AdS Locality and the Conformal Bootstrap

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based on:
1703.00278;
1711.02031 with Fernando Alday;
work in progress with:
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- Alex Maloney, Zarah Zarahee, Yan Gobeil

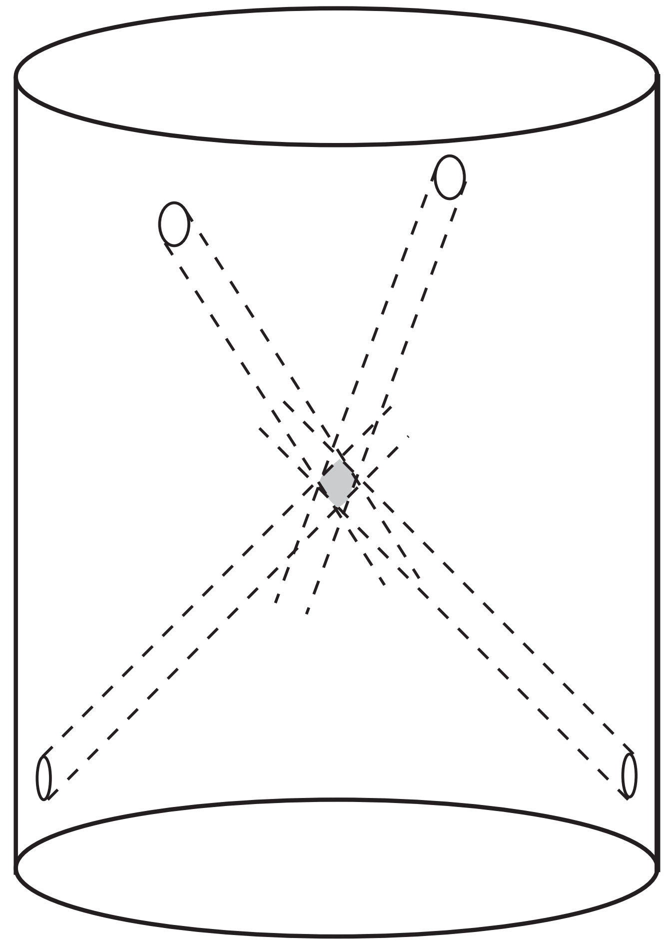
AdS/CFT@ 20 and beyond: ICTS Bangalore

How does (sharp) AdS locality emerges from CFT?

Kinematic holography: in any QFT, processes localize to within ~AdS radius (because different length scales decouple)

AdS/CFT: in some strongly coupled QFTs, physics localizes sharply in AdS (to $\ell_{\rm s} \ll R_{\rm AdS}$)

We'll study CFT correlation functions



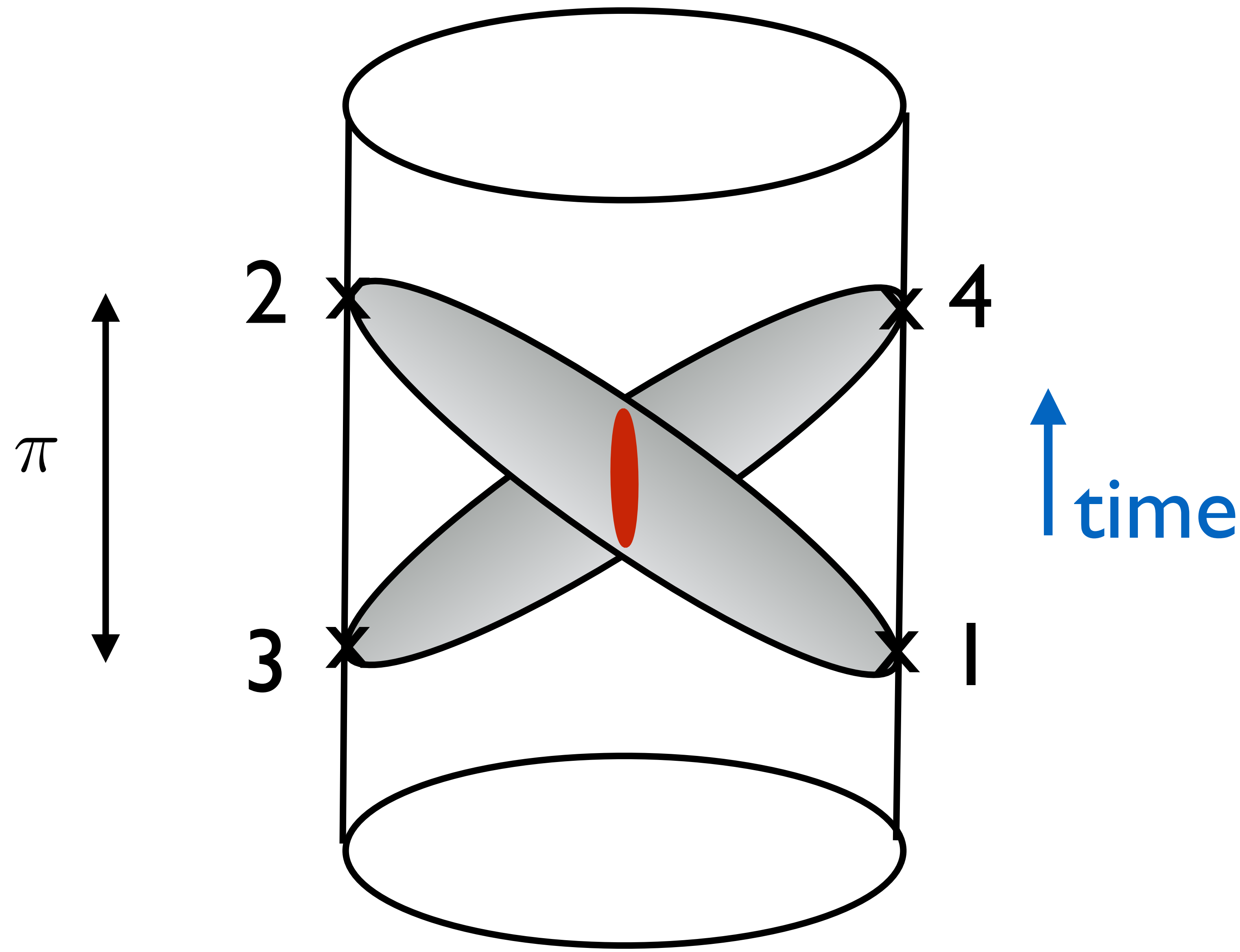
[fig: Heemskerk,Penedones, Polchinski& Sully]

AdS locality: probe by focusing wave-packets into bulk

(sharpest signal: (d+2)-point correlator)

[Maldacena, Simmons-Duffin& Zhiboedov '15]
[…, Engelhardt& Fischetti '17]

This talk: 4 points, only locality in time (``causality”)



Regge limit *(large boost)*:

-spreads transversely over AdS_{d-2}

-localizes in time (in two null directions)

[Cornalba, Costa, Penedones ’06-…]

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Mathematically:

causality ⇒ analyticity in E ⇒ dispersion relation

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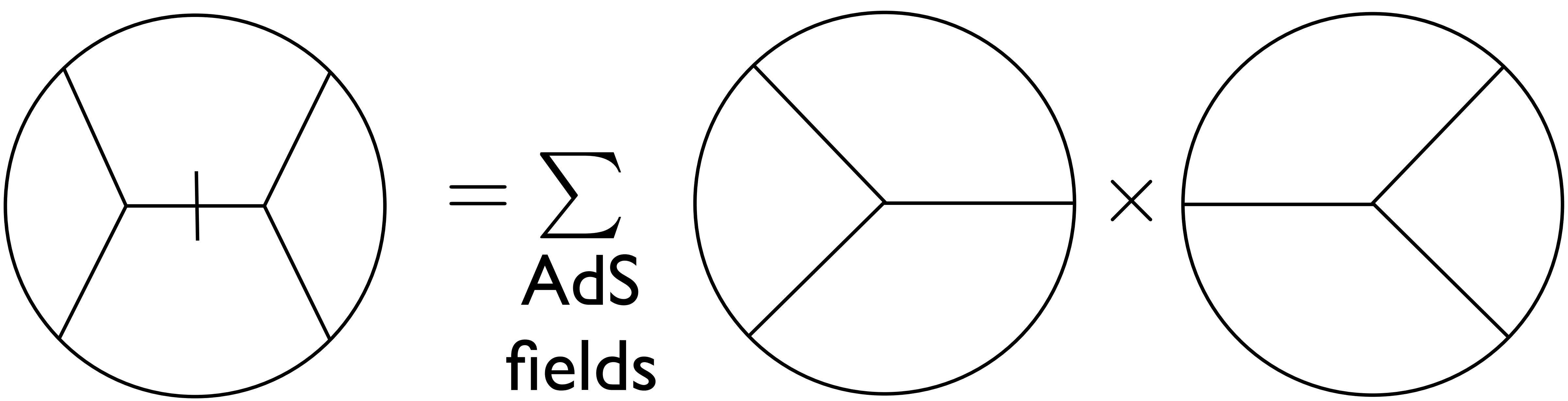


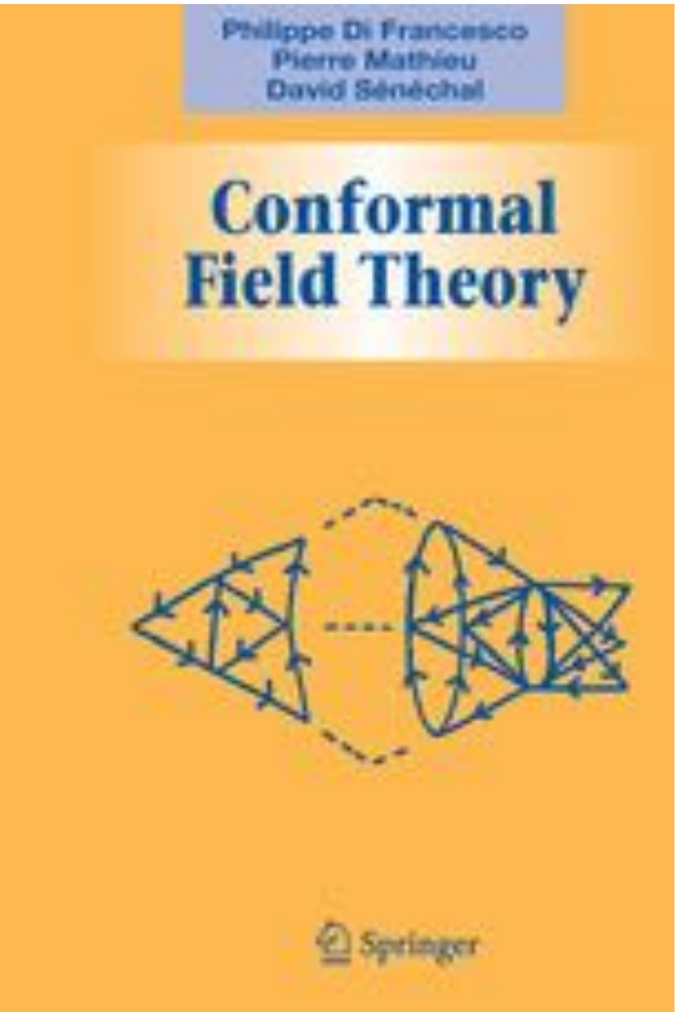
HPPS Conjecture:

Any CFT with:

1. Large-N expansion
2. Large gap in operator dimensions

has a bulk dual, local to lengths $\ell_{\rm AdS}/\Delta_{\rm gap}$.

[Heemskerk,Penedones,Polchinski& Sully '09]



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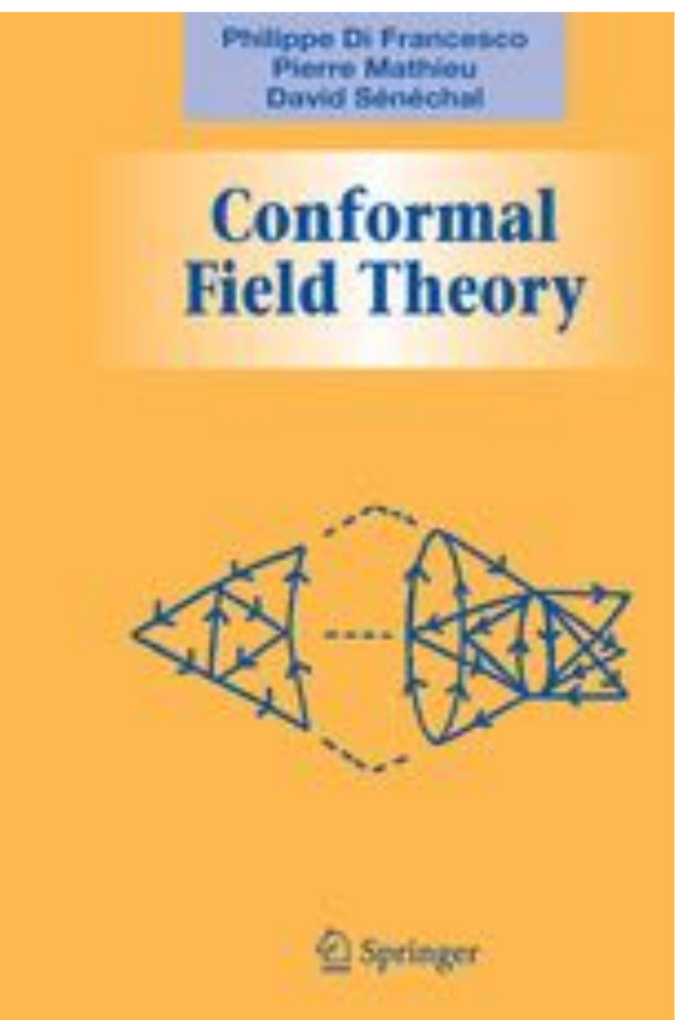

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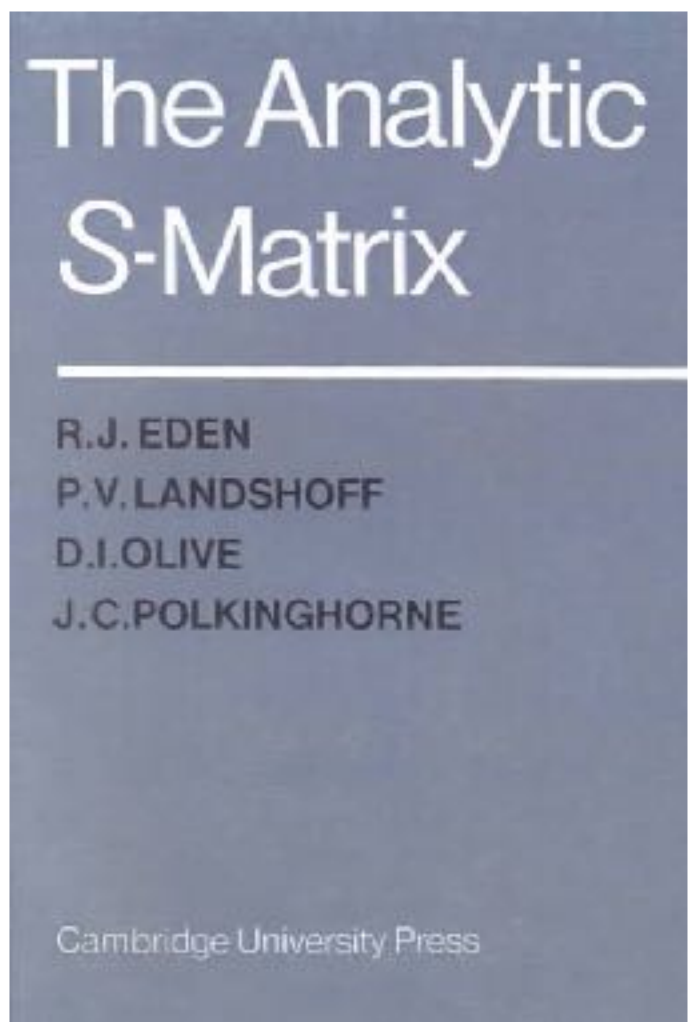


= New tool: CFT dispersion relation

Outline

1. Lorentzian inversion formula
   -setup and Regge limit of correlator
   -analyticity in spin
   -'Absorptive part' in CFT

2. AdS locality
   -CFT derivation of bulk EFT
   -concrete tree and one-loop Witten diagrams

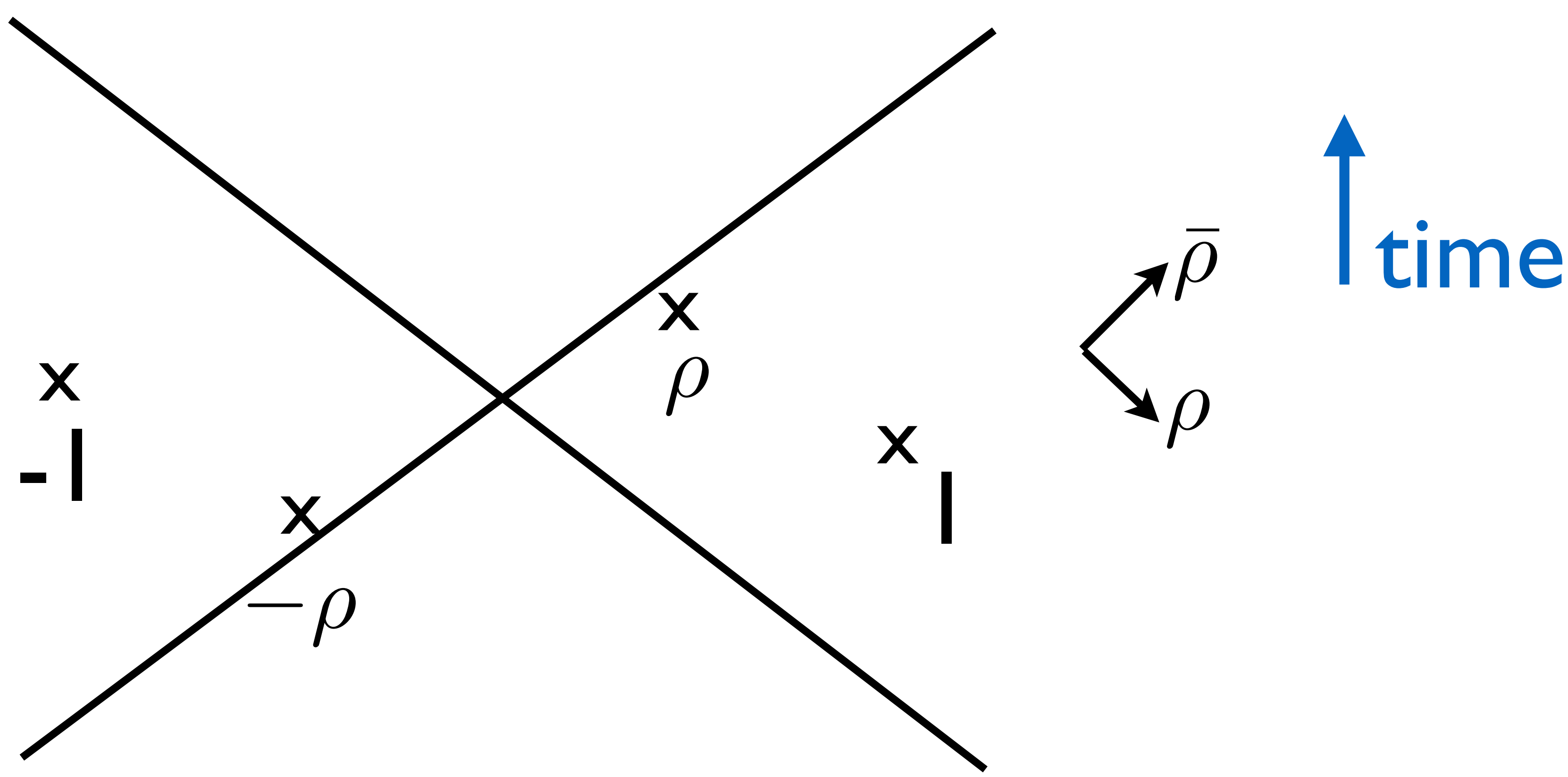
3. *More should be true!*
   -Pushing 1/J expansions to spin J=0
   -Bulk point limit

We'll study Lorentzian 4-point correlator in CFT_d

$z \neq \bar{z}$ = independent light-cone coordinates

Symmetrical parametrization:

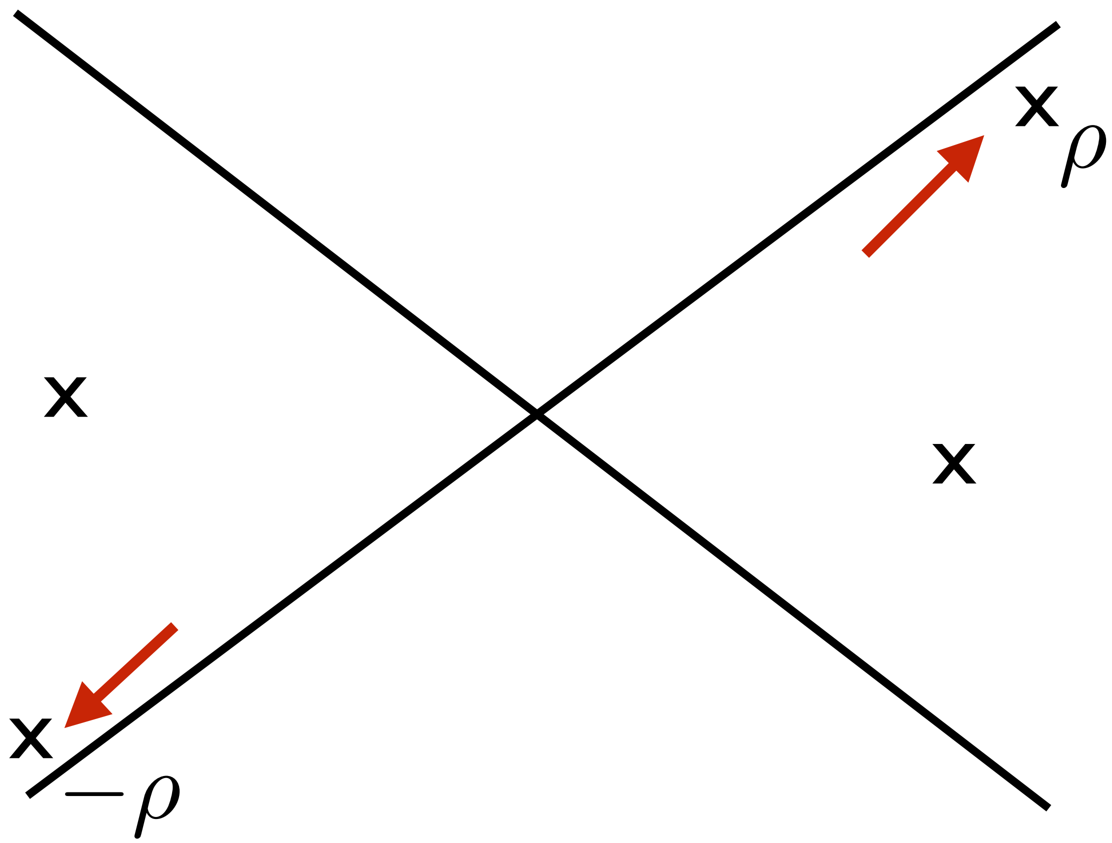
[Rychkov& Hogervorst '13]

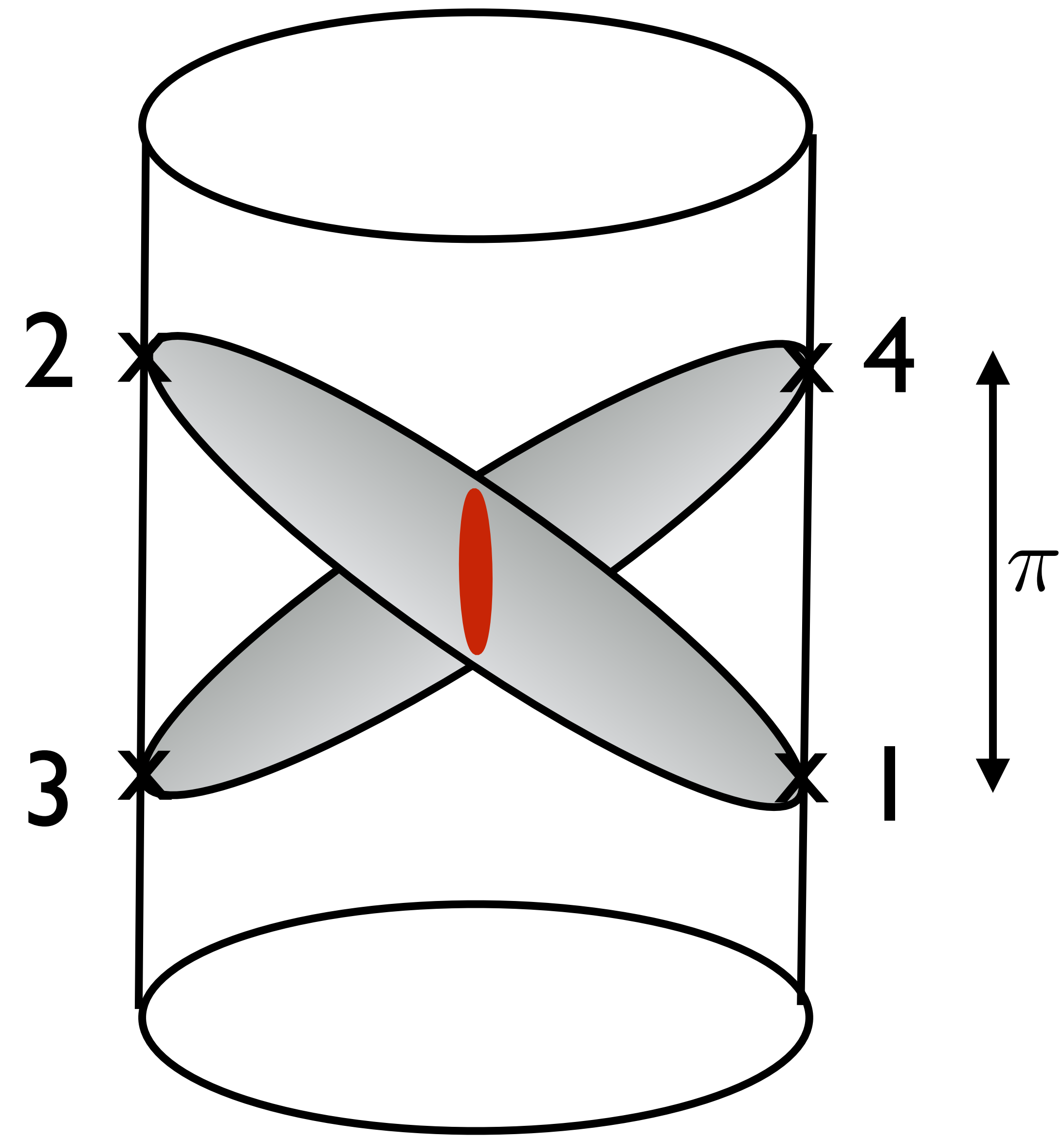


$$z = \frac{4\rho}{(1+\rho)^2}$$

$$\bar{z} = \frac{4\bar{\rho}}{(1+\bar{\rho})^2}$$

Regge limit:

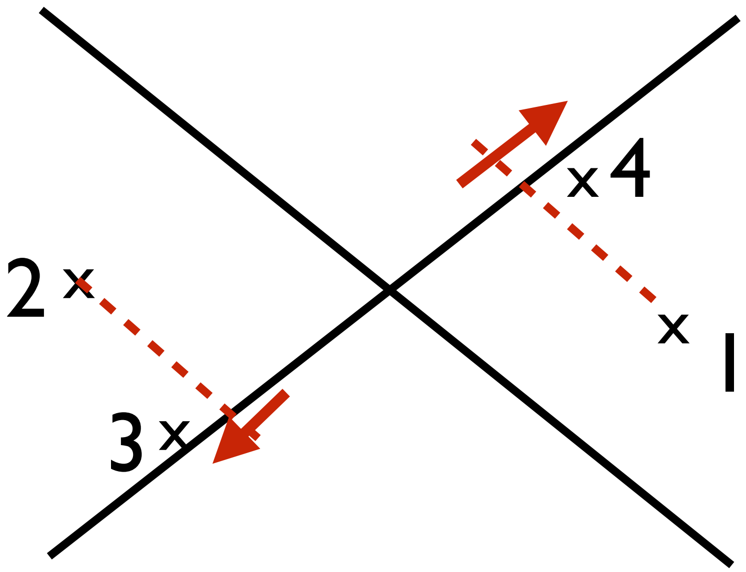




« large modular time $\log\sqrt{\bar{\rho}/\rho}$ for half-plane in O(-1)O(1) state »

(⇒ correlator bounded)

s-channel OPE (1,2) diverges after crossing light-cone (ρ>1)



Options:

1. So what: just use the t-channel OPE (1,4) instead
2. Figure out how to resum s-channel OPE ✓

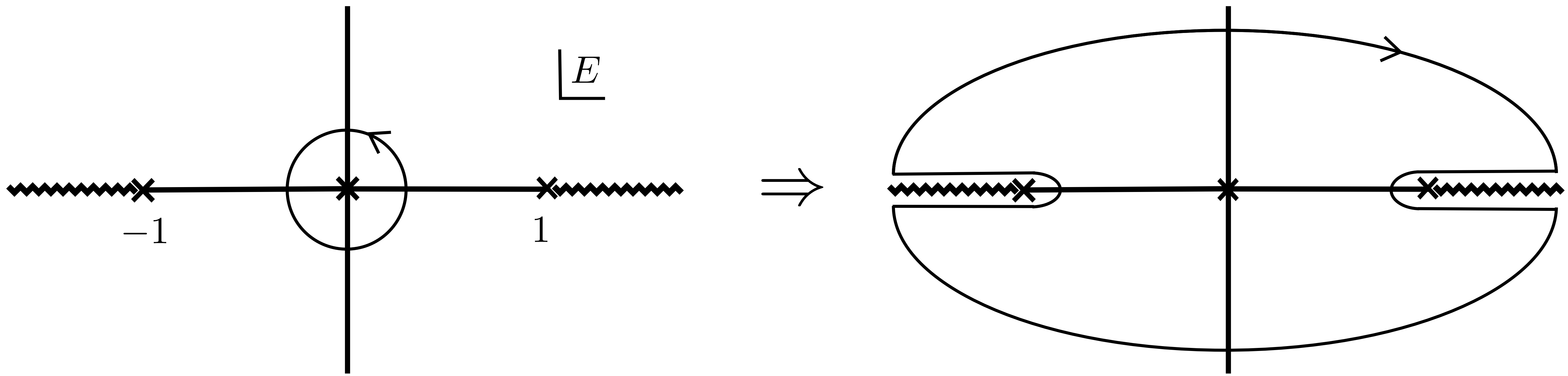
Toy model: amplitude f(E) that's:

1. Analytic in cut plane
2. |f(E)/E| bounded at large |E|
3. Has Taylor series at small E

$$f(E) = \sum_{J=0}^{\infty} f_J E^J$$

(=Correlator with $E = \sqrt{\bar{\rho}/\rho} = \exp(\text{modular time})$)

Q: What does 'nice behavior' at large |E| implies for series?



⇒ can write coefficients as integral over branch cut

Cauchy: $$f_J \equiv \frac{1}{2\pi i}\oint_{|E|<1} \frac{dE}{E} E^{-J} f(E)$$

$$= \frac{1}{2\pi}\int_1^\infty \frac{dE}{E} E^{-J} \left(\text{Disc}\, f(E) + (-1)^J \text{Disc}\, f(-E)\right) \qquad (J>1),$$

⇒ coefficients f_J are analytic in J (& bounded at large Im J) f(E) extends nicely beyond |E|<1 ⇔ f_J is analytic in spin

It works both ways: Watson-Sommerfeld resummation

$$f(E) = \sum_{J=0}^{\infty} f_J E^J$$

$$= \int_{-i\infty}^{i\infty} \frac{dJ}{1-e^{2\pi i J}} E^J \left[f_J^t + f_J^u e^{i\pi J}\right]$$

resums the OPE into the full cut plane

(note: changing any *single* f_J would radically change large-E)

Regge theory: Euclidean ⇔ Lorentzian OPEs

partial waves: $$a_j(s) = \int_{-\pi}^{\pi} d\theta \cos(j\theta) \mathcal{M}(s, t(\cos\theta))$$

+

disp. relation: $$\mathcal{M}(s,t) = \int \frac{dt'}{\pi(t-t')} \mathrm{Im}\,\mathcal{M}(s,t') + (t \leftrightarrow u)$$

=

analyticity in spin $$a_j(s) = \int_{\eta_0}^{\infty} d\eta\, e^{-j\eta} \mathrm{Im}\mathcal{M}(s, t(\cosh(\eta))) + (-1)^j (t \leftrightarrow u)$$

[Froissart-Gribov, ~'60]

Equivalent to dispersion relation

Claim:

1. OPE data of unitary CFTs is similarly analytic in spin

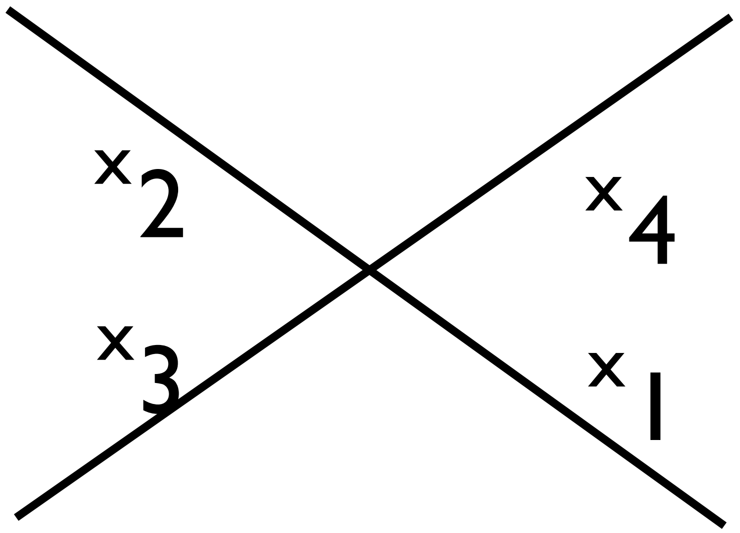
$$G(z,\bar{z}) = \sum_{J,\Delta} c_{J,\Delta} G_{J,\Delta}(z,\bar{z})$$

2. determined by 'absorptive part' of correlator

$$c(J,\Delta) = \int_{\Diamond} \left[\text{Inverse block}\right] \times \left[\text{dDisc}\, G\right]$$

[SCH '17]

$$\text{dDisc}\, G \equiv \tfrac{1}{2}\langle 0|[\phi_2,\phi_3][\phi_1,\phi_4]|0\rangle$$



Positive & bounded

cf: [Maldacena, Shenker&Stanford 'bound on chaos']
[Hartman,Kundu&Tajdini 'proof of ANEC']

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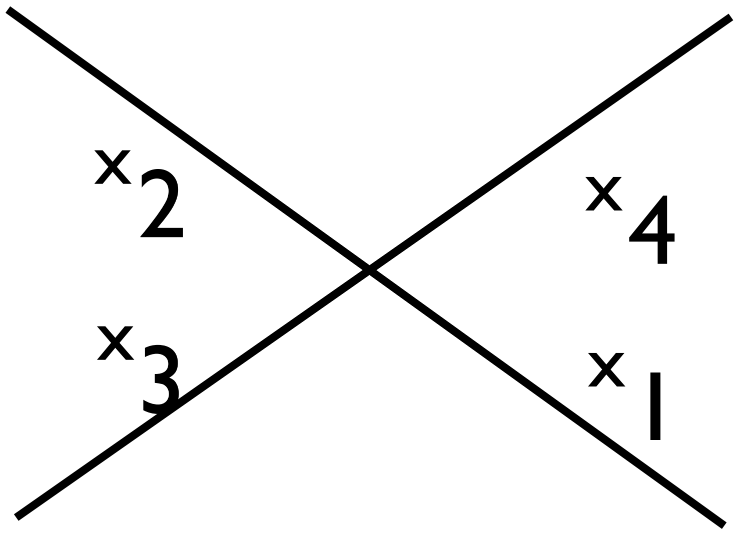

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Intuition: correlator ⇒ scattering amplitude

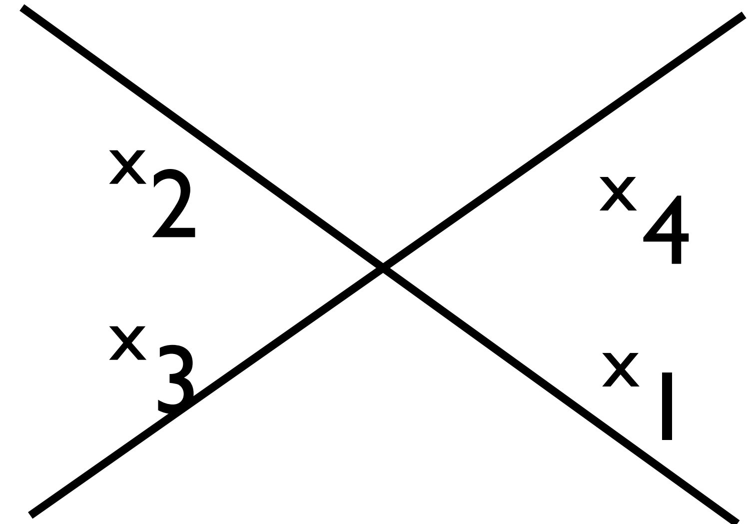
$$\langle 0|T\phi_1\cdots\phi_4|0\rangle \equiv S = G_E + i\mathcal{M}$$

$$\langle 0|\bar{T}\phi_1\cdots\phi_4|0\rangle \equiv S^* = G_E - i\mathcal{M}^*$$

$$\langle 0|\phi_2\phi_3\phi_1\phi_4|0\rangle \equiv G_E$$

⇒ dDisc G is CFT version of Im M!!

large-N: saturated by single-traces!



$$\phi_1\phi_4 \sim \sum_{j,\Delta} c_{j,\Delta}((x_1-x_4)^2)^{\frac{\Delta-\Delta_1-\Delta_4}{2}}$$

integer $+\gamma/N_c^2$

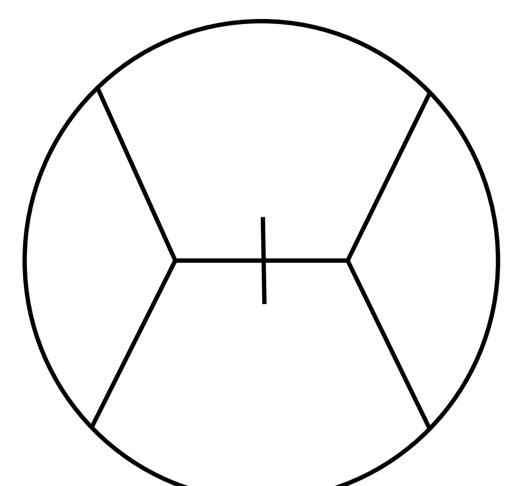
$$[\phi_1,\phi_4] \sim \sum_{j,\Delta} c_{j,\Delta}|(x_1-x_4)^2|^{\frac{\Delta-\Delta_1-\Delta_4}{2}} \sin(\pi\frac{\Delta-\Delta_1-\Delta_4}{2})$$

Thus:

~1/N$_c$^4

$$[\phi_2,\phi_3][\phi_1,\phi_4] \sim \sum_{t-\text{channel}} (\ldots)\sin(\pi\frac{\Delta-\Delta_1-\Delta_4}{2})\sin(\pi\frac{\Delta-\Delta_2-\Delta_3}{2})$$

⇒ AdS Cutkowski rules



Method of the missing box

	Euclidean		Lorentzian	
Taylor series:				
	E^J		E^{-J}	
Rotation symmetry:				
SO(2)	$\cos(j\theta)$	⟶	$e^{-j\eta}$	SO(1,1)
SO(3)	$P_j(\cos\theta)$	⟶	$Q_j(\cosh\eta)$	SO(2,1)
Conformal symmetry:				
SO(d+1,1)	$G_{j,\Delta}(z,\bar{z})$	⟶	???	SO(d,2)

The « conformal blocks » for four-point function solve some some second-order (and fourth-order) Casimir eqs.

$$G_{J,\Delta}(z,\bar{z}) = \frac{z\bar{z}}{\bar{z}-z}\left[k_{\Delta-J-2}(z)k_{\Delta+J}(\bar{z}) - k_{\Delta+J}(z)k_{\Delta-J-2}(\bar{z})\right]$$

$$k_\beta(z) = \bar{z}^{\beta/2}\,{}_2F_1(\beta/2+a, \beta/2+b, \beta, z)\,.$$

$$\sim (z\bar{z})^{\Delta/2}(z/\bar{z})^{J/2}$$

The Lorentzian ‘inverse block’

$$\mu(z,\bar{z})G_{\Delta+1-d,J+d-3} \sim (z\bar{z})^{(J-1)/2}(z/\bar{z})^{(\Delta-1)/2}$$

basically same, with spin and dimension interchanged!

[SCH ’17]

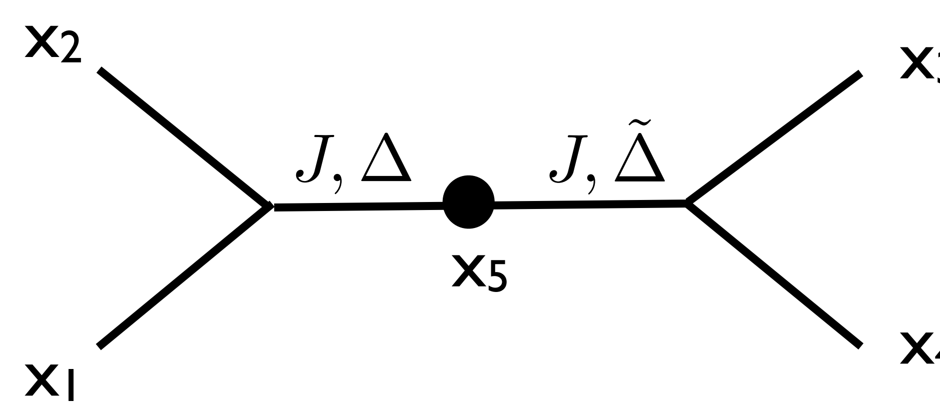
Simpler derivation

[Simmons-Duffin, Stanford&Witten '17]
[Simmons-Duffin& Kravchuk '18]

Start from Euclidean Plancherel formula for harmonic fcts:

$$c(J,\Delta) = \int \frac{d^dx_1 \cdots d^dx_5}{\mathrm{vol(SO(d,2))}} \langle O_1 \cdots O_4 \rangle$$

x (3-point functions)



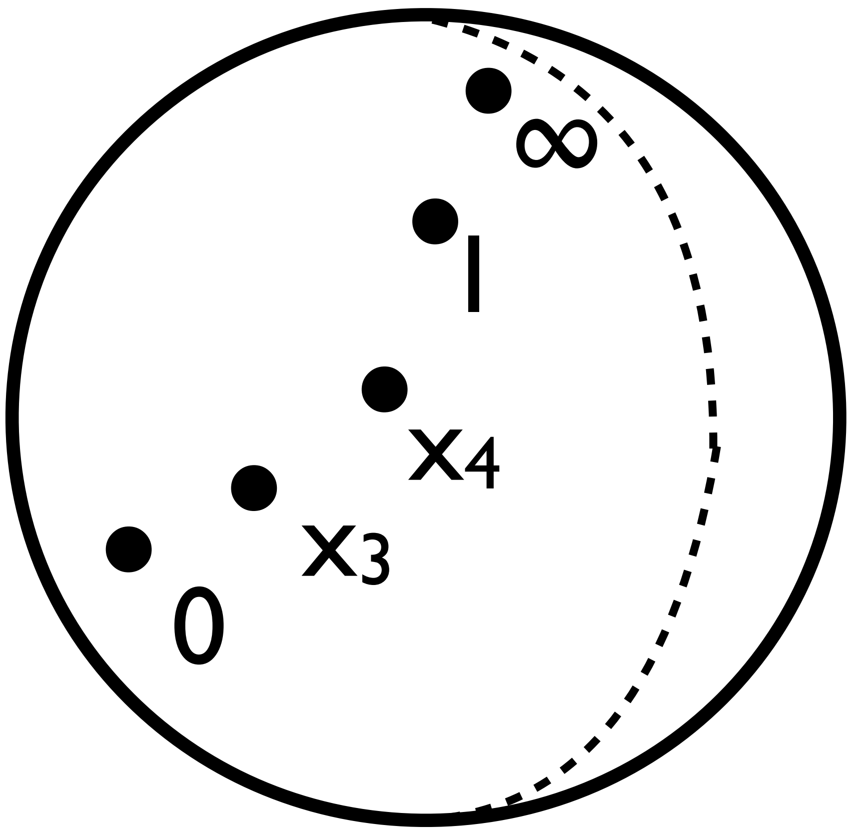
(this gives generating function of all OPE data:

$$\lim_{\Delta \to \Delta'} c(J,\Delta) = \frac{f^2_{OO\to J,\Delta}}{\Delta' - \Delta} \quad)$$

[Ferrara, Gato, Grillo 70's]

The trick.

1. Gauge-fix to $(0,1,x_3,x_4,\infty)$
   then Wick-rotate (x_3,x_4)



2. Deform time contours to
   pick time-like double-commutators

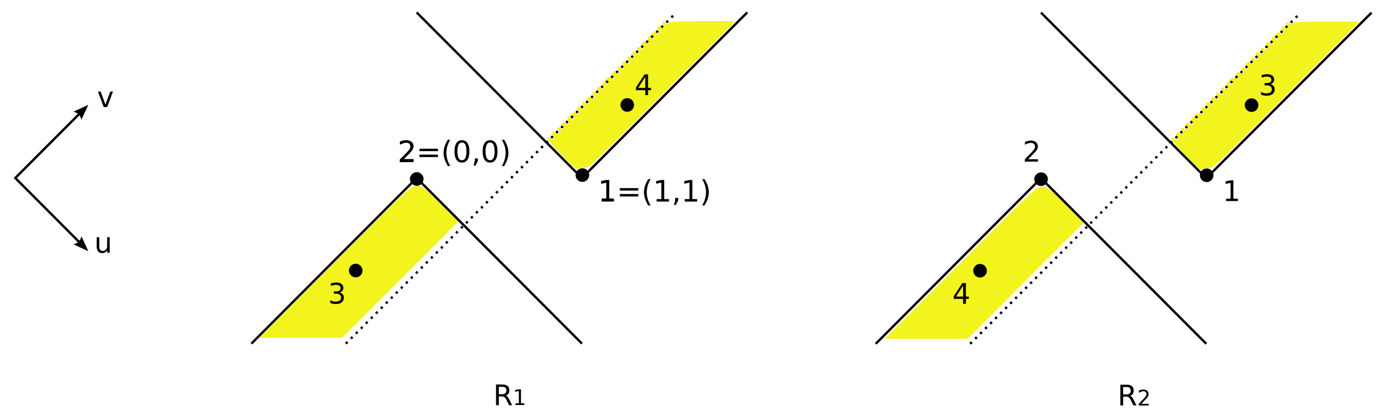


Figure 2: We show typical configurations for points 3 and 4 within regions R_1 and R_2. The dotted line is not fixed in place, it is only to emphasize that points 3 and 4 must be spacelike separated. Time goes up.

(cf KLT trick: [5-point closed string]=[open string]^2)

Result: Froissart-Gribov formula

$$c(J,\Delta) = \int_{\Diamond} \left[\text{Inverse block}\right] \times \left[\text{dDisc}\, G\right]$$

s-channel
OPE coefficients

block with
j and Δ
exchanged

convergent
t-channel sum

converges for j>1 (boundedness in Regge limit)

A (boring) test: 2D Ising

$$G(\rho,\bar\rho) = \left|\frac{1}{(1-\rho^2)^{1/4}}\right|^2 + \left|\frac{\sqrt{\rho}}{(1-\rho^2)^{1/4}}\right|^2$$

- Double discontinuity:

$$\frac{1-\frac{1}{\sqrt{2}}(\sqrt{\rho}+\sqrt{\bar\rho})+\sqrt{\rho\bar\rho}}{(1-\rho^2)^{1/4}(1-\bar\rho^2)^{1/4}} > 0$$

- Factorized integral against 2d (global) blocks

$$c_{j,\Delta} = f_0(j+\Delta)f_0(j+2-\Delta) - \tfrac{1}{2}f_{1/4}(j+\Delta)f_0(j+2-\Delta)+\ldots$$

$$f_p(\alpha) = 2^{a-3+2p}\frac{\Gamma(\frac{7}{4})\Gamma(p+\frac{\alpha-2}{4})}{\Gamma(p+\frac{\alpha+5}{4})}\,{}_3F_2\big(\tfrac{1}{2},\tfrac{\alpha}{2},p+\tfrac{\alpha-2}{4};\tfrac{a+1}{2},p+\tfrac{\alpha+5}{4};1\big)\,. \qquad \text{(B.6)}$$

- Residues at all poles do match global OPE! ✓

$$C_{j,\Delta} = -K_{j,\Delta} \mathrm{Res}_{\Delta'=\Delta} c(j, \Delta')$$

$$C_{0,1} = \frac{1}{4}, \qquad C_{2,2} = \frac{1}{64}, \qquad C_{4,4} = \frac{9}{40960}, \qquad C_{0,4} = \frac{1}{4096}$$

$$C_{4,5} = \frac{1}{65536}, \qquad C_{6,6} = \frac{35}{3670016} \qquad C_{2,6} = \frac{9}{2621440}, \qquad C_{6,7} = \frac{1}{1310720}, \ldots$$

Outline

1. Lorentzian inversion formula:
   -setup and Regge limit of correlator
   -analyticity in spin
   -'Absorptive part' in CFT ✓

2. AdS locality
   -CFT derivation of bulk EFT
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3. *More should be true!*
   -Pushing 1/J expansions to spin J=0
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AdS locality

recall HPPS conjecture:
Any large-N CFT with a large gap of operator dimension has an AdS dual, down to lengths $\ell_{\rm AdS}/\Delta_{\rm gap}$

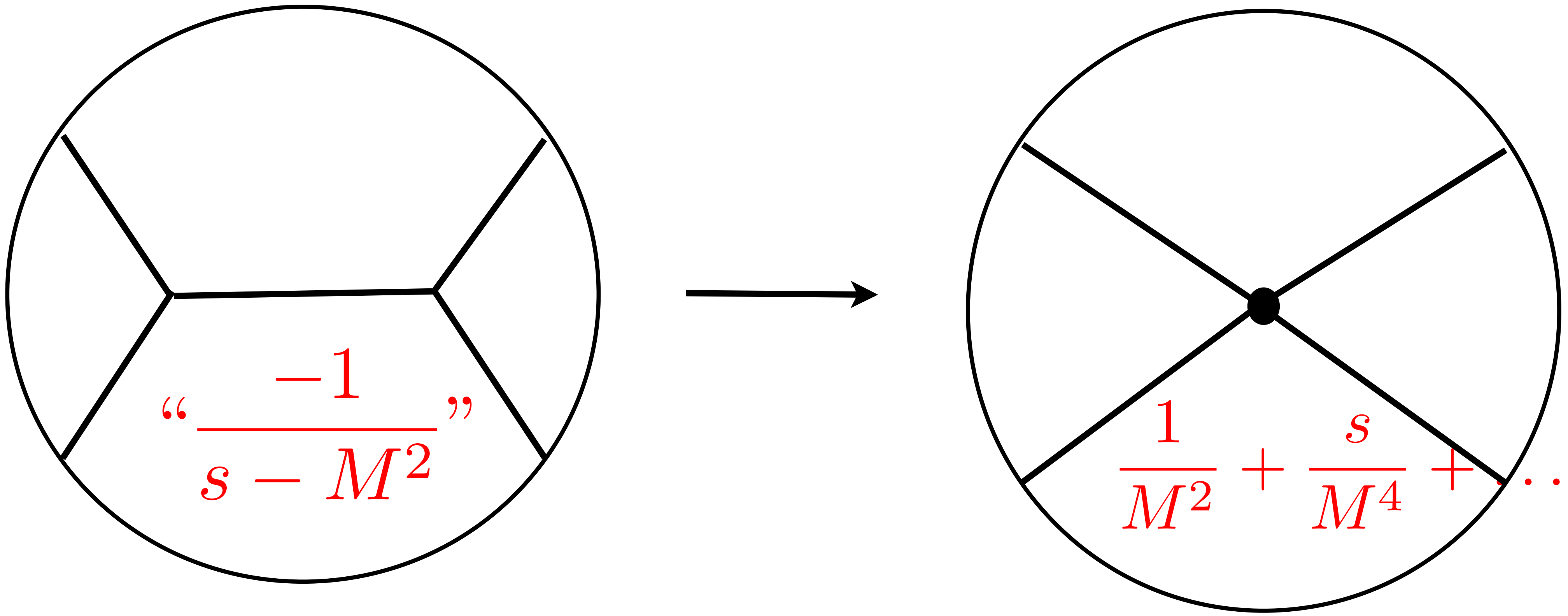
[Heemskerk,Penedones,Polchinski& Sully '09]

They proved: solutions to crossing using finite-spin double-traces

$\longleftrightarrow$ derivative interactions in AdS

But why are higher-derivatives suppressed by powers of Δ_{gap}?

- Effective field theory in AdS:



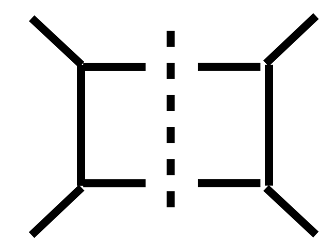
- In AdS5xS5, for example, $1/M^2 \sim \alpha' \ll L^2_{\rm AdS}$
- ‘dispersion relation’ clarifies that:

$$\mathcal{M}(s)\Big|_{\rm heavy} \sim \int_{M^2}^{\infty} \frac{ds'}{s'-s}{\rm Im}\mathcal{M}(s') \sim \frac{1}{M^2}+\frac{s}{M^4}+\dots$$

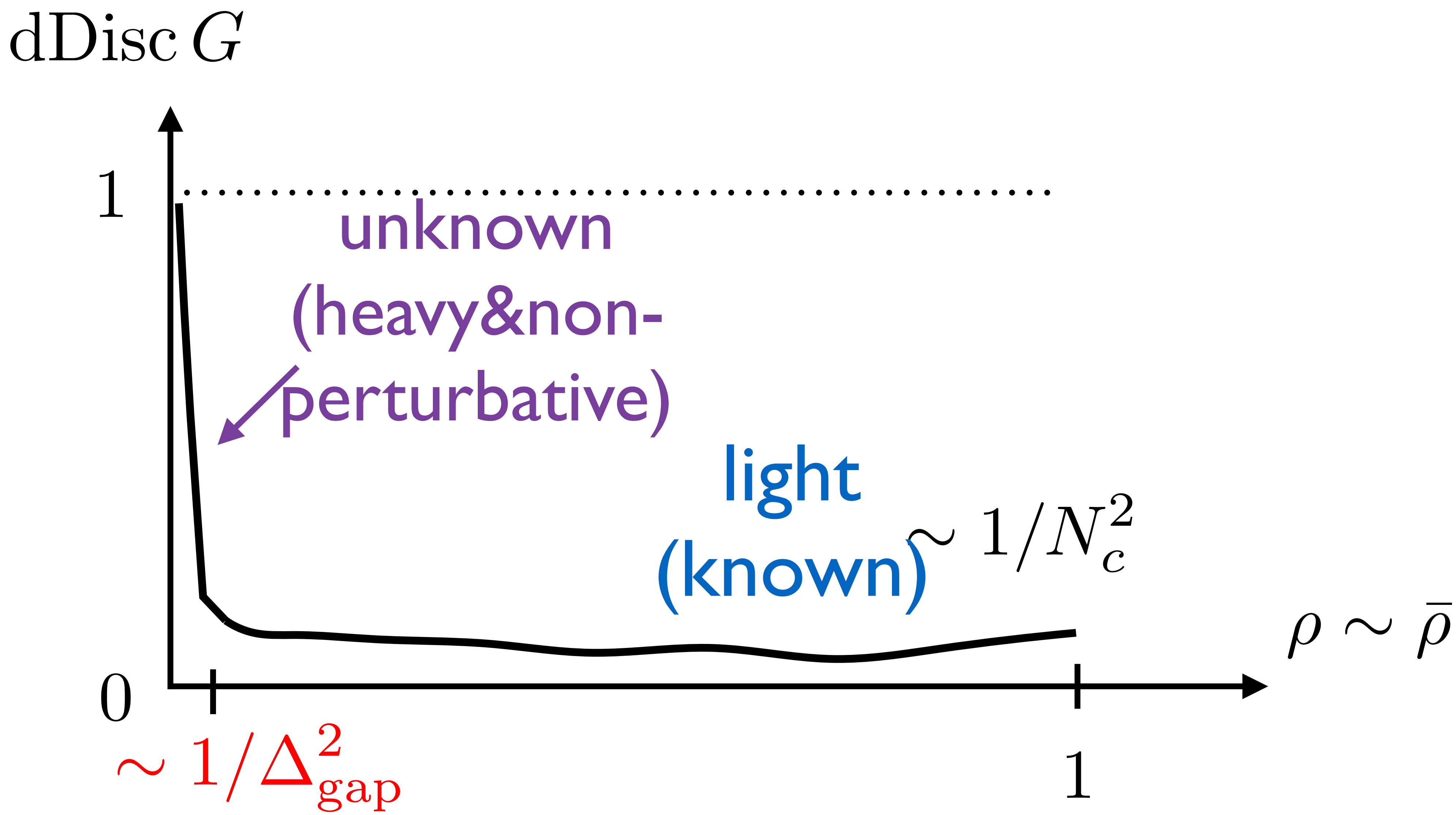
dDisc from the cross-channel

$$\mathrm{dDisc}\, G = \sum_{J',\Delta'} \sin^2\left(\tfrac{\pi}{2}(\Delta' - 2\Delta)\right) \left(\frac{1-\sqrt{\rho}}{1+\sqrt{\rho}}\right)^{\Delta'+J'} \left(\frac{1-\sqrt{\bar\rho}}{1+\sqrt{\bar\rho}}\right)^{\Delta'-J'}$$

- Double-traces killed at large N
- Heavy operators killed unless $\rho, \bar\rho < \Delta_{\mathrm{gap}}^2$



AdS dual is local (according to HPPS) $\Leftrightarrow$ dDisc saturated by few light primaries



$$c_{j,\Delta} = \int F_{j,\Delta}\,\mathrm{dDisc}\,G \quad = c_{j,\Delta}\Big|_{\text{light}} + c_{j,\Delta}\Big|_{\text{heavy}}$$

'minimal solution'

correction small for j>2

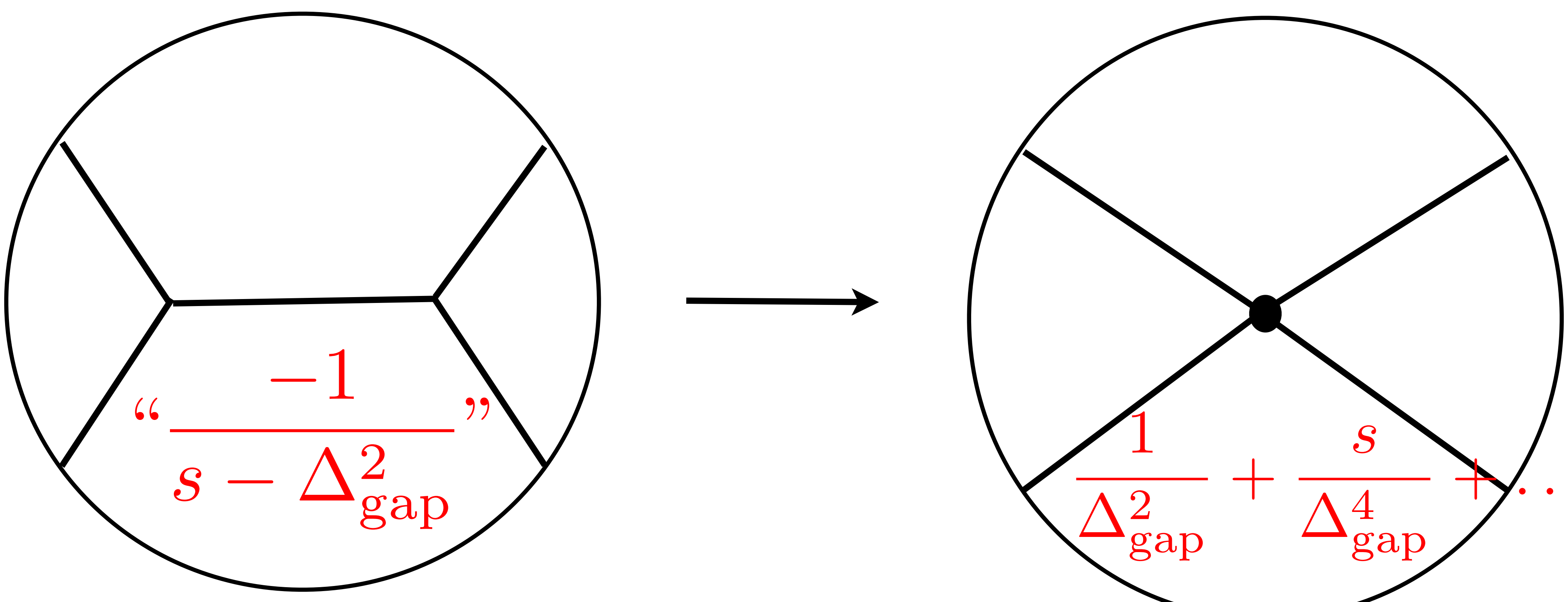
[see also: Alday, Bissi & Perlmutter; Li, Meltzer & Poland]

'Heavy' part depends on nonperturbative UV completion.

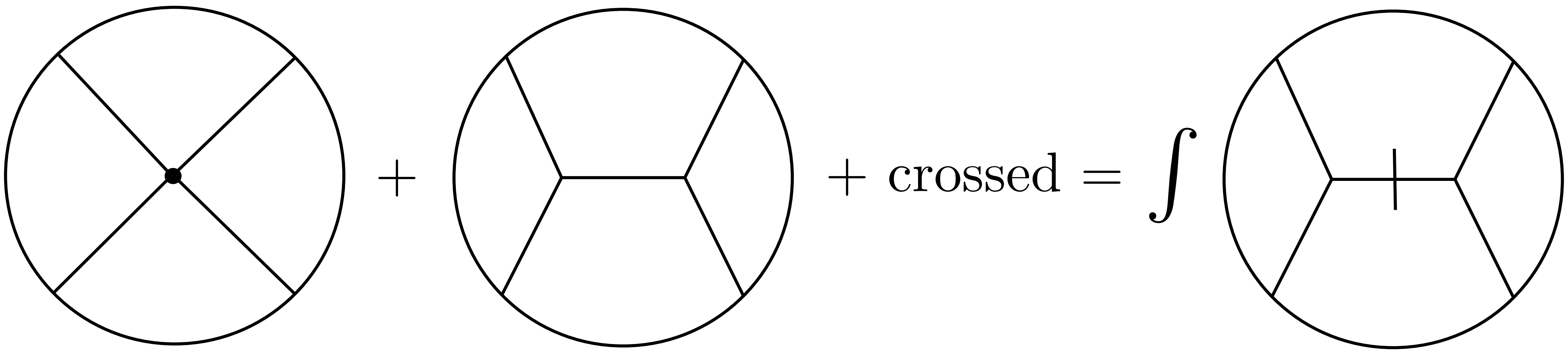
It's weighed by $\sim (\rho\bar{\rho})^{J/2}$. Use positivity + boundedness:

$$\left|c(j, \tfrac{d}{2}+i\nu)_{\text{heavy}}\right| \leq \frac{1}{c_T}\frac{\#}{(\Delta_{\text{gap}}^2)^{j-2}}$$

This establishes, from CFT, an EFT power-counting in AdS.



Ex.: tree-level AdS gravity

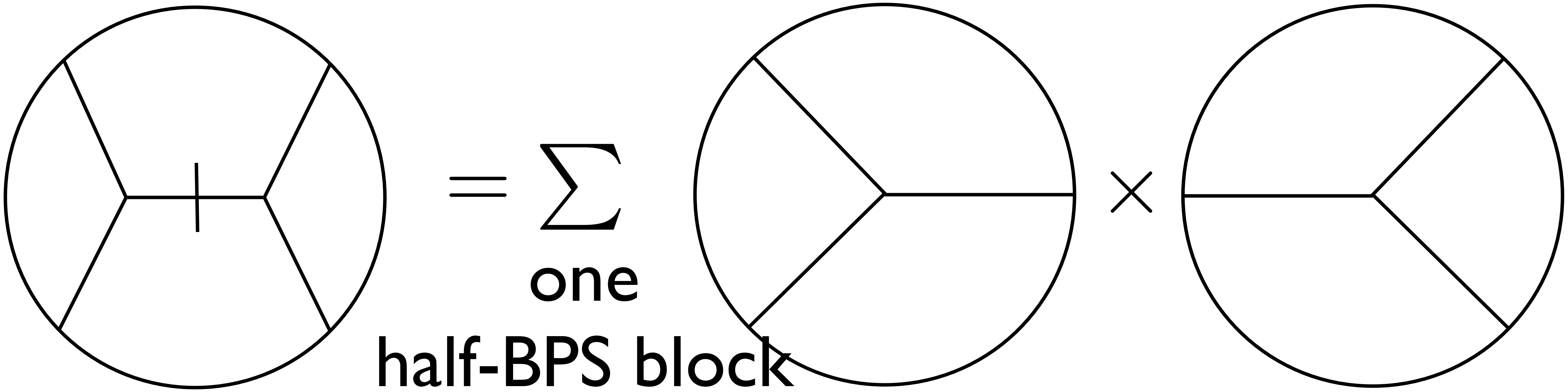


+ contact terms with j≤2

with SUSY, of course, contact terms are restricted.

In N=4 SYM, susy-delta function shifts J by 4:
tree-level converges for J≥-2, no ambiguities at all!

Correlator of Tr[Z^2] N=4 SYM:



[Alday& SCH '17]

dDisc is just the polar part as v->0 of one block:

$$\mathcal{G}(u,v) = \frac{1}{v^2} + \frac{2u^2\log u - 3u^2 + 4u - 1}{v(u-1)^3}\,\frac{1}{c}$$

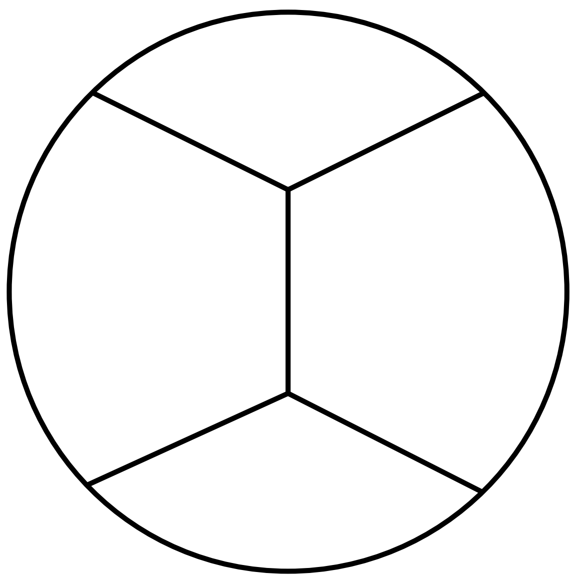
Plug into inversion integral gives all OPE data!!

$$\langle a^{(0)} \rangle_{n,\ell} = 2(\ell+1)(6+\ell+2n)$$

$$\langle a^{(0)}\gamma^{(1)} \rangle_{n,\ell} = -(n+1)(n+2)(n+3)(n+4)$$

$$\langle a^{(1)} \rangle_{n,\ell} = \frac{1}{2}\partial_n \langle a^{(0)}\gamma^{(1)} \rangle_{n,\ell}$$

Result matches perfectly supergravity Witten diagrams



+…

$$\mathcal{G} = 1 + \frac{1}{v^2} + \frac{1}{c}\left(\frac{1}{v} - u^2 \bar{D}_{2,4,2,2}(z,\bar{z})\right) + O(1/c^2)$$

[D'Hoker, Freedman, Mathur, Matusis& Rastelli, ~99]

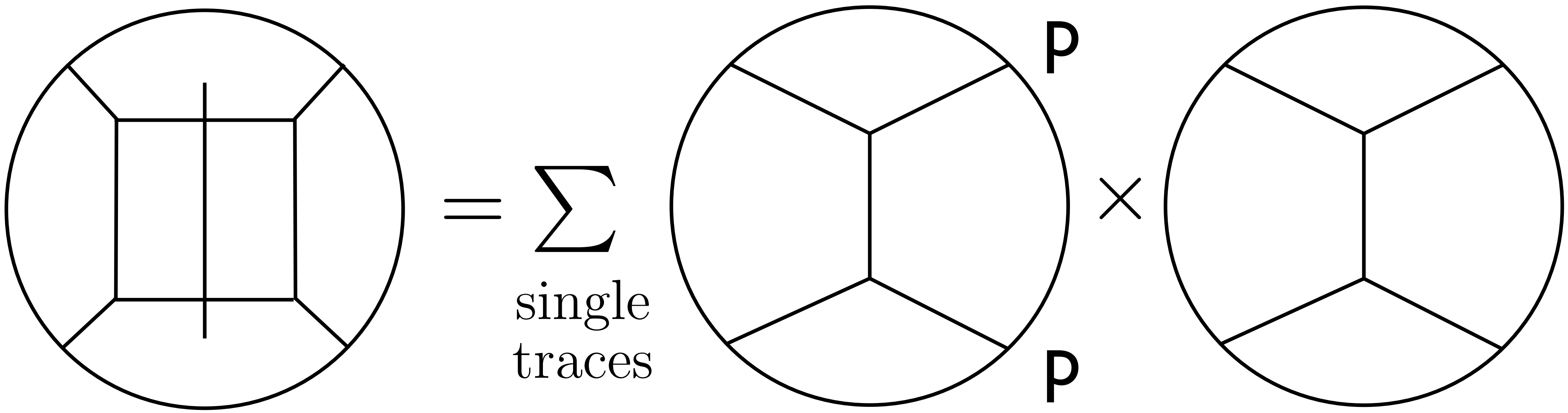
- Same answer also rederived recently using 'bootstrap' ideas (Mellin space, or making functional ansatz)

[Rastelli&Zhou 16, Drummond et al. 17]

- New: control over ambiguities ('cR^4') from CFT only

  susy ties it to a spin 4 effect: $\Rightarrow c < \frac{1}{\Delta_{\text{gap}}^2} \sim \frac{1}{\sqrt{\lambda}}$

One-loop supergravity:



Product of trees: $\langle O_2 O_2 O_p O_p \rangle \times \langle O_p O_p O_2 O_2 \rangle$

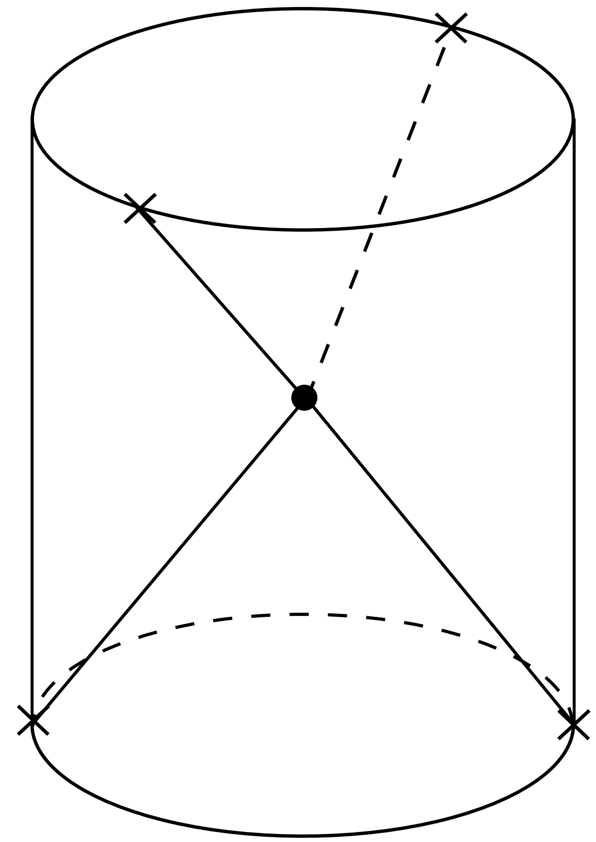
Accounts for mixing between $O_2 \Box^n O_2$ and $O_p \Box^{n'} O_p$

dDisc at one-loop was computed last summer by two groups

[Alday&Bissi, Aprile,Drummond,Heslop&Paul]

We studied the ‘bulk point’ limit of the CFT 1/N correction

[Alday,SCH ’17]



large-Δ OPE data ⇔ flat-space partial waves:

$$\lim_{n\to\infty} \frac{\left\langle a e^{-i\pi\gamma} \right\rangle_{n,\ell}}{\left\langle a^{(0)} \right\rangle_{n,\ell}} = b_\ell(s) \qquad \sqrt{s} = 2n/L$$

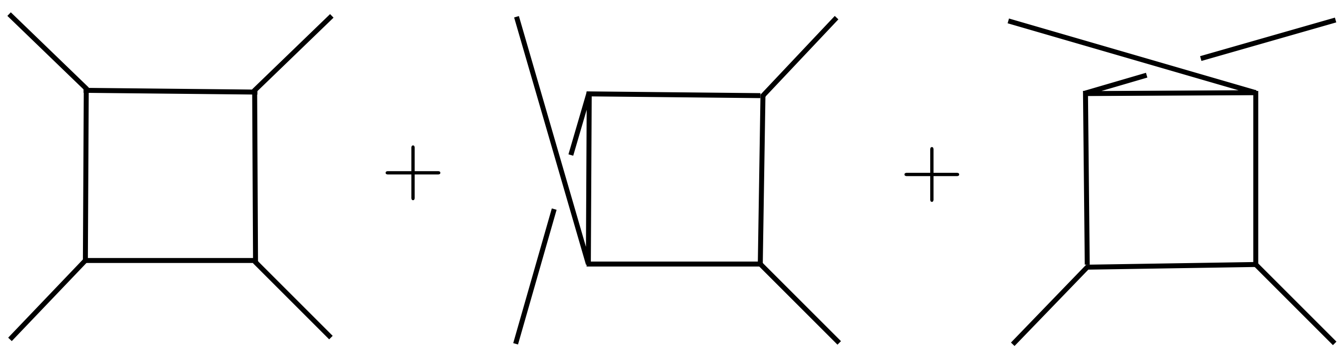
[HPPS]

Note: In AdS/CFT there is (never?) a local 5D theory

Here only flat 10-dimensional type-IIB supergravity

one-loop 10D amplitude is simple:

$$A_{10}^{sugra}(s,t) = 8\pi G_N \frac{s^3}{tu} + \frac{(8\pi G_N)^2}{(4\pi)^5}\left(I_{box}(s,t) + I_{box}(s,u) + I_{box}(t,u)\right)$$



We expand it over 5D partial waves: $A_5 = A_{10}/\mathrm{vol}\ \mathrm{S}_5$

$$A_5(s,t) = \frac{128\pi}{\sqrt{s}} \sum_{\ell\ \mathrm{even}} (\ell+1)^2 b_\ell(s)\, P_\ell(\cos\theta) \qquad P_\ell(\theta) = \frac{\sin(\ell+1)\theta}{(\ell+1)\sin\theta}$$

- We find perfect match!

$$\lim_{n\to\infty} \frac{\left\langle a e^{-i\pi\gamma} \right\rangle_{n,\ell}}{\left\langle a^{(0)} \right\rangle_{n,\ell}} = b_\ell(s)$$

- The mechanism is interesting, and becomes obvious using flat space Froissart-Gribov formula:

$$b_\ell(s) = 1 + \frac{1}{c} \frac{i\pi\,(\sqrt{s}L/2)^3}{2(\ell+1)} + \frac{1}{c^2} \frac{i\pi\,(\sqrt{s}L/2)^{11}}{\ell+1} \int_0^1 \frac{d\bar{z}}{\bar{z}^2} \left(\frac{1-\sqrt{1-\bar{z}}}{1+\sqrt{1-\bar{z}}} \right)^{\ell+1} \mathrm{Disc}_t f^{(2)}(1/\bar{z}),$$

$$\mathrm{Disc}_t f^{(2)}(1/z) = \frac{z(1-z)^2}{480} \left(\left(1 - \frac{1}{z^5} \right) \log(1-z) + \frac{1}{z} - \frac{1}{z^4} + \frac{1}{2z^2} - \frac{1}{2z^3} + i\pi - \log(z) \right)$$

correlator
$\mathcal{G}(z, \overline{z})$

$\longrightarrow$ flat space

amplitude
$A_5(s, t)$

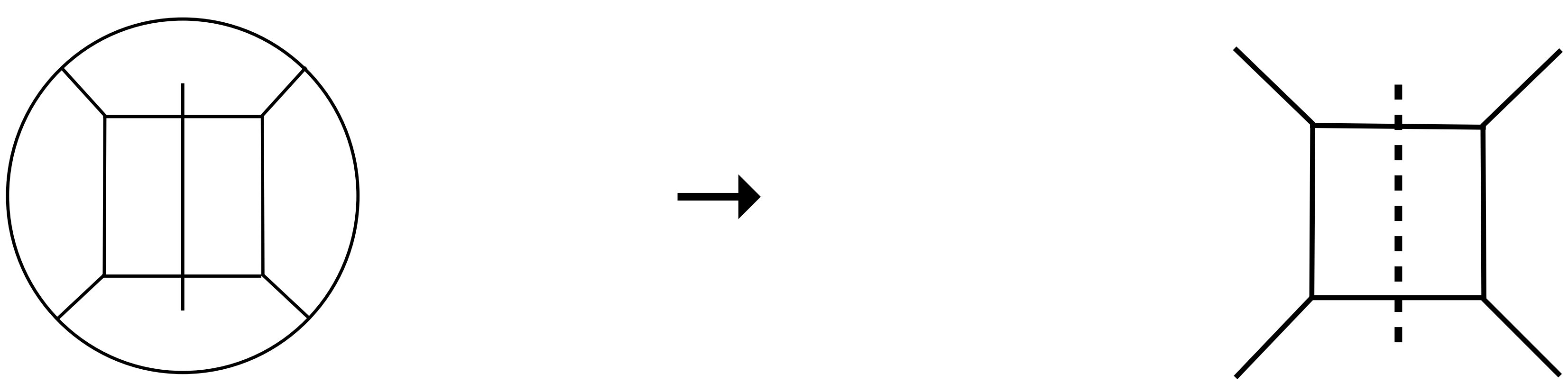
inversion integral $\Uparrow$

dispersion relation $\Uparrow$

double-disc.
$\mathrm{dDisc}\,[\mathcal{G}]$

$\longrightarrow$ flat space

discontinuity
$\mathrm{Disc}\,[A_5]$



UV completion details can affect j=0 at $1/(N^2\Delta_{\rm gap}^4)$

⇒one-loop UV divergence, reflecting R^4 counter-term

Since full integrand is positive definite, this can't be canceled:

$$C_{R^4} \geq \Lambda_{\rm UV}^2 \equiv (\Delta_{\rm gap}/L_{\rm AdS})^2$$

consistent with IIB string theory effective action

Minimal subtraction is in the swampland

More on tree-level:

-conjectured eigenvalues of anomalous dimension matrix:

$$\gamma^{(1)}_{n,\ell} = -\frac{1}{c}\frac{\Delta^{(8)}(n,\ell)}{(j+1)_6}$$

[Aprile,Drummod,Heslop&Paul '18]
[Rastelli& Zhou,'17]

-match 10D flat space amplitude (in 10D Gegenbauer's)

$$\frac{1}{\pi}\delta^{(1)} = -\frac{1}{c}\frac{(L\sqrt{s}/2)^8}{(j+1)_6}$$

Explanation& proof: $AdS_5 x S_5$ is conformally flat.
IIB tree amplitude accidentally conformal: $\sim \delta^{16}(Q)/(stu)$

⇒All S_5 harmonics form a single 10D object

[SCH& Anh-Khoi Trinh, to appear]

Outline

1. Lorentzian inversion formula:
   -setup and Regge limit of correlator
   -analyticity in spin
   -'Absorptive part' in CFT ✓

2. AdS locality
   -CFT derivation of bulk EFT
   -concrete tree and one-loop Witten diagrams ✓

3. *More should be true!*
   -Pushing 1/J expansions to spin J=0
   -Bulk point limit

Large-spin bootstrap

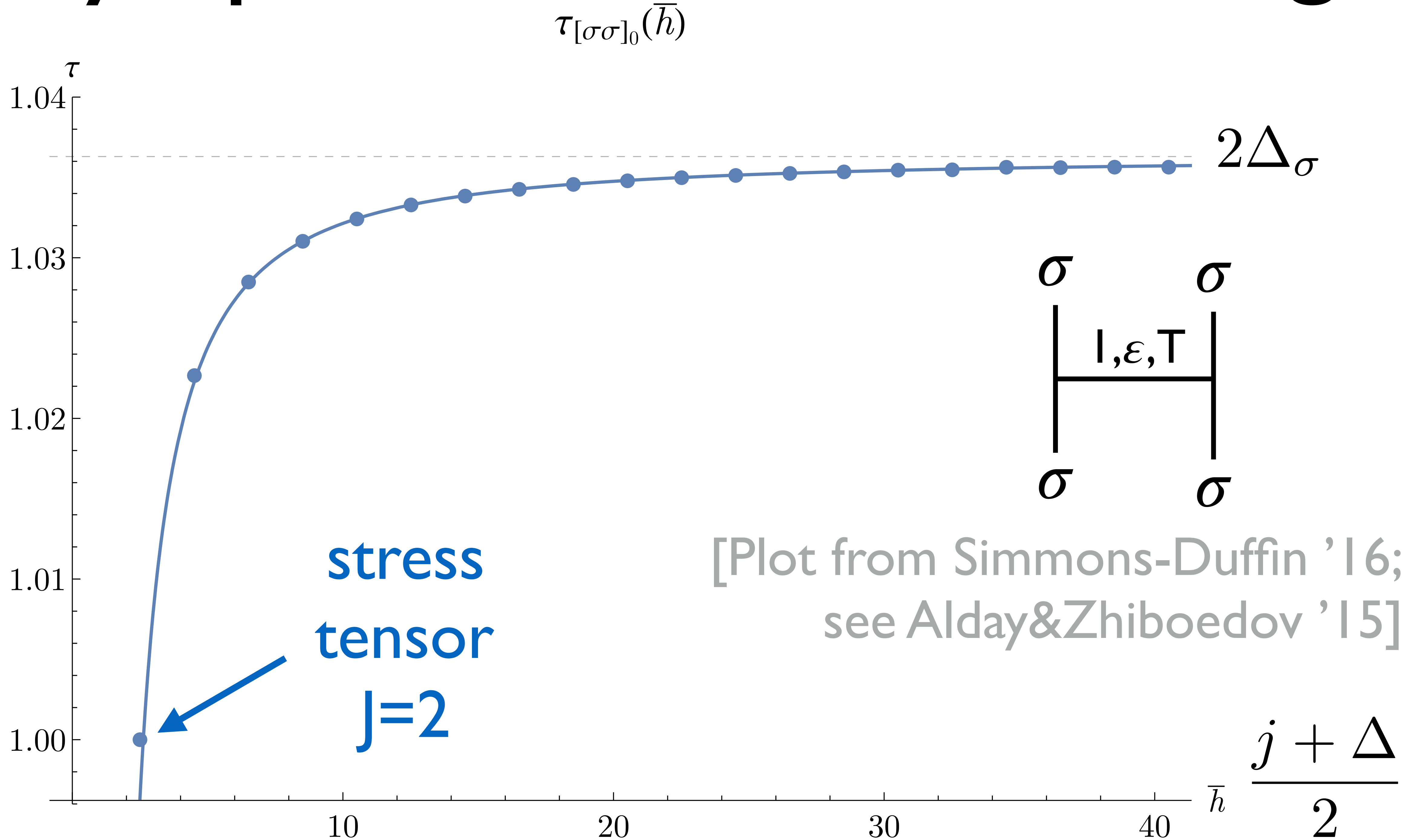
Large-J pushes inversion integral to corner $(z, \bar{z}) \to (0, 1)$

large spin in s-channel $\leftrightarrow$ low twist in t-channel

$\Rightarrow$ Solve crossing in asymptotic series in 1/J

[Komargodski&Zhiboedov,
Fitzpatrick,Kaplan,Poland&Simmons-Duffin,
Alday&Bissi&…,
Kaviraj,Sen,Sinha&…,
Alday,Bissi,Perlmutter&Aharony,…]

Asymptotic series in 3D Ising



[Plot from Simmons-Duffin '16;
see Alday&Zhiboedov '15]

All states (at least with J>1) have to lie on Regge traj.

All trajectories seem multi-twists of σ (ε)

Recipe for Ising's AdS dual:

-Take a *single* bulk scalar field σ (ε)

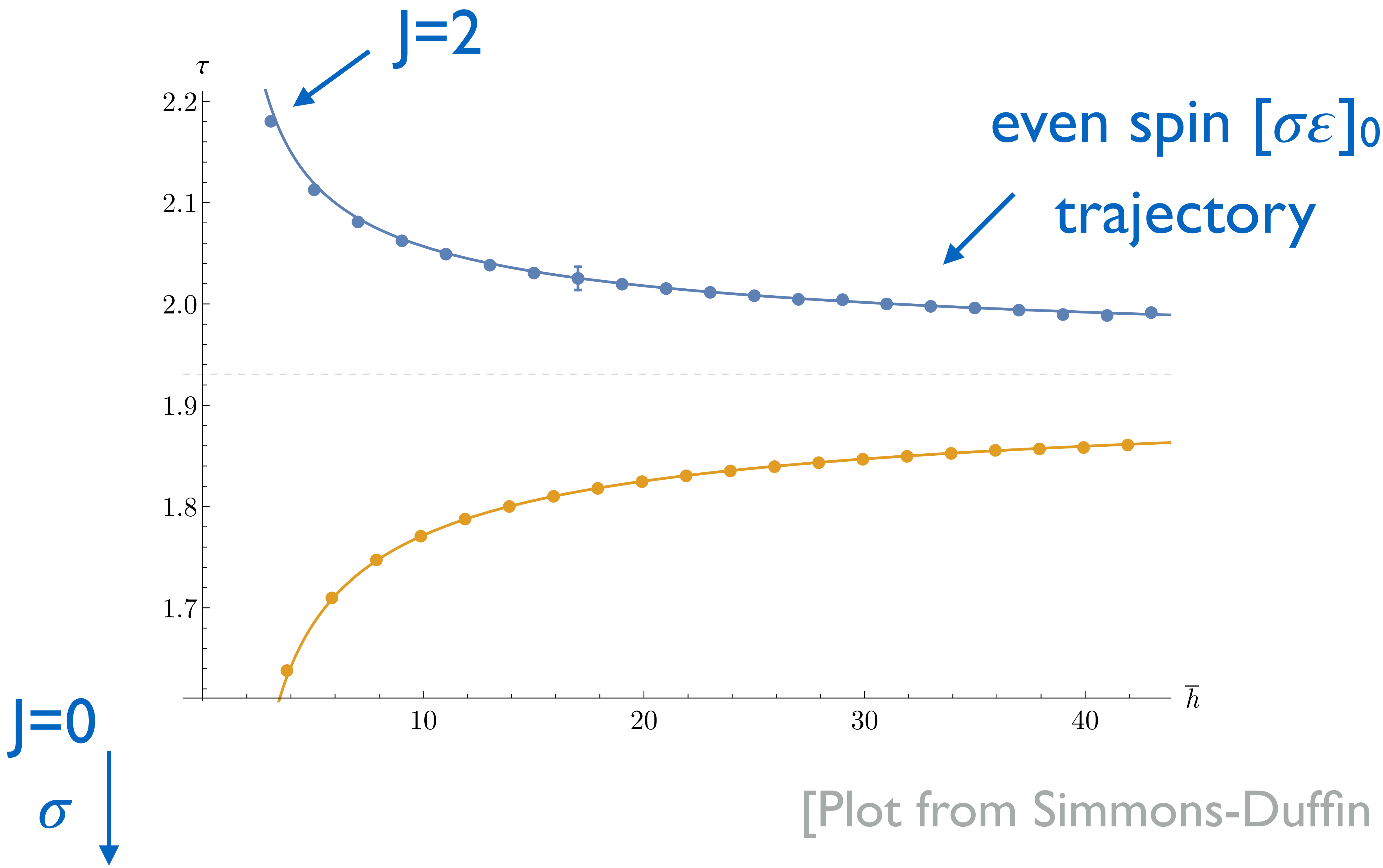
-Make Fock space of composites: size ~J R_{AdS}

-Large-J expansion= EFT on L>>R_{AdS} distances (!?)

-Graviton comes for free as a 'large' composite.
No gravity at sub-AdS distances.

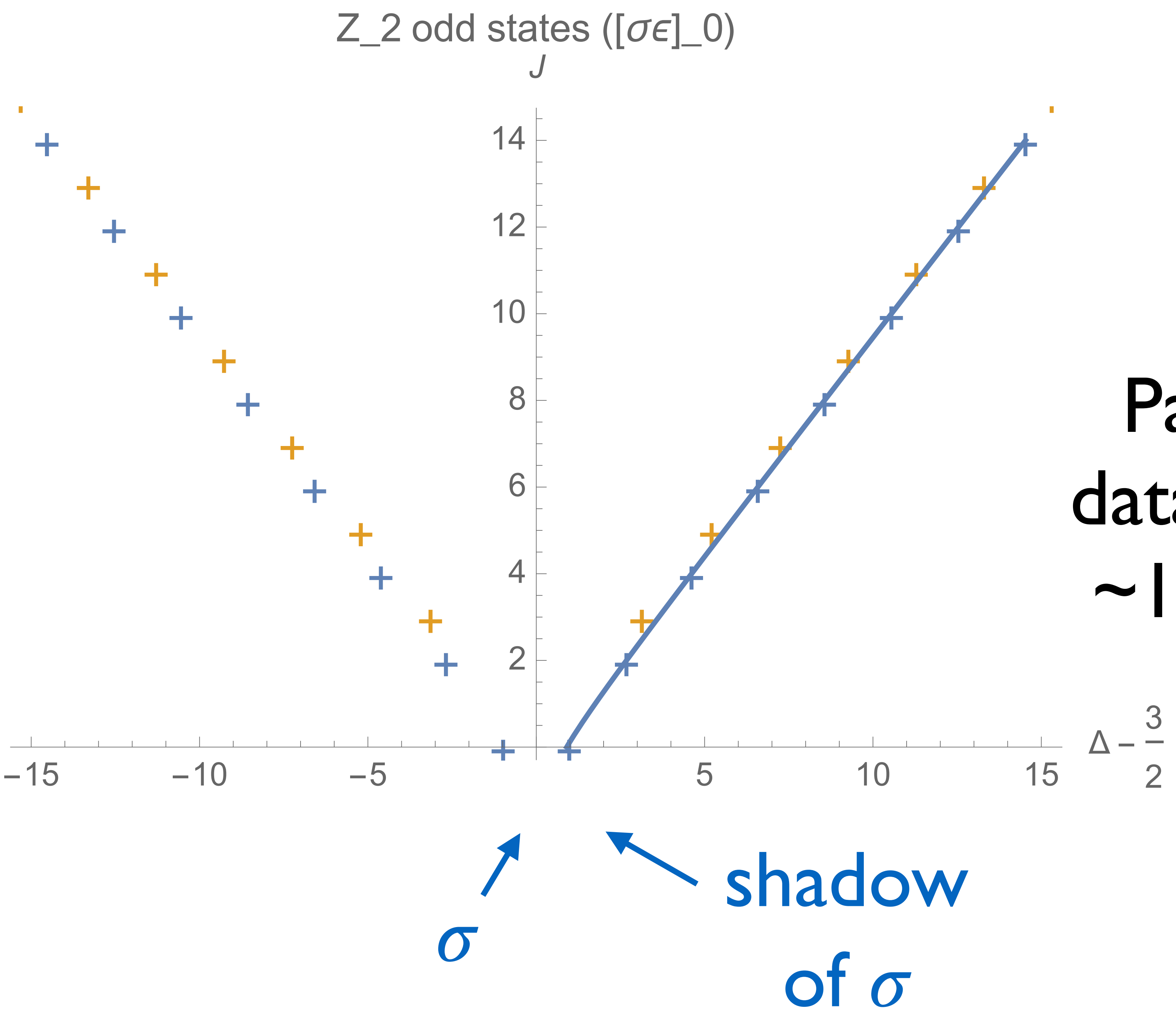
Relation to higher-spins in AdS_4?

What about spin 0?

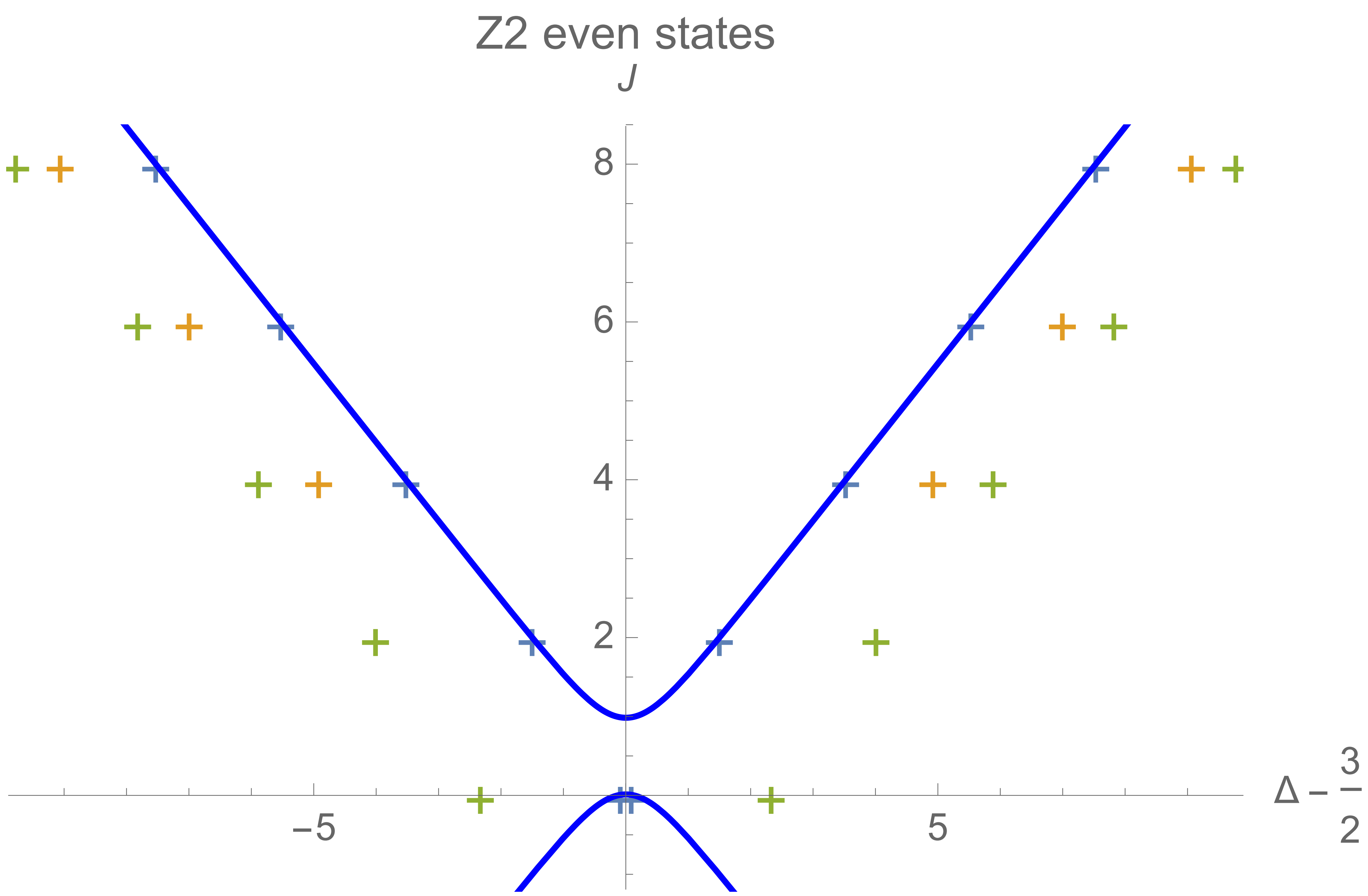


[Plot from Simmons-Duffin '16]

what's analytic is c(j,Δ): its poles come shadow-symmetric pairs $(\Delta, d-\Delta)$



Pade fit of J≥2 data passes within ~15% of shadow



(fit accounts for possible square-root branch point)

Conjectures:

[work in progress w/ Gobeil, Maloney, Zahraee]

In 3D Ising:

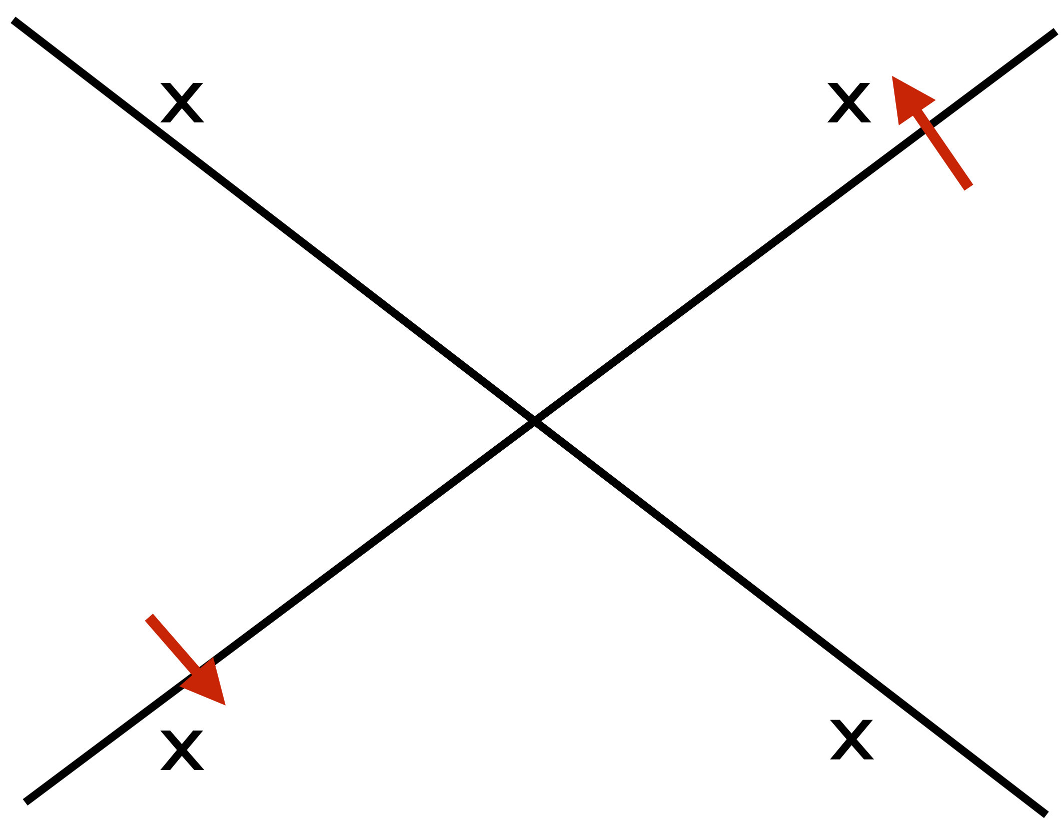
1. the shadow of σ is on the $[\sigma\varepsilon]_0^+$ trajectory, that of ε on (the continuation of) $[\sigma\sigma]_0$

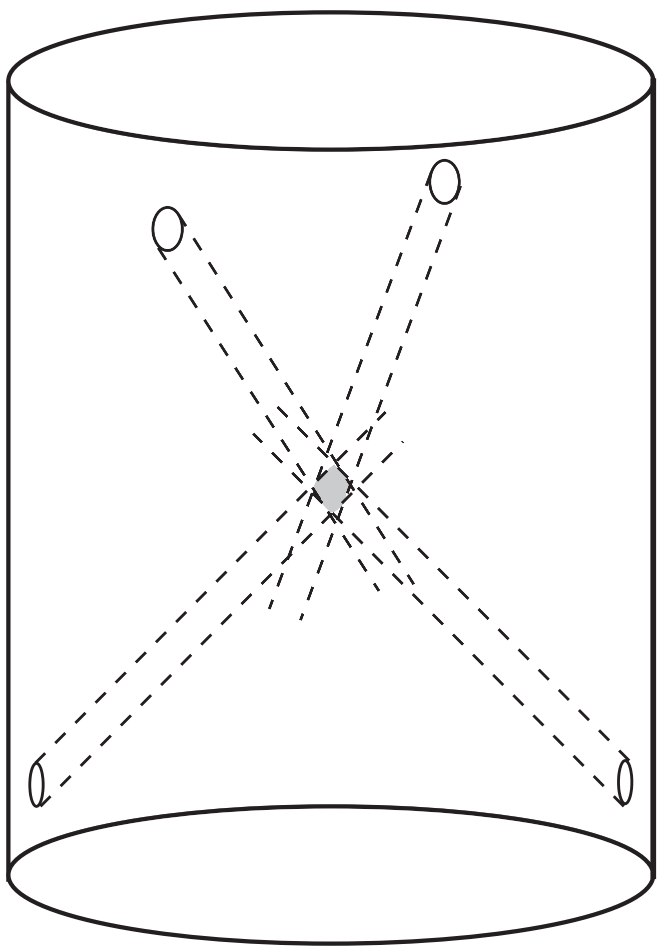
2. Residue of $[\sigma\varepsilon]_0^-$ has fine-tuned zero at J=1

3. Intercept $J^*<1$: dDisc->0 in Regge limit (corrollary: spectrum is regular (non-chaotic))

$$\lim_{\Delta\to\infty} \frac{\langle a \sin^2(\pi\gamma)\rangle_\Delta}{\langle a\rangle_\Delta} \to 0$$

Beyond Rindler: hard scattering





Analyticity in spin extends s-channel OPE to Rindler wedge:

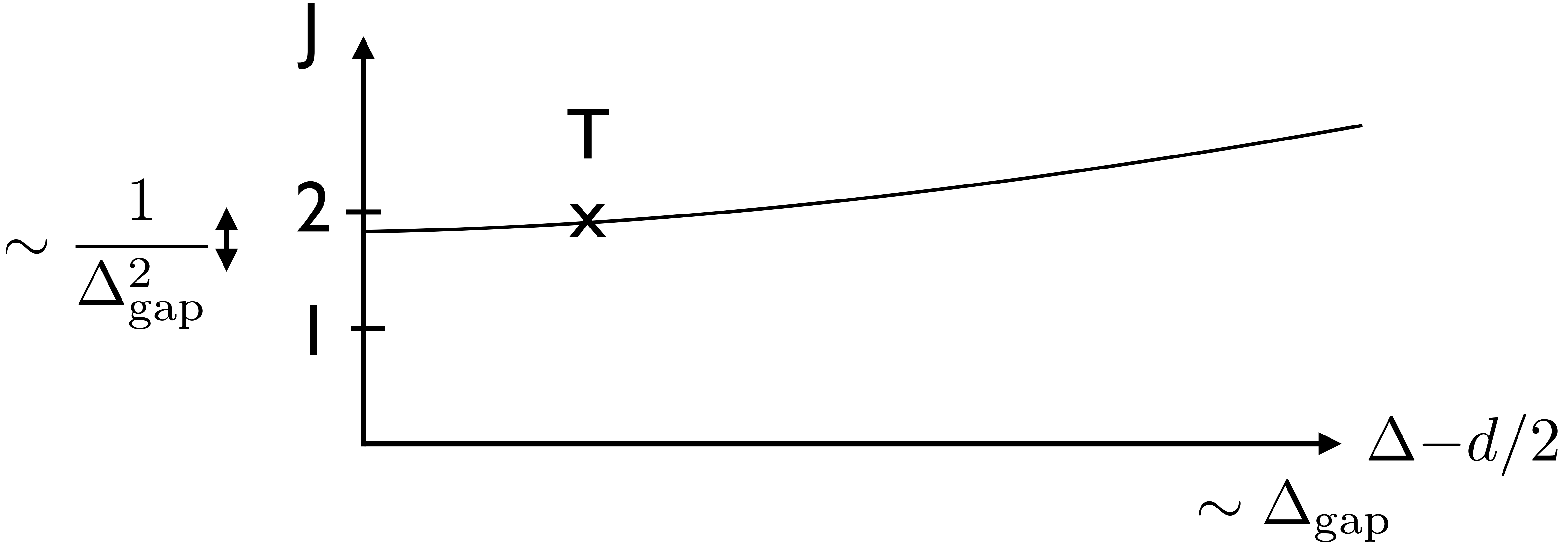
$$G(z,\bar{z}) = \int_{-i\infty}^{i\infty} dJ\ d\Delta\ \frac{c_{J,\Delta}}{1-e^{2\pi i J}} F_{J,\Delta}(z,\bar{z}) + (\mathrm{u-channel})$$

[Costa, Goncalves& Penedones '12]

Leaving Rindler, $e^{-i\pi\Delta}$ can make integral diverge!

In examples, seems saturated by leading *single-trace* traj.

$$\mathcal{B}(s,b) = \int_{-i\infty}^{i\infty} d\Delta\, r(\Delta) \frac{(-s)^{j(\Delta)-1}}{\sin(\pi J(\Delta))} \Pi_\Delta(b)$$



Theorem: Im B≥0. Imaginary part has stringy peak:

$$\mathrm{Im}\ \mathcal{B}(s,b) \sim \exp\left(-\frac{x_\perp^2}{\alpha' \log|s|}\right)$$

[Camanho, Edelstein,Maldacena&Zhiboedov '14]
[Costa, Hansen& Penedones '17]
[Kulaxizi, Parnachev& Zhiboedov '17]
[Afkhami-Jeddi, Hartman, Kundu& Tajdini '17]

signature of AdS locality

Summary

- Dispersion relation for OPE coefficients:

$$c(j,\Delta) \equiv \int_0^1 d\rho d\bar{\rho}\, g_{\Delta,j}\, \mathrm{dDisc}\, G$$

s-channel cross-channels

- Input: CFT unitarity ⇒ analyticity&positivity

- -AdS/CFT correlators from light exchanged fields
  -Heavy primaries 'integrated out' → bulk locality

- *More should be true*!

- A challenge: CFT 4-pt function ⇒ classical metrics?

(Spin versus dimension)

- Consider an AdS interaction with flat-space limit:

$$stu$$

- This has spin two in the Regge limit in all channels:

$$stu = st(s+t) \sim s^2 \equiv s^j \qquad (s \to \infty, t \text{ fixed})$$

- Not constrained (Regge limit only localizes in time!). All else with more derivatives is constrained.
- For TT$\phi\phi$, only one unconstrained spin-2 contact interaction. For TTTT, none!