A Fermionic Tensor Model in $2 - \epsilon$ dimensions

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Outline of talk

- Motivation and review of tensor models [based on Klebanov Tarnopolsky, 2016]

- Formal large *N* solution of a *d*-dimensional fermionic tensor model [Based on arXiv: 1710.09357 with Ritam Sinha]

- Studying 1-*d* tensor models at finite *N* via a 2-ε dimensional expansion. [Work in progress with Ritam Sinha+review of Giombi, Klebanov, Tarnopolsky 2017]

Tensor Models and SYK

- The Sachdev-Ye-Kitaev (SYK) has attracted a great deal of attention recently as a possibly-simple model of holography.

- Witten and then Klebanov and Tarnopolsky showed that tensor models have similar dynamics to the SYK model, in terms of melonic dominance. (But see S. Choudhury *et al.* and K. Bulycheva, I. R. Klebanov, A. Milekhin, G. Tarnopolsky)

Large *N* limits: the good, the bad and the ugly

- **Vector Models:** fields with one index: gives rise to a large *N* limit in which leading-order diagrams can be explicitly summed: dynamics fairly trivial: dual to higher spin gauge theory

- **Matrix Models:** fields with two indices: gives rise to a large *N* limit in which leading-order diagrams cannot be explicitly summed: dynamics non-trivial: dual to Einstein gravity (sometimes)

- **Melonic Tensor Models:** fields with three (or more) indices: can give rise to a large N limit in which all leading diagrams can be explicitly summed. dynamics non-trivial: holographic dual? Often (in $d>1$) this large *N* limit actually doesn't exist.

What is the relation between these large N limits

- Via ABJ(M) theory, we learned that there is a natural physical connection between the vector model large N limit and the matrix model large N limit.

- One of the open questions we hope to address, perhaps indirectly, is what is the relation between the melonic large N limit and the other two limits.

M/N Expansion

$U(N) \times U(M)$ *ABJ* theory clarifies the relation between vector model and matrix model large N limits:

- When M/N is small the theory is a vector model, and a small summable subset of planar diagrams are dominant. We can in principle solve the theory.

- When M/N corrections are taken into account, more and more planar diagrams are included. At any finite order of M/N these are in principle summable, and the theory can be solved.

- When $M/N \sim 1$, all planar diagrams are dominant, and we cannot solve the theory.

M/N Expansion

$U(N)\times U(M)$ *ABJ* theory clarifies the relation between vector model and matrix model large N limits:

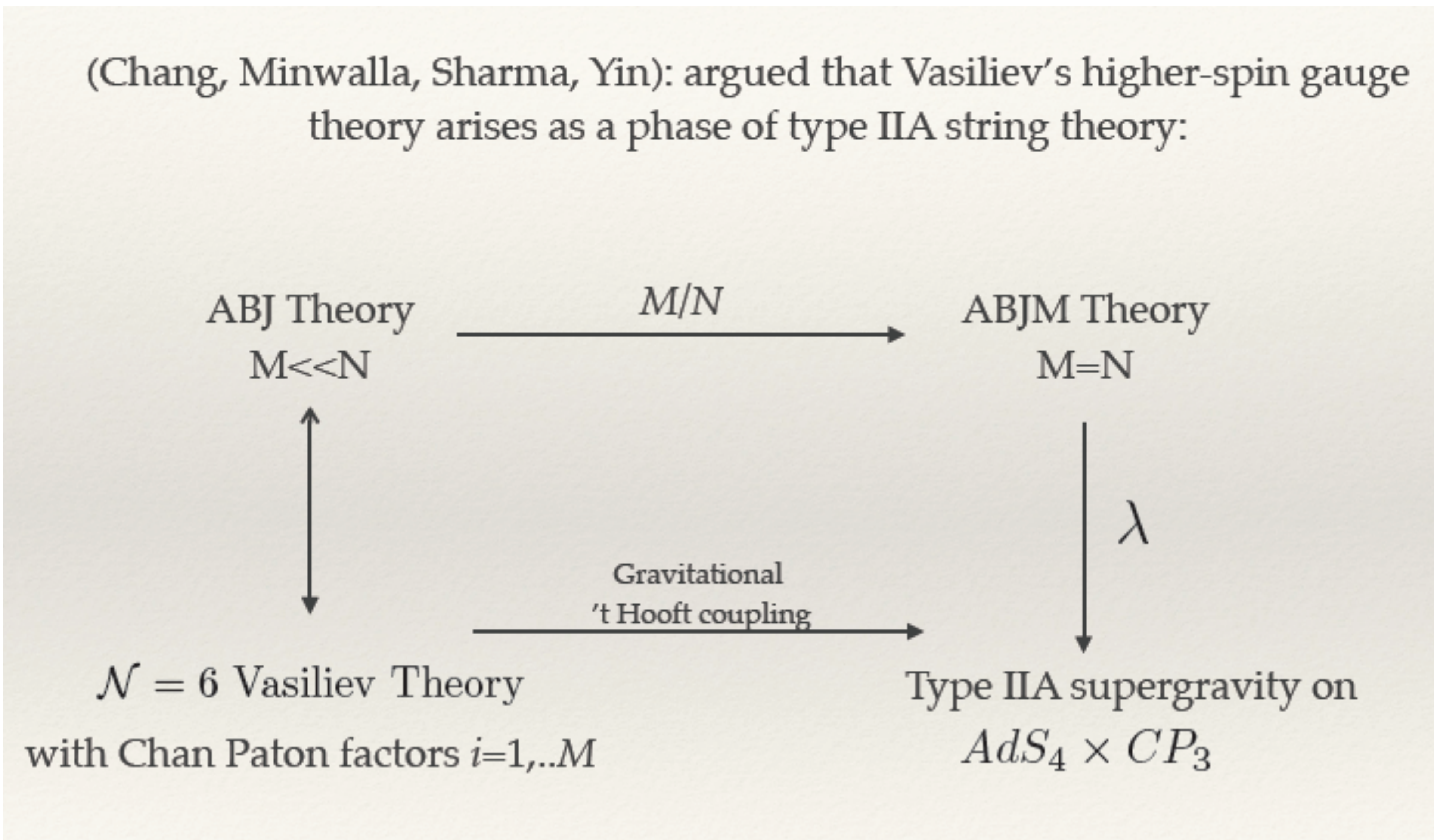
- When M/N is small the spectrum of single-trace primary operators is very small (essentially ~1 for each spin *s*). Their scaling dimensions are protected via a simple argument from conformal representation theory. [Giombi Minwalla SP Trivedi Wadia Yin, Aharony Gur-Ari Yacoby]

- When M/N corrections are taken into account, the spectrum of operators increases, and the anomalous dimensions get corrections proportional to powers of M/N.

- When $M/N\sim 1$, there are a large number of single-trace primary operators, which are strings of matrices of the form *tr(AB), tr (ABAB)* etc. Their anomalous dimensions are not protected, and may diverge at strong coupling.

M/N Expansion

- From the perspective of a holographic dual:

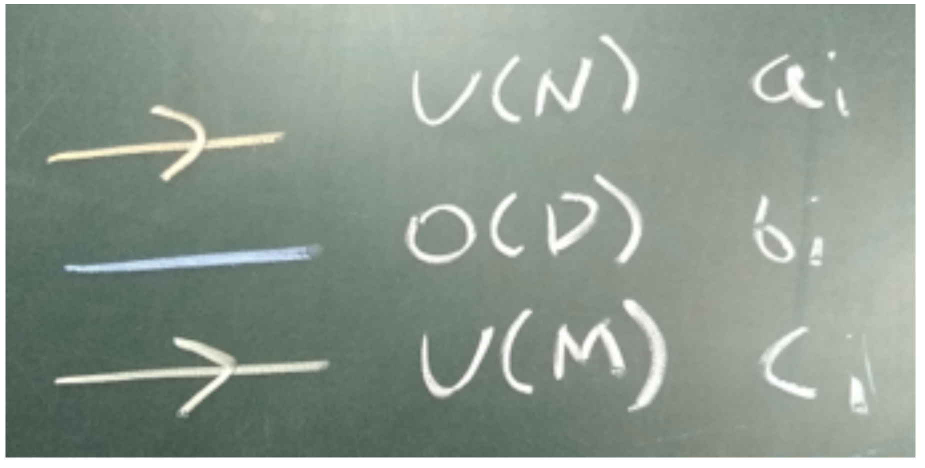


Tensor Models

- It is clear that the holographic duals of melonic tensor models may be “ugly.”
- In higher-dimensions, as we will see, this large N limit generically doesn’t exist. (But we would like to understand this better.)
- However, it would be nice if we could better understand how the tensor model large N limit relates to the other models.

Tensor Models - Quick Review

- [Gurau, and many others] Consider fields with three indices: ψ^{abc}
- We take the indices transform in *U(N)×O(D)×U(M),* though other possibilities exist*.* For the most part, we will take *M=D=N.*
- Diagrams can be drawn in a 't Hooft triple-line notation, with each line of a different colour.

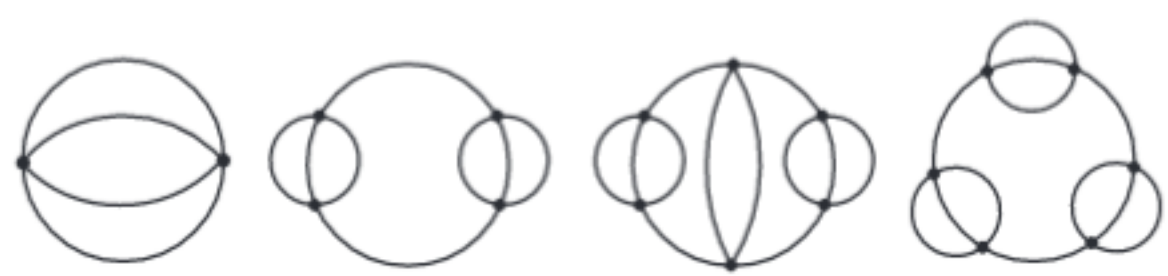


Klebanov-Tarnopolsky Model

- KT define a 1-dimensional model with the four fermion *tetrahedron* coupling.

$$S = \int dt \Big(\frac{i}{2} \psi^{abc} \partial_t \psi^{abc} + \frac{1}{4} g \psi^{a_1 b_1 c_1} \psi^{a_1 b_2 c_2} \psi^{a_2 b_1 c_2} \psi^{a_2 b_2 c_1} \Big)$$

- 't Hooft limit with *g*²*NDM* held fixed.
- only melonic diagrams (a subset of planar diagrams) contribute.



1-dimensional fermionic KT model

- For the 1-dimensional fermionic model: KT found that the spectrum of fermion bilinears was identical to that of the SYK model.
- However, there are a huge number of gauge-invariant, single trace operators. These mean that the holographic dual is quite different from SYK. (Choudhury, …, Minwalla, *et al.* and Bulycheva, Klebanov, Milekhin, Tarnopolsky)

Klebanov-Tarnopolsky Tensor Models

Klebanov and Tarnopolsky 2016, and later Giombi, Klebanov and Tarnopolsky 2017 studied the following d-dimensional bosonic generalization:

$$S = \int d^d x \Big(\frac{1}{2} \partial_\mu \phi^{abc} \partial^\mu \phi^{abc} + \frac{1}{4} g \phi^{a_1 b_1 c_1} \phi^{a_1 b_2 c_2} \phi^{a_2 b_1 c_2} \phi^{a_2 b_2 c_1} \Big)$$

d-dimensional bosonic tensor model: Summary of results

- The *d*-dimensional bosonic model is not well-defined because the interaction term actually contains a negative direction. But it can still be studied formally in perturbation theory (like ϕ^3 theory in 6 dimensions).

- GKT found that in generic dimensions the spectrum of the theory contained a complex scaling dimension for the lowest mass bilinear.

- In a holographic dual, this complex scaling dimension maps to a field with mass below the BF bound, and indicates that the large *N* saddle point is unstable. (Or more simply, it indicates that the large *N* fixed point doesn't exist.)

Our fermionic generalisation

- We consider perhaps the most naive *d* dimensional generalisation of KT model— a tensor model with melonic dominance based on three-index Dirac fermions in *d* dimensions.

$$S = \int d^dx \left(i\bar{\psi}_a{}^b{}_c \partial\!\!\!/ \psi^{abc} + \frac{1}{2} g_1 \bar{\psi}_{a_1}{}^{b_1}{}_{c_1} \psi^{a_1 b_2 c_2} \bar{\psi}_{a_2}{}^{b_1}{}_{c_2} \psi^{a_2 b_2 c_1} + \frac{1}{2} g_2 \bar{\psi}_{a_1}{}^{b_1}{}_{c_1} \psi^{a_2 b_2 c_1} \bar{\psi}_{a_2}{}^{b_1}{}_{c_2} \psi^{a_1 b_2 c_2} \right)$$

- One motivation: Unlike the bosonic model it doesn't have a negative direction, so one might hope that it is has no complex scaling dimensions…

Formal Large *N* solution

- In the large *N* limit, with $\lambda_i = g_i N^{3/2}$ held fixed, the theory is also dominated by melonic diagrams which can be explicitly summed for arbitrary *d.*

- When *d<2,* the theory is an infrared fixed point.

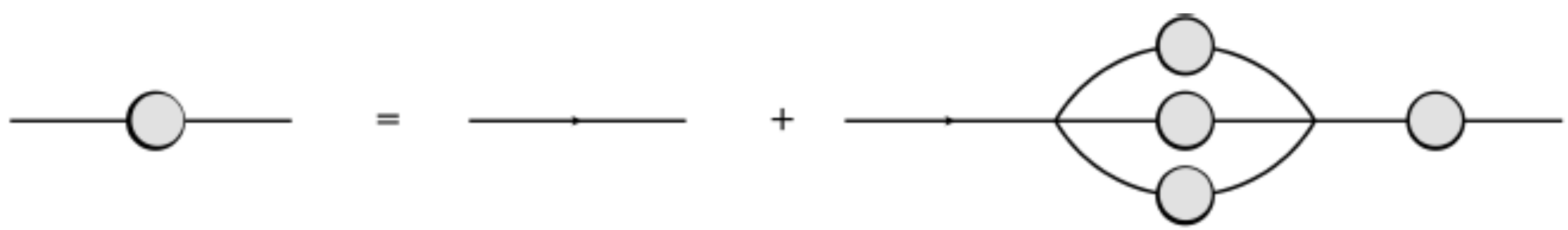
- When *d>2* the theory is not renormalizable at finite *N*, but may define a formal UV fixed point at large *N*.

    - Recall that the vectorial *Gross-Neveu* (GN) model apparently defines a fixed point in *d=3,* even at finite *N.* This can be studied using the GN model in 2+ε dimensions, or the Gross-Neveu-Yukawa (GNY) model in 4-ε dimensions. However, a tensorial GNY model is not known to us.

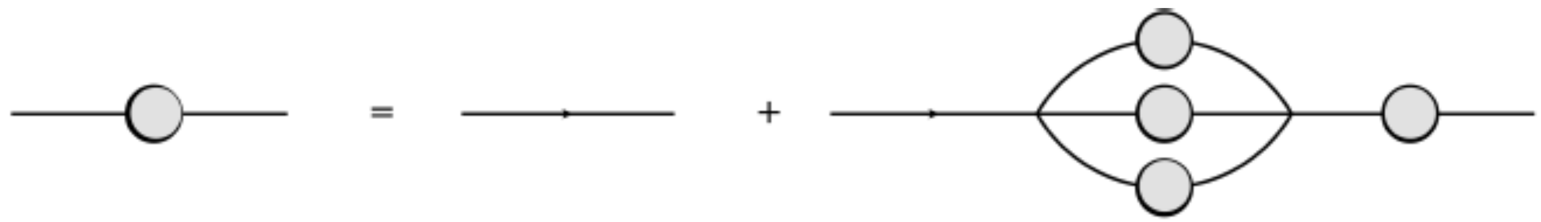
- The case of *d=2* and mass generation is studied in a related paper by Benedetti, Carrozza, Gurau, and Sfondrini.

Gap Equation for the Exact Propagator

$$\langle\psi(x)\bar{\psi}(y)\rangle = G(x,y)$$



Solving the Gap Equation



$$G(p) = G_0(p) - (\lambda_1^2 + \lambda_2^2) \int \frac{d^d q \, d^d r}{(2\pi)^{2d}} G_0(p) G(p-q+r) \mathrm{Tr}[G(q)G(r)] G(p)$$
$$+ 2\lambda_1 \lambda_2 \int \frac{d^d q \, d^d r}{(2\pi)^{2d}} G_0(p) G(p-q+r) G(r) G(q) G(p).$$

Strong Coupling:

$$G(p) = -\lambda^{-1/2} \frac{i\not{p}}{(p^2)^{d/4+1/2}} \left[\frac{d_\gamma}{(4\pi)^d} \frac{\Gamma(1/2 - d/4)}{\Gamma(3d/4 + 1/2)} \right]^{-1/4}.$$

$$\lambda^2 = (\lambda_1^2 + \lambda_2^2) - 2\lambda_1 \lambda_2 / d_\gamma.$$

- Translating to position space, we find the fermion has mass dimension *d/4* in the strong coupling fixed point.

Spectrum of Bilinear Operators

- We next calculate the spectrum of single-trace bilinears primary operators with spin-0.

- These are of the schematic form:

$$\bar{\psi}(\gamma^\mu \partial_\mu)^n \psi$$

  - If *n* is odd, this is a parity-even scalar (at least in d=3)

  - If *n* is even, this is a parity-odd scalar

Exact Three-Point Function in the Strong-Coupling limit

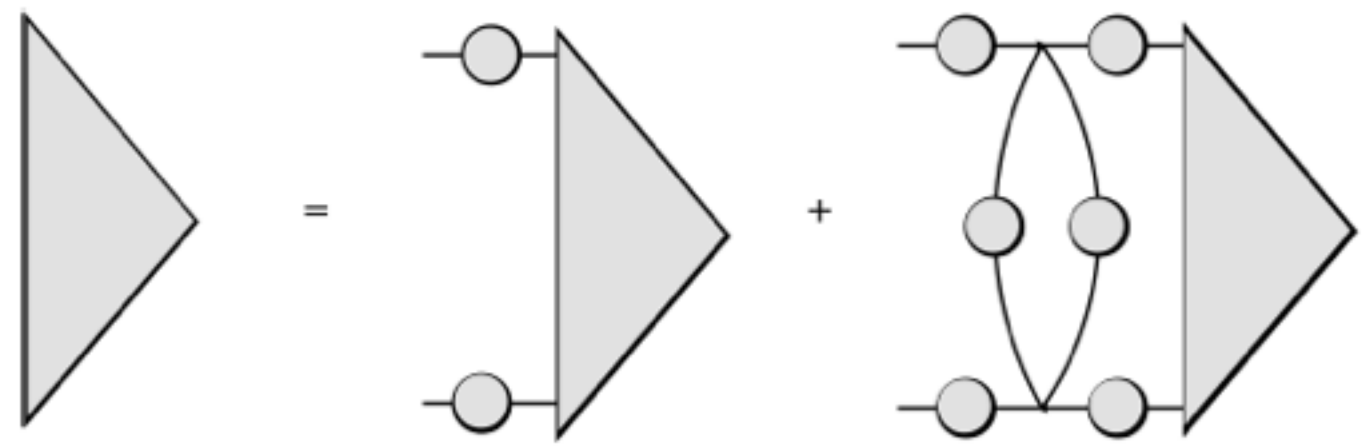
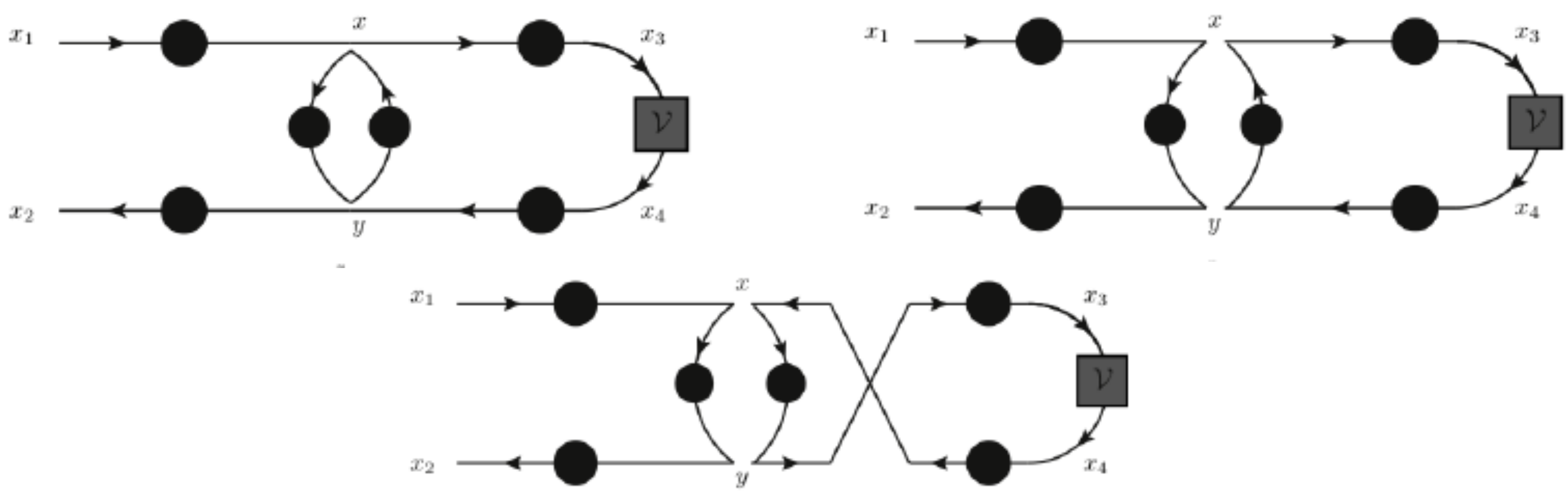


Figure 2: The Schwinger-Dyson Equation for the exact three point function $\langle\psi(x_1)O_s(x_3,\epsilon_3)\bar{\psi}(x_2)\rangle$.

- In the strong-coupling limit, we drop the first term on the RHS, and this becomes an eigenvalue equation: $K[v_{d,\tau}(x,y);x_1,x_2] = g(d,\tau)v_{d,\tau}(x_1,x_2)$

Integration Kernel



$$
\begin{aligned}
K[v(x,y);x_1,x_2] = \int d^{d}x d^{d}y \Big(& -(\lambda_1^2+\lambda_2^2)G(x_1,x)v(x,y)G(y,x_2)\mathrm{tr}\ [G(x,y)G(y,x)] \\
& -(\lambda_1^2+\lambda_2^2)G(x_1,x)G(x,y)G(y,x_2)\mathrm{tr}\ [G(y,x)v(x,y)] \\
& -(\lambda_1^2+\lambda_2^2)G(x_1,y)G(y,x)G(x,x_2)\mathrm{tr}\ [G(y,x)v(x,y)] \\
& +2\lambda_1\lambda_2 G(x_1,x)G(x,y)G(y,x)v(x,y)G(y,x_2) \\
& +2\lambda_1\lambda_2 G(x_1,x)v(x,y)G(y,x)G(x,y)G(y,x_2) \\
& +2\lambda_1\lambda_2 G(x_1,y)G(y,x)v(x,y)G(y,x)G(x,x_2)\Big)
\end{aligned}
$$

Determining the scaling dimensions for single-trace bilinear operators from eigenvalues of the integration kernel

- Their 3-point functions take the following form for parity-even and parity-odd scalars:

$$\langle (\bar{\xi}_1 \psi(x_1))(\bar{\psi}(x_2)\xi_2)\mathcal{O}_s(x_3)\rangle = \frac{aP_3 + b(S_3/P_3)}{|x_{31}|^{\tau}|x_{12}|^{2\Delta_\psi - 1 - \tau}|x_{23}|^{\tau}}$$

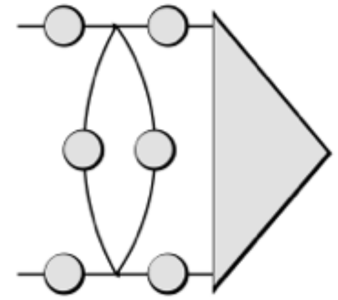
$$P_3 \sim \bar{\xi}_1 \not{x}_{12} \xi_2$$

$$(S_3/P_3) \sim \frac{\bar{\xi}_1 \not{x}_{13} \not{x}_{32} \xi_2}{|x_{12}||x_{31}||x_{23}|}$$

- We plug in these two forms into the eigenvalue equation to determine *g.* The value of twist for which *g=1,* determines the allowed scaling dimensions.

$$K[v_{d,\tau}(x,y); x_1, x_2] = g(d,\tau) v_{d,\tau}(x_1, x_2)$$

Determining the scaling dimensions for single-trace bilinear operators from eigenvalues of the integration kernel



$$K[v_{d,\tau}(x,y);x_1,x_2] = g(d,\tau)v_{d,\tau}(x_1,x_2)$$

We find

$$g_{\text{even}}(d,\tau) = -\frac{3\cos\left(\frac{\pi d}{4}\right)\Gamma\left(\frac{3d}{4}+\frac{1}{2}\right)\Gamma\left(\frac{d+2}{4}\right)\sec\left(\frac{1}{4}\pi(d-2\tau)\right)}{\Gamma\left(\frac{1}{4}(3d-2\tau+2)\right)\Gamma\left(\frac{1}{4}(d+2\tau+2)\right)}$$

$$g_{\text{odd}}(d,\tau) = -\frac{\Gamma(\frac{3d}{4}+\frac{1}{2})\Gamma(\frac{d}{4}-\frac{\tau}{2})\Gamma(\frac{\tau}{2}-\frac{d}{4})}{\Gamma(\frac{1}{2}-\frac{d}{4})\Gamma(\frac{d}{4}+\frac{\tau}{2})\Gamma(\frac{3d}{4}-\frac{\tau}{2})}.$$

Numerical Spectrum

Solving the equation

$$g_{\rm odd}(d,\tau) = 1, \tag{4.2}$$

for τ will give us the scaling dimensions $\tau_n^{(\rm odd)}$ of operators of the schematic form $\bar{\psi}\partial\!\!\!/^{2n}\psi$. We expect $\tau_n^{(\rm odd)} = 2n + 2\Delta_\psi + \delta_n = 2n + d/2 + 2\delta_n$ where $\delta_n \to 0$ as $n \to \infty$.

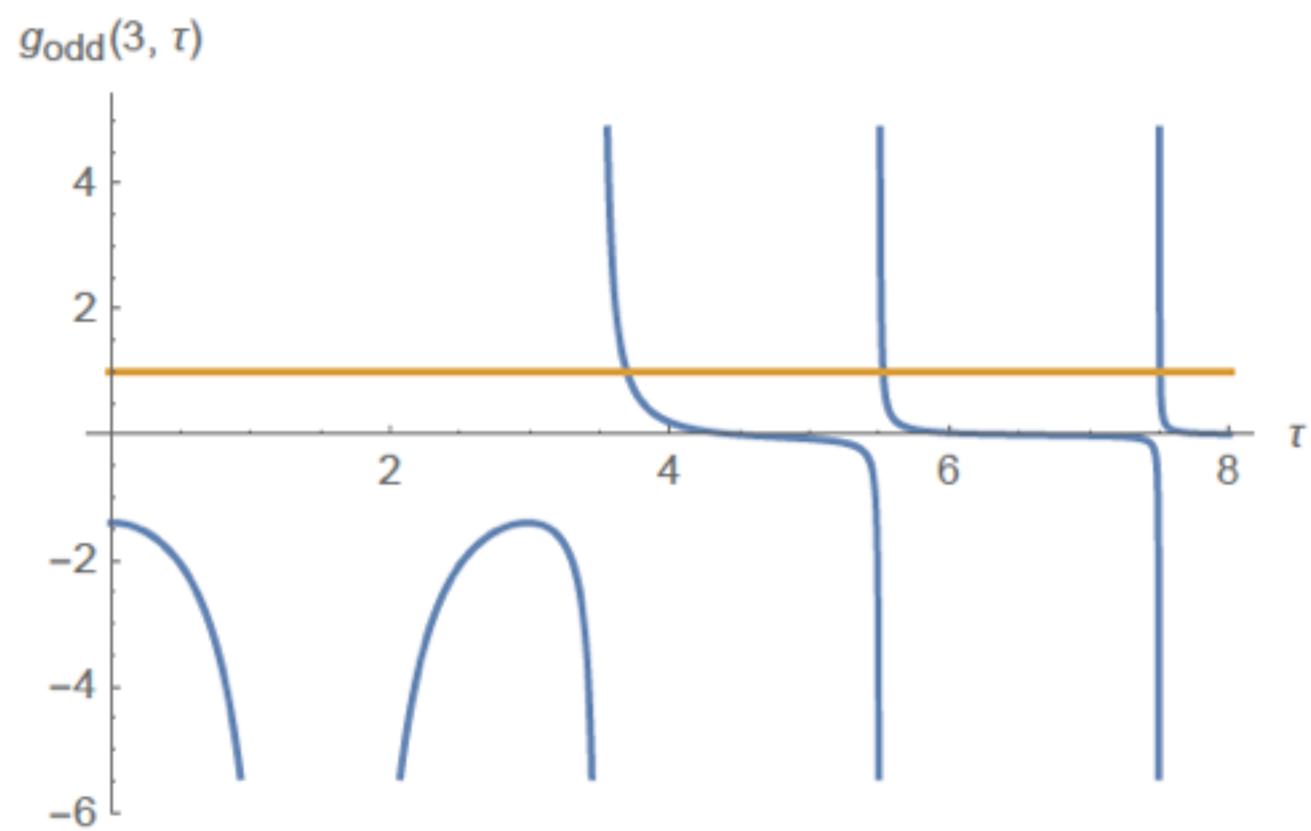


Figure 5: A plot $g_{\rm odd}(3,\tau)$ and 1 for $d = 3$.

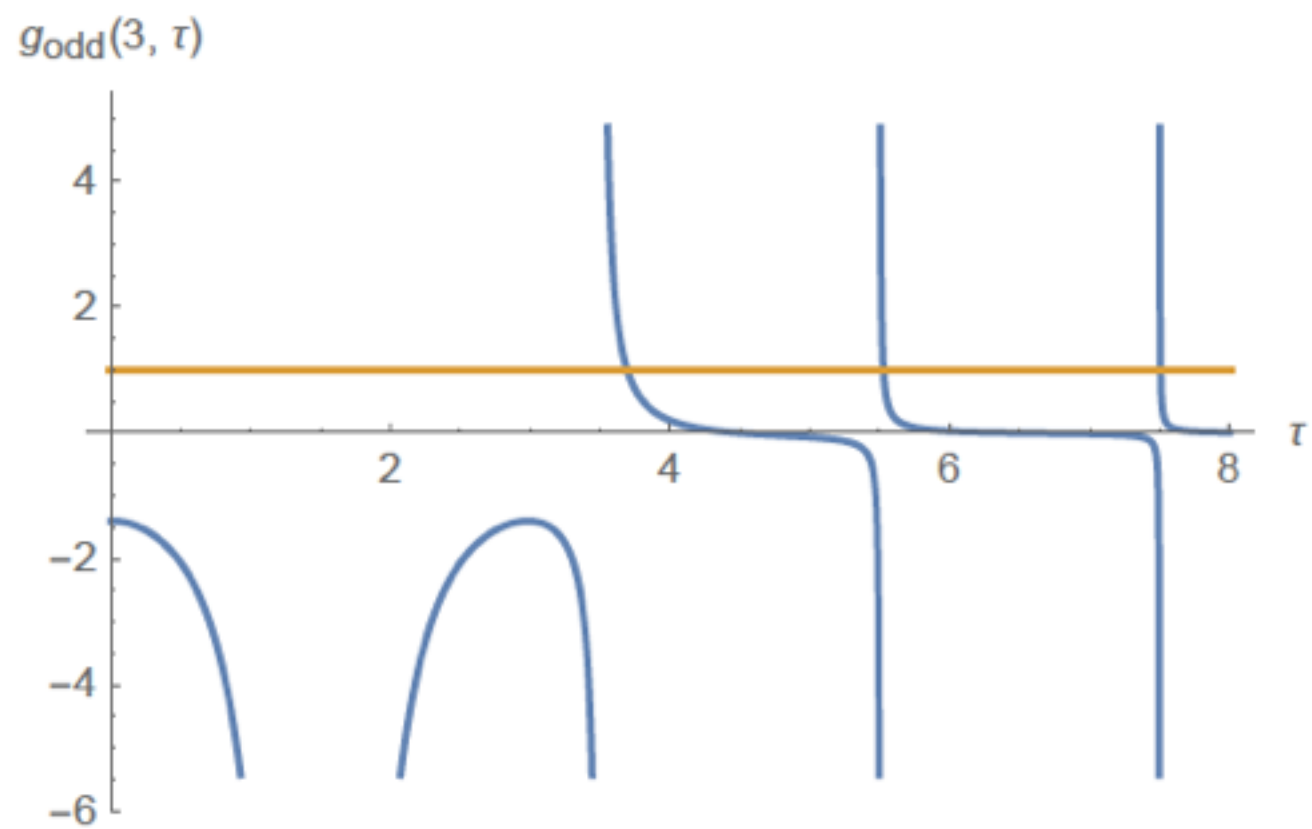
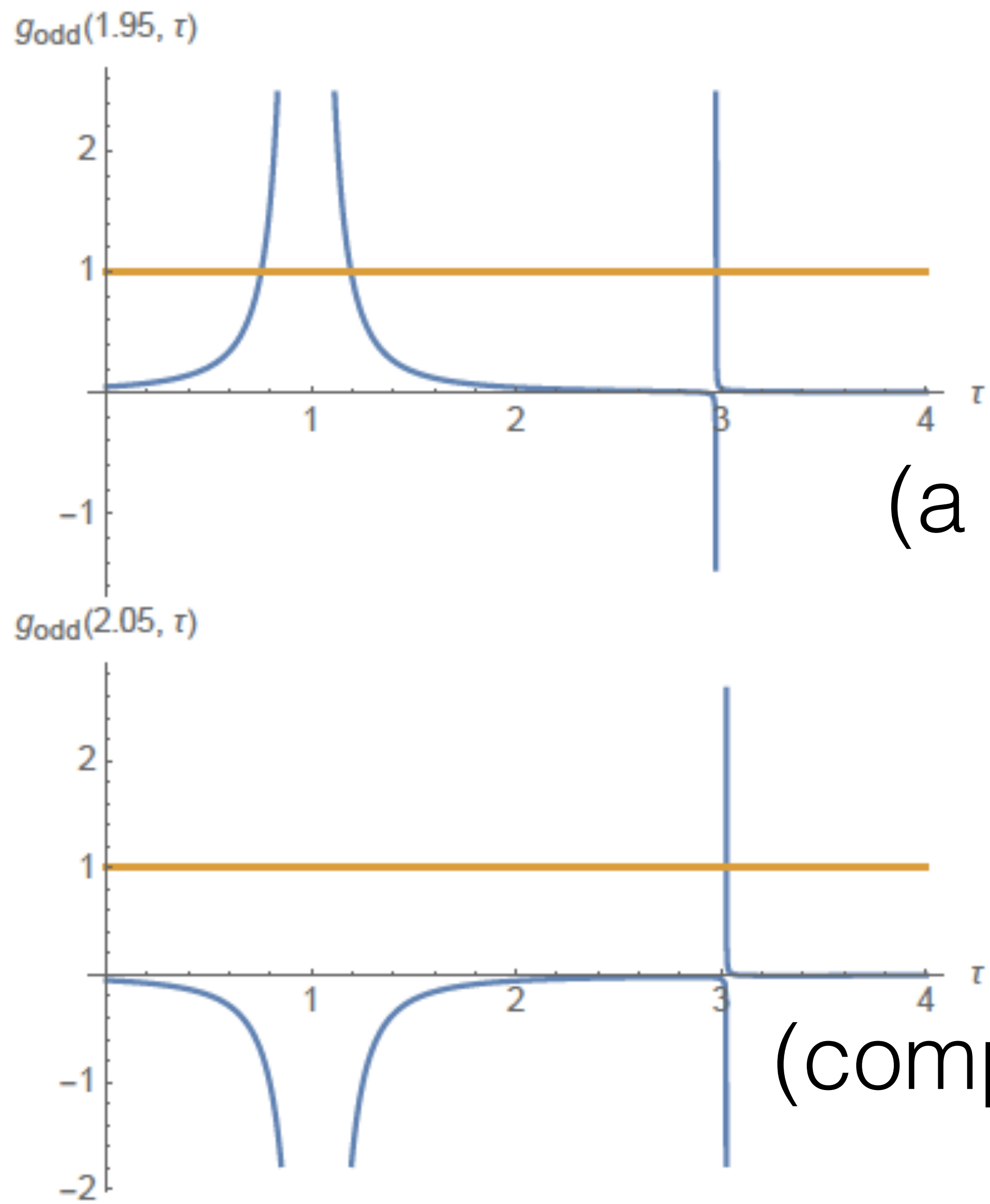


Figure 5: A plot $g_{odd}(3, \tau)$ and 1 for $d = 3$.

- We find the scaling dimensions: 3.69, 5.53, 7.51, 9.50, ….
- These approach 2n+1.5 as expected.
- A real eigenvalue corresponding to n=0 is not present in the spectrum. This instead corresponds to the complex eigenvalue 1.5 +1.7*i*

Complex Eigenvalue if d>2



d<2
(a real spectrum)

d>2
(complex mode exists)

Figure 6: A plot $g_{\rm odd}(\tau)$ and 1 for $d = 1.95$ (top) and $d = 2.05$ (below).

Complex Eigenvalue if $d>2$

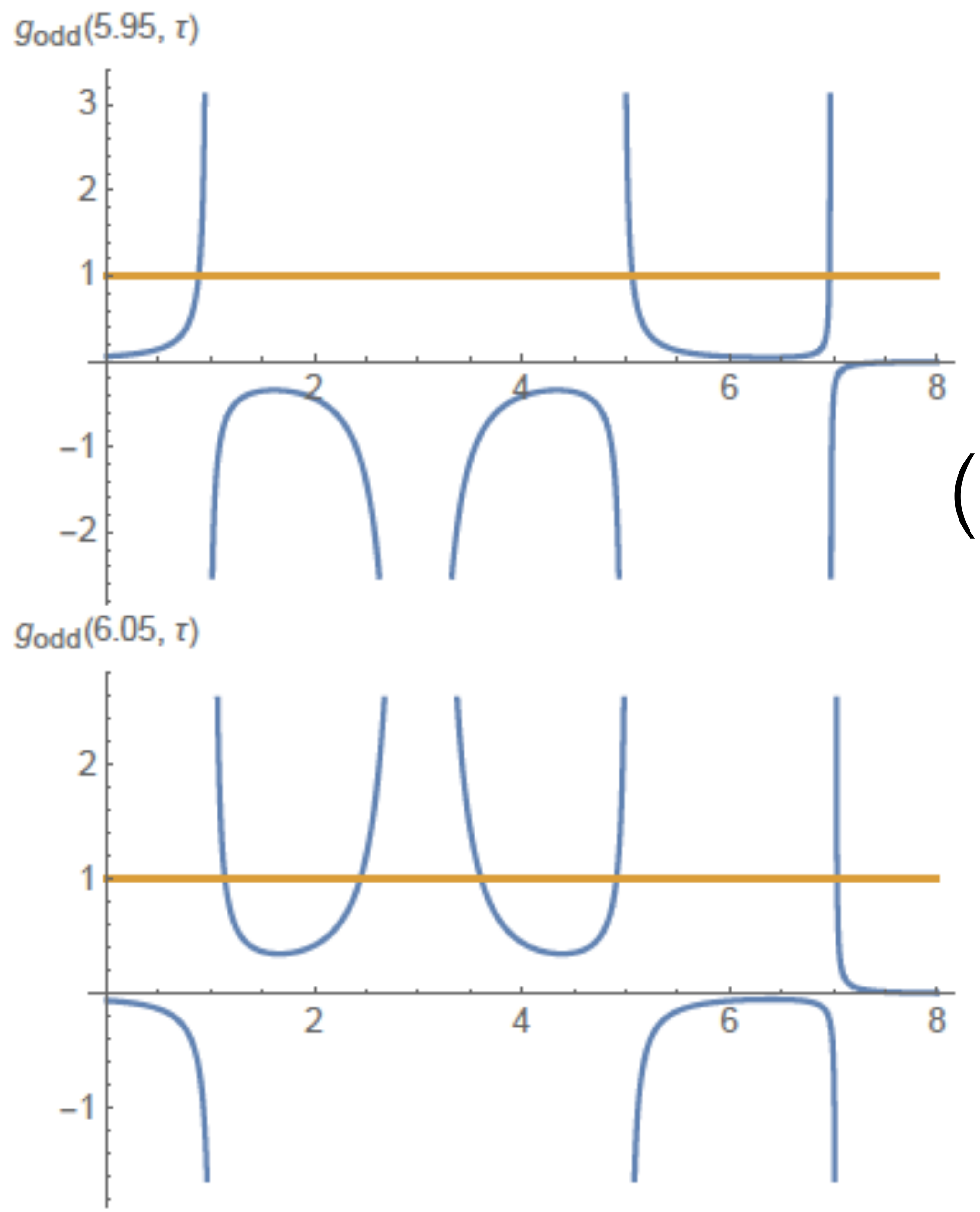
- In general we find an operator of complex scaling dimension exists in the spectrum for most values of $d>2$.

- However, for $d<2$, the bilinear spectrum is purely real.

    - Also, near $d=2+4n$, we find a window with a real spectrum.

    - In particular, we find a real spectrum for $6<d<6.14$.

Real Spectrum in d>6



d<6
(a complex mode)

d>6
real spectrum

Figure 7: A plot $g_{\text{odd}}(\tau)$ and 1 for $d = 5.95$ (top) and $d = 6.05$ (below). We see that some eigenvalues become real as one crosses $d = 6$.

Large N scaling dimensions in d=2-ε

- We can also solve the large N eigenvalue equation for the spectrum perturbatively in ε in d=2-ε dimensions.

Solving $g_{\text{odd}}(2-\epsilon,\tau)=1$, we find the parity-odd scalar spectrum in $d=2-\epsilon$ is:

$$\tau_{0,\pm}^{(\text{odd})}=1\pm\sqrt{\epsilon}-\frac{1}{2}\epsilon\pm\frac{3}{8}\epsilon^{3/2}\pm\left(\frac{\zeta(3)}{8}+\frac{9}{128}\right)\epsilon^{5/2}+O\left(\epsilon^{3}\right) \tag{5.1}$$

$$\tau_{1}^{(\text{odd})}=3-\frac{1}{2}\epsilon+\frac{1}{4}\epsilon^{2}+0\epsilon^{3}-\frac{7}{64}\epsilon^{4}+O\left(\epsilon^{5}\right) \tag{5.2}$$

$$\begin{aligned}\tau_{n}^{(\text{odd})}=&(2n+1)-\frac{1}{2}\epsilon+\frac{1}{4n^{2}}\epsilon^{2}+\frac{\left(4n^{2}H_{n-1}+(2-3n)n+1\right)}{16n^{4}}\epsilon^{3}\\&+\frac{\left(8n^{4}H_{n-1}^{2}-6n^{3}-2n^{2}+4((2-3n)n+2)n^{2}H_{n-1}+1\right)}{64n^{6}}\epsilon^{4}+O\left(\epsilon^{5}\right)\ \text{ for } n\geq 1.\end{aligned} \tag{5.3}$$

Large *N* scaling dimensions in *d=2-ε*

- The parity-even spectrum is:

$$\tau_0^{(\text{even})} = 2 + \epsilon - \frac{3\epsilon^2}{2} + \frac{3\epsilon^3}{2} + \left(-\frac{3\zeta(3)}{4} - \frac{21}{8}\right)\epsilon^4 + O\left(\epsilon^5\right) \tag{5.4}$$

$$\begin{aligned}\tau_n^{(\text{even})} &= 2(n+1) - \frac{1}{2}\epsilon + \frac{3}{4n(n+1)}\epsilon^2 + \frac{3\left(4(n+1)nH_{n-1} - 3n^2 + n + 5\right)}{16n^2(n+1)^2}\epsilon^3 \\ &+ \frac{3\left(8(n+1)^2n^2H_{n-1}^2 + 4(n(9-n(3n+2))+8)nH_{n-1} - 12n^3 - 24n^2 + 19\right)}{64n^3(n+1)^3}\epsilon^4 + O\left(\epsilon^5\right), \text{ for } n \geq 1.\end{aligned} \tag{5.5}$$

Large N scaling dimensions in d=6+ε

- We can also do this in d=6+ε:

$$\tau_{-1}^{(\text{odd})} = 1 + \frac{5}{2}\epsilon + \frac{107}{24}\epsilon^2 + \frac{3047}{192}\epsilon^3 + \left(\frac{15\zeta(3)}{8} + \frac{484679}{6912}\right)\epsilon^4 + O\left(\epsilon^5\right) \tag{5.6}$$

$$\tau_{0,\pm}^{(\text{odd})} = 3 \pm \sqrt{6}\sqrt{\epsilon} + \frac{1}{2}\epsilon \pm \frac{35}{16}\sqrt{\frac{3}{2}}\epsilon^{3/2} \pm \frac{(1536\sqrt{6}\zeta(3) + 4799\sqrt{6})}{2048}\epsilon^{5/2} + O\left(\epsilon^{7/2}\right) \tag{5.7}$$

$$\tau_{1}^{(\text{odd})} = 5 - \frac{3}{2}\epsilon - \frac{107}{24}\epsilon^2 - \frac{3047}{192}\epsilon^3 + \left(-\frac{15\zeta(3)}{8} - \frac{484679}{6912}\right)\epsilon^4 + O\left(\epsilon^5\right) \tag{5.8}$$

$$\tau_{2}^{(\text{odd})} = 7 + \frac{3}{4}\epsilon + \frac{43}{384}\epsilon^2 - \frac{301}{6144}\epsilon^3 + \left(\frac{63713}{3538944} - \frac{3\zeta(3)}{256}\right)\epsilon^4 + O\left(\epsilon^5\right) \tag{5.9}$$

$$\begin{aligned}\tau_{2+n}^{(\text{odd})} &= (2n+7) + \frac{1}{2}\epsilon - \frac{6\Gamma(n)}{(n+2)\Gamma(n+5)}\epsilon^2 \\ &+ \frac{6\left(\frac{1}{2}\left(\frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \frac{1}{n+4} + \frac{1}{n}\right) + H_{n-1} - \frac{6\Gamma(n)\Gamma(n+2)}{\Gamma(n+3)\Gamma(n+5)} - \frac{29}{16}\right)}{n(n+1)(n+2)^2(n+3)(n+4)}\epsilon^3 + O\left(\epsilon^4\right), \text{ for } n \geq 1.\end{aligned} \tag{5.10}$$

$$\tau_{-2}^{(\text{even})} = 2\epsilon + \frac{13\epsilon^2}{8} + \frac{67\epsilon^3}{24} + \left(\frac{3\zeta(3)}{4} + \frac{54401}{9216}\right)\epsilon^4 + O\left(\epsilon^5\right) \tag{5.11}$$

$$\tau_{-1}^{(\text{even})} = 2 - \frac{5\epsilon}{2} + \frac{17\epsilon^2}{16} - \frac{3635\epsilon^3}{384} + \left(\frac{50759}{2304} - \frac{105\zeta(3)}{16}\right)\epsilon^4 + O\left(\epsilon^5\right) \tag{5.12}$$

$$\tau_{0}^{(\text{even})} = 4 + \frac{7\epsilon}{2} - \frac{17\epsilon^2}{16} + \frac{3635\epsilon^3}{384} + \left(\frac{105\zeta(3)}{16} - \frac{50759}{2304}\right)\epsilon^4 + O\left(\epsilon^5\right) \tag{5.13}$$

$$\tau_{1}^{(\text{even})} = 6 - \epsilon - \frac{13\epsilon^2}{8} - \frac{67\epsilon^3}{24} + \left(-\frac{3\zeta(3)}{4} - \frac{54401}{9216}\right)\epsilon^4 + O\left(\epsilon^5\right) \tag{5.14}$$

$$\tau_{2}^{(\text{even})} = 8 + \frac{4\epsilon}{5} + \frac{197\epsilon^2}{2000} - \frac{32581\epsilon^3}{600000} + \left(\frac{8429}{281250} - \frac{3\zeta(3)}{250}\right)\epsilon^4 + O\left(\epsilon^5\right) \tag{5.15}$$

$$\tau_{2+n}^{(\text{even})} = 8 + 2n + \frac{\epsilon}{2} - \frac{18\epsilon^2\Gamma(n)}{\Gamma(n+6)} + \frac{9\epsilon^3\Gamma(n)\left(8H_{n+5}\Gamma(n+6) - 288\Gamma(n) + \Gamma(n+6)(8H_{n-1} - 29)\right)}{8\Gamma(n+6)^2} + O\left(\epsilon^4\right) \tag{5.16}$$

Towards a finite-*N* *epsilon*-expansion in $d=2-\varepsilon$

- These large *N* results suggest that the theory is weakly interacting in $d=2-\varepsilon$, *so* we can develop an epsilon-expansion to study the 1-dimensional Klebanov Tarnopolsky tensor model at finite *N*, (analogous to the $4-\varepsilon$ expansion for Wilson-Fisher fixed point, or GNY model).

- This requires carefully renormalizing the theory in $2-\varepsilon$ dimensions, and solving for fixed points of the beta function.

- There may be subtleties in interpreting the results because the 1d-theory is not strictly conformal…

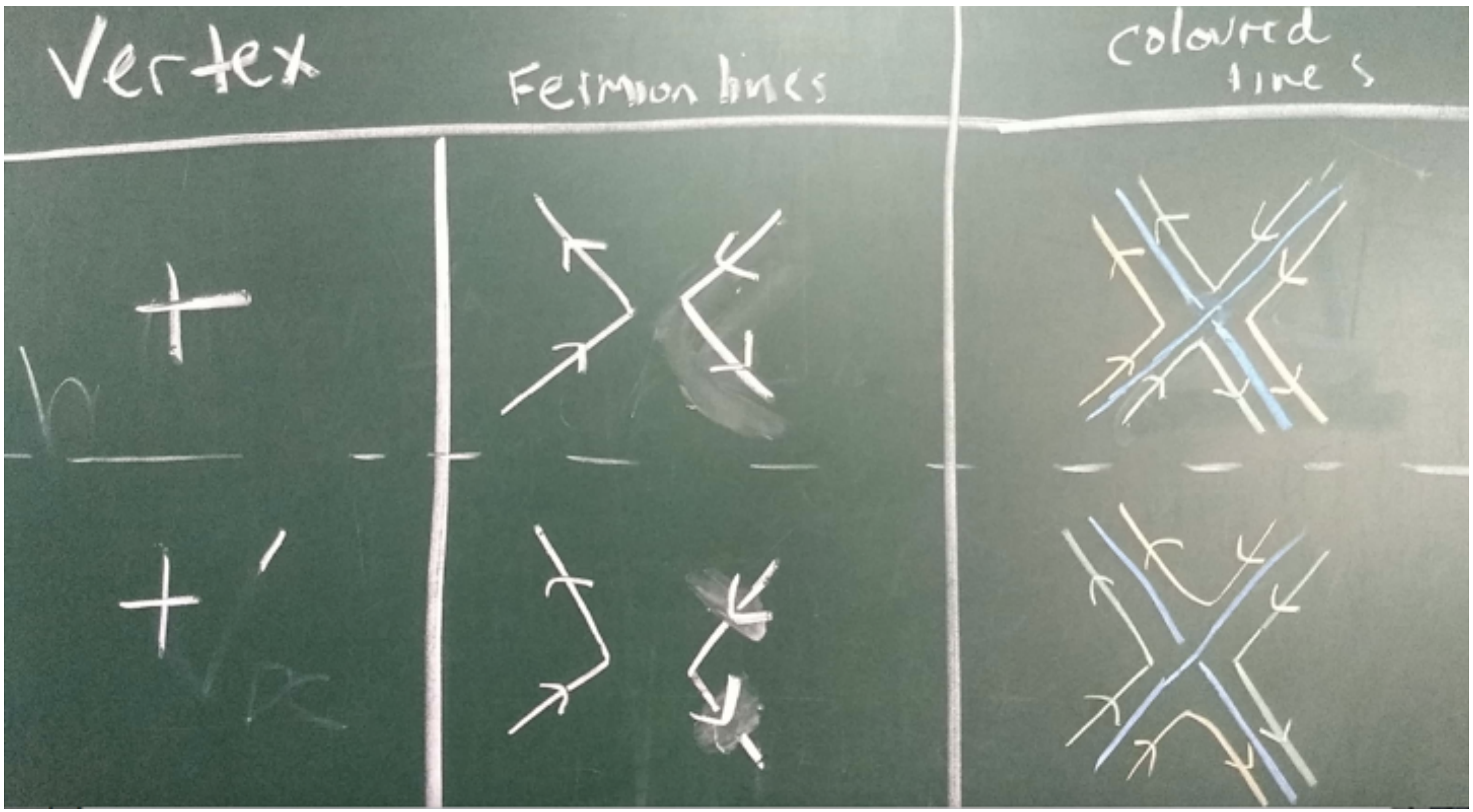
The most general renormalizable action:

$$S_{\rm int} = \int d^{2-\epsilon}x \ \mu^{\epsilon} Z_{\psi}^2 \mathcal{L} + \mathcal{L}',$$

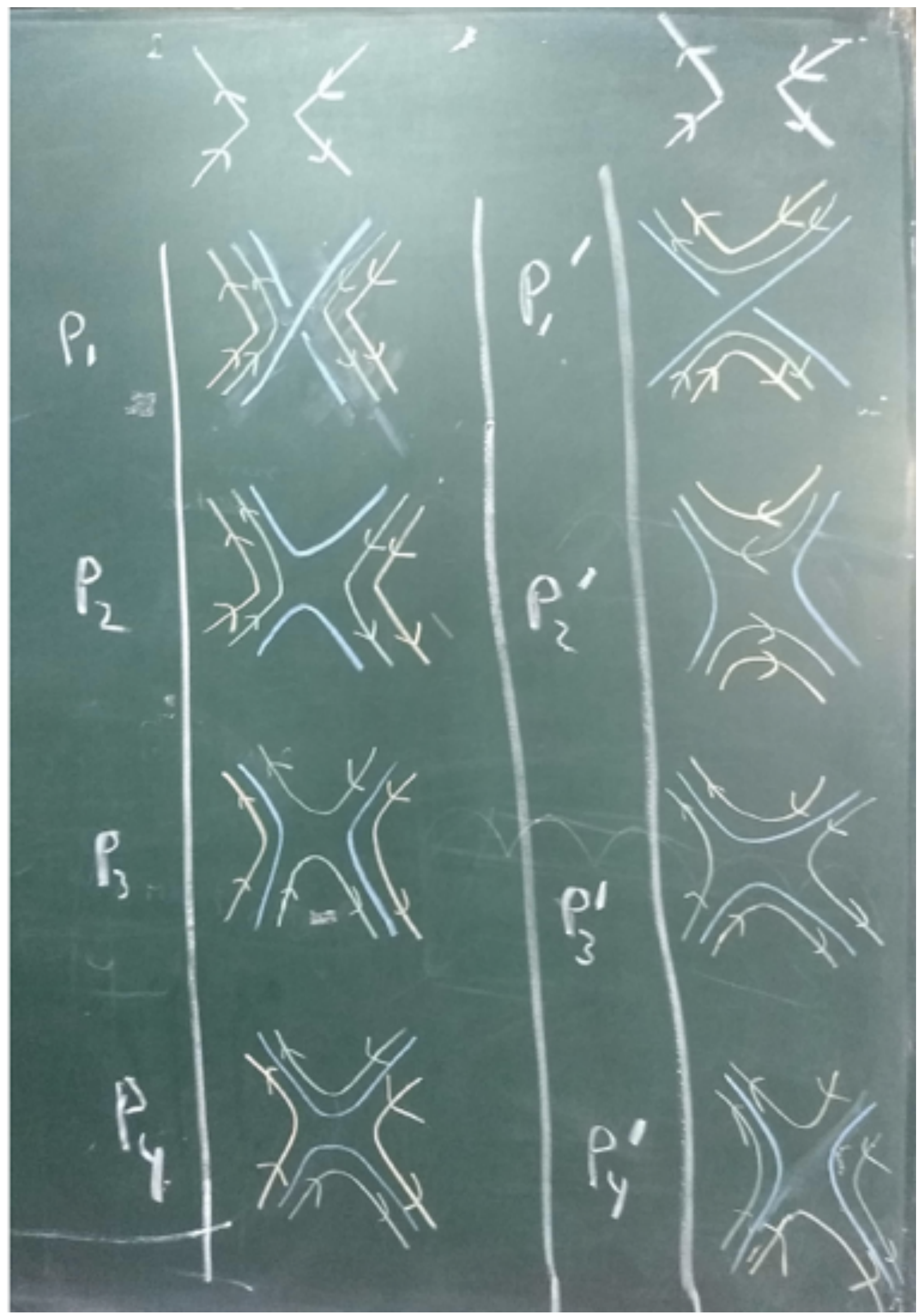
- To renormalize the tensor model, we need three different types of 4-fermion interactions:
    - tetrahedron
    - pillow-interactions
    - vector-interactions
- These combined with different spinor index contractions generate many interaction terms.

$$\begin{aligned}\mathcal{L} =& Z_t t \bar{\psi}_{a_1}{}^{b_1}{}_{c_1} \psi^{a_1 b_2 c_2} \bar{\psi}_{a_2}{}^{b_1}{}_{c_2} \psi^{a_2 b_2 c_1} + \\ & Z_{p_1} p_1 \bar{\psi}_{a_1}{}^{b_1}{}_{c_1} \psi^{a_1 b_2 c_1} \bar{\psi}_{a_2}{}^{b_1}{}_{c_2} \psi^{a_2 b_2 c_2} + \\ & Z_{p_2} p_2 \bar{\psi}_{a_1}{}^{b_1}{}_{c_1} \psi^{a_1 b_2 c_1} \bar{\psi}_{a_2}{}^{b_2}{}_{c_2} \psi^{a_2 b_1 c_2} + \\ & Z_{p_3} p_3 \bar{\psi}_{a_1}{}^{b_1}{}_{c_1} \psi^{a_1 b_1 c_2} \bar{\psi}_{a_2}{}^{b_2}{}_{c_2} \psi^{a_2 b_2 c_1} + \\ & Z_{p_4} p_4 \bar{\psi}_{a_1}{}^{b_1}{}_{c_1} \psi^{a_1 b_2 c_2} \bar{\psi}_{a_2}{}^{b_2}{}_{c_2} \psi^{a_2 b_1 c_1} + \\ & Z_v v \bar{\psi}_{a_1}{}^{b_1}{}_{c_1} \psi^{a_1 b_1 c_1} \bar{\psi}_{a_2}{}^{b_2}{}_{c_2} \psi^{a_2 b_2 c_2},\end{aligned}$$

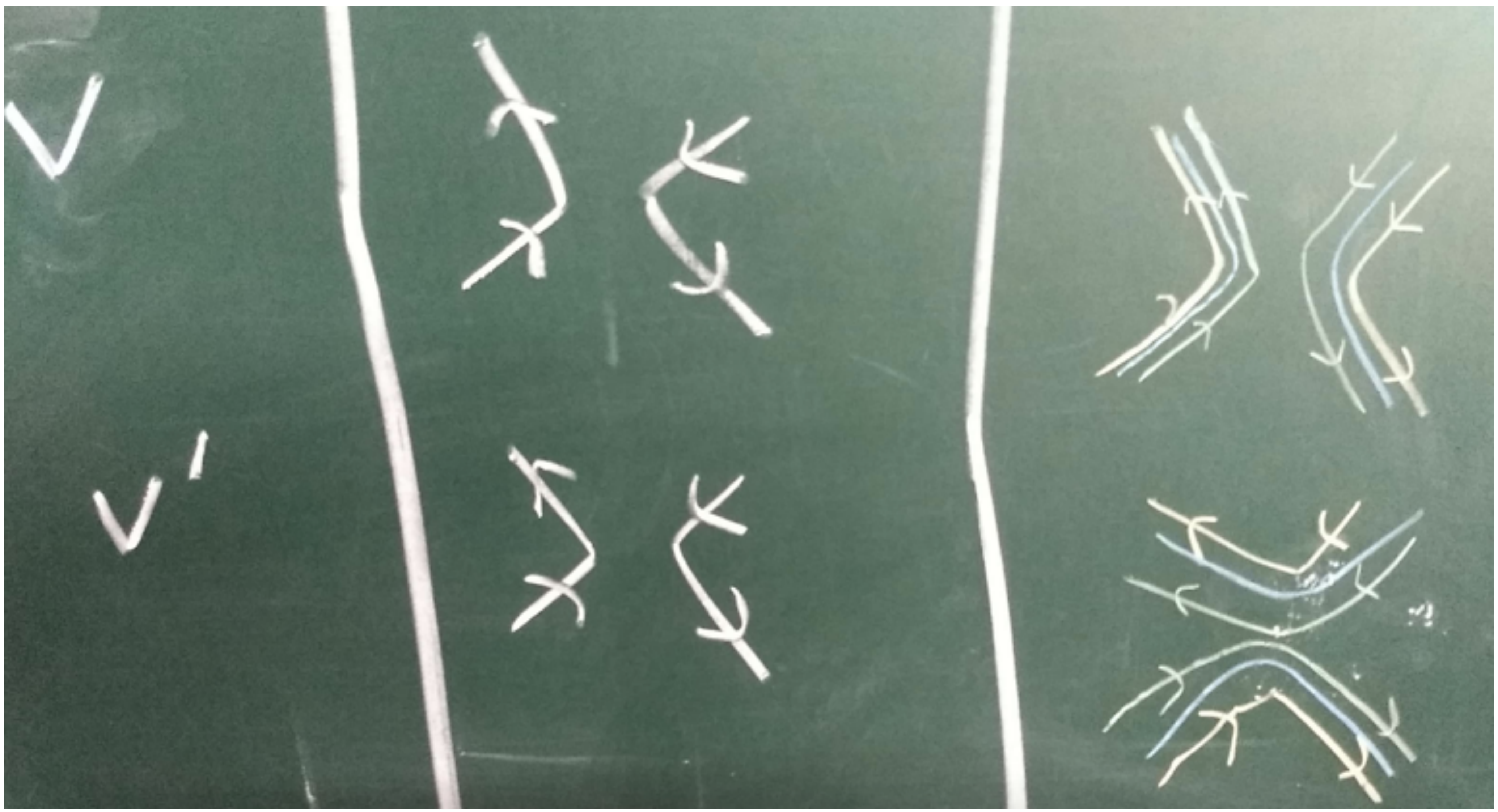
Tetrahedron Interactions



Pillow Interactions



Vector Interactions



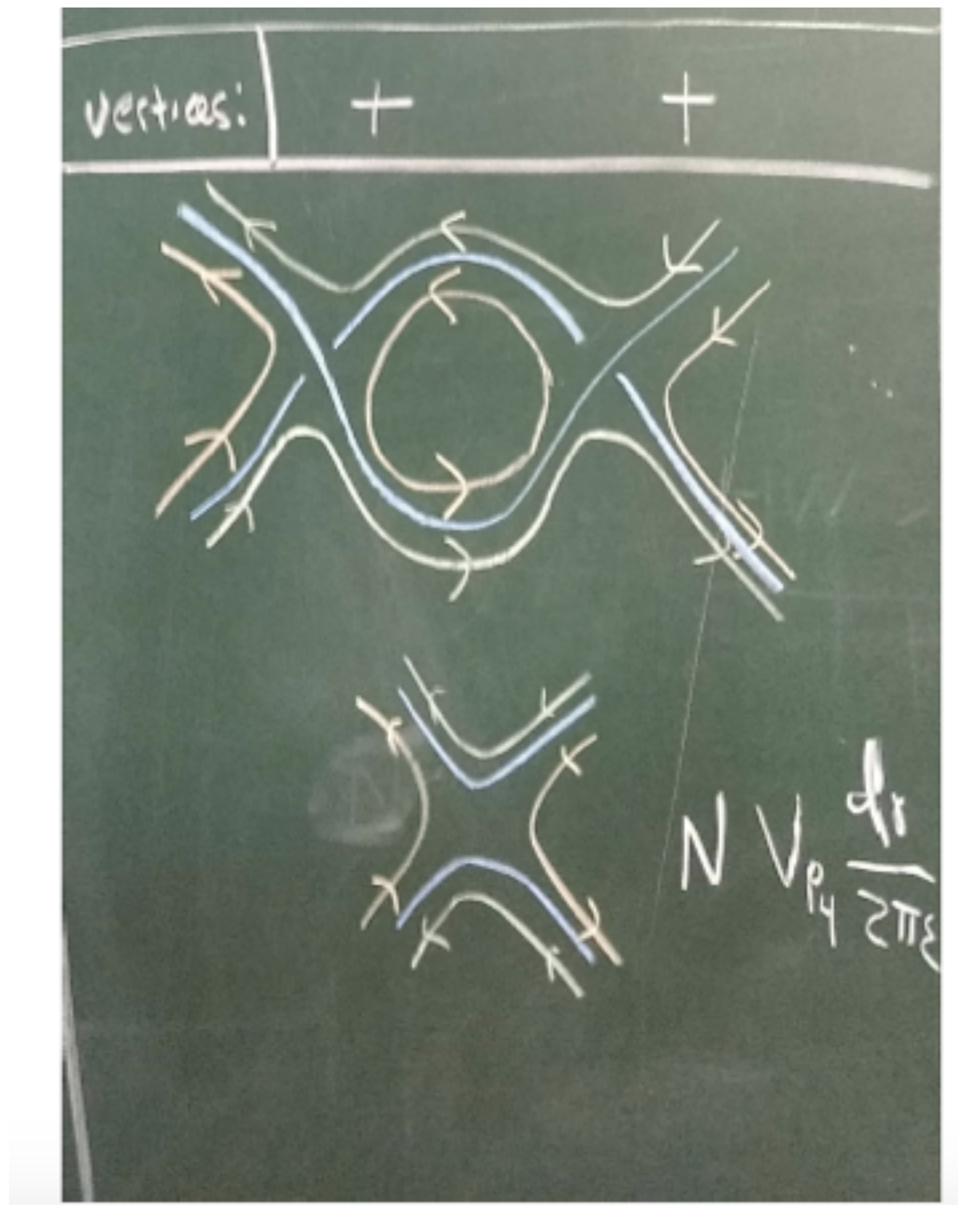
Hierarchy of Interactions:

- Vector interactions only generate vector interactions.

- Pillow interactions can generate vector interactions or other pillow interactions (at large *N*)

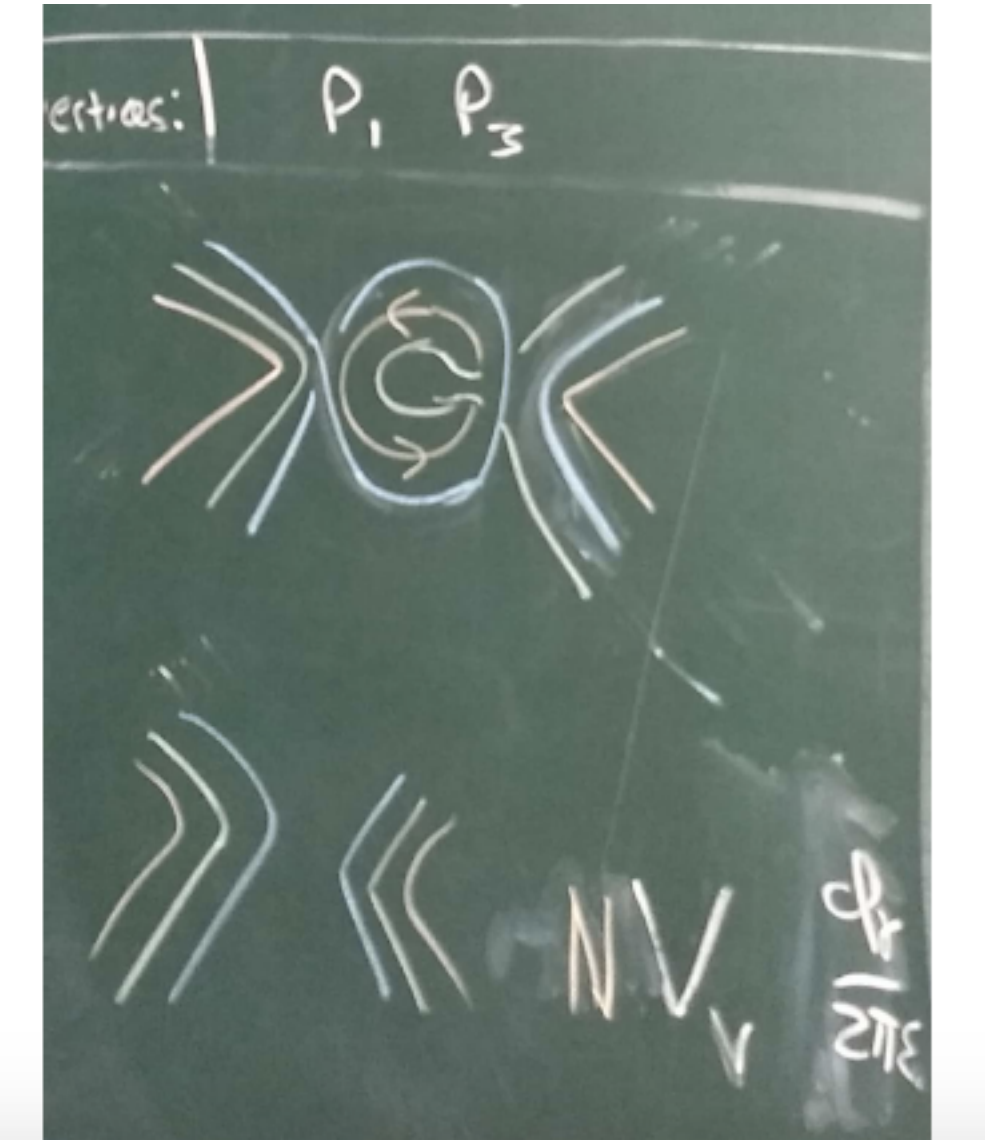
- Tetrahedron interactions can generate all three interactions.

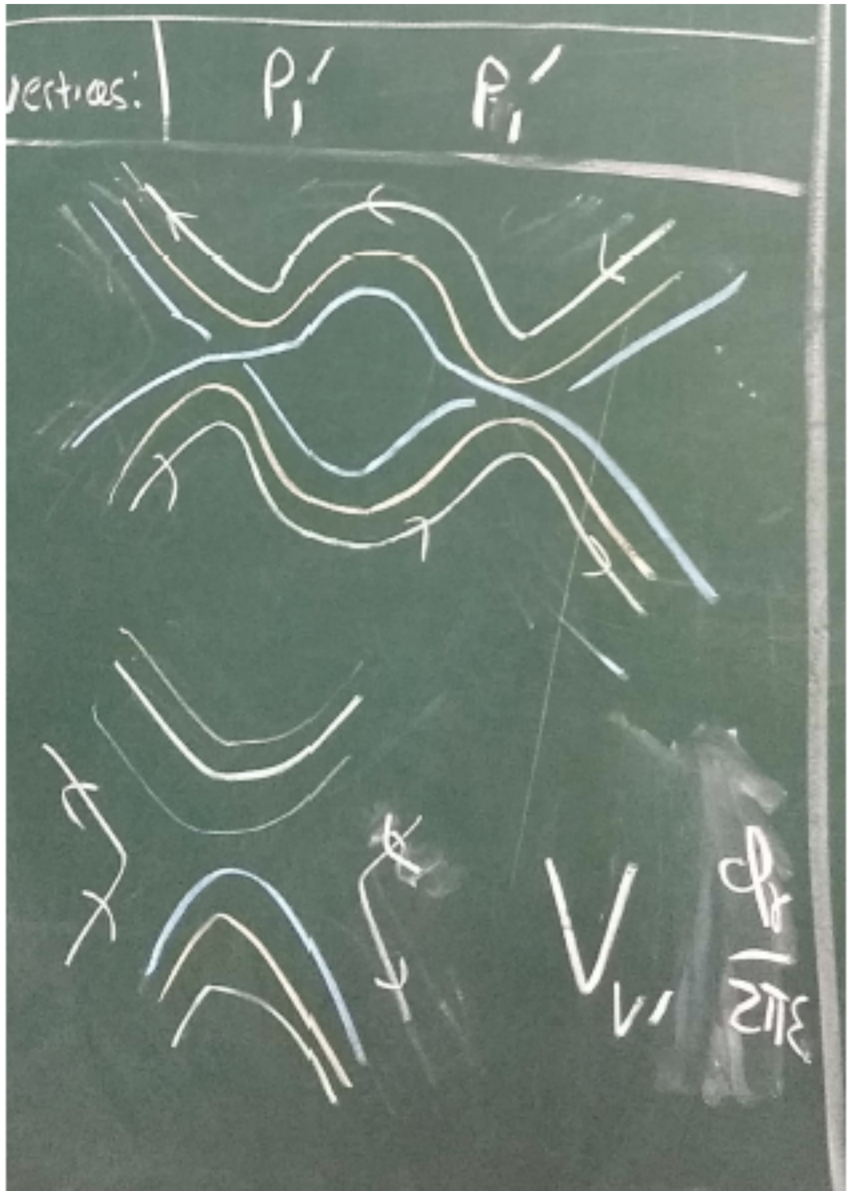
Pillow interaction from two tetrahedron vertices



Pillow interactions from two pillow vertices

Vector interactions from two pillow vertices





Beta functions for fermionic tensor model:

We calculate the beta functions as usual:

Let us define $G_t \equiv \log(Z_t)$, and $\beta_t \equiv \frac{dt}{d\log\mu}$ etc.

$$\left(1 + t\frac{\partial}{\partial t}G_t\right)\beta_t = -\epsilon t$$

$$p_i\beta_t\frac{\partial}{\partial t}G_{p_i} + \sum_j p_i\beta_{p_j}\frac{\partial}{\partial p_j}G_{p_i} + \beta_{p_i} = -\epsilon p_i \text{ (no sum on } i\text{)}$$

$$v\beta_t\frac{\partial}{\partial t}G_v + \sum_j v\beta_{p_j}\frac{\partial}{\partial p_j}G_v + (1 + v\frac{\partial}{\partial_v}G_v)\beta_v = -\epsilon v.$$

We expect that we will find a purely real fixed point for d<2. In the large *N* limit, $t \sim \varepsilon^{1/2}$ so we need to go to two loops.

Physical Expectations

- Unfortunately the calculation is still in progress…
- Once complete, we hope it will enable us to study the 1-d tensor model at finite N, and understand the relation between the various large N limits.
- To see the sort of questions we hope to address, we can look at similar computations in theory of bosonic tensor models in d=4-ε

Bosonic Tensor Model in 4-ε dimensions

- Giombi, Klebanov and Tarnopolsky studied melonic phi^4 theory in 4-ε dimensions.

$$S = \int d^d x \Big(\frac{1}{2}\partial_\mu \phi^{abc} \partial^\mu \phi^{abc} + \frac{1}{4} g \phi^{a_1 b_1 c_1} \phi^{a_1 b_2 c_2} \phi^{a_2 b_1 c_2} \phi^{a_2 b_2 c_1} \Big)$$

- In the large *N* limit, the lowest scalar bilinear has a complex dimension for d<4.

- They can study the theory at finite *N* using beta functions already calculated by Mack and Osborne in the 90s.

- We can ask the similar questions in this case…

Beta functions for tensor and pillow vertices in bosonic tensor models in d=4-ε

$$\beta_t = -\epsilon g_1 + \frac{4}{3(4\pi)^2}\Big(3g_1g_2(N+1) + 18g_1g_3 + 2g_2^2\Big)$$
$$+ \frac{2}{9(4\pi)^4}\Big(9(N^3-15N-10)g_1^3 - 36g_1^2\big((N^2+4N+13)g_2 + 15Ng_3\big)$$
$$-3g_1\big((N^3+15N^2+93N+101)g_2^2 + 12(5N^2+17N+17)g_2g_3 + 6(5N^3+82)g_3^2\big)$$
$$-4g_2^2\big((2N^2+13N+24)g_2 + 72g_3\big)\Big)\,, \tag{3.6}$$

$$\beta_p = -\epsilon g_2 + \frac{2}{3(4\pi)^2}\Big(9g_1^2(N+2) + 12g_2g_1(N+2) + g_2^2(N^2+5N+12) + 36g_2g_3\Big)$$
$$-\frac{2}{9(4\pi)^4}\Big(108(N^2+N+4)g_1^3 + 9g_1^2\big((N^3+12N^2+99N+98)g_2 + 72(N+2)g_3\big)$$
$$+36g_1g_2\big((4N^2+18N+29)g_2 + 3(13N+16)g_3\big) + g_2\big((5N^3+45N^2+243N+343)g_2^2$$
$$+36(7N^2+15N+29)g_2g_3 + 18(5N^3+82)g_3^2\big)\Big)\,, \tag{3.7}$$

from Giombi, Klebanov, Tarnopolsky

Beta function for bosonic tensor models in 4-ε dimensions in the large *N limit*

$$g_1 = \frac{(4\pi)^2\tilde{g}_1}{N^{3/2}}, \quad g_2 = \frac{(4\pi)^2\tilde{g}_2}{N^2}, \quad g_3 = \frac{(4\pi)^2\tilde{g}_3}{N^3},$$

$$\tilde{\beta}_t = -\epsilon\tilde{g}_1 + 2\tilde{g}_1^3,$$

$$\tilde{\beta}_p = -\epsilon\tilde{g}_2 + \left(6\tilde{g}_1^2 + \frac{2}{3}\tilde{g}_2^2\right) - 2\tilde{g}_1^2\tilde{g}_2,$$

$$\tilde{\beta}_{ds} = -\epsilon\tilde{g}_3 + \left(\frac{4}{3}\tilde{g}_2^2 + 4\tilde{g}_2\tilde{g}_3 + 2\tilde{g}_3^2\right) - 2\tilde{g}_1^2(4\tilde{g}_2 + 5\tilde{g}_3).$$

$$\tilde{g}_1^* = (\epsilon/2)^{1/2}, \quad \tilde{g}_2^* = \pm 3i(\epsilon/2)^{1/2}, \quad \tilde{g}_3^* = \mp i(3 \pm \sqrt{3})(\epsilon/2)^{1/2}.$$

from Giombi, Klebanov, Tarnopolsky

- Suppose we study an *O(N)*×*O(D)*×*O(M)* tensor model with the ratios *M/N, D/N* held fixed. For any finite value of these ratios, we find a complex fixed point.

- We expect that, for the fermionic tensor model in 2-ε dimensions, we will find a real fixed point instead. Calculations in progress.

Bifundamental Critical ϕ^4 Theory

- Is the absence of a real fixed point a special feature of tensor models?
- What about a matrix-valued theory: ϕ^4 theory with scalars in the bifundamental representation *O(N)*×*O(M)*?
- When *M=1,* the theory is a well defined large *N* fixed point in 3 dimensions.

$$S = \int d^d x \Big(\frac{1}{2}\partial_\mu \phi^{ab} \partial^\mu \phi^{ab} + \frac{1}{4} g_1 O_{st}(x) + \frac{1}{4} g_2 O_{dt}(x) \Big) ,$$

$$O_{st}(x) = \phi^{ab}\phi^{cb}\phi^{cd}\phi^{ad} = \mathrm{Tr}\phi\phi^T\phi\phi^T , \qquad O_{dt}(x) = \phi^{ab}\phi^{ab}\phi^{cd}\phi^{cd} = \mathrm{Tr}\phi\phi^T \mathrm{Tr}\phi\phi^T .$$

$$g_1 = \frac{(4\pi)^2 \tilde{g}_1}{N}, \quad g_2 = \frac{(4\pi)^2 \tilde{g}_2}{N^2} ,$$

Bi-fundamental critical ϕ^4 Theory

- When *M*=*N*, one can check that the theory has a complex fixed point in 4-ε dimensions. (Osborn and Stergiou studied this some time ago)

$$S = \int d^d x \Big(\frac{1}{2} \partial_\mu \phi^{ab} \partial^\mu \phi^{ab} + \frac{1}{4} g_1 O_{st}(x) + \frac{1}{4} g_2 O_{dt}(x) \Big) ,$$

$$O_{st}(x) = \phi^{ab} \phi^{cb} \phi^{cd} \phi^{ad} = \mathrm{Tr} \phi \phi^T \phi \phi^T ,$$

$$O_{dt}(x) = \phi^{ab} \phi^{ab} \phi^{cd} \phi^{cd} = \mathrm{Tr} \phi \phi^T \mathrm{Tr} \phi \phi^T .$$

$$g_1 = \frac{(4\pi)^2 \tilde{g}_1}{N}, \quad g_2 = \frac{(4\pi)^2 \tilde{g}_2}{N^2} ,$$

$$\tilde{g}_1^* = \frac{\epsilon}{4} + \frac{3\epsilon^2}{32}, \quad \tilde{g}_2^* = -\frac{1}{4} \left(1 \pm i\sqrt{2} \right) \epsilon - \frac{1}{32} \left(1 \mp 2i\sqrt{2} \right) \epsilon^2$$

Bifundamental critical-ϕ^4 theory Beta function in large *N* limit when *M/N* <1

$$S = \int d^d x \Big(\frac{1}{2} \partial_\mu \phi^{ab} \partial^\mu \phi^{ab} + \frac{1}{4} g_1 O_{st}(x) + \frac{1}{4} g_2 O_{dt}(x) \Big) ,$$

$$\beta_{\tilde{g}_1} = -(3\alpha + 3)\tilde{g}_1^3 + (2\alpha + 1)\tilde{g}_1^2 - \tilde{g}_1 \epsilon$$

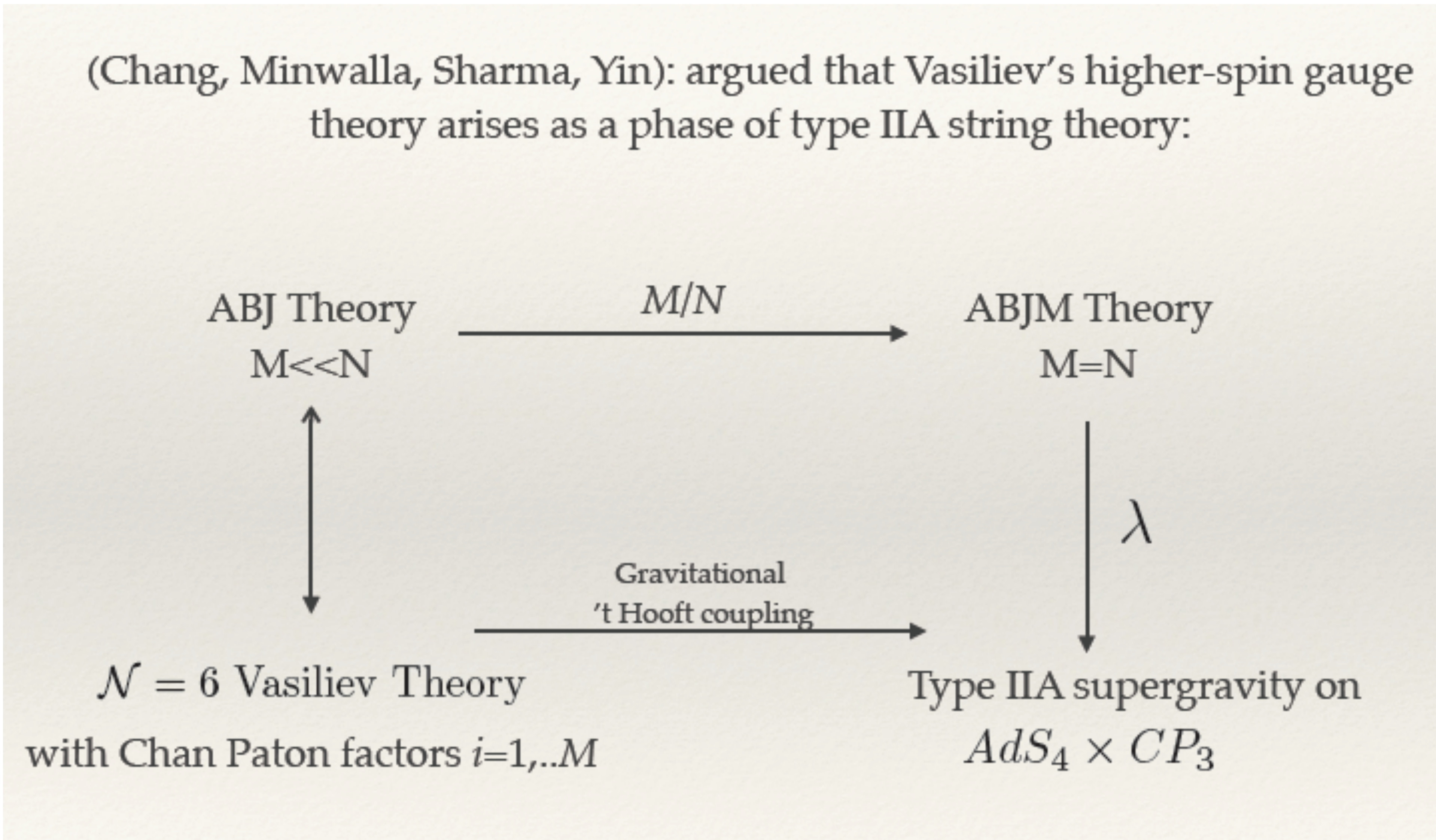
$$\beta_{g_2} = -12(\alpha + 1) g_1^3 - 10\alpha g_1^2 g_2 + 6 g_1^2 + 2(2\alpha + 2) g_1 g_2 + 2\alpha g_2^2 - g_2 \epsilon$$

$$\alpha \equiv M/N$$

- The fixed point is real only when

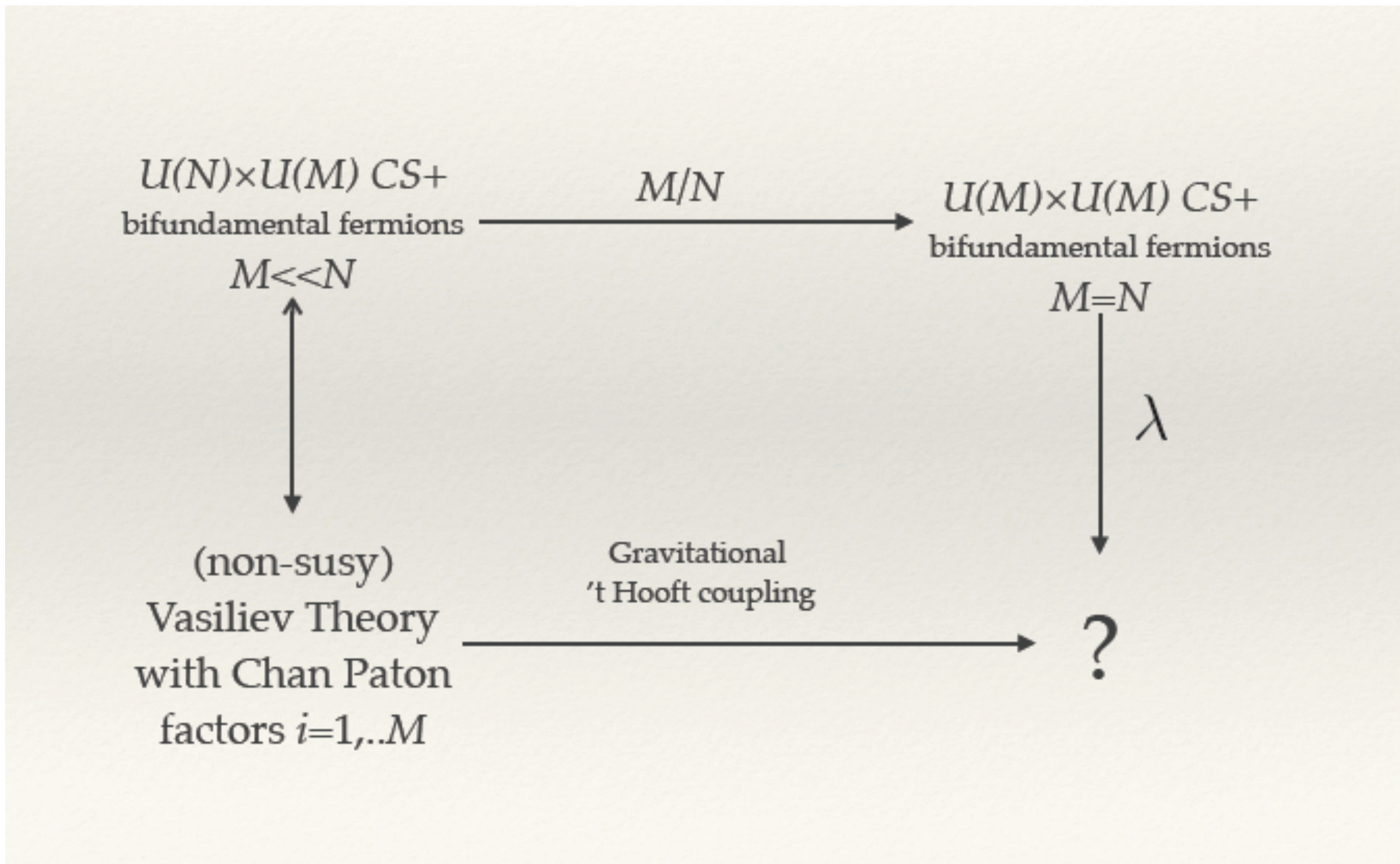
$$M/N < 5 - 2\sqrt{6} \approx .1$$

Aside: Comments on *M/N* expansion and non-supersymmetric AdS_4/CFT_3



- What role does supersymmetry play in this diagram?

Aside: Comments on *M/N* expansion and non-supersymmetric AdS_4/CFT_3



- Giombi Minwalla, SP, Trivedi, Wadia, Yin; Gurucharan, SP;

Aside: Comments on *M/N* expansion and non-supersymmetric AdS_4/CFT_3

- While it is difficult to study the above conjecture: we can instead consider bi-fundamental critical -ϕ^4 theory in d=3.

- The calculations in epsilon expansion suggest somehow the conformal ϕ^4 theory stops existing once *M/N*=0.1, at a finite value of gravitational 't Hooft coupling in dual HS theory.

- We expect similar behaviour in d=3 GN/GNY model, though calculations are still in progress.

Generalization to $O(M)\times O(N)\times O(D)$

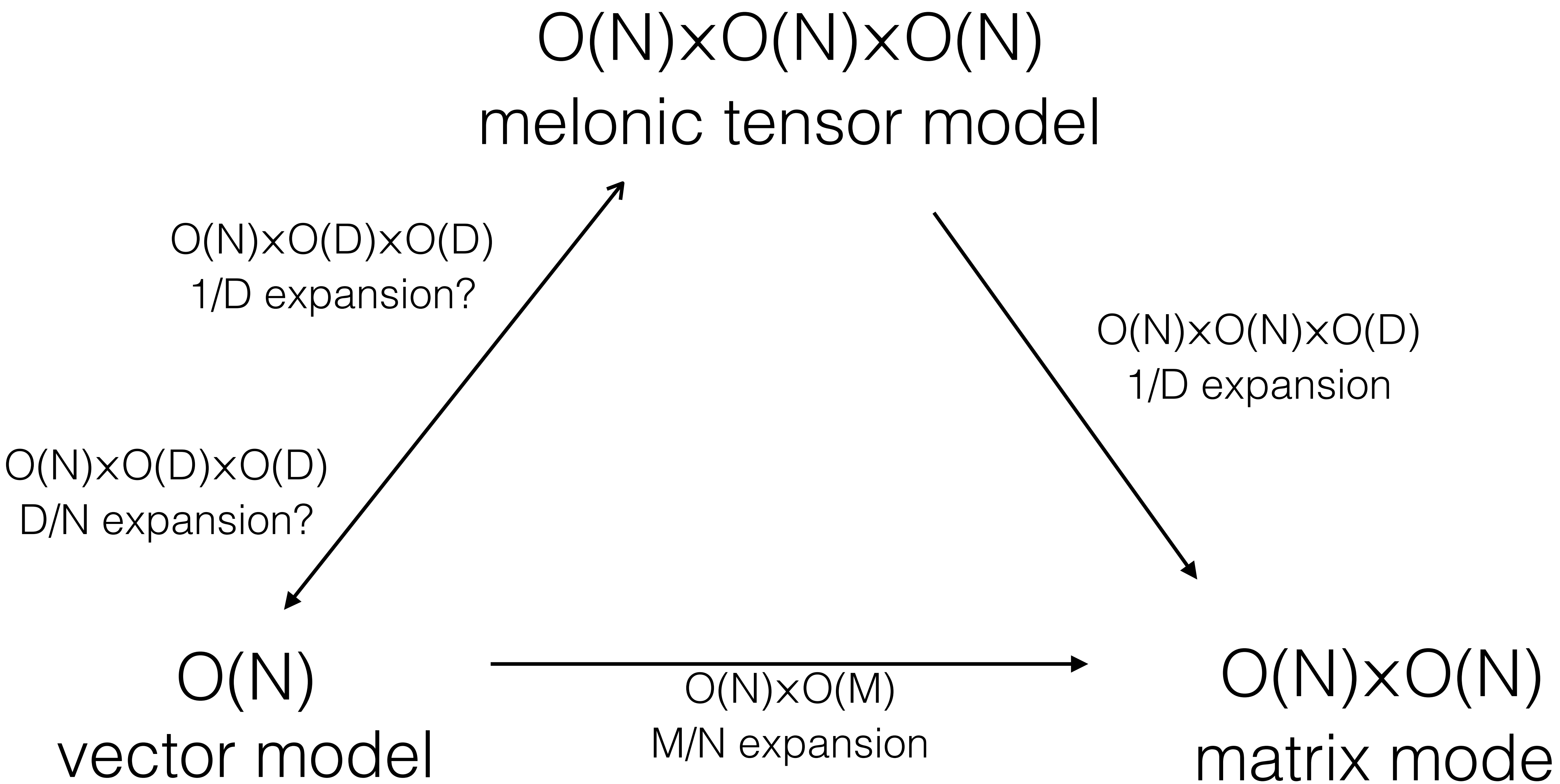
- The bifundamental model can be thought of as a O(N)×O(M)×O(D) theory, with D=1 but M and N large..

- As D increases (staying order 1), the critical value of M/N for which a real fixed point exist will decrease.

- If D is order N, then one returns to the melonic regime, studied earlier.

- Suggests that 1/D is the correct expansion to relate melonic tensor models to matrix models.

Conjectured relation between the three large *N* limits



1-d tensor models

- One could also ask whether a 1-dimensional fermionic matrix model exists. Such a theory would naturally have a simpler holographic dual than tensor models.

- Klebanov-Milekhin-Popov-Tarnopolsky recently showed by exact solution that the simplest such theories ($U(N) \times U(N) \times O(D)$ for $D=1, 2$) do not give rise to conformal quantum mechanics (a finite gap between the ground state and first excited state in standard ’t Hooft limit).

- It would be interesting to see this in epsilon-expansion, and consider larger values of D or $D \sim N^x$ $(x < 1)$

Other tensor models in *d* dimensions

- In general it seems difficult to construct theories with melonic dominance in higher dimensions.

- The exception is a supersymmetric tensor model in 2d, studied by Murugan, Witten, and Stanford (as a random SYK like model), which may have a real spectrum and can also be studied in 3-epsilon dimensions.

- There are a few other non-supersymmetric melonic tensor models one can write down with bosons and fermions in higher dimensions, and it remains to classify whether any give a real spectrum.