The Universe before the hot Big Bang

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Basic properties of the present Universe:

- Visible Universe is large

  Size of the visible part of the Universe is
  15 Gigaparsec ≈ 45 billion light years

  1 Mpc $= 3 \cdot 10^6$ light yrs $= 3 \cdot 10^{24}$ cm

- The Universe is old

  Its lifetime is at least 13.8 billion years

- Visible Universe is homogeneous
  on large scales ($\gtrsim 200$ Mpc):

  different parts of the Universe look the same.

Deep surveys of galaxies and quasars $\Longrightarrow$
map of a good part of visible Universe

Basic properties of the present Universe:

- The Universe expands

  Space stretches out

  Distances between galaxies increase in time.

  Wavelength of a photon also increases.

  If emitted at time t with wavelength λ,
  it comes to us with longer wavelength

$$\lambda_0 = (1+z)\lambda$$

  $z = z(t)$: redshift, directly measurable.

Basic properties of the present Universe:

- 3d space is Euclidean (observational fact!)

  Sum of angles of a triangle = 180^o, even for triangles as large as the size of the visible Universe.

- All of above is encoded in space-time metric (Friedmann–Lemâitre–Robertson–Walker)

$$ds^2 = dt^2 - a^2(t)\mathbf{dx}^2$$

$\mathbf{x}$: comoving coordinates, label distant galaxies.

$a(t)dx$: physical distances.

$a(t)$: scale factor, grows in time.
Set its present value to 1, then $a < 1$ in the past

$$H(t) = \frac{\dot{a}}{a} : \quad \text{Hubble parameter, expansion rate}$$

- Present value

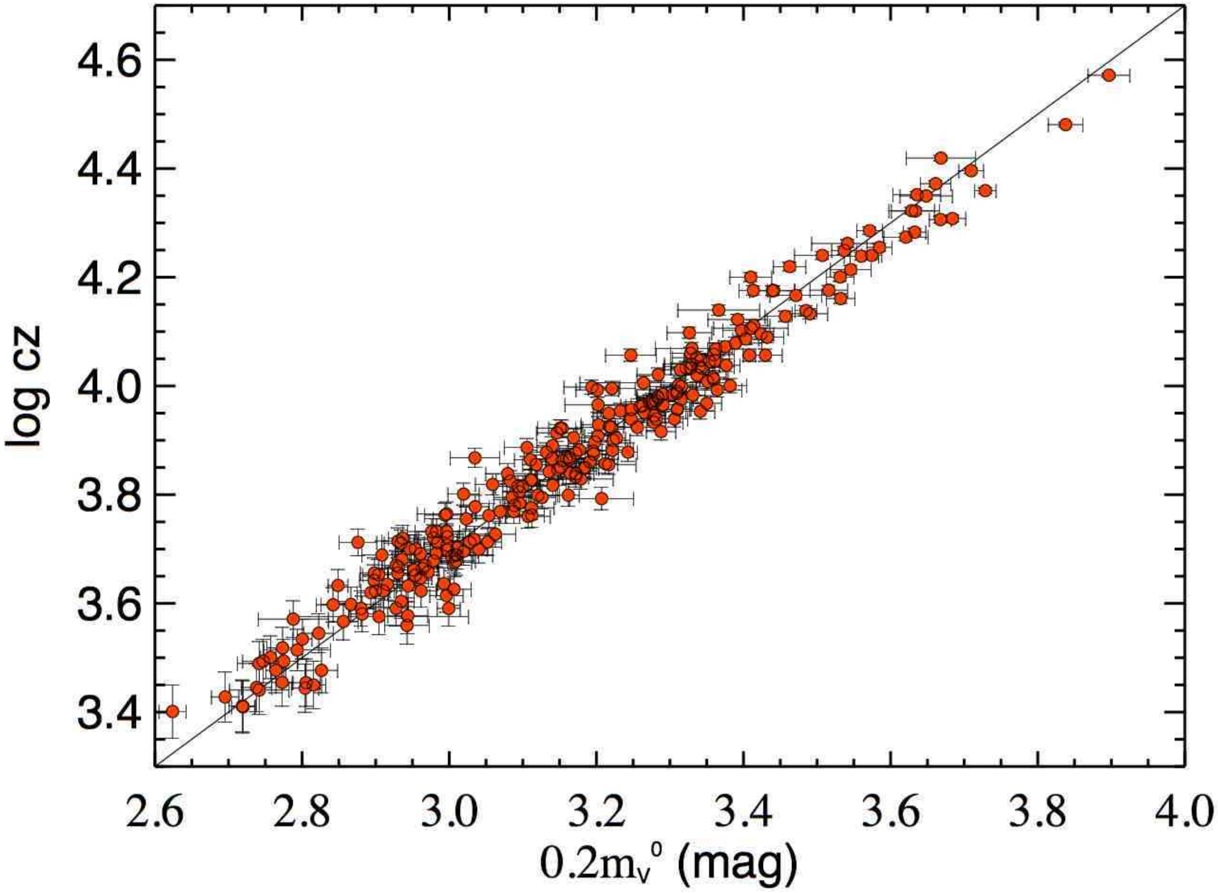
$$H_0 = (67.3 \pm 1.2)\ \frac{\text{km/s}}{\text{Mpc}} = (14 \cdot 10^9\ \text{yrs})^{-1}$$

$1\ \text{Mpc} = 3 \cdot 10^6\ \text{light yrs} = 3 \cdot 10^{24}\ \text{cm}$

- Hubble law (valid at $z \ll 1$)

$$z = H_0 r$$

Hubble diagram



$\text{mag} = 5\log_{10} r + \text{const}$

- The Universe is warm. It is filled with Cosmis Microwave Background: photons that were thermally produced when the Universe was young and hot.

CMB temperature today

$$T_0 = 2.7255 \pm 0.0006 \text{ K}$$

Fig.

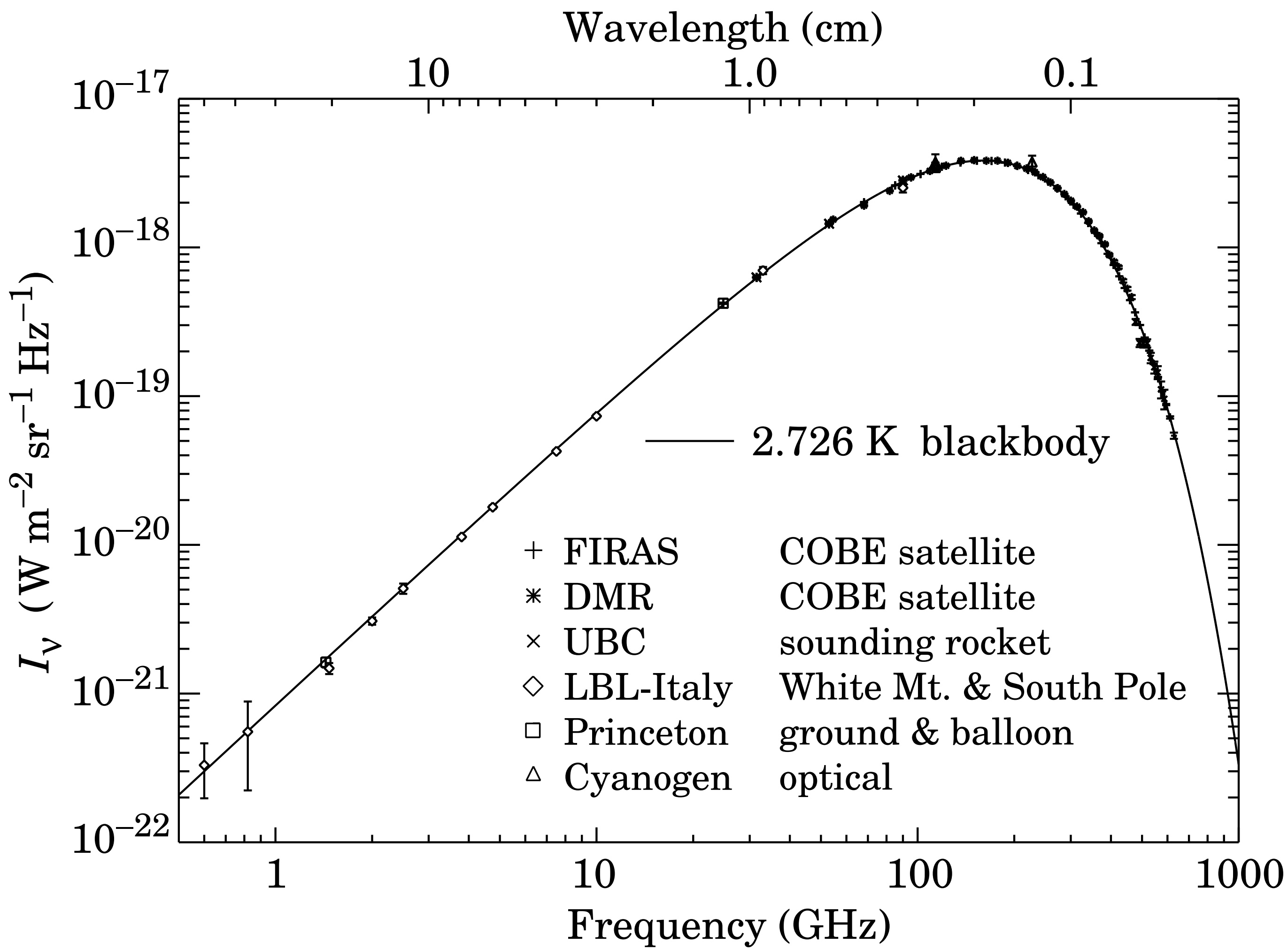
It was denser and warmer at early times.

It also expanded a lot faster at early times:
according to General Relativity, expansion rate is determined by Friedmann equation

$$H^2 = \frac{8\pi}{3} G\rho$$

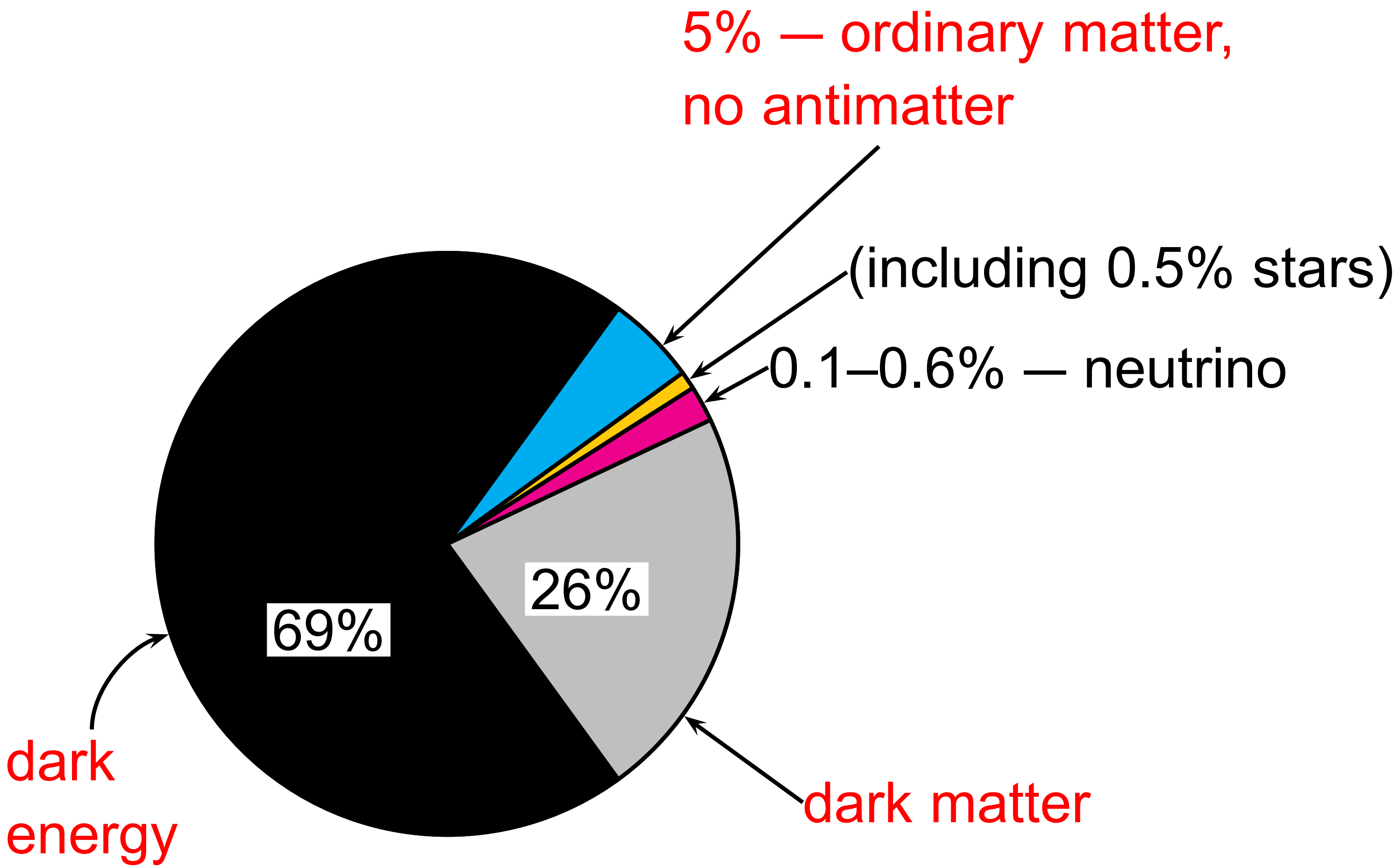
where ρ is energy density, G is Newton's gravity constant.

CMB spectrum



$T = 2.726$ K

Basic properties of the present Universe: unknowns



Cornerstones of thermal history

- Recombination, transition from plasma to gas.

  $z = 1090$, $T = 3000$ K, $t = 380\,000$ years

  Last scattering of CMB photons

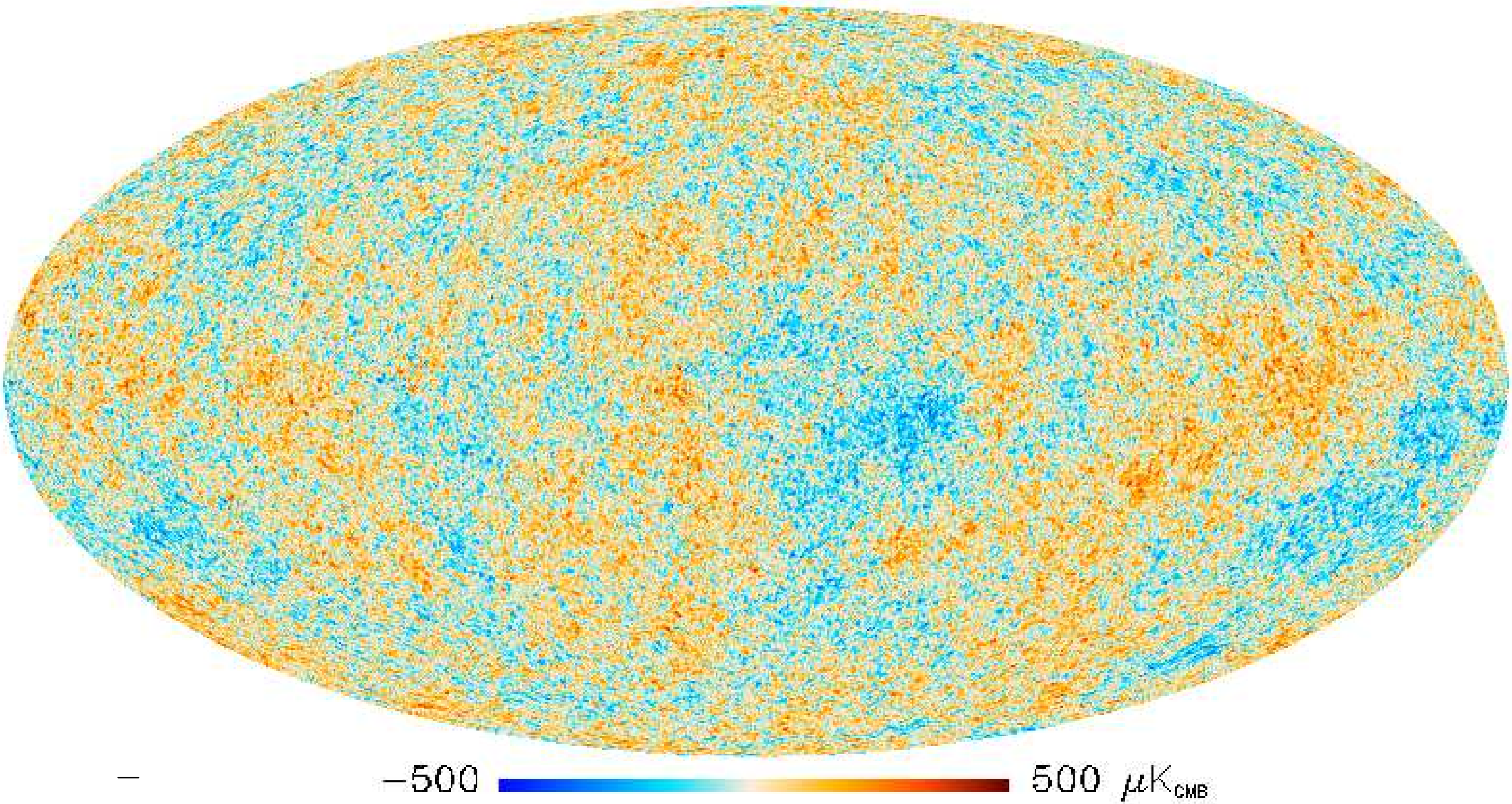
  Photographic picture (literally!) of the Universe at that epoch Fig.

  The Universe was much more homogeneous: the inhomogeneities were at the level

  $$\frac{\delta\rho}{\rho} \sim 10^{-4} - 10^{-5}$$

  But this photo tells a lot!

$$T = 2.726^{\circ}K, \quad \frac{\delta T}{T} \sim 10^{-4} - 10^{-5}$$



Planck

- Big Bang Nucleosynthesis, epoch of thermonuclear reactions

$$
\begin{aligned}
p+n &\rightarrow {}^2H \\
{}^2H+p &\rightarrow {}^3He \\
{}^3He+n &\rightarrow {}^4He \\
\text{up to} & \quad\;\; {}^7Li
\end{aligned}
$$

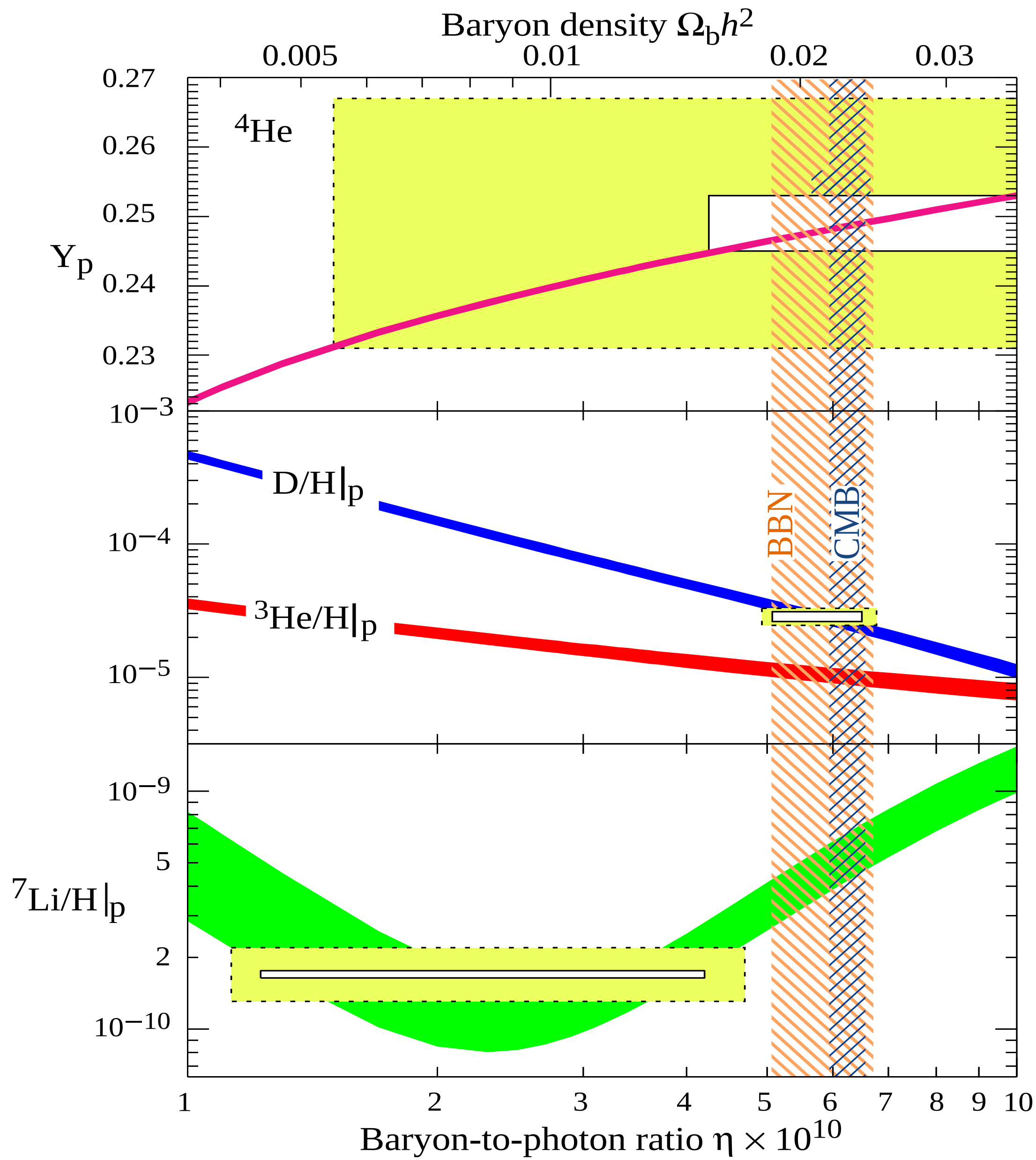
Abundances of light elements: measurements vs theory

$$T = 10^{10} \rightarrow 10^9 \text{ K}, \quad t = 1 \rightarrow 500 \text{ s}$$

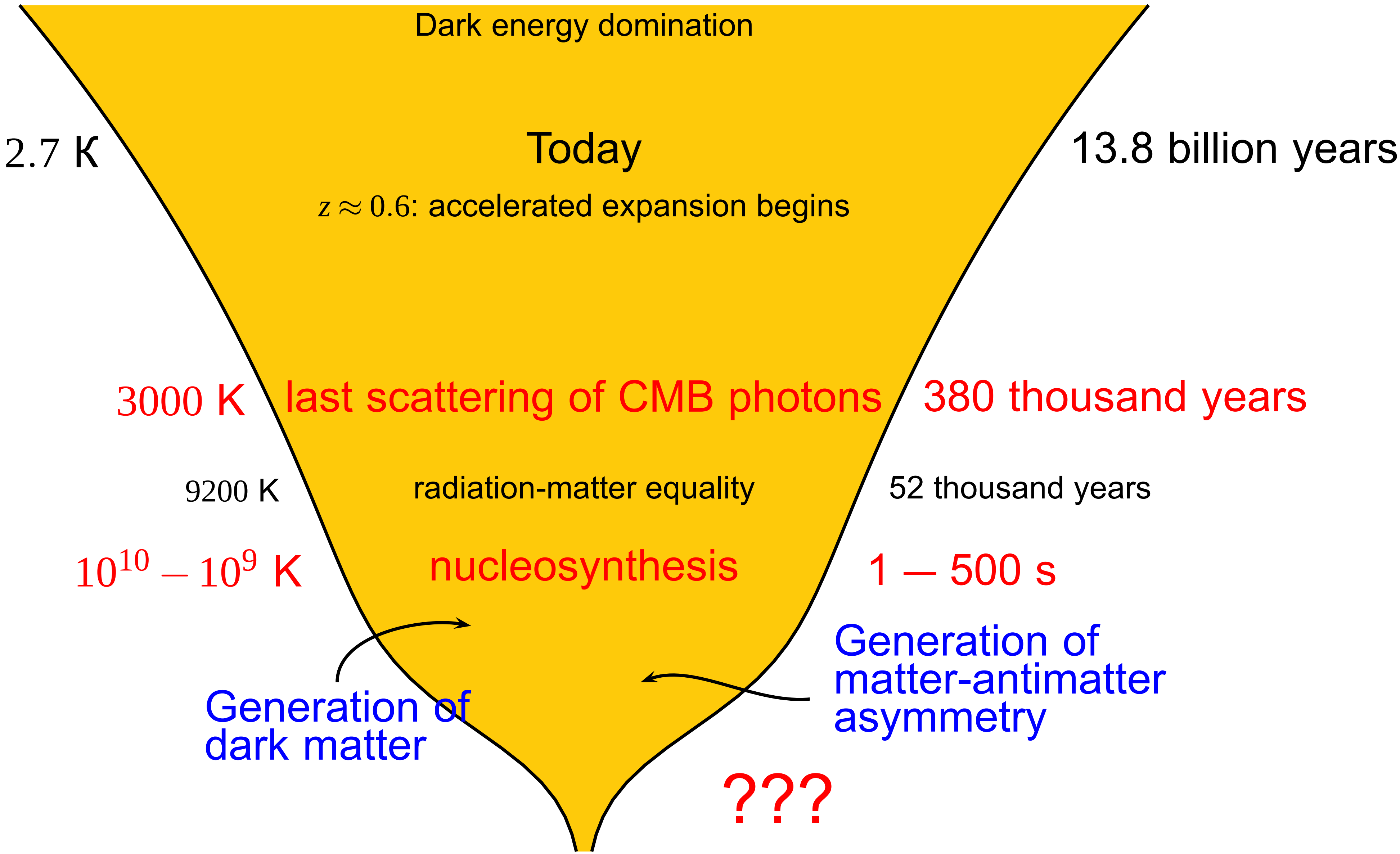
Earliest time in thermal history probed so far Fig.

- Neutrino decoupling: $T = 2-3$ MeV $\sim 3 \cdot 10^{10}$K, $t \sim 0.1 - 1$s
- Generation of dark matter*
- Generation of matter-antimatter asymmetry*

*may have happend before the hot Big Bang epoch



$\eta_{10} = \eta \cdot 10^{-10}$ = baryon-to-photon ratio. Consistent with CMB determination of η



Dark energy domination
2.7 K
Today
13.8 billion years
$z \approx 0.6$: accelerated expansion begins
3000 K
last scattering of CMB photons
380 thousand years
9200 K
radiation-matter equality
52 thousand years
$10^{10} - 10^{9}$ K
nucleosynthesis
1 — 500 s
Generation of matter-antimatter asymmetry
Generation of dark matter
???

With Big Bang nucleosynthesis theory and observations we are confident of the theory of the early Universe at temperatures up to $T \simeq 1$ MeV, age $t \simeq 1$ second

With the Large Hadron Collider, we hope to be able to go up to temperatures $T \sim 100$ GeV, age $t \sim 10^{-10}$ second

Are we going to have a handle on even earlier epoch?

Key: cosmological perturbations

Our Universe is not exactly homogeneous.

Inhomogeneities: ⊙ density perturbations and associated gravitational potentials (3d scalar), observed;

⊙ gravitational waves (3d tensor), not observed (yet? – what about BICEP-2?).

Today: inhomogeneities strong and non-linear

In the past: amplitudes small,

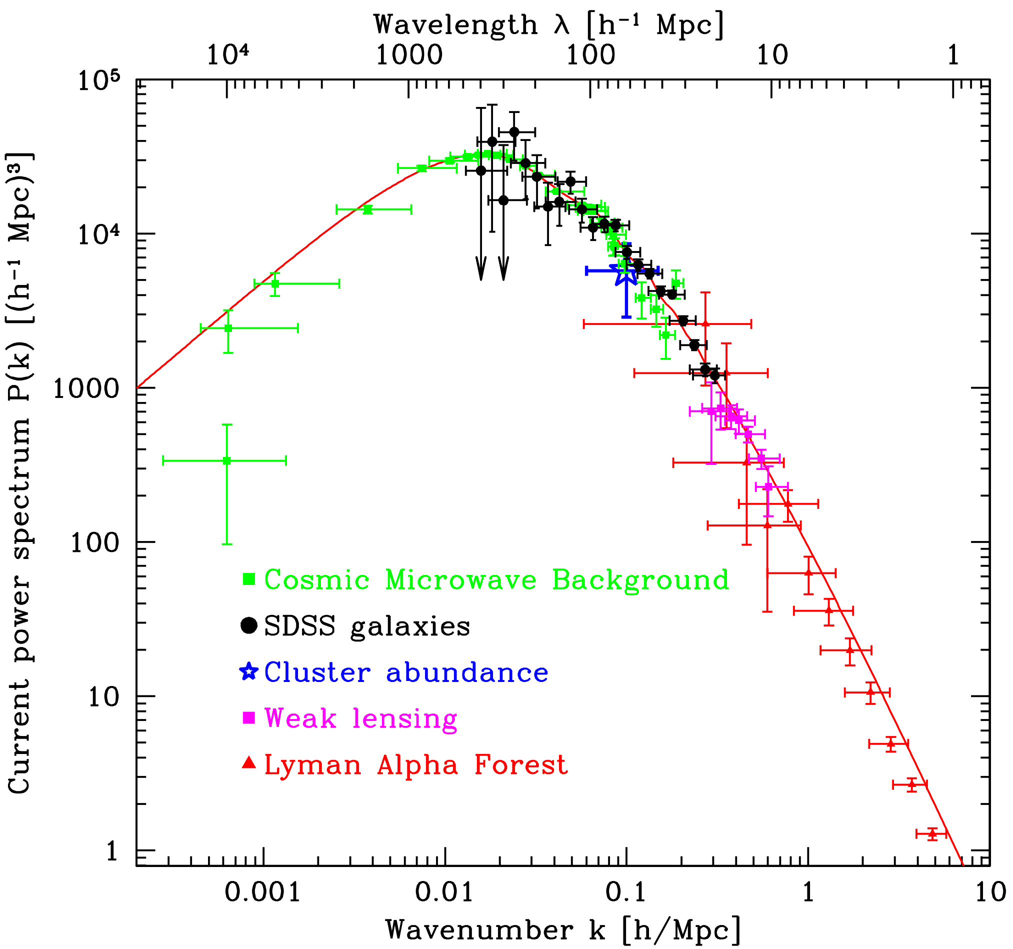
$$\frac{\delta\rho}{\rho} = 10^{-4} - 10^{-5}$$

Linear analysis appropriate.

Wealth of data

- Cosmic microwave background: photographic picture of the Universe at age 380 000 yrs, $T = 3000$ K
  - Temperature anisotropy
  - Polarization
- Deep surveys of galaxies and quasars
- Gravitational lensing, etc.

Overall consistency



NB: density perturbations = random field.

k = wavenumber

$P(k)$ = power spectrum transfered to present epoch using linear theory

We have already learned a number of fundamental things

Extrapolation back in time with known laws of physics and known elementary particles and fields $\Longrightarrow$ hot Universe, starts from Big Bang singularity (infinite temperature, infinite expansion rate)

We now know that this is not the whole story.

Key point: causality

Friedmann–Lemaître–Robertson–Walker metric:

$$ds^2 = dt^2 - a^2(t)d\vec{x}^{\,2}$$

Expanding Universe:

$a(t) \propto t^{1/2}$ at "radiation domination epoch", before $T \simeq 1$ eV, $t \simeq 50$ thousand years

$a(t) \propto t^{2/3}$ later, until recently.

Cosmological horizon (assuming that nothing preceeded hot epoch): length that light travels from Big Bang moment,

$$l_H(t) = (2-3)ct$$

Wavelength of perturbation grows as $a(t)$.
E.g., at radiation domination

$$\lambda(t) \propto t^{1/2} \quad \text{while} \quad l_H \propto t$$

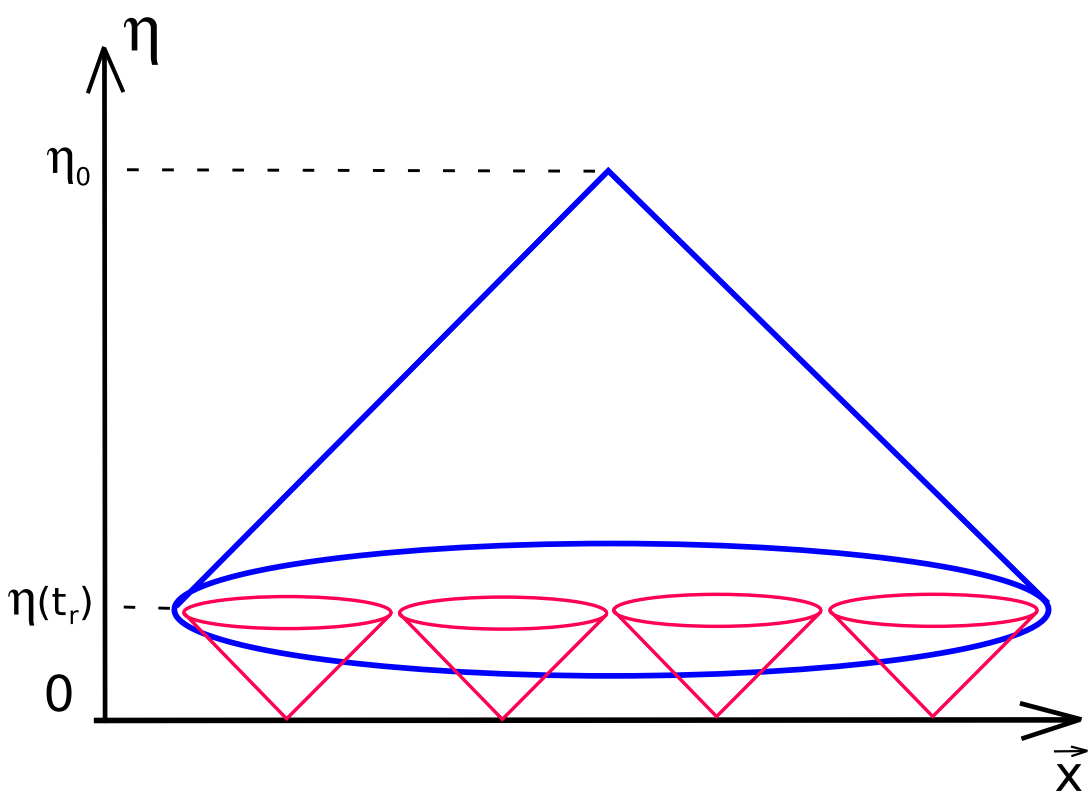
Today $\lambda < l_H$, subhorizon regime

Early on $\lambda(t) > l_H$, superhorizon regime.

NB: Horizon entry occured after Big Bang Nucleosynthesis for perturbations of all relevant wavelengths $\Longleftrightarrow$ no guesswork.

Causal structure of space-time in hot Big Bang theory (i.e., assuming that the Universe started right from the hot epoch)

$$\eta = \int \frac{dt}{a(t)}\,, \quad \text{conformal time}$$



Angular size of horizon at recombination $\approx 2^o$.

Horizon problem

Today our visible Universe consists of $50^3 \sim 10^5$ regions which were causally disconnected at recombination.

Why are they exacly the same?

May sound as a vague question.

But

Properties of perturbations make it sharp.

Major issue: origin of perturbations

Causality $\Longrightarrow$ perturbations can be generated only when they are subhorizon.

Off-hand possibilities:

- Perturbations were never superhorizon, they were generated at the hot cosmological epoch by some causal mechanism.

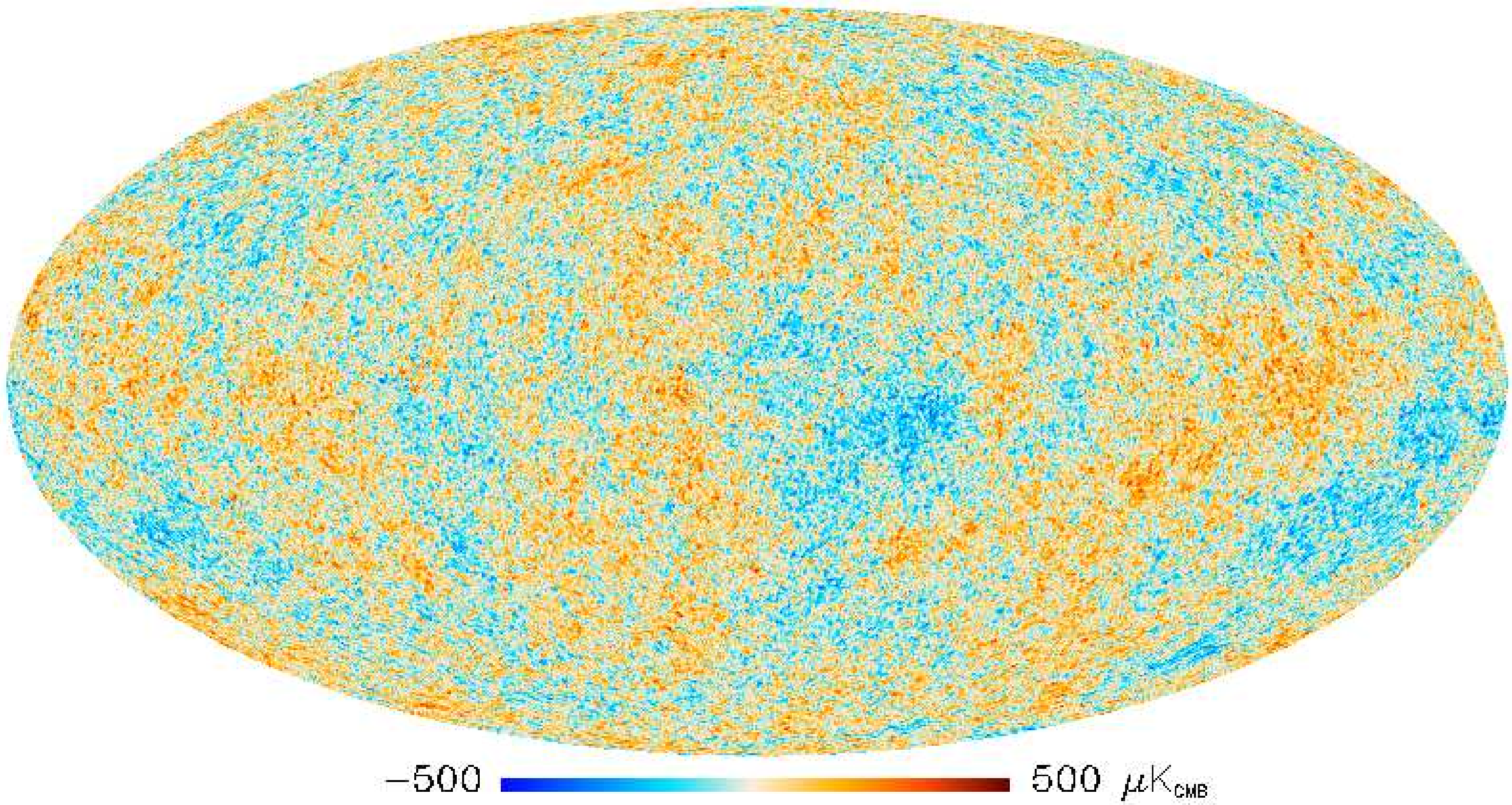
  E.g., seeded by topological defects (cosmic strings, etc.)

  N. Turok et.al.' 90s

  The only possibility, if expansion started from hot Big Bang.

  Not an option

- Hot epoch was preceeded by some other epoch. Perturbations were generated then.



There are perturbations which were superhorizon at the time of recombination, angular scale $\gtrsim 2^o$. Causality: they could not be generated at hot epoch!

Shorter wavelengths: perturbations in baryon-photon plasma = sound waves.

If they were superhorizon, they started off with one and the same phase.

Reason: solutions to wave equation in superhorizon regime in expanding Universe

$$\frac{\delta\rho}{\rho} = \text{const} \qquad \text{and} \qquad \frac{\delta\rho}{\rho} = \frac{\text{const}}{t^{3/2}}$$

Assume that modes were superhorizon. Consistency of the picture: the Universe was not very inhomogeneous at early times, the initial condition is (up to amplitude),

$$\frac{\delta\rho}{\rho} = \text{const} \implies \frac{d}{dt}\frac{\delta\rho}{\rho} = 0$$

Acoustic oscillations start after entering the horizon at zero velocity of medium $\implies$ phase of oscillations well defined.

Perturbations develop different phases by the time of photon last scattering (= recombination), depending on wave vector:

$$\frac{\delta\rho}{\rho}(t_r) \propto \cos\left(\int_0^{t_r} dt\ v_s\ \frac{k}{a(t)}\right)$$

(v_s = sound speed in baryon-photon plasma)

cf. Sakharov oscillations' 1965

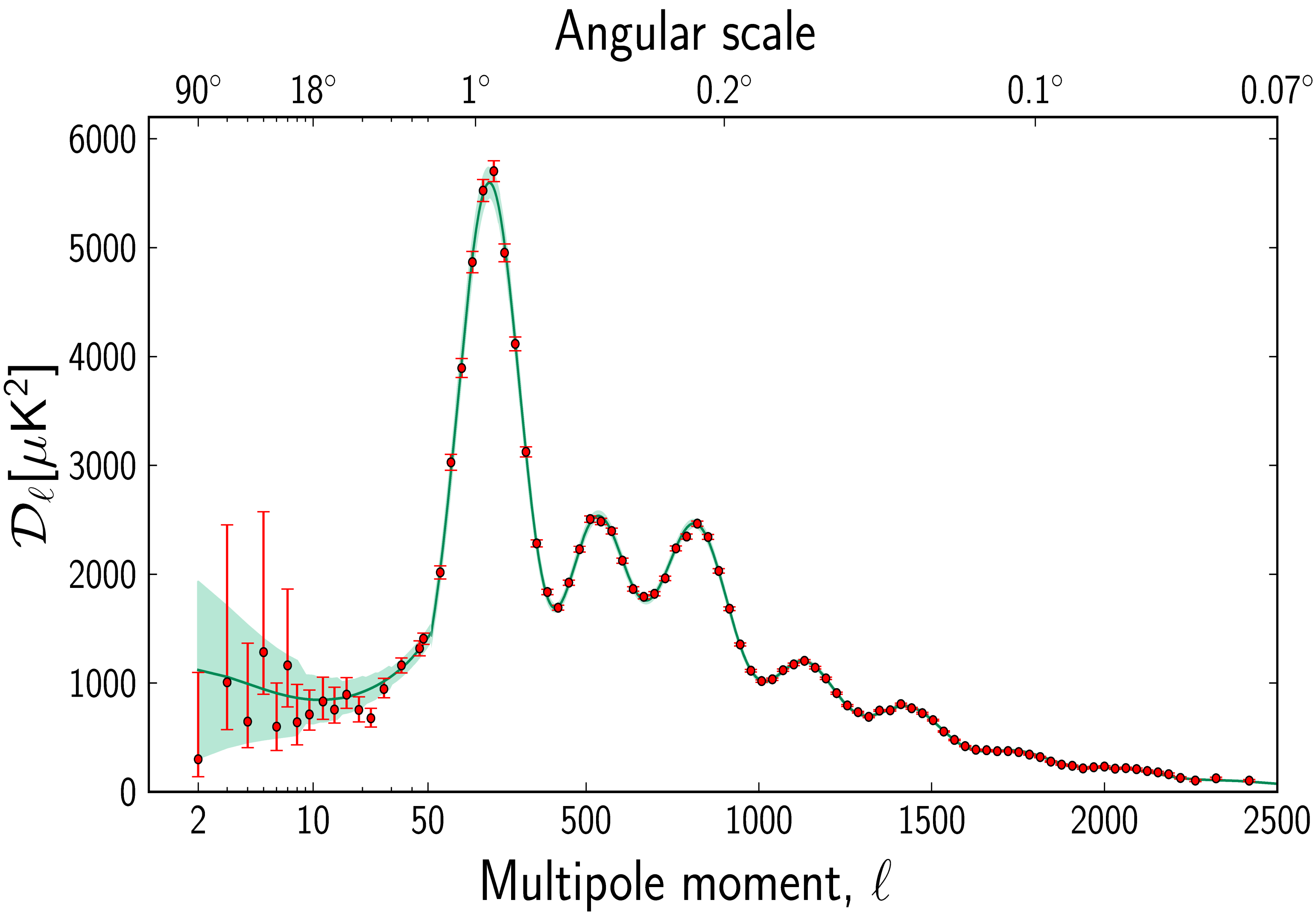
Oscillations in CMB temperature angular spectrum

Fourier decomposition of temperatue fluctuations:

$$\delta T(\theta,\varphi) = \sum_{l,m} a_{lm} Y_{lm}(\theta,\varphi)$$

$\langle a^*_{lm} a_{lm}\rangle = C_l$, temperature angular spectrum;

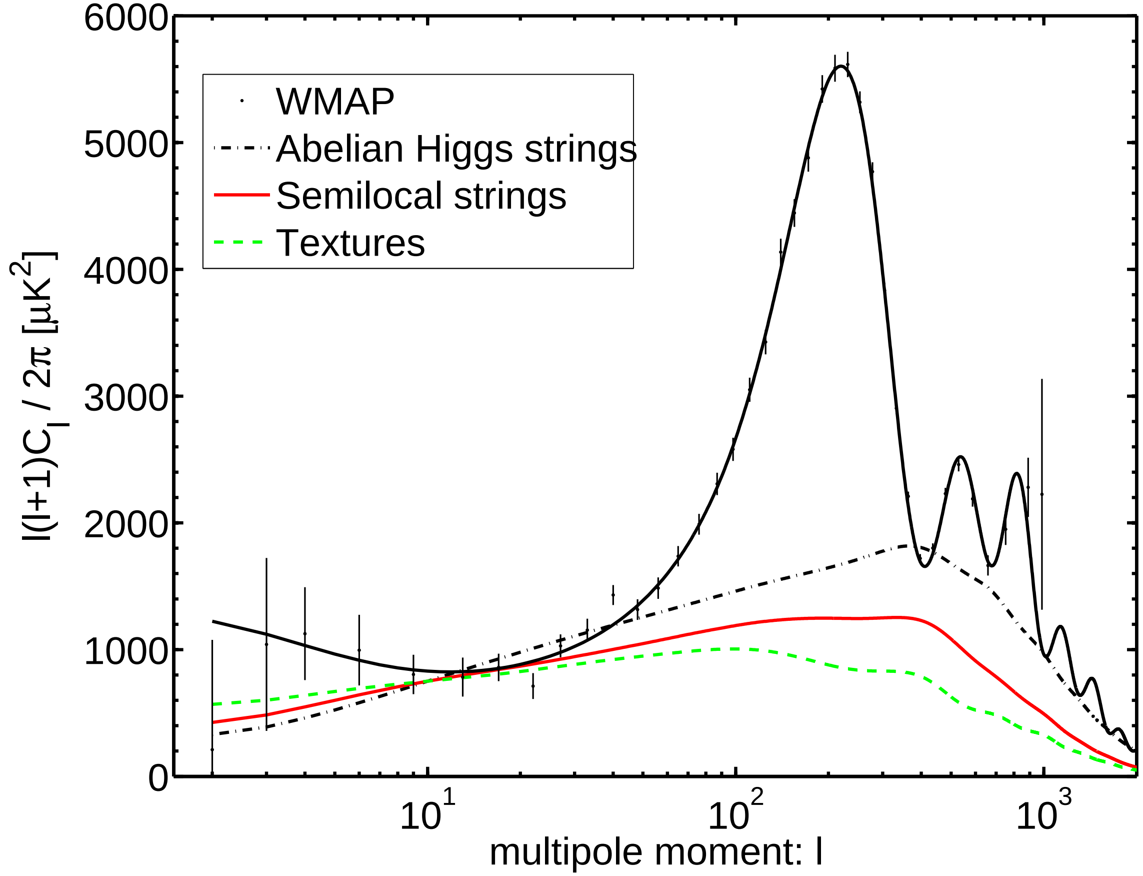
larger $l \Longleftrightarrow$ smaller angular scales, shorter wavelengths



Planck

$$\mathscr{D}_l = \frac{l(l+1)}{2\pi} C_l$$

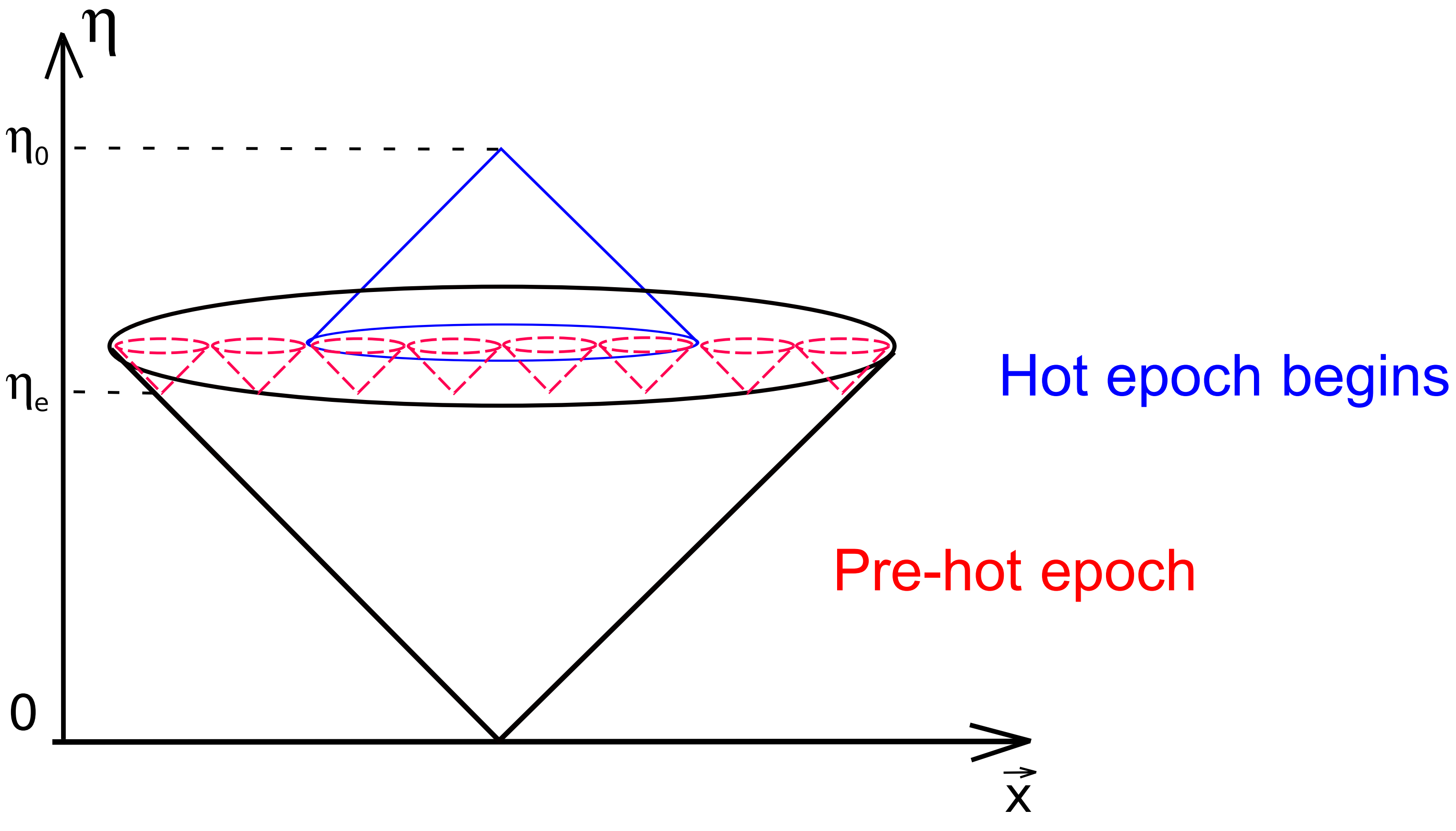
These properties would not be present if perturbations were generated at hot epoch in causal manner.



Primordial perturbations were generated at some yet unknown epoch before the hot expansion stage.

This is true also for perturbations of smaller wavelengths/angular scales (acoustic peaks in CMB angular spectrum)

That epoch must have been long (in conformal time) and unusual: perturbations were subhorizon early at that epoch, our visible part of the Universe was in a causally connected region.



Excellent guess: inflation

Starobinsky'79; Guth'81; Linde'82; Albrecht and Steinhardt'82

Exponential expansion with almost constant Hubble rate,

$$a(t) = \mathrm{e}^{\int H dt}\,, \qquad H \approx \text{const}$$

- Initially Planck-size region expands to entire visible Universe in $t \sim 100\ H^{-1} \Longrightarrow$ for $t \gg 100\ H^{-1}$ the Universe is VERY large

- Perturbations subhorizon early at inflation:

$$\lambda(t) = 2\pi \frac{a(t)}{k} \ll H^{-1}$$

since $a(t) \propto \mathrm{e}^{Ht}$ and $H \approx$ const;
wavelengths gets redshifted, the Hubble parameter stays constant

Alternatives to inflation:

- Bouncing Universe: contraction — bounce — expansion
- “Genesis”: start up from static state

Creminelli et.al.’06; ’10

Difficult, but not impossible. Einstein equations (neglecting spatial curvature)

$$H^2 = \frac{8\pi}{3} G\rho$$

$$\frac{dH}{dt} = -4\pi(\rho + p)$$

$\rho = T_{00}$ energy density, $p = T_{11} = T_{22} = T_{33}$ effective pressure.

Bounce, start up scenarios $\Longrightarrow \frac{dH}{dt} > 0 \Longrightarrow \rho > 0$ and $p < -\rho$

Very exotic matter or modified General Relativity.
Yet there are examples (e.g., galileon field theories) with no obvious pathologies like ghosts, gradient instabilities.

Other suggestive observational facts about density perturbations
(valid within certain error bars!)

- Primordial perturbations are Gaussian. Gaussianity = Wick theorem for correlation functions

  This suggests the origin: enhanced vacuum fluctuations of weakly coupled quatum field(s)

  NB: Linear evolution does not spoil Gaussianity.

  - Inflation does the job very well: vacuum fluctuations of all light fields get enhanced greatly due to fast expansion of the Universe.

    Including the field that dominates energy density (inflaton) $\Longrightarrow$ perturbations in energy density.

    Mukhanov, Chibisov'81; Hawking'82; Starobinsky'82; Guth, Pi'82; Bardeen et.al.'83

  - Enhancement of vacuum fluctuations is less automatic in alternative scenarios

Primordial power spectrum is almost flat: no length scale

Homogeneity and anisotropy of Gaussian random field:

$$\langle \frac{\delta\rho}{\rho}(\vec{k})\frac{\delta\rho}{\rho}(\vec{k}')\rangle = \frac{1}{4\pi k^3}\mathscr{P}(k)\delta(\vec{k}+\vec{k}')$$

$\mathscr{P}(k) =$ power spectrum, gives fluctuation in logarithmic interval of momenta,

$$\langle\left(\frac{\delta\rho}{\rho}(\vec{x})\right)^2\rangle = \int_0^\infty \frac{dk}{k}\,\mathscr{P}(k)$$

Flat spectrum: $\mathscr{P}$ is independent of k

Harrison' 70; Zeldovich' 72, Peebles,Yu' 70

Parametrization

$$\mathscr{P}(k) = A\left(\frac{k}{k_*}\right)^{n_s-1}$$

$A =$ amplitude, $(n_s-1) =$ tilt, $k_* =$ fiducial momentum (matter of convention). Flat spectrum $\Longleftrightarrow n_s = 1$.
Experiment: $n_s = 0.96 \pm 0.01$

There must be some symmetry behind flatness of spectrum

- Inflation: symmetry of de Sitter space-time $SO(4,1)$

$$ds^2 = dt^2 - \mathrm{e}^{2Ht} d\vec{x}^{\,2}$$

Relevant symmetry: spatial dilatations supplemented by time translations

$$\vec{x} \to \lambda \vec{x}\,, \quad t \to t - \frac{1}{2H} \log \lambda$$

- Alternative: conformal symmetry $SO(4,2)$

Conformal group includes dilatations, $x^\mu \to \lambda x^\mu$.
$\Longrightarrow$ No scale, good chance for flatness of spectrum

First mentioned by Antoniadis, Mazur, Mottola' 97
Concrete models: V.R.' 09;
Creminelli, Nicolis, Trincherini' 10.

NB: (Super)conformal symmetry has long been discussed in the context of Quantum Field Theory and particle physics.

Large and powerful symmetry behind, e.g., adS/CFT correspondence and a number of other QFT phenomena

It may well be that ultimate theory of Nature is (super)conformal

What if our Universe started off from or passed through an unstable (super)conformal state and then evolved to much less symmetric state we see today?

Exploratory stage: toy models + general arguments so far.

General setting:

Hinterbichler, Khouri' 12

- Effectively Minkowski space-time
- Conformally invariant theory
- Field ρ of conformal weight $\Delta \neq 0$
- Instability
  of conformally invariant background $\rho = 0$

  Homogeneous classical solution

$$\rho_c(t) = \frac{\text{const}}{(t_* - t)^\Delta}$$

by conformal invariance.

NB: Spontaneous breaking of conformal symmetry:
$O(4,2) \to O(4,1)$

- Another scalar field θ of effective conformal weight 0 (in rolling background); analog of curvaton in inflationary models
- Kinetic term dictated by conformal invariance (modulo field rescaling)

$$L_\theta = \rho^{2/\Delta} (\partial_\mu \theta)^2$$

- Assume potential terms negligible $\Longrightarrow$

$$L_\theta = \frac{1}{(t_* - t)^2} \cdot (\partial_\mu \theta)^2$$

Exactly like scalar field minimally coupled to gravity in de Sitter space, with $t =$ conformal time, $a(t) = \text{const}/(t_* - t)$.

θ develops perturbations with flat power spectrum.

There are various ways to reprocess perturbations of field θ into density perturbations, e.g., at hot epoch. Density perturbations inherit shape of power spectrum and correlation properties from $\delta\theta$, plus possible additional non-Gaussianity.

Can one tell?

More intricate properties of cosmological perturbations

Not detected yet.

- Primordial gravitational waves predicted by simplest hence plausible) inflationary models, but not alternatives to inflation

  Huge wavelengths, from 100 Mpc to size of visible Universe

  Sizeable amplitudes, $h \sim 10^{-5} - 10^{-6}$
  (cf. $h \lesssim 10^{-22}$ for gravity waves of astrophysical origin)

  Almost flat power spectrum

  May make detectable imprint on CMB temperature anisotropy

  V.R., Sazhin, Veryaskin' 82; Fabbri, Pollock' 83; ...

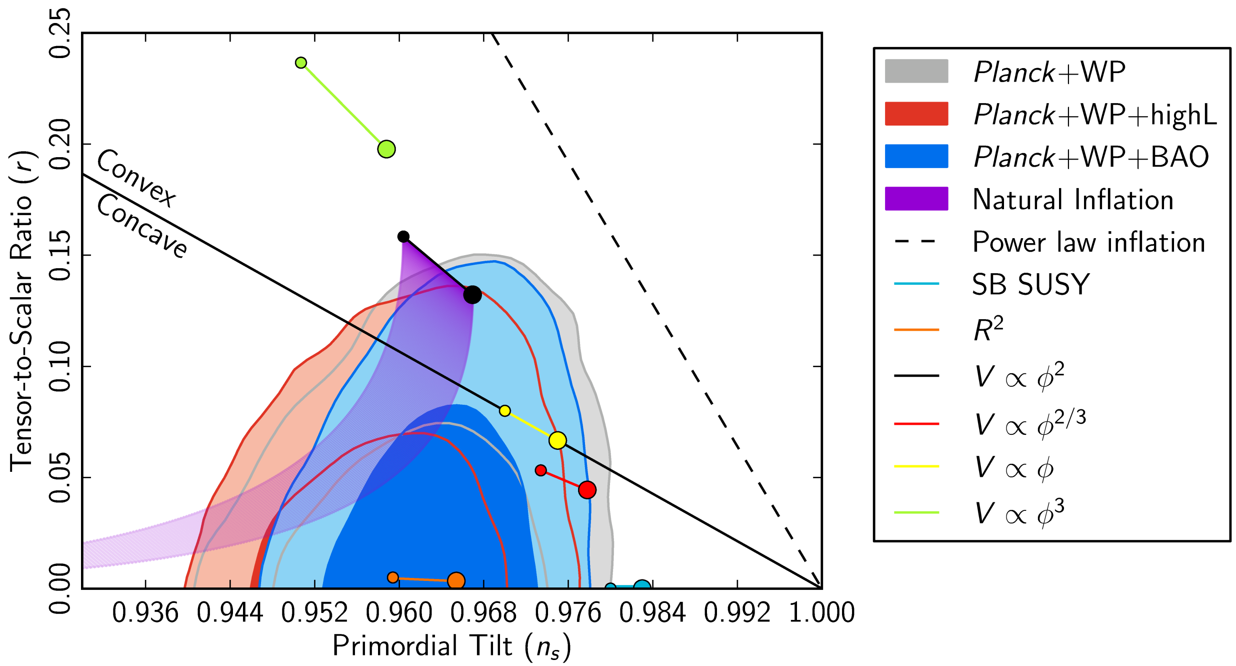
  and especially on CMB polarization

  Basko, Polnarev' 1980; Polnarev' 1985; Sazhin, Benitez' 1995
  Kamionkowski, Kosowsky, Stebbins' 96; Seljak, Zaldarriaga' 96; ...

Smoking gun for inflation

Planck-2013 + everybody else

Scalar spectral index vs. power of tensors



$$r = \left(\frac{\text{amplitude of gravity waves}}{\text{amplitude of density perturbations}} \right)^2$$

Claim of BICEP-2: $r \approx 0.2$ — but problems with emission from Galaxy

- Non-Gaussianity: hot topic
  - Very small in the simplest inflationary theories
  - Sizeable in more contrived inflationary models and in alternatives to inflation. Often begins with bispectrum

$$\langle \frac{\delta\rho}{\rho}(\mathbf{k_1})\frac{\delta\rho}{\rho}(\mathbf{k_2})\frac{\delta\rho}{\rho}(\mathbf{k_3})\rangle = \delta(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})G(k_i^2,\mathbf{k_1}\mathbf{k_2},\mathbf{k_1}\mathbf{k_3})$$

    Shape of $G(k_i^2,\mathbf{k_1}\mathbf{k_2},\mathbf{k_1}\mathbf{k_3})$ different in different models $\Longrightarrow$ potential discriminator.

  - Sometimes bispectrum vanishes, e.g., due to some symmetries. But trispectrum (connected 4-point function) may be measurable.
  - Very specific shape of trispectrum in conformal models

Libanov, Mironov, V.R.' 10, 11

- Statistical anisotropy

$$\mathscr{P}(\mathbf{k}) = \mathscr{P}_0(k)\left(1 + w_{ij}(k)\frac{k_i k_j}{k^2} + \dots\right)$$

  - Anisotropy of the Universe at pre-hot stage
  - Possible in inflation with strong vector fields (rather contrived)

    Ackerman, Carroll, Wise' 07; Pullen, Kamionkowski' 07; Watanabe, Kanno, Soda' 09

  - Natural in conformal models

    Libanov, V.R.' 10; Libanov, Ramazanov, V.R.' 11

To summarize:

- No doubt there was an epoch preceding the hot Big Bang. The question is what was that epoch?
- Inflation is consistent with all data. But there are competitors: the data may rather point towards (super)conformal beginning of the cosmological evolution.

More options:

Matter bounce, Finelli, Brandenberger' 01.

Negative exponential potential, Lehners et. al.' 07;
Buchbinder, Khouri, Ovrut' 07; Creminelli, Senatore' 07.

Lifshitz scalar, Mukohyama' 09

- Only very basic things are known for the time being.
- To tell, we need to discover

more intricate properties of cosmological perturbations

- Primordial tensor modes = gravitational waves

  Sizeable amplitude, (almost) flat power spectrum predicted by simplest (and hence most plausible) inflationary models
  but not alternatives to inflation

  - Together with scalar and tensor tilts $\Longrightarrow$ properties of inflaton

- Non-trivial correlation properties of density perturbations (non-Gaussianity) $\Longrightarrow$ potential discriminator between scenarios
  Very small in single field inflation.

  - Shape of non-Gaussianity: function of invariants $(\vec{k}_1 \cdot \vec{k}_2)$, etc.

- Statistical anisotropy $\Longrightarrow$ anisotropic pre-hot epoch.

  - Shape of statistical anisotropy $\Longrightarrow$ specific anisotropic model

At the eve of new physics

LHC $\Longleftrightarrow$ Planck,
dedicated CMB polarization experiments,
data and theoretical understanding
of structure formation ...

chance to learn
what preceeded the hot Big Bang epoch

Barring the possibility that Nature is dull

Backup slides

A toy model:

V.R.' 09; Libanov, V.R.' 10
Hinterbichler, Khouri' 11

Complex $clasical$ scalar field χ with negative quartic potential (to mimic instability of conformally invariant state)

$$S = \int \sqrt{-g}\left[g^{\mu\nu}\partial_\mu\chi^*\partial_\nu\chi - (-h^2|\chi|^4)\right]$$

Conformal symmetry in 4 dimensions. Global symmetry $U(1)$ (to mimic other symmetries of conformal theory).

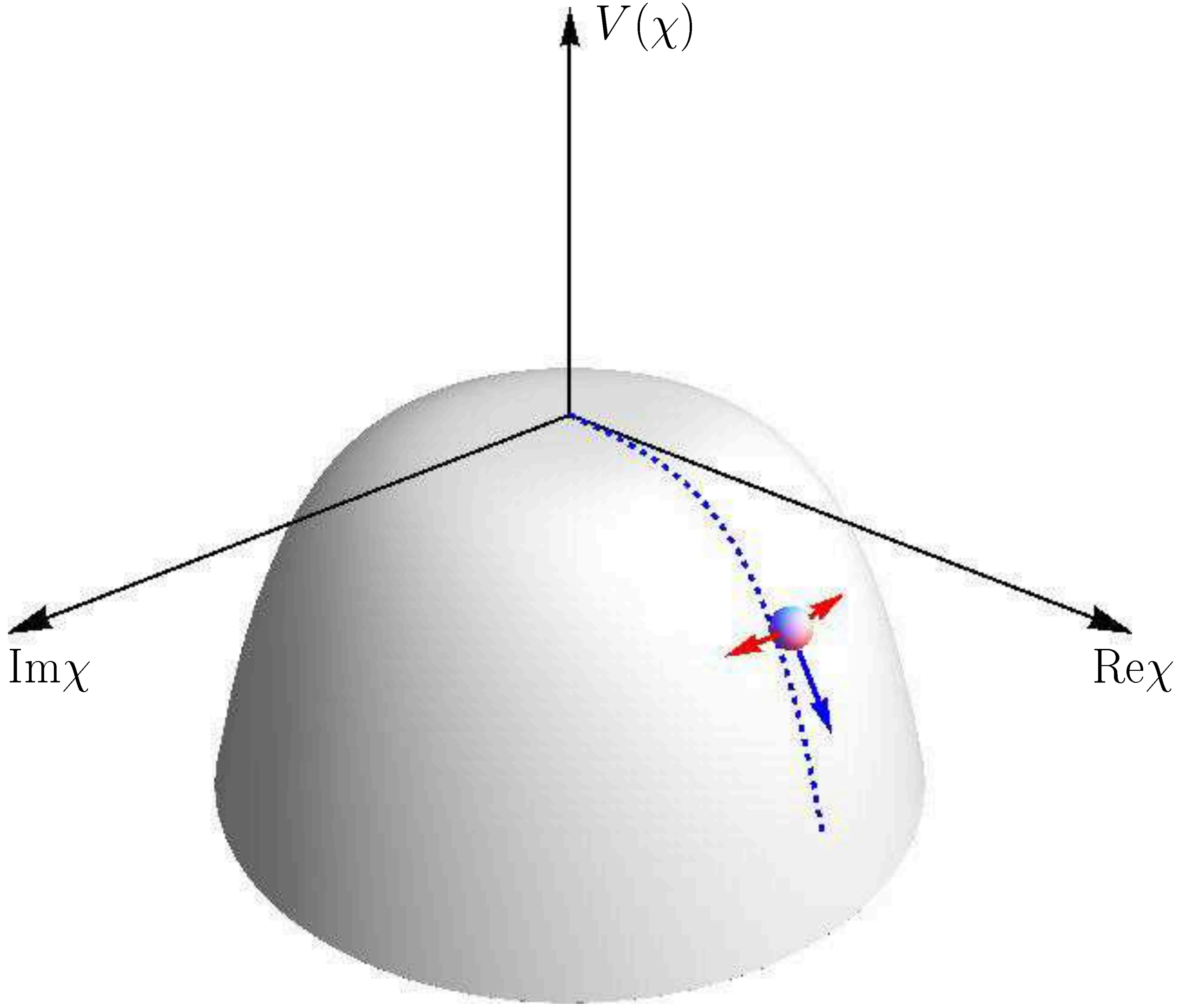
Homogeneous isotropic evolution (real χ_c w/o loss of generality):

$$\chi_c(t) = \frac{1}{h(t_* - t)}$$

Dictated by conformal invariance. t runs from large negative value

NB: gravity assumed negligible or irrelevant

Conformal rolling



Fluctuations of $\theta = \text{Arg}\,\chi$

automatically have flat power spectrum

Linearized equation for fluctuation $\delta\chi_2 \equiv \text{Im}\chi$. Mode of 3-momentum k:

$$\frac{d^2}{dt^2}\delta\chi_2 + k^2\delta\chi_2 - 2h^2\chi_c^2\delta\chi_2 = 0$$

[recall $h\chi_c = 1/(t_* - t)$]

Regimes of evolution:

- Early times, $k \gg 1/(t_* - t)$, short wavelength regime, χ_c negligible, free Minkowskian field

$$\delta\chi_2 = \frac{1}{(2\pi)^{3/2}\sqrt{2k}}\text{e}^{-ikt}A_{\vec{k}} \ + \ \text{h.c.}$$

- Late times, $k \ll 1/(t_* - t)$, long wavelength regime,
  term with χ_c dominates,

$$\delta\chi_2 = \frac{1}{(2\pi)^{3/2}\sqrt{2k}} \cdot \frac{1}{k(t_* - t)} \cdot A_{\vec{k}} \quad + \quad \text{h.c.}$$

- Phase of the field χ freezes out:

$$\delta\theta = \frac{\delta\chi_2}{\chi_c} = \frac{1}{(2\pi)^{3/2}\sqrt{2k}} \cdot \frac{h}{k} \cdot A_{\vec{k}} \quad + \quad \text{h.c.}$$

- Power specrum of phase is flat:

$$\langle \delta\theta^2 \rangle = \frac{h^2}{2(2\pi)^3} \int \frac{d^3k}{k^3} \quad \Longrightarrow \quad \mathscr{P}_{\delta\theta} = \frac{h^2}{(2\pi)^2}$$

- This is automatic consequence of global $U(1)$
  and conformal symmetry

To see this, consider long wavelength regime:

$\vec{k}$ negligible, equation for $\delta\chi_2$ is equation for spatially homogeneous perturbation. χ_c is solution to full field equation, $\mathrm{e}^{i\alpha}\chi_c$ also $\Longrightarrow$ $\delta\chi = i\alpha\chi_c$ is solution to perturbation equation $\Longrightarrow$

$$\delta\chi_2 : \quad \mathrm{e}^{-ikt} \quad \Longrightarrow \quad C(k)\chi_c(t) = \frac{1}{k(t_* - t)}$$

NB: $1/k$ on dimensional grounds.

NB: In fact, equation for $\delta\chi_2$ is precisely the same as equation for minimally coupled massless scalar field in inflating Universe

Comments:

- Mechanism requires long cosmological evolution: need

$$(t_* - t) \gg 1/k$$

early times, short wavelength regime,
well defined vacuum of the field $\delta\chi_2$.

For $k \sim H_0$ this is precisely the requirement that the horizon problem is solved, at least formally.

This is a pre-requisite for most mechanisms that generate density perturbations

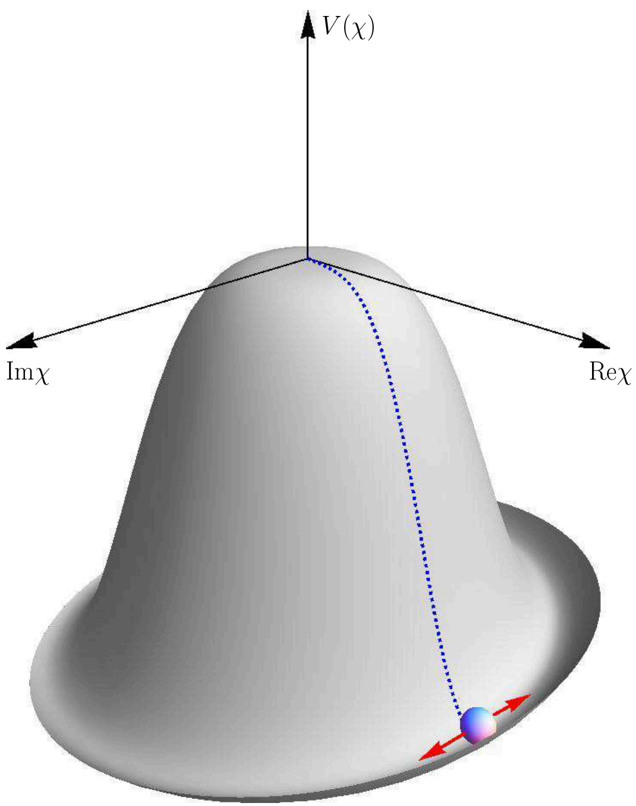
- Small explicit breaking of conformal invariance $\Longrightarrow$ tilt of the spectrum

Osipov, V.R.

Depends both on the way conformal invariance is broken and on the evolution of scale factor

Conformal rolling ends at some point

Toy model: scalar potential actually has a minimum at large field.
Modulus of χ freezes out at this minimum.



Perturbations of the phase get reprocessed into density perturbations. This can happen in a number of ways

Inflationary context: Linde, Mukhanov' 97; Enqvist, Sloth' 01; Moroi, Takahashi' 01; Lyth, Wands' 01; Dvali, Gruzinov, Zaldarriaga' 03; Kofman' 03

Toy model # 2

Creminelli, Nicolis, Trincherini '10

Galilean Genesis

Begin with galileon field π, Lagrangian

$$L_\pi = -f^2 e^{2\pi} \partial_\mu \pi \partial^\mu \pi + \frac{f^3}{\Lambda^3} \partial_\mu \pi \partial^\mu \pi \cdot \Box \pi + \frac{f^3}{2\Lambda^3} (\partial_\mu \pi \partial^\mu \pi)^2$$

Despite higher derivatives in L, field eqn. 2nd order.
Conformally invariant. Under dilatations

$$e^{\pi(x)} \to \lambda e^{\pi(\lambda x)}$$

Universe begins from Minkowski space-time. Galileon rolls as

$$e^{\pi_c} = \frac{1}{H_G(t_* - t)}, \qquad t < t_* ,$$

where $H_G^2 = \frac{2\Lambda^3}{3f}$. Again dictated by conformal invariance.

Initial energy density is zero, then it slowly builds up,

$$H(t) = \frac{1}{3}\frac{f^2}{M_{Pl}^2}\frac{1}{H_G^2(t_* - t)^3}$$

until $(t_* - t_e) \sim H_G^{-1} \cdot f/M_{PL}$. NB: Hubble parameter grows in time. Violation of all energy conditions. Yet consistent theory, no ghosts, tachyons, other pathologies.

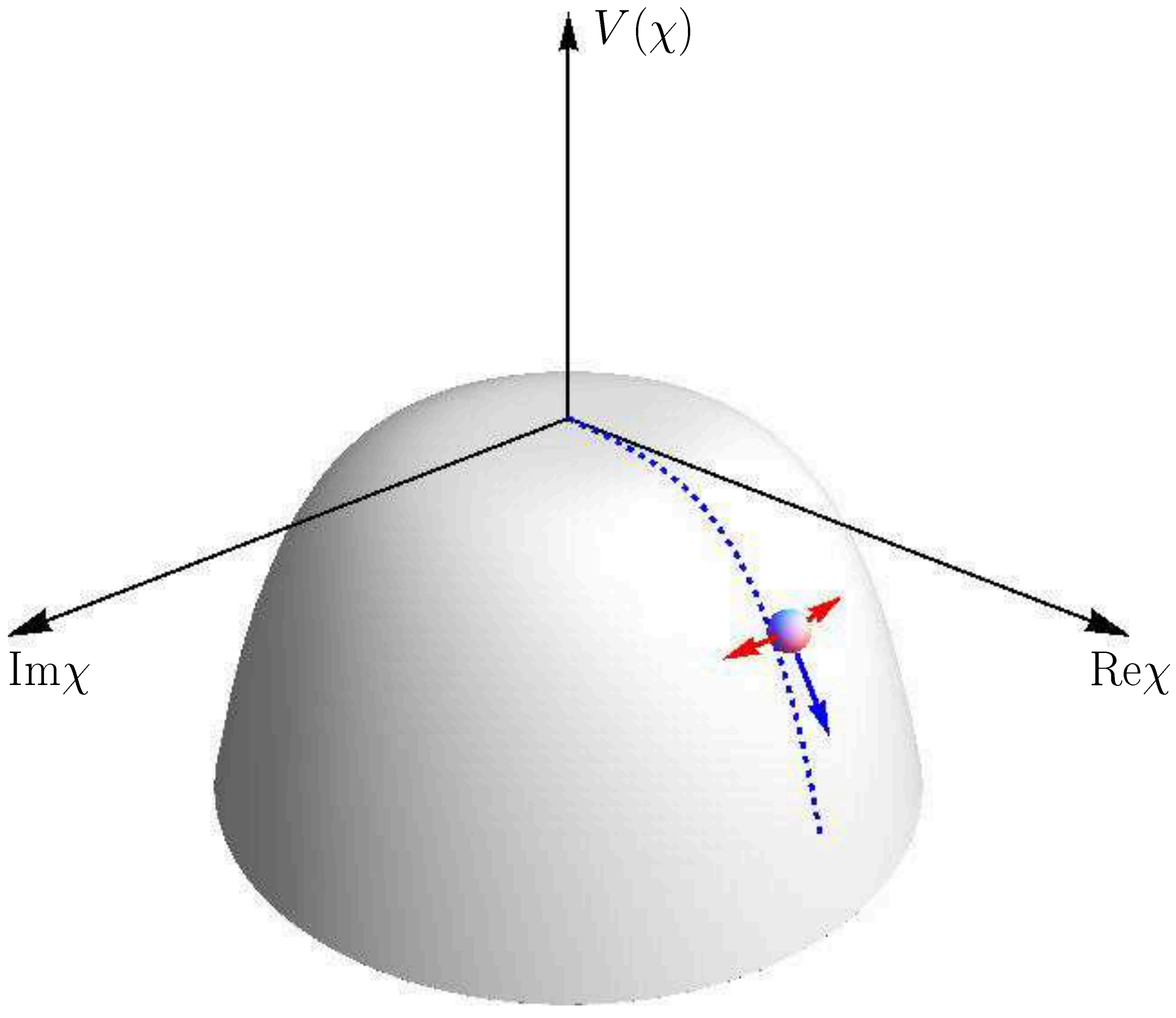
At some point galileon is assumed to transmit its energy to conventional matter, hot epoch begins.

Galileon perturbations are not suitable for generating scalar perturbations.
Introduce another field θ of conformal weight 0,

$$L_\theta = \mathrm{e}^{2\pi}(\partial_\mu \theta)^2 \implies L_\theta = \frac{\text{const}}{(t_* - t)^2} \cdot (\partial_\mu \theta)^2$$

Dynamics of perturbations $\delta\theta$ in background π_c is exactly the same as in conformal rolling model.

Non-linear effects: back to conformal evolution



Peculiarity: raidial perturbations.

Linear analysis of perturbations of $\chi_1 = \mathrm{Re}\chi$ about the homogeneous real solution χ_c:

$$\frac{d^2}{dt^2}\delta\chi_1 + k^2\delta\chi_1 - 6h^2\chi_c^2\delta\chi_1 = 0$$

[recall $h\chi_c = 1/(t_* - t)$].
Again initial condition

$$\delta\chi_1 = \frac{1}{(2\pi)^{3/2}\sqrt{2k}} \mathrm{e}^{i\vec{k}\vec{x}-ikt} B_{\vec{k}} \ + \ \text{h.c.}$$

But now the solution is

$$\delta\chi_1 = \frac{1}{4\pi}\sqrt{\frac{t_*-t}{2}} H_{5/2}{}^{(1)}\left[k(t_*-t)\right] \cdot B_{\vec{k}} \ + \ \text{h.c.}$$

- In long wavelength regime, $k \ll 1/(t_* - t)$,

$$\delta\chi_1 = \frac{3}{4\pi^{3/2}} \frac{1}{k^2\sqrt{k}(t_* - t)^2} B_{\vec{k}} \quad + \quad \text{h.c.}$$

- Red spectrum:

$$\langle \delta\chi_1^2 \rangle \propto \int \frac{d^3k}{k^5}$$

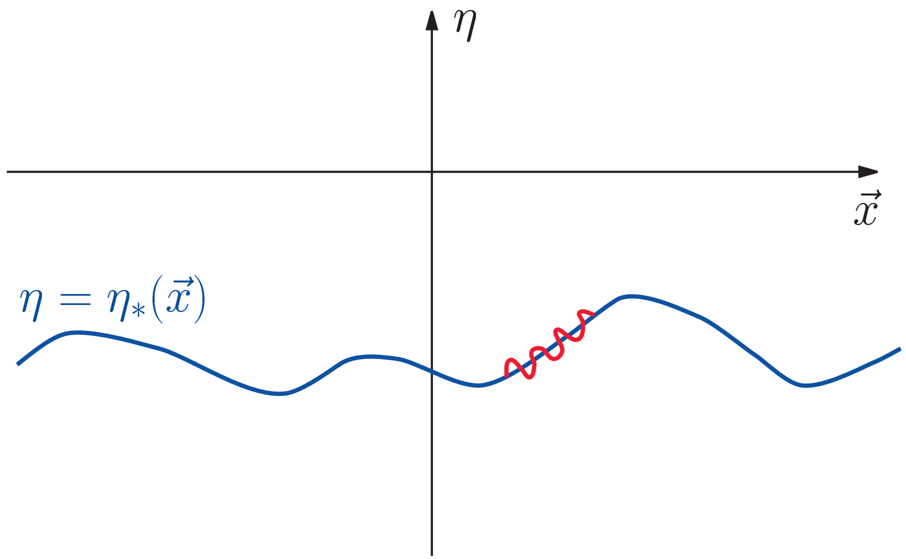
- Again by symmetry: now time translations: $\chi_c \propto 1/(t_* - t) \Longrightarrow$ spatially homogeneous solution to perturbation equation $\delta\chi = \partial_t \chi_c$.

Interpretation: inhomogeneous time shift $t_* \longrightarrow t_* + \delta t_*(\vec{x})$

$$\text{Re}\chi = \chi_c(t) + \delta\chi_1(t,\vec{x}) = \frac{1}{t_* - t} + \frac{F(\vec{x})}{(t_* - t)^2} = \frac{1}{t_* + \delta t_*(\vec{x}) - t}$$

Background for perturbations $\delta\chi_2 = \text{Im}\chi$ (in other words, for phase θ) is no longer spatially homogeneous.

Interaction between $\delta\theta$ and δt_* induces non-Gaussianity and statistical anisotropy.



- Sub-scenario # 1. Phase perturbations superhorizon in conventional sense after end of conformal rolling stage

Libanov, V.R.

- Sub-scenario # 2 (more natural in bouncing models, less natural in Genesis): Phase perturbations sub-horizon in conventional sense after end of conformal rolling stage

Libanov, Ramazanov, V.R.

Statistical anisotopy:

Effect on $\delta\theta$ of long modes of δt_*, superhorizon today.

Separation of scales between $\delta\theta$ and δt_*: evolution of $\delta\theta$ in background, slowly varying in space

$$\mathrm{Re}\chi = \frac{1}{t_* + \delta t_*(\vec{x}) - t}$$

- Sub-scenario # 1.
  Scalar power spectrum ($n_{k\,i} = k_i/k =$ unit vector along $\vec{k}$)

$$\mathscr{P}(\vec{k}) = \mathscr{P}_0(k)\left(1 - \frac{\pi}{k} n_{k\,i} n_{k\,j} \partial_i \partial_j t_*\right)$$

Statistical anisotropy due to constant in space tensor $\partial_i \partial_j t_*|_{\mathrm{long\ \ wavelengths}}$

NB: general property of conformal mechanisms

Creminelli, Joyce, Khoury, Simonovic

NB: Power spectrum of $\partial^2 t_*$ is blue $\Longrightarrow$

$$\langle (\pi \partial_i \partial_j t_*)^2 \rangle_{\text{long wavelengths}} \simeq \frac{9h^2}{4} \int_0^{H_0} k dk \simeq h^2 H_0^2$$

Statistical anisotropy effect on perturbations of wave vector k:

$$\mathscr{P}(\vec{k}) = \mathscr{P}^{(0)}(k) \left(1 + \frac{hH_0}{k} w_{ij} n_{ki} n_{kj} \right)$$

(with $w_{ij} w_{ij} = 1$) $\Longrightarrow$ effect on CMB behaves as $1/l$

Difficult case: only low CMB multipoles feel the effect, large cosmic variance.

Planck data: $h^2 < 0.5$

Ramazanov, Rubtsov' 14

- Sub-scenario # 2: Phase perturbations sub-horizon in conventional sense after end of conformal rolling stage

  Natural for contracting (e.g., ekpyrotic) Universe.

  $\delta\theta$ evolves non-trivially before it becomes super-horizon and freezes out again.

  - For given $\vec{k}$, phase perturbation after second freeze-out is a linear combination of waves coming from direction of $\vec{k}$ and from opposite direction and traveling large distance r $\Longrightarrow$ Imprint on $\delta\theta(\vec{k})$ of random field $t_*(\pm\vec{n}_k r)$, which depends on $\vec{n}_k$ only

$$\mathscr{P}_{\delta\theta}(\vec{k}) = \mathscr{P}_0\{1 - n_{ki}[\partial_i t_*(+\vec{n}_k r) + \partial_i t_*(-\vec{n}_k r)]\} \ ,$$

    Non-trivial dependence on $\vec{n}_k$. Statistical anisotropy with all even multipoles.

Resulting statistical anisotropy

$$\mathscr{P}_\zeta(\vec{k}) = \mathscr{P}_\zeta^{(0)}(k)\left[1 + \mathscr{Q}\cdot w_{ij}\left(n_{ki}n_{ki} - \frac{1}{3}\delta_{ij}\right) + \text{higher multipoles}\right]$$

with $w_{ij}w_{ij} = 1$ and

$$\langle\mathscr{Q}^2\rangle = \frac{675}{32\pi^2}h^2 \ .$$

NB: multipoles $\mathscr{Q}$, etc., are independent of $k \Longrightarrow$ no suppression of effect on CMB at large l, unlike in sub-scenario # 1.

Easier to search in data.

Towards being falsified: analysis of Planck data $\Longrightarrow h^2 \leq 1.3\cdot 10^{-3}$

Ramazanov, Rubtsov' 14,

cf. Kim, Komatsu' 13

Non-Gaussianity to order h^2

Over and beyond non-Gaussianity which can be generated when perturbations in θ are converted into adiabatic perturbations.

Invariance $\theta \to -\theta \Longrightarrow$ bispectrum vanishes.

Sub-scenario #1: trispectrum fully calculated.

Libanov, Mironov, V.R.

Most striking property: singularity in folded limit:

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \zeta_{\vec{k}_4} \rangle = c \cdot \delta \left(\sum_{i=1}^{n} \vec{k}_i \right) \cdot \frac{1}{k_{12} k_1^4 k_3^4} \left[1 - 3 \left(\frac{\vec{k}_{12} \cdot \vec{k}_1}{k_{12} k_1} \right)^2 \right] \left[1 - 3 \left(\frac{\vec{k}_{12} \cdot \vec{k}_3}{k_{12} k_3} \right)^2 \right]$$

$$\vec{k_{12}} = \vec{k}_1 + \vec{k}_2 \to 0$$

This is in contrast to single field inflationary models.

Origin: infrared enhancement of radial perturbations $\delta\chi_1$

NB: again general property of conformal mechanisms

Creminelli, Joyce, Khoury, Simonovic

Sub-scenario # 2: tri-spectrum depends on directions of momenta

$$\langle \zeta(\vec{k})\zeta(\vec{k}')\zeta(\vec{q})\zeta(\vec{q}')\rangle = \frac{\mathscr{P}_\zeta^{(0)}(k)}{4\pi k^3}\frac{\mathscr{P}_\zeta^{(0)}(q)}{4\pi q^3}\delta(\vec{k}+\vec{k}')\delta(\vec{q}+\vec{q}')\cdot[1+F_{NG}(\vec{n}_k,\vec{n}_q)]$$

$$+ \text{ permutations}$$

with

$$F_{NG} = \frac{3h^2}{\pi^2}\log\frac{\text{const}}{|\vec{n}_k - \vec{n}_q|}$$

NB: flat power spectrum of $\vec{\partial}t_* \Longrightarrow$ log behavior of F_{NG}.

BICEP-2 saga

Power spectra of tensor (gravity waves) and scalar perturbations (per log interval of momenta=wave numbers)

$$\mathscr{P}_T = \frac{16}{\pi}\frac{H_{infl}^2}{M_{Pl}^2} = \frac{128}{3}\frac{\rho_{infl}}{M_{Pl}^4}\,, \qquad \mathscr{P}_s = 2.5\cdot 10^{-9}$$

Notation: tensor-to-scalar ratio

$$r = \frac{\mathscr{P}_T}{\mathscr{P}_s}$$

Scalar spectral index

$$\mathscr{P}_s(k) = \mathscr{P}_s(k_*)\cdot\left(\frac{k}{k_*}\right)^{n_s-1}$$

Predictions of inflationary models

Assume power-law inflaton potential $V(\phi) = g\phi^n$. Then

$$r = \frac{4n}{N_e} \qquad n_s - 1 = -\frac{n+2}{2N_e}$$

$$N_e = \ln\frac{a_e}{a_\times} = 50 - 60$$

$a_e =$ scale factor at the end of inflation

$a_\times =$ scale factor at the time when our visible Universe exits the horizon at inflation.

Tensor perturbations = gravity waves

Metric perturbations

$$ds^2 = dt^2 - a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j$$

$h_{ij} = h_{ij}(\vec{x}, t)$, $h_i^i = \partial_i h_j^i = 0$, spin 2.

Gravity waves: effects on CMB

- Temperature anisotropy (in addition to effect of scalar perturbations)

V.R., Sazhin, Veryaskin' 1982; Fabbri, Pollock' 83

WMAP, Planck

NB: gravity wave amplitudes are time-independent when superhorizon and decay as $h_{ij} \propto a^{-1}(t)$ in subhorizon regime.

Strongest contribution to δT at large angles

$$\Delta\theta \gtrsim 2^o, \quad l \lesssim 50, \quad \text{Present wavelengths } \sim 1 \text{ Gpc}$$

- Polarization

Basko, Polnarev' 1980; Polnarev' 1985; Sazhin, Benitez' 1995

especially B-mode

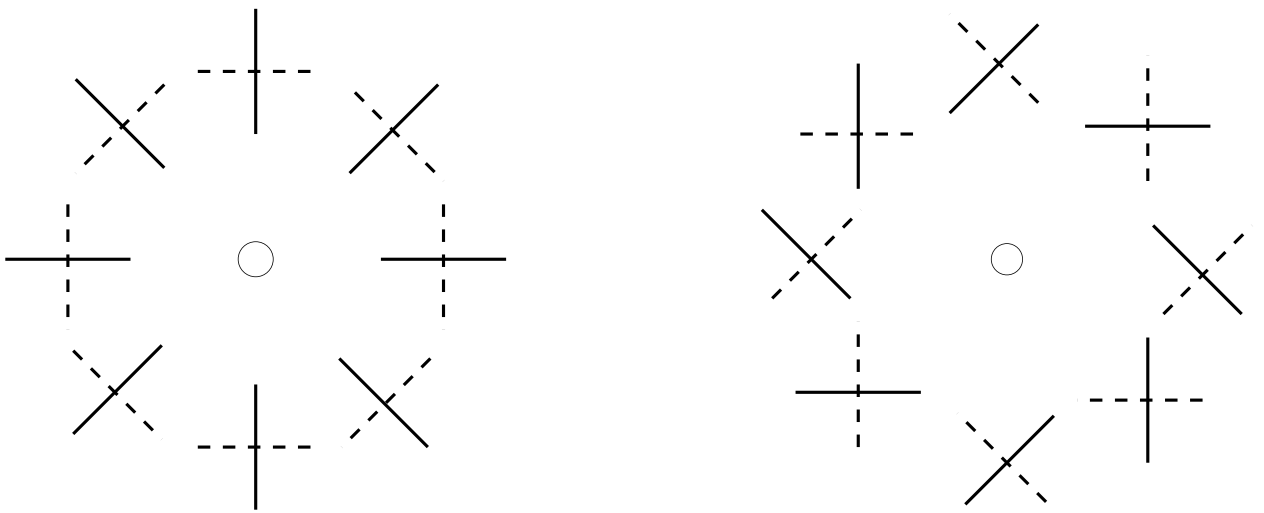
Kamionkowski, Kosowski, Stebbins' 1997; Seljak, Zaldarriaga' 1997

Weak signal, degree of polarization $P(l) \propto l$ at $l \lesssim 50$ and decays with l at $l > 50$.

Amplitude at $r = 0.2$:

$$P(l \sim 30) \sim 3 \cdot 10^{-8} \Longrightarrow P \cdot T \sim 0.1 \ \mu\text{K}$$

Linear polariation: E- and B-modes



E-mode, parity even

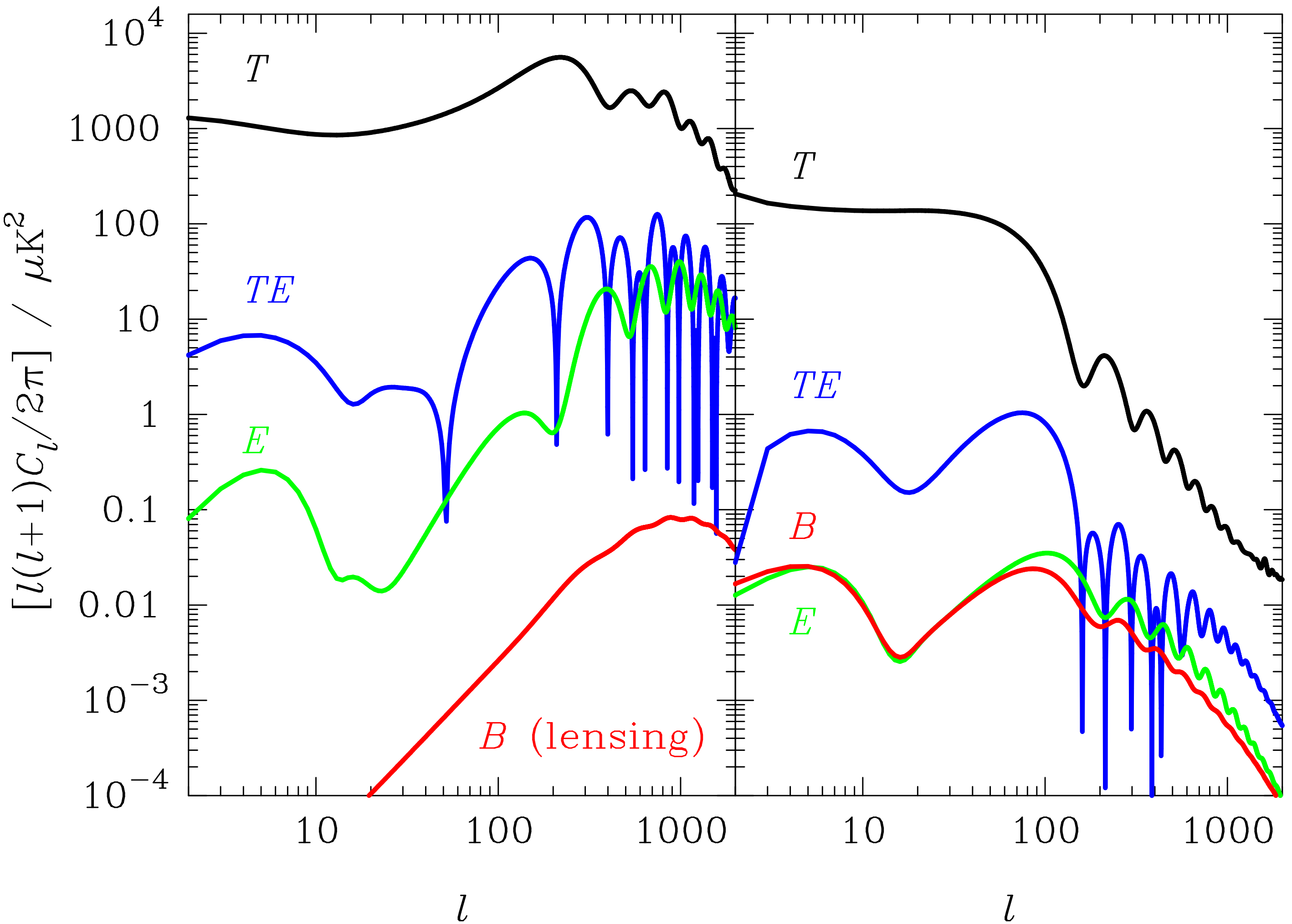
From both scalar
and tensor perturbations

B-mode, parity odd

From tensors only

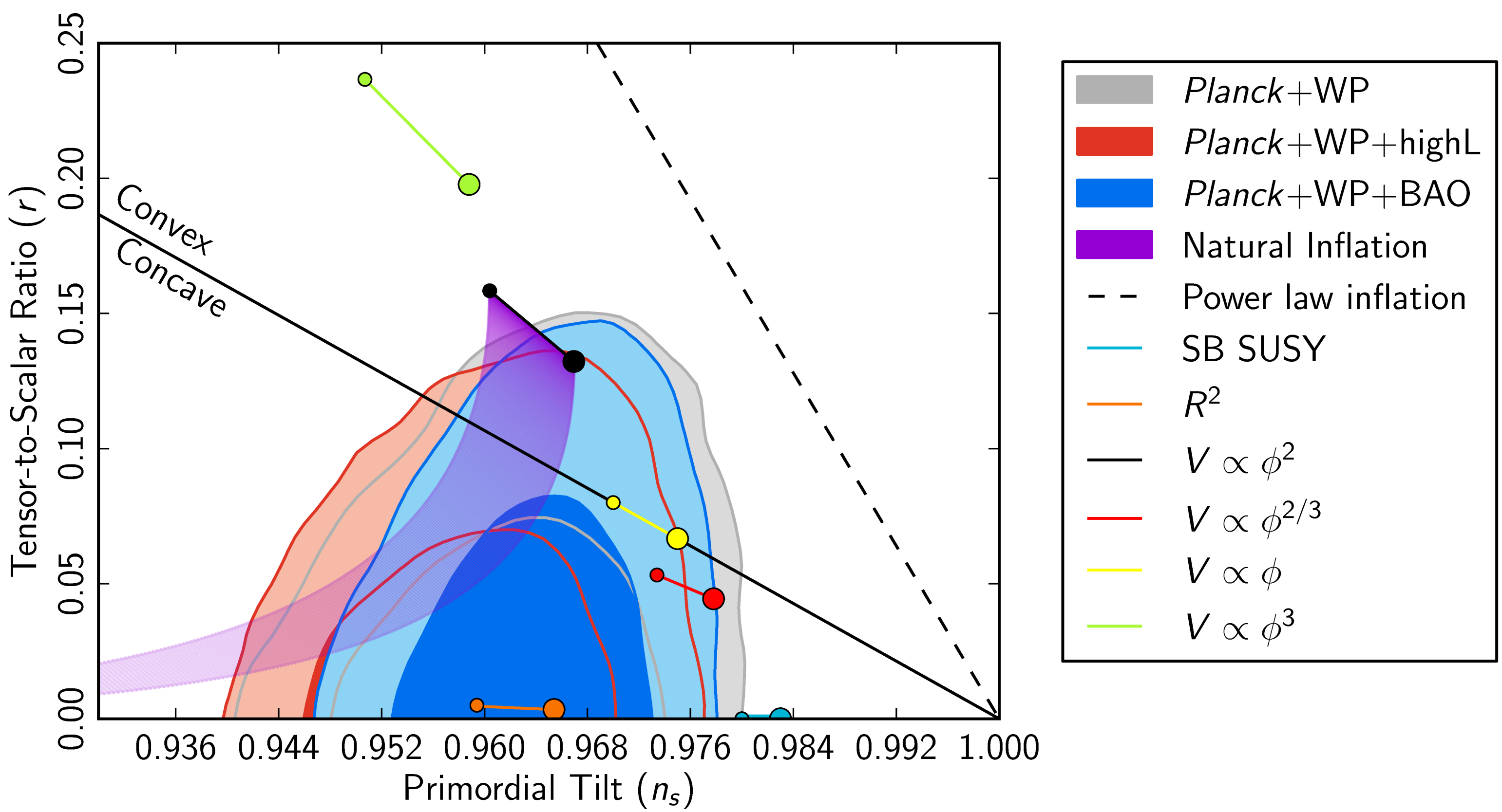
(+ lensing by structures
at relatively small angular scales)

Effects of scalars (left) and tensors (right)



Planck-2013 + everybody else

Scalar spectral index vs. power of tensors

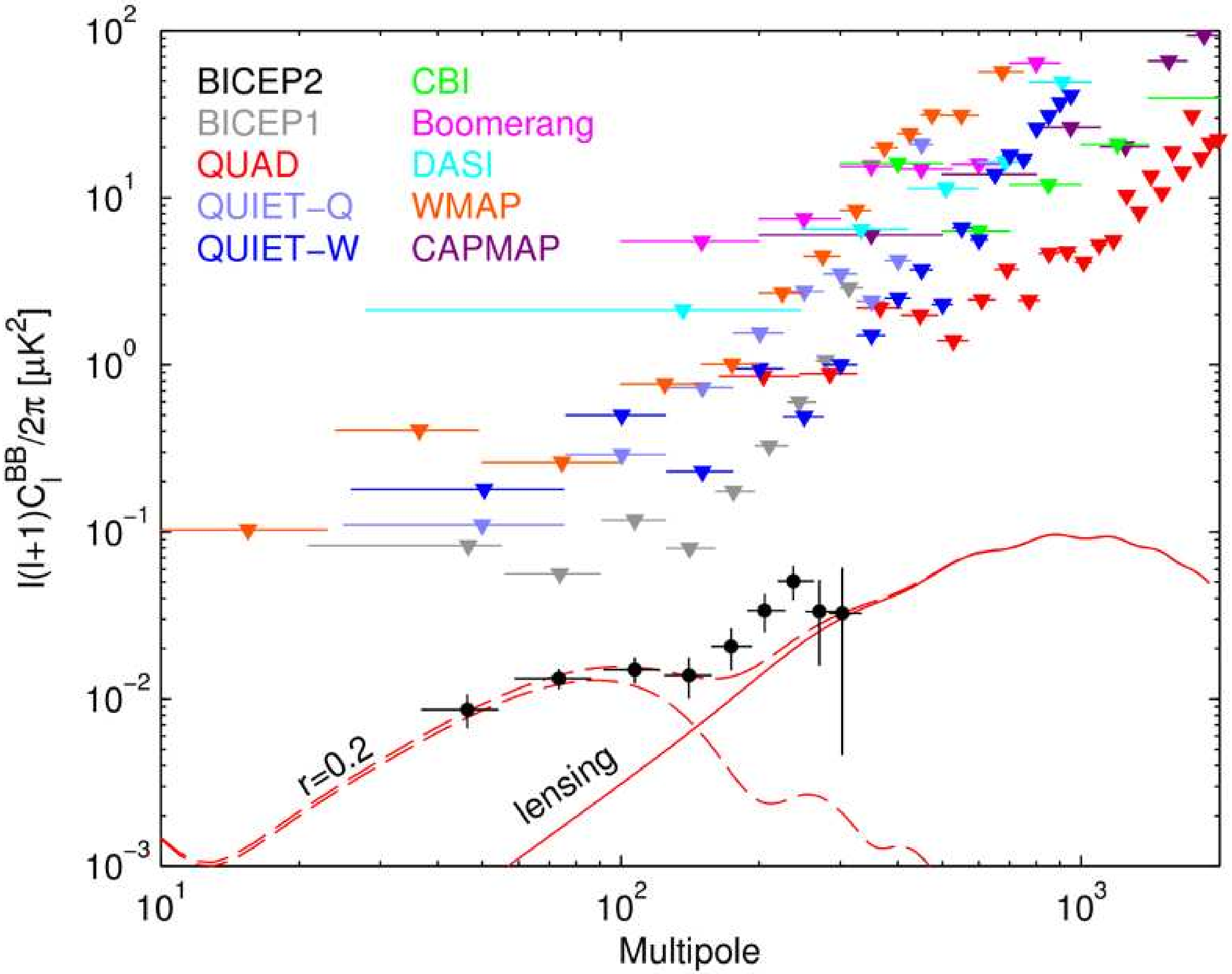


BICEP-2 at South pole

- 590 days of data taking
- Sky region of 390 square degrees towards Galactic pole
- One frequency 150 GHz
- March 2013: claim of discovery of CMB polarization generated by relic gravity waves

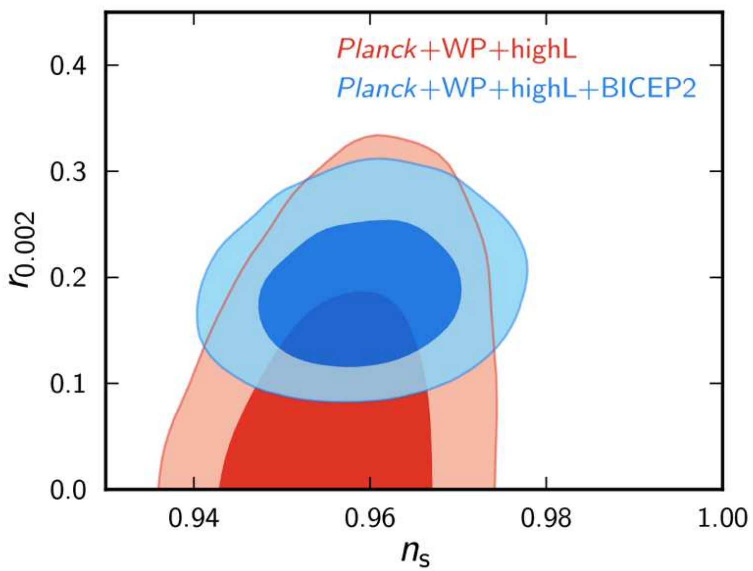
BICEP-2 result

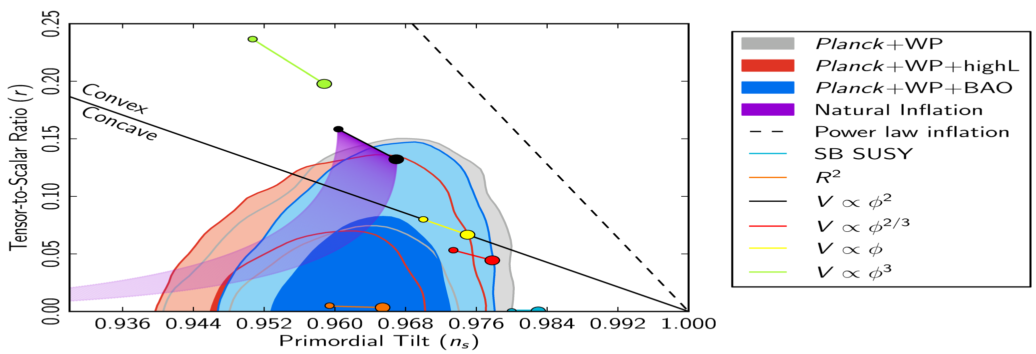
- $30 < l < 150, \quad r = 0.2^{+0.07}_{-0.05}, \quad r \neq 0 \ > 5\sigma$



Tension between BICEP-2 and Planck:

- $r = 0.2$ is large: 10% contribution to δT at low multipoles $l \lesssim 30$.





BICEP-2 and Planck with
$dn_s/d\ \ln k = -0.02$ (very large!)
Inflation: $dn_s/d\ \ln k \approx -0.001$

Planck + others

Were this the discovery, then

- Proof of inflation
- $\rho_{infl}^{1/4} = 2 \cdot 10^{16}$ GeV
- Experimental proof of linearized quantum gravity (no wonder!)

In future:

Tensor spectral index $\Longrightarrow$ consistency relation in single field inflation

$$n_T = -\frac{r}{8}$$

Signal is there.

Are there relic gravity waves???

Dangerous "foreground": polarized dust in our Galaxy, $r \sim 0.1\ \mu$m

Oriented by Galactic magnetic field, emits polarized radiation (way to study magnetic fields in our Galaxy)

Dominates completely at high frequencies

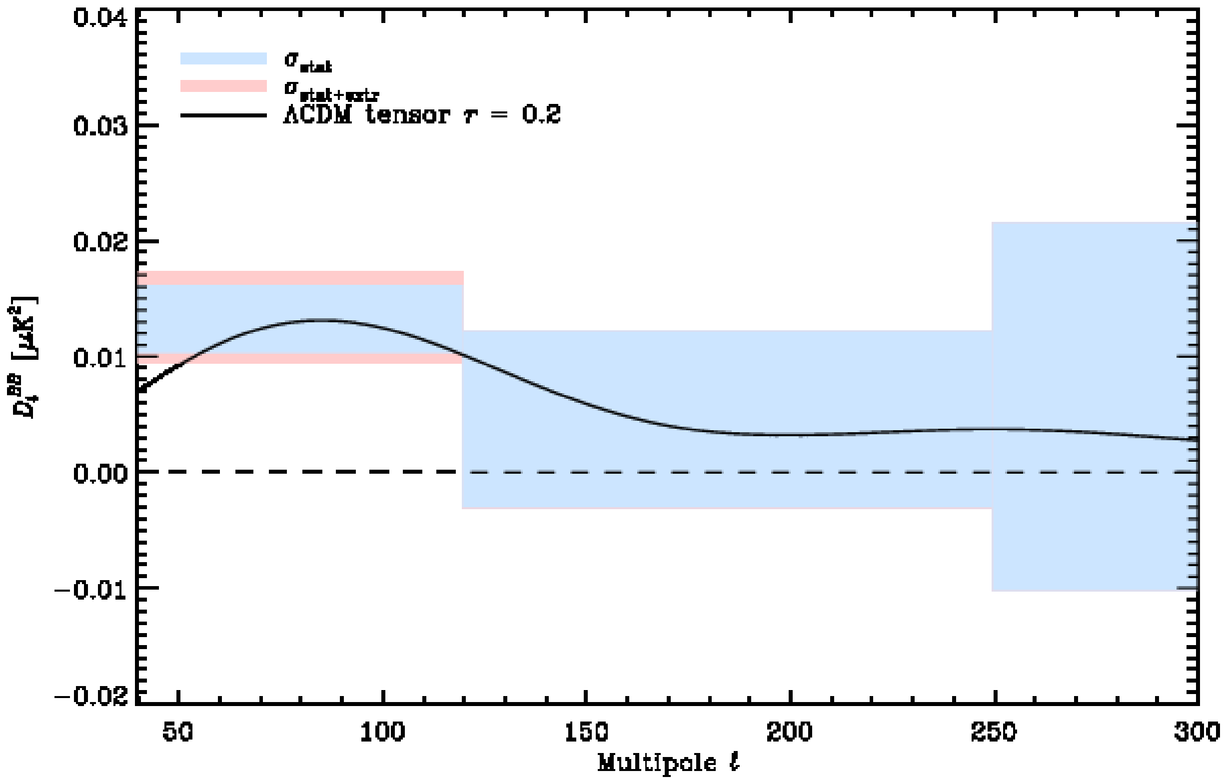
Poorly known until very recently

Prejudice: negligible at Galactic polar regions.

And what's the reality?

Planck, September 2014: analyzed dust contribution to polarization

Planck-2014



Extrapolation of dust contribution from 353 GHz to 150 GHz (shaded regions)

Solid line: expected gravity wave signal at $r = 0.2$

NB: Same patch of the sky as used by BICEP-2

Smells like dust, looks like dust, tastes like dust...

Discovery postponed – too bad!

Hard task for experimentalists: extract signal from relic gravity waves from dust foreground

Especially if $r < 0.1$