INTERNATIONAL CENTRE for THEORETICAL SCIENCES
TATA INSTITUTE OF FUNDAMENTAL RESEARCH

AdS/CFT at 20 and Beyond @ ICTS, May , 2018

Aspects of Entanglement Entropy under Local Excitations in CFTs

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Mainly Based on our recent papers:

[1] JHEP 1801 (2018) 115 [arXiv:1711.09913]

with Yuya Kusuki (YITP, Kyoto)

[2] J.Phys. A50 (2017) 24, 244001 [arXiv:1701.03110]

with Pawel Caputa (YITP, Kyoto), Yuya Kusuki (YITP, Kyoto)

and Kento Watanabe (Tokyo)

Also thanks to collaborators in our earlier works:

Jyotirmoy Bhattacharrya (IIT Kharagpur)

Song He (AEI, Potsdam)

Alexander Jahn (Berlin Free U.)

Masahiro Nozaki (Chicago)

Tokiro Numasawa (McGill)

Tomonori Ugajin (OIST)

① Introduction

AdS/CFT [Maldacena 1997, Gubser-Klebanov-Polyakov, Witten 1998,.....]

Emergence of Gravity from Dynamics in CFTs

⇔ Emergent Spacetime from Quantum Entanglement

AdS/CFT in GR limit highly relies on the special feature of **holographic CFTs** (large N and strongly coupled).

→ **Maximally Chaotic** [e.g. Maldacena-Stanford-Shenker 2015]

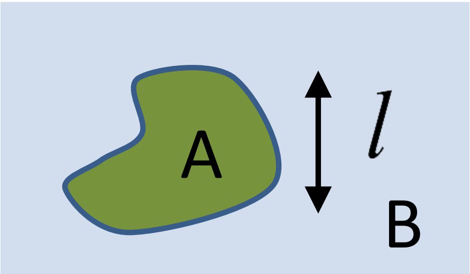
⇒ In this talk, we want to see special features of holographic CFTs via quantum entanglement dynamics.

➡ The time evolution of (Renyi) Entanglement Entropy (for locally excites states) classifies CFTs ?

We will discuss 4 difference classes.

n-th Renyi entanglement entropy (REE)

$$S_A^{(n)} = \frac{1}{1-n} \cdot \log \mathrm{Tr}\left[(\rho_A)^n\right].$$



$$H_{tot} = H_A \otimes H_B \quad .$$

$$\rho_A = \mathrm{Tr}_B |\Psi\rangle\langle\Psi| \; .$$

$$\lim_{n\to 1} S_A^{(n)} = -\mathrm{Tr}[\rho_A \log \rho_A] = S_A \quad . \qquad (\mathrm{Tr}[\rho_A] = 1).$$

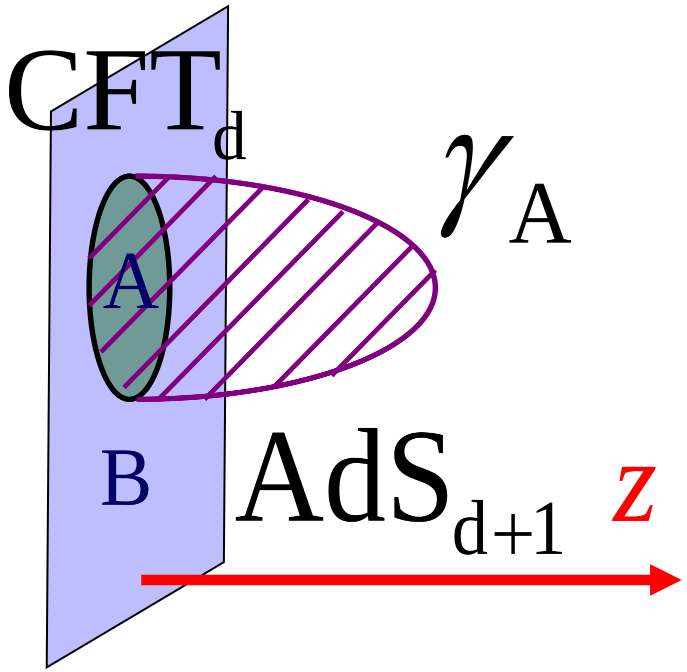
In **AdS/CFT**, we can calculate the entanglement entropy geometrically as an area of **minimal area surface in AdS**.

Holographic EE (HEE)

[Ryu-TT 06, Hubeny-Rangamani-TT 07]

$$S_A = \underset{\gamma_A}{\mathrm{Min}} \left[\frac{\mathrm{Area}(\gamma_A)}{4G_N} \right]$$

$$ds^2_{AdS} = R^2 \cdot \frac{dz^2 - dt^2 + dx^2}{z^2}$$



$$\partial \gamma_A = \partial A$$

$$\gamma_A \approx A \text{ (homologous)}$$

Consider excited states defined by local operators:

$$|O(x)\rangle \equiv \underline{e^{-\varepsilon H}} \cdot O(x)|0\rangle.$$

UV regularization

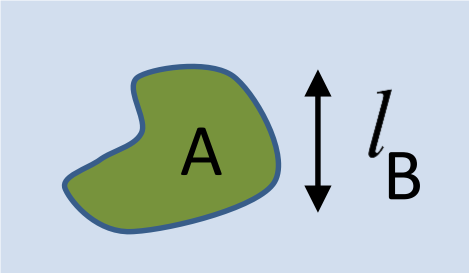
$$\Delta S_A^{(n)} \equiv S_A^{(n)}\left[|O(x)\rangle\right] - S_A^{(n)}\left[|0\rangle\right].$$

We study

$S_A^{(n)}$ ~ Loss of information when we assume that the region B is invisible.

$\Delta S_A^{(n)}$ ~ ``degrees of freedom'' of the operator O.

Two limits



(1) $l \rightarrow 0$ limit ($\approx$ small energy limit)

In this case, we find a property analogous to the first law of thermodynamics:

$$\Delta S_A[|O\rangle] \propto \Delta E_A$$

[Bhattacharya-Nozaki-Ugajin-TT 12, Blanco-Casini-Hung-Myers 13, Wong-Klich-Pando Zayas-Vaman 13 …,]

(2) $l \rightarrow \infty$ limit ($\approx$ large energy limit)

This leads to a very `entropic' quantity !

⇒ The main purpose of this talk.

[Nozaki-Numasawa-TT 14,….]

Contents

② Calculations of Renyi EE for locally excited states

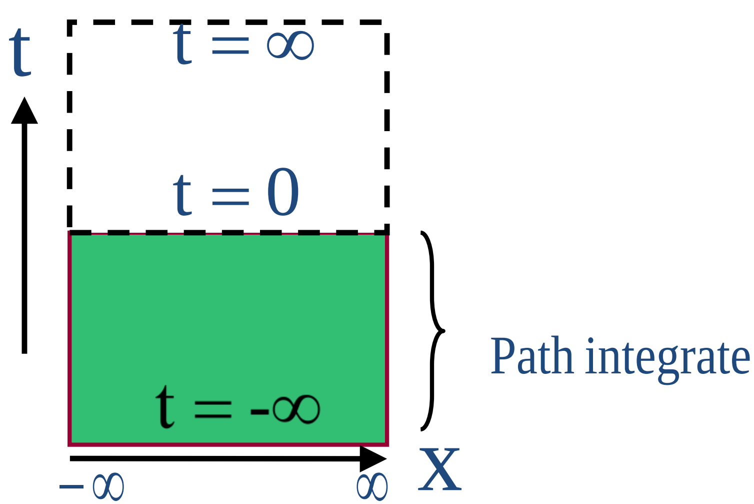
(2-1) Replica method for ground states

A basic method to find EE in QFTs is the **replica method**.

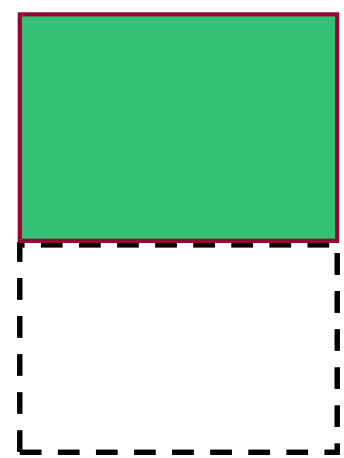
$$S_A = -\frac{\partial}{\partial n} \log \mathrm{Tr}_A (\rho_A)^n |_{n=1} \quad .$$

In the path-integral formalism, the ground state wave function $|\Psi\rangle$ can be expressed as follows:

$|\Psi\rangle =$

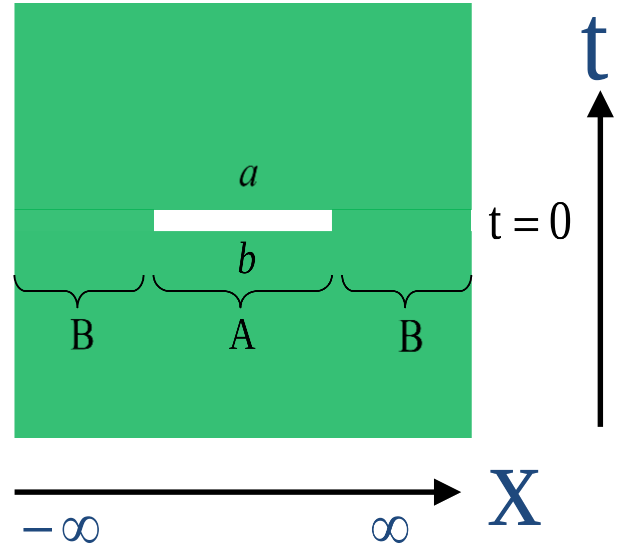


, $\langle\Psi| =$



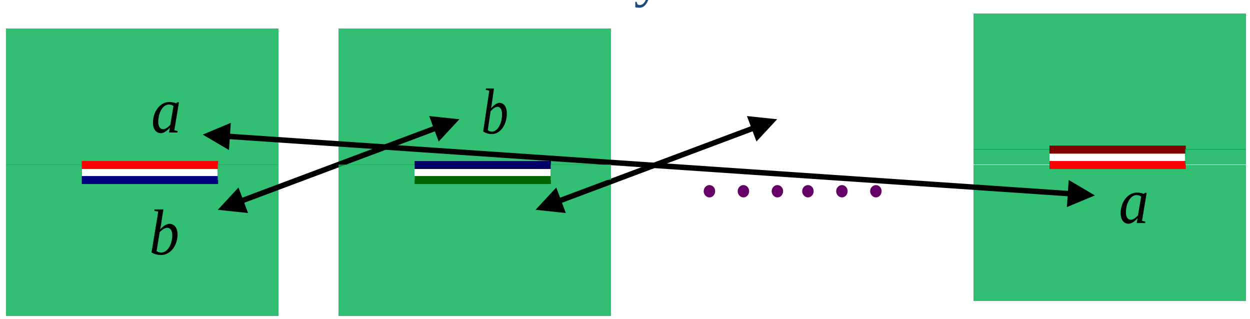
Then we can express

$\rho_A = \mathrm{Tr}_B |\Psi\rangle\langle\Psi|$ as follows: $[\rho_A]_{ab} =$



Glue each boundaries successively.

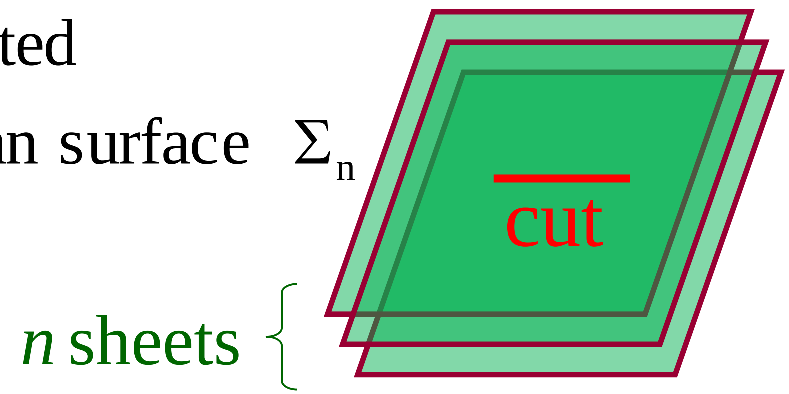
$\mathrm{Tr}(\rho_A)^n =$



$$= \frac{Z(\Sigma_n)}{Z(\Sigma_1)^n}.$$

n-sheeted

Riemann surface Σ_n



(2-2) Replica Method for Excited States

We want to calculate $\mathrm{Tr}\left(\rho_A\right)^n$ for

$$\rho_A(t,x) = e^{-iHt} e^{-\varepsilon H} O(x)\left|0\right\rangle\left\langle 0\right| O(x) e^{-\varepsilon H} e^{iHt}$$

$$= O(\tau_e, x)\left|0\right\rangle\left\langle 0\right| O(\tau_l, x),$$

$$(\tau_e \equiv -\varepsilon - it, \qquad \tau_l \equiv -\varepsilon + it),$$

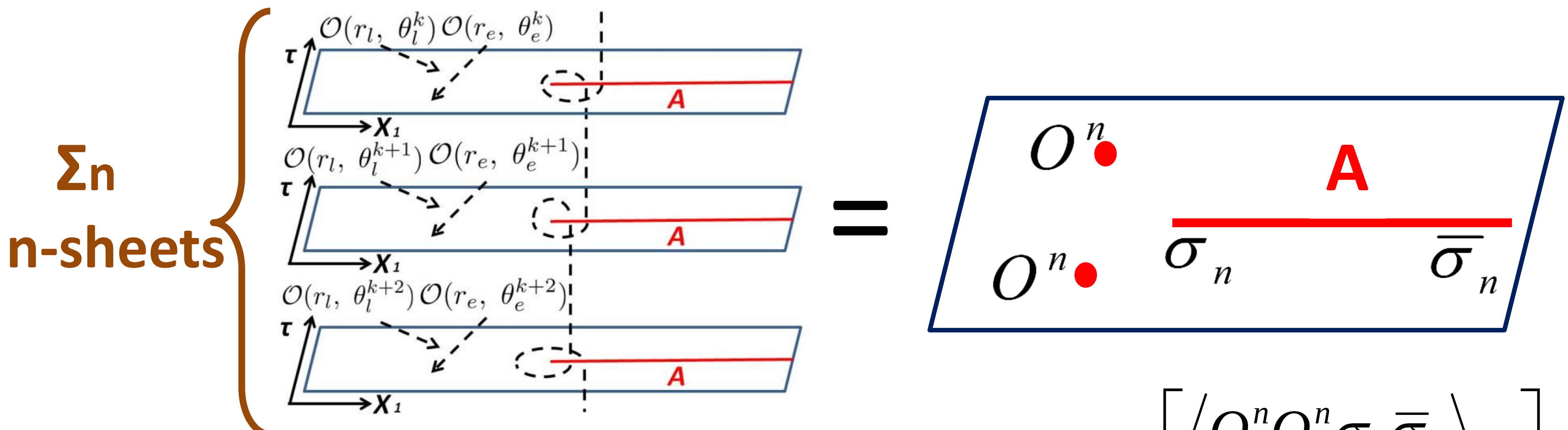
where ε is the UV regulator for the operator.

Here we consider a $d+1$ dim. CFT on R^{d+1}.

$(\tau, x_1, x_2, \cdots, x_d) \in \mathrm{R}^{d+1} \Rightarrow$ We set $x_1 + i\tau = re^{i\theta}$.

In this way, the Renyi EE can be expressed in terms of correlation functions (2n-point function etc.) on Σn :

$$\Delta S_A^{(n)} = \frac{1}{1-n}\cdot\Big[\log\big\langle O(r_l,\theta_l^n)O(r_e,\theta_e^n)\cdots O(r_l,\theta_l^1)O(r_e,\theta_e^1)\big\rangle_{\Sigma_n} - n\cdot\log\big\langle O(r_l,\theta_l)O(r_e,\theta_e)\big\rangle_{\Sigma_1}\Big].$$



$$\Delta S_A^{(n)} = \frac{1}{1-n}\cdot\log\left[\frac{\langle O^n O^n \sigma_n \bar{\sigma}_n\rangle_{\Sigma_n}}{\langle OO\rangle^n\cdot\langle\sigma_n\bar{\sigma}_n\rangle}\right].$$

③ Case 1: Free scalar CFTs [Numasawa-Nozaki-TT 14]

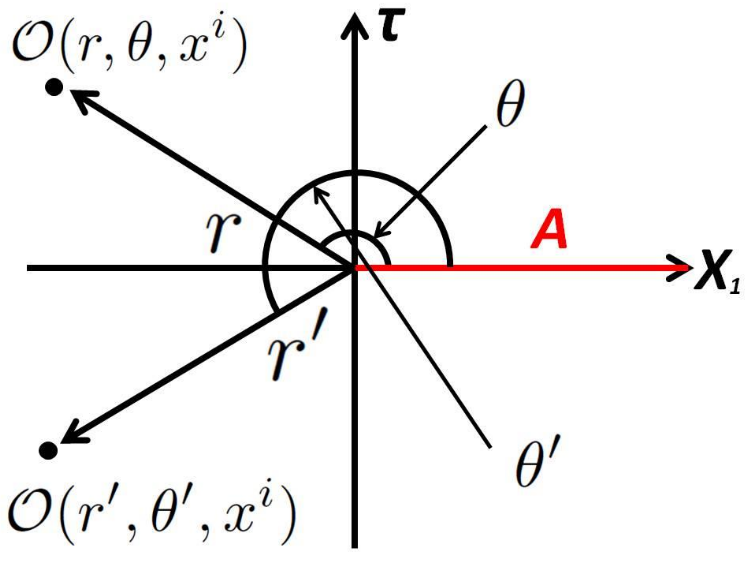
We focus on the free massless scalar field theory on Σn

$$S = \int d^{d+1}x\left[\partial_\mu \phi \partial^\mu \phi\right]$$

and calculate 2n-pt functions using the Green function:

$$G^{d=3}_{\Sigma n}[(r,\theta,\vec{x});(s,\varphi,\vec{y})] = \frac{1}{4n\pi^2 rs(a-1/a)} \cdot \frac{a^{1/n}-a^{-1/n}}{a^{1/n}+a^{-1/n}-2\cos((\theta-\varphi)/n)},$$

where $$\frac{a}{1+a^2} \equiv \frac{rs}{|\vec{x}-\vec{y}|^2+r^2+s^2}.$$



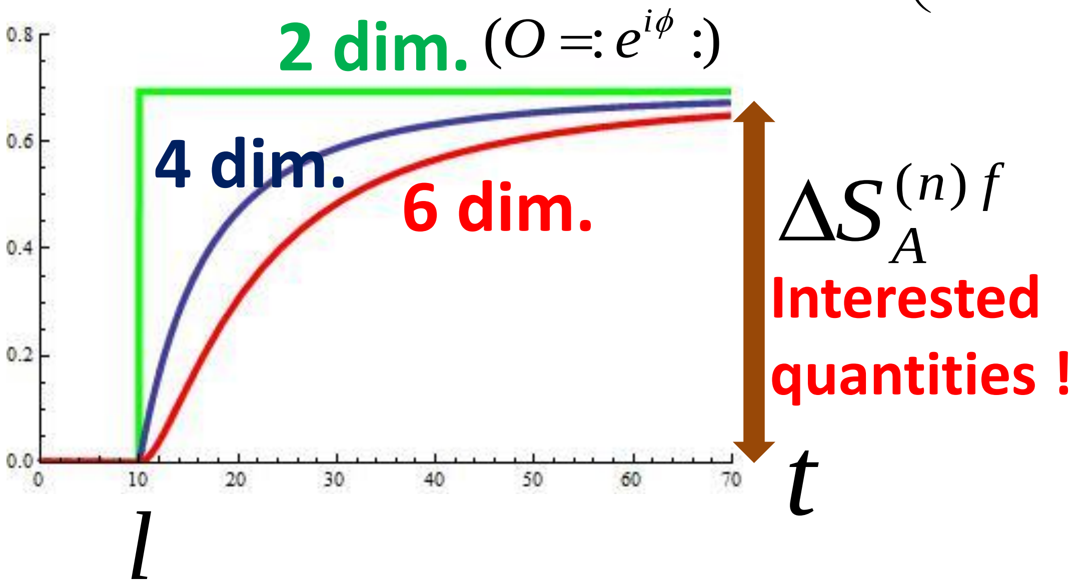
The operator O is chosen as

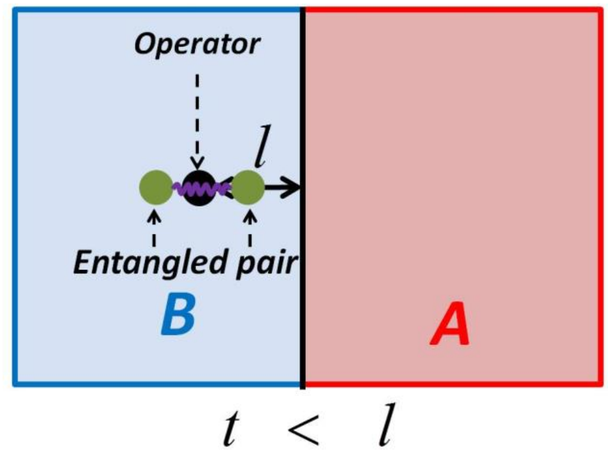
$$O_k = :\phi^k: \quad .$$

Time evolution in free massless scalar theory

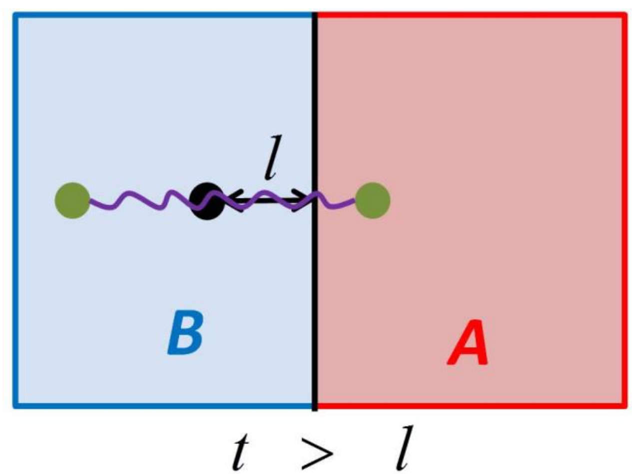
$\Delta S_A^{(2)}$ for $O =: \phi :$ (i.e. $k = 1$)

$$\left(\begin{array}{l} \text{We chose } x_1 = -l \text{ with } l = 10 \\ \text{and } x_2 = \cdots = x_d = 0. \end{array} \right)$$





$$E.g. \quad \Delta S_{A(4\mathrm{dim})}^{(2)} = \log\left(\frac{2t^2}{t^2 + l^2}\right).$$



Note :

$\Delta S_A^{(n)f}$ is `topologically invariant'

under deformations of A.

$$\Delta S_A^{(n)f} \quad \text{for} \quad O = \phi^k \quad \text{in } \ d+1>2 \,\text{dim.}$$

TABLE I. $\Delta S_A^{(n)f}$ and $\Delta S_A^f \left(= \Delta S_A^{(1)f}\right)$ for free massless scalar field theories in dimensions higher than two ($d > 1$).

	n	$k=1$	$k=2$	$\cdots$	$k=l$
$\Delta S_A^{(n)f}$ Renyi Entropy	2	$\log 2$	$\log \frac{8}{3}$	$\cdots$	$-\log\left(\frac{1}{2^{2l}}\sum_{j=0}^{l}({}_lC_j)^2\right)$
	3	$\log 2$	$\frac{1}{2}\log\frac{32}{5}$	$\cdots$	$\frac{-1}{2}\log\left(\frac{1}{2^{3l}}\sum_{j=0}^{l}({}_lC_j)^3\right)$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	m	$\log 2$	$\frac{1}{m-1}\log\frac{2^{2m-1}}{2^{m-1}+1}$	$\cdots$	$\frac{1}{1-m}\log\left(\frac{1}{2^{ml}}\sum_{j=0}^{l}({}_lC_j)^m\right)$
ΔS_A^f EE	1	$\log 2$	$\frac{3}{2}\log 2$	$\cdots$	$l\log 2 - \frac{1}{2^l}\sum_{j=0}^{l}{}_lC_j\log{}_lC_j$

EPR state !

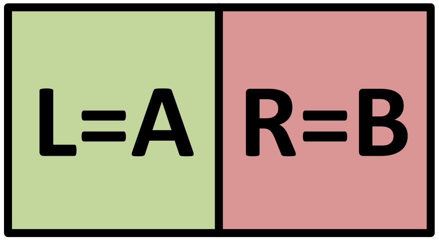
[For a proof: Nozaki 14]

$${}_nC_k \equiv \frac{n!}{k!(n-k)!}$$

Heuristic Explanation

First , notice that in free CFTs, there are definite (quasi) particles moving at the speed of light.

$$\Rightarrow \quad \phi \approx \underset{\text{left-moving}}{\phi_L} + \underset{\text{right-moving}}{\phi_R} \; .$$



$$\phi^k |\mathrm{vac}\rangle \approx \sum_{j=0}^{k} {}_kC_j \cdot (\phi_L)^j \cdot (\phi_R)^{k-j} |\mathrm{vac}\rangle$$

$$= 2^{-k/2} \sum_{j=0}^{k} \sqrt{{}_kC_j}\, |j\rangle_L |k-j\rangle_R . \qquad {}_nC_k \equiv \frac{n!}{k!(n-k)!}$$

$$\Rightarrow \quad \Delta S_A^{(n)f} = \frac{1}{1-n} \log\left[2^{-nk} \sum_{j=0}^{k} ({}_kC_j)^n\right],$$

$$\Delta S_A^f = k \log 2 - 2^{-k} \sum_{j=0}^{k} {}_kC_j \cdot \log[{}_kC_j].$$

Agree with replica Calculations !

④ Case 2: Rational 2d CFTs [He-Numasawa-Watanabe-TT 14]

(4-1) Free Scalar CFT in 2d

Consider following two operators in the free scalar CFT:

(i) $O_1 =: e^{i\alpha\phi} : \quad \Rightarrow \quad \Delta S_A^{(n)f} = 0.$

$$|O_1\rangle = e^{i\alpha\phi_L}|0\rangle_L \otimes e^{i\alpha\phi_R}|0\rangle_R$$ **⇒Direct product state**

(ii) $O_2 =: e^{i\alpha\phi} : + : e^{-i\alpha\phi} : \quad \Rightarrow \quad \Delta S_A^{(n)f} = \log 2.$

$$|O_2\rangle = e^{i\alpha\phi_L}|0\rangle_L \otimes e^{i\alpha\phi_R}|0\rangle_R + e^{-i\alpha\phi_L}|0\rangle_L \otimes e^{-i\alpha\phi_R}|0\rangle_R$$

$$\approx |\uparrow\rangle_L|\uparrow\rangle_R + |\downarrow\rangle_L|\downarrow\rangle_R$$ **⇒ EPR state**

(4-2) General Results for 2d Rational CFTs

First, focus on 2nd REE. Using the conformal map: $z = \sqrt{w} = \sqrt{re^{i\theta}}$,

It is straightforward to rewrite the 2nd REE in terms of 4-pt functions on $\Sigma_1 = \mathrm{C}$:

$$\left\langle O(w_1,\overline{w}_1)O(w_2,\overline{w}_2)O(w_3,\overline{w}_3)O(w_4,\overline{w}_4)\right\rangle_{\Sigma_2} = | z_{13}z_{24} |^{-4\Delta_O} \cdot G_O(z,\overline{z}).$$

$$w_1 = i(\varepsilon - it) - l, \quad \overline{w}_1 = -i(\varepsilon - it) - l,$$
$$w_2 = -i(\varepsilon + it) - l, \quad \overline{w}_2 = i(\varepsilon + it) - l.$$
$$w_3 = e^{2\pi i} w_1, \quad w_4 = e^{2\pi i} w_2.$$

$$\left(z \equiv \frac{z_{12}z_{34}}{z_{13}z_{24}}, \quad z_{ij} \equiv z_i - z_j. \right)$$

We can show that the limit $\varepsilon \rightarrow 0$ leads to

(i) Early time: $0 < t < l$ $\quad (z,\overline{z}) \approx (O(\varepsilon^2), O(\varepsilon^2)) \rightarrow (0,0).$ $\quad \rightarrow \Delta S_A^{(n)} = 0$

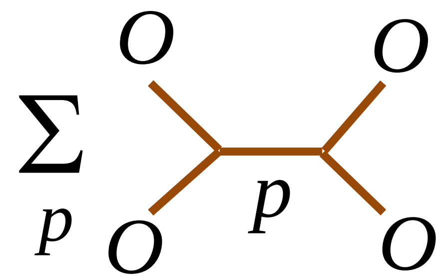
(ii) Late time: $t \geq l$

Chiral Fusion Transformation z→1-z

$$(z,\overline{z}) \approx (1 + O(\varepsilon^2), O(\varepsilon^2)) \rightarrow (1,0).$$

In terms of conformal block, we find at late time:

$$G_O(z,\bar{z}) = \sum_p \left(C_{OO}^{p}\right)^2 \cdot F_O(p\,|\,z)\cdot \overline{F}_O(p\,|\,\bar{z})$$



$$\underset{(z,\bar{z})\to(1,0)}{\cong} F_O(I\,|\,z)\cdot \overline{F}_O(I\,|\,\bar{z})$$

$$\cong F_{I,I}[O]\cdot(1-z)^{-2\Delta_O}\cdot \bar{z}^{-2\Delta_O}.$$

$F_{p,q}[O]$: fusion matrix

Then the n=2 REE is expressed at late time:

$$d_O \equiv \frac{S_{I,O}}{S_{I,I}} = \frac{1}{F_{I,I}[O]}.$$

Quantum Dimension

[Moore-Seiberg 89]

$$\Delta S_A^{(2)f} = -\log F_{I,I}[O] = \log d_O.$$

More generally we find $\Delta S_A^{(n)f} = \log d_O$.

Ex. Ising CFT: $\Delta S_A^{(n)}[I] = \Delta S_A^{(n)}[\varepsilon] = 0,\quad \Delta S_A^{(n)}[\sigma] = \log\sqrt{2}.$

States in RCFT $= \bigoplus_i \left[H_L^{(i)} \otimes H_R^{(i)}\right]$ ➡ **Entanglement Entropy**

⑤ Case 3: Large c CFTs (Holographic CFTs)

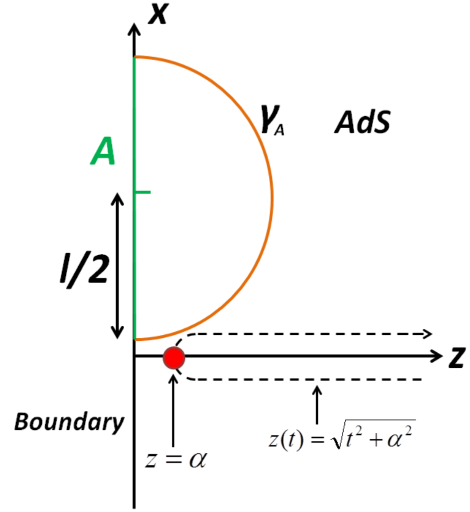
(5-1) Holographic Entanglement Entropy

For n=1 (EE), we can employ the HEE to find $\Delta S_A^{(1)}$ directly.

For 2d CFT (AdS_3/CFT_2),

$$\Delta S_A^{(1)} \approx \frac{c}{6}\log\left(\frac{t}{\varepsilon}\right).$$

[Nozaki-Numasawa-TT 13]

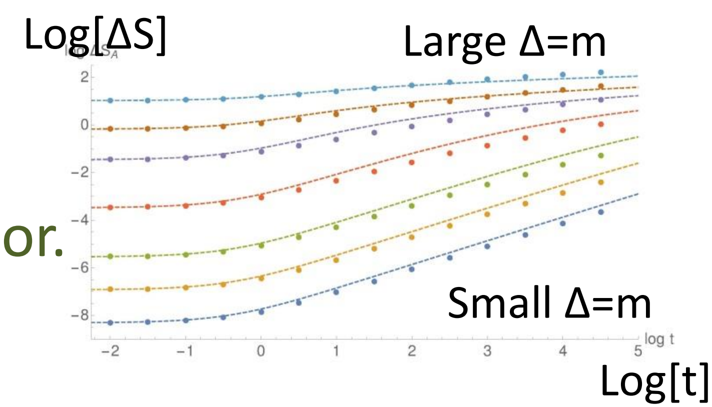


⇒ Reproduced by the large c CFT

[Asplund-Bernamonti-Galli-Hartman 14]

For higher dim. CFTs, it is still too early to say final results. Our numerical results for AdS4/CFT3, implies ~Log[t] like behavior.

[Jahn-TT 17]



(5-2) Renyi EE in Large c CFTs

Holographic CFTs (we focus 2d) are characterized by the large central charge and sparse spectrum.

[Heemskerk-Penedones-Polchinski-Sully 09, ...]

This allows us to approximate four point functions by vacuum conformal blocks. [Hartman 13, Asplund-Bernamonti-Galli-Hartman 14]

For Renyi EE calculations for locally excited states, we obtain

$$\Delta S_A^{(n)} = \frac{1}{1-n}\cdot\log\left[\frac{\left\langle O^n O^n \sigma_n \bar{\sigma}_n \right\rangle_{\Sigma_n}}{\left\langle OO\right\rangle^n \cdot \left\langle \sigma_n \bar{\sigma}_n \right\rangle}\right] \qquad z(t) = \frac{2i\epsilon}{l-t+i\epsilon}, \quad \bar{z}(t) = \frac{-2i\epsilon}{l+t-i\epsilon}$$

$$= \frac{1}{1-n}\cdot\log\left[|z|^{4h_{\sigma_n}} \sum_p \left(C_{OO}^{p}\right)^2 \cdot F_{OO}^{OO}(p\,|\,z)\cdot \bar{F}_{OO}^{OO}(p\,|\,\bar{z})\right]$$

$$\approx \frac{1}{1-n}\cdot\log\left[|z|^{4h_{\sigma_n}} \left(C_{OO}^{I}\right)^2 \cdot F_{OO}^{OO}(I\,|\,z)\bar{F}_{OO}^{OO}(I\,|\,\bar{z})\right]$$

If we consider n=1 (von-Neumann EE), the twist operator σn becomes light because $h_{\sigma_n} = \frac{c}{24}(n-1/n)$.

⇒ We can apply the HHLL conformal block calculation.

This leads to $\Delta S_A^{(1)} \approx \frac{c}{6}\log\left(\frac{t}{\varepsilon}\right)$. [Asplund-Bernamonti-Galli-Hartman 14]

On the other hand, if the excitation O(x) is light, then we find the Renyi EE behaves as

$$\Delta S_A^{(n)} \approx \frac{2nh_O}{n-1}\log\left(\frac{t}{\varepsilon}\right) .$$ [Caputa-Nozaki-TT 14]

➡ These behaviors look different: how are they related ?

What happen if σn and O(x) are both heavy ?

Motivated by this, we performed full analysis by using the Zamolodchikov's recursion relation [Kusuki-TT 17] .

[Zamolodchikov 1984, …., Chen-Hussong-Kaplan-Li 17]

The conformal block for $\langle O_A(0) O_A(z) O_B(1) O_B(\infty)\rangle$ is given by

$$F^{h_A h_A}_{h_B\, h_B}(0|z)$$
$$= (16q)^{-\frac{c-1}{24}} z^{\frac{c-1}{24}} (1-z)^{\frac{c-1}{24}-h_A-h_B} \theta_3(q)^{\frac{c-1}{2}-8(h_A+h_B)} \times H^{h_A,h_B}(q).$$

$$H^{h_A,h_B}(q) = 1 + \sum_{m=1}^{\infty} c_m\, q^{2m}$$

Here we introduced $q \equiv e^{-\pi\frac{K(1-z)}{K(z)}}$.

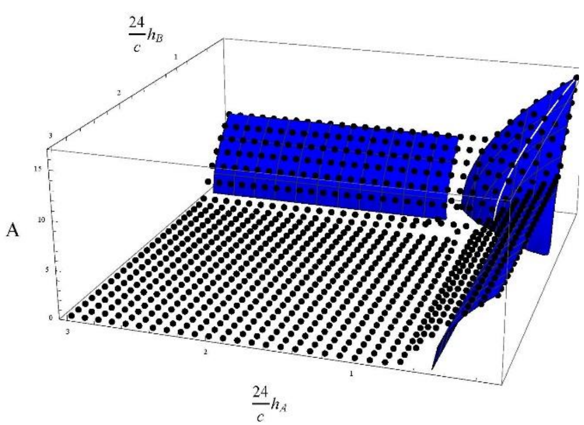
Late time limit of Renyi EE : $q \approx i \cdot e^{\frac{\pi^2}{4\log[t/\varepsilon]}} \to i.$

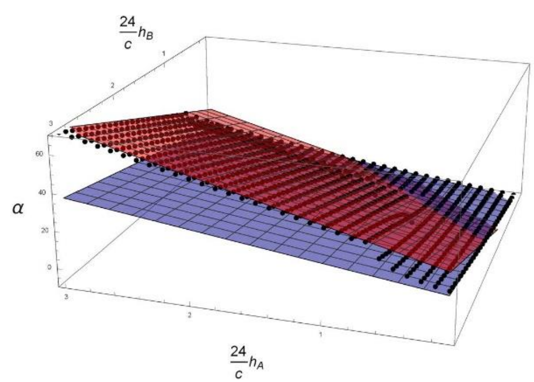
We numerically solved the Zamolodchikov recursion relation up to m≈1000.

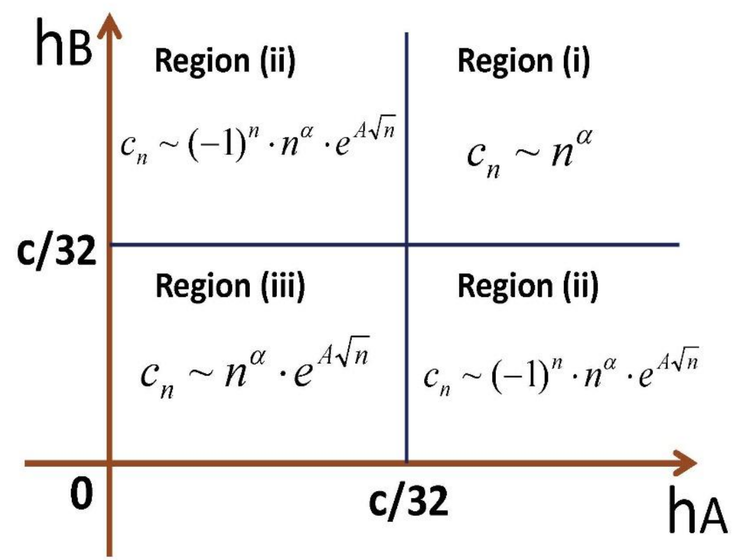
For large m, we find the Cardy formula –like behavior:

$$|c_m| \approx \beta \cdot m^{\alpha} \cdot e^{A\sqrt{m}} \quad (m \to \infty).$$

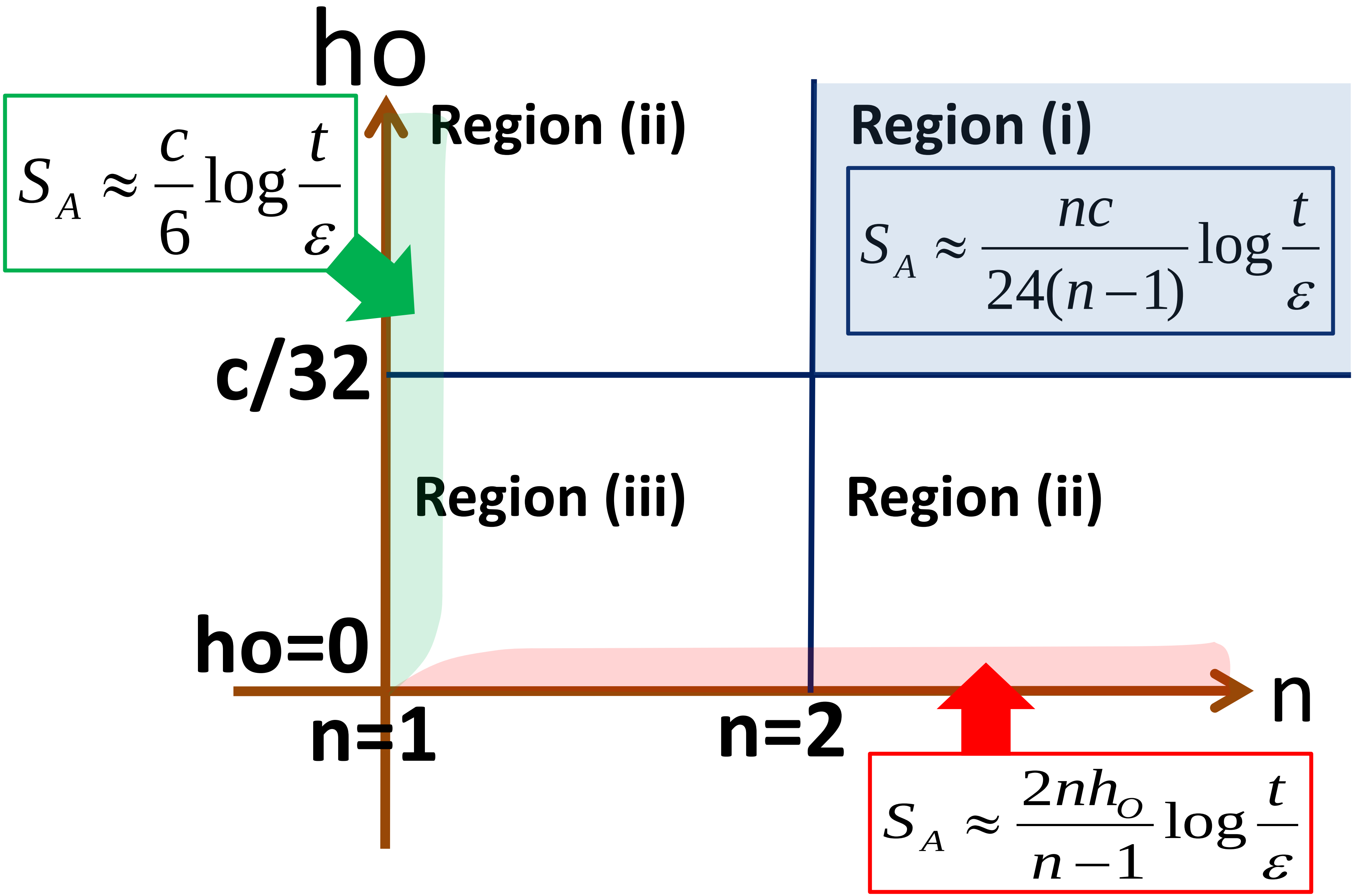
Such behaviors are sensitive to the dimensions hA and hB.





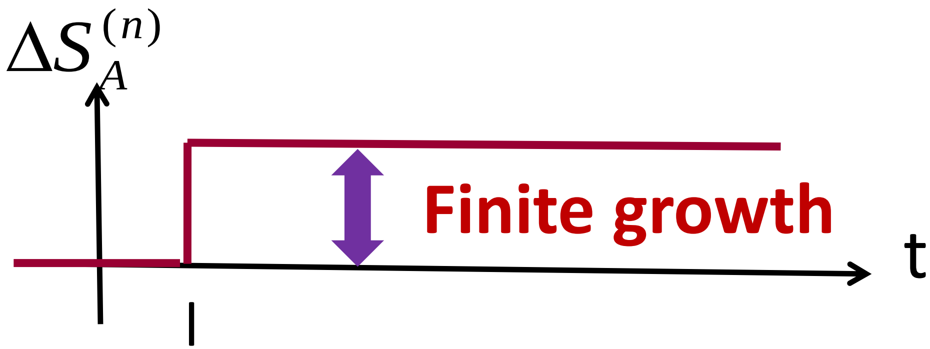


Our Final Result on Renyi EE in Large c CFT



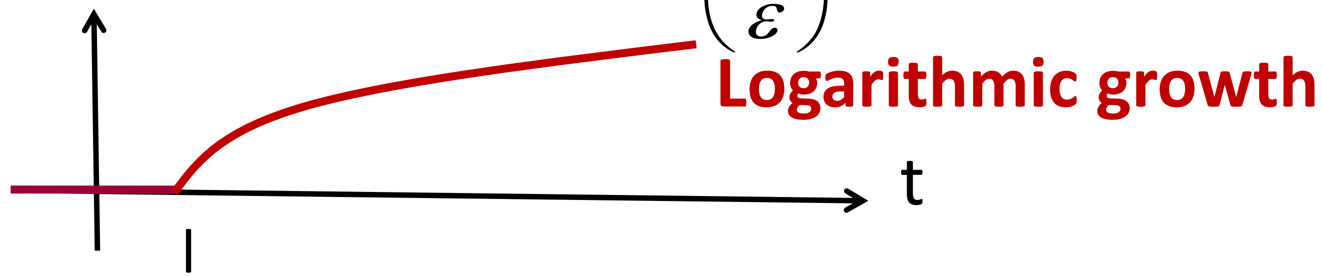
Summary so far (We focus on 2d CFTs)

(i) RCFT (Integrable and Rational)



(ii) Holographic CFTs (Large c CFTs) ~Maximally Chaotic

$$\Delta S_A^{(n)} \approx B(c, h_O, n) \cdot \log\left(\frac{t}{\varepsilon}\right)$$



Are there any intermediate behaviors ?

⑥ Case 4: Cyclic Orbifold CFTs

[Caputa-Kusuki-Watanabe-TT 17]

We consider a class of cyclic (Zn) orbifold CFTs (c=2n):

$$\left(T^2\right)^n / Z_n , \qquad (Z_n = \text{symmetric group}).$$

The T^2 CFT is described by two real scalar X and Y, which are compactified as X~X+2πR , Y~Y+2πR.

It is useful to define $\eta=R^2$. ($\eta=1$: self-dual radius)

(a) If $\eta=p/q$ (rational), the torus CFT becomes rational.

(b) If η is irrational, the torus CFT becomes irrational.

Integrable but Irrational !

A twisted sector of Sn orbifold

$$(X^{(i)}, Y^{(i)}) \to (X^{(i+1)}, Y^{(i+1)}), \quad i = 1,2,..,n$$

The corresponding primary operator (twist operator) is denoted by σ_n . Conformal dim.: $\Delta = \frac{c}{24}(n - n^{-1})$.

We are interested in the local excitation by this twist operator:

$$|\Psi\rangle \equiv e^{-\varepsilon H} \cdot \sigma_n(x)|0\rangle.$$

UV regularization

The four point function of twist operators

[Calabrese-Cardy-Tonni 09]

$$\left\langle \sigma_n \bar{\sigma}_n \sigma_n \bar{\sigma}_n \right\rangle \propto F_n(z, \bar{z}),$$

$$F_n(z,\bar{z}) = \frac{2^{n-1}\eta^{n-1}}{\prod_{k=1}^{n} I_{k/n}(z,\bar{z})} \cdot \Theta(\eta\Gamma)^2,$$

$$\Theta(\Gamma) \equiv \sum_{m \in Z^{n-1}} e^{i\pi m^T \cdot \Gamma \cdot m}, \qquad (\Gamma \text{ is a certain rank } 2(n-1) \text{ matrix})$$

$$I_{k/n}(z,\bar{z}) = {}_2F_1\left(\frac{k}{n}, 1-\frac{k}{n}, 1; z\right) \cdot {}_2F_1\left(\frac{k}{n}, 1-\frac{k}{n}, 1; 1-\bar{z}\right) + (c.c.).$$

Normalized s.t. $F_n(0,0) = 1.$

(a) Rational case (η=p/q)

In the rational case we find

$$F_n(1,0) = \frac{1}{(2pq)^n}.$$

Thus the growth of 2nd Renyi entropy is finite:

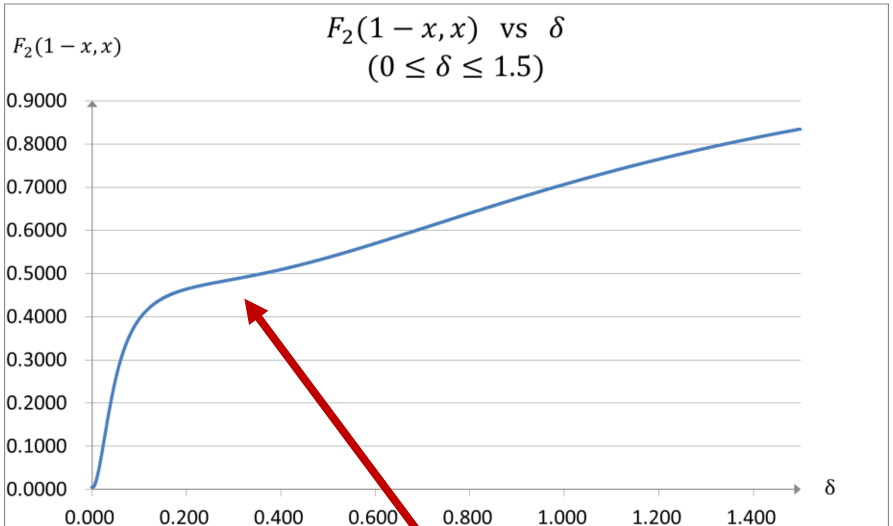
$$\Delta S_A^{(2)} = (n-1) \cdot \log(2pq) \sim (c/2) \log(2pq).$$

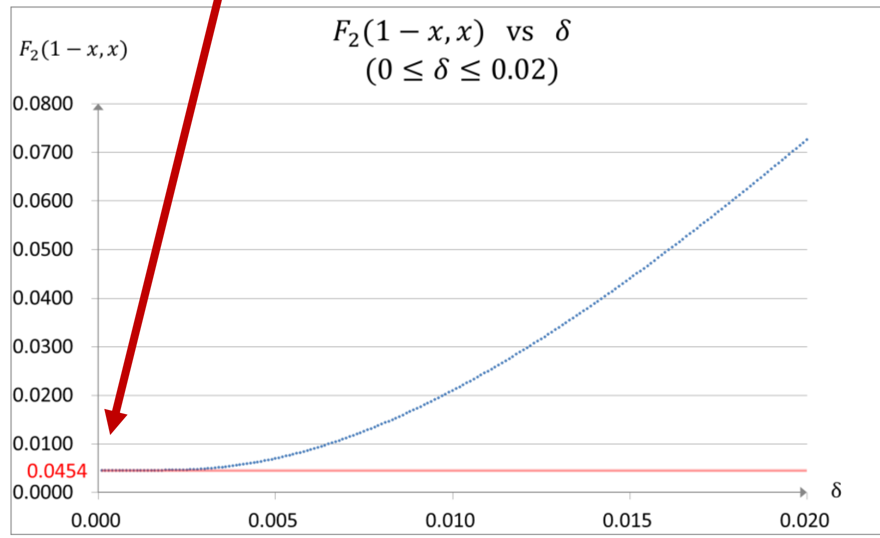
Indeed this agrees with the quantum dimension:

$$d_\sigma = \frac{S_{0\sigma}}{S_{00}} = \frac{1}{(s_{00})^{n-1}} = (2pq)^{n-1}.$$

Numerical Plot for η=10/11

Correct Plateau F2=1/220





Plateau corresponding to η≒1 ⇒ F2≒1/2.

$$\text{We defined: } \delta = \frac{\pi}{2\log(2t/\varepsilon)} \quad \left(\underset{t\to\infty}{\longrightarrow} 0\right)$$

(b) Irrational case (η≠p/q)

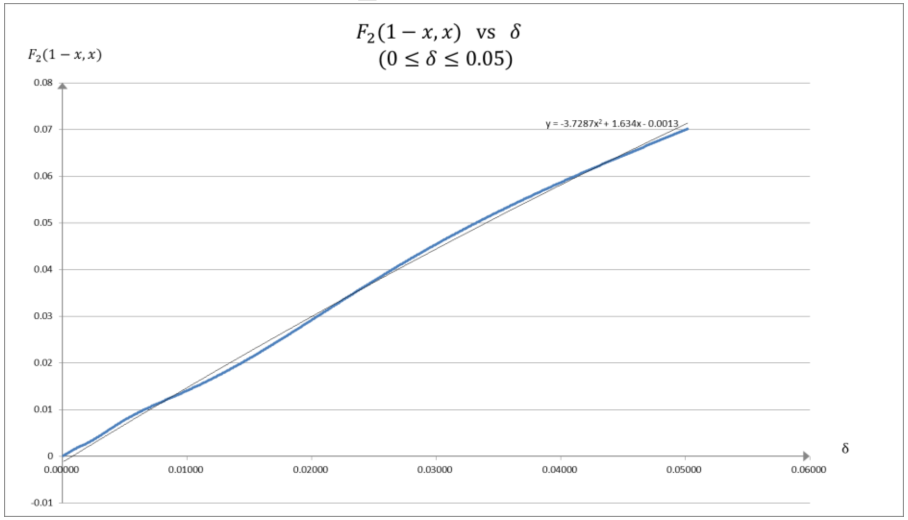
First of all we get $F_n(1,0)=0. \Rightarrow \Delta S_A^{(2)}(t=\infty)=\infty.$

As we sweep δ or the time t, we picks up infinitely many plateau contributions (⇔ rational values η≒p/q).

Note: Continued fractions describe irrational numbers.

eg. $\sqrt{2}=1+\cfrac{1}{2+\cfrac{1}{2+\dots}}.$

Plot of F_2 for $\eta=\sqrt{2}$



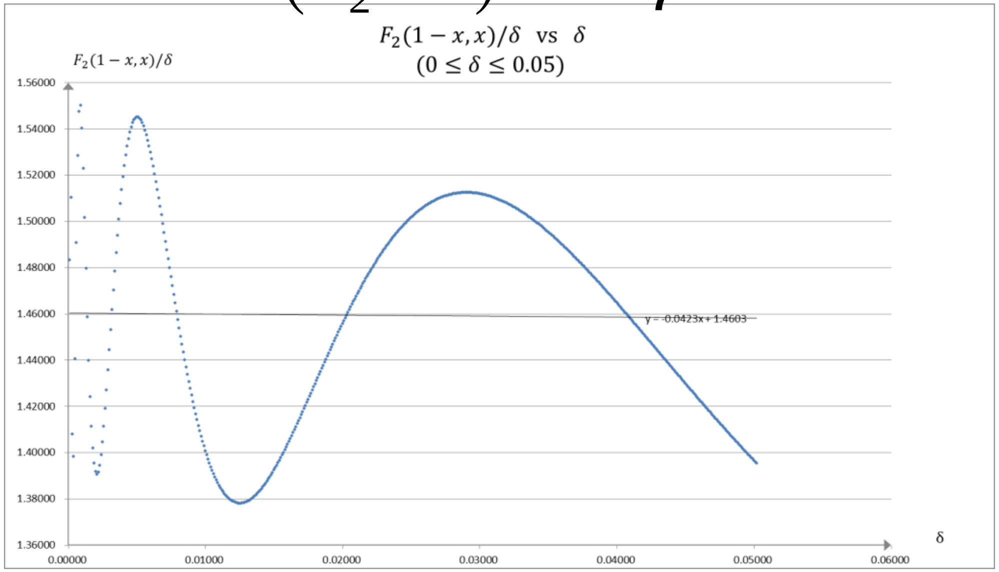
F2 behaves as follows for small δ (or large t) :

$$1 \leq \frac{F_2\left(1 - \frac{\varepsilon^2}{4t^2}, \frac{\varepsilon^2}{4t^2}\right)}{\delta} \leq A(\eta).$$

This leads to the log(log) growth of Renyi EE !

$$\Delta S_A^{(2)} \approx \log\left(\log\left(\frac{t}{\varepsilon}\right)\right).$$

Plot of $\left(F_2 / \delta\right)$ for $\eta = \sqrt{2}$



⑦ Conclusions

We summarize the time evolutions of (Renyi) EEs $\Delta S_A^{(n)}$ for locally excited states in various CFTs.

(a) Free CFTs in any dim. or Rational CFTs in 2d

$$\Delta S_A^{(n)} \text{ shows a finite growth} = \log[\text{quantum dim.}].$$

(b) Holographic CFTs in 2d $\Delta S_A^{(n)} \propto B(c, h_O, n) \cdot \log\left(\frac{t}{\varepsilon}\right)$

(c) Irrational but integrable 2d CFT (sym. orbifold CFTs)

$$\Delta S_A^{(n)} \propto \log\log\left(\frac{t}{\varepsilon}\right)$$

(d) Higher dim. Hol. CFTs ⇒ At least sublinear [Jahn-TT, 2017]

These results naturally lead us to the following conjecture:

Our Conjecture

Maximal growth of EE *at late time* under a local excitation will be logarithmic with the coefficient for large c CFTs:

$$\Delta S_A^{(n)}(t) \leq B(c, h_O, n) \cdot \log\left(\frac{t}{\varepsilon}\right).$$