 07 (2003) 028.

∴ Properly incorporating these resummation effects will be important for other heavy particles predicted by the theories of New Physics that may be observed at LHC.

In these two lectures we briefly address the questions

1. What is resummation?
2. Where do the large logs come from?
3. IR power counting & get soft factorization
4. How resummation follows from factorization
5. What are recoil & threshold resummation.

Introduction to Resummation

QCD : Theory of strong interaction has properties

- Confinement : Quarks & gluons are confined in hadrons. Physical particles are bound states - not accessible to perturbation theory
- Asymptotic Freedom.

↓

justifies the use of QCD as a P.T.

- QCD is renormalizable : UV divergences can be removed after renormalization
- Apart from UV divergences, there are also divergences due to IR region of soft momenta & collinear momenta.

IR div. : occur when at least one of the particles is massless.

- ~~due~~ occur due to degeneracy of states since we cannot distinguish soft emission & collinear splittings from situations where these are absent.

Expt : finite energy & angular resolutn of particle detector $\Longrightarrow$ physical x-sectns are always inclusive over arbitrarily soft produced particles.

After renormalizatⁿ procedure, the remaining divergences in any physical observable are

- soft (due to parton radiation with small 4-momentum)

and/or

- collinear (due to emission of ~~particles~~ parton moving in parallel to the emitting ones)

IR div. appear on a diagram by diagram basis but they cancel in properly averaged quantities

Need to sum over all distinguishable states

<u>QED</u> : <u>Block Nordsieh Thᵐ (1937)</u>

Cancellation of IR divergences in QED when summation over final states is performed

<u>QCD</u> : complicated due to self coupling of gluons.

BN thᵐ breaks due to non-cancellatⁿ of collinear div. in transitⁿ rates

<u>Kinoshita-Lee-Nauenberg Thᵐ (KLN)</u>

In a theory with massless fields, transition rates are free of IR (soft & collinear) divergence if the summation over initial & final degenerate states is carried out. (5)

But
Expt: can't prepare initial states in collision expts which satisfy conditions of KLN Theorem.

Yet → collinear divergences can be absorbed in the scale dependence of parton densities & thus they factorize from hard scattering process

→ Factorization Theorem → enable us to use pQCD as a predictive calculation tool
↳ say its possible to separate SD & LD physics in physical qties.

$$\sigma^{H_1H_2} = \underbrace{f_{a/H} \otimes f_{b/H}}_{\text{describe LD dynamics}} \otimes \underbrace{\hat{\sigma}_{ab}}_{S.D.}$$

pdf's
- cannot be determined from pQCD
- to be fitted to experimental data
- evolution of these can be determined by pQCD
- are process independent

Partonic x-sect^n — quantifies interact^n of partons in hard process

$\hat{\sigma} \to$ IR safe & calculable with PT
provided the coefficients of perturbative expansion are small.

$$\hat{\sigma} = \hat{\sigma}_0 + \alpha_s \hat{\sigma}^{(1)} + \alpha_s^2 \hat{\sigma}^{(2)} + \ldots$$

Ideally the series converges

However ... the coefficients are usually enhanced at the kinematic edge of phase space.

Why ?

Cancellation of real & virtual corrections can leave remnants that become large in certain kinematic region

Integration in virtual corrections spans all momenta — soft, hard & $\underset{\hookrightarrow \text{ removed}}{\underline{UV}}$

In specific kinematic config. real & virtual corrections can be highly unbalanced spoiling the cancellation mechanism.

$$\left[1 - z = 1 - \frac{Q^2}{s}, \text{ as } \hat{s} \to Q^2, z \to 1\right]$$

Thus

$$\sigma^* = \sigma_0 \left[1 + \sum_{n=1}^{\infty} \alpha_n^n \left(C_{2n}^{(n)} L^{2n} + C_{2n-1}^{n} L^{2n-1} + \ldots\right)\right]$$

$L \to$ large log

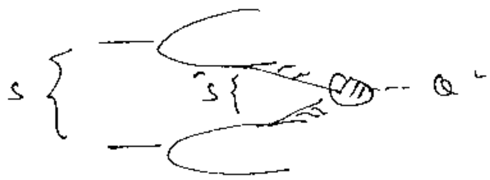
- Recoil logs. $L^2 = \ln^2\left(\frac{p_T^2}{M_Z^2}\right)$ in p_T distribution of Z boson in $p\bar{p}$ collision

~~These~~ double log (Z boson gets p_T from recoiling against soft gluon emission)

- Threshold logs.

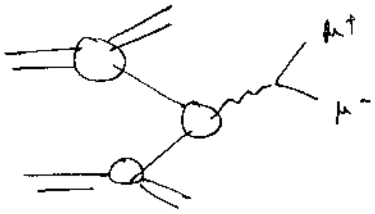
$p\bar{p} \to \gamma^* + X$

$L^2 = \ln^2\left(1 - \frac{Q^2}{s}\right)$



Drell Yan process

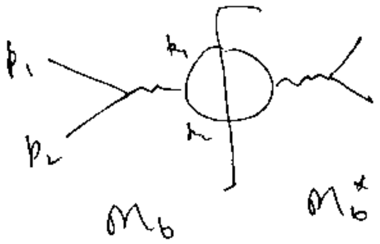
$A(p_1) + B(p_2) \to \mu^+(k_1) + \mu^-(k_2) + X$



Born x-sectⁿ

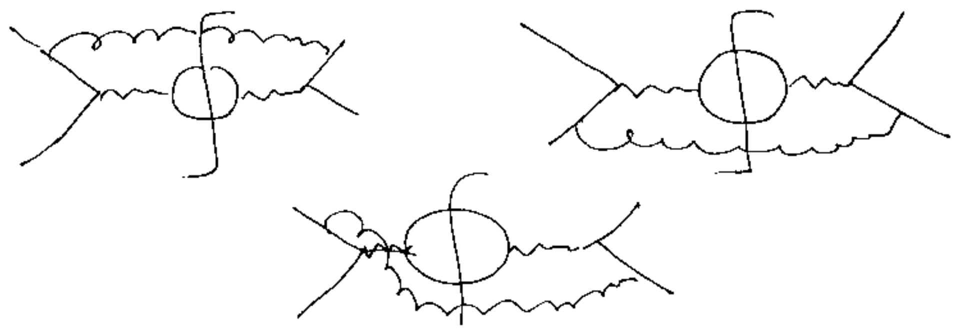
$$\sigma^b = \sigma_B\, \delta\left(1 - \frac{Q^2}{s}\right)$$

Ex. $\boxed{\sigma_B = \frac{4\pi\alpha^2}{9\, Q^2 s}\, Q_f^2}$



$O(\alpha_s)$ corrections (include real & virtual corrections)

Real (we'll consider $q\bar{q} \to g\,\mu^+\mu^-$)



$$\frac{\alpha_s}{\pi}\frac{d\sigma^r}{dQ^2} = \cancel{4\sigma_0 C_F \mu^{2\epsilon}}$$

$$= \sigma_B \frac{\alpha_s}{\pi} C_F \left(\frac{4\pi\mu^2}{Q^2}\right)^{\epsilon} \frac{z^{\epsilon}(1-\epsilon)^{1-2\epsilon}}{\Gamma(1-\epsilon)}$$

$$\int_0^1 dy\, [y(1-y)]^{-\epsilon} \times (1-\epsilon) \left[\frac{2z}{(1-z)^2 y(1-y)}\right.$$

$$\left. + (1-\epsilon)\left(\frac{1-y}{y} + \frac{y}{1-y}\right)(-2\epsilon)\right]$$

~~Note~~ Note: For $\epsilon \to 0$, the term in square bracket are divergent for $y \to 0$ and $y \to 1$ so that the integral over phase space variables would be infinite

However, the ~~singular~~ singular terms from the integrals can be extracted if ϵ is kept finite

Ex. Use the following integrals

$$\int_0^1 dy \frac{[y(1-y)]^{-\epsilon}}{y(1-\epsilon)} = \frac{\Gamma^2(1-\epsilon)}{\Gamma(-2\epsilon)} = -\frac{2\Gamma^2(1-\epsilon)}{\epsilon\,\Gamma(1-2\epsilon)}$$

$$\int_0^1 dy\,[y(1-y)]^{-\epsilon} = \frac{1}{1-2\epsilon}\,\frac{\Gamma^2(1-2\epsilon)}{\Gamma(1-2\epsilon)}$$

$$\int_0^1 dy\,[y(1-y)]^{-\epsilon}\,\frac{y}{1-y} = \int_0^1 dy\,[y(1-y)]^{-\epsilon}\,\frac{1-y}{y}$$

$$= -\frac{(1-\epsilon)\,\Gamma^2(1-2\epsilon)}{\epsilon\,(1-2\epsilon)\,\Gamma(1-2\epsilon)}.$$

2 Show that

$$\boxed{\frac{\alpha_s}{\pi}\,\frac{d\sigma^r}{dQ^2} = -\sigma_B\,\frac{\alpha_s}{\pi}\,C_F\,D(\epsilon)\,\frac{2z^\epsilon}{\epsilon}\Big[2(1-z)^{-1-2\epsilon} + (1-z)^{1-2\epsilon}\Big]}$$

where $D(\epsilon) = \left(\frac{4\pi\mu^2}{Q^2}\right)^\epsilon \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \xrightarrow{\epsilon\to 0} 1$

Only the first term in the square bracket is singular for $z \to 1$ in the case $\epsilon \to 0$.

To get a finite answer, we will make use of plus distributions defined by

$$\int_0^1 dx\, f_+(x)\,g(x) = \int_0^1 f(x)\,[g(x) - g(1)]$$

Note that the first term in square bracket in $\frac{d\sigma^r}{dQ^2}$ above, when convoluted with a test function gives

$$\int_0^1 \frac{f(z)\,dz}{(1-z)^{1+2\epsilon}} = \int_0^1 dz\,\frac{f(z) - f(1) + f(1)}{(1-z)^{1+2\epsilon}}$$

$$= \int_0^1 dz\,\frac{f(z)-f(1)}{(1-z)^{1+2\epsilon}} + f(1)\underbrace{\int_0^1 \frac{dz}{(1-z)^{1+2\epsilon}}}_{\frac{\Gamma(1)\Gamma(-2\epsilon)}{\Gamma(1-2\epsilon)} = -\frac{1}{2\epsilon}}$$

expand $(1-z)^{-1-2\epsilon} = \frac{1}{1-z} e^{-2\epsilon \ln(1-z)}$

$$= \frac{1}{1-z}\left[1 - 2\epsilon \ln(1-z) + \dots \right]$$

$$\Rightarrow \int_0^1 \frac{f(z)\,dz}{(1-z)^{1+2\epsilon}} = \int_0^1 dz\, \frac{f(z)-f(1)}{(1-z)} - 2\epsilon \int_0^1 dz \left[f(z)-f(1)\right]\frac{\ln(1-z)}{1-z} - \frac{1}{2\epsilon}\int_0^1 dz\, f(z)\,\delta(1-z)$$

$$\Rightarrow \frac{1}{(1-z)^{1+2\epsilon}} = -\frac{1}{2\epsilon}\delta(1-z) + \left(\frac{1}{1-z}\right)_+ - 2\epsilon\left(\frac{\ln(1-z)}{1-z}\right)_+ + O(\epsilon^2)$$

Substitute in $\frac{d\sigma^r}{dQ^2}$ ~~and~~ expand the second term in [ ] also to ϵ

$$\frac{\alpha_s}{\pi}\frac{d\sigma^r}{dQ^2} = \sigma_B \frac{\alpha_s}{\pi} C_F\, D(\epsilon)\left[\frac{2}{\epsilon^2}\delta(1-z) - \frac{2(1+z^2)}{\epsilon(1-z)_+} + 4(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ \underbrace{-2\left(\frac{1+z^2}{1-z}\right)_+ \ln z}_{\text{this term does not give a singularity for } z\to 1 \text{ since } \lim_{z\to 1}\frac{\ln z}{(1-z)} \text{ is constant.}}\right]$$