New Perspective on Bulk Reconstruction

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Based on work with Dan Kabat

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Introduction

- What does AdS/CFT tell us about quantum gravity

- One way to explore this is to ask how do the degrees of freedom of the CFT arrange into bulk degrees of freedom?

- More specific: What are the CFT operators whose CFT correlation function reproduce (in some approximation) bulk theory correlation functions.

- Hopefully they have a natural CFT origin.

Bulk reconstruction

We want to find CFT operators that obey

$$< \phi(z_1, x_1) \cdots \phi(z_n, x_n) >_{CFT} = bulk\ result$$

In perturbation theory in 1/N we start with 2-point functions (early work BDHM, BKLT, Bena). A suitable expression is

$$\Phi^{(0)}(x,z) = \int dx' K(x,z|x')\mathcal{O}(x')$$

Where $K(x,z|x')$ is a smearing function determined by solving the bulk free equation of motion with $\phi(z,x) \sim_{z\to 0} z^{\Delta}\mathcal{O}(x)$.
For example in AdS_3 a good expression is (HKLL)

$$\Phi^{(0)}(Z,X,T) = \frac{\Delta - 1}{\pi} \int_{y'^2+t'^2<Z^2} dt'dy' \left(\frac{Z^2 - y'^2 - t'^2}{Z}\right)^{\Delta-2} \mathcal{O}(T+t', X+iy')$$

One can get corrections by solving the bulk equations of motion to the next order (KLL, HMPS) ,

$$(\nabla^2 - m^2)\phi = \frac{1}{N}\phi^2$$

$$\phi = \Phi^{(0)} + \frac{1}{N}\Phi^{(1)} + \cdots$$

Solving the equations perturbativly using a space-like Greens function will give the bulk field in terms of boundary data. But for this one needs to know the bulk equations of motion.

Alternativly, Inserting $\Phi^{(0)}$ into 3-point functions one gets an expression that does not obey bulk locality.

$$\int d^d x' K_i(x,z|x')\langle \mathcal{O}_i(x')\mathcal{O}_j(y_1)\mathcal{O}_k(y_2)\rangle = \frac{1}{(y_1-y_2)^{2\Delta_j}}\left[\frac{z}{z^2+(x-y_2)^2}\right]^{\Delta_k-\Delta_j} I_{ijk}(\chi)$$

$$I_{ijk}(\chi) = c_{ijk}\left(\frac{1}{\chi-1}\right)^{\Delta_*} F(\Delta_*, \Delta_* - \frac{d}{2}+1, \Delta_i - \frac{d}{2}+1, \frac{1}{1-\chi})$$

$$\Delta_* = \tfrac{1}{2}(\Delta_i + \Delta_j - \Delta_k) \qquad \chi = \frac{[(x-y_1)^2+z^2][(x-y_2)^2+z^2]}{(y_1-y_2)^2 z^2}$$

This result is gotten by computing things at large chi and analytically continuing. Importantly, it has a singularity at chi=1 and a branch cut 0<chi<1, as well as a singularity at chi=0.

The chi=1 singularity occurs when the bulk operator hits the bulk geodesic connecting the two boundary points. More generally (in AdS_3) when it hits the horizon whose bifurcation sphere is the RT surface. In higher dimension , at a fixed time T, it is a shell of radius T around the geodesic.

This is simplest seen by using the interpretation of OPE blocks (Ferrara, Grillo, Parisi-1972; Czech, Lamprou, McCandlish, Mosk, Sully; Carneiro da Cunha, Guica; de Boer, Hael, Heller, Myers-2016)

$$\mathcal{O}(y_1)\mathcal{O}(y_2) = \sum_k c_k \int_\gamma ds \Phi_k^{(0)}$$

This can be corrected by defining an improved bulk operator(KLL)

$$\phi_i(x,z) = \int d^d x' K_{\Delta_i}(x,z|x')\mathcal{O}_i(x') + \frac{1}{N}\sum_n a_n^{CFT} \int d^d x' K_{\Delta_n}(x,z|x')\mathcal{O}_n(x')$$

where $\mathcal{O}_n(x')$ are a tower of higher dimension primary double trace operators.

The coefficients a_n^{CFT} are chosen such that inserted in the 3-point function bulk locality is restored.

Turns out this also gives the correct equation of motion for the bulk operator, correct transformation laws, and includes the freedom of bulk field re-definitions (KL 2015)

This is not a satisfactory story.

- We need to know the bulk metric to compute the smearing function.
- We are using bulk argument (bulk locality) as a criterion, but why is that relevant from the CFT point of view.
- While the preceding story can be done for scalars interacting with gravity or gauge fields, they do not obey bulk locality due to Gauss constraints, so bulk locality is not a good excuse (especially for bulk gauge or gravity fields).
- Also, why did it work this way. $\Phi^{(0)}$ could have just been wrong without giving clues to what is right.
- What about entanglement, seems to play no role.

New perspective

Modular Hamiltonian

Given a density matrix one can define a modular Hamiltonian which generates a modular flow

$$H_{mod} = -\log \rho$$

For a region A in the CFT we can define a density matrix by tracing over the compliment region, so we get $H_{mod,A}$

One can similarly do for the compliment region of A, and get $H_{mod,\bar{A}}$

One defines the total modular hamiltonian as

$$\tilde{H}_{mod} = H_{mod,A} - H_{mod,\bar{A}}$$

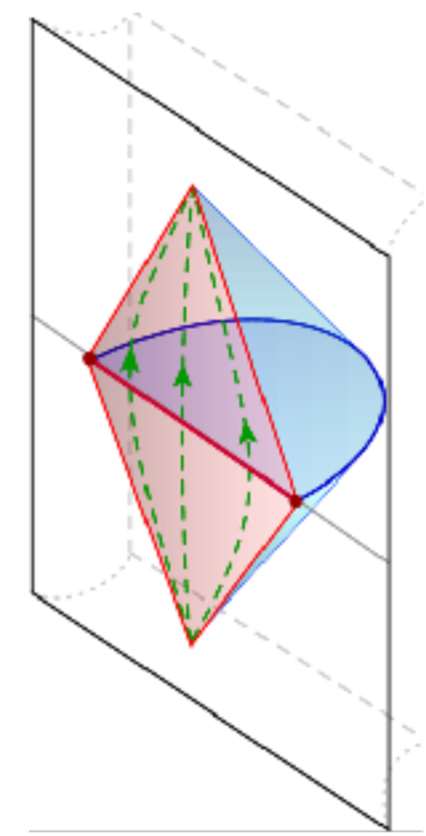
For spherical regions in the CFT ground state the expression for the modular Hamiltonian is known (Casini, Huerta, Myers)

Given a gravity dual and a region A on the boundary, one has an RT surface in the bulk which is the minimal surface in the bulk whose boundary is A.

The RT surface separates the bulk into two, and one can define a bulk modular Hamiltonain by tracing over one of the bulk regions.

In this way a bulk total bulk modular Hamiltonian can be constructed.

The action of the bulk and boundary modular Hamiltonians should be identified (JLMS)



The RT surface serve as a horizon for the modular evolution which is a fixed point of the modular flow, so one has for scalar bulk objects on the RT surface

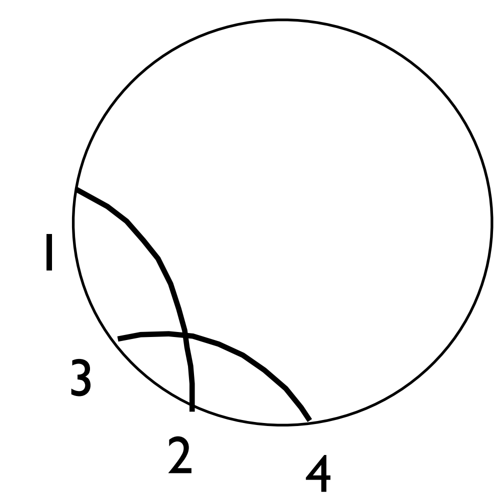
$$[\tilde{H}^{bulk}_{mod}, \Phi] = 0$$

Thus a CFT operator which represents a local scalar bulk objects inside the bulk should obey

$$[\tilde{H}^{CFT}_{mod}, \Phi] = 0$$

For any modular Hamiltonian whose RT surface goes through that point.

In AdS_3 it is enough to look at two surfaces



$$[\tilde{H}^{12}_{mod}, \Phi] = 0 \qquad [\tilde{H}^{34}_{mod}, \Phi] = 0$$

$$\tilde{H}_{mod} = \frac{2\pi}{y_2 - y_1}(Q_0 + y_1 y_2 P_0 + (y_1 + y_2) M_{01})$$

A solution to these equation is given for AdS_3, up to a space time dependent constant (which can be fixed) by (KL2017)

$$\Phi^{(0)}(Z,X,T) = \frac{\Delta-1}{\pi}\int_{y'^2+t'^2<Z^2} dt'dy' \left(\frac{Z^2-y'^2-t'^2}{Z}\right)^{\Delta-2} \mathcal{O}(T+t', X+iy')$$

for any scalar primary $\mathcal{O}$ with any dimension Δ, and any linear combination of such expressions.

This generalized earlier work by Miyaji,Numasawa,Shiba,Takayanagi,Watanabe; Nakayama and Ooguri; H.Verlinde, which relied on symmetry arguments.

So what goes wrong?

The singularity and branch cut at chi=1are obstacles for interpreting $\Phi^{(0)}$ as a local bulk operator. This was the guide to the reconstruction procedure.

But from the derivation using the modular Hamiltonian this seems puzzling. Since we know the modular Hamiltonian in the vacuum exactly why are there corrections to the locality equations.

We will show that the 0<chi<1 branch cut has a problem from the CFT point of view. (KL2018)

Claim: $$\Phi^{(0)}(x,z) = \int dx' K(x,z|x')\mathcal{O}(x')$$

is not a good CFT operator. This happens since the 3-point function becomes ill defined due to the chi=1 singularity, where the i-epsilon prescription becomes ambiguous.

It is best to look at a concrete example

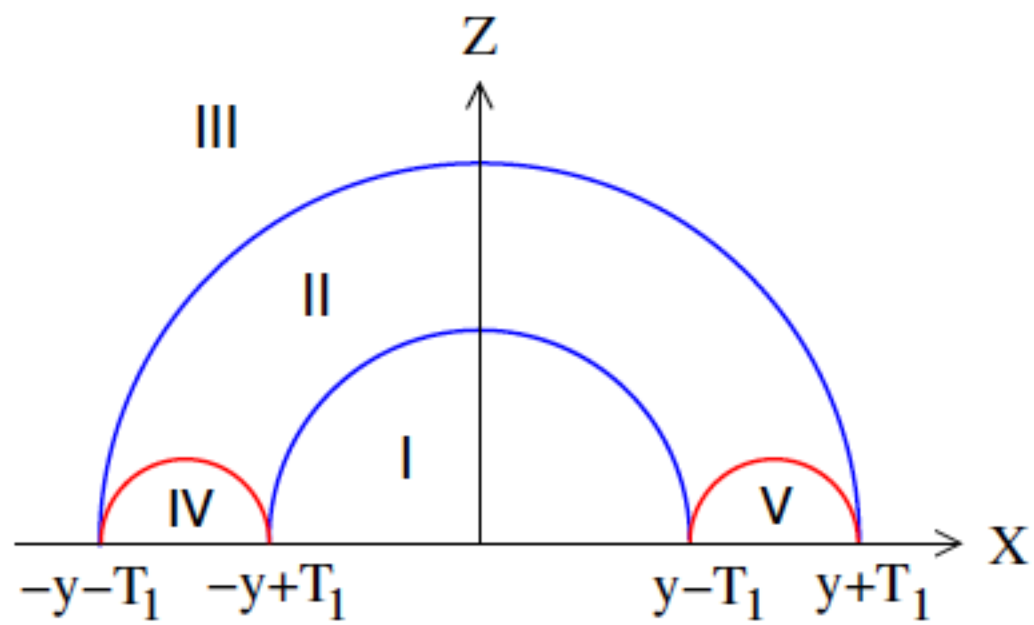
AdS_3, $\Delta = \Delta_2 = \Delta_3 = 1$, largest epsilon is to the left.

$$\langle \phi^{(0)} \mathcal{O}_2 \mathcal{O}_3 \rangle = \frac{\gamma}{2\left(T_{23}^{+} T_{23}^{-}\right)^2} \log \frac{AB}{CD}$$

where

$$\begin{aligned}
A &= -T_1^2 + (X_1 + y)^2 + Z_1^2 + 2i\epsilon_{12} T_1 \\
B &= -T_1^2 + (X_1 - y)^2 + Z_1^2 + 2i\epsilon_{13} T_1 \\
C &= -(T_1 + y)^2 + X_1^2 + Z_1^2 + i\epsilon_{12}(T_1 + y - X_1) + i\epsilon_{13}(T_1 + y + X_1) \\
D &= -(T_1 - y)^2 + X_1^2 + Z_1^2 + i\epsilon_{12}(T_1 - y + X_1) + i\epsilon_{13}(T_1 - y - X_1)
\end{aligned}$$

$(T_2 = 0,\ X_2 = -y)$ and $(T_3 = 0,\ X_3 = +y)$ $\qquad$ $T^{\pm} = T \pm X$ and $T_{ij} = T_i - T_j$



start from region	cross	resulting $i\,\mathrm{Im\,Arg}$
IV	$\chi = 0$ at $A = 0$	$-2i\epsilon_{12}T_1$
V	$\chi = 0$ at $B = 0$	$-2i\epsilon_{13}T_1$
I	$\chi = 1$ at $D = 0$	$+i\epsilon_{12}(T_1 - y + X_1) + i\epsilon_{13}(T_1 - y - X_1)$
III	$\chi = 1$ at $C = 0$	$-i\epsilon_{12}(T_1 + y - X_1) - i\epsilon_{13}(T_1 + y + X_1)$

Starting in any of the regions near the boundary and analytically continuing into region II can give a different sign for the i-epsilon, making the result in region II ill defined.

This happens only if the smeared operator is in the middle of the 3-point function, where $sign(\epsilon_{12}) \neq sign(\epsilon_{13})$

Implications

$\Phi^{(0)}$ is ill defined as a CFT operator.

Why does this happen ?, (non conventional smearing !!)

$$\phi^{(0)} = \int K\mathcal{O} = \frac{Z^{\Delta}}{2}\sum_{m=0}^{\infty}\frac{\Gamma(\Delta-1)Z^{2m}}{\Gamma(m+1)\Gamma(m+\Delta)}(\partial_+\partial_-)^m\mathcal{O}(T^+,T^-)$$

Inside the 3-point function the sum does not converge for all parameters, so it has to be defined via analytical continuation. But the i-epsilon prescription used to define the order of the operators can give different answers depending on how the analytical continuation was done

So the operator is ill defined as a CFT operator, since without giving more information, it does not give well defined correlation functions.

The problem is that the chi=1 singularity depends on the labels of the three operators, unlike the chi=0 singularity. To make the smeared operator well defined one needs to cancel this type of singularity by changing the smeared operator into a CFT operator which does not have a singularity at chi=1, inside correlation functions.

This is now an arguments un-related to bulk locality, but is totally within the CFT framework. As such it gives a complete CFT reasoning for the bulk reconstruction procedure.

This also puts the scalar interacting with gauge or gravity fields on the same footing, since Gauss law constraint make things non-local, but still well defined.

Associativity and bulk quantum mechanics

$\Phi^{(0)}$ is an infinite sum of well defined linear local CFT operators. The singularity and resulting analytical continuation can be interpreted as resulting in the breakdown of one of the properties of a linear operator, i.e associativity.

The ambiguity, on which of the branches of the singularity one ends on, is interpreted as a choice in the order of multiplication for a non associative operator

$$\langle\left(\mathcal{O}_2\phi^{(0)}\right)\mathcal{O}_3\rangle \qquad \text{and} \qquad \langle\mathcal{O}_2\left(\phi^{(0)}\mathcal{O}_3\right)\rangle$$

One evidence for this comes from taking OPE limits in which one of the boundary operators is far away, and the order of multiplication is natural.

As a non associative operator it treats the other two operators similarly

$$\langle \left([\mathcal{O}_2, \phi^{(0)}] \right) \mathcal{O}_3 \rangle \neq 0$$
$$\langle \mathcal{O}_2 \left([\phi^{(0)}, \mathcal{O}_3] \right) \rangle \neq 0$$

The bulk reconstruction procedure produces an operator in 1/N perturbation theory which is associative (lacking the chi=1 singularity), and surprisingly obeys a variety of bulk properties.

However beyond perturbation theory this will not be possible, and the best bulk operators we can write are non-associative. This means that a regular bulk Hilbert space interpretation is not possible beyond perturbation theory around a fixed background.
(see also Papadodimas and Raju)

Bulk quantum mechanics is at best a perturbative approximation to something else. Thinking about bulk quantum gravity as a regular quantum theory may not be correct.