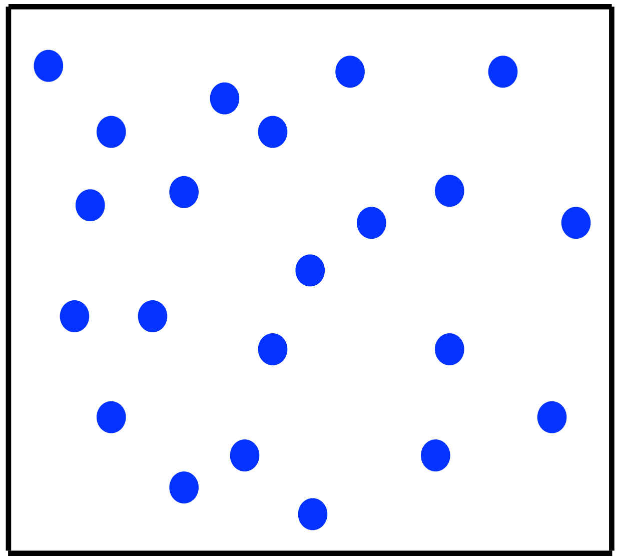
New constraints on transport from Schwinger Keldysh theory

Amos Yarom

Together with: K. Jensen, N. Pinzani, R. Marjieh

See also: Haehl, Loganayagam, Rangamani
together with Geracie, Narayan, Nizami, Ramirez
and: Crossley, Glorioso, Liu
together with Gao, Rajagopal

The second law of thermodynamics



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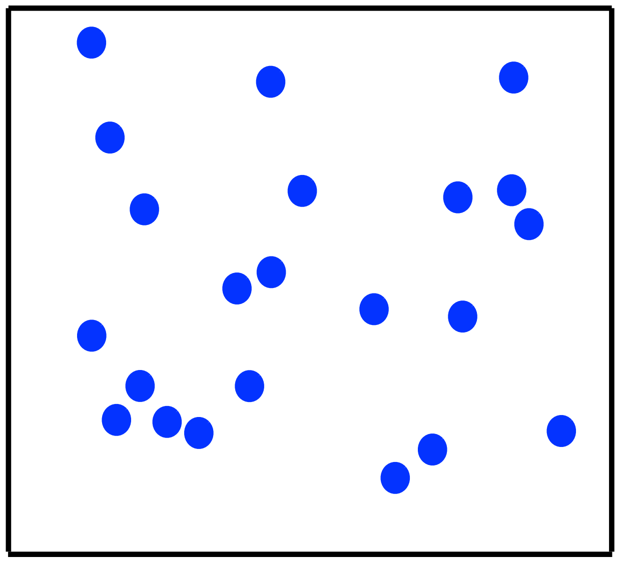

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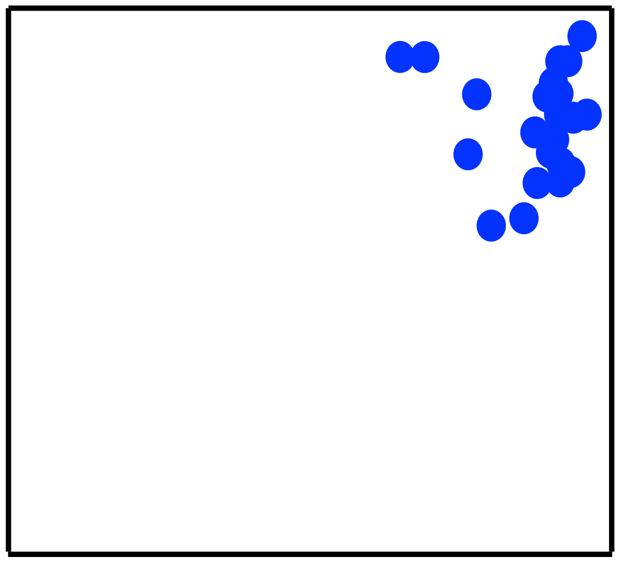

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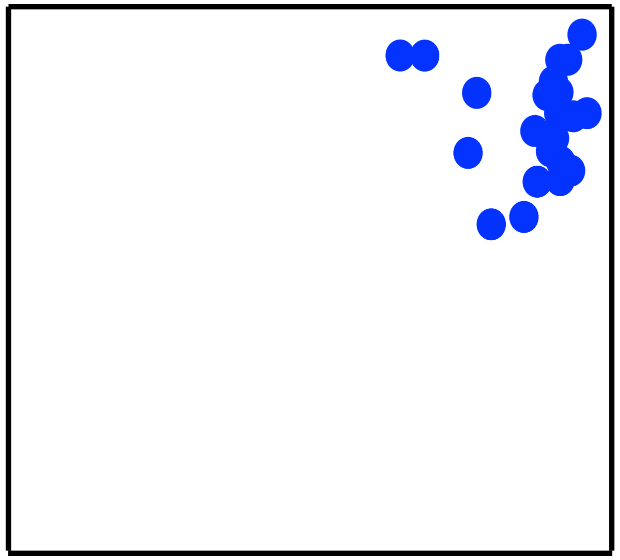
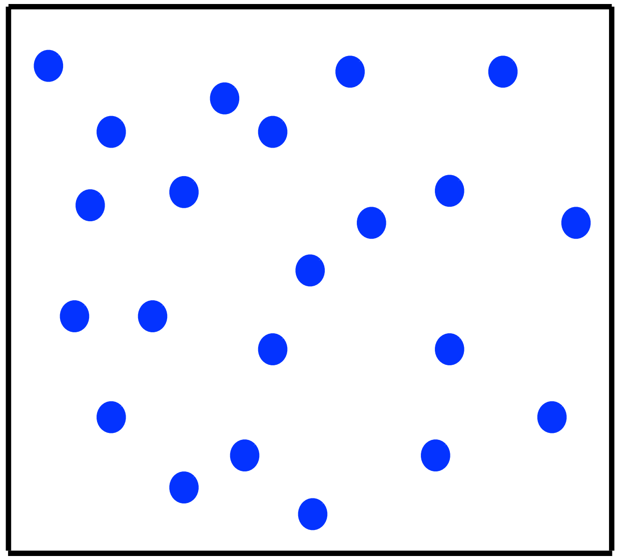


$$\Delta S \geq 0$$

The second law of thermodynamics



The second law of thermodynamics

$$\Delta S_{there} > 0$$

$$\Delta S_{here} < 0$$

$$\Delta S_{total} = \Delta S_{here} + \Delta S_{there} \geq 0$$

A local second law

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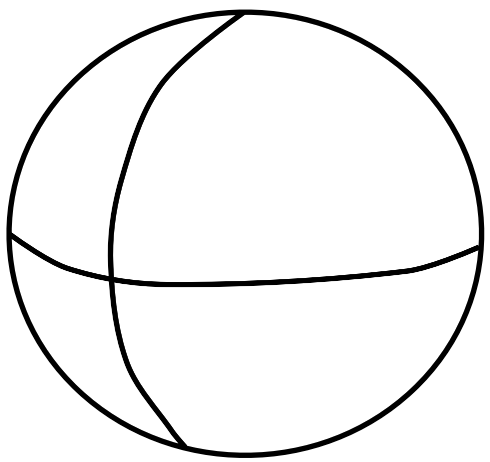
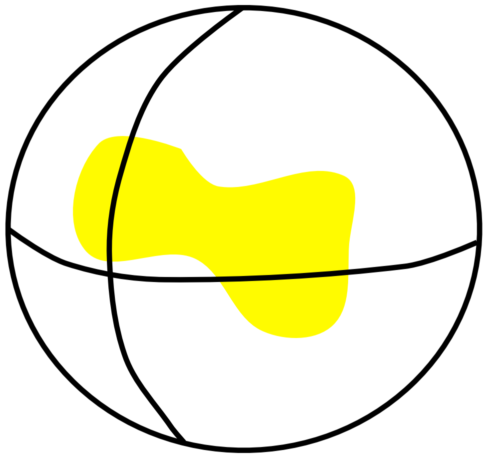


A local second law

$$\frac{dS}{dt}$$



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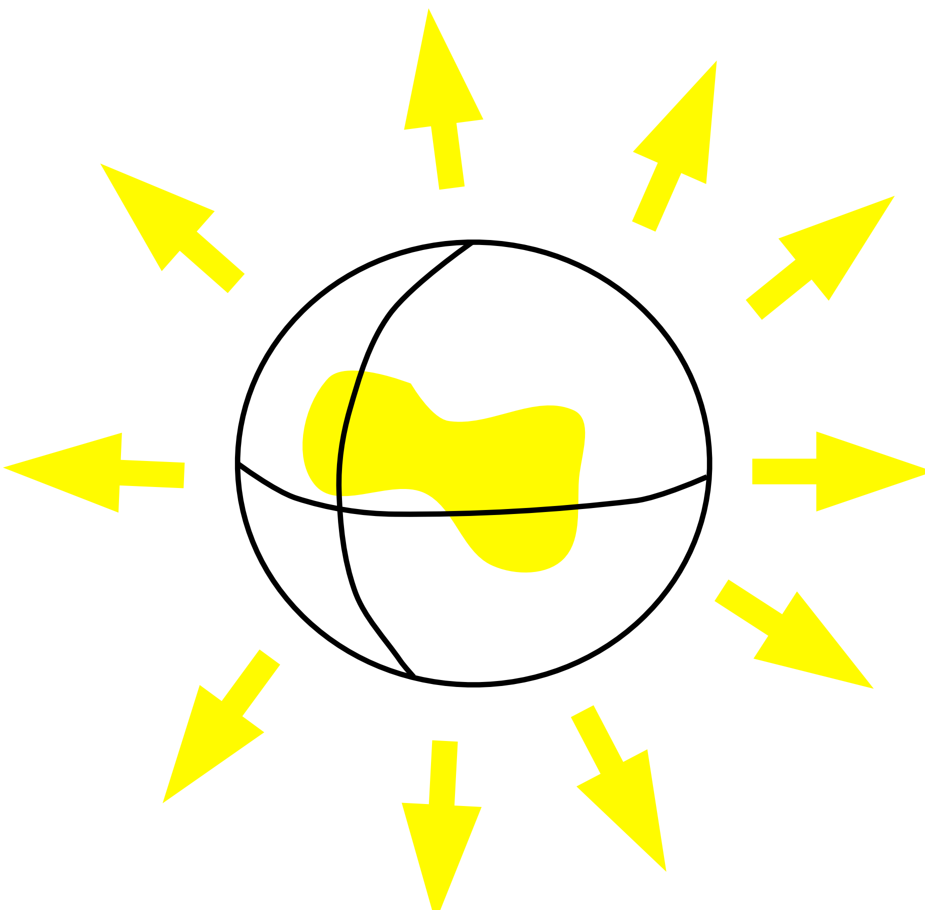
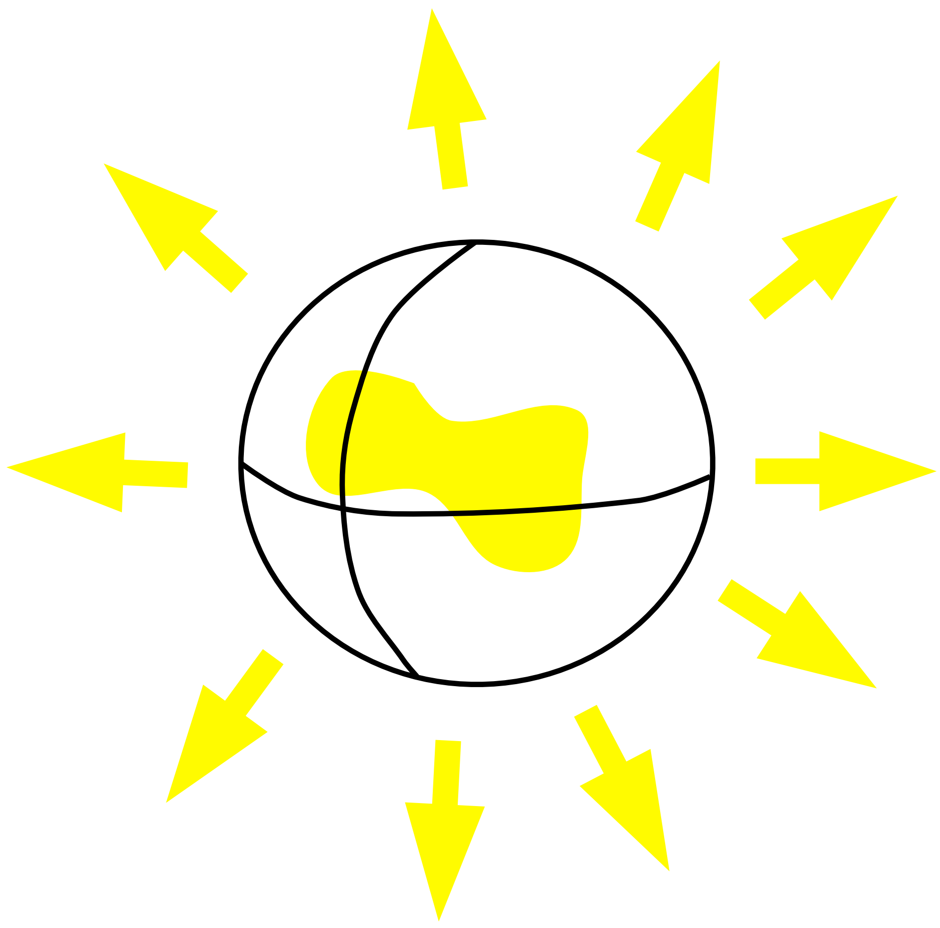


$$\frac{dS}{dt} + \int \vec{S} \cdot d\vec{a}$$

A local second law



$$\frac{dS}{dt} + \int \vec{S} \cdot d\vec{a} \geq 0$$

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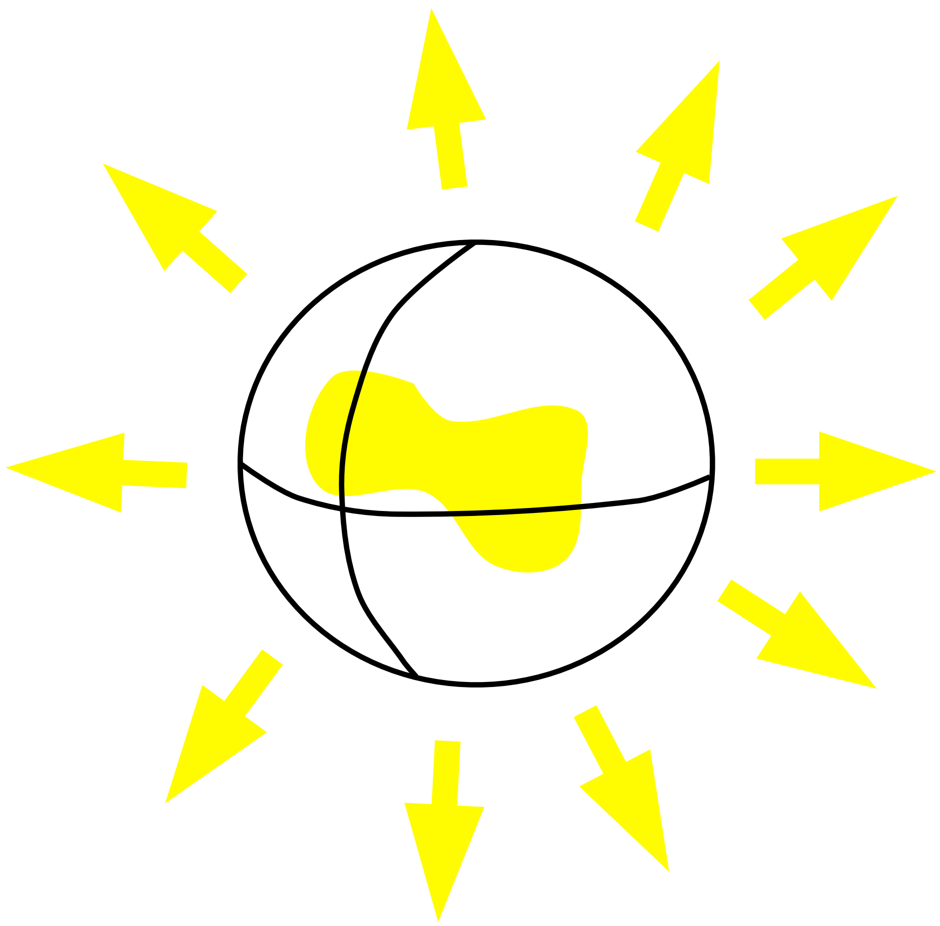
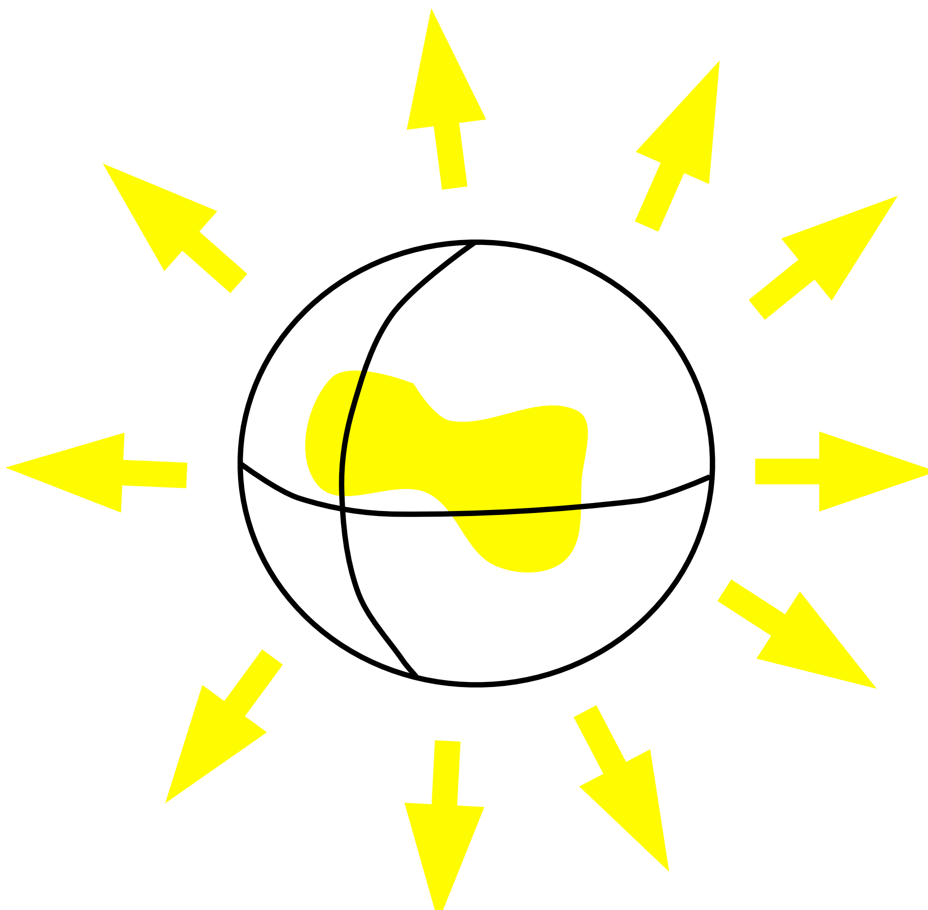


$$\frac{dS}{dt} + \int \vec{S} \cdot d\vec{a} \quad \geq 0$$

Local version of the 2nd law:

1) $\partial_\mu J_s^\mu \geq 0$

A local second law



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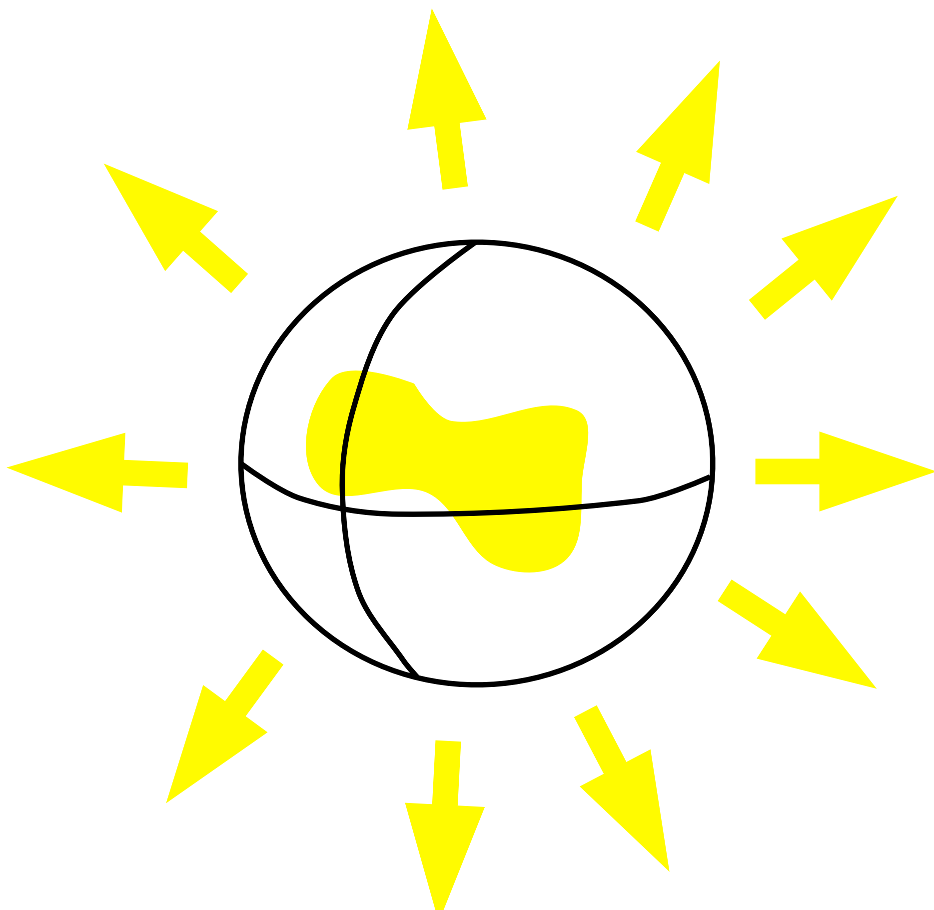

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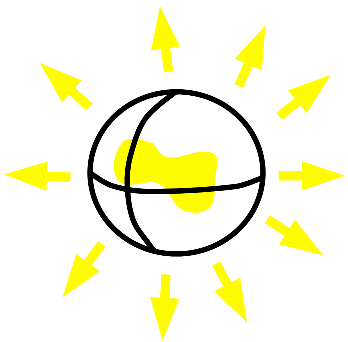

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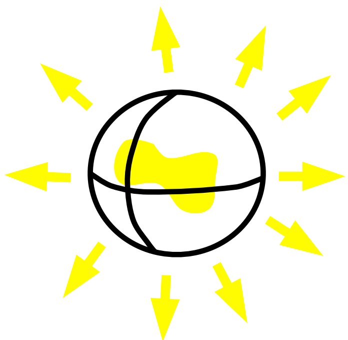
Implications

1) Constraints on transport

2) Interesting interplay with AdS/CFT computations



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Implications

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Open ends

1) Does it exist?

2) Is it sufficient?

Outline

- The Schwinger Keldysh effective action.
- The entropy current.
- New constraints on hydrodynamics

Schwinger-Keldysh

Recall that:

$$\langle 0| \underbrace{J \ldots J}_{n} |0\rangle \sim \frac{\delta^n}{\delta A^n} \ln Z[A]$$

Schwinger-Keldysh

Recall that:

$$\langle 0|\underbrace{J\ldots J}_{n}|0\rangle \sim \frac{\delta^n}{\delta A^n}\ln Z[A]$$

But also

$$\mathrm{Tr}\left(e^{-\beta H}\underbrace{J\ldots J}_{n}\right) \sim \frac{\delta^n}{\delta A^n}\ln Z_{SK}[A]$$

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Recall that:

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Defining

$$A_r = \frac{1}{2}(A_1 + A_2) \qquad A_a = A_1 - A_2$$

We find, for instance,

$$\left.\frac{\delta^2}{\delta A_a(t_1)\delta A_a(t_2)} i \ln Z_{SK}\right|_{A_1=A_2} = G_{sym}(t_1, t_2)$$

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Schwinger-Keldysh

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Symmetries:

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Schwinger-Keldysh

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- Reality & positivity

$$\begin{aligned} Z_{SK}[A_1, A_2]^* &= \mathrm{Tr}\left(U^*[A_1^*]e^{-\beta H^*}U^T[A_2^*]\right) \\ &= \mathrm{Tr}\left(\left(U^*[A_1^*]e^{-\beta H^*}U^T[A_2^*]\right)^T\right) \\ &= Z_{SK}[A_2^*, A_1^*] \end{aligned}$$

Schwinger-Keldysh

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- KMS (Kubo-Martin-Schwinger)

$$Z_{SK}[A_1, A_2] = Z_{SK}[\eta_{A_1} A_1(-t_1), \eta_{A_2} A_2(-t_2 - i\beta)]$$

Schwinger-Keldysh

Degrees of freedom.

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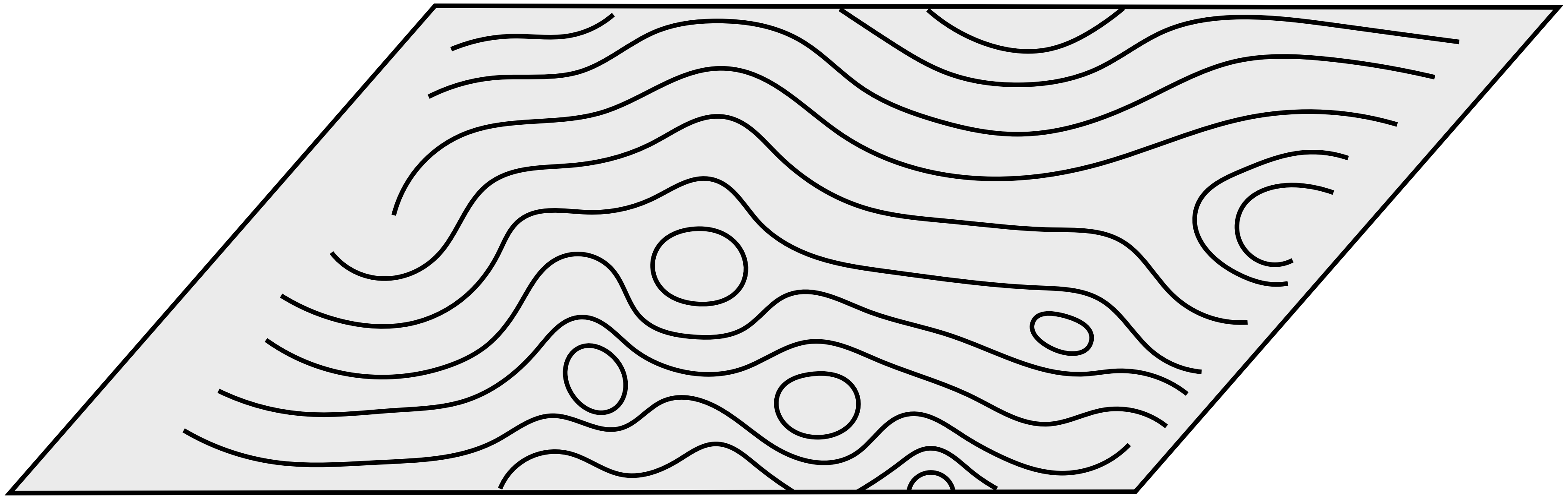
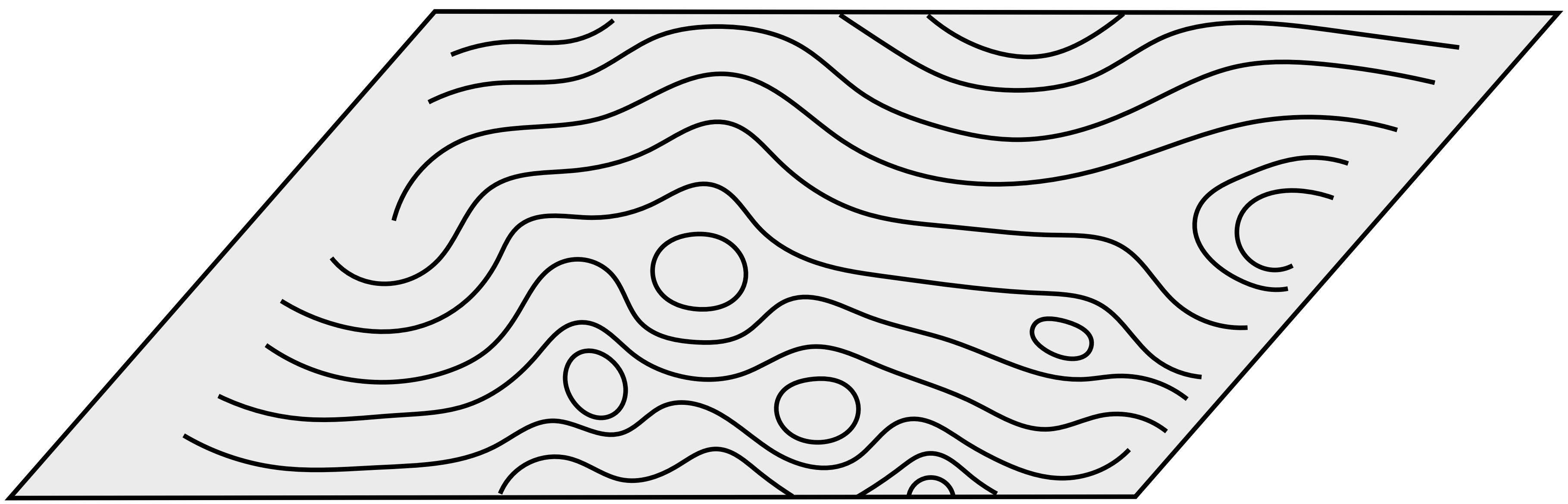


Schwinger-Keldysh

Degrees of freedom.

- Motivation 1 (fluid variables)



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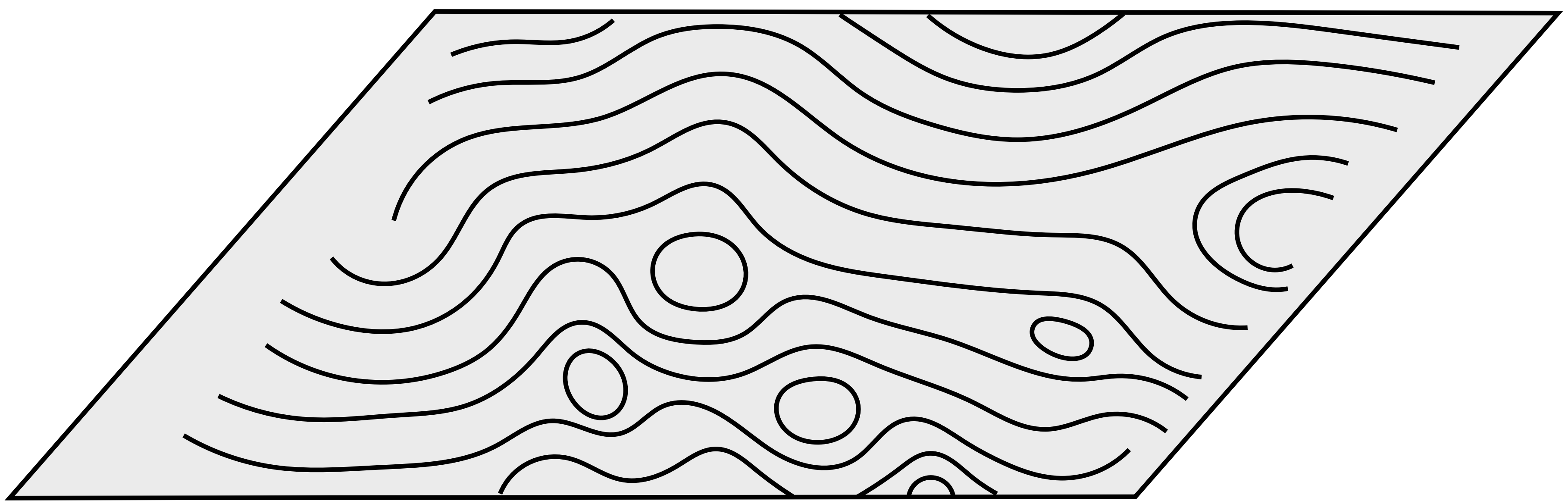
  Euler description of fluids:



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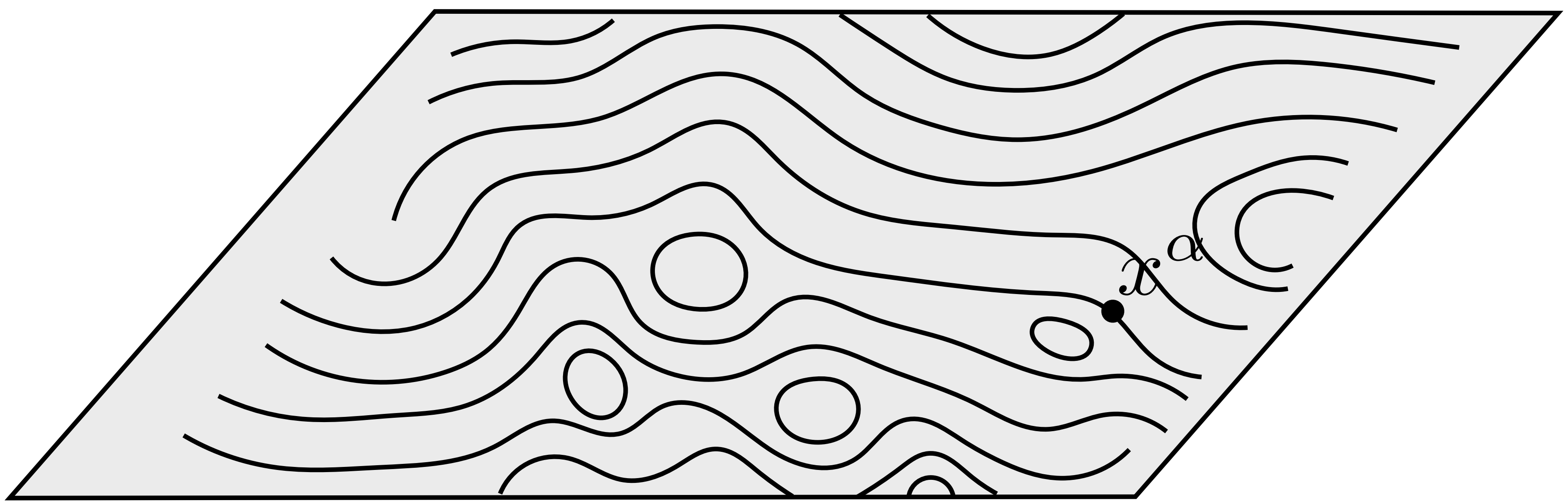

$u^\mu(x^\alpha) \quad T(x^\alpha)$

Schwinger-Keldysh

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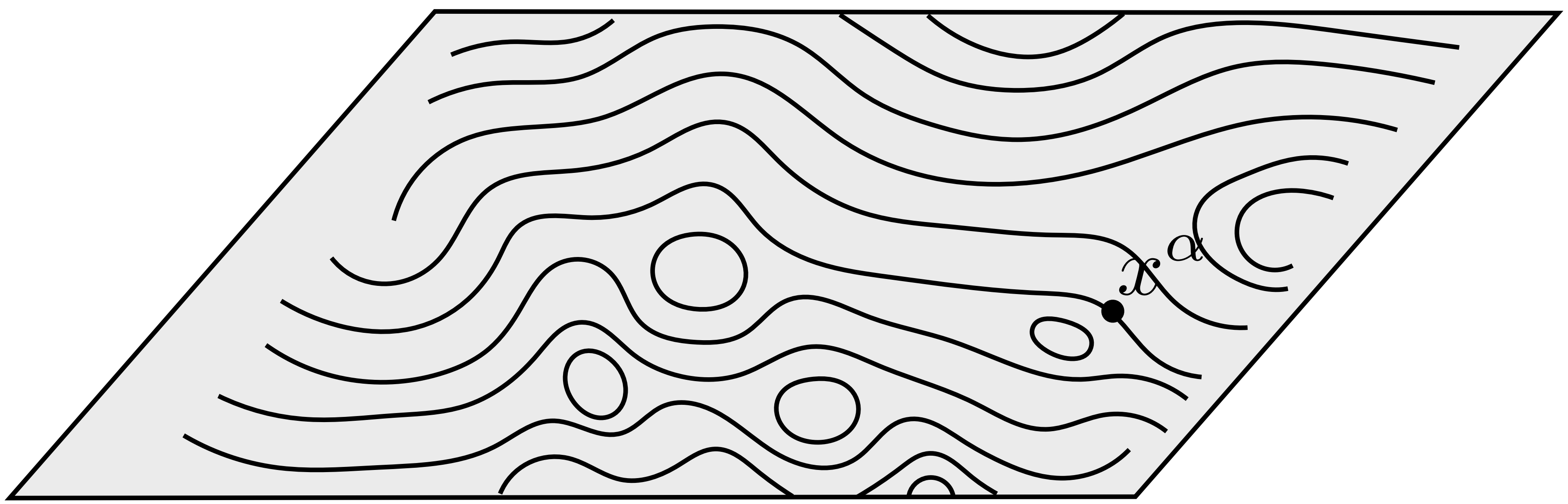

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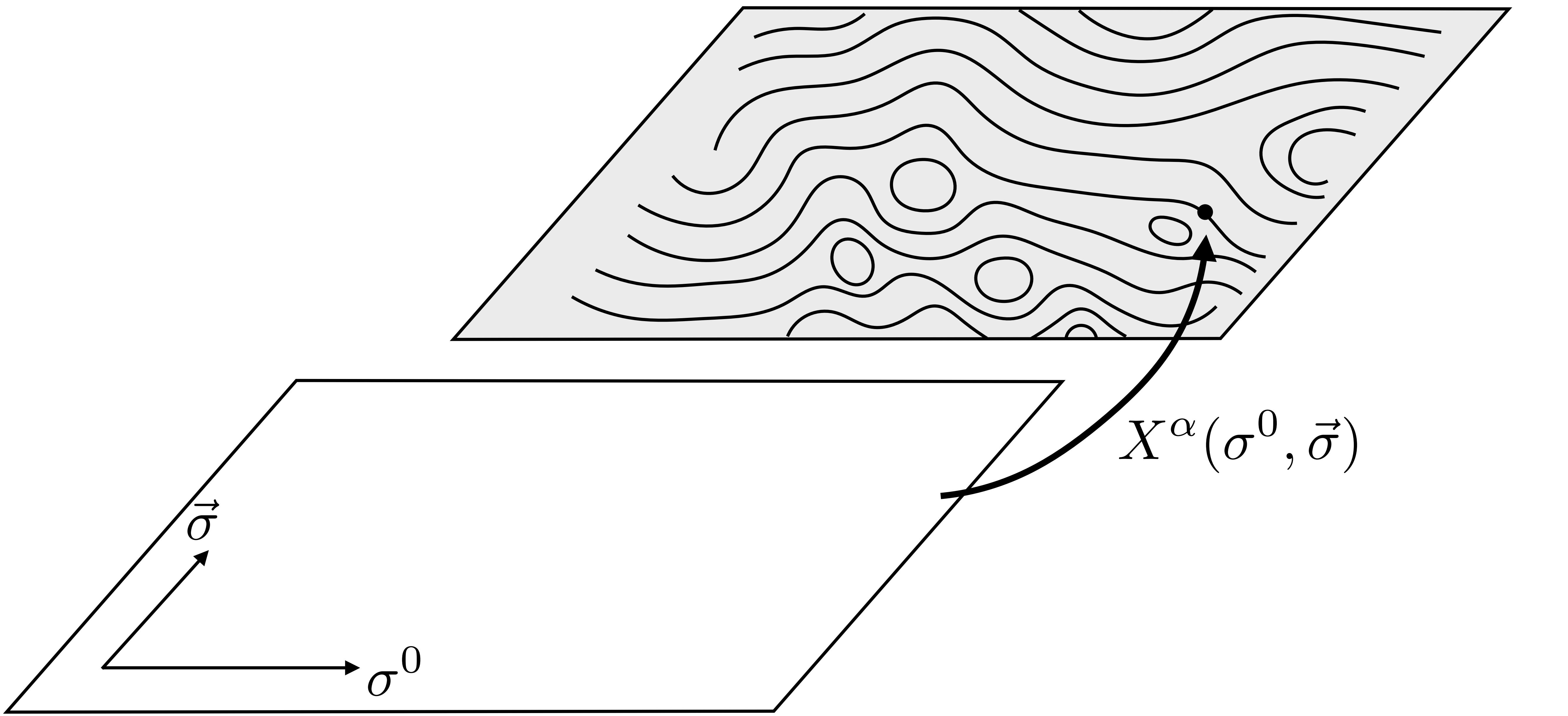

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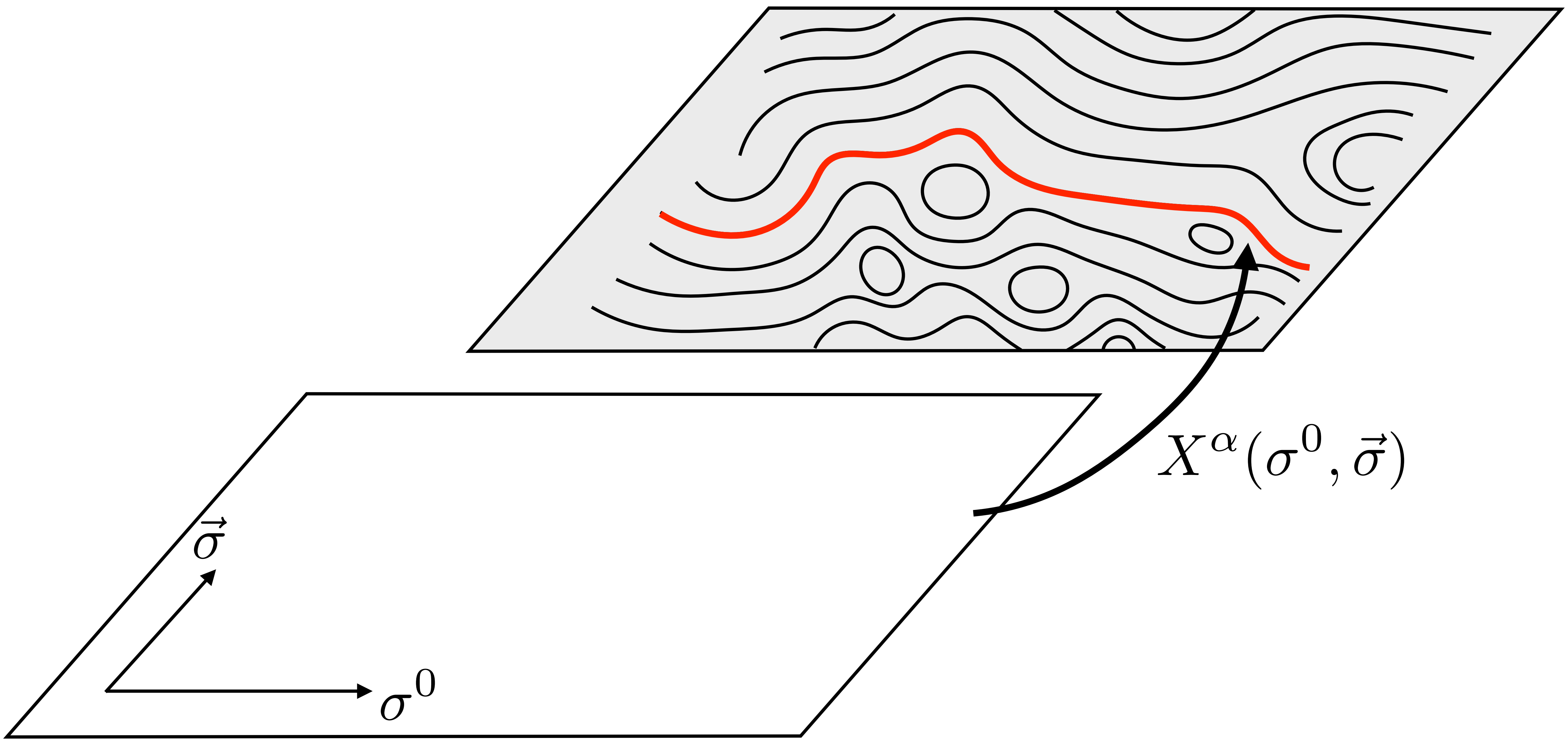

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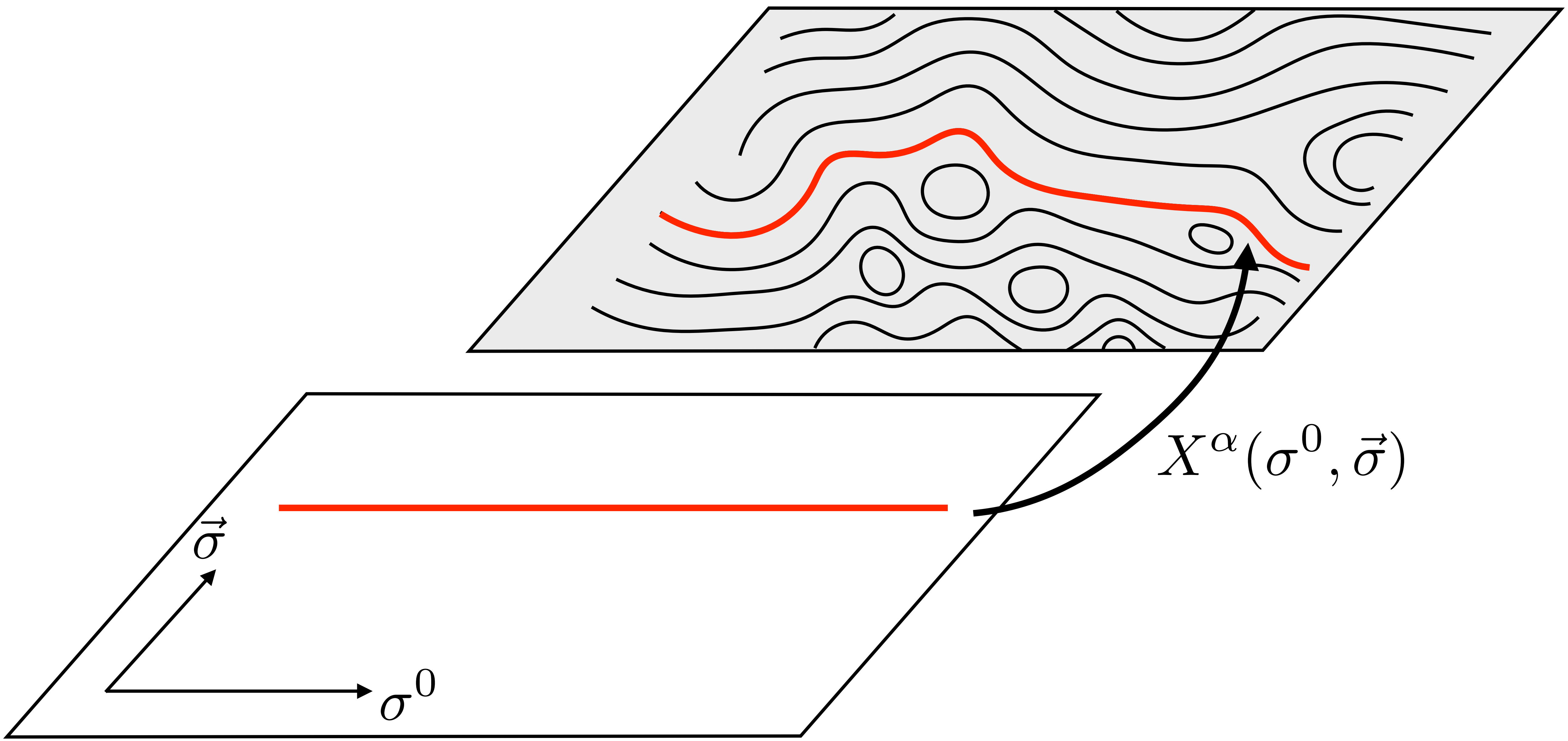

Schwinger-Keldysh

Degrees of freedom. $X^\alpha(\sigma^0, \vec{\sigma})$

- Motivation 1 (fluid variables)

Schwinger-Keldysh

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- Motivation II (fluid equations of motion)

$$\partial_\mu T^{\mu\nu} = 0$$

Schwinger-Keldysh

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$$\delta_X S = \ldots = \int d^d\sigma \sqrt{-g} \nabla_i \left(\frac{\delta L}{\delta g_{ij}} \right) \partial_j X^\mu g_{\mu\nu} \delta X^\nu$$

Schwinger-Keldysh

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$$S = \int d^d\sigma \sqrt{-g} L(g_{ij}) \qquad g_{ij} = \partial_i X^\mu \partial_j X^\nu g_{\mu\nu}(X^\alpha)$$

$$\delta_X S = \ldots = \int d^d\sigma \sqrt{-g} \nabla_i \left(\frac{\delta L}{\delta g_{ij}} \right) \partial_j X^\mu g_{\mu\nu} \delta X^\nu$$

$$\frac{2}{\sqrt{-|g_{\mu\nu}|}} \frac{\delta S}{\delta g_{\mu\nu}} = \ldots = \left(\frac{\delta L}{\delta g_{ij}} \right) \partial_i X^\mu \partial_j X^\nu$$

Schwinger-Keldysh

Degrees of freedom. $X^\alpha(\sigma^0, \vec{\sigma})$

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Schwinger-Keldysh

$$Z_{SK}[A_1, A_2] \underset{\frac{\mu}{\Lambda} \ll 1}{\rightarrow} \int D\xi_1 D\xi_2 e^{\frac{i}{\hbar} S_{eff}}$$

Our goal is to find S_{eff} .

Symmetries:

- $Z_{SK}[A_1 + d\Lambda_1, A_2] = Z_{SK}[A_1, A_2 + d\Lambda_2] = Z_{SK}[A_1, A_2]$
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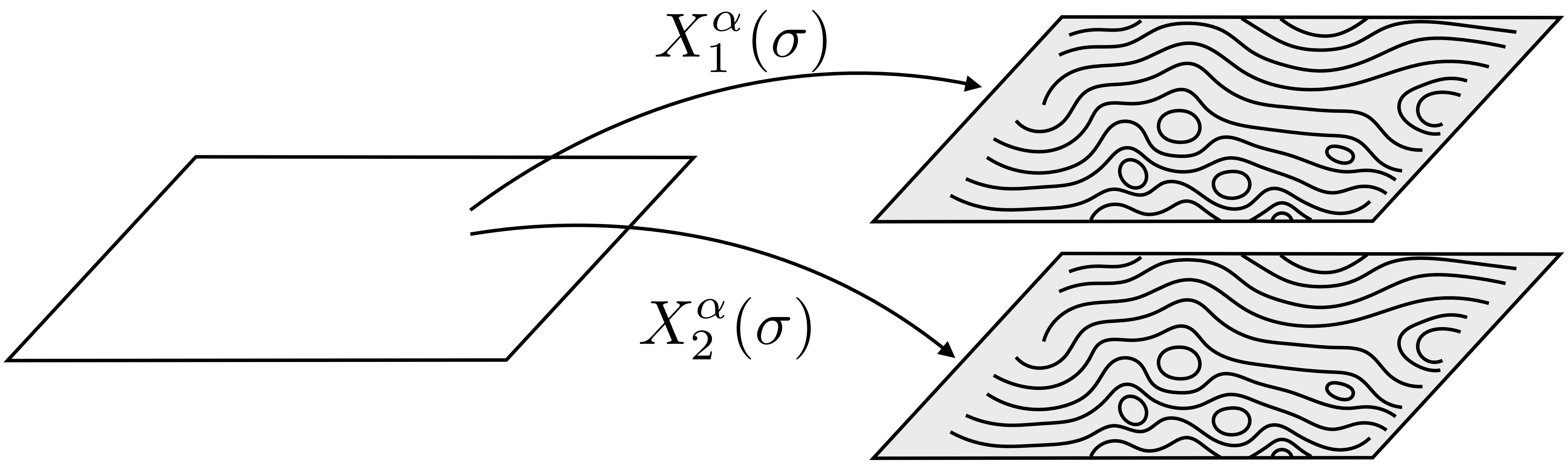
Schwinger-Keldysh



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Recall that the KMS symmetry is a $\mathbb{Z}_2$ symmetry:

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Let us define the action of the $\mathbb{Z}_2$ symmetry on fields as K .

$\widetilde{\mathcal{L}}$ is the $\mathbb{Z}_2$ transform of $\mathcal{L}$: $K(\mathcal{L}) = \widetilde{\mathcal{L}}$

Schwinger-Keldysh

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Schwinger-Keldysh

SK symmetry:

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Recall that:

$$A_r = \frac{1}{2}\left(A_1 + A_2\right) \qquad A_a = A_1 - A_2$$

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Or:

$$\left.\frac{\delta^n}{\delta A_r^n} \ln Z_{SK}\right|_{A_1 = A_2} = 0$$

Schwinger-Keldysh

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There are 2 equivalent ways to generate this symmetry:

1. Think of it as a topological symmetry that emerges once the sources are aligned.

Schwinger-Keldysh

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In both cases the generator is a nilpotent operator, Q.

Schwinger-Keldysh

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To realize the symmetry we use superspace:

$$\mathbb{X}_a = X_g + \theta(X_1 - X_2) \qquad \mathbb{X}_r = \frac{1}{2}(X_1 + X_2) + \theta X_{\bar{g}}$$

$$Q\mathbb{X}_{r/a} = \frac{\partial}{\partial\theta}\mathbb{X}_{r/a}$$

$$S = \int d^d\sigma d\theta L(\mathbb{X}_r, \mathbb{X}_a, \partial_\theta)$$

Schwinger-Keldysh

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Lie derivative along β

$$S = \int d^d\sigma d\theta L(\mathbb{X}_r, \mathbb{X}_a, \partial_\theta)$$

$$\beta^i = \delta^i_0 \beta$$

Due to its appearance in:

$$Z_{SK} = \mathrm{Tr}\left(U_1 e^{-\beta H} U_2^\dagger\right)$$

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Schwinger-Keldysh

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Schwinger-Keldysh

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- Double diff

Schwinger-Keldysh

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- SK+KMS

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- SK+KMS+diff

Schwinger-Keldysh

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*Infinitesimal coordinate transformations do not commute with Q. So far we have only retained all the symmetries in a limit where the sources are almost aligned and the temperature is large, or in a probe limit.

Schwinger-Keldysh

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$$\mathbb{D}_i \mathbb{V}_k = \partial_i \mathbb{V}_k - \Gamma^k_{ij} \mathbb{V}_k$$

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$$Z_{SK}[A_1, A_2] \underset{\frac{\mu}{\Lambda} \ll 1}{\longrightarrow} \int D\xi_1 D\xi_2 e^{\frac{i}{\hbar} S_{eff}}$$

Our goal is to find S_{eff} .

End result:

$$S_{eff} = \int d^d\sigma d\theta d\bar{\theta} \left(\mathcal{L} + \widetilde{\mathcal{L}} \right)$$

where:

$$\mathcal{L} = \sqrt{-\mathbb{g}}\mathcal{L}(\mathbb{g}_{ij}, \mathbb{D}_i, iD_\theta, D_{\bar{\theta}}, \beta^i) \qquad\qquad \mathcal{L} \in \mathbb{R}$$

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a result of:

$$Z_{SK}[A_1, A_2]^* = Z_{SK}[A_2^*, A_1^*]$$

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In addition we impose

$$\mathrm{Im} S_{eff} \geq 0$$

due to

$$|Z_{SK}[A_1, A_2]|^2 \leq 1$$

Schwinger-Keldysh

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Example:

$$\mathcal{L} = \sqrt{-\mathbb{g}}P(\beta^i \mathbb{g}_{ij} \beta^j)$$

Schwinger-Keldysh

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Leads to:

$$T^{ij} = \epsilon u^i u^j + (g^{ij} + u^i u^j)P$$

Schwinger-Keldysh

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where:

$$\epsilon = \frac{\partial P}{\partial T} \qquad T\beta^i = u^i \qquad u^i u_i = -1$$

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Example:

$$\mathcal{L} = \sqrt{-\mathbb{g}} \left(P - \eta \mathbb{g}^{ik} \mathbb{g}^{jl} D_\theta \mathbb{g}_{ij} D_{\bar{\theta}} \mathbb{g}_{kl} \right)$$

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Leads to:

$$T^{ij} = \epsilon u^i u^j + (g^{ij} + u^i u^j)P - \eta\sigma^{ij}$$

Entropy current

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Consider:

$$S_{eff} = \int d^d\sigma d\theta d\bar{\theta} \left(\mathcal{L} + \widetilde{\mathcal{L}} \right)$$

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Note that:

$$\{Q, \bar{Q}\} = i\pounds_\beta$$

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Note that:

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We would like to find a current associated with $\pounds_\beta$

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Start with a toy model:

$$S = \int d^d\sigma \sqrt{-g} L(\phi, g_{\mu\nu})$$

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A transformation

$$\delta_\beta \phi = \pounds_\beta \phi \qquad\qquad \delta_\beta g_{\mu\nu} = \pounds_\beta g_{\mu\nu}$$

is not necessarily a symmetry.

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Consider a "gauged" version of the transformation

$$\delta_T \phi = \Lambda_T \pounds_\beta \phi \qquad \delta_T g_{\mu\nu} = \Lambda_T \pounds_\beta g_{\mu\nu}$$

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First, introduce a connection:

$$\partial_\mu \to \partial_\mu + A_\mu \pounds_\beta \qquad\qquad \delta_T A_\mu = \Lambda_T \pounds_\beta A_\mu - A_\mu \pounds_\beta \Lambda_T - \partial_\mu \Lambda_T$$

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This ensures that:

$$\delta_T L(\phi, g_{\mu\nu}, A) = \Lambda_T \delta_\beta L(\phi, g_{\mu\nu}, A)$$

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This ensures that:

$$\delta_T L(\phi, g_{\mu\nu}, A) = \Lambda_T \delta_\beta L(\phi, g_{\mu\nu}, A)$$

Next consider:

$$\delta_T \frac{\sqrt{-g}}{1+\beta^\mu A_\mu} = \beta^\mu \partial_\mu \frac{\sqrt{-g}\Lambda_T}{1+\beta^\mu A_\mu} + \frac{\sqrt{-g}\Lambda_T}{1+\beta^\mu A_\mu} \partial_\mu \beta^\mu$$

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This ensures that:

$$\delta_T \left(\frac{\sqrt{-g} L}{1 + \beta^\mu A_\mu} \right) = \partial_\mu \left(\frac{\Lambda_T \beta^\mu L}{1 + \beta^\mu A_\mu} \right)$$

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So that under $\sqrt{-g} \to \dfrac{\sqrt{-g}}{1 + \beta^\mu A_\mu}$

$$\delta_T S[\phi, g_{\mu\nu}, A] = 0$$

Entropy current

Start with a toy model:

$$S = \int d^d\sigma \sqrt{-g} L(\phi, g_{\mu\nu})$$

Consider a "gauged" version of the transformation

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Therefore

$$\delta_T S = \int d^d\sigma \sqrt{-g} \left(E_\phi \delta_T \phi + \frac{1}{2} T^{\mu\nu} \delta_T g_{\mu\nu} - \delta_T A_\mu S^\mu \right) = 0$$

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And we get the on-shell relation:

$$\nabla_\mu S^\mu \Big|_{A=0} = \frac{1}{2} T^{\mu\nu} \pounds_\beta g_{\mu\nu}$$

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And we get the on-shell relation: $\left(\delta_T A_\mu = \Lambda_T \pounds_\beta A_\mu - A_\mu \pounds_\beta \Lambda_T - \partial_\mu \Lambda_T\right)$

$$\nabla_\mu S^\mu \Big|_{A=0} = \frac{1}{2} T^{\mu\nu} \pounds_\beta g_{\mu\nu}$$

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Start with a toy model:

$$S = \int d^d\sigma \sqrt{-g} L(\phi, g_{\mu\nu})$$

Our model:

$$S_{eff} = \int d^d\sigma d\theta d\bar{\theta} \left(\mathcal{L} + \widetilde{\mathcal{L}}\right)$$

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$$S^\mu = \left. \frac{\delta S}{\delta A_\mu} \right|_{A=0}$$

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$$\mathbb{S}^I = \left.\frac{\delta S_{eff}}{\delta \mathbb{A}_I}\right|_{\mathbb{A}_i=0}$$

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The bottom component of this equation is

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The bottom component of this equation is

$$\nabla_i S^i + S^\theta_{\bar{g}} + S^{\bar{\theta}}_g = 0$$

or

$$\nabla_i S^i = -S^\theta_{\bar{g}} - S^{\bar{\theta}}_g$$

Entropy current

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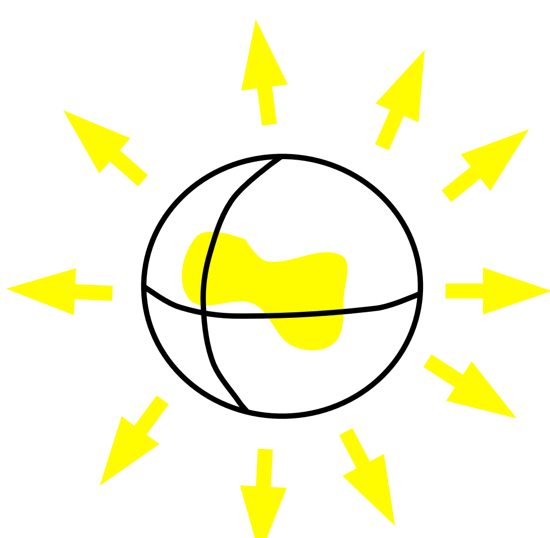
with

$$\nabla_i S^i = -S^{\theta}_{\bar{g}} - S^{\bar{\theta}}_{g}$$

Recall that the entropy current has 2 defining properties:



1) In equilibrium: $J^\mu_s = (s, 0, 0, 0)$

2) $\partial_\mu J^\mu_s \geq 0$

Entropy current

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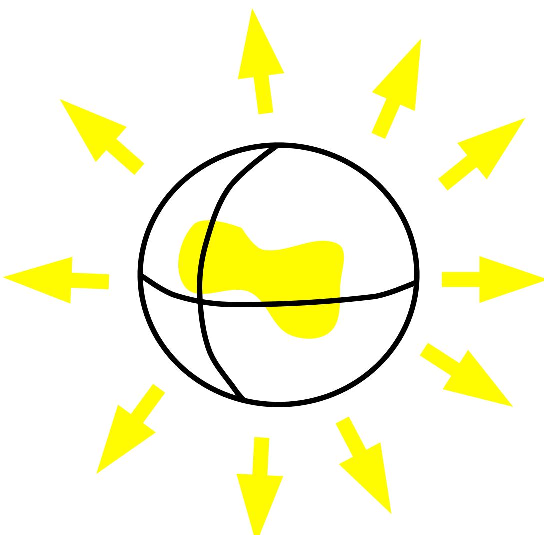

1) In equilibrium: $J_s^\mu = (s, 0, 0, 0)$

2) $\partial_\mu J_s^\mu \geq 0$

We can show that:

1) $S^i = s u^i + \mathcal{O}(\partial)$

2) $Im(S_{eff}) \geq 0 \implies \int d^d\sigma (S_g^{\bar{\theta}} + S_{\bar{g}}^{\theta}) \geq 0$

Entropy current

1) $S^i = su^i + \mathcal{O}(\partial)$

We use:

$$\mathcal{L} = \sqrt{-\mathbb{g}}P(\beta^i \mathbb{g}_{ij}\beta^j)$$

and compute S^i explicitly.

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$$\mathcal{L} = \sqrt{-\mathbb{g}} \Big(L_0(\mathbb{g}) + L^{ijkl}(\mathbb{g}) D_\theta \mathbb{g}_{ij} D_{\bar{\theta}} \mathbb{g}_{kl} + L^{ijklmn}(\mathbb{g}) D_\theta \mathbb{g}_{ij} D_{\bar{\theta}} \mathbb{g}_{kl} D_\theta D_{\bar{\theta}} \mathbb{g}_{mn} + \ldots \Big)$$

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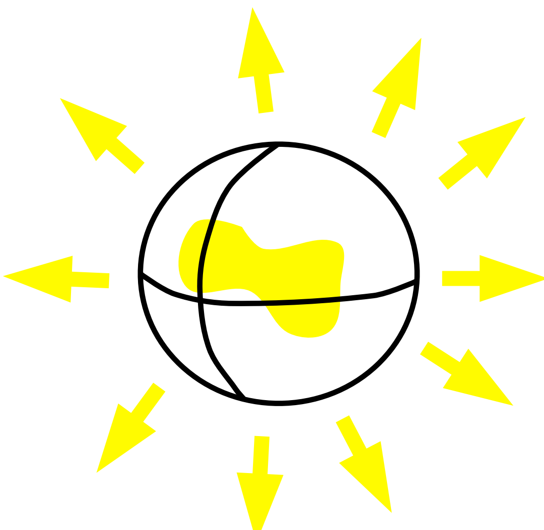
and compute $Im(S_{eff})$ and $S_g^{\bar{\theta}} + S_{\bar{g}}^{\theta}$ explicitly.

Entropy current

Recall that the entropy current has 2 defining properties:



1) In equilibrium: $J_s^\mu = (s,0,0,0)$

2) $\partial_\mu J_s^\mu \geq 0$

In our model:

1) $S^i = s u^i + \mathcal{O}(\partial)$

2) $Im(S_{eff}) \geq 0 \implies \int d^d\sigma (S_g^{\bar\theta} + S_{\bar g}^{\theta}) \geq 0$

Classification

Recall:

$$\mathcal{L} = \sqrt{-\mathfrak{g}}\left(P - \eta \mathfrak{g}^{ik}\mathfrak{g}^{jl} D_\theta \mathfrak{g}_{ij} D_{\bar{\theta}} \mathfrak{g}_{kl}\right)$$

Leads to:

$$T^{ij} = \epsilon u^i u^j + (g^{ij} + u^i u^j)P - \eta\sigma^{ij}$$

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More generally, we can write:

$$\mathcal{L} = \sqrt{-\mathfrak{g}}\Big(L_0(\mathfrak{g}) + L^{ijkl}(\mathfrak{g}) D_\theta \mathfrak{g}_{ij} D_{\bar{\theta}} \mathfrak{g}_{kl} \\ + L^{ijklmn}(\mathfrak{g}) D_\theta \mathfrak{g}_{ij} D_{\bar{\theta}} \mathfrak{g}_{kl} D_\theta D_{\bar{\theta}} \mathfrak{g}_{mn} + \ldots\Big)$$

And classify terms according to how they contribute to entropy production

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- Scalars

Classification

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And classify terms according to how they contribute to entropy production

- Scalars $\qquad \nabla \cdot J_S = 0$

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And classify terms according to how they contribute to entropy production

- Scalars $\quad \nabla \cdot J_S = 0 \qquad \notin Im(S_{eff})$
- Dissipative $\quad \nabla \cdot J_S > 0 \qquad \in Im(S_{eff})$

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And classify terms according to how they contribute to entropy production

- Scalars $\nabla \cdot J_S = 0 \quad \notin Im(S_{eff})$
- Dissipative $\nabla \cdot J_S > 0 \quad \in Im(S_{eff})$
- Non dissipative $\nabla \cdot J_S = 0 \quad \notin Im(S_{eff})$
- Exceptional/Pseudo-dissipative $\nabla \cdot J_S = 0 \quad \in Im(S_{eff})$

Exceptional transport

As an example consider:

$$T^{\mu\nu} = (\epsilon + P)\, u^\mu u^\nu + P g^{\mu\nu} + \gamma \left((u^\mu u^\nu + g^{\mu\nu})\sigma^2 - 2\nabla_\alpha u^\alpha \sigma^{\mu\nu} \right)$$

We find:

$$J_S^\mu = s u^\mu$$

But:

$$\gamma = 0$$

Summary

$$Z_{SK}[A_1, A_2] \underset{\frac{\mu}{\Lambda} \ll 1}{\longrightarrow} \int D\xi_1 D\xi_2 e^{\frac{i}{\hbar} S_{eff}}$$

Our goal is to find S_{eff} .

Symmetries:

- $Z_{SK}[A_1 + d\Lambda_1, A_2] = Z_{SK}[A_1, A_2 + d\Lambda_2] = Z_{SK}[A_1, A_2]$
- $Z_{SK}[A, A] = 1$
- $Z_{SK}[A_1, A_2]^* = Z_{SK}[A_2^*, A_1^*] \qquad |Z_{SK}[A_1, A_2]|^2 \leq 1$
- $Z_{SK}[A_1, A_2] = Z_{SK}[\eta_{A_1} A_1(-t_1), \eta_{A_2} A_2(-t_2 - i\beta)]$

Degrees of freedom:

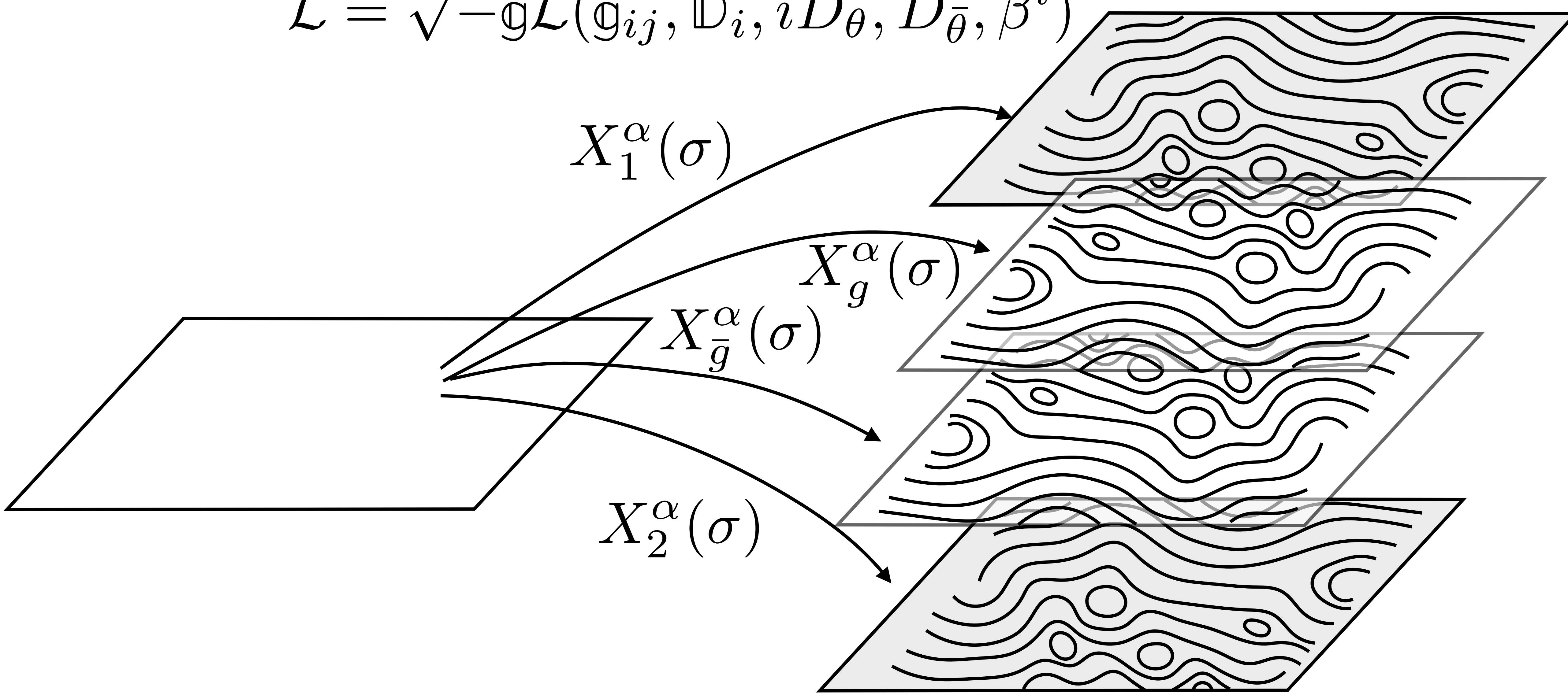
- $X_1^\alpha \qquad X_2^\alpha$

Summary

We found:

where:

$$\mathcal{L} = \sqrt{-\mathbb{g}}\mathcal{L}(\mathbb{g}_{ij}, \mathbb{D}_i, iD_\theta, D_{\bar{\theta}}, \beta^i)$$



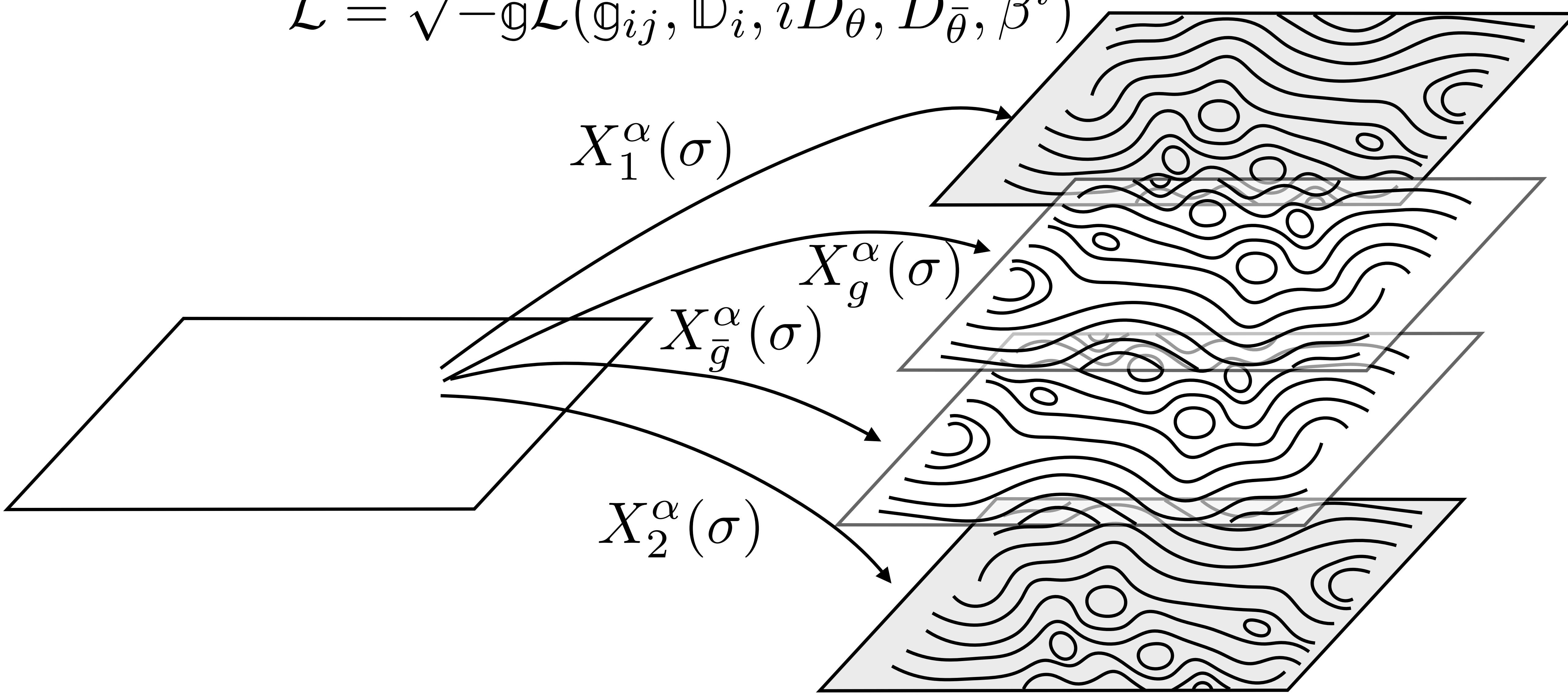
Summary

We found:

$$S_{eff} = \int d^d\sigma d\theta d\bar{\theta} \left(\mathcal{L} + \widetilde{\mathcal{L}} \right)$$

where:

$$\mathcal{L} = \sqrt{-\mathbb{g}}\,\mathcal{L}(\mathbb{g}_{ij}, \mathbb{D}_i, iD_\theta, D_{\bar{\theta}}, \beta^i)$$



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We found:

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$$\mathcal{L} = \sqrt{-\mathbb{g}}\mathcal{L}(\mathbb{g}_{ij}, \mathbb{D}_i, iD_\theta, D_{\bar{\theta}}, \beta^i)$$

It is equipped with a current $\mathbb{S}^I$ which satisfies

1. $S^i = su^i + \mathcal{O}(\partial)$
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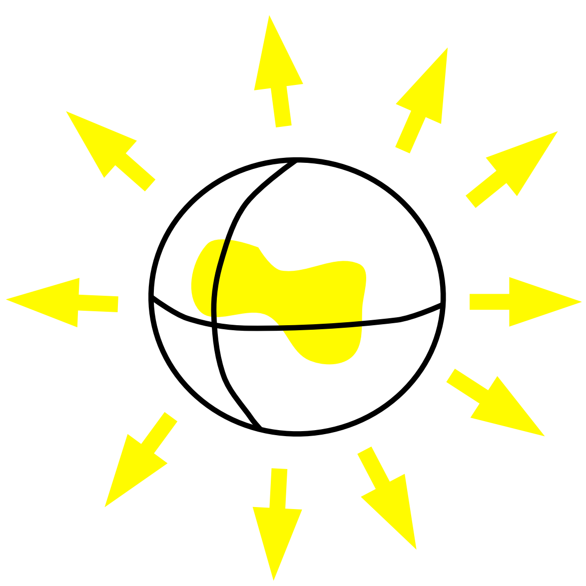

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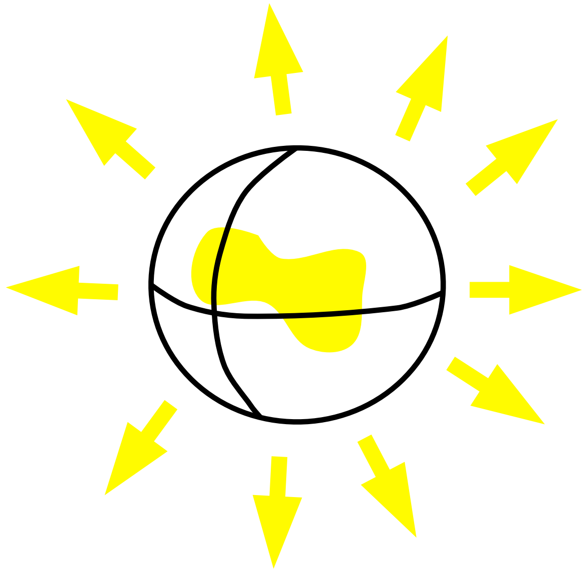

Summary

Given:

$$\mathcal{L} = \sqrt{-g}\Big(\mathbb{L}_0 + L^{AB} D_\theta \mathbb{F}_A D_{\bar\theta} \mathbb{F}_B$$

$$+ L^{ABC} D_\theta \mathbb{F}_A D_{\bar\theta} \mathbb{F}_B D_\theta D_{\bar\theta} \mathbb{F}_C + \ldots\Big)$$

We have found:

- Scalars $\nabla \cdot J_S = 0$
- Dissipative $\nabla \cdot J_S > 0 \quad \in Im(S_{eff})$
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Schwinger-Keldysh

Symmetries:

- Doubled gauge/diff invariance.

- Schwinger-Keldysh symmetry.

- Reality & positivity

$$Z_{SK}[A_1, A_2]^* = Z_{SK}[A_2^*, A_1^*] \qquad |Z_{SK}[A_1, A_2]|^2 \leq 1$$

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$$A = \mathrm{Tr}\left(U^\dagger \rho V\right) \qquad \rho = \sum_n r_n |n\rangle\langle n|$$

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$$\left|\mathrm{Tr}(U\rho V^\dagger)\right|^2 \leq 1$$

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$$Z_{SK}[A_1(t_1), A_2(t_2)] = \mathrm{Tr}\left(U[A_1(t_1)]e^{-\beta H}U^\dagger[A_2(t_2)]\right)$$

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$$= \mathrm{Tr}\left(U^\dagger[A_2(t_2 - i\beta)]e^{-\beta H}U[A_1(t_1)]\right)$$

$$= \mathrm{Tr}\left((CPT)U^\dagger[A_2(t_2 - i\beta)]e^{-\beta H}U[A_1(t_1)](CPT)\right)^*$$

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Schwinger-Keldysh

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$$= \mathrm{Tr}\left(U[\eta_{A_1}A_1(-t_1)]e^{-\beta H}U^\dagger[A_2(-t_2 - i\beta)]\right)$$

$$= Z_{SK}[\eta_{A_1}A_1(-t_1), \eta_{A_2}A_2(-t_2 - i\beta)]$$

Schwinger-Keldysh

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$$Z_{SK}[A_1, A_2] = Z_{SK}[\eta_{A_1} A_1(-t_1), \eta_{A_2} A_2(-t_2 - i\beta)]$$