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ENTROPY FROM SUPERSPACE INFLOW

HOW ENTROPY IS NOETHERIAN

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Related work

- ★ Crossley, Glorioso, Liu, Gao
- ★ Jensen, Marjieh, Pinzani-Fokeeva, Yarom

Prologue: A preview of things to come...

ENTROPY PRODUCTION VIA INFLOW

- For systems in local equilibrium the Noether current for thermal diffeomorphisms is the macroscopic free energy current (Legendre transform of entropy current).

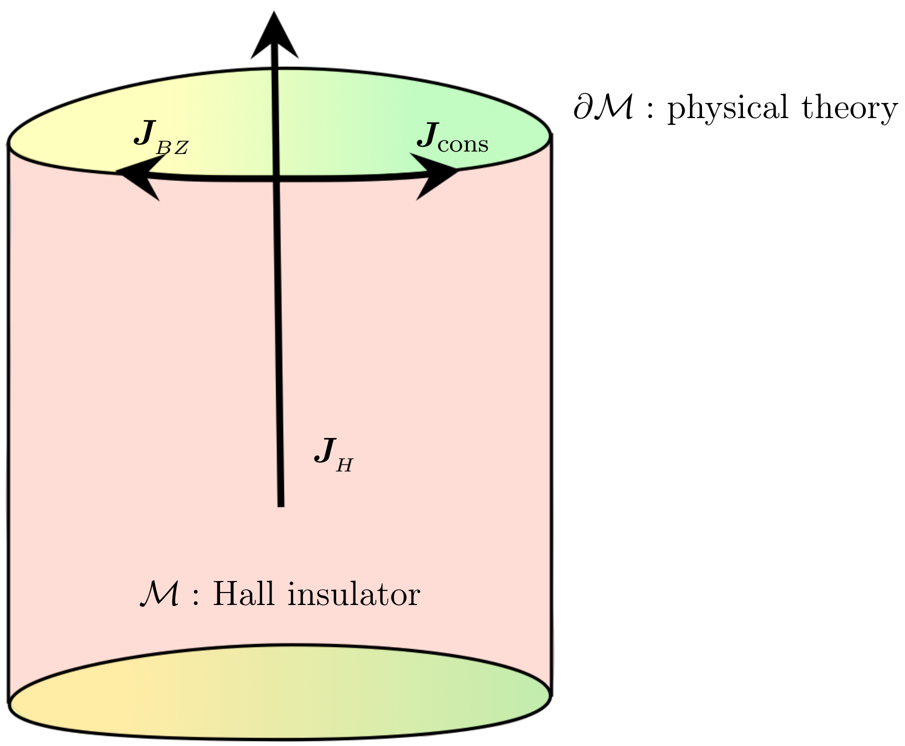
- Local equilibrium is characterized by an emergent topological/BRST supersymmetry wherein diffeomorphisms along the Euclidean thermal circle are gauged (thermal equivariance).

- Net entropy is conjugate to the gauged thermal diffeomorphisms & is conserved. Physical entropy production happens by virtue of it being sourced in the superspace directions, i.e., there is an inflow of entropy from superspace.

$$\mathring{\mathcal{D}}_I \mathring{\mathbf{N}}^I - \frac{1}{2}\, \mathring{\mathbf{T}}^{IJ}\, \pounds_{\boldsymbol{\beta}} \mathring{\mathrm{g}}_{IJ} = 0 \qquad \Longrightarrow \qquad \mathcal{D}_a \mathbf{N}^a - \frac{1}{2}\, \mathbf{T}^{ab}\, \pounds_{\boldsymbol{\beta}} \mathbf{g}_{ab} = \Delta \geq 0$$

ENTROPY INFLOW

- While the inflow mechanism for entropy arises from the superspace, it is morally similar to the manner in which the inflow mechanism operates in the context of Hall insulators & chiral edge states ('t Hooft anomalies).



Callan, Harvey (1985)

anomaly inflow: coupling to a topological sector
with physical entropy being sourced in superspace

Act I

in which we recall why entropy is Noetherian in gravitational theories

THERMODYNAMIC ENTROPY

- Microcanonical picture: counting statistics for microstates
- Canonical picture: von Neumann entropy for Gibbs state

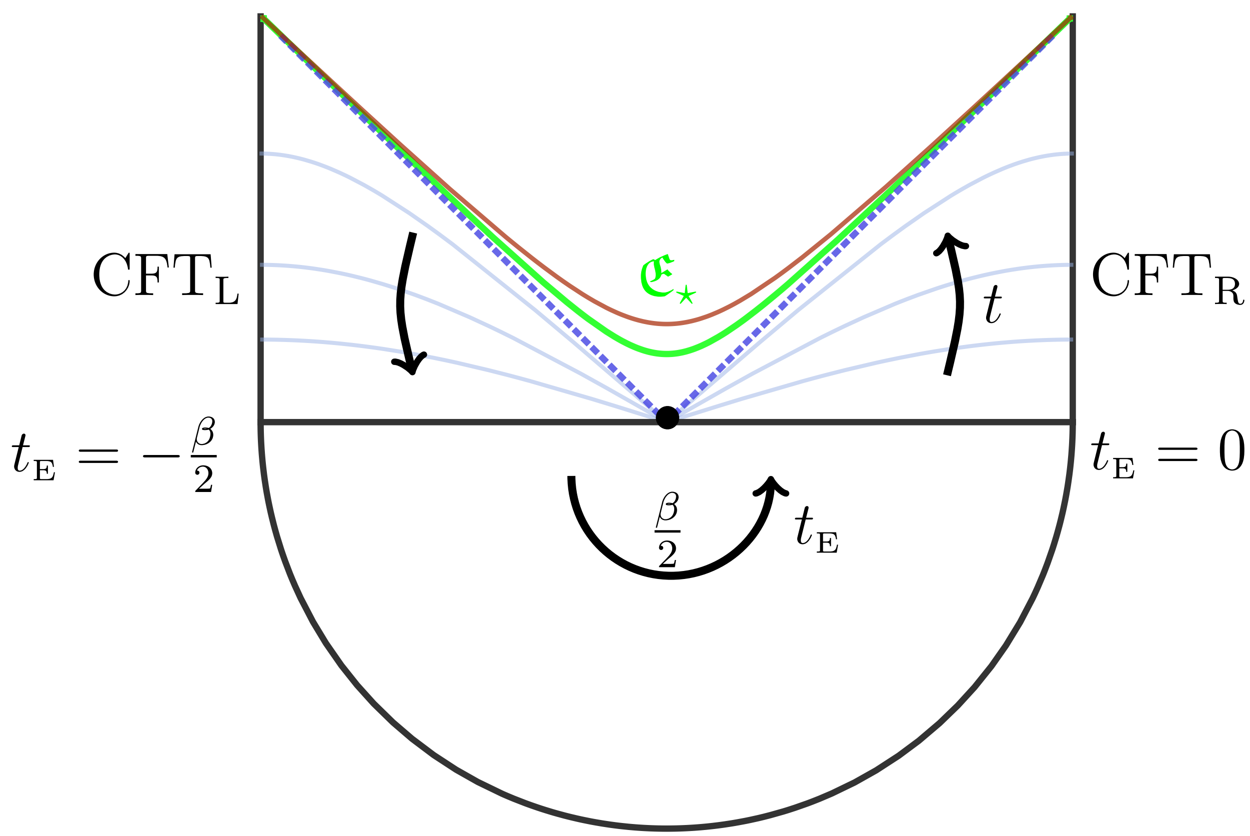
$$\rho_\beta = e^{-\beta\, H} \qquad\qquad S = -\mathrm{Tr}\left(\rho_\beta \log \rho_\beta\right)$$

- One way to interpret this is to purify the Gibbs state via the thermofield double:

$$|\,\mathsf{TFD}\rangle_\beta = \frac{1}{\sqrt{Z(\beta)}} \sum_k e^{-\frac{1}{2}\,\beta\, E_k}\; |\,\mathfrak{r}_k\rangle \otimes |\,\mathfrak{l}_k\rangle\,, \qquad Z(\beta) = \sum_k e^{-\beta\, E_k}$$

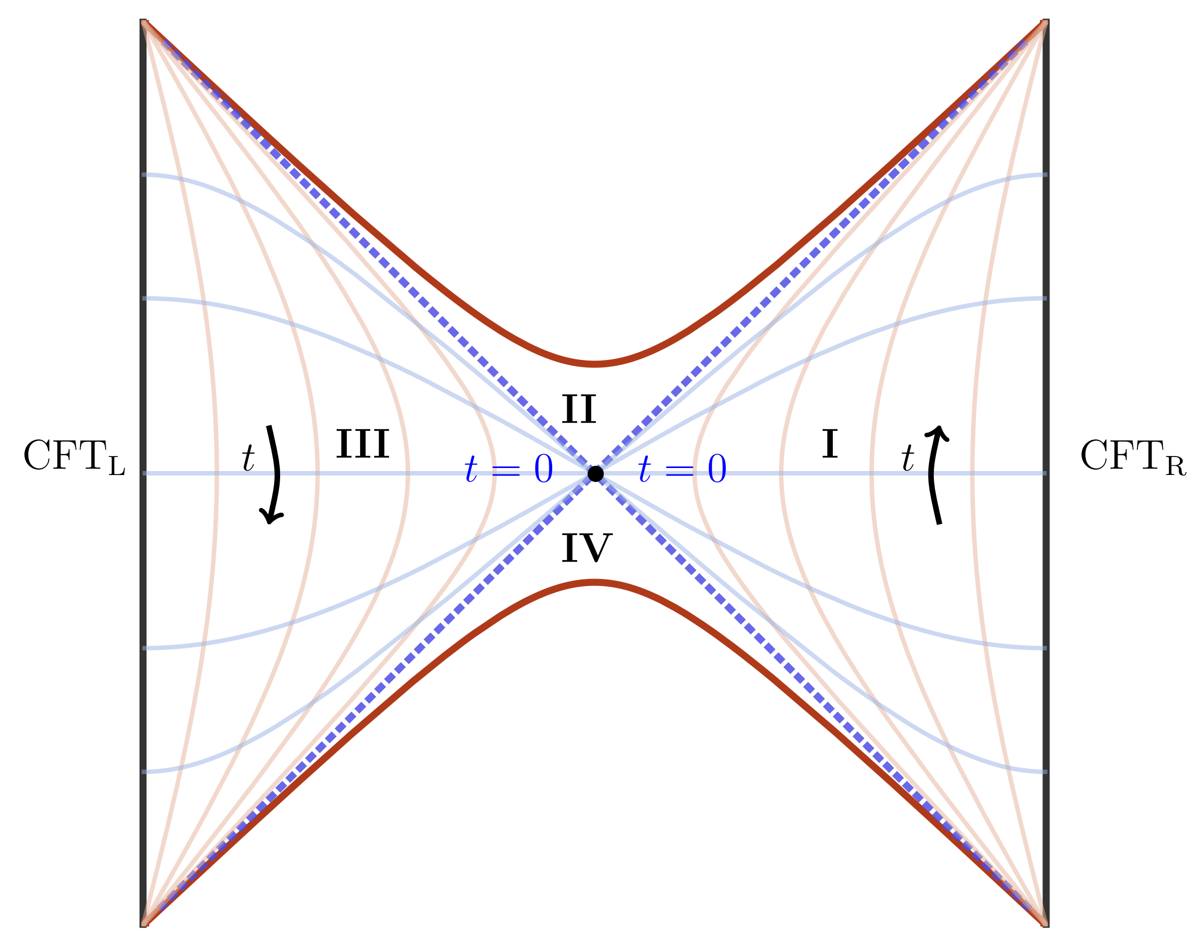
- In this picture the von Neumann entropy is the entanglement between the two sets of degrees of freedom.

GEOMETRIZING THERMOFIELD



State preparation via Euclidean functional integral

Lorentzian (real-time) picture as an eternal black hole.



ENTROPY IN GEOMETRY

- Geometrically the entropy is encoded in a geometric functional – the area in the case of Einstein-Hilbert dynamics for black holes.
- More generally, one can show that this equilibrium entropy in gravitational systems is a Noether charge.

Iyer-Wald [gr-qc/9403028]

$$\delta \mathbf{L} = \mathbf{E}_\phi \cdot \delta\phi + d\mathbf{\Theta}(\phi, \delta\phi)$$

variation = eqs of motion + boundary terms

$$\mathbf{J} = \mathbf{\Theta}(\phi, \mathcal{L}_\xi \phi) - \iota_\xi \mathbf{L}$$

symplectic potential = Legendre transform of boundary terms

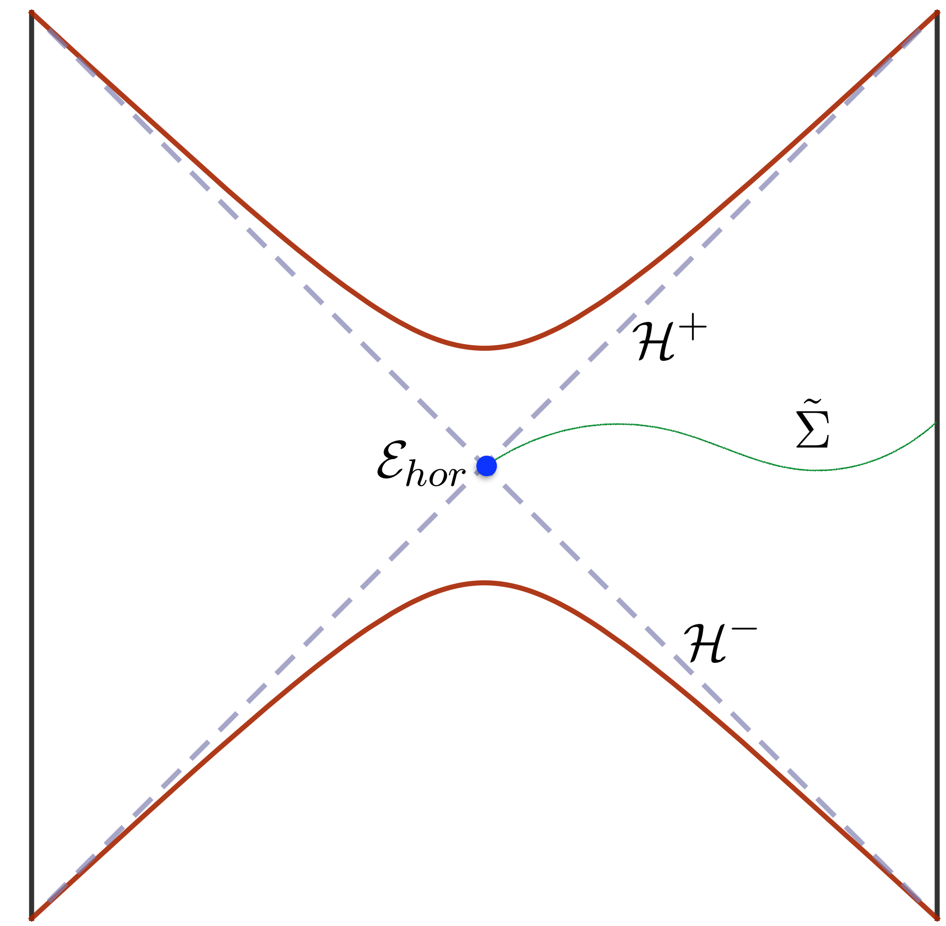
$$\nabla_A J^A = d\mathbf{J}[\xi] = d\mathbf{\Theta}(\phi, \mathcal{L}_\xi\phi) - d(\iota_\xi \mathbf{L}) \\ = -\mathbf{E}_\phi \cdot \delta\phi$$

symplectic potential conserved modulo eqs of motion

ENTROPY IN GEOMETRY

$$d\mathbf{J}[\xi] \overset{\mathbf{E}}{=} 0 \implies \mathbf{J}[\xi] \overset{\mathbf{E}}{=} d\mathbf{Q}[\xi]$$

$$S_{bh} = \int_{\mathcal{E}_{hor}} \mathbf{Q}[\xi_{hor}]$$



- The Noether charge that captures entropy involves diffeomorphisms along the horizon.
- In the Euclidean picture these are rotations along the thermal circle.
- Thermal diffeomorphisms are responsible for entropy (or free energy).

NOETHERIAN ENTROPY & ADS/CFT

- Given that black hole entropy is Noetherian, it must be that the dual field theory can also be interpreted similarly in equilibrium.
- Likewise for a subclass of field theory reduced density matrices the entanglement entropy is also Noetherian (eg. spherical domains in the vacuum state of a CFT).
- Another example is the ground state entropy of the SYK model where the emergent SL(2) symmetry appears to control the degeneracy.

Questions:

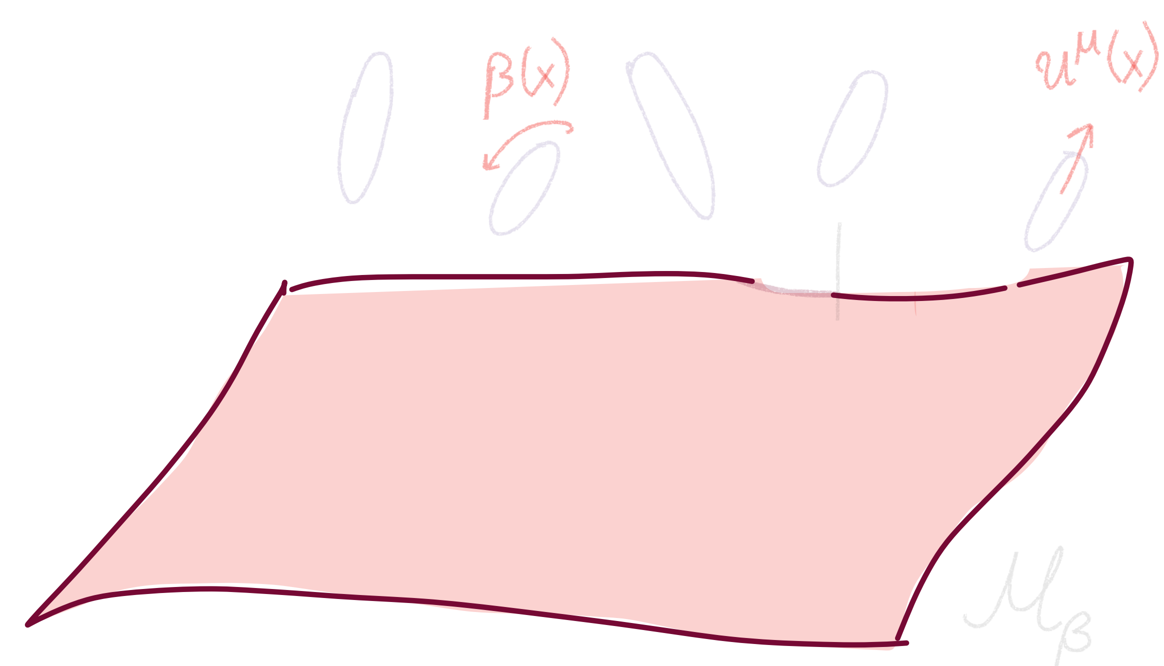
- How far do these statements generalize?
- Can entropy always be given a Noetherian interpretation?

Act II

in which we infer some salient facts from hydrodynamics

ENTROPY IN EQUILIBRIUM

- AdS/CFT maps the gravitational dynamics into a boundary QFT.
- Eternal black holes describe thermal (Gibbsian) states of the QFT.
- How does the Noetherian feature of entropy show up in the QFT?
- Hydrostatic free energy current is a Noether current for thermal diffeomorphisms.



- Thermal equilibrium can be attained in any stationary spacetime with a timelike Killing field.
- Observables can be obtained from a statistical field theory on a Riemannian space $\mathcal{M}_{\boldsymbol{\beta}}$: the thermal circle fibration over a the spatial geometry.

ENTROPY IN EQUILIBRIUM

- The hydrostatic partition function on $\mathcal{M}_{\boldsymbol{\beta}}$ allows for computation of equal time correlation function of conserved currents.

Banerjee et al [1203.3544]

Jensen et al [1203.3566]

- Fluid view of such hydrostatic configurations when gradients are small:
  - local temperature is fixed to be the size of the thermal circle.
  - fluid velocity is oriented along the Killing field (the fibre).

$$\boldsymbol{\beta}^\mu = \frac{u^\mu}{T}$$

thermal vector

$$W[g_{\mu\nu}] = \int_{\mathcal{M}_{\boldsymbol{\beta}}} d^d x \sqrt{-g}\; P_s[\boldsymbol{\beta}^\alpha, g_{\mu\nu}]$$

- It then follows that the equilibrium entropy current is the Noether current for thermal diffeomorphisms (same argument as before).

$$(J^\mu_S)_{eq} = -\boldsymbol{\beta}_\nu\, T^{\mu\nu} + \boldsymbol{\beta}^\mu P_s[\boldsymbol{\beta}^\alpha, g_{\alpha\beta}] - \Theta^\mu_{PS}$$

$$N^\mu = J^\mu_S + \boldsymbol{\beta}_\nu\, T^{\mu\nu}$$

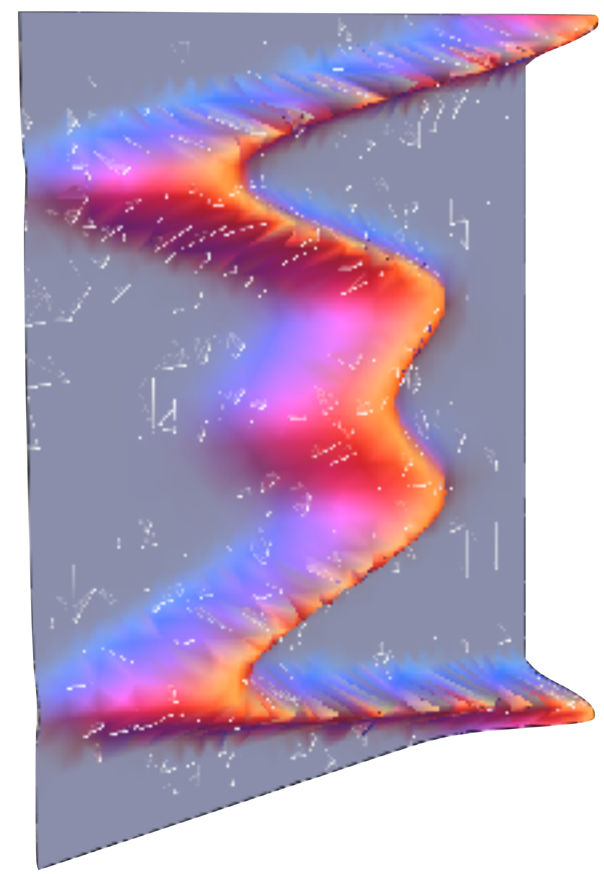
$$\nabla_\mu J^\mu_S = 0 \implies \nabla_\mu N^\mu = 2\, T^{\mu\nu} \nabla_{(\mu}\boldsymbol{\beta}_{\nu)} = T^{\mu\nu} \pounds_{\boldsymbol{\beta}} g_{\mu\nu} = 0$$

HLR [1502.00636]

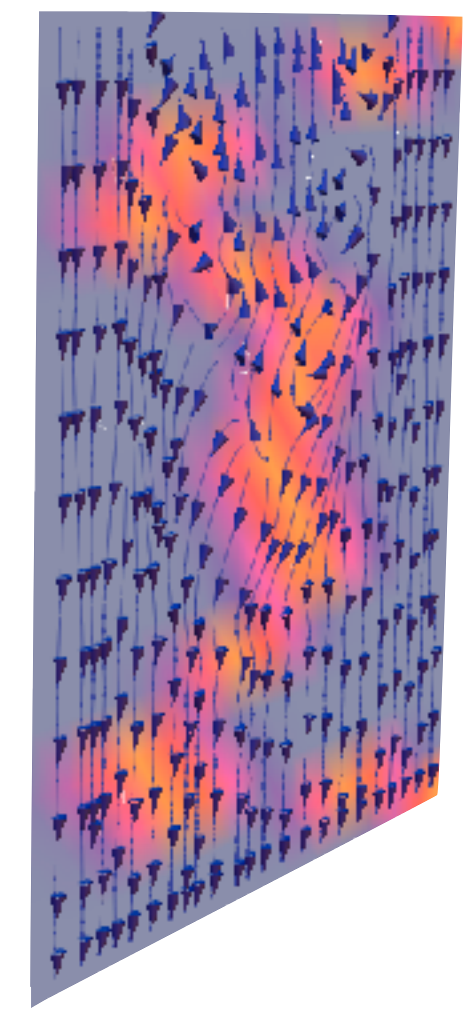
BEYOND EQUILIBRIUM

- Deviations from thermal state operate on long spatio-temporal scales are captured by hydrodynamics.
- In the hydrodynamic limit, dissipation leads to entropy production.
- Geometrically, stuff falls behind the black hole, the horizon grows, and remarkably the local area functional leads to a notion of entropy current.

$$\nabla_\mu J^\mu_S = \frac{1}{T}\,\left(2\,\eta\,\sigma_{\mu\nu}\sigma^{\mu\nu} + \zeta\,\Theta^2\right)$$



entropy (horizon)



conserved charges (boundary)

$$T^{\mu\nu} = \epsilon\, u^\mu\, u^\nu + p\, P^{\mu\nu} - 2\,\eta\,\sigma^{\mu\nu} - \zeta\,\Theta\, P^{\mu\nu}$$

ENTROPY NEAR-EQUILIBRIUM

- **Bhattacharyya's Theorem:** There exists a hydrodynamic entropy current satisfying the local form of second law, as long as leading order dissipative terms are sign-definite (e.g., viscosities & conductivities are non-negative), provided hydrostatics is consistent with the equilibrium partition function.

Bhattacharyya [1403.7639]

- The allowed class of hydrodynamic constitutive relations fall into 8 distinct classes of solutions to the *adiabaticity equation:*

$$\nabla_\mu N^\mu - \frac{1}{2} T^{\mu\nu}\, \pounds_{\boldsymbol{\beta}} g_{\mu\nu} = \Delta \geq 0 \qquad\qquad N^\mu = J^\mu_S + \boldsymbol{\beta}_\nu\, T^{\mu\nu}$$

- One of the 8 classes captures dissipation and entropy production, and is parameterized by a 4-tensor which gives a non-negative definite inner product on the space of symmetric two-tensors:

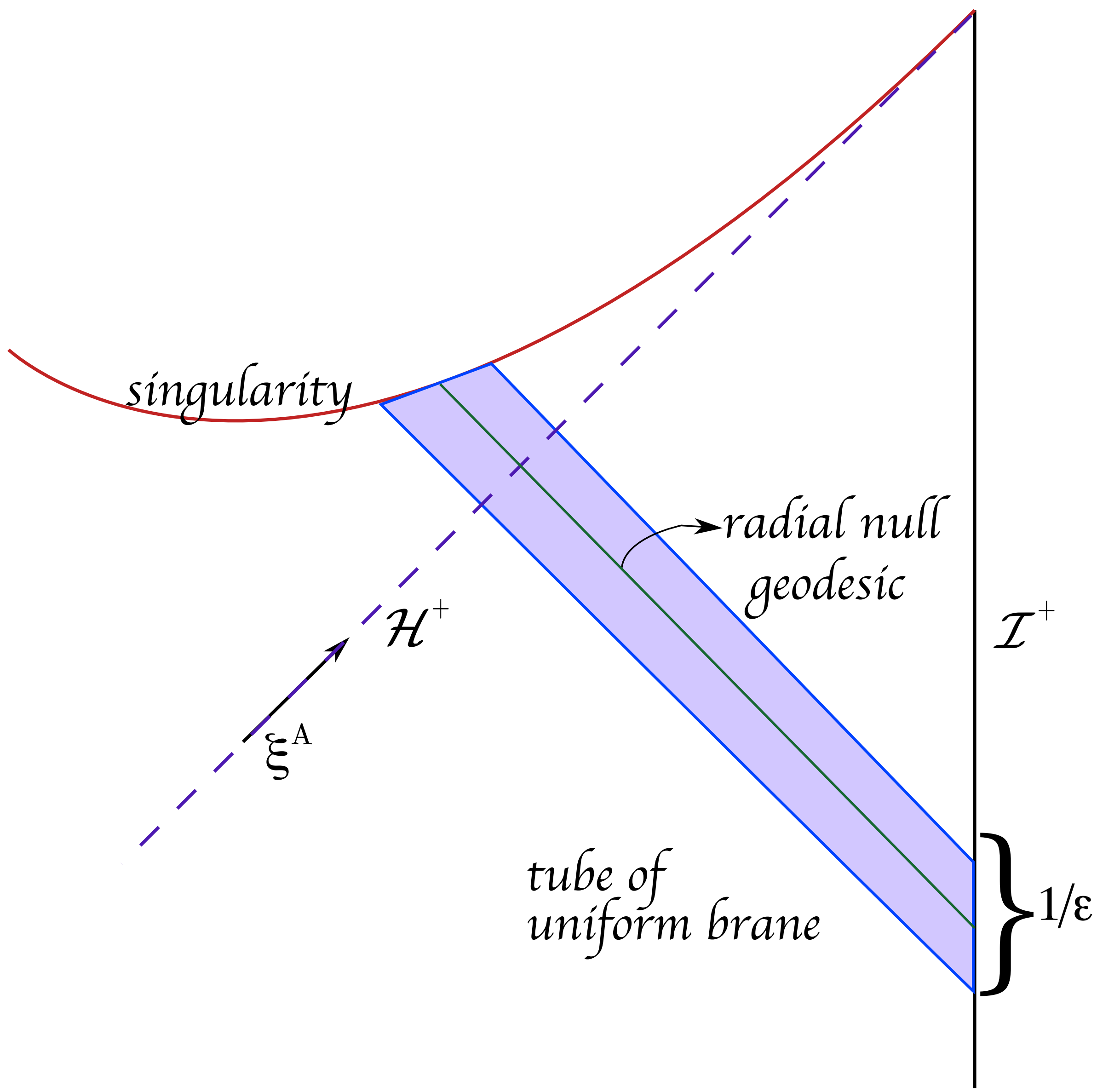
$$T^{\mu\nu}_{diss} = \frac{1}{2}\, \boldsymbol{\eta}^{(\mu\nu)(\rho\sigma)}\, \pounds_{\boldsymbol{\beta}} g_{\rho\sigma} \qquad\qquad \Delta = \frac{1}{4}\, \boldsymbol{\eta}^{(\mu\nu)(\rho\sigma)}\; \pounds_{\boldsymbol{\beta}} g_{\mu\nu}\; \pounds_{\boldsymbol{\beta}} g_{\rho\sigma}$$

HLR [1502.00636]

cf., Glorioso, Liu [1612.07705]

HYDRODYNAMIC ENTROPY CURRENT

- Unlike other conserved charges, the role of entropy current is somewhat mysterious, as it emerges in the IR without being manifest in the UV, in near-thermal systems.



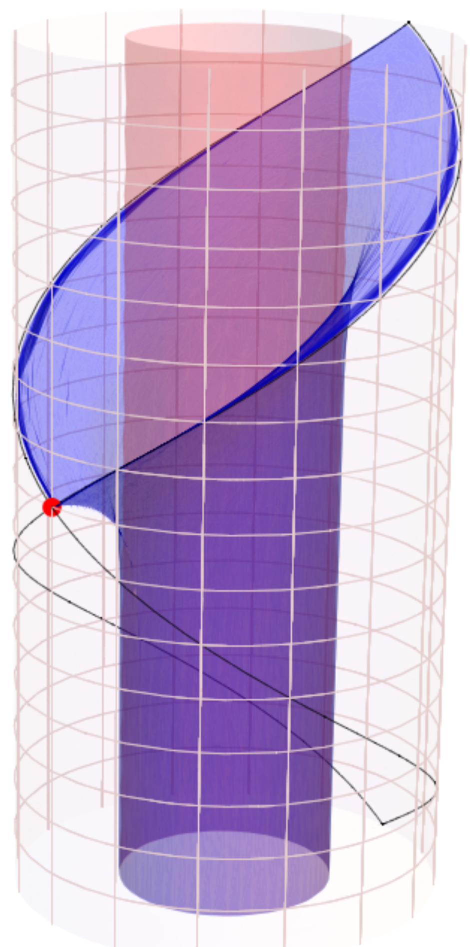
- Very clear in AdS/CFT where entropy is a property of horizons while conserved charges live on the boundary.

- Eg., in fluid/gravity the hydrodynamic entropy current is the pullback of the horizon area/ entropy (or generalizations thereof).

Bhattacharyya et al [0803.2526]

WHEREFROM ENTROPY?

- ✦ Unlike other conserved charges, the role of entropy current is somewhat mysterious, as it emerges in the IR without being manifest in the UV, in near-thermal systems.



- ✦ Similar statement may plausibly hold in out of equilibrium scenarios.
- ✦ Thermodynamic entropy is the pullback of apparent horizon area, as opposed to event horizon area (latter being teleological).
- ✦ In fluid/gravity limit the long-wavelength blurs the distinction between the two.

Chesler, Yaffe [0812.2053]

Figueras, Hubeny, Rangamani, Ross [0902.4696]

cf., Engelhardt, Wall [1706.02038]

ADIABATIC FLUIDS

- Adiabaticity equation admits 7 inequivalent classes of adiabatic constitutive relations (eg., equilibrium data, anomalies, Hall transport etc) which are non-entropy producing.

- Attempting to understand this conservative sub-sector of near equilibrium dynamics led to two empirical observations:

  - One needs two sets of hydrodynamic degrees of freedom with the average values appearing in the constitutive relations.

  - The adiabaticity equation holds as a Bianchi identity off-shell once one introduces a KMS gauge symmetry which gauges thermal diffeos.

HLR [1502.00636]

- The free energy current is the Noether current for the thermal KMS gauge symmetry.

- These statements can be rephased in the framework of *thermal equivariance*.

NON-EQUILIBRIUM EFFECTIVE DYNAMICS

- What is the framework for a consistent Wilsonian treatment of low energy dynamics in mixed states of a QFT?

- Hydrodynamics should emerge in interacting quantum systems by integrating out irrelevant high energy modes.
- Low energy modes should be universal.
- CPT breaking, dissipation, ideally should arise from a dynamical principle.
- Effective theory should capture stochastic and quantum fluctuations.

  - Integrating out high energy modes starting from microscopic Schwinger-Keldysh leads to coupling between L and R encoded in influence functionals.

- What influence functionals are consistent with microscopic unitarity?

Act III

in which we pause to reflect upon non-equilibrium effective field theories and learn how one might constrain their dynamics to respect microscopic unitary requirements

HYDRODYNAMICS: TRANSPORT, FLUCTUATIONS

- Hydrodynamics: low energy dynamics of conserved currents in near equilibrium situations.

- Transport is captured by response functions: these are the first non-trivial correlators involving 1-average and rest difference operators.

- KMS relations relate response functions to fluctuations, e.g., and embody the fluctuation-dissipation theorem:

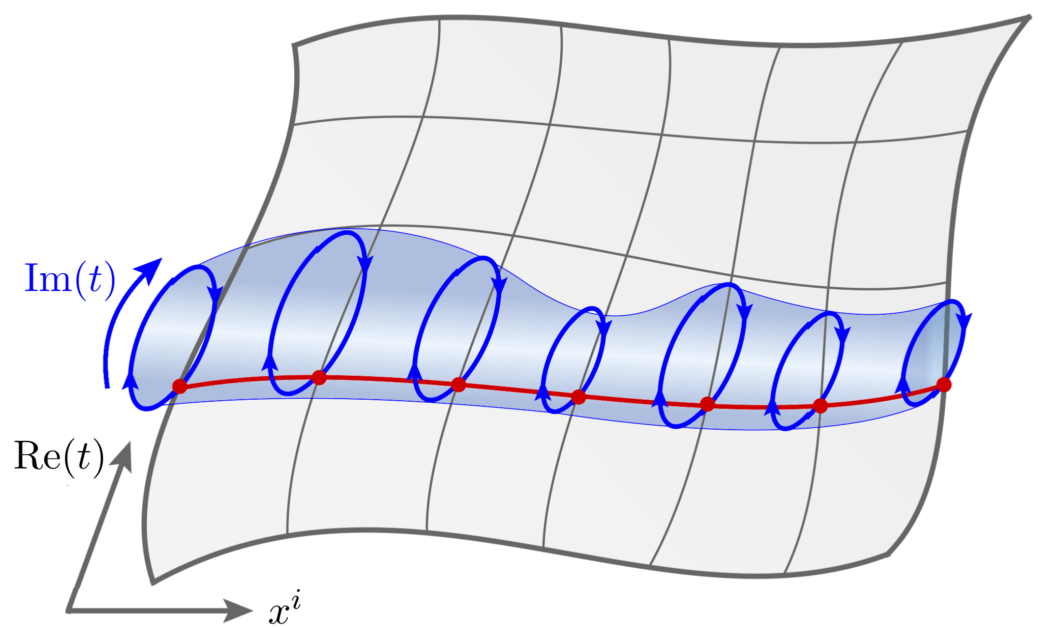
$$\mathrm{Tr}\left(\widehat{\mathbb{A}}(t_{_A})\,\widehat{\mathbb{B}}(t_{_B})\,\hat{\rho}_{_T}\right) = \mathrm{Tr}\left(\widehat{\mathbb{B}}(t_{_B} - i\,\beta)\,\widehat{\mathbb{A}}(t_{_A})\right)$$

$$\Longrightarrow \quad \langle\{\widehat{\mathbb{A}},\widehat{\mathbb{B}}\}\rangle = -\coth\left(\frac{1}{2}\,\beta\,\omega_{_B}\right)\,\langle[\widehat{\mathbb{A}},\widehat{\mathbb{B}}]\rangle$$

- Look to constructing an effective field theory that captures all hydrodynamic transport & attendant fluctuations.

THE TOPOLOGICAL SECTOR

- Framework for encoding the constraints of Schwinger-Keldysh functional integrals and KMS relations involves topological gauge theory of thermal diffeomorphisms.



$$(\Lambda_1, \Lambda_2)_{\boldsymbol{\beta}} = \Lambda_1 \pounds_{\boldsymbol{\beta}} \Lambda_2 - \Lambda_2 \pounds_{\boldsymbol{\beta}} \Lambda_1$$

THE SCHWINGER-KELDYSH SUPER-QUARTET

- Macroscopic theory involves two sets of degrees of freedom
  - the average or classical fields (which dissipate)
  - the difference or quantum/stochastic fluctuation fields
- Consistent couplings are dictated by the ghost fields that are now part of the BRST multiplet. All of this structure can be nicely captured in a superspace that is locally $\mathbb{R}^{(d-1,1)|2}$ coordinatized by $z^I = \{\sigma^a, \theta, \bar{\theta}\}$.
- Advantage: ensure that low energy influence functionals are consistent with microscopic unitarity and fluctuation-dissipation relations.
- The BRST symmetry ensures both the largest time equation (difference operators can't be future-most) & the KMS relations.

THE MALLICK-MOSHE-ORLAND LIMIT

- The thermal superalgebra involves supercharges obeying

$$\mathcal{Q}^2 = (\mathring{\mathfrak{F}}_{\bar\theta\bar\theta}|_{\bar\theta=\theta=0})\,\mathcal{L}_{KMS}\,, \qquad \overline{\mathcal{Q}}^2 = (\mathring{\mathfrak{F}}_{\theta\theta}|_{\bar\theta=\theta=0})\,\mathcal{L}_{KMS}\,, \qquad \left[\mathcal{Q}, \overline{\mathcal{Q}}\right]_\pm = (\mathring{\mathfrak{F}}_{\theta\bar\theta}|_{\bar\theta=\theta=0})\,\mathcal{L}_{KMS}$$

- The universal gauge multiplet has exactly one component with vanishing ghost number: the bottom component of $\mathring{\mathfrak{F}}_{\theta\bar\theta}$.

- Assumption: The topological gauge dynamics is consistent with the existence of a vacuum where $\langle \mathring{\mathcal{F}}_{\theta\bar\theta}| \rangle = -i$ & CPT is spontaneously broken.

$$\mathcal{Q}^2 = 0\,, \qquad \overline{\mathcal{Q}}^2 = 0\,, \qquad \left[\mathcal{Q}, \overline{\mathcal{Q}}\right]_\pm = -i\,\mathcal{L}_{KMS} \mapsto i\pounds_{\boldsymbol{\beta}}$$

- This algebra has appeared in the statistical mechanics literature in the context of stochastic Langevin dynamics & is the high temperature version of the algebra derived in related works: *Mallick, Moshe, Orland [1009.4800]*

$$\delta^2 = \bar\delta^2 = 0, \qquad \{\delta, \bar\delta\} = 2\,\tanh\left(\frac{i}{2}\,\beta\,\partial_t\right) \simeq i\,\beta\,\partial_t$$

Crossley, Glorioso, Liu [1511.03646]

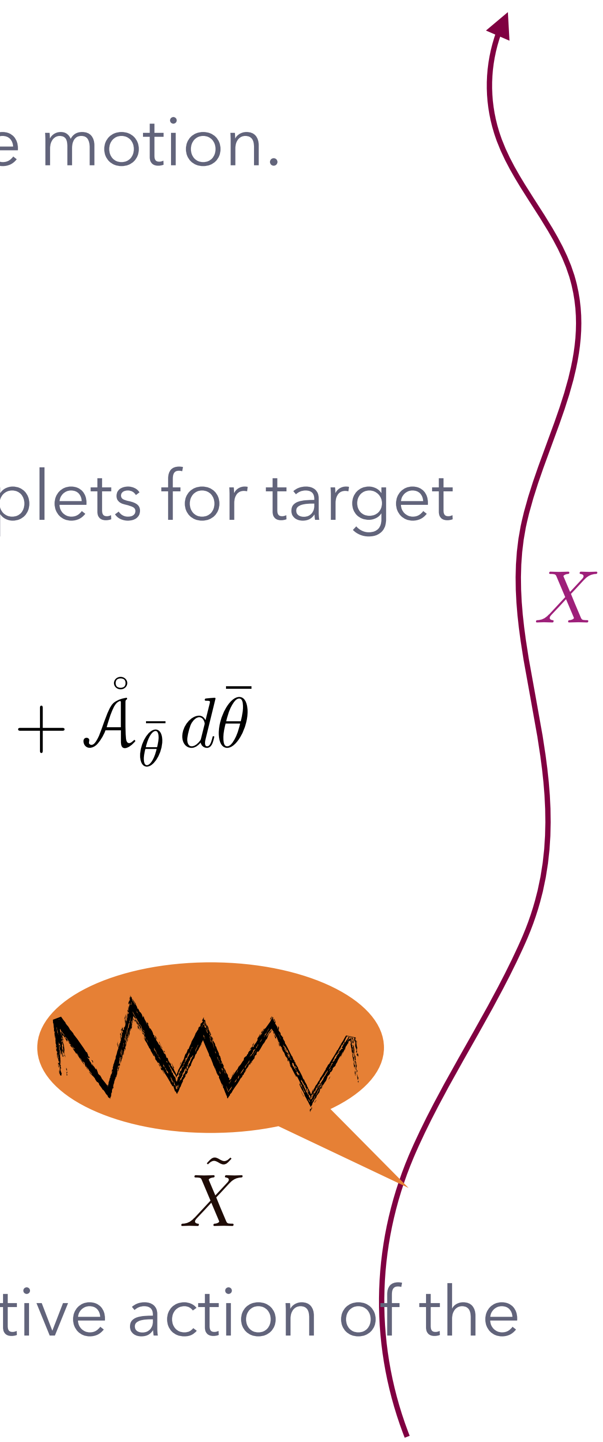
BROWNIAN PARTICLE

- Brownian particle immersed in a fluid undergoes dissipative motion.
- Langevin effective action: worldvolume B0-brane theory.
- Data for the worldvolume theory: thermal equivariant multiplets for target space coordinate map and thermal gauge field data.

$$\mathring{X} = \{X, X_\psi, X_{\bar\psi}, \tilde{X}\} \qquad \qquad \mathring{\mathcal{A}} \equiv \mathring{\mathcal{A}}_t\, dt + \mathring{\mathcal{A}}_\theta\, d\theta + \mathring{\mathcal{A}}_{\bar\theta}\, d\bar\theta$$

- Effective action with the symmetries is simply.

$$S_{\text{B0}} = \int dt\, d\theta\, d\bar\theta \left\{ \frac{m}{2} \left(\mathring{\mathcal{D}}_t \mathring{X}\right)^2 - U(\mathring{X}) - i\,\nu\, \mathring{\mathcal{D}}_\theta \mathring{X}\, \mathring{\mathcal{D}}_{\bar\theta} \mathring{X} \right\}$$



- MSR action follows as the basic thermal gauge invariant effective action of the worldline theory after CPT breaking

$$(\mathring{\Lambda}, \mathring{X})_{\boldsymbol{\beta}} = \mathring{\Lambda}\, \pounds_{\boldsymbol{\beta}} \mathring{X} = \mathring{\Lambda}\, \Delta_{\boldsymbol{\beta}} \mathring{X} = \mathring{\Lambda}\, \boldsymbol{\beta} \frac{d}{dt} \mathring{X} \qquad \qquad \mathring{\mathcal{D}}_I = \partial_I + [\mathring{\mathcal{A}}_I,\ \cdot\ ]$$

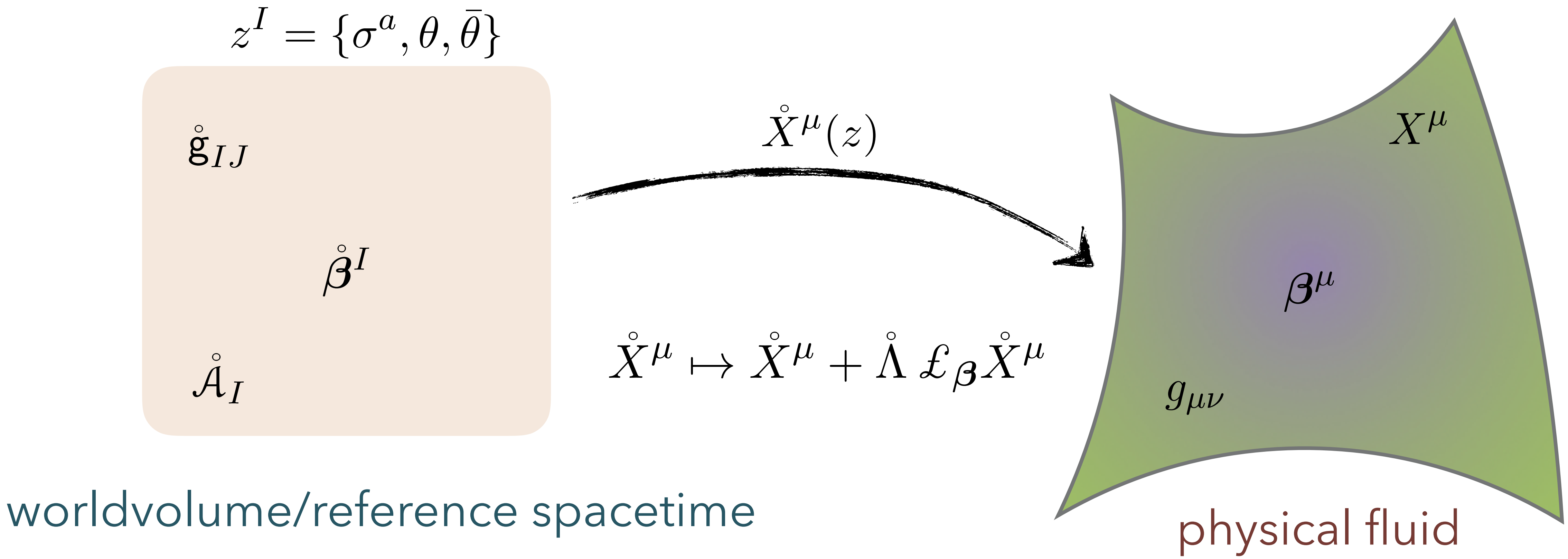
Martin, Siggia, Rose (1973)
Parisi, Sourlas (1982)

Act IV

in which we finally meet the effective field theories

capturing hydrodynamic dissipation & fluctuations

HYDRODYNAMIC SIGMA MODELS



$$\mathring{X}^\mu = X^\mu + \theta\, X^\mu_{\bar{\psi}} + \bar{\theta}\, X^\mu_{\psi} + \bar{\theta}\theta \left(\tilde{X}^\mu - \Gamma^\mu_{\rho\sigma}\, \overline{\psi}^\rho\, \psi^\sigma \right)$$

✦ Hydrodynamic action is a deformation of a topological field theory reflecting the near-topological nature of the observables.

$$\mathring{\mathrm{g}}_{IJ}(z) \;\to\; \mathring{\mathrm{g}}_{IJ}(z) + \bar{\theta}\,\theta\; \mathsf{h}_{IJ}(\sigma)$$

EFFECTIVE ACTIONS

- Hydrodynamic effective actions can be constructed as superspace integrals

$$S_{wv} = \int d^d\sigma \, \mathcal{L}_{wv} \,, \qquad \mathcal{L}_{wv} = \int d\theta \, d\bar{\theta} \, \frac{\sqrt{-\mathring{\mathrm{g}}}}{\mathring{\mathbf{z}}} \, \mathring{\mathcal{L}}[\mathring{\mathrm{g}}_{IJ}, \boldsymbol{\beta}^a, \mathring{\mathfrak{D}}_I, \mathring{\mathfrak{g}}^{(\bar{\psi})}_{IJ}, \mathring{\mathfrak{g}}^{(\psi)}_{IJ}] \,,$$

$$\mathring{\mathbf{z}} = 1 + \mathring{\boldsymbol{\beta}}^I \, \mathring{\mathcal{A}}_I \qquad \mathring{\mathfrak{g}}^{(\bar{\psi})}_{IJ} \equiv \mathring{\mathcal{D}}_\theta \mathring{\mathrm{g}}_{IJ} \,, \qquad \mathring{\mathfrak{g}}^{(\psi)}_{IJ} \equiv \mathring{\mathcal{D}}_{\bar{\theta}} \mathring{\mathrm{g}}_{IJ}$$

HLR [1511.07809,1803.11155]

Symmetries constraining the action:

- Invariance under thermal diffeos and SK-KMS BRST symmetries.
- Target spacetime superdiffeomorphisms
- Restricted worldvolume diffeomorphisms
- CPT & ghost charge conservation.

Crossley, Glorioso, Liu [1511.03646,1701.07817]
Gao, Liu [1701.07445]
Jensen et al [1701.07436,1804.04654]

EFFECTIVE ACTIONS

- Ideal fluid is obviously captured by the pressure super-potential functional

$$\mathring{\mathcal{L}}^{(\text{ideal})} = \frac{\sqrt{-\mathring{\mathrm{g}}}}{\mathring{\mathbf{z}}}\ \mathring{\mathfrak{f}}(\mathring{T})$$

- Dissipative terms are captured by a an appropriate 4-tensor inspired coupling that involves the superderivatives of the metric:

$$\mathcal{L}_{\text{wv, diss}} = \int d\theta\, d\bar{\theta}\ \frac{\sqrt{-\mathring{\mathrm{g}}}}{\mathring{\mathbf{z}}} \left(-\frac{i}{4}\right) \mathring{\boldsymbol{\eta}}^{IJKL}\ \mathring{\mathfrak{g}}^{(\overline{\psi})}_{IJ}\ \mathring{\mathfrak{g}}^{(\psi)}_{KL}$$

$$\mathring{\boldsymbol{\eta}}^{IJKL} = \mathring{\zeta}(\mathring{T})\, \mathring{T}\, \mathring{P}^{IJ}\, \mathring{P}^{KL} + 2\, \mathring{\eta}(\mathring{T})\, \mathring{T}\, (-)^{K(I+J)}\, \mathring{P}^{K\langle I}\, \mathring{P}^{J\rangle L}$$

HLR [1511.07809]

- Positivity of entropy production follows on demanding the imaginary part is positive definite (which reduces us back to the remit of Bhattacharyya's theorem).

$$\Delta\ = \frac{1}{4}\, \boldsymbol{\eta}^{abcd}\, \pounds_{\boldsymbol{\beta}}\mathrm{g}_{ab}\, \pounds_{\boldsymbol{\beta}}\mathrm{g}_{cd} + \text{fluctuations} + \text{ghost-bilinears}$$

Glorioso, Liu [1612.07705]

Act V

in which entropy gets produced by inflow

INFLOW FROM DISSIPATIVE ACTIONS

- Thermal diffeomorphism symmetry implies that the *super-adiabaticity equation* is satisfied as a Bianchi identity:

$$\mathring{\mathcal{D}}_I \mathring{\mathbf{N}}^I - \frac{1}{2}\,\mathring{\mathbf{T}}^{IJ}\,\pounds_{\boldsymbol{\beta}}\mathring{\mathrm{g}}_{IJ} = 0\,.$$

- This in turn leads to the picture of entropy inflow:

$$\underbrace{\left(\mathring{\mathcal{D}}_a \mathring{\mathbf{N}}^a - \frac{1}{2}\,\mathring{\mathbf{T}}^{ab}\,\pounds_{\boldsymbol{\beta}}\,\mathring{\mathrm{g}}_{ab}\right)\Bigg|}_{\text{classical + fluctuations}} = -\underbrace{\left(\mathring{\mathcal{D}}_\theta \mathring{\mathbf{N}}^\theta + \mathring{\mathcal{D}}_{\bar{\theta}} \mathring{\mathbf{N}}^{\bar{\theta}} + \mathring{\mathbf{T}}^{a\theta}\,\pounds_{\boldsymbol{\beta}}\,\mathring{\mathrm{g}}_{a\theta} + \mathring{\mathbf{T}}^{a\bar{\theta}}\,\pounds_{\boldsymbol{\beta}}\,\mathring{\mathrm{g}}_{a\bar{\theta}} + \mathring{\mathbf{T}}^{\theta\bar{\theta}}\,\pounds_{\boldsymbol{\beta}}\,\mathring{\mathrm{g}}_{\theta\bar{\theta}}\right)\Bigg|}_{\text{entropy inflow}}$$

- Switching off the fluctuation fields leads to physical entropy flowing from superspace:

$$\Delta = -\left(\mathcal{D}_\theta \mathbf{N}^\theta + \mathcal{D}_{\bar{\theta}} \mathbf{N}^{\bar{\theta}}\right) + \text{ghosts bilinears}$$

HLR [1803.08490,1803.11155]
Jensen et al [1803.07070]

NO INFLOW OF ENERGY-MOMENTUM

- Target space diffeomorphisms ensure that the dynamical content of the effective action is simply super-energy momentum conservation.

$$\mathring{\mathfrak{D}}_I \left(\mathring{\mathbf{T}}^{IJ} \, \mathring{\mathfrak{D}}_J \mathring{X}^\mu \right) = 0$$

- This by itself would be problematic, since we would learn that the equations are contaminated by the presence of super-components which turn out to include physical degrees of freedom (not ghosts or fluctuations).

- However, superspace components of energy-momentum tensor conspire to mutually cancel out and do not modify dynamical equations.

$$\mathring{\mathfrak{D}}_a \left(\mathring{\mathbf{T}}^{ab} \, \mathring{\mathfrak{D}}_b \mathring{X}^\mu \right) \Big| \; = \nabla_\mu T^{\mu\nu} + \text{ghost bilinears} + \text{fluctuations} \; = 0$$

FLUCTUATION DISSIPATION AS CPT BREAKING

- Stochasticity and dissipation arises because of spontaneous CPT symmetry breaking.
- The Ward identities following from CPT convolved with a thermal gauge transformation results in the Jarzynski work relation for the Brownian particle

$$S_{\mathrm{B0}} \mapsto S_{\mathrm{B0}} - i\,\langle \mathring{\mathcal{F}}_{\theta\bar{\theta}}| \rangle\, \boldsymbol{\beta}\, (\Delta G + W) \quad \Longrightarrow \quad \left\langle e^{-\beta W} \right\rangle = e^{-\beta\,\Delta G}$$

Mallick, Moshe, Orland 2010

- The CPT symmetry in our construction is implemented as R-parity in superspace and its breaking encoded in the vev for the ghost number zero field strength: $\langle \mathring{\mathcal{F}}_{\theta\bar{\theta}}| \rangle = -i$
- Expect similar statements to hold in hydrodynamic effective field theories.

Epilogue: Whence we take stock and try to speculate on what might lie ahead...

ENTROPY PRODUCTION VIA INFLOW

- For systems in local equilibrium the Noether current for thermal diffeomorphisms is the macroscopic free energy current (Legendre transform of entropy current).

- Local equilibrium is characterized by an emergent topological/BRST supersymmetry wherein diffeomorphisms along the Euclidean thermal circle are gauged (thermal equivariance).

- Net entropy is conjugate to the gauged thermal diffeomorphisms & is conserved. Physical entropy production happens by virtue of it being sourced in the superspace directions, i.e., there is an inflow of entropy from superspace.

$$\mathring{\mathcal{D}}_I \mathring{\mathbf{N}}^I - \frac{1}{2}\,\mathring{\mathbf{T}}^{IJ}\,\pounds_{\boldsymbol{\beta}}\mathring{\mathrm{g}}_{IJ} = 0 \qquad \Longrightarrow \qquad \mathcal{D}_a \mathbf{N}^a - \frac{1}{2}\,\mathbf{T}^{ab}\,\pounds_{\boldsymbol{\beta}}\mathbf{g}_{ab} = \Delta \geq 0$$

LOOKING AHEAD…

- Near-equilibrium dynamics appears to be under control but what about non-equilibrium?
- Open quantum systems & renormalization
  Avinash, Jana, Loganayagam, Rudra [1704.08335]
- How does thermal equivariance extend to include non-stochastic fluctuations? Deformation quantization? *Basart, Flato, Lichnerowicz, Sternheimer 1984*
- Microscopic unitary which enforces fluctuation-dissipation etc., is upheld thanks to the BRST + thermal gauge symmetry. Lessons for gravity? Connections to SL(2,R) symmetry in discussions of chaos?
- What is the analogous story for higher out-of-time-order correlators?
- Are the similar statements for modular evolutions (equivalent in some contexts), and if so what does it imply for geometry = entanglement?

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Haehl, Loganayagam, MR (HLR)

- ✤ Classification of solutions to hydro axioms: [1412.1090] [1502.00636]
- ✤ Basic philosophy: [1510.02494]
- ✤ Dissipative hydrodynamic actions: [1511.07809] & [1803.11155]
- ✤ Origins in Schwinger-Keldysh: [1610.01940]
- ✤ Thermal Equivariance: [1610.01941]
- ✤ Overview of related works: [1701.07896]
- ✤ Inflow mechanism [1803.08490]

Related work

- ★ Hydrodynamic effective actions: Crossley, Glorioso, Liu [1511.03646]
- ★ Second Law: Glorioso, Liu [1612.07705]

- ★ Hydrodynamic effective actions II: Crossley, Glorioso, Liu [1701.07817]
- ★ Superspace formalism: Gao, Liu [1701.07445]

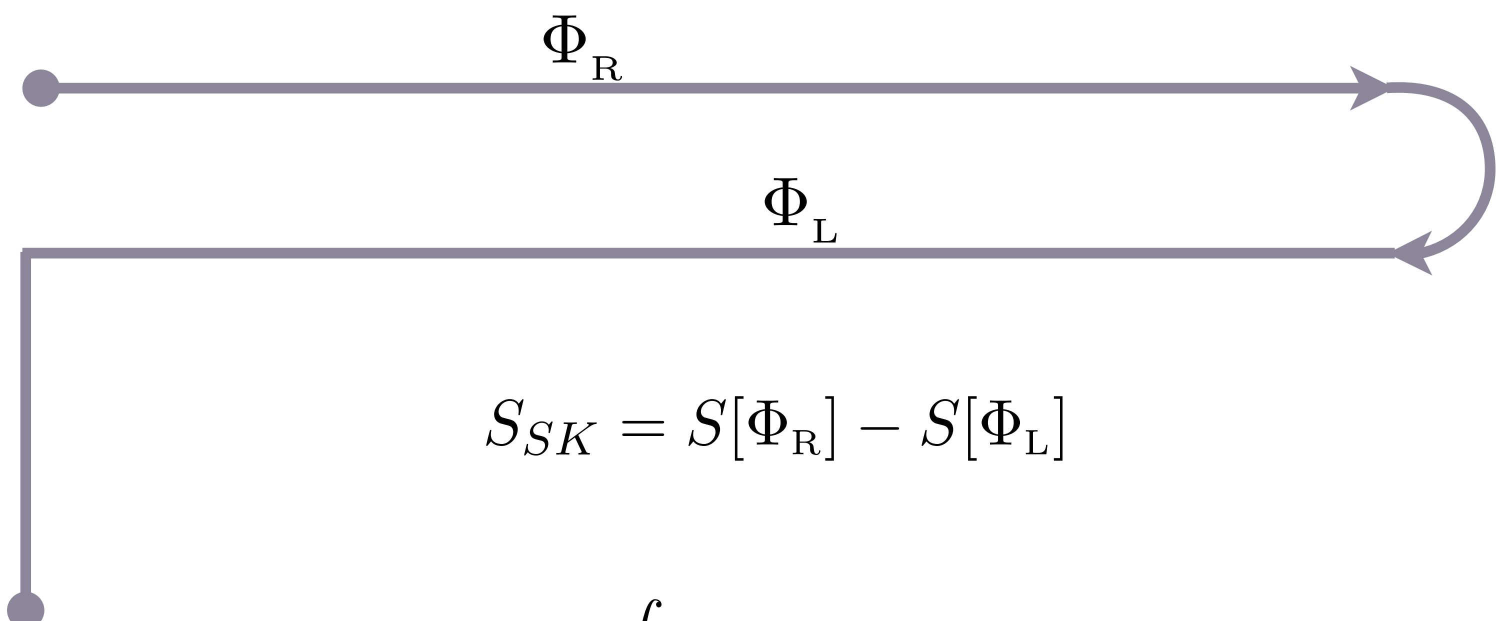
- ★ Jensen, Pinzani-Fokeeva, Yarom [1701.07436]
- ★ Inflow type picture + hydro actions: Jensen, Marjieh, Pinzani-Fokeeva, Yarom [1803.07070] & [1804.04654]

Thank You!

SCHWINGER-KELDYSH FORMALISM

- The Schwinger-Keldysh formalism computes singly out-of-time ordered correlation functions in a generic (mixed) state.

thermal density matrix



turning point identity operator

$$S_{SK} = S[\Phi_{\rm R}] - S[\Phi_{\rm L}]$$

$$\delta S_{SK} = \int d^d x \sqrt{-g}\, (\mathcal{J}_{\rm R}\, \mathbb{O}_{\rm R} - \mathcal{J}_{\rm L}\, \mathbb{O}_{\rm L})$$

Generating functional

$$\mathcal{Z}_{SK}[J_{\rm R}, J_{\rm L}] \equiv \text{Tr}\Big\{ U[J_{\rm R}]\ \hat{\rho}_{\text{initial}}\ (U[J_{\rm L}])^\dagger \Big\}$$

Time ordered correlations

$$\text{Tr}\Big(\ \hat{\rho}_{\text{initial}}\ \bar{\mathcal{T}}\Big(U^\dagger \mathbb{O}_{\rm L} U^\dagger \mathbb{O}_{\rm L} \ldots\Big)\ \mathcal{T}\,(U\mathbb{O}_{\rm R} U \mathbb{O}_{\rm R} \ldots)\ \Big)$$

TWO SUM RULES

- *Unitarity* of Schwinger-Keldysh path integral implies vanishing difference operator correlators:

$$\langle \mathcal{T}_{SK} \prod_{k=1}^{n} \left(\mathbb{O}_{\rm R}^{(k)}(t_k) - \mathbb{O}_{\rm L}^{(k)}(t_k) \right) \rangle = 0$$

- The KMS condition translates into a second sum rule for thermal differences: *Weldon '05*

$$\langle \mathcal{T}_{SK} \prod_{k=1}^{n} \left(\mathbb{O}_{\rm R}^{(k)}(t_k) - \mathbb{O}_{\rm L}^{(k)}(t_k - i\,\beta) \right) \rangle = 0$$

- Keldysh (light-cone) basis $\qquad \mathbb{O}_{dif} \equiv \mathbb{O}_{\rm R} - \mathbb{O}_{\rm L}\ , \qquad \mathbb{O}_{av} \equiv \frac{1}{2}\left(\mathbb{O}_{\rm R} + \mathbb{O}_{\rm L}\right)$

- Adv-Ret basis $\qquad \mathbb{O}_{adv} \equiv \mathbb{O}_{\rm R} - \mathbb{O}_{\rm L}\ , \qquad i\Delta_{\boldsymbol{\beta}}\mathbb{O}_{ret} = \mathbb{O}_{\rm R}(t) - \mathbb{O}_{\rm L}(t - i\beta)$

$$i\,\Delta_{\boldsymbol{\beta}}\mathbb{O}(t) = \mathbb{O}(t) - \mathbb{O}(t - i\,\beta) \sim -i\,\beta\partial_t\mathbb{O}$$

- Furthermore, *a largest time* and *thermal smallest time* equations hold.

THE SK-KMS ALGEBRA

- Superderivations: BRST charges associated with Schwinger-Keldysh construction.
- Thermal equivariance: implements KMS condition by gauging thermal diffeomorphisms & takes care of the fluctuation/dissipation constraints.

$$
\begin{gathered}
\mathcal{Q}_{SK}^2 = \overline{\mathcal{Q}}_{SK}^2 = \mathcal{Q}_{KMS}^2 = \overline{\mathcal{Q}}_{KMS}^2 = 0 \,, \\
\left[\mathcal{Q}_{SK}, \mathcal{Q}_{KMS}\right]_{\pm} = \left[\overline{\mathcal{Q}}_{SK}, \overline{\mathcal{Q}}_{KMS}\right]_{\pm} = \left[\overline{\mathcal{Q}}_{SK}, \mathcal{Q}_{SK}\right]_{\pm} = \left[\mathcal{Q}_{KMS}, \overline{\mathcal{Q}}_{KMS}\right]_{\pm} = 0 \,, \\
\left[\mathcal{Q}_{SK}, \overline{\mathcal{Q}}_{KMS}\right]_{\pm} = \left[\overline{\mathcal{Q}}_{SK}, \mathcal{Q}_{KMS}\right]_{\pm} = \mathcal{L}_{KMS} \,, \\
\left[\mathcal{Q}_{KMS}, \mathcal{Q}^0_{KMS}\right]_{\pm} = \left[\overline{\mathcal{Q}}_{KMS}, \mathcal{Q}^0_{KMS}\right]_{\pm} = 0 \,, \\
\left[\mathcal{Q}_{SK}, \mathcal{Q}^0_{KMS}\right]_{\pm} = \mathcal{Q}_{KMS} \,, \qquad \left[\overline{\mathcal{Q}}_{SK}, \mathcal{Q}^0_{KMS}\right]_{\pm} = -\overline{\mathcal{Q}}_{KMS} \,.
\end{gathered}
$$

$$
\mathcal{L}_{KMS} \equiv \Delta_{\boldsymbol{\beta}}
$$