A Critique of the Fuzzball Program

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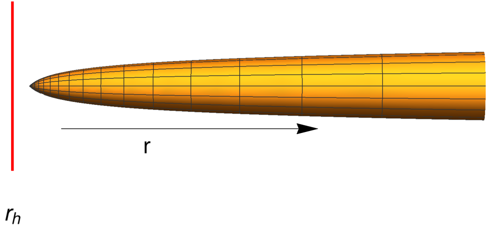
Outline

Fuzzballs

- Fuzzballs are classical solutions with the same charges as the black-hole.
- Look like black-holes at long-distances. But, fuzzballs have no horizon.
- Avoid no-hair theorem, because an extra-dimension shrinks to zero before we reach the horizon.



The Fuzzball proposal

- Fuzzballs have structure; the extra-dimension can shrink to zero, in various ways.
- Claim is that these distinct solutions are the true microstates of the black-hole.
- Fuzzball program also claims that black-holes have no interior. (This feature also suggested as resolution to information paradox.)

Summary: viability of the fuzzball proposal

We will argue that

- General statistical mechanics considerations + simple physical assumptions $\implies$ black-hole microstates cannot be represented by distinct, reliable, geometries.
- As a corollary, fuzzballs cannot serve as reliable indicators of the nature of the black-hole interior.

These general arguments are backed by specific calculations in various sets of fuzzball solutions.

Outline

Typical States

- Let H_E be a subspace of the Hilbert space; $\dim(H_E) = e^S$.

- **Result:** Typical states picked with the Haar measure, $d\mu_\psi$, on this space are exponentially close to the maximally mixed state.

$$\int \langle \Psi | A | \Psi \rangle d\mu_\psi = \mathrm{Tr}(\rho A),$$

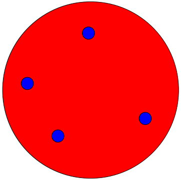
where $\rho = e^{-S} P_E$

- Deviations are exponentially suppressed

$$\int \left(\langle \Psi | A | \Psi \rangle - \mathrm{Tr}(\rho A) \right)^2 d\mu_\psi \leq \frac{\sigma_{\text{ens}}}{e^S + 1}$$

with $\sigma_{\text{ens}} = \left(\mathrm{Tr}(\rho A^2) - (\mathrm{Tr}\rho A)^2 \right)$.

Typicality of most states



Most states are very close to typical. Volume of atypical states is exponentially suppressed.

Implications for the fuzzball program

- Almost all black-hole microstates correspond to some universal average geometry
- What is this universal geometry?

Universal average geometry

Expectation: The conventional black-hole geometry — after incorporating classical string-theory corrections — correctly computes the average value of bulk observables such as the metric and correlation functions of the metric as long as we are not within Planck length of the horizon.

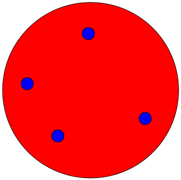
Average Geometry

What if the average geometry is not the conventional black hole? Then

- Many correlators obtained naturally from continuing Euclidean computation. So, if average geometry $\neq$ conventional black hole then even the Euclidean saddle-point is not good.
- Correlation functions in the CFT at finite temperature should be compared to this other universal geometry and not the black-hole.

We will not consider this possibility further.

Fuzzballs as a basis



- Perhaps fuzzballs form an atypical basis?
- But even a basis cannot be too atypical.

Limits on atypicality of a basis

- With
$$\sigma_{\text{ens}} = \mathrm{Tr}(\rho A^2) - \left[\mathrm{Tr}(\rho A)\right]^2$$
consider an observable A where
$$\frac{\sigma_{\text{ens}}}{\langle A \rangle} = \mathrm{O}\left(\frac{1}{S^{\alpha}}\right)$$
for some positive number α.

- Let $|v_{\alpha_1}\rangle \ldots |v_{\alpha_M}\rangle$ be those elements of a basis where $\frac{\langle v_{\alpha_j}|A|v_{\alpha_j}\rangle - \langle A \rangle}{\langle A \rangle}$ remains finite in the thermodynamic limit.

- Then $\frac{M}{e^S}$ vanishes at least as fast as $\mathrm{O}\left(\frac{1}{S^{2\alpha}}\right)$.

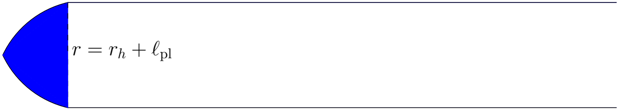
Estimating σ_{ens}

- The conventional black hole can be used to estimate σ_{ens}.
- Take A to be the metric operator at a point well separated from the horizon.
- Then $\frac{\sigma_{\text{ens}}(A)}{\langle A \rangle} \sim \frac{1}{\sqrt{S}}$.

Limits on atypicality

So, even if fuzzballs form an atypical basis, almost all fuzzball states must have metric expectation values within $O\left(\frac{1}{\sqrt{S}}\right)$ of the black-hole away from the horizon.

So if fuzzballs are microstates, typical fuzzballs must resemble a black-hole almost exactly up to the horizon; and possibly deviate a Planck distance away.



Eigenstate thermalization

- Eigenstate thermalization:

$$\langle v_j | A | v_i \rangle = A_i \delta_{ij} + B e^{\frac{-S}{2}} R_{ij}$$

- Requires that typical fuzzball states resemble black hole closely for all simple probes even for operators where σ_{ens} is large.
- We will NOT assume the ETH.

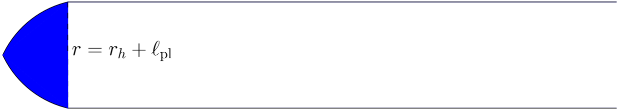
Small energy gap

- We have e^S states in a band $[E_0 - \Delta, E_0 + \Delta] \implies$ Gap between neighbouring energy eigenstates is e^{-S}.
- Gap can be detected by perturbing a given energy eigenstate
$$A|E_i\rangle = \sum_{i,j} A_{ij}|E_j\rangle$$
- In the dual geometry, gap can be detected by perturbing the geometry with a massive particle.
- Black holes allow a continuous spectrum because of the horizon.

Independent argument that the fuzzball metric must "almost" have a horizon.

Planck-scale structure

Can classical solutions be used to argue for such Planck-scale structure?



Difference and Quantumness Parameters

Let $\hat{O}(r)$ be a bulk observables with classical expectation value $O^{\text{bh}}(r)$ in the conventional geometry. In a fuzzball state

$$\langle f|\hat{O}(r)|f\rangle = O^{\text{fuzz}}(r).$$
$$\sigma^2(r) = \langle f|\hat{O}(r)^2|f\rangle - \langle f|\hat{O}(r)|f\rangle^2,$$

Define quantumness parameter

$$\mathrm{q}_O(r) = \left|\frac{\sigma(r)}{O^{\text{fuzz}}(r)}\right|.$$

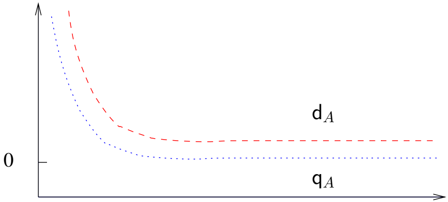
and difference parameter

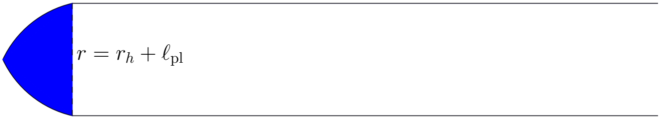
$$\mathrm{d}_O(r) = \left|\frac{O^{\text{bh}}(r) - O^{\text{fuzz}}(r)}{O^{\text{fuzz}}(r)}\right|.$$

These measure how reliable a classical solution is and how distinguishable it is from the black-hole.

Planck scale structure?

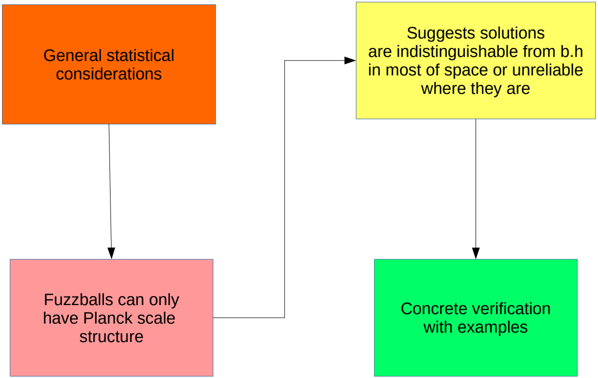
Expect that





So the solution is either indistinguishable from the conventional black-hole or unreliable.

Summary of logic



Summary of rest of talk

We now verify these expectations in

1. Original Lunin-Mathur two-charge solutions
2. Recently discovered multi-charge solutions

[Bena et al., 2016–17]

Outline

Lunin-Mathur Geometries

Claimed to be dual to Ramond ground states of the D1-D5 CFT.

$$ds^2 = e^{-\frac{\phi}{2}} ds^2_{\text{str}}, \quad e^{-\phi} = \frac{f_5}{f_1},$$

$$ds^2_{\text{str}} = \frac{1}{\sqrt{f_1 f_5}} \left(-(dt + A)^2 + (dy + B)^2 \right) + \sqrt{f_1 f_5} d\vec{x}^2 + \sqrt{\frac{f_1}{f_5}} d\vec{z}^2,$$

$$f_5 = 1 + \frac{Q_5}{L} \int_0^L \frac{ds}{|\vec{x} - \vec{F}(s)|^2}; \quad f_1 = 1 + \frac{Q_5}{L} \int_0^L \frac{|\vec{F}'(s)|^2}{|\vec{x} - \vec{F}(s)|^2}$$

$$A_i = \frac{Q_5}{L} dx^i \int_0^L \frac{F_i'(s)}{|\vec{x} - \vec{F}(s)|^2} ds; \quad dB = *_4 dA$$

$$C = \frac{1}{f_1} (dt + A) \wedge (dy + B) + \mathcal{C}; \quad d\mathcal{C} = - *_4 df_5.$$

$$W_i = \frac{Q_5}{L} \int_0^L \frac{F_i(s)}{|\vec{x} - \vec{F}(s)|^2} ds.$$

Conventional solution

- Conventional solution obtained by

$$f_1 \to 1 + \frac{Q_1}{\vec{x}^2}; \quad f_5 \to 1 + \frac{Q_5}{\vec{x}^2}$$

with

$$ds^2_{\text{str}} = \frac{1}{\sqrt{f_1 f_5}}\left(-dt^2 + dy^2\right) + \sqrt{f_1 f_5}\, d\vec{x}^2 + \sqrt{\frac{f_1}{f_5}}\, d\vec{z}^2,$$

- Conventional solution has vanishing horizon; different setting compared to macroscopic black holes.

Quantization of Lunin-Mathur Solutions

- Upon quantization

$$F^k(s) = \mu \sum_{n>0} \frac{1}{\sqrt{2n}} \left(a_n^k e^{\frac{-2\pi i n s}{L}} + (a_n^k)^\dagger e^{\frac{2\pi i n s}{L}} \right),$$

[Rychkov, 2005]

- States must satisfy $H = \sum n a_n^\dagger a_n = N_1 N_5$. Also

$$\mu = \frac{g_s}{R\sqrt{V_4}}, \quad L = \frac{2\pi Q_5}{R} \quad Q_5 = g_s N_5; \quad Q_1 = g_s N_1 / V_4$$

$$S_{\text{fuzz}}(E) = 2\pi \sqrt{\frac{2 N_1 N_5}{3}}$$

Excitations on compact-manifold can reproduce full entropy.

[C. Krishnan, A. Raju, 2015]

Physical Quantities Computed

- We will compute

$$\langle f_5 \rangle, \quad \langle f_1 \rangle, \quad \langle A_i \rangle, \quad \langle W_i \rangle$$

and

$$\langle f_5^2 \rangle, \quad \langle f_1^2 \rangle, \quad \langle W_i W_j \rangle$$

- We can compute "thermal" expectations

$$\langle O \rangle_\beta = \mathrm{Tr}\left(e^{-\beta H} O \right)$$

where $$\beta = \left(\frac{2\pi^2}{3 N_1 N_5} \right)^{\frac{1}{2}}$$

is the inverse-"temperature" at which $\langle H \rangle = N_1 N_5$.

- Precisely verify our expectations of d (difference parameter) and q (quantumness parameter)

Quantum Expectation Values

For one-point functions we find

$$\langle f_5 - 1\rangle_\beta = Q_5 \frac{1 - e^{-\frac{r^2}{\tau}}}{r^2}$$

$$\langle f_1 - 1\rangle_\beta = Q_1 \frac{\left(1 - e^{-\frac{r^2}{\tau}}\right)}{r^2}$$

$$\langle A_i\rangle_\beta = 0.$$

$$\langle W_i\rangle_\beta = -\frac{Q_5}{r^4}\left(\tau x_i e^{-\frac{r^2}{\tau}}\left(1 - e^{\frac{r^2}{\tau}} + \frac{r^2}{\tau}\right)\right)$$

where

$$\tau = \frac{\pi^2 \mu^2}{3\beta}$$

Implications of one-point functions

- The "average" geometry differs from conventional geometry when $r^2 = \tau$.

- At $r^2 = \tau$, compact-direction radius is $(\alpha' = 1)$

$$R^2_{\text{stretched}} = \frac{\pi}{2}\sqrt{\frac{2}{3}}\left(\frac{Q_1}{Q_5}\right)^{\frac{1}{4}} \frac{\ell_{\text{pl}}^4}{\sqrt{V_{\text{com}}}}$$

- $V_{\text{com}} \equiv$ volume of the compact-manifold in string-frame. So, we need $V_{\text{com}} \geq 1$. Dilaton should not blow up: $\frac{Q_1}{Q_5} = \mathrm{O}(1)$.

$$\implies R_{\text{stretched}} \ll \ell_{\text{pl}}!$$

So the "quantum-corrected" fuzzball geometry deviates from conventional geometry after compact direction has shrunk below Planck scale!

Expectations for d and q

$$\langle f_5 \rangle_\beta = 1 + Q_5 \frac{1 - e^{-\frac{r^2}{\tau}}}{r^2}$$

$$\langle f_5 \rangle_{\text{bh}} = 1 + \frac{Q_5}{r^2}$$

- Away from $r^2 = \tau$, geometry is indistinguishable from the conventional geometry.
- Close to $r^2 = \tau$, quantum fluctuations expected to be large, so geometry is unreliable.

Quantum Fluctuations in f_5

We find

$$\langle (f_5 - 1)^2 \rangle = \int_0^L \frac{ds}{L}\frac{d\tilde{s}}{L}\frac{e^{-\frac{r^2}{c}}}{c^2}\left[\mathrm{Ei}\left(\frac{r^2}{c}\right) - 2\mathrm{Ei}\left(\frac{(\tau - c)r^2}{\tau c}\right) + \mathrm{Ei}\left(\frac{(\tau - c)r^2}{c(\tau + c)}\right)\right] + \frac{2\tau e^{-\frac{r^2}{\tau}}}{cr^2(\tau - c)} - \frac{(\tau + c)e^{-\frac{2r^2}{\tau + c}}}{cr^2(\tau - c)} - \frac{1}{cr^2}$$

where

$$c = \frac{\mu^2}{\beta}\left[Li_2\left(e^{-\frac{2i\pi(s-\tilde{s})}{L}}\right) + Li_2\left(e^{\frac{2i\pi(s-\tilde{s})}{L}}\right)\right]$$

and

$$Ei(x) = -\int_{-x}^{\infty} e^{-t}\mathcal{P}\left(\frac{1}{t}\right) dt$$

Integral over $s, \tilde{s}$ must be done numerically.

Quantumness and Deviation Parameters for f_5: small r

$$\mathrm{d}_5 = \left| \frac{\langle (f_5 - 1) \rangle_\beta - f_5^{\mathrm{bh}} + 1}{\langle (f_5 - 1) \rangle_\beta} \right|$$

$$\mathrm{q}_5 = \left| \frac{\left(\langle (f_5 - 1)^2 \rangle_\beta - \langle (f_5 - 1) \rangle_\beta^2 \right)^{\frac{1}{2}}}{\langle (f_5 - 1) \rangle_\beta} \right|$$

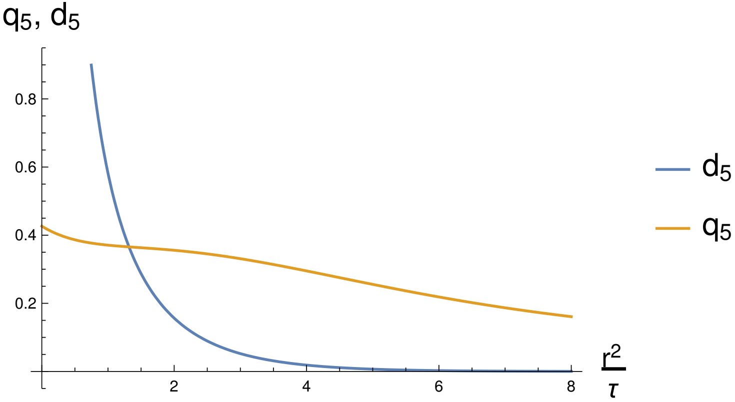
- Solution differs from the black-hole only around $r^2 = \mathrm{O}(\tau)$. For small r

$$\mathrm{d}_5 = -\frac{\tau}{r^2} + \frac{1}{2}; \quad \mathrm{q}_5 = 0.426 - 0.119 \frac{r^2}{\tau};$$

- So, quantum fluctuations are $\mathrm{O}(1)$ at $r = 0$.

Results for Difference and Quantumness Parameters

Can compute fluctuations for larger *r* numerically.



Precisely as expected, solution is either indistinguishable from the conventional solution or unreliable.

Quantum fluctuations in W_i

$$\frac{1}{Q_5^2}\langle W_i W_j\rangle = \mathcal{A}\delta_{ij} + \mathcal{B}x_i x_j$$

where

$$\begin{aligned}\mathcal{A} =& \frac{(c-\tau)e^{-\frac{r^2}{c}}\left(r^2(c-\tau)+3c(\tau+c)\right)\left(-2\mathrm{Ei}\left(\frac{(\tau-c)r^2}{\tau c}\right)+\mathrm{Ei}\left(\frac{(\tau-c)r^2}{c(\tau+c)}\right)+\mathrm{Ei}\left(\frac{r^2}{c}\right)\right)}{12c^4}\\ &+\frac{\tau e^{-\frac{r^2}{\tau}}\left(-\tau c^2(\tau+3c)+r^4(\tau-c)^2+cr^2(c-\tau)(2\tau+3c)\right)}{6c^3r^4(\tau-c)}\\ &+\frac{(\tau+c)e^{-\frac{2r^2}{\tau+c}}\left(c^2r^2(\tau+c)^2+r^6\left(-(\tau-c)^2\right)+2cr^4(\tau-c)(\tau+c)\right)}{12c^3r^6(\tau-c)}\end{aligned}$$

and

$$\begin{aligned}\mathcal{B} =& \frac{\left(\tau^2+4\tau c+c^2\right)e^{-\frac{r^2}{c}}\left(-2\mathrm{Ei}\left(\frac{(\tau-c)r^2}{\tau c}\right)+\mathrm{Ei}\left(\frac{(\tau-c)r^2}{c(\tau+c)}\right)+\mathrm{Ei}\left(\frac{r^2}{c}\right)\right)}{6c^4}\\ &+\frac{\tau e^{-\frac{r^2}{\tau}}\left(\tau^2\left(2c^2+cr^2+r^4\right)+\tau c\left(6c^2+5cr^2+4r^4\right)+c^2r^2\left(6c+r^2\right)\right)}{3c^3r^6(\tau-c)}\\ &-\frac{(\tau+c)e^{-\frac{2r^2}{\tau+c}}\left(\tau^2\left(2c^2+cr^2+r^4\right)+2\tau c\left(2c^2+3cr^2+2r^4\right)+c^2\left(2c^2+5cr^2+r^4\right)\right)}{6c^3r^6(\tau-c)}\end{aligned}$$

Difference and Quantumness Parameters for W_i

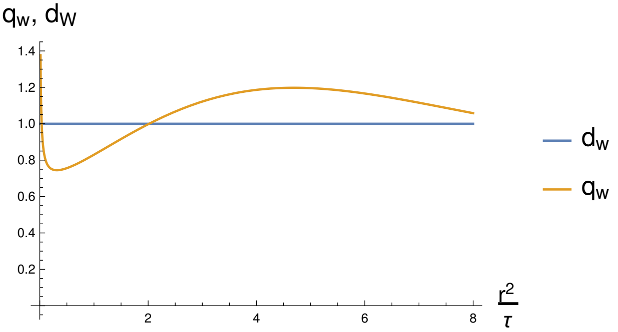
$$
\begin{aligned}
\mathrm{d}_W &= 1 \\
\mathrm{q}_W &= \frac{\left(\hat{x}^i \hat{x}^j \langle W_i W_j \rangle_\beta - \hat{x}^i \hat{x}^j \langle W_i \rangle_\beta \langle W_j \rangle_\beta\right)^{\frac{1}{2}}}{\hat{x}^i \hat{x}^j \langle W_i \rangle_\beta \langle W_j \rangle_\beta}
\end{aligned}
$$

At small r, we have

$$
\mathrm{q}_W = 0.140 \frac{\sqrt{\tau}}{r} + 1.587 \frac{r}{\sqrt{\tau}}
$$

Difference and Quantumness Parameters for W_i

For larger r, we can plot



Results: 2-charge fuzzballs

- We can compute the average fuzzball geometry for two-charge solutions.
- Differs from the conventional geometry at the Planck scale.
- The average geometry is unreliable where it is interesting.

Entropy puzzle?

- Most of the entropy comes from Planck-sized fuzzballs: typical size of F is $\tau^{\frac{1}{2}}$.

$$\sum_i \int_0^L \langle : F^i(s) F^i(s) :\rangle_\beta \frac{ds}{L}. = 2\tau$$

$$\sum_{i,j} \int_0^L \langle : F^i(s) F^i(s) :: F^j(\tilde{s}) F^j(\tilde{s}) :\rangle_\beta \frac{ds}{L}\frac{d\tilde{s}}{L} = \frac{22}{5}\tau^2.$$

- How does right entropy emerge from unreliable solutions?

- Guess: Solutions are reliable for large $F(s)$. In this region of phase space, solutions can be counted reliably. This answer can be extrapolated to all of phase space to get correct total entropy.

Outline

Multi-charge solutions

- Lunin-Mathur geometries correspond to solutions that have no horizon classically.
- Several solutions with same charges as macroscopic black-holes have been found.
- A recent larger class was found by Bena, Giusto, Martinec, Russo, Shigemori, Turton, Warner (2016–17).

Multi-charge solutions

$$ds_6^2 = -\frac{2}{\sqrt{\mathcal{P}}}(dv + \beta)(du + \omega + \frac{1}{2}\mathcal{F}(dv + \beta)) + \sqrt{\mathcal{P}} ds_4^2$$

$$u = (t - y)/\sqrt{2}; v = (t + y)/\sqrt{2}; y \sim y + 2\pi R_y;$$

$$ds_4^2 = \frac{\Sigma dr^2}{r^2 + a^2} + \Sigma d\theta^2 + (r^2 + a^2)\sin^2\theta d\phi^2 + r^2\cos^2\theta d\psi^2;$$

$$\mathcal{P} = Z_1 Z_2 - Z_4^2; \quad \beta = \frac{a^2 R_y}{\sqrt{2}\Sigma}(\sin^2\theta d\phi - \cos^2\theta d\psi);$$

$$\Sigma = (r^2 + a^2\cos^2\theta)$$

Solutions are asymptotically AdS and labeled by integers n, m, k and parameters a, b, R_y. We only consider $k = 1, m = 0$, arbitrary n.

Multi-charge solutions

- Charges are

$$J_L = \frac{\mathcal{N}}{2}\left(a^2 + \frac{m}{k}b^2\right); \quad J_R = \frac{\mathcal{N}}{2}a^2;$$
$$M = P_y = \frac{\mathcal{N} n}{2R_y}b^2.$$

with $\mathcal{N} = \frac{n_1 n_5}{a^2+b^2/2}$. We will denote $\kappa = \frac{b}{a}$.

- The asymptotic AdS radius is

$$\frac{\lambda^4}{R_y^2} = a^2 + b^2/2.$$

- Useful to think of $b \sim \mathrm{O}(\lambda)$. Then "$a$" controls the size of the fuzzball.

Scalar Wightman Function

- We will compute

$$G(\omega, \gamma) = \int \langle \Psi | O(t, y) O(0, 0) | \Psi \rangle e^{\frac{i \omega t}{R_y}} e^{-\frac{i \gamma y}{R_y}} dt dy$$

for a marginal scalar operator $O(t, y)$ on the boundary.

- Note this is a Wightman function.

Physical Quantity of Interest: Large γ Behaviour

- At large-γ one can prove for the thermal Wightman function that
$$\lim_{\gamma \to \infty} \frac{-\log |G_{\omega,\gamma}|}{\gamma} \geq \frac{\beta}{2}$$
Here $\beta = \min(\beta_L, \beta_R)$.

- Black holes saturate this bound.

- Physically, the near-horizon region allows arbitrarily spacelike modes to propagate.

Do fuzzball solutions saturate this bound?

Physical Quantity of Interest: Gap between successive excitations

- If the gap between successive excitations is $O\left(e^{-S}\right)$, the thermal Wightman function should have an effectively continuous spectrum.

$$G_{\mathfrak{F}}(\omega_0, \gamma) = \int d\omega G(\omega, \gamma) \mathfrak{F}(\omega_0, \omega) d\omega$$

  If $\mathfrak{F}(\omega_0, \omega)$ is any smearing function with width larger than e^{-S} then $G_{\mathfrak{F}}$ has support for continuous ranges of ω_0.

- True even in integrable systems; stronger expectation than eigenstate thermalization hypothesis.

- Only free-theories with degeneracy violate this expectation.

Propagation of a massless scalar

- Wave-equation $\Box\phi = 0$ is separable!

  [Bena et. al, Tyukov et. al., 2017]

- We will consider propagation with no angular momentum on S^3 for simplicity.

- We set

$$\phi(r,t,y) = \frac{\psi_{\omega,\gamma}(r)}{\sqrt{r(r^2+a^2)}} e^{-i\frac{\omega t}{R_y}} e^{\frac{i\gamma y}{R_y}}$$

Wave Equation

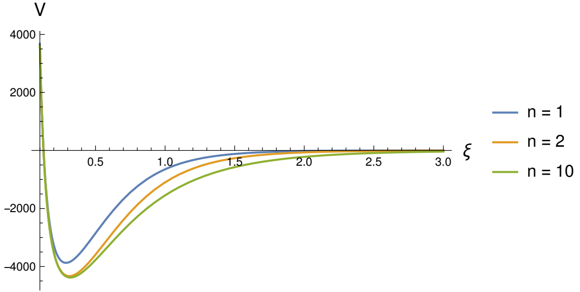
With $\xi = \frac{r}{a}$ and $b = a\kappa$, we have

$$\psi''_{\omega,\gamma}(\xi) - V(\xi)\psi_{\omega,\gamma}(\xi) = 0$$

with

$$V(\xi) = \frac{1}{4\left(\xi^2+1\right)^2}\Big[6 + \frac{4\gamma^2-1}{\xi^2} + 4\gamma^2 + 3\xi^2 + \kappa^2\left(\kappa^2+2\right)(\omega-\gamma)^2\frac{\xi^{2n}}{\left(\xi^2+1\right)^n} - \left(\kappa^2(\omega-\gamma)+2\omega\right)^2\Big]$$

WKB Potential



- A graph of $V(\xi)$ vs ξ with $\gamma = 10, \omega = 0, \kappa = 4$ and different values of *n*.
- Black-hole potential would keep dropping to $-\infty$ near $\xi = 0$.

Energy gap

- At large γ, we can use the WKB approximation. We get the standard quantization condition

$$2\int_{\xi_1}^{\xi_2} |V(\zeta)|^{\frac{1}{2}} d\zeta = (2m+1)\pi$$

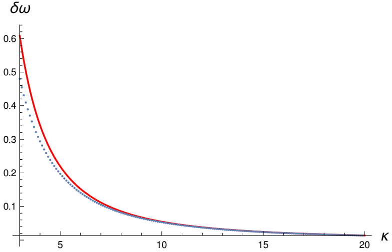
- At large κ we get

$$(\delta\omega)\kappa^2 g_n = \pi$$

where $g_n = \{0.5, 0.574, 0.610, 0.632, 0.648, \ldots\}$.

Numerical calculation of the energy gap

We can calculate the energy-gap by solving the scalar equation numerically. WKB approximation is excellent at large γ.



(Comparison between a numerical calculation (dots) of the gap between the first two allowed frequencies and analytic formula for $\gamma = 100$, $n = 2$.)

Energy-gap conclusions

The energy gap between successive excitations is $O(1)$ and too large for these states to be microstates of the black hole. $O(1)$ gap is suggestive of a phase of zero-entropy.

Large-γ falloff

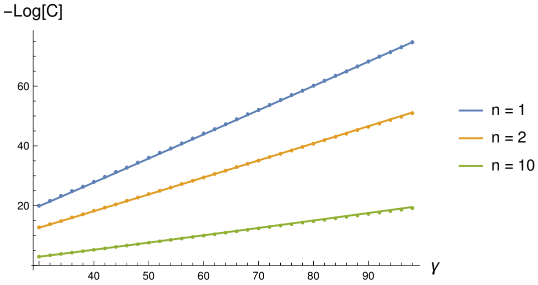
- At large γ, the Wightman function falls off faster than the black-hole. Does not saturate large-γ bound.

$$\lambda_{\text{fuzz}} = \lim_{\gamma \to \infty} \frac{-\log |G_{\omega,\gamma}|}{\gamma} = \frac{\pi}{2\sqrt{n}} + \frac{(11n-1)\pi}{16 n^{\frac{3}{2}} \kappa^2}$$

- $$\lambda_{\text{fuzz}} - \frac{1}{2}\beta_L = \frac{\pi(3n+7)}{16\kappa^2 n^{3/2}} + \text{O}\left(\frac{1}{\kappa^4}\right)$$

Large-γ falloff

Asymptotic falloff can also be verified numerically



Comparison between a numerical calculation (dots) of the asymptotic value of the wave-function with the analytic formula for different values of n, γ with $\kappa = 5$.

Large-γ falloff conclusions

- For $\kappa = \frac{b}{a} = O(1)$, the Wightman function falls off too fast at large-γ; implies that if these fuzzballs are microstates, they violate eigenstate thermalization.

- If these fuzzballs are microstates, some other fuzzballs must "oversaturate" the large-γ bound. We do not know of any geometry that oversaturates the bound.

Outline

Conclusions

- Fuzzballs that vary at $O(1)$ distance from the b.h. horizon cannot represent b.h. microstates.
- If fuzzballs are to represent even a basis of black-hole microstates, typical fuzzballs can vary from the conventional black-hole only Planck-length outside the horizon.
- But, in such geometries, quantum fluctuations become large near horizon. So the classical solution is unreliable where it is interesting.

Fuzzballs as stars

- All these problems arise, only if we insist on fuzzballs as black-hole microstates.
- If we think of fuzzballs as stars in string-theory, they constitute an interesting class of solutions, which deserve investigation.

Additional Slides

$a \to 0$ limit

- What if $a = \ell_{\text{pl}}$? Then $\kappa \to \infty$ the large-γ falloff tend to the black-hole answer. (Energy-gap is still too large!)
- The $a \to 0$ solutions represent only a small class of microstates, since $J_R \propto a^2$.
- But can these solutions be microstates of the non-rotating D1-D5 system?

$a \to 0$ limit

First note that if we stay away from $r \sim O(a)$, then

$$ds_6^2 \xrightarrow[a\to 0]{} \frac{(b^2 n - 2r^2)}{\sqrt{2}bR_y} dt^2 + \frac{(b^2 n + 2r^2)}{\sqrt{2}bR_y} dy^2 + \frac{bR_y}{\sqrt{2}r^2} dr^2 + \frac{\sqrt{2}bn}{R_y} dt dy + \frac{bR_y \cos^2(\theta)}{\sqrt{2}} d\psi^2 + \frac{bR_y \sin^2(\theta)}{\sqrt{2}} d\phi^2 + \frac{bR_y}{\sqrt{2}} d\theta^2$$

Change of variables to

$$\rho = \left(r^2 + \frac{b^2 n}{2}\right)^{\frac{1}{2}}$$

shows this is the metric of an extremal BTZ black hole.

$a \to 0$ limit

- But if we take $r = a\xi$ and expand around $\xi = 0$, we find a different metric! eg. for $n = 2$,

$$\sqrt{-g} = a^2 \lambda^2 \xi \sqrt{1 - \xi^4} \cos(\theta) \sin(\theta)$$

- Now, if $a \sim \mathrm{O}\left(\ell_{\mathrm{pl}}\right)$ then $\delta a \sim a$. [ensemble fluctuations.]
- So we expect

$$\mathrm{q} \sim \frac{\delta g}{g} \sim \frac{\delta g}{g \delta a} \delta a = \mathrm{O}\left(1\right)$$

if $\frac{\delta g}{\delta a} \sim \frac{g}{a}$ and $\frac{\delta a}{a} = \mathrm{O}\left(1\right)$.