Tutorial – Monte Carlo and Importance Sampling —— July 9, 2011

1. Consider the Normal, Exponential and Gamma distributions from last time. Use Monte Carlo simulation (in the simplest sense) to **estimate** the mean and variance of each distribution using $N = 1000$ random variables. Recall, the sample mean and sample variance that you'll calculate are also random variables. So, repeat this process $M = 100$ times and present your results with a histogram. Repeat the experiment for $N = 10000$. *What relationship do you see between the sample variance, s^2, the variance, σ^2 and N?* Note the MatLab command 'sum' will calculate the sum of a vector (array) for you.

2. Consider the following integral we saw in the last lecture

$$\int_0^2 x^3 dx.$$

   Recall, we could break up the integrand in multiple ways and we considered these two:

   (a) $f(x) = 1/2$

   (b) $f(x) = \frac{3}{8}x^2$

   In each case, verify the number of samples, N needed to calculate this integral with probability .99 of an error tolerance of 0.1 (exactly the case we considered in the lecture). For each case, esitimate this integral (drawing samples from the respective $f(x)$ above and using the correpsonding number of required samples in each case).

3. Use importance sampling to calculate $P(Z > 4.5)$ where Z is a standard Normal, e.g. $Z \sim N(0, 1)$. To do so draw random varianbes from the biasing distribution choosen in the last lecture

$$\tilde{f}(x) = \frac{\exp(-(x - 4.5))}{\int_{4.5}^{\infty} \exp(-(x - 4.5))dx}.$$

   The perform Monte Carlo simulatins as above, but recall for each $X_n \sim \tilde{f}$, you'll need to calculate the likilihood ratio

$$f(X_n)/\tilde{f}(X_n),$$

   where the distribution of a standard normal is given by

$$f(x) = \frac{1}{\sqrt{2\pi}} \exp(-x^2/2).$$