Order α_s QCD radiative corrections to scalar particle production at Hadron Colliders

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Abstract

We describe the computation of next to leading order Quantum Chromodynamics radiative corrections to resonant production of a scalar particle ϕ at Hadron colliders such as the Tevatron at Fermilab and the LHC at CERN. We use dimensional regularisation to regulate ultraviolet, soft and collinear singularies that are present in various parton level sub process contributions. We show how soft singularities cancel between virtual and real emission subprocesses. The remaining UV and collinear singularities are removed using $\overline{MS}$ scheme . Finally, we demonstrate how next to leading corrections reduce renormalisation and factorisation scale dependence of production cross section making our theoretical predictions more stable under perturbation.

Based on talks given at "Advanced School on Radiative Corrections for the LHC", April 4-11, 2011, SINP, Kolkata. This article is dedicated to our colleague Prof Rahul Basu who passed away recently.

1 Introduction

Leading order predictions of observables at hadron colliders often suffer from various theoretical uncertainties. These uncertainties arise from missing higher order perturbative corrections, choice of renormalisation, factorisation scales and parton distribution functions and experimental inputs that serve as boundary conditions for renormalisation group equations. In this article we discribe how next to leading order QCD corrections can improve the theoretical predictions with respect to renormalisation and factorisation scale choices. We have considered total cross section for resonant production of a scalar particle as our observable to demostrate this.

2 Resonant production of a scalar particle

In this section, we explain in detail, the computation of NLO (order α_s ($\alpha_s = g_s^2/4\pi$)) QCD radiative corrections to resonant production of a neutral scalar particle ϕ in hadron colliders. The hadronic reaction is given by

$$H_1(P_1) + H_2(P_2) \to \phi(p) + X \,, \tag{1}$$

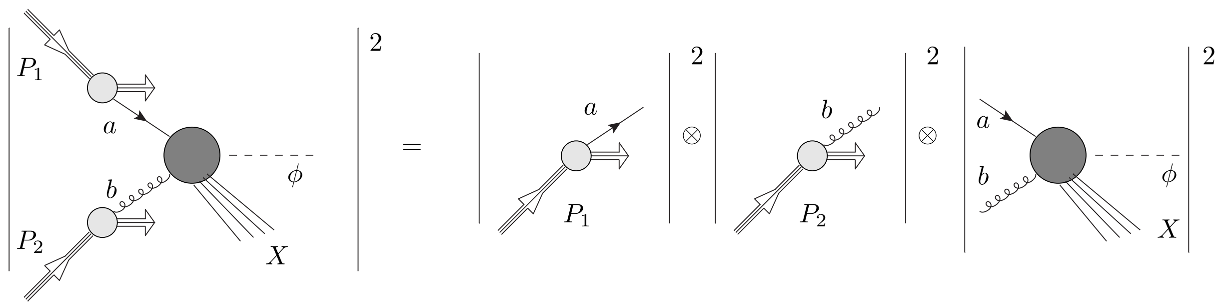
where H_1 and H_2 denote the incoming hadrons, $P_i (i = 1, 2)$ their momenta and X represents an inclusive hadronic state. The action that describes the interaction of $\hat{\phi}$ with the quark and anti-quark fields is given by Yukawa interaction:

$$S_Y = \int d^4\xi \; \mathcal{L}_{int}(\xi) = \hat{Y} \int d^4\xi \; \hat{\phi}(\xi) \; \overline{\hat{\psi}}(\xi)\hat{\psi}(\xi) \tag{2}$$

where $\hat{Y}$ is the bare coupling strength of the interaction.

In the parton model, we can express the hadronic cross section, $\sigma^{H_1 H_2}$ for the reaction given in eqn.(1) in terms of ultraviolet (UV) renormalised partonic cross sections $\hat{\sigma}_{ab}$, $a, b = q, \overline{q}, g$ and bare parton distribution functions $\hat{f}_c(x_i)$, $i = 1, 2$ and $c = q, \overline{q}, g$ of colliding partons as follows:

$$\sigma^{H_1 H_2}(S, m^2) = \int dx_1 \int dx_2 \hat{f}_a(x_1)\hat{f}_b(x_2)\hat{\sigma}_{ab}(\hat{s}, m^2) \tag{3}$$



where $S = (P_1 + P_2)^2$ and $\hat{s} = (x_1 P_1 + x_2 P_2)^2$ are the center of mass energies of incoming hadrons and partons respectively. m is the mass of the final state scalar particle. The sum over a and b is implied for the repeated indices. The bare parton distribution function $\hat{f}_c(x)$ describes the probability of finding a parton of type c which carries a momentum fraction x of the hadron. Since the partonic cross sections are often singular due to collinear kinematics of massless partons we have put "hats" on parton distribution functions $f_c(x_i)$ ($c = a, b$ and $i = 1, 2$) and on partonic cross sections. The parton distribution functions describe long distance part of the hadronic cross section and hence they are not computable in perturbative QCD. On the other hand, the bare partonic cross sections $\hat{\sigma}_{ab}$ that describe the short distance part of the reaction can be computed in QCD perturbation theory using

$$\hat{\sigma}_{ab}(\hat{s}, m^2) = \frac{1}{2\hat{s}} \int [dPS_m] \, \overline{\Sigma} |M_{ab}|^2 \tag{4}$$

as a series expansion in strong coupling constant g_s. In the above equation M_{ab} is the amplitude for the partonic scattering reaction

$$a(p_a) + b(p_b) \to \phi + \sum_c c(p_c), \qquad a,b,c = q,\overline{q},g \tag{5}$$

with p_a, p_b and p_c are the momenta of partons a, b and c respectively. The symbol $\overline{\sum}$ means suming over spin/polarisation and color of final state partons c and averaging the spin/polarisation and color of incoming partons a and b. The phase space integration is given by

$$\int [dPS_m^{ab}] = \int \prod_{i=1}^{m} \left(\frac{d^{n-1}p_i}{(2\pi)^{n-1} 2p_i^0} \right) (2\pi)^n \delta^n (\sum_{j=1}^{m} p_j - p_a - p_b) \tag{6}$$

Defining

$$\hat{\Delta}(\hat{\tau}, m^2) \equiv \hat{s}\hat{\sigma}_{ab}(\hat{s}, m^2), \qquad \hat{\tau} = \frac{m^2}{\hat{s}} \tag{7}$$

and using the identity

$$\int d\hat{\tau}\delta(\tau - \hat{\tau}x_1x_2) = \frac{S}{\hat{s}}, \qquad \tau = \frac{m^2}{\hat{S}} \tag{8}$$

we can express the hadronic cross section, eqn.(3) as

$$\sigma^{H_1H_2}(S, m^2) = \frac{1}{S} \int dx_1 \int dx_2 \int d\hat{\tau} \hat{f}_a(x_1) \hat{f}_b(x_2) \hat{\Delta}_{ab}(\hat{\tau}, m^2) \delta(\tau - \hat{\tau}x_1x_2) \tag{9}$$

The above expression can be written in "convolution" notation as follows:

$$\sigma^{H_1H_2}(S, m^2) = \frac{1}{S} \hat{f}_a \otimes \hat{f}_b \otimes \hat{\Delta}_{ab}(m^2) \tag{10}$$

Beyond the leading order in perturbative QCD, the partonic cross sections $\hat{\sigma}_{ab}$ get contributions from subprocesses involving loops with virtual partons as well as from emission of additional real partons. These subprocesses often suffer from various singularities resulting from ultraviolet, soft and collinear regions of loop integrations and also from soft and collinear regions of phase space integrations. In the following we will regulate all these singularities using dimensional regularisation, where the number of space time dimensions is taken to be $n = 4 + \varepsilon$ with ε being a complex number. The singularities in the resulting subprocess partonic cross sections will appear as poles in $1/\varepsilon^\alpha$ where $\alpha = 1, 2$. The ultraviolet singularies can be systematically removed by using renormalisation prescription namely $\overline{MS}$. The soft singularities that arise due to massless gluons present in the virtual as well as real emission subprocesses cancel among themselves thanks to Kinoshita-Lee-Nauenberg (KLN) theorem. The collinear singularities arise when two or more massless partons become collinear to each other. They often do not cancel and hence the partonic cross sections are collinear singular beyond leading order. We will describe how these singularities are removed in the following.

It is often contivenient to work with bare strong constant coupling $\hat{\alpha}_s$ and bare Yukawa coupling $\hat{Y}$ and finally to replace them by renormalised ones. That is, we expand UV as well as soft finite bare partonic cross sections interms of bare coupling constants $\hat{\alpha}_s$ and $\hat{Y}$ as follows

$$\hat{\Delta}_{ab}(\hat{\tau}, m^2, \varepsilon) = \hat{\Delta}_{ab}^{(0)}(\hat{\tau}, m^2, \varepsilon) + \frac{\hat{\alpha}_{s,\varepsilon}}{4\pi} S_\varepsilon \hat{\Delta}_{ab}^{(1)}(\hat{\tau}, m^2, \varepsilon) + \mathcal{O}(\hat{\alpha}_{s,\varepsilon}^2) \tag{11}$$

where the perturbative coefficients $\hat{\Delta}_{ab}^{(i)}(\hat{\tau}, m^2, \varepsilon)$ are UV divergent but they will become finite after coupling constant renormalisations at the renormalisation scale μ_R:

$$\hat{\alpha}_{s,\varepsilon} S_\varepsilon \quad = \quad Z_{\alpha_s}(\mu_R^2, \varepsilon) \alpha_s(\mu_R^2) \left(\frac{1}{\mu_R^2} \right)^{\frac{\varepsilon}{2}} \tag{12}$$

and

$$\hat{Y}_\varepsilon^2 S_\varepsilon \quad = \quad Z_Y^2(\mu_R^2,\varepsilon) Y^2(\mu_R^2) \left(\frac{1}{\mu_R^2}\right)^{\frac{\varepsilon}{2}} \tag{13}$$

where

$$\hat{\alpha}_{s,\varepsilon} = \frac{\hat{g}_{s,\varepsilon}^2}{4\pi}, \qquad \hat{g}_{s,\varepsilon} = \hat{g}_s \left(\frac{1}{\mu}\right)^{\frac{\varepsilon}{2}}, \qquad \hat{Y}_{s,\varepsilon} = \hat{Y} \left(\frac{1}{\mu}\right)^{\frac{\varepsilon}{2}} \tag{14}$$

with μ being the scale introduced in order to make $\hat{g}_s$ and $\hat{Y}$ dimensionless in n dimensions. The factor S_ε results from angular averaging in n dimensions and is given by

$$S_\varepsilon = \exp\left(\frac{\varepsilon}{2}\left(\gamma_E - \ln(4\pi)\right)\right) \tag{15}$$

where $\gamma_E = 0.577215665$ is Euler-Mascheroni constant.

The collinear singularities present in $\hat{\Delta}(\hat{\tau}, m^2, \varepsilon)$ will go away only if we sum over all the degenerate initial states. This is achieved by the procedure called mass factorisation wherein one redefines the bare parton distribution functions $\hat{f}_c(x)$ at a scale called factorisation scale μ_F in such a way that the collinear singularities in the bare parton cross sections $\hat{\Delta}(\hat{\tau}, m^2, \varepsilon)$ are removed. Such a collinear renormalised parton distribution function $f_a(\tau, \mu_F^2)$ is given by,

$$f_a(\tau, \mu_F^2) = \Gamma_{ab}(\tau, \mu_F^2, \varepsilon) \otimes \hat{f}_b(\tau) \tag{16}$$

where $\Gamma_{ab}(\tau, \mu_F^2, \varepsilon)$ are renormalisation kernels defined in $\overline{MS}$ scheme. They are expanded in powers of strong coupling constant as

$$\Gamma_{ab}(\tau, \mu_F^2, \varepsilon) = \delta_{ab}\delta(1-\tau) + \frac{\hat{\alpha}_{s,\varepsilon}}{4\pi} S_\varepsilon \frac{(\mu_F^2)^{\frac{\varepsilon}{2}}}{\varepsilon} P_{ab}^{(0)}(\tau) + (\mathcal{O}(\hat{\alpha}_{s,\varepsilon}^2)) \tag{17}$$

$P_{ab}^{(0)}(\tau)$ are called Altarelli-Parisi splitting functions. These kernels contain right poles in $1/\varepsilon$ to cancel the collinear singularities in the bare partonic suprocess cross sections. This procedure is called mass factorisation. Note that this introduces a scale μ_F parametrising the arbitrariness inherent in the removal of collinear singularity in the parton cross section through the redefinition of bare parton distribution functions.

Treating $\hat{f}_a$ and f_a as components column vectors $\hat{f}$ and f respectively and Γ_{ab} the ab components of a matrix Γ, eqn.(10) can be written as

$$\sigma^{H_1 H_2}(S, m^2) \quad = \quad \frac{1}{S} \hat{f}^T \otimes \hat{\Delta}(m^2, \varepsilon) \otimes \hat{f} \tag{18}$$

where the argument τ is supressed as it is understood in convolution notation. Substituting eqn.(16) in eqn.(18) we obtain,

$$\sigma^{H_1 H_2}(S, m^2) \quad = \quad \frac{1}{S}\Big(f^T(\mu_F^2) \otimes \left[\Gamma^{-1}(\mu_F^2, \varepsilon)\right]^T\Big) \otimes \hat{\Delta}(m^2, \varepsilon) \otimes \Big(\Gamma^{-1}(\mu_F^2, \varepsilon) \otimes f(\mu_F^2)\Big) \tag{19}$$

which can be written as

$$\sigma^{H_1 H_2}(\tau, m^2) = \frac{1}{S} f^T(\mu_F^2) \otimes \Delta(m^2, \mu_F^2) \otimes f(\mu_F^2) \tag{20}$$

where the renormalised parton cross section $\Delta(\hat{\tau}, m^2, \mu_F^2)$ in the limit $\varepsilon \to 0$ is given by

$$\Delta(\hat{\tau}, m^2, \mu_F^2) \equiv \left[\Gamma^{-1}(\mu_F^2, \varepsilon)\right]^T \otimes \hat{\Delta}(m^2) \otimes \Gamma^{-1}(\mu_F^2, \varepsilon) \tag{21}$$

Expanding the above equation in powers of $\hat{\alpha}_s$ as

$$\Delta_{ab}(\hat{\tau}, m^2, \mu_F^2) = \Delta_{ab}^{(0)}(\hat{\tau}, m^2, \mu_F^2) + \frac{\hat{\alpha}_{s,\varepsilon}}{4\pi} S_\varepsilon \Delta_{ab}^{(1)}(\hat{\tau}, m^2, \mu_F^2) + \mathcal{O}(\hat{\alpha}_{s,\varepsilon}^2) \tag{22}$$

and substituting eqn. (16) in eqn.(18) we obtain

$$\sigma^{H_1H_2}(S,m^2) = \frac{1}{S}\Bigg[f_q \otimes f_{\overline{q}} \otimes \Delta^{(0)}_{q\overline{q}} + f_{\overline{q}} \otimes f_q \otimes \Delta^{(0)}_{\overline{q}q}$$
$$+\frac{\hat{\alpha}_{s,\varepsilon}}{4\pi} S_\varepsilon \left(f_q \otimes f_{\overline{q}} \otimes \Delta^{(1)}_{q\overline{q}} + f_{\overline{q}} \otimes f_q \otimes \Delta^{(1)}_{\overline{q}q}\right)$$
$$+\frac{\hat{\alpha}_{s,\varepsilon}}{4\pi} S_\varepsilon \left(\sum_{a=q,\overline{q}} f_a \otimes f_g \otimes \Delta^{(1)}_{ag} + \sum_{a=q,\overline{q}} f_g \otimes f_a \otimes \Delta^{(1)}_{gq}\right) + \mathcal{O}(\alpha^2_{s,\varepsilon})\Bigg] \tag{23}$$

where

$$\Delta^{(1)}_{q\overline{q}} = \hat{\Delta}^{(1)}_{q\overline{q}} - (P^{(0)}_{qq} + P^{(0)}_{\overline{q}\overline{q}}) \otimes \hat{\Delta}^{(0)}_{q\overline{q}} \left(\frac{1}{\varepsilon} + \frac{1}{2}\ln(\mu_F^2)\right) \tag{24}$$

$$\Delta^{(1)}_{ag} = \hat{\Delta}^{(1)}_{ag} - P^{(0)}_{\overline{a}g} \otimes \hat{\Delta}^{(0)}_{a\overline{a}} \left(\frac{1}{\varepsilon} + \frac{1}{2}\ln(\mu_F^2)\right) \tag{25}$$

$$\Delta^{(1)}_{ga} = \hat{\Delta}^{(1)}_{ga} - P^{(0)}_{\overline{a}g} \otimes \hat{\Delta}^{(0)}_{\overline{a}a} \left(\frac{1}{\varepsilon} + \frac{1}{2}\ln(\mu_F^2)\right) \qquad a = q,\overline{q} \tag{26}$$

are finite parts of the partonic cross sections defined in $\overline{MS}$ scheme. Finally we replace $\hat{\alpha}_{s,\varepsilon}$ by $\alpha_s(\mu_R^2)$ in the eqn.(125) because

$$\hat{\alpha}_{s,\varepsilon} S_\varepsilon = Z_{\alpha_s}(\varepsilon)\alpha_s(\mu_R^2)\left(\frac{1}{\mu_R^2}\right)^{\frac{\varepsilon}{2}}$$
$$= \left(1 + \frac{\alpha_s(\mu_R^2)}{4\pi}\frac{2\beta_0}{\varepsilon} + \mathcal{O}(\alpha_s^2)\right)\alpha_s(\mu_R^2)\left(\frac{1}{\mu_R^2}\right)^{\frac{\varepsilon}{2}}$$
$$= \alpha_s(\mu_R^2) + \mathcal{O}(\alpha_s^2(\mu_R^2)) \tag{27}$$

Generalising to all orders, we can express the hadronic cross section as follows:

$$\sigma^{H_1H_2}(S,m^2) = f_a(\tau,\mu_F^2) \otimes f_b(\tau,\mu_F^2) \otimes \Delta_{ab}(m^2,\mu_F^2)$$
$$= \sum_{a,b=q,\bar{q},g} \int_\tau^1 \frac{dx_1}{x_1} \int_{\tau/x_1}^1 \frac{dx_2}{x_2} f_a(x_1,\mu_F^2)\, f_b(x_2,\mu_F^2)\, \Delta_{ab}\left(\frac{\tau}{x_1\, x_2}, m^2, \mu_F^2\right) \tag{28}$$

where the finite Δ_{ab} are given by

$$\Delta_{ab}\left(\hat{\tau}, m^2, \mu_F^2\right) = \sum_{i=0}^{\infty} \frac{\alpha_s^i(\mu_R^2)}{4\pi} \Delta^{(i)}_{ab}\left(\hat{\tau}, m^2, \mu_F^2, \mu_R^2\right) \tag{29}$$

3 Renormalisation Group Invariance

Recall that the strong coupling constant $\hat{a}_s$ and the Yukawa coupling constant $\hat{Y}$ are renormalised by the renormalisation constants $Z_Y(\mu_R^2)$ and $Z_{a_s}(\mu_R^2)$ respectively at the renormalisation scale μ_R as

$$\begin{aligned} \hat{a}_s \left(\frac{1}{\mu^2}\right)^{\frac{\varepsilon}{2}} S_\varepsilon &= Z_{a_s}(\mu_R^2) a_s(\mu_R^2) \left(\frac{1}{\mu_R^2}\right)^{\frac{\varepsilon}{2}} \\ \hat{Y}^2 \left(\frac{1}{\mu^2}\right)^{\frac{\varepsilon}{2}} S_\varepsilon &= Z_Y^2(\mu_R^2) Y^2(\mu_R^2) \left(\frac{1}{\mu_R^2}\right)^{\frac{\varepsilon}{2}} \end{aligned} \tag{30}$$

The fact that $\hat{a}_s$ and $\hat{Y}$ do not depend on the renormalisation scale μ_R implies

$$\begin{aligned} \mu_R^2 \frac{d}{d\mu_R^2} \ln a_s(\mu_R^2) &= \frac{\varepsilon}{2} - \mu_R^2 \frac{d}{d\mu_R^2} \ln Z_{a_s}(\mu_R^2) = \frac{\varepsilon}{2} + \frac{1}{a_s(\mu_R^2)} \beta(a_s(\mu_R^2)) \\ &= \frac{\varepsilon}{2} - \sum_{i=0}^{\infty} a_s^{i+1}(\mu_R^2)\beta_i \\ \mu_R^2 \frac{d}{d\mu_R^2} \ln Y^2(\mu_R^2) &= \frac{\varepsilon}{2} - \mu_R^2 \frac{d}{d\mu_R^2} \ln Z_Y^2(\mu_R^2) = \frac{\varepsilon}{2} + 2\gamma(a_s) \\ &= \frac{\varepsilon}{2} - 2\sum_{i=0}^{\infty} a_s^{i+1}(\mu_R^2)\gamma_i \end{aligned} \tag{31}$$

where LO and NLO coefficients β_0,β_1 are

$$\begin{aligned} \beta_0 &= \frac{11}{3} C_A - \frac{4}{3} T_F n_f \\ \beta_1 &= \frac{34}{3} C_A^2 - 4 T_F n_f C_F - \frac{20}{3} T_F n_f C_A \,, \end{aligned} \tag{32}$$

the LO and NLO coefficients γ_0 and γ_1 are

$$\begin{aligned} \gamma_0 &= 3C_F \\ \gamma_1 &= \frac{3}{2} C_F^2 + \frac{97}{6} C_F C_A - \frac{10}{3} C_F T_F n_f \end{aligned} \tag{33}$$

and the color factors for $SU(N)$ QCD are given by

$$C_A = N, \qquad C_F = \frac{N^2-1}{2N}, \qquad T_F = \frac{1}{2} \tag{34}$$

and n_f is the number of active flavours. Using $da_s/\beta(a_s) = d\mu_R^2/\mu_R^2$, in the limit $\varepsilon \to 0$ we find

$$\frac{d}{da_s} \ln Y(a_s) = \frac{\gamma(a_s)}{\beta(a_s)} \tag{35}$$

The solution to the above equation is given by

$$Y(a_s(\mu_R^2)) = Y(a_s(\mu_0^2)) \exp\left(\int_{a_s(\mu_0^2)}^{a_s(\mu_R^2)} da_s \frac{\gamma(a_s)}{\beta(a_s)}\right) \tag{36}$$

Expanding the integrand in powers of a_s, the integration in the exponential can be performed easily to obtain

$$Y(a_s(\mu_R^2)) = Y(a_s(\mu_o^2)) \frac{C(a_s(\mu_R^2))}{C(a_s(\mu_0^2))} \tag{37}$$

where

$$C(a_s) = 1 + a_s^{\frac{\gamma_0}{\beta_0}} \left[1 + a_s \left(\frac{\gamma_1}{\beta_0} - \frac{\beta_1 \gamma_0}{\beta_0^2} \right) + \mathcal{O}(a_s^2) \right] \tag{38}$$

In fig. (1), we have plotted LO as well as NLO evolution of $\alpha_s(\mu_R^2)$ and $Y(\mu_R^2)$ as a function of renromalisation scale μ_R. Eventhough the parton distribution functions are independent of μ_R, the LO prediction of the hadronic cross section depends on the scale μ_R through partonic cross section which is proportional to $Y^2(\mu_R^2)$ for resonant production of a scalar particle.

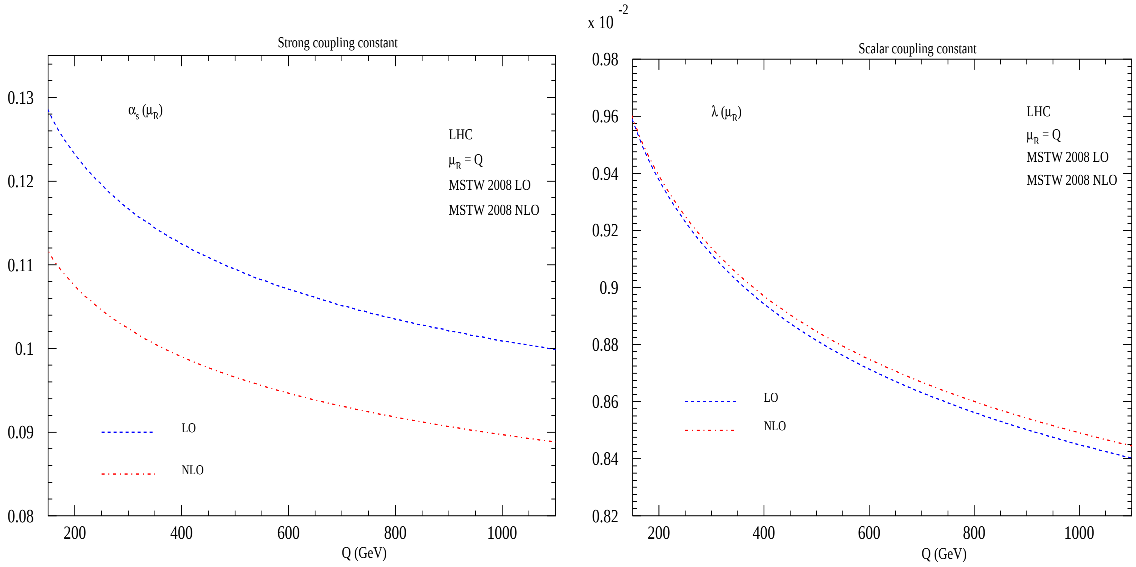


Figure 1: *α_s and Y are plotted as a function for renormalisation scale μ_F using LO and NLO evolution equations.*

The mass factorisation/renormalisation of parton distribution functions leads to renormalisation group equations with respect to factorisation scale μ_F. Recall that

$$f_a(\tau, \mu_F^2) \quad = \quad \Gamma_{ab}(\tau, \mu_F^2) \otimes \hat{f}_b(\tau) \tag{39}$$

Demanding that $\hat{f}$ is μ_F independent, we obtain

$$\begin{aligned} \mu_F^2 \frac{df}{d\mu_F^2} &= \mu_F^2 \frac{d}{d\mu_F^2} \Gamma(\mu_F^2) \otimes \hat{f} \\ &= \mu_F^2 \frac{d}{d\mu_F^2} \Gamma(\mu_F^2) \otimes \Gamma^{-1}(\mu_F^2) \otimes f(\mu_F^2) \\ &= \frac{1}{2} P_{ab}(\mu_F^2) \otimes f(\mu_F^2) \end{aligned} \tag{40}$$

where P_{ab} are the Altarelli-Parisi splitting functions and are defined by

$$P_{ab}(\tau, \mu_F^2) = \lim_{\varepsilon \to 0} 2\mu_F^2 \frac{d}{d\mu_F^2} \Gamma(\mu_F^2) \otimes \Gamma^{-1}(\mu_F^2) \tag{41}$$

The perturbative expansion for P_{ab} is given by

$$P_{ab}(\tau, \mu_F^2) \quad = \quad \sum_{i=0}^{\infty} a_s^{i+1}(\mu_F^2) P_{ab}^{(i)}(\tau) \tag{42}$$

Hence the renormalisation group evolution equation for the parton distribution function can be written as

$$\mu_F^2 \frac{d}{d\mu_F^2} f_a(\tau, \mu_F^2) = \frac{1}{2} \int_\tau^1 \frac{dz}{z} P_{ac}(z, \mu_F^2) f_c(\frac{\tau}{z}, \mu_F^2) \tag{43}$$

These are called Altarelli-Parisi evolution equations. Since the hadronic cross section is proportional to the partonic flux namely

$$\Phi_{ab}(\tau, \mu_F^2) \equiv f_a(\tau, \mu_F^2) \otimes f_b(\tau, \mu_F^2) = \int_\tau^1 \frac{dz}{z} f_a(z, \mu_F^2) f_b\left(\frac{\tau}{z}, \mu_F^2\right), \tag{44}$$

we have plotted them in fig.(2) as a function of m for two choices of μ_F viz $m = \mu_F/10$ and $m = 10\mu_F$.

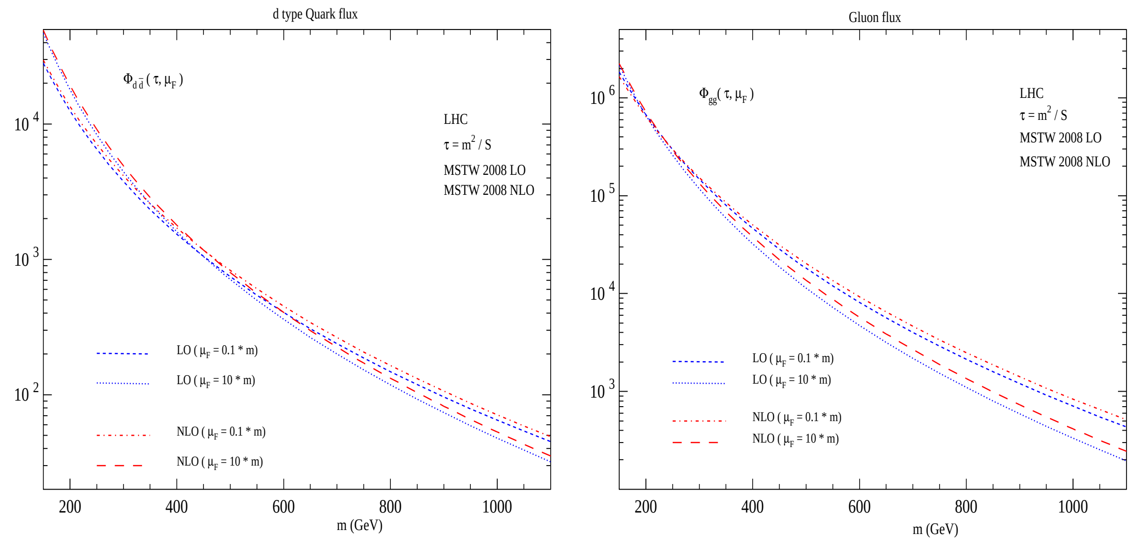


Figure 2: $\Phi_{d\bar{d}}, \Phi_{gg}$ *are plotted as a function for renormalisation scale* m *using LO and NLO evolution equations for two choices of* μ_F *viz* $\mu_F = m/10, \mu_F = 10m$.

$\Delta_{ab}^{(i)}$'s beyond leading order (i.e., $i > 0$) depend on μ_F. But μ_F dependence cancels with that coming from partonic distribution functions $f_c(\tau, \mu_F^2)$ after convolutions with $\Delta_{ab}^{(i)}$s and summed to all orders in perturbation theory. This is due to the fact that the observable $\sigma^{H_1 H_2}(S, m^2)$ is renormalisation group invariant with respect to μ_F

$$\mu_F^2 \frac{d}{d\mu_F^2} \sigma^{H_1 H2}(S, m^2) \quad = \quad 0\,. \tag{45}$$

This implies

$$\mu_F^2 \frac{d}{d\mu_F^2} \sigma^{H_1 H_2}(S, m^2) = \mu_F^2 \frac{d}{d\mu_F^2} \left(f^T(\mu_F^2) \otimes \Delta(\mu_F^2) \otimes f(\mu_F^2) \right) = 0 \tag{46}$$

which gives

$$\begin{aligned} 0 \quad = \quad & \left(\mu_F^2 \frac{df^T(\mu_F^2)}{d\mu_F^2} \right) \otimes \Delta(\mu_F^2) \otimes f(\mu_F^2) + f(\mu_F^2) \otimes \left(\mu_F^2 \frac{d\Delta(\mu_F^2)}{d\mu_F^2} \right) \otimes f(\mu_F^2) \\ & + f(\mu_F^2) \otimes \Delta(\mu_F^2) \otimes \left(\mu_F^2 \frac{df(\mu_F^2)}{d\mu_F^2} \right) \end{aligned} \tag{47}$$

Substituting eqn.(40) in the above equation, we obtain the following renormalisation group equation for $\Delta(\mu_F^2)$s:

$$\mu_F^2 \frac{d}{d\mu_F^2}\Delta(\hat{\tau}, m^2, \mu_F^2) = -\frac{1}{2}\left[P^T(\mu_F^2)\otimes\Delta(m^2,\mu_F^2) + \Delta(m^2,\mu_F^2)\otimes P(\mu_F^2)\right] \tag{48}$$

Note that in the leading order hadronic cross section, the factorisation scale enters only in the parton distributions functions namely through Φ_{ab} and the LO partonic cross sections are independent of μ_F. Hence, predictions at leading order will be sensitive to the choice of factorisation scale which is unphysical. Beyond leading order, partonic cross sections will also depend on the factorisation scale (see eqn.(24,25,26)) due to mass factorisation.

While each term $\Delta_{ab}^{(i)}$ for $i > 0$ in the perturbative expansion of eqn.(29) is dependent on the UV renormalisation scale μ_R, the sum to all orders in perturbation theory is μ_R independent:

$$\mu_R^2 \frac{d}{d\mu_R^2}\Delta_{ab}\left(\hat{\tau}, m^2, \mu_F^2\right) = 0 \tag{49}$$

This is the statement of renormalisation group invariance.

The renormalisation group equations of α_s, Y and f_a (see eqns.(31,43)) determine the scale dependence of fixed perturbative results. Hence, the hadronic cross sections computed using truncated perturbative results often give renormalisation and factorisation scale dependent predictions. They can spoil the reliability of the theoretical predictions. In the following sections we will explicitly demonstrate how both these scale dependences in the hadronic cross sections get reduced as we include next to leading order contributions $\Delta_{ab}^{(1)}(\hat{\tau}, m^2)$.

4 Subprocess contributions

In this section we describe the computational details of $\Delta_{ab}^{(0)}$ and $\Delta_{ab}^{(1)}$ that contribute at next to leading order in perturbative QCD to the resonant production of scalar particle in hadron colliders.

4.1 Leading Order (LO) contribution from $q + \overline{q} \to \phi$

The leading order partonic sub process that contributes to resonant production of ϕ proceeds through annihilation of quark and anti-quark as follows:

$$\bar{q}(p') + q(p) \to \phi(q) \tag{50}$$

where p and p' are the momenta of incoming quark and anti-quark respectively and q is the momentum of the outgoing scalar.

The amplitude for this process is given by

$$M_{q\overline{q}}^{(0)} = -i\hat{Y}_\varepsilon \; \bar{v}_i(p') \; u_i(p) \qquad \text{with} \qquad \hat{Y}_\varepsilon = \frac{\hat{Y}}{(\mu^2)^{\frac{\varepsilon}{2}}} \tag{51}$$

the subscript i denotes the color index of the quark and anti-quark. Since we consider only light quarks in the initial state, we have

$$(p')^2 = p^2 = 0 \tag{52}$$

whereas the scalar has a mass m. The cross-section for the process is

$$\hat{\sigma}_{q\overline{q}}^{(0)}(\hat{\tau}, m^2) = \frac{1}{4\,(p.p')}\int\left[dPS_1^{q\overline{q}}\right]\frac{1}{4N^2}\sum_{spin,color}|M_{q\overline{q}}^{(0)}|^2 \tag{53}$$

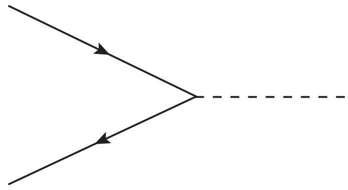


Figure 3: Subprocess $q_i + \bar{q}_j \to \phi$ at $\mathcal{O}(\alpha_s^0)$.

where

$$\left[dPS_1^{q\overline{q}}\right] = \frac{d^{n-1}q}{(2\pi)^{n-1}\, 2\, q^0}\ (2\,\pi)^n\ \delta^n(p'+p-q)\,, \tag{54}$$

the factors $1/4$ and $1/N^2$ arise from the spin and color averaging respectively for the incoming quark and anti-quark and $4\ p\cdot p'$ is the flux factor. We can simplify the phase space integration using the identity

$$\int \frac{d^{n-1}q}{(2\pi)^{n-1}\, 2\, q_0} \quad = \quad \int \frac{d^n q}{(2\pi)^n} 2\pi\delta(q^2-m^2)\theta(q^0)\,, \tag{55}$$

to obtain

$$\int \frac{d^{n-1}q}{(2\pi)^{n-1}\, 2\, q^0}(2\,\pi)^n\ \delta^n(p'+p-q) = 2\pi\delta(\hat{s}-m^2) \tag{56}$$

The square of the matrix element after suming over spin and color becomes

$$\sum_{spin,color} |M_{q\overline{q}}^{(0)}|^2 = 4N\hat{Y}_\varepsilon^2 p\cdot p' \tag{57}$$

Substituting eqns.(56,57) in eqn.(53) we obtain

$$\begin{aligned} \Delta_{q\overline{q}}^{(0)}(\hat{\tau},m^2) \quad &= \quad \hat{\Delta}_{q\overline{q}}^{(0)}(\hat{\tau},m^2) \\ &= \quad \Delta_0\ \delta(1-\hat{\tau}), \qquad\qquad \Delta_0 \equiv \hat{Y}_\varepsilon^2\ \frac{\pi}{2N\hat{s}} \end{aligned} \tag{58}$$

4.2 Next to Leading Order (NLO) contributions

The QCD correction to the process of interest has contributions from two different, but related, sources. First, the quark-pair-initiated process itself receives radiative correction.

$$\mathcal{O}(\alpha_s) \quad \text{corrections} \quad \text{to} \quad q+\overline{q}\to\phi \tag{59}$$

To this must be added the contribution arising from radiating off a soft gluon:

$$q+\overline{q}\to\phi+g \tag{60}$$

And secondly, since our true initial state is not quarks, but (anti-)protons, we must include possible contributions from "initial-state" gluons as well.

$$\begin{aligned} &q+g\to\phi+g \\ &\overline{q}+g\to\phi+g \end{aligned} \tag{61}$$

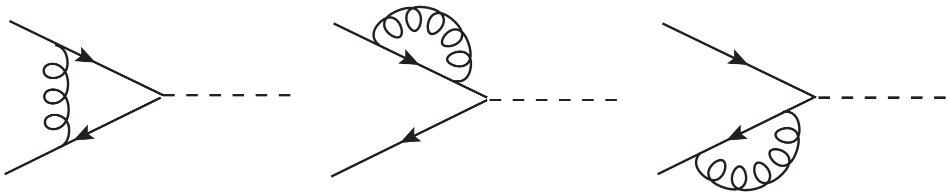


Figure 4: $\mathcal{O}(\alpha_s)$ virtual gluon corrections to $q_i + \bar{q}_j \to \phi$.

4.2.1 $\mathcal{O}(\alpha_s)$ correction to $q + \overline{q} \to \phi$

To calculate the QCD radiative correction to this process, we start by computing the $\mathcal{O}(g_s^2)$ corrections to the vertex function and the self energy, where g_s is the QCD strong coupling constant. A prime ingredient for this is the calculation of the corresponding renormalisation constants $Z_{1,Y}$ and Z_2. Even on regulating the ultraviolet (UV) singularities, we would, expectedly, be left with soft and collinear singularities. Throughout our calculation we shall use dimensional regularisation to regulate any singlarity and the $\overline{MS}$ prescription for renormalising UV singularities.

Let us first consider the vertex function M^V upto order g_s^2. This can be expanded as

$$M^V_{q\overline{q}} = M^{(0)}_{q\overline{q}} + M^{(1),V}_{q\overline{q}} \tag{62}$$

where $M^{(0)}_{q\overline{q}} = -i\hat{Y}_\varepsilon v(p')\ u(p)$, and

$$M^{(1),V}_{q\overline{q}} = -\hat{Y}_\varepsilon\ \hat{g}^2_{s,\varepsilon} \int \frac{d^n k}{(2\pi)^n}\ \frac{\bar{v}(p')\ t^a\ t^a \left[\ \gamma^\mu(-\not{p}' + \not{k})(\not{p} + \not{k}\,)\,\gamma_\mu\ \right] u(p)}{[(p'-k)^2 + i\eta\ ]\,[(p+k)^2 + i\eta\ ]\,[k^2 + i\eta\ ]}\ . \tag{63}$$

In 4-dimensions, when the components of the momentum k become large, the integral becomes ultraviolate singular due terms of the form

$$\int dk \frac{g(k)}{k}\ :\ \text{for regular function } g(x)\text{, it diverges logarithmically in the limit } k \to \infty. \tag{64}$$

$$\tag{65}$$

In addition to UV singularities, we encounter soft singularities when the gluon in the loop becomes soft, i.e the momentum of the gluon becomes zero. Also, we find that when the gluon in the loop becomes collinear to either quark or anti quark, the integral becomes singular which goes by the name collinear singularity. That is,

$$\int_0^\infty \frac{d|\vec{k}|}{|\vec{k}|} \qquad :\ \text{diverges logarithmically in the soft limit } |\vec{k}| \to 0. \tag{66}$$

$$\int_{-1}^1 \frac{d\text{cos}\theta}{1-\cos^2\theta} \qquad :\ \text{diverges logarithmically in the collinear limit } \theta \to 0. \tag{67}$$

The soft and collinear singularies are often called infra-red singularities. The integral that appear in vertex function has both UV and infra-red singularities but it divides into sum of integrals having disjoint regions of convergence in $4+\varepsilon$ dimension. In other words, integrals which are singular in the UV region but convergent in infra-red region have regions of convergence distinct from those which are convergent in the UV region but singular in infra-red region. Ofcourse the full integral is not convergent for any value of ε. We can treat each part separately and for each part there is some domain of $4+\varepsilon$ in which it exists and from which it may be analytically continued in the complex $4+\varepsilon$ plane. Hence, we can work in n dimensional space time where UV,soft and collinear singularities are regulated simultaneously.

The matrices t^a are the Gell-Man matrices and satisfy $(t^a t^a)_{ij} = C_F \delta_{ij}, C_F = (N^2-1)/2N$. Using

$$\gamma_\mu \not{a} \not{b} \gamma^\mu = 4a \cdot b + (n-4) \not{a} \not{b} \tag{68}$$

and the equations of motion, namely

$$\not{p} u(p) = 0, \qquad \overline{v}(p') \not{p}' = 0 \tag{69}$$

we get

$$M_{q\overline{q}}^{(1),V} = -\hat{Y}_\varepsilon \; \hat{g}_{s,\varepsilon}^2 \int \frac{d^n k}{(2\pi)^n} \frac{\overline{v}(p') \; t^a \; t^a \left[-4p \cdot p' + 4(p-p') \cdot k + nk^2 \right] u(p)}{[(p'-k)^2 + i\eta \;] \, [(p+k)^2 + i\eta \;] \, [k^2 + i\eta \;]} \tag{70}$$

which can be written as

$$M_{q\overline{q}}^{(1),V} \; = \; \bar{v}(p') \; (-i \; \hat{Y}_\varepsilon) \; \bar{\Gamma}^{(1)} \; u(p) \; , \tag{71}$$

where

$$\bar{\Gamma}^{(1)} = -i \; C_F \hat{g}_{s,\varepsilon}^2 \; \left[-2m^2 \; I_1 + 4 \; (p-p')^\mu \; I_\mu + n \; g^{\mu\nu} \; I_{\mu\nu} \; \right] , \tag{72}$$

with

$$I_{\{1, \; \mu, \; \mu\nu\}} \quad = \quad \int \frac{d^n k}{(2\pi)^n} \frac{\left\{ 1, \; k_\mu, \; k_\mu \; k_\nu \right\}}{[(p'-k)^2 + i\eta \;] \, [(p+k)^2 + i\eta \;] \, [k^2 + i\eta \;]} \; . \tag{73}$$

Naive power counting shows that $I_{\mu\nu}$ is logarithmically divergent in 4-dimensions, while the other two are convergent. The integrals can be evaluated explicitly (for example, using Feynman parametrisation) and the results can be expressed in terms of Gamma functions:

$$\begin{aligned} I_1 &= K_\varepsilon(q) \left(\frac{1}{q^2} \right) \left[\frac{4}{\varepsilon^2} (1+\varepsilon) \right] \\ I_\mu &= K_\varepsilon(q) \left(\frac{(p'-p)_\mu}{q^2} \right) \left[\frac{2}{\varepsilon} \right] \\ I_{\mu\nu} &= K_\varepsilon(q) \Bigg(-g_{\mu\nu} + \frac{p_\mu p_\nu + p'_\mu p'_\nu}{q^2} (2+\varepsilon) - \frac{p_\mu p'_\nu + p_\nu p'_\mu}{q^2} \varepsilon \Bigg) \left[\frac{2}{\varepsilon} \right] \end{aligned} \tag{74}$$

where

$$K_\varepsilon(q) = \frac{1}{16\pi^2} \left(-\frac{q^2}{4\pi} \right)^{\frac{\varepsilon}{2}} \frac{\Gamma^2 \left(1 + \frac{\varepsilon}{2} \right) \Gamma \left(1 - \frac{\varepsilon}{2} \right)}{\Gamma \left(2 + \varepsilon \right)} \qquad q = p + p' \tag{75}$$

The resultant vertex function is then ($\epsilon \equiv n-4$)

$$\bar{\Gamma}^{(1)} \quad = \quad \frac{\hat{\alpha}_{s,\varepsilon}}{4\pi} C_F \left(\frac{-m^2}{4\pi} \right)^{\frac{\epsilon}{2}} \frac{\Gamma^2 \left(1 + \frac{\epsilon}{2} \right) \Gamma \left(1 - \frac{\epsilon}{2} \right)}{\Gamma(2+\epsilon)} \left[\; -\frac{2}{\epsilon^2} \; (2+\epsilon)^2 \; \right] , \tag{76}$$

with $\hat{\alpha}_{s,\varepsilon} \equiv \hat{g}_{s,\varepsilon}^2 / 4\pi$.

The renormalisation constant $Z_{1,Y}$ is defined through the relation

$$Z_{1,Y}^{-1} = 1 + \bar{\Gamma}^{(1)}|_{UV} \tag{77}$$

and, of course, depends on the way the ultraviolet divergent part $\Gamma^{(1)}|_{UV}$ is isolated. Within the $\overline{MS}$ scheme, it can easily be ascertained to be

$$Z_{1,Y} = 1 + \frac{\hat{\alpha}_{s,\varepsilon}}{4\pi} S_\varepsilon \left(\mu_R^2 \right)^{\frac{\varepsilon}{2}} C_F \; \left(\; \frac{8}{\epsilon} \; \right) \; , \tag{78}$$

The self energy correction to the Born amplitude (say, to the quark only) can be expressed as

$$M_{q\bar{q}}^{(1),S} = -i\ \hat{Y}_\varepsilon \bar{v}(p') \frac{1}{\not p} \overline{\Sigma}(\not p) u(p)\ , \tag{79}$$

where

$$\overline{\Sigma}(\not p) = -i\ \hat{g}_{s,\varepsilon}^2\ C_F \int \frac{d^n k}{(2\,\pi)^n} \frac{\gamma^\mu\ (\not p + \not k)\ \gamma_\mu}{[k^2 + i\eta]\,[(p+k)^2 + i\eta]}\ . \tag{80}$$

Notice that $\Sigma(\not p)$ does not contribute to the amplitude given in eqn.(50) due to the massless nature of the light quarks. On the other hand, the above equation can be used to determine the wave function renormalisation constant Z_2. In the $\overline{MS}$ scheme, this can be rewritten as

$$Z_2 = 1 + \frac{\hat{\alpha}_{s,\varepsilon}}{4\pi} S_\varepsilon \left(\mu_R^2\right)^{\frac{\varepsilon}{2}} C_F \left(\frac{2}{\epsilon}\right)\ . \tag{81}$$

Our next task is to compute the virtual contributions to the process given in eqn.(50). In order to do this, we have to redefine the fields and the coupling constants in terms of the renormalised ones (and, of course, the renormalisation constants Z_2 and $Z_{1,Y}$.) This is equivalent to adding UV counter terms corresponding to the vertex function and the wave function renormalisation constant.

$$M_{q\bar{q}}^{(0)+V+CT} = M_{q\bar{q}}^{(0)} + M_{q\bar{q}}^{(1),V} + M_{q\bar{q}}^{(1),CT}\ . \tag{82}$$

It turns out that the effect of the counter term(CT) is

$$M_{q\bar{q}}^{(0)} + M_{q\bar{q}}^{(1),CT} = \left(-i\ Y_\varepsilon(\mu_R^2)\ \frac{Z_{1,Y}}{Z_2}\right)\ \bar{v}(p')\ u(p) \tag{83}$$

$Y_\varepsilon(\mu_R^2) = Y(\mu_R^2)/(\mu_R)^{\frac{\varepsilon}{2}}$ and the renormalised coupling Y is related to the unrenormalised one through

$$Y(\mu_R^2) = \frac{Z_2}{Z_{1,Y}} S_\varepsilon^{\frac{1}{2}} \left(\frac{\mu_R}{\mu}\right)^{\frac{\varepsilon}{2}} \hat{Y}\ , \qquad Z_Y = \frac{Z_{1,Y}}{Z_2} \tag{84}$$

The virtual and counter term contribution to the Born diagram can be expressed as

$$\begin{aligned}\hat{\sigma}^{(0)+V+CT} &= \frac{1}{4(p\cdot p')} \int \frac{d^{n-1}q}{(2\pi)^{n-1} 2q_0} \frac{1}{4N^2} \sum_{spin,color} \left(|M_{q\bar{q}}^{(0)}|^2 + 2\mathrm{Re}\left(M_{q\bar{q}}^{(0)*} M_{q\bar{q}}^{(1),V+CT}\right)\right) \\ &\times (2\pi)^n \delta^n(p+p'-q)\ .\end{aligned} \tag{85}$$

$$M_{q\bar{q}}^{(1),V+CT} = M_{q\bar{q}}^{(1),V} + M_{q\bar{q}}^{(1),CT}\ . \tag{86}$$

Performing the phase space integration, we find

$$\hat{\sigma}^{(0)+V+CT} = \frac{\pi}{2Ns} Y_\varepsilon^2(\mu_R^2) \left(1 + \frac{\hat{\alpha}_{s,\varepsilon}}{4\pi} S_\varepsilon C_F F^{(1)}\right) \delta(1-\hat{\tau})\ , \tag{87}$$

where

$$F^{(1)} = \frac{4\pi}{\hat{\alpha}_{s,\varepsilon} S_\varepsilon C_F} \left(\frac{Z_{1,Y}^2}{Z_2^2} + 2\overline{\Gamma}^{(1)} - 1\right)\ . \tag{88}$$

The function $F^{(1)}$ does not contain any UV singularities due to the presence of counter terms defined through the renormalisation contants $Z_{1,Y}, Z_2$. Using the identities

$$\begin{aligned} Re\left[(-1)^{\frac{\varepsilon}{2}}\right] &= 1 - \frac{3}{4}\zeta(2)\varepsilon^2 + \mathcal{O}(\varepsilon^3) \\ \Gamma(1+a\varepsilon) &= exp\left(-a\varepsilon\gamma_E + \sum_{k=2}^{\infty} \frac{\zeta(k)}{k}(-a\varepsilon)^k\right) \end{aligned} \tag{89}$$

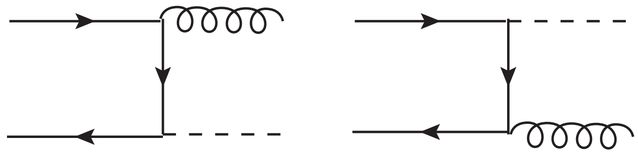


Figure 5: $\mathcal{O}(\alpha_s)$ real gluon emission subprocess: $q_i + \bar{q}_j \to \phi + g$.

we can simplify $\overline{\Gamma}^{(1)}$ as

$$\overline{\Gamma}^{(1)} = \frac{\hat{\alpha}_{s,\varepsilon}}{4\pi} S_\varepsilon C_F (m^2)^{\frac{\varepsilon}{2}} \left[-\frac{8}{\varepsilon^2} - 2 + 7\zeta(2) + \mathcal{O}(\varepsilon) \right] \tag{90}$$

to obtain $F^{(1)}$:

$$F^{(1)} = \left[-\frac{16}{\epsilon^2} + \frac{12}{\epsilon} - 4 + 14\zeta(2) - 6\ln\left(\frac{m^2}{\mu_R^2}\right) \right] . \tag{91}$$

where $\zeta(2) = \pi^2/6$. Hence, we find

$$\begin{aligned} \hat{\Delta}^{(0)+V+CT} &= \hat{s}\hat{\sigma}^{(0)+V+CT} \\ &= \hat{\Delta}^{(0)}_{q\overline{q}} \left(1 + \frac{\hat{\alpha}_{s,\varepsilon}}{4\pi} S_\varepsilon C_F (m^2)^{\frac{\varepsilon}{2}} \left[-\frac{16}{\epsilon^2} + \frac{12}{\epsilon} - 4 + 14\zeta(2) - 6\ln\left(\frac{m^2}{\mu_R^2}\right) \right] \right) . \end{aligned} \tag{92}$$

The double pole term in ε is the result of overlaping of soft and collinear regions in the loop integration and the simple pole arises from collinear region.

4.2.2 $\mathcal{O}(\alpha_s)$ contribution from $q + \overline{q} \to \phi + g$

We now compute the contribution coming from the gluon bremsstrahlung to $\mathcal{O}(\alpha_s)$:

$$q(p) + \overline{q}(p') \to \phi(q) + g(k). \tag{93}$$

Here, k is the momentum of the out going gluon. The corresponding cross-section is

$$\hat{\sigma}^r_{q\overline{q}} = \frac{1}{4(p \cdot p')} \int \left[dPS_2^{q\overline{q}} \right] \mathcal{K}_{q\overline{q}} \sum_{s,c} |M^r_{q\overline{q}}|^2 \tag{94}$$

where the spin and color average factor is given by

$$\mathcal{K}_{q\overline{q}} = \frac{1}{4N^2} \tag{95}$$

and the two body phase space is

$$\left[dPS_2^{q\overline{q}} \right] = \frac{d^{n-1}k}{(2\,\pi)^{n-1}\, 2\, k^0} \frac{d^{n-1}q}{(2\,\pi)^{n-1}\, 2\, q^0} (2\,\pi)^n\, \delta^n(\, k + q - p - p'\,) \tag{96}$$

and the amplitude $M^r_{q\overline{q}}$ is given by

$$M^r_{q\overline{q}} = -\hat{Y}_\varepsilon\, \hat{g}_{s,\varepsilon}\, \bar{v}(p')\, t^a \Big[\frac{\gamma^\mu\, (\not{k} - \not{p}')}{(k - p')^2} + \frac{(\not{p} - \not{k})\, \gamma^\mu}{(p - k)^2} \Big] u(p)\, \varepsilon^*_\mu(k) . \tag{97}$$

The phase space integration can be easily done in the centre of mass frame of incoming quarks wherein the momenta of the particles can be parametrised as

$$\begin{aligned} p &= \frac{\sqrt{s}}{2}\Big(1,0,\vec{0},1\Big), & p' &= \frac{\sqrt{s}}{2}\left(1,0,\vec{0},-1\right) \\ k &= |q|\Big(1,\sin\theta,\vec{0},\cos\theta\Big), & q &= \Big(q_0,-|q|\sin\theta,\vec{0},-|q|\cos\theta\Big) \\ q_0 &= \frac{\sqrt{s}}{2}(1+\hat{\tau}) & |q| &= \frac{\sqrt{s}}{2}(1-\hat{\tau}), \end{aligned} \tag{98}$$

The square of the matrix element is found to be

$$\begin{aligned} \mathcal{K}_{q\overline{q}}\sum_{s,c}|M^r_{q\overline{q}}|^2 &= \hat{Y}^2_\varepsilon \hat{g}^2_{s,\varepsilon}\frac{C_F}{N}\Bigg[2(n-2)p\cdot kp'\cdot k\Bigg(\frac{1}{(k-p')^4}+\frac{1}{(p-k)^4} \\ &\quad +\frac{4}{(k-p')^2(p-k)^2}\Bigg)+\frac{8p\cdot p'(p\cdot p'-p\cdot k-p'\cdot k)}{(p-k)^2(k-p')^2}\Bigg] \end{aligned} \tag{99}$$

The two body phase space integration reduces to

$$\begin{aligned} \int\Big[dPS_2^{q\overline{q}}\Big] &= 2^{4-2n}\frac{\pi^{1-\frac{n}{2}}}{\Gamma\left(\frac{n}{2}-1\right)}\left(\frac{\hat{s}-m^2}{\hat{s}}\right)^{n-3}\hat{s}^{\frac{n}{2}-2}\int_0^\pi d\theta(\sin(\theta))^{n-3} \\ &= \frac{1}{8\pi}\left(\frac{m^2}{4\pi}\right)^{\frac{\varepsilon}{2}}\frac{\hat{\tau}^{-\frac{\varepsilon}{2}}(1-\hat{\tau})^{1+\varepsilon}}{\Gamma\left(1+\frac{\varepsilon}{2}\right)}\int_0^1 dy\Big[y(1-y)\Big]^{\frac{\varepsilon}{2}}, \qquad 1+\cos(\theta)=2y \end{aligned} \tag{100}$$

and

$$\begin{aligned} \int\Big[dPS_2^{q\overline{q}}\Big]\frac{1}{(k-p')^{2\alpha}(p-k)^{2\beta}} &= \frac{1}{8\pi}\left(\frac{m^2}{4\pi}\right)^{\frac{\varepsilon}{2}}\frac{\hat{\tau}^{-\frac{\varepsilon}{2}}(1-\hat{\tau})^{1+\varepsilon}}{\Gamma\left(1+\frac{\varepsilon}{2}\right)}\left(-\frac{\hat{\tau}}{m^2(1-\hat{\tau})}\right)^{\alpha+\beta} \\ &\quad\times\frac{\Gamma\left(1-\alpha+\frac{\varepsilon}{2}\right)\Gamma\left(1-\beta+\frac{\varepsilon}{2}\right)}{\Gamma\left(2-\alpha-\beta+\varepsilon\right)} \end{aligned} \tag{101}$$

Note that the phase space integrations over the square of the matrix elements develop soft and collinear singularities in 4-dimensions. They appear as poles in $1/\varepsilon$ after the phase space integrations are carried out.

$$\begin{aligned} \int\Big[dPS_2^{q\overline{q}}\Big]\mathcal{K}_{q\overline{q}}|M^r_{q\overline{q}}|^2 &= \hat{Y}^2_\varepsilon\hat{g}^2_{s,\varepsilon}\frac{C_F}{N}\left(\frac{1}{8\pi}\left(\frac{m^2}{4\pi}\right)^{\frac{\varepsilon}{2}}\Gamma\left(1+\frac{\varepsilon}{2}\right)\frac{\hat{\tau}^{-\frac{\varepsilon}{2}}(1-\hat{\tau})^{\varepsilon}}{\Gamma\left(2+\varepsilon\right)}\right) \\ &\quad\times\left[\frac{16}{\varepsilon}\Big(-1-\hat{\tau}+\frac{2}{1-\hat{\tau}}\Big)-8-24\hat{\tau}+\frac{32}{1-\hat{\tau}}+8\varepsilon(1-\hat{\tau})\right] \end{aligned} \tag{102}$$

The term $1/(1-\hat{\tau})$ present in the above expression is not integrable in because it diverges as $\hat{\tau}\to 1$ or equivalently $\hat{s}\to m^2$, a configuration that results from soft gluons. On the other hand, in $4+\varepsilon$ dimensions the factor that results from phase space measure, $(1-\hat{\tau})^\varepsilon$, regulates integral to produce

$$\int_0^1 dz\frac{(1-z)^\varepsilon}{1-z}=\frac{1}{\varepsilon}\quad\text{for}\quad\varepsilon>0 \tag{103}$$

For any regular function $f(\hat\tau)$ (regular as $\hat\tau \to 1$), we find

$$\begin{aligned}\int_0^1 d\hat\tau f(\hat\tau)\frac{(1-\hat\tau)^\varepsilon}{1-\hat\tau} &= \int_0^1 d\hat\tau\Big(f(\hat\tau)-f(1)\Big)\frac{(1-\hat\tau)^\varepsilon}{1-\hat\tau} + f(1)\int_0^1 d\hat\tau\frac{(1-\hat\tau)^\varepsilon}{1-\hat\tau} \\ &= \int_0^1 d\hat\tau\Big(f(\hat\tau)-f(1)\Big)\sum_{i=0}^\infty \frac{\varepsilon^i}{i!}\frac{\ln^i(1-\hat\tau)}{1-\hat\tau} + \frac{f(1)}{\varepsilon}\int d\hat\tau \delta(1-\hat\tau) \\ &= \int_0^1 d\hat\tau f(\hat\tau)\sum_{i=0}^\infty \frac{\varepsilon^i}{i!}\left(\frac{\ln^i(1-\hat\tau)}{1-\hat\tau}\right)_+ + \frac{f(1)}{\varepsilon}\int d\hat\tau\delta(1-\hat\tau) \qquad (104)\end{aligned}$$

In the above, the " + " prescription for a function $\ln^i(1-\hat\tau)/(1-\hat\tau)$ is defined as

$$\int_0^1 d\hat\tau f(\hat\tau)\left(\frac{\ln^i(1-\hat\tau)}{1-\hat\tau}\right)_+ = \int_0^1 d\hat\tau\Big(f(\hat\tau)-f(1)\Big)\frac{\ln^i(1-\hat\tau)}{1-\hat\tau}\ . \qquad (105)$$

From eqn.(104), for any orbitrary regular function, we find

$$\frac{(1-\hat\tau)^\varepsilon}{1-\hat\tau} = \frac{1}{\varepsilon}\delta(1-\hat\tau) + \frac{1}{(1-\hat\tau)_+} + \varepsilon\left(\frac{\ln(1-\hat\tau)}{1-\hat\tau}\right)_+ + \mathcal{O}(\varepsilon^2) \qquad (106)$$

The above representation is useful to isolate soft singularities as poles in $1/\varepsilon$ in the eqn.(102) to get $\hat\Delta^r_{q\overline{q}} = \hat s\hat\sigma^r_{q\overline{q}}$:

$$\begin{aligned}\hat\Delta^r_{q\overline{q}} &= \Delta_0\frac{\hat\alpha_{s,\varepsilon}}{4\pi}S_\varepsilon(m^2)^{\frac{\varepsilon}{2}}C_F\Bigg[\left(\frac{16}{\varepsilon^2}-6\zeta(2)\right)\delta(1-\hat\tau)+\frac{8}{\varepsilon}\left(-1-\hat\tau+\frac{2}{(1-\hat\tau)_+}\right) \\ &\quad +4\left(1+\hat\tau-\frac{2}{1-\hat\tau}\right)\ln(\hat\tau) - 8(1+\hat\tau)\ln(1-\hat\tau) \\ &\quad +16\left(\frac{\ln(1-\hat\tau)}{1-\hat\tau}\right)_+ + 4(1-\hat\tau)\Bigg] \qquad (107)\end{aligned}$$

Note that soft singularity resulting from $\hat\tau \to 1$ overlaps with the singularity coming from collinear configurations in the two body phase space giving double poles $(1/\varepsilon^2)$ in the above expression. The single pole contribution proportional to $(-1-\hat\tau+2/(1-\hat\tau))$ is due to collinear configurations of the emitted gluon with the incoming quark/anti-quark states. Additing contributions coming from the virtual subprocesses and UV counter terms, we find

$$\begin{aligned}\hat\Delta^{(1)}_{q\overline{q}} &= \hat\Delta^{(1),V+CT}_{q\overline{q}} + \hat\Delta^r_{q\overline{q}} \\ &= \Delta_0(m^2)^{\frac{\varepsilon}{2}}C_F\Bigg[\left(\frac{12}{\varepsilon}-4+8\zeta(2)-6\ln\left(\frac{m^2}{\mu_R^2}\right)\right)\delta(1-\hat\tau) \\ &\quad +\frac{8}{\varepsilon}\left(-1-\hat\tau+\frac{2}{(1-\hat\tau)_+}\right)\ln(\hat\tau)+4\left(1+\hat\tau-\frac{2}{1-\hat\tau}\right)\ln(\hat\tau)-8(1+\hat\tau)\ln(1-\hat\tau) \\ &\quad +16\left(\frac{\ln(1-\hat\tau)}{1-\hat\tau}\right)_+ + 4(1-\hat\tau)\Bigg] \qquad (108)\end{aligned}$$

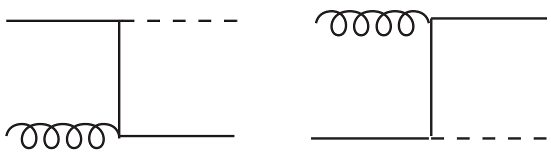


Figure 6: $\mathcal{O}(\alpha_s)$ quark (antiquark) gluon initiated subprocess: $q_i(\overline{q}_i) + g \to \phi + q_j(\overline{q}_j)$.

The finite partonic cross section defined by $\Delta^{(1)}_{q\overline{q}}$ in eqn.(24) is given by

$$\Delta^{(1)}_{q\overline{q}} = \hat{\Delta}^{(1)}_{q\overline{q}} - \Delta_0 2\Bigg(4C_F\Bigg[-1-\hat{\tau}+\frac{2}{(1-\hat{\tau})_+}+\frac{3}{2}\delta(1-\hat{\tau})\Bigg]\Bigg)\Bigg(\frac{1}{\varepsilon}+\frac{1}{2}\ln(\mu_F^2)\Bigg) \tag{109}$$

where we have substituted Altarelli-Parisi plitting function given by

$$P^{(0)}_{qq} = P^{(0)}_{\overline{q}\overline{q}} = 4C_F\Bigg[-1-\hat{\tau}+\frac{2}{(1-\hat{\tau})_+}+\frac{3}{2}\delta(1-\hat{\tau})\Bigg] \tag{110}$$

in eqn.(24). Substituting eqn.(108) in the eqn.(109), we obtain

$$\begin{aligned}\Delta^{(1)}_{q\overline{q}} = \Delta_0 C_F\Bigg[&\Bigg(-4+8\zeta(2)+6\ln\left(\frac{\mu_R^2}{\mu_F^2}\right)\Bigg)\delta(1-\hat{\tau}) \\ &+4\Bigg(-1-\hat{\tau}+\frac{2}{(1-\hat{\tau})_+}\Bigg)\ln\left(\frac{m^2}{\mu_F^2}\right)+4\Bigg(1+\hat{\tau}-\frac{2}{1-\hat{\tau}}\Bigg)\ln(\hat{\tau}) \\ &-8(1+\hat{\tau})\ln(1-\hat{\tau})+16\left(\frac{\ln(1-\hat{\tau})}{1-\hat{\tau}}\right)_+ +4(1-\hat{\tau})\Bigg]\end{aligned} \tag{111}$$

Note that $\Delta^{(1)}_{q\overline{q}}$ is free of UV, soft and collinear singularities and hence suitable for any further study.

4.2.3 $\mathcal{O}(\alpha_s)$ contribution from $q + g \to \phi + q$

We now compute the final piece, namely the contribution of the Compton-like process to order α_s:

$$q(p) \; + \; g(k) \to \phi(q) \; + \; q(p'). \tag{112}$$

The partonic cross-section is given by

$$\hat{\sigma}_{qg} = \frac{1}{4(p\cdot k)}\int \; [dPS_2^{qg}] \; \mathcal{K}_{qg}\sum_{s,c}|M_{qg}|^2 \tag{113}$$

the spin and color average keeping in my that the gluons have $n-2$ polarisation states in n dimensions gives the factor

$$\mathcal{K}_{qg} = \frac{1}{2(n-2)N(N^2-1)}\,. \tag{114}$$

and the two body phase space is

$$[dPS_2^{qg}] = \frac{d^{n-1}p'}{(2\,\pi)^{n-1}\,2\,p'^0}\;\frac{d^{n-1}q}{(2\,\pi)^{n-1}\,2\,q^0}\;(2\,\pi)^n\;\delta^n(\,q+p'-k-p\,) \tag{115}$$

The matrix element is given by

$$M_{qg} = -\hat{Y}_\varepsilon\,\hat{g}_{s,\varepsilon}\bar{u}(p')\ t^a\Big[\ \frac{\gamma^\mu\ (\not{p}'-\not{k})}{(p'-k)^2}\ +\ \frac{(\not{p}+\not{k})\ \gamma^\mu}{(p+k)^2}\ \Big]\ u(p)\ \varepsilon_\mu(k)\ . \tag{116}$$

$$\begin{aligned}\mathcal{K}_{qg}\sum_{s,c}|M_{qg}|^2 &= \hat{Y}_\varepsilon^2\hat{g}_{s,\varepsilon}^2\frac{2T_F}{N(n-2)}\Bigg[2(n-2)p\cdot kp'\cdot k\Bigg(\frac{1}{(p'-k)^4}+\frac{1}{(p+k)^4}\\ &\quad+\frac{4}{(p'-k)^2(k+p)^2}\Bigg)+\frac{8p\cdot p'(-p\cdot p'+p\cdot k-p'\cdot k)}{(k+p)^2(p'-k)^2}\Bigg]\end{aligned} \tag{117}$$

where $T_F = 1/2$.

$$\begin{aligned} p &= \frac{\sqrt{s}}{2}\Big(1,0,\vec{0},1\Big), & k &= \frac{\sqrt{s}}{2}\,(1,0,\vec{0},-1)\\ p' &= |q|\Big(1,\sin\theta,\vec{0},\cos\theta\Big), & q &= \Big(q_0,-|q|\sin\theta,\vec{0},-|q|\cos\theta\Big)\\ q_0 &= \frac{\sqrt{s}}{2}(1+\hat{\tau}) & |q| &= \frac{\sqrt{s}}{2}(1-\hat{\tau}),\end{aligned} \tag{118}$$

$$\begin{aligned}\int[dPS_2^{qg}]\,\frac{1}{(p'-k)^{2\alpha}(p+p')^{2\beta}} &= \frac{1}{8\pi}\left(\frac{m^2}{4\pi}\right)^{\frac{\varepsilon}{2}}\frac{\hat{\tau}^{-\frac{\varepsilon}{2}}(1-\hat{\tau})^{1+\varepsilon}}{\Gamma\left(1+\frac{\varepsilon}{2}\right)}\Big(-1\Big)^{\alpha}\left(\frac{\hat{\tau}}{m^2(1-\hat{\tau})}\right)^{\alpha+\beta}\\ &\quad\times\frac{\Gamma\left(1-\alpha+\frac{\varepsilon}{2}\right)\Gamma\left(1-\beta+\frac{\varepsilon}{2}\right)}{\Gamma\left(2-\alpha-\beta+\varepsilon\right)}\end{aligned} \tag{119}$$

On performing the angular integration, we get

$$\begin{aligned}\int[dPS_2^{qg}]\,\mathcal{K}_{qg}\sum_{s,c}|M_{qg}|^2 &= \hat{Y}_\varepsilon\hat{g}_{s,\varepsilon}^2\frac{T_F}{N}\left(\frac{1}{8\pi}\left(\frac{m^2}{4\pi}\right)^{\frac{\varepsilon}{2}}\Gamma\left(1+\frac{\varepsilon}{2}\right)\frac{\hat{\tau}^{\frac{-\varepsilon}{2}}(1-\hat{\tau})^{1+\varepsilon}}{\Gamma\left(2+\varepsilon\right)}\right)\\ &\quad\times\frac{2}{2+\varepsilon}\Bigg[-\frac{3}{2}-\frac{5}{2}\hat{\tau}+\frac{3}{1-\hat{\tau}}\\ &\quad+\frac{1}{\varepsilon}\left(\frac{2}{1-\hat{\tau}}-4\hat{\tau}\right)+\varepsilon\left(-\frac{3}{4}-\frac{\hat{\tau}}{4}+\frac{1}{1-\hat{\tau}}\right)\Bigg]\end{aligned} \tag{120}$$

Substituting the above expression in eqn.(113) and expanding around $\varepsilon = 0$, we find $\hat{\Delta}_{qg} = \hat{s}\hat{\sigma}_{qg}$:

$$\begin{aligned}\hat{\Delta}_{qg} &= \Delta_0\frac{\alpha_{s,\varepsilon}}{4\pi}S_\varepsilon(m^2)^{\frac{\varepsilon}{2}}T_F\Bigg[\frac{4}{\varepsilon}\Big(\hat{\tau}^2+(1-\hat{\tau})^2\Big)+(1-\hat{\tau})(7\hat{\tau}-3)\\ &\quad-2\Big(\hat{\tau}^2+(1-\hat{\tau})^2\Big)\ln(\hat{\tau})+4\Big(\hat{\tau}^2+(1-\hat{\tau})^2\Big)\ln(1-\hat{\tau})\Bigg]\end{aligned} \tag{121}$$

Note that the above expression has only single pole $1/\varepsilon$ which results from the outgoing quark becoming collinear to initial gluon state. The finite partonic cross section defined by $\Delta_{qg}^{(1)}$ in eqn.(25) is given by

$$\Delta_{qg}^{(1)} = \hat{\Delta}_{qg}^{(1)}-\Delta_0\Bigg(4T_F\Big[\hat{\tau}^2+(1-\hat{\tau})^2\Big]\Bigg)\left(\frac{1}{\varepsilon}+\frac{1}{2}\ln(\mu_F^2)\right) \tag{122}$$

where we have substituted Altarelli-Parisi splitting function given by

$$P^{(0)}_{qg} = P^{(0)}_{\overline{q}g} = 4T_F\Big[\hat\tau^2 + (1-\hat\tau)^2\Big] \tag{123}$$

in eqn.(25). Substituting eqn.(121) in the eqn.(122), we obtain

$$\begin{aligned}\Delta^{(1)}_{qg} &= \hat\Delta^{(1)}_{qg} - \Delta_0\Bigg(4T_F\Big(\hat\tau^2+(1-\hat\tau)^2\Big)\Bigg)\Bigg(\frac{1}{\varepsilon}+\frac{1}{2}\ln(\mu_F^2)\Bigg)\\ &= \Delta_0 T_F\Bigg[(1-\hat\tau)(7\hat\tau-3)+2\Big(\hat\tau^2+(1-\hat\tau)^2\Big)\ln\left(\frac{m^2}{\mu_F^2}\right)\\ &\quad -2\Big(\hat\tau^2+(1-\hat\tau)^2\Big)\ln(\hat\tau)+4\Big(\hat\tau^2+(1-\hat\tau)^2\Big)\ln(1-\hat\tau)\Bigg]\end{aligned} \tag{124}$$

Note that both the quark and gluon initiated processes, after the mass factorisation, are free of collinear singularity. We use these results for our further analysis after folding with appropriate parton distributions.

5 Results

Having obtained the analytic expressions in the last section, we now endeavour to see the impact of these corrections to resonant production of a scalar. In order to demonstrate the role of these corrections, we restrict ourselves to case where the scalar couples only to d type quarks and anti-quarks. To this end, one needs to define the leading-order and the next-to-leading-order cross-sections for hadronic collider:

- $\sigma_{\rm LO}$: convolute the cross-section $\Delta^{(0)}_{q\overline{q}}$ with the appropriate LO evolved quark, anti-quark distribution functions;
- $\sigma_{\rm NLO}$: convolute the cross-sections $\Delta^{(0)}_{q\overline{q}} + \frac{\alpha_s}{4\pi}\Delta^{(1)}_{q\overline{q}}$, $\frac{\alpha_s}{4\pi}\Delta^{(1)}_{ag}$ and $\frac{\alpha_s}{4\pi}\Delta^{(1)}_{ga}$ $(a=q,\overline{q})$ with the appropriate NLO evolved quark,anti-quark and gluon distributions functions.

Explicitly, we find

$$\sigma^{H_1H_2}_{LO}(S,m^2) = \frac{1}{S}\Bigg[f^{LO}_d(\mu_F^2)\otimes f^{LO}_{\overline{d}}(\mu_F^2)\otimes\Delta^{(0)}_{d\overline{d}}(\mu_R^2) + f^{LO}_{\overline{d}}(\mu_F^2)\otimes f^{LO}_d(\mu_F^2)\otimes\Delta^{(0)}_{\overline{d}d}(\mu_R^2)\Bigg]$$

$$\begin{aligned}\sigma^{H_1H_2}_{NLO}(S,m^2) &= \frac{1}{S}\Bigg[f^{NLO}_d(\mu_F^2)\otimes f^{NLO}_{\overline{d}}(\mu_F^2)\otimes\Delta^{(0)}_{d\overline{d}}(\mu_R^2) + f^{NLO}_{\overline{d}}(\mu_F^2)\otimes f^{NLO}_d(\mu_F^2)\otimes\Delta^{(0)}_{\overline{d}d}(\mu_R^2)\\ &\quad +\frac{\alpha_s^{NLO}(\mu_R^2)}{4\pi}\Bigg(f^{NLO}_d(\mu_F^2)\otimes f^{NLO}_{\overline{d}}(\mu_F^2)\otimes\Delta^{(1)}_{d\overline{d}}(\mu_R^2,\mu_F^2)\\ &\quad +f^{NLO}_{\overline{d}}(\mu_F^2)\otimes f^{NLO}_d(\mu_F^2)\otimes\Delta^{(1)}_{\overline{d}d}(\mu_R^2,\mu_F^2)\Bigg)\\ &\quad +\frac{\alpha_s^{NLO}(\mu_R^2)}{4\pi}\Bigg(\sum_{a=d,\overline{d}} f^{NLO}_a(\mu_F^2)\otimes f^{NLO}_g(\mu_F^2)\otimes\Delta^{(1)}_{ag}(\mu_R^2,\mu_F^2)\\ &\quad +\sum_{a=d,\overline{d}} f^{NLO}_g(\mu_F^2)\otimes f^{NLO}_a(\mu_F^2)\otimes\Delta^{(1)}_{gq}(\mu_R^2,\mu_F^2)\Bigg)\Bigg]\end{aligned} \tag{125}$$

For our numerical analysis, we choose to work with

$$\begin{aligned} Y(M_Z) &= 0.01, \quad \text{at} \quad \text{LO, NLO} \\ \alpha_s(M_Z) &= 0.13939 \quad \text{at} \quad \text{LO}, \qquad \alpha_s(M_Z) = 0.12018 \quad \text{at} \quad \text{NLO} \end{aligned} \tag{126}$$

and **MSTW 2008** LO and NLO parton distribution functions.

To begin with, we concentrate on the resonance production of a scalar starting with a $d\bar{d}$ initial state (at the Born level). In Fig.8, we plot both the LO and the NLO cross-sections as a function of scalar mass for two choices of factorisation scale $\mu_F = 0.1\ m$ and $\mu_F = 10\ m$. The renormalisation scale μ_R has been chosen to be the same as the scalar mass. As is clear from the plots, the inclusion of NLO contributions reduce the factorisation scale uncertainty considerably. The fall in the parton-level cross-sections is due to the rapid decrease of the parton densities at high momentum fractions.

To parametrise the effect of the NLO corrections, it is common to introduce the K-factor:

$$K \equiv \frac{\sigma_{\text{NLO}}^{H_1 H_2}(S, m^2)}{\sigma_{\text{LO}}^{H_1 H_2}(S, m^2)}\ , \tag{127}$$

which we plot in Fig.9. As is clear from the plots the 30% to 65% corrections results from the NLO QCD corrections and the precise value depends on the mass of the scalar and center of mass energy of the machine and type of incoming parton fluxes.

Having explored the dependence of the K-factor on the scalar mass we now turn to the renormalisation and factorisation scales. In top left (right) panel of Fig.10, we plot the variation of both LO and NLO cross sections for the LHC with respect to μ_F/m (μ_R/m) while keeping scale $\mu_R = m$ ($\mu_F = m$) The bottom panels corresspond to the case at Tevatron. We have set $m = 800$ GeV for LHC and $m = 300$ GeV for Tevatron. We find the NLO corrected cross sections are less sensitive to both factorisation and renormalisation scales.

6 Conclusions

We have systematically described the formalism to compute QCD higher order radiative corrections to resonant production of a scalar particle in hadron colliders. We have shown how various singularities can arise when we include corrections beyond the leading order in QCD perturbation theory and show how they can be removed. Dimensional regularisation has been used throughout to regulate all the singularities. We have used $\overline{MS}$ scheme to renormalise UV and collinear singularities. The numerical importance of such corrections to production cross section and the role of these corrections in reducing the theoretical uncertainities resulting from renormalisation and factorisation scales are demonstrated in detail.

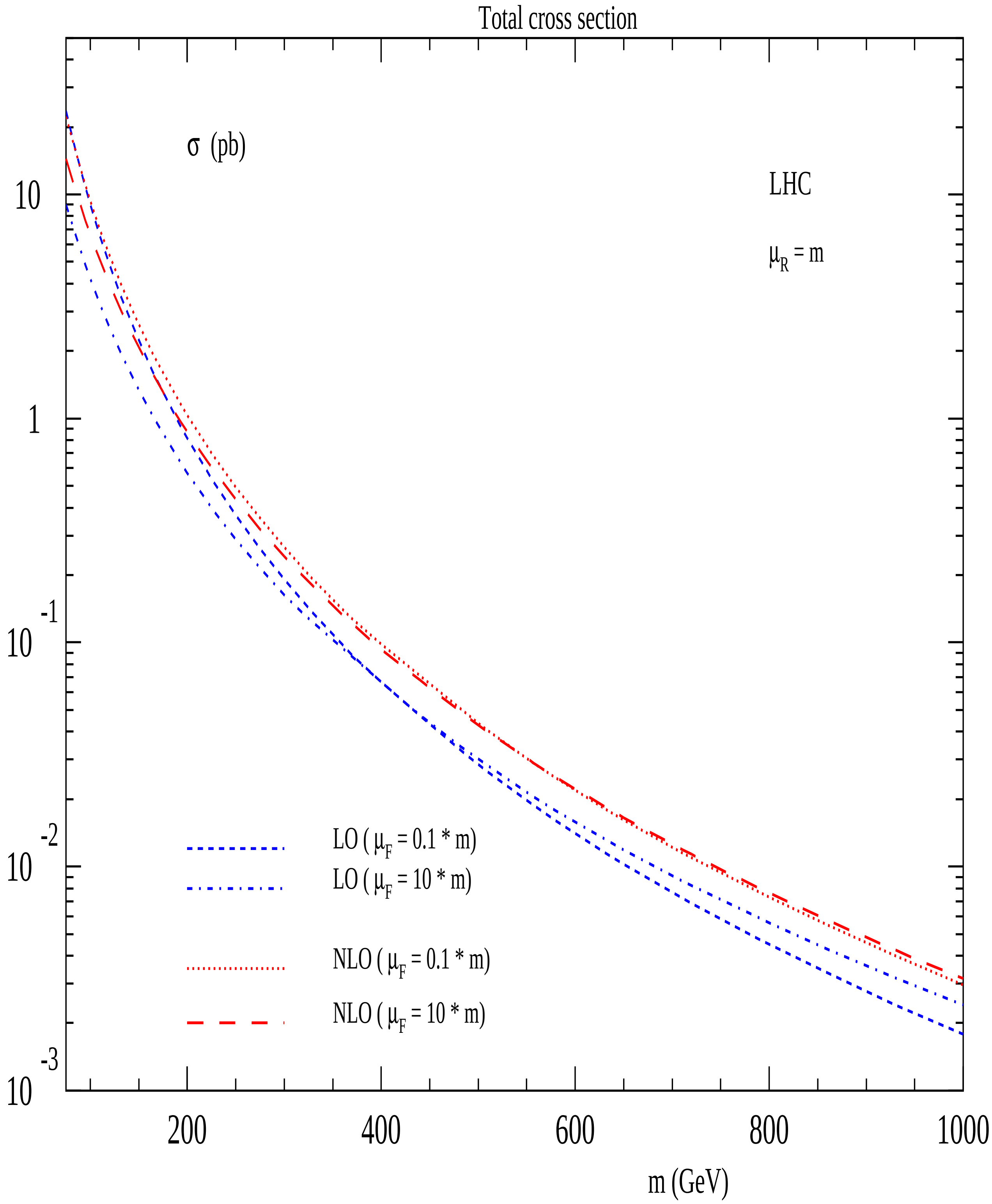


Figure 7: *Cross-section for resonant scalar production at the LHC.* $Y(M_Z)$ *has been set to 0.01. We have used MSTW 2008 LO and NLO parton distribution functions.*

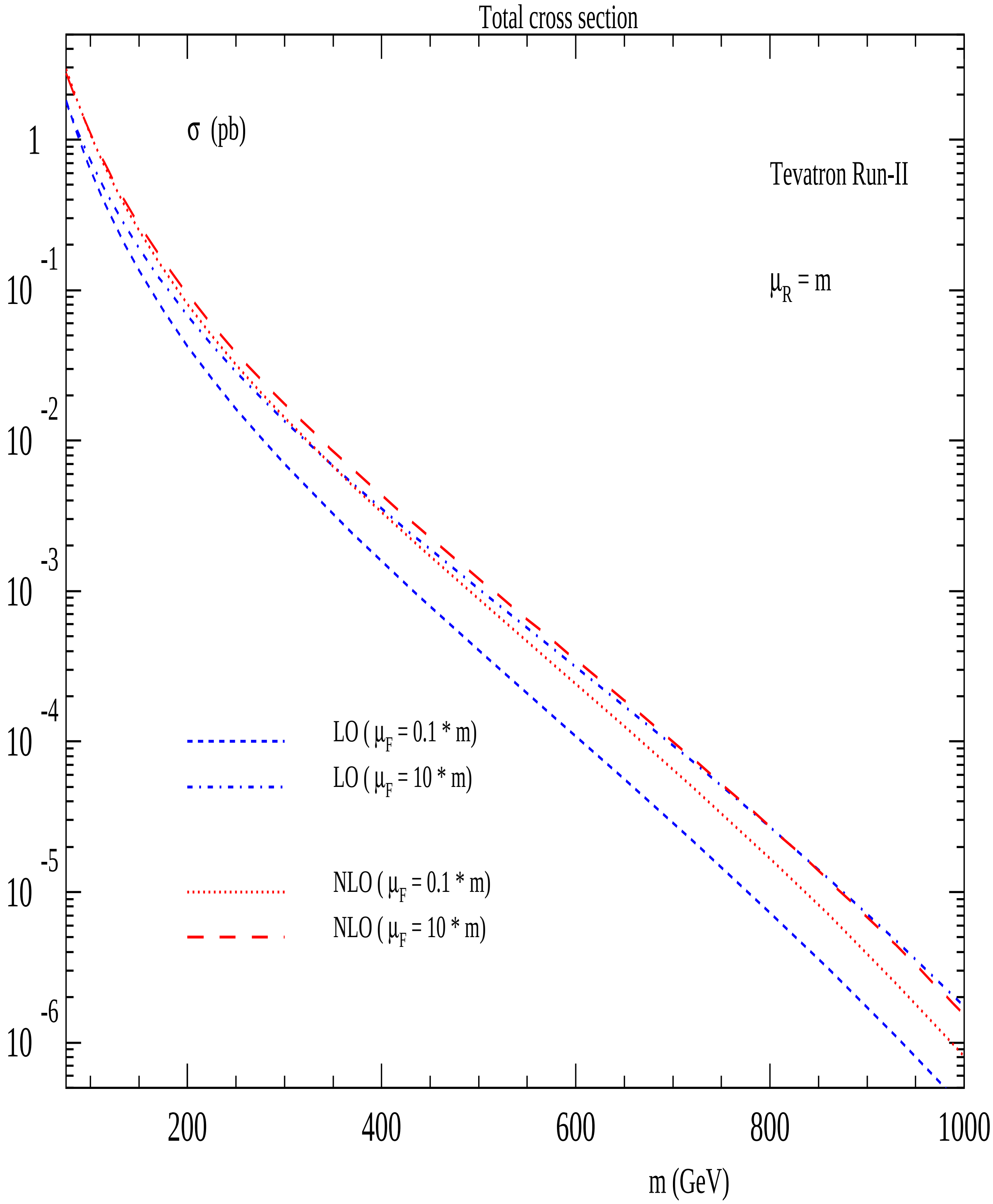
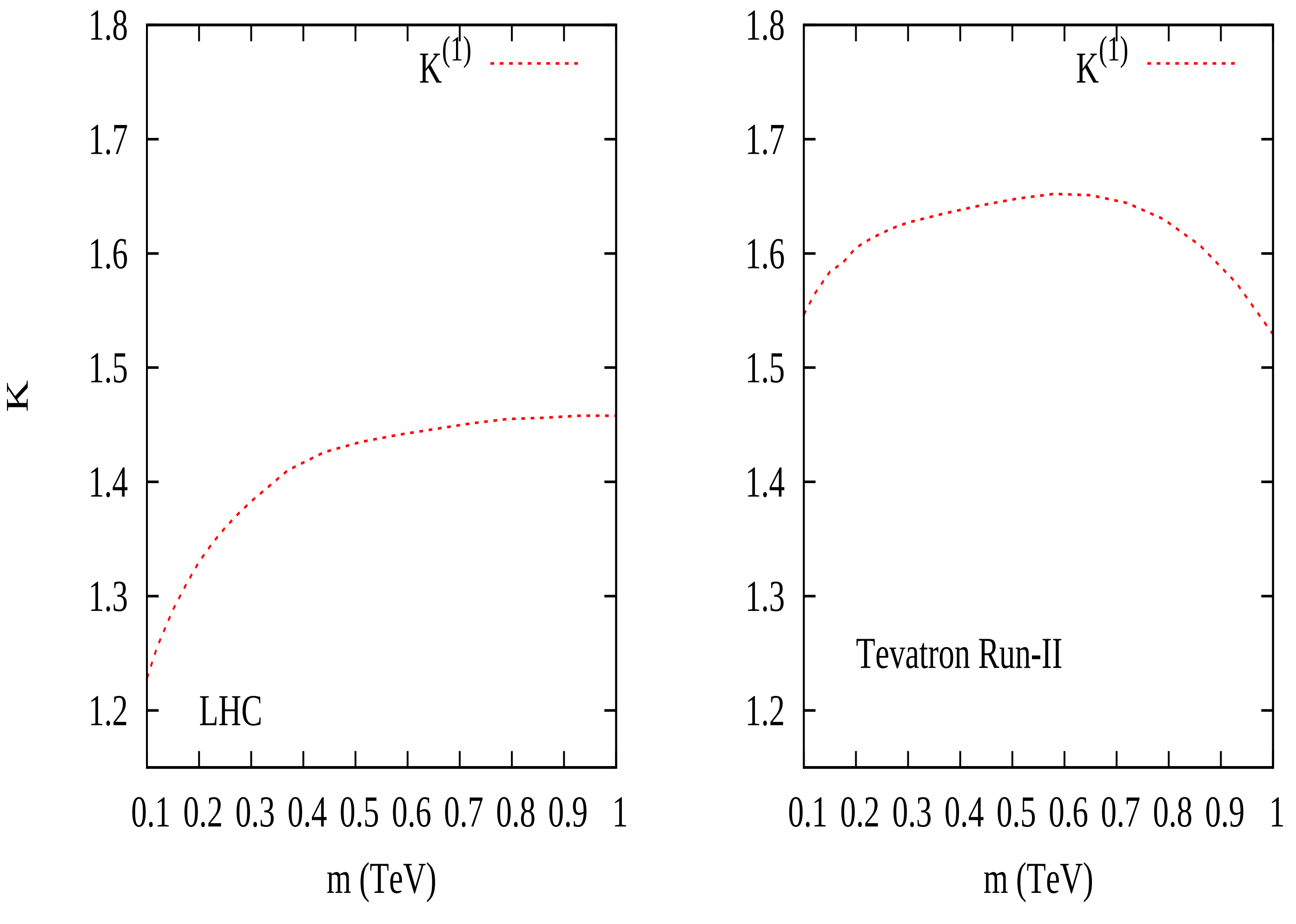


Figure 8: *Cross-section for resonant scalar production at the Tevatron.* $Y(M_Z)$ *has been set to 0.01. We have used MSTW 2008 LO and NLO parton distribution functions.*

$\mu_R = m_{\tilde{\nu}}$



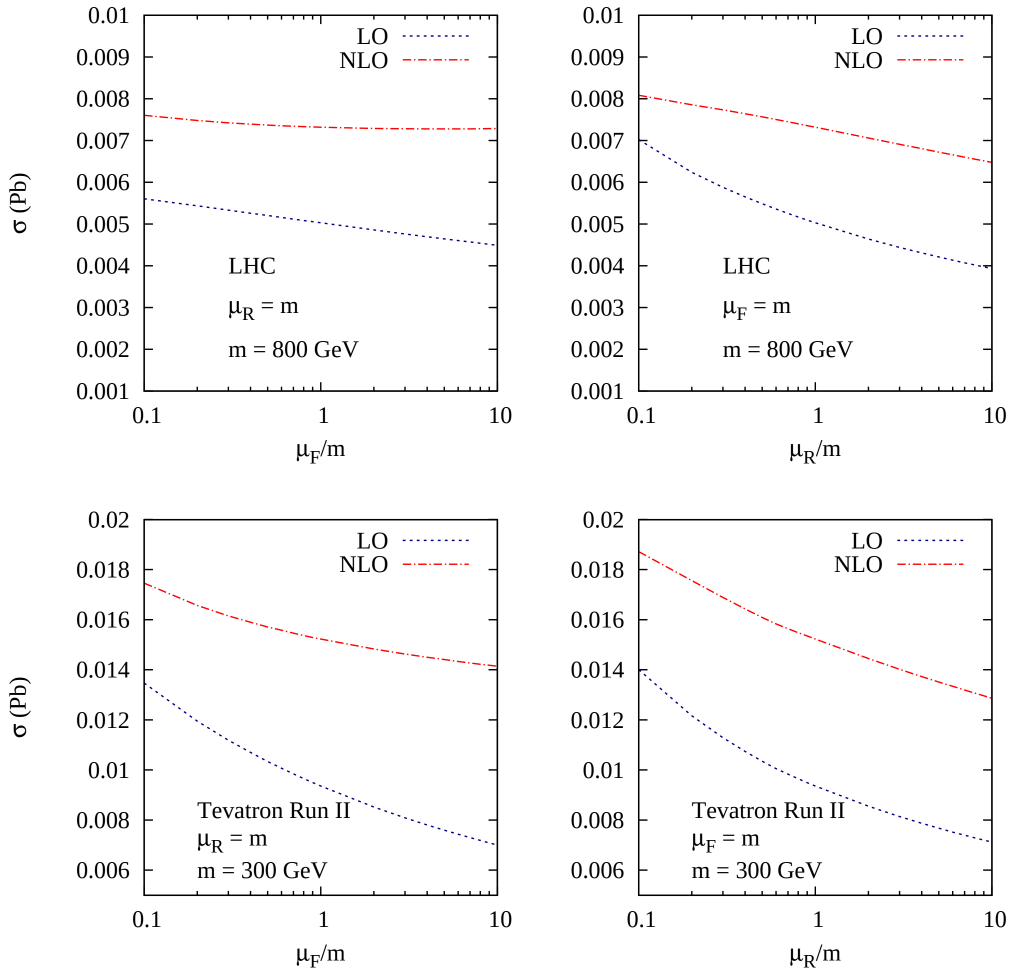


Figure 10: *The dependence of the cross-sections at the LHC on the value of the factorisation scale* μ_F *and the renormalisation scale* μ_R*. The top and bottom panels correspond to the LHC and the Tevatron cross-sections respectively.*