A Paradigm for Dark Matter

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Dark Matter-Energy

Numerous distinct observations at many scales (sub-galactic to cosmological) involving very different astrophysical and cosmological phenomena all lead to the same conclusion–

FACT: There is Missing Energy Momentum Tensor in the Universe.

Only $\sim$5% of the stress tensor is visible matter.

Around and above galactic scales, this missing stress tensor can be modeled as pressureless dust plus dark energy. This is the Λ-CDM model and it does a good job of explaining cosmological data (from WMAP).

This success does not really require detailed models of dark matter/energy. Even structure formation fits with this picture for the most part, when we treat dark matter as collisonless particles (aka dust) and run simulations.

In other words, any theory that can reproduce this stress tensor is largely on the same observational footing as Λ-CDM.

As I mentioned, there are many distinct pieces of evidence for dark matter at all astrophysical and cosmological scales.

But all of this evidence so far is gravitational.

In other words, it should be emphasized that we really only know dark stuff as a missing stress tensor piece in the Einstein equations.

The underlying theory/model behind the stress tensor is mostly not (yet) observationally accessible.

There are two approaches when faced with gravitational anomalies in astronomy. One is to postulate missing matter, the other is to modify the theory of gravity.

The current majority view is that this missing “matter" should be viewed as arising from dark matter of some kind, for example the lightest supersymmetric particle (or axions, or many other things).

In this talk we will suggest that dark matter can be viewed as a sort-of Janus object with properties of both matter as well as gravity.

The Dark Connection

More specifically- We will argue that a background non-Riemannian piece in the affine connection (whcih defines covariant derivatives on fields) looks exactly like a stress tensor piece, at the level of the field equations.

We will further argue that this modification of the connection is not in tension with tests of general relativity or current bounds on the principle of equivalence.

It could lead to dark matter strength scattering of ordinary fermions with this background connection, which is in principle detectable. This is the analogue of a direct detection experiment in this paradigm.

The (dark) stress tensor in this picture is algebraically determined in terms of the dark connection, and it is easily seen that it is possible to choose it so that it reproduces Λ-CDM (or galactic dark matter).

A paradigm, not a model: operationally what this means, is we won't try to write down any microscopic theory for the dark connection. The dynamics of the dark connection is controlled by demanding that the theory be diff invariant, aka conservation laws.

But it seems surprising that it is not ruled out, as far as I can tell.

Why do this?

There is an urban legend that gravity modification cannot explain dark matter.

Our construction will be an explicit demosntration that such blanket statements require various qualifications.

Because we can simulate the stress tensor using the dark connection, things like Bullet cluster, dark-matter-less galaxies, etc are all entirrely consistent with our approach.

My original motivation for considering this was the fact that MOND seems unreasonably successful in those circumstances when it is actually successful. Usually in galactic dynamics of (many, but not all) galaxies.

What is the most natural way to understand MOND in a covariant theory?

The less distateful way to view MOND is as a modification of the Newtonian gravitational potential instead of as a modification of Newtonian dynamics. If one adopts this view, a natural approach towards a covariant version of MOND would be to consider the possibility of a modified connection.

In any event, without further ado...

The Dark Connection

What we do is quite simple, and various things of this type have been explored in various contexts for a 100 years.

In general relativity the connection is Riemannian:

$$\left\{ {a \atop bc} \right\} = \frac{1}{2} g^{ak} \left(g_{kb,c} + g_{kc,b} - g_{bc,k} \right) \tag{1}$$

Here instead, we allow a more general connection

$$\Gamma_{bc}{}^{a} = \left\{ {a \atop bc} \right\} + \Delta_{bc}{}^{a} \tag{2}$$

We will treat Δ as a background.

A standard approach to deal with non-Riemannian connection is that of Palatini or metric-affine theory, where one treats the whole connection Γ as an independent field.

Here instead we introduce a new tensor field Δ, which we will call the dark connection. Note that a general connection can always be broken down in this way because the difference bwtween two connections is always a tensor.

Now we consider the Einstein-Hilbert-Palatini action with the modified connection:

$$S[\Delta] = \int \left[\frac{1}{2\kappa} R(\Gamma) + \mathcal{L}_{\mathrm{M}} \right] \sqrt{-g}\, \mathrm{d}^4 x \tag{3}$$

with $\kappa = 8\pi G$, $c = 1$, and $R = g^{ab} R_{ab}$.

Let us emphasize the trivial but crucial fact that the Riemann tensor does not depend on the metric, only on the connection.

Plugging in equation (2) into the Ricci tensor, it follows immediately that

$$R_{ab}(\Gamma) = R_{ab}(\{\}) + \tilde{R}_{ab}(\Delta) \tag{4}$$

where the $R_{ab}(\{\})$ is the standard Ricci tensor computed with the Levi-Civita connection, and

$$\tilde{R}_{ab}(\Delta) \equiv \nabla^{LC}_c \Delta_{ba}{}^c - \nabla^{LC}_b \Delta_{ca}{}^c + \Delta_{cd}{}^c \Delta_{ba}{}^d - \Delta_{bd}{}^c \Delta_{ca}{}^d . \tag{5}$$

Here it is useful to emphasize that ∇^{LC} is the covariant derivative with the Levi-Civita connection.

It is easy to check that the first two terms of equation (5) enter the Lagrangian as a total derivative. This means that upon ignoring the boundary terms that arise from them, our action becomes

$$S = \int \left[\frac{1}{2\kappa} g^{ab} \left(R_{ab}(\{\}) + \Delta_{cd}{}^{c} \Delta_{ba}{}^{d} - \Delta_{bd}{}^{c} \Delta_{ca}{}^{d} \right) + \mathcal{L}_{\mathrm{M}} \right] \sqrt{-g}\, \mathrm{d}^4 x \tag{6}$$

This is exactly of the Einstein-Hilbert form, except for the additional term containing

$$\tilde{\mathcal{T}}_{ab} \equiv \Delta_{cd}{}^{c} \Delta_{ba}{}^{d} - \Delta_{bd}{}^{c} \Delta_{ca}{}^{d} \tag{7}$$

The equation of motion for the metric from this form of the action is

$$R_{ab}(\{\}) - \frac{1}{2} g_{ab} R(\{\}) = 8\pi G (T_{ab} + \mathcal{T}_{ab}) \tag{8}$$

with the usual matter stress tensor

$$T_{ab} = \frac{-2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_{\rm M})}{\delta g^{ab}}. \tag{9}$$

But note that we also have a new term

$$\mathcal{T}_{ab} = \frac{1}{2} g_{ab} g^{cd} \tilde{\mathcal{T}}_{cd} - \tilde{\mathcal{T}}_{ab}. \tag{10}$$

This term is (a) symmetric in the two indices, and (b) is covariantly conserved with respect to the metric. This is our candidate for the dark sector stress tensor in the dark connection approach.

Dark Connection as a Proxy for Dark Matter

It is straightforward to argue that this type of a stress tensor indeed can reproduce the standard fluid stress tensors in cosmology and galactic physics.

The standard fluid in cosmology can be motivated by saying that it is the most general symmetric tensor that respects the isometries of the background, that one can construct (at lowest order in derivatives) with the background (FRW) metric g^{FRW}_{ab}, and the velocity vector U^a of the "matter":

$$T_{ab} = p g^{FRW}_{ab} + (p+\rho) U_a U_b \tag{11}$$

We can apply the same philosophy here in order to fix the most general form of the dark connection in the FRW background. The result is:

$$\Delta_{bc}{}^{a} = \alpha\, U^a g^{FRW}_{bc} + \beta\, (U_c \delta^a_b + U_b \delta^a_c) + \gamma\, U^a U_b U_c \tag{12}$$

Since there are 3 scalar fields in the dark connection, generically one would expect to reproduce the two scalar fields of the fluid-stress tensor. Indeed one can check numerically that the condition for this, namely

$$\begin{aligned} P = & \frac{1}{2}\left(-\alpha^2+\alpha(\gamma-6\beta)+3\beta(\gamma-\beta)\right), \\ \rho = & \frac{3}{2}\left(\alpha^2+\alpha(2\beta-\gamma)+\beta(\gamma-\beta)\right) \end{aligned} \tag{13}$$

has huge regions of the (real) α, β, γ parameter space as allowed solutions, for various types of equations of state.

This includes dust, Λ, dust plus Λ, etc which are the types of stress tensors one uses typically for dark sector models.

What is the Catch?

So far things look simple to the point of trivial, but then why has nobody tried this as an approach for modeling dark matter?

(At least it seems so, within our googling powers.)

The trouble is that if we mindlessly modify the connection, parallel transport and covariant derivatives both change: it is not just the Einstein equation that defines general relativity, it is also geodesic equation and field propagation.

In other words, one cannot be glib and just declare that the dark connection is just another tensor field and therefore the tests of general relativity will be fine.

It is indeed a tensor field, but it could affect local parallelism.

Weird choices of the connection will destroy the Principle of Equivalence, and essentially all the succesful tests of (general) relativity.

In particular, it is absolutely crucial for the successful tests (including things like PSR1913+16) that the post-Newtonian approximation work!

Post-Newtonian approximation crucially needs the (point) sources of gravity to move along metric geodesics.

We believe this is the reason why people have not investigated this as an approach to dark matter, even though various very closely related things have been discussed a lot in the GR literature.

So is everything lost?

Not quite! The reason is that just because we have changed the covariant derivatives acting on fields on the manifold, does not imply that the geodesics of photons and massive test particles will also follow the same notion of gedoesic.

Particle motion should be derived from the underlying field theory, and we will argue that in the particle limit of field theory (even with a dark connection), it is distance minimization that emerges as the physical notion of geodesic and not affine parallelism with respect to the original (total) connection.

We will motivate (we cannot derive, because the full theory is obviously not known) this by arriving at particle geodesics in three different ways, starting from an underlying field theory.

Firstly, we note that an eikonal/WKB limit is a standard approach to get to geodesics from field equations.

Crucially, for photons (and for scalar fields), the matter part of the action does not contain the connection at all, and the geodesic emerges purely from the metric terms that automtically arise in the field equations (ie., Maxwell and Klein-Gordon equations in curved space).

So their eikonal limit paths are the same as in general relativity!

In particular, this guarantees that photons traverse the usual null geodesics of general relativity.

The WKB approach is cleanest for massless particles: for massive partciles we will have to scale the mass as well, and even though this is a mathematically well-defined process and leads to the metric geodesics we want, its physical significance for understanding macroscopic test particles, starting from the field theory, is less clear.

A second argument we will adopt is that of Papapetrou and Dixon.

They showed that for any matter distrubution defined by a stress tensor that satisfies the covariant conservation with respect to the metric, the center of mass of that distribution will traverse a geodesic of that metric.

Their goal was to derive a mutlipole expansion for motion of matter in general relativity starting from stress tensor conservation. What we need is just the monopole part: we have checked that their calculation goes through self-consistently, as long as the stress tensor is covariantly conserved with respect to the metric.

As we emphasized, this is true for the stress tensor even with the dark connection turned on, so the Papapetrou-Dixon argument for the center of mass of matter distributions goes through here as well.

In particular, this approach is true for both massless and massive trajectories. The (largely only techincal in my view) drawback here is that it is a statement about the center of mass, and not about point particles.

A third argument we will present is a theorem due to Geroch and Jang.

Let (M, g_{ab}) be an orientable spacetime, and $\gamma : I \to M$ be a smooth curve. Suppose that given any open subset $\mathcal{O}$ of M containing $\gamma[I]$, there exists a smooth symmetric field T_{ab} with the following properties.

- T_{ab} satisfies a fairly strong timelike energy condition.
- T_{ab} is covariantly conserved with respect to the metric.
- T_{ab} is supported within $\mathcal{O}$.
- T_{ab} is not identically zero everywhere on $\mathcal{O}$.

then, γ must be a timelike geodesic.

Again, it is possible to check that the theorem does not rely on the choice of the connection, as long as item 2 holds.

The strength of the Geroch-Jang theorem over the Papapetrou-Dixon argument is that it formalizes our idea of a "point" particle, by looking at infinitesimal neighborhoods of curves.

Its weakness however is that it requires an energy condition.

In any event, we will take these facts collectively as showing that macroscopic test prticles aand light traverse metric geodesics as expected in general relativity, even when there is a dark connection.

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Conclusions and Speculations

- We have argued that a background non-Riemannian piece in the affine connection might be indistinguishable from dark matter according to present day tests.
- One of our goals in doing this was to see how stringently gravity modfiation is ruled out as an approach to dark matter. Answer seems to be, not very strongly.
- One thing we have not emphasized in the talk is that for fermionic particles (and more generally particles other than photons and scalars) the dark connection will give rise to a 3-pt interaction piece in the Lagrangian. This will in principle be of about the same strength as dark matter direct detection tests, and can possibly be viewed as a fifth force. It will be interesting to see if it has detectable signatures.

Thank You!