# N- extended Super $BMS_3$ Algebras

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Based on arXiv:1609.09210 [1]
arXiv:1706.02922 [2]

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[1] JHEP11(2016)059 with N. Banerjee, D. Jatkar, I. Lodato, S. Mukhi
[2] PhysRevD.96.066029 with N. Banerjee, I. Lodato

# Plan of the Talk:

- Brief Introduction to $BMS_3$ Symmetry
- N=2 $SBMS_3$ Algebra by Contraction
- N=4 $SBMS_3$ Algebra by Contraction
- Asymptotic Symmetry Analysis
- Conclusions

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Besides from actual symmetry analysis, $BMS_3$ algebra has also been derived as a suitable contraction of two copies of Virasoro algebra.

The algebra for the $N = 1$ supersymmetric version is also known. Here we extend the analysis to the $N = 2$ and $N = 4$ supersymmetric $BMS_3$ algebras.

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For correct contraction and identification of the resulting algebra as the super-BMS algebra, we will impose the following conditions:

- None of the terms should blow up!
- The resulting algebra must contain the corresponding super-Poincare algebra as a subalgebra.
- The anticommutators of a supercharge with its Hermitian conjugate must be proportional to a translation generator with a positive coefficient.

# N=2 $SBMS_3$ Algebra by Contraction:

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$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}n(n^2-1)\delta_{n+m,0}$$
$$[L_n, Q_r] = \left(\frac{n}{2} - r\right) Q_{n+r}$$
$$\{Q_r, Q_s\} = L_{r+s} + \frac{c}{6}\left(r^2 - \frac{1}{4}\right)\delta_{r+s,0}$$

$$[\bar{L}_n, \bar{L}_m] = (n-m)\bar{L}_{n+m} + \frac{\bar{c}}{12}n(n^2-1)\delta_{n+m,0}$$
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The scaling of the Virasoro part is already a known result (asymmetric, with mixing of modes).

We found that the only way to get the correct physical algebra is to scale the supercharges symmetrically.

$$P_m = \lim_{\epsilon \to 0} \epsilon \left( L_m + \bar{L}_{-m} \right) \qquad J_m = \lim_{\epsilon \to 0} \left( L_m - \bar{L}_{-m} \right)$$

$$c_1 = \lim_{\epsilon \to 0} (c - \bar{c}) \qquad c_2 = \lim_{\epsilon \to 0} \epsilon (c + \bar{c})$$

$$\mathcal{Q}^1_r = \lim_{\epsilon \to 0} \sqrt{\epsilon} Q_r \qquad \mathcal{Q}^2_r = \lim_{\epsilon \to 0} \sqrt{\epsilon} \bar{Q}_{-r}$$

This results in the following algebra:

$$[J_n, J_m] = (n-m)J_{n+m} + \frac{c_1}{12} n(n^2-1)\delta_{n+m,0}$$
$$[J_n, P_m] = (n-m)P_{n+m} + \frac{c_2}{12} n(n^2-1)\delta_{n+m,0}$$
$$[J_n, \mathcal{Q}^a_r] = \left(\frac{n}{2} - r\right)\mathcal{Q}^a_{n+r}$$
$$\{\mathcal{Q}^a_r, \mathcal{Q}^b_s\} = \frac{1}{2}\delta^{ab}\left[P_{r+s} + \frac{c_2}{6}\left(r^2 - \frac{1}{4}\right)\delta_{r+s,0}\right]$$

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We can identify the Super-Poincare group as $\{J_0, J_{\pm 1}, P_0, P_{\pm 1}, \mathcal{Q}^a_{\pm\frac{1}{2}}\}$.

# N=4 Super $BMS_3$ Algebra by Contraction:

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$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}n(n^2-1)\delta_{n+m,0}$$

$$[L_n, Q_r^{\pm}] = \left(\frac{n}{2} - r\right) Q_{n+r}^{\pm} \qquad [L_n, R_m] = -mR_{n+m}$$

$$\{Q_r^+, Q_s^-\} = L_{r+s} + \frac{1}{2}(r-s)R_{r+s} + \frac{c}{6}\left(r^2 - \frac{1}{4}\right)\delta_{r+s,0}$$

$$\{Q_r^+, Q_s^+\} = 0 \qquad \{Q_r^-, Q_s^-\} = 0$$

$$[R_n, R_m] = \frac{c}{3}n\delta_{n+m,0} \qquad [R_n, Q_r^{\pm}] = \pm Q_{n+r}^{\pm}$$

and an exactly identical copy of the same algebra, with barred generators and central charge.

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The Virasoro part is contracted in the usual way.

The supercharges also scale in the same way as the previous case, albeit there are more copies now.

$$\mathcal{Q}_r^{1,\pm} = \lim_{\epsilon\to 0} \sqrt{\epsilon} Q_r^{\pm} \qquad \mathcal{Q}_r^{2,\pm} = \lim_{\epsilon\to 0} \sqrt{\epsilon} \bar{Q}_{-r}^{\pm}$$

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We found that only the asymmetrical scaling of the R-symmetries (with mixing of modes) gives the correct algebra satisfying all the physical requirements:

$$\mathcal{R}_m = \lim_{\epsilon\to 0}(R_m - \bar{R}_{-m}) \qquad \mathcal{S}_m = \lim_{\epsilon\to 0} \epsilon(R_m + \bar{R}_{-m})$$

This gives the following algebra:

$$[J_n, J_m] = (n-m)J_{n+m} + \frac{c_1}{12}n(n^2-1)\delta_{n+m,0}$$

$$[J_n, P_m] = (n-m)P_{n+m} + \frac{c_2}{12}n(n^2-1)\delta_{n+m,0}$$

$$[J_n, \mathcal{Q}_r^{a,\pm}] = \left(\frac{n}{2} - r\right)\mathcal{Q}_{n+r}^{a,\pm}$$

$$[J_n, \mathcal{R}_m] = -m\mathcal{R}_{n+m} \qquad [J_n, \mathcal{S}_m] = -m\mathcal{S}_{n+m}$$

$$[\mathcal{R}_n, \mathcal{R}_m] = \frac{c_1}{3}n\delta_{n+m,0} \qquad [\mathcal{R}_n, \mathcal{S}_m] = \frac{c_2}{3}n\delta_{n+m,0}$$

$$[\mathcal{S}_n, \mathcal{S}_m] = 0 \qquad [P_n, \mathcal{R}_m] = -m\mathcal{R}_{n+m}$$

$$[\mathcal{R}_m, \mathcal{Q}_r^{1,\pm}] = \pm\mathcal{Q}_{m+r}^{1,\pm} \qquad [\mathcal{R}_m, \mathcal{Q}_r^{2,\pm}] = \mp\mathcal{Q}_{m+r}^{2,\pm}$$

$$\{\mathcal{Q}_r^{1,+}, \mathcal{Q}_s^{1,-}\} = \frac{1}{2}\left[P_{r+s} + \frac{1}{2}(r-s)\mathcal{S}_{r+s} + \frac{c_2}{6}\left(r^2 - \frac{1}{4}\right)\delta_{r+s,0}\right]$$

$$\{\mathcal{Q}_r^{2,+}, \mathcal{Q}_s^{2,-}\} = \frac{1}{2}\left[P_{r+s} - \frac{1}{2}(r-s)\mathcal{S}_{r+s} + \frac{c_2}{6}\left(r^2 - \frac{1}{4}\right)\delta_{r+s,0}\right]$$

This is a version of the $N = 4$ $\mathrm{SBMS}_3$ algebra.

The anticommutator of each supercharge with its Hermitian conjugate closes into a linear combination of $P$ and $\mathcal{S}$ plus a central term.

However the algebra can be more generalised!

This is a version of the $N = 4$ SBMS$_3$ algebra.

The anticommutator of each supercharge with its Hermitian conjugate closes into a linear combination of $P$ and $\mathcal{S}$ plus a central term.

However the algebra can be more generalised!
Note that in the above algebra the central charges appearing in the $[J_n, J_m]$ and $[\mathcal{R}_n, \mathcal{R}_m]$ are related.

This feature turns out to be too restrictive! Jacobi identity puts no such restriction on these two central charges, hence one can write a more generic version of the algebra with the two central charges to be independent.

The algebra finally becomes:

$$[J_n, J_m] = (n-m)J_{n+m} + \frac{c_1}{12}n(n^2-1)\delta_{n+m,0}$$

$$[J_n, P_m] = (n-m)P_{n+m} + \frac{c_2}{12}n(n^2-1)\delta_{n+m,0}$$

$$[J_n, \mathcal{Q}_r^{a,\pm}] = \left(\frac{n}{2} - r\right)\mathcal{Q}_{n+r}^{a,\pm}$$

$$[J_n, \mathcal{R}_m] = -m\mathcal{R}_{n+m} \qquad [J_n, \mathcal{S}_m] = -m\mathcal{S}_{n+m}$$

$$[\mathcal{R}_n, \mathcal{R}_m] = \frac{c_3}{3}n\delta_{n+m,0} \qquad [\mathcal{R}_n, \mathcal{S}_m] = \frac{c_2}{3}n\delta_{n+m,0}$$

$$[\mathcal{S}_n, \mathcal{S}_m] = 0 \qquad [P_n, \mathcal{R}_m] = -m\mathcal{R}_{n+m}$$

$$[\mathcal{R}_m, \mathcal{Q}_r^{1,\pm}] = \pm\mathcal{Q}_{m+r}^{1,\pm} \qquad [\mathcal{R}_m, \mathcal{Q}_r^{2,\pm}] = \mp\mathcal{Q}_{m+r}^{2,\pm}$$

$$\{\mathcal{Q}_r^{1,+}, \mathcal{Q}_s^{1,-}\} = \frac{1}{2}\left[P_{r+s} + \frac{1}{2}(r-s)\mathcal{S}_{r+s} + \frac{c_2}{6}\left(r^2 - \frac{1}{4}\right)\delta_{r+s,0}\right]$$

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However, we later found out that the extended superconformal algebra contains non-linear terms! [Henneaux, Maoz, Schwimmer: hep-th/9910013v2].

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We are therefore trying to find the correct algebra by direct asymptotic analysis of both the ADS and the flat case.

# N=4 $SBMS_3$ by Asymptotic Symmetry Analysis

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Three dimensional gravity is equivalent to a very specific gauge theory called Chern-Simons theory.

We start with a set of global or bulk generators.

We then expand the Chern-Simons gauge field $\mathcal{A} = A_\mu dx^\mu$ in terms of these global symmetry generators.

The dynamical fields are then studied at asymptotic infinity.

Using the canonical formulation, we then find the asymptotic algebra.

As we are interested in null infinity we will work in the BMS coordinates $(u, r, \varphi)$, with the radial direction $r$ being null.

Let us start with the global generators $\mathcal{J}_a, P_a$ and $\mathcal{Q}_\alpha^{1,\pm}, \mathcal{Q}_\alpha^{2,\pm}$ where $(a = 0, 1, 2)$ and $(\alpha = \pm\frac{1}{2})$. They satisfy the algebra

$$[\mathcal{J}_a, \mathcal{J}_b] = \epsilon_{abc}\mathcal{J}^c \qquad \left[\mathcal{J}_a, \mathcal{Q}_\alpha^{1\pm}\right] = \frac{1}{2}(\Gamma^a)_\alpha^\beta \mathcal{Q}_\beta^{1,\pm}$$

$$[\mathcal{J}_a, P_b] = \epsilon_{abc}P^c \qquad \left[\mathcal{J}_a, \mathcal{Q}_\alpha^{2\pm}\right] = \frac{1}{2}(\Gamma^a)_\alpha^\beta \mathcal{Q}_\beta^{2\pm}$$

$$\{\mathcal{Q}_\alpha^{1,\pm}, \mathcal{Q}_\beta^{1,\mp}\} = -\frac{1}{2}(C\Gamma^a)_{\alpha\beta}P_a \mp \frac{1}{2}C_{\alpha\beta}\mathcal{S} \qquad [\mathcal{R}, \mathcal{Q}_\alpha^{1\pm}] = \pm\frac{1}{2}\mathcal{Q}_\alpha^{1\pm}$$

$$\{\mathcal{Q}_\alpha^{2,\pm}, \mathcal{Q}_\beta^{2,\mp}\} = -\frac{1}{2}(C\Gamma^a)_{\alpha\beta}P_a \pm \frac{1}{2}C_{\alpha\beta}\mathcal{S} \qquad [\mathcal{R}, \mathcal{Q}_\alpha^{2\pm}] = \mp\frac{1}{2}\mathcal{Q}_\alpha^{2\pm}$$

We also construct the supertrace elements:

$$< \mathcal{J}_a, P_b >= \eta_{ab}, \quad < \mathcal{Q}_\alpha^{1,2\pm}, Q_\beta^{1,2\mp} >= C_{\alpha\beta}, \quad < \mathcal{R}, \mathcal{S} >= -1$$

The gauge field is expanded in terms of the basis generators:

$$\mathcal{A} = e^a P_a + \omega^a \mathcal{J}_a + \sum_{\alpha=\pm} \psi_{\pm}^{1\alpha} \mathcal{Q}_{\alpha}^{1\pm} + \sum_{\alpha=\pm} \psi_{\pm}^{2\alpha} \mathcal{Q}_{\alpha}^{2\pm} + \upsilon \mathcal{R} + \sigma \mathcal{S}$$

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Here $e^a$ is the vielbein field, $\omega^a$ is the corresponding dual spin connection, $\psi^{1\alpha}_{\pm}, \psi^{2\alpha}_{\pm}$ are Majorana gravitini and $\upsilon, \sigma$ are internal gauge fields.

Using the form of the Chern-Simons action

$$I[\mathcal{A}] = \frac{k}{4\pi} \int \left\langle \mathcal{A}, d\mathcal{A} + \frac{2}{3}\mathcal{A}^2 \right\rangle$$

we can write it explicitly in terms of the generators and fields and show its invariance under supersymmetry.

Now we perform a change of basis to a new set of generators: $\{M_n, \mathcal{L}_n, q_\alpha^{1,2\pm}, \mathcal{R}, \mathcal{S}\}$ related to the old basis as:

$$M_n = P_a U_n^a, \quad \mathcal{L}_n = \mathcal{J}_a U_n^a, \quad q_\alpha^{1\pm} = \sqrt{2}\mathcal{Q}_\alpha^{1\pm}, \quad q_\alpha^{2\pm} = \sqrt{2}\mathcal{Q}_\alpha^{2\pm}$$

where $n, m = \{-1, 0, 1\}$ and $\alpha = \{\frac{1}{2}, -\frac{1}{2}\}$.

The metric also changes from $\eta_{ab}$ from $\gamma_{nm}$.

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In terms of these new generators the super Poincare algebra reads:

$$[\mathcal{L}_n, \mathcal{L}_m] = (n-m)\mathcal{L}_{n+m} \qquad \left[\mathcal{L}_n, q_\alpha^{1\pm}\right] = \left(\frac{n}{2} - \alpha\right) q_{n+\alpha}^{1\pm}$$

$$[\mathcal{L}_n, M_m] = (n-m)M_{n+m} \qquad \left[\mathcal{L}_n, q_\alpha^{2\pm}\right] = \left(\frac{n}{2} - \alpha\right) q_{n+\alpha}^{2\pm}$$

$$\{q_\alpha^{1,\pm}, q_\beta^{1,\mp}\} = M_{\alpha+\beta} \pm (\alpha-\beta)\mathcal{S} \qquad [\mathcal{R}, q_\alpha^{1\pm}] = \pm\frac{1}{2} q_\alpha^{1\pm}$$

$$\{q_\alpha^{2,\pm}, q_\beta^{2,\mp}\} = M_{\alpha+\beta} \mp (\alpha-\beta)\mathcal{S} \qquad [\mathcal{R}, q_\alpha^{2\pm}] = \mp\frac{1}{2} q_\alpha^{2\mp}$$

We now choose the gauge field in the boundary in the radial gauge

$$\mathcal{A} = b^{-1}(a + d)b, \qquad b = exp\left(\frac{r}{2}M_{-1}\right)$$

where now $a(u, \varphi) = a_\varphi d\varphi + a_u du$ with

$$\begin{aligned} a_u &= M_1 - \frac{1}{4}\mathcal{M}M_{-1} - \frac{i\rho}{2}\mathcal{S} \\ a_\varphi &= \mathcal{L}_{-1} - \frac{1}{4}\mathcal{M}\mathcal{L}_{-1} - \frac{1}{4}\mathcal{N}M_{-1} - \frac{i\phi}{2}\mathcal{S} - \frac{i\rho}{2}\mathcal{R} \\ &\quad -\frac{1}{4}\left(\Psi^1_+ q^{1+}_- - \Psi^1_- q^{1-}_-\right) + \frac{1}{4}\left(\Psi^2_+ q^{2+}_- - \Psi^2_- q^{2-}_-\right) \end{aligned}$$

where the various charges are functions of $u$ and $\varphi$.

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$$a_\varphi = \mathcal{L}_{-1} - \frac{1}{4}\mathcal{M}\mathcal{L}_{-1} - \frac{1}{4}\mathcal{N}M_{-1} - \frac{i\phi}{2}\mathcal{S} - \frac{i\rho}{2}\mathcal{R}$$
$$-\frac{1}{4}\left(\Psi^1_+ q^{1+}_- - \Psi^1_- q^{1-}_-\right) + \frac{1}{4}\left(\Psi^2_+ q^{2+}_- - \Psi^2_- q^{2-}_-\right)$$

where the various charges are functions of $u$ and $\varphi$.

The boundary gauge field so chosen satisfies the condition that it reproduces the asymptotically flat metric

$$ds^2 = \gamma_{nm}e^n e^m = \mathcal{M}du^2 - 2dudr + \mathcal{N}dud\varphi + r^2 d\varphi^2$$

Let us consider the most general gauge transformations:

$$\Lambda = \Upsilon^n \mathcal{L}_n + \xi^n M_n + +\lambda_R \mathcal{R} + \lambda_S \mathcal{S}$$
$$+ \zeta_+^{1\alpha} q_\alpha^{1+} + \zeta_-^{1\alpha} q_\alpha^{1-} + \zeta_+^{2\alpha} q_\alpha^{2+} + \zeta_-^{2\alpha} q_\alpha^{2-}$$

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Now we solve the equations of motion: $d\mathcal{A} + \frac{1}{2}[\mathcal{A}, \mathcal{A}] = 0$

and construct the set of gauge transformations which leave the aymptotic condition invariant: $\delta\mathcal{A} = d\Lambda + [\mathcal{A}, \Lambda]$

Then we calculate the variation of the charges using the relation:

$$\delta\mathcal{C}[\Lambda] = -\frac{k}{2\pi} \int < \Lambda, \delta\mathcal{A}_\varphi > d\varphi$$

and get the charge as

$$\mathcal{C} = -\frac{k}{4\pi} \int d\varphi \Big( \Upsilon \mathcal{J} + T\mathcal{M} + \Psi_+^1 \zeta_-^1 - \Psi_-^1 \zeta_+^1 \\ - \Psi_+^2 \zeta_-^2 + \Psi_-^2 \zeta_+^2 + i\lambda_{\mathcal{R}} \phi + i\lambda_{\mathcal{S}} \rho \Big)$$

where $\mathcal{N} = \mathcal{J}(\varphi) + u\delta_\varphi \mathcal{M}$.

Now from the relation $\delta_{\Lambda_1}\mathcal{C}[\Lambda_2] = \{\mathcal{C}[\Lambda_1], \mathcal{C}[\Lambda_2]\}$ we can calculate the Poisson brackets.

Before that let us define the modes as Fourier transforms of the fields:

$$\mathcal{J}_n = \frac{k}{4\pi}\int d\varphi e^{in\varphi}\mathcal{J}, \qquad \mathcal{M}_n = \frac{k}{4\pi}\int d\varphi e^{in\varphi}\mathcal{M},$$
$$\mathcal{R}_n = \frac{k}{4\pi}\int d\varphi e^{in\varphi}\phi, \qquad \mathcal{S}_n = \frac{k}{4\pi}\int d\varphi e^{in\varphi}\rho,$$
$$\Psi_r^{1,2\pm} = \frac{k}{4\pi}\int d\varphi e^{ir\varphi}\Psi^{1,2\pm}, \qquad [\Psi^{1,2\pm}\mathcal{S}]_r = \frac{k}{4\pi}\int d\varphi e^{ir\varphi}\Psi^{1,2\pm}\phi,$$
$$[\mathcal{S}\mathcal{S}]_\alpha = \frac{k}{4\pi}\int d\varphi e^{i\alpha\varphi}\phi\phi, \qquad \delta_{n,0} = \frac{1}{2\pi}\int d\varphi e^{in\varphi}.$$

The inverse Fourier transform is given, for example, as

$$\mathcal{J}(\varphi) = \frac{2}{k}\sum_n e^{-in\varphi}\mathcal{J}_n$$

In terms of modes, the Poisson brackets are:

$$i\{\mathcal{J}_n, \mathcal{J}_m\}_{PB} = (n-m)\mathcal{J}_{n+m} \qquad i\{\mathcal{R}_n, \mathcal{S}_m\}_{PB} = \frac{c_M}{12} n\delta_{n+m,0}$$

$$i\{\mathcal{J}_n, \mathcal{M}_m\}_{PB} = (n-m)\mathcal{M}_{n+m} + \frac{c_M}{12} n^3\delta_{n+m,0}$$

$$i\{\mathcal{J}_n, \Psi_r^{1,2\pm}\}_{PB} = \left(\frac{n}{2} - r\right)\Psi_{r+n}^{1,2\pm} \mp \frac{1}{4}\left[\Psi^{1,2\pm}\mathcal{S}\right]_{n+r}$$

$$i\{\mathcal{R}_n, \Psi_r^{1\pm}\}_{PB} = \pm\frac{1}{2}\Psi_{r+n}^{1\pm} \qquad i\{\mathcal{R}_n, \Psi_r^{2\pm}\}_{PB} = \mp\frac{1}{2}\Psi_{r+n}^{2\pm}$$

$$\{\Psi_r^{1+}, \Psi_s^{1-}\}_{PB} = \mathcal{M}_{r+s} + (r-s)\mathcal{S}_{r+s} + \frac{1}{4}[\mathcal{S}\mathcal{S}]_{r+s} + \frac{c_M}{6} r^2\delta_{r+s,0}$$

$$\{\Psi_r^{2+}, \Psi_s^{2-}\}_{PB} = \mathcal{M}_{r+s} - (r-s)\mathcal{S}_{r+s} + \frac{1}{4}[\mathcal{S}\mathcal{S}]_{r+s} + \frac{c_M}{6} r^2\delta_{r+s,0}$$

where $c_M = 12k$.

Note the presence of spurious non-linear terms in the $\{\mathcal{J}, \Psi\}_{PB}$ and $\{\Psi, \Psi\}_{PB}$ Poisson brackets!

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So we add a Sugawara-like term to $\mathcal{J}_n$:

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Finally, we also need to perform a shift in $\mathcal{M}_n$: (Justification for this will become clear later)

$$\hat{\mathcal{M}}_n = \mathcal{M}_n + \frac{1}{4}(\mathcal{S}\mathcal{S})_n$$

With these modifications all the non-linear terms get cancelled or absorbed in the modified generators.

Also the Poisson brackets of $\hat{\mathcal{J}}_n$ with $\mathcal{R}_m$ and $\mathcal{S}_m$ give the correct RHS.

In addition we get an extra non-zero Poisson bracket:

$$i\{\hat{\mathcal{M}}_n, \mathcal{R}_m\} = -m\mathcal{S}_{n+m}$$

Finally we quantize the algebra following the rule:

$$i\{,\}_{PB} \to [,] \quad \text{and} \quad \{,\}_{PB} \to \{,\}$$

and get the algebra in terms of (anti-)commutators:

# Final Algebra after Central Extension:

$$[\hat{\mathcal{J}}_n, \hat{\mathcal{J}}_m] = (n-m)\hat{\mathcal{J}}_{n+m} + \frac{c_J}{12}\, n^3 \delta_{n+m,0}$$

$$[\hat{\mathcal{J}}_n, \hat{\mathcal{M}}_m] = (n-m)\hat{\mathcal{M}}_{n+m} + \frac{c_M}{12}\, n^3 \delta_{n+m,0}$$

$$[\hat{\mathcal{J}}_n, \mathcal{R}_m] = -m\,\mathcal{R}_{n+m}\ , \qquad [\hat{\mathcal{J}}_n, \mathcal{S}_m] = -m\,\mathcal{S}_{n+m}$$

$$[\hat{\mathcal{M}}_n, \mathcal{R}_m] = -m\,\mathcal{S}_{n+m}, \qquad [\mathcal{R}_n, \Psi^{2\pm}_r] = \mp\tfrac{1}{2}\Psi^{2\pm}_{r+n}$$

$$[\hat{\mathcal{J}}_n, \Psi^{1,2\pm}_r] = (\frac{n}{2} - r)\Psi^{1,2\pm}_{r+n}, \qquad [\mathcal{R}_n, \Psi^{1\pm}_r] = \pm\tfrac{1}{2}\Psi^{1\pm}_{r+n}$$

$$[\mathcal{R}_n, \mathcal{R}_m] = \frac{c_J}{12}\, n\,\delta_{n+m,0}, \qquad [\mathcal{R}_n, \mathcal{S}_m] = \frac{c_M}{12}\, n\,\delta_{n+m,0}$$

$$\{\Psi^{1+}_r, \Psi^{1-}_s\} = \hat{\mathcal{M}}_{r+s} + (r-s)\mathcal{S}_{r+n} + \frac{c_M}{6}\, r^2 \delta_{r+s,0}$$

$$\{\Psi^{2+}_r, \Psi^{2-}_s\} = \hat{\mathcal{M}}_{r+s} - (r-s)\mathcal{S}_{r+n} + \frac{c_M}{6}\, r^2 \delta_{r+s,0}$$

# Energy Bound and Killing Spinors:

Considering all possible positive-definite combinations of the supercharges $\Psi^{1,2\pm}_{\pm\frac{1}{2}}$, we get the bound on the shifted charge $\hat{\mathcal{M}}_0$.

$$\hat{\mathcal{M}}_0 = \frac{1}{4}\sum_{i,\alpha}\left[\Psi^{i,+}_{\alpha}\Psi^{i,-}_{-\alpha} + \Psi^{i,-}_{-\alpha}\Psi^{i,+}_{\alpha}\right] - \frac{k}{2} \geq -\frac{k}{2} = -\frac{1}{8G}$$

We then found the asymptotic supersymmetries that preserve the asymptotically flat backgrounds.

Imposing that both the gravitinos and their generic variations are zero at infinity, we get the asymptotic Killing spinor equation, which (under some assumptions) has the solutions:

$$\zeta_+^i = e^{-i\frac{\rho}{4}} \left( c_1^i e^{\frac{\sqrt{\mathcal{M}}}{2}\varphi} + c_2^i e^{-\frac{\sqrt{\mathcal{M}}}{2}\varphi} \right)$$

$$\zeta_-^i = e^{i\frac{\rho}{4}} \left( d_1^i e^{\frac{\sqrt{\mathcal{M}}}{2}\varphi} + d_2^i e^{-\frac{\sqrt{\mathcal{M}}}{2}\varphi} \right)$$

for arbitrary $c_{1,2}^i$ and $d_{1,2}^i$ constatnt spinors ($i = 1, 2$).

The solutions are well-defined, given the periodicity of $\varphi$ only when $\mathcal{M} = -n^2$ and $n > 0$, a strictly positive integer without loss of generality.

For $n = 1, \rho = 0$ we find the Killing spinors for the Minkowski vacuum, $\mathcal{M} = -1$. For $n > 1$, the energy bound is violated and we have angular defect solutions.

# Global Killing Spinors:

These describe the globally defined supersymmetry transformations that leave the pure bosonic solutions in the asymptotic region invariant.

The global Killing spinor equation is given as

$$D\zeta^1_\pm = \left(d + \omega \pm \frac{1}{2}\upsilon\right)\zeta^1_\pm = 0$$

Now from the gauge fields $a_u$ and $a_\phi$ we can find the values of the spin connection and the R-gauge field.

Using these we can solve the above equation and get the final solution for the global Killing spinor:

$$\zeta^1_{\pm gen} = \Lambda^{-1}\tilde{\zeta}^1 e^{\mp i\frac{1}{2}\phi\varphi}$$

Here $\tilde{\zeta}^1$ is a constant spinor.

$$\Lambda = exp\left[\frac{1}{2}\left(\tilde{\Gamma}_{+1} - \frac{\mathcal{M}}{4}\tilde{\Gamma}_{-1}\right)\varphi\right]$$

$$\tilde{\Gamma}_{-1} = \begin{pmatrix} 0 & -2 \\ 0 & 0 \end{pmatrix} \qquad \tilde{\Gamma}_{1} = \begin{pmatrix} 0 & 0 \\ 2 & 0 \end{pmatrix}$$

Like the asymptotic case, the Killing spinors are globally well-defined when $\mathcal{M} = -n^2$, with $n$ being a positive integer.

# Asymptotic Analysis of $AdS_3$ :

# Asymptotic Analysis of $AdS_3$ :

There are two inequivalent locally supersymmetric extensions of $AdS_3$ theories containing an R-symmetry: the $(2,0)$ and the $(0,2)$ theories.

We will denote these as the unbarred and barred sectors respectively.

The action is a functional of two independent connections:

$$I = I[A_+] + [\bar{A}_-]$$

Here we have defined $x^{\pm} = \frac{u}{l} \pm \varphi$, where $l$ is the identical AdS radius in both the sectors.

The $(2,0)$ sector asymptotically only depends on $x^+$ and the $(0,2)$ sector on $x^-$.

The global generators for the unbarred sector $L_n, \mathcal{Q}^{\pm}_{\alpha}$ and $R$ $\left(a = 0, 1, 2; \alpha = \pm\frac{1}{2}\right)$ and satisfy the algebra:

$$[L_n, L_m] = (n-m)L_{n+m} \qquad [L_n, \mathcal{Q}^{\pm}_{\alpha}] = \left(\frac{n}{2} - 1\right)\mathcal{Q}^{\pm}_{n+\alpha}$$

$$\{\mathcal{Q}^{+}_{\alpha}, \mathcal{Q}^{-}_{\beta}\} = L_{\alpha+\beta} + (\alpha-\beta)R \qquad \{\mathcal{Q}^{\pm}_{\alpha}, \mathcal{Q}^{\pm}_{\beta}\} = 0$$

and the supertrace elements:

$$\langle L_n, L_m\rangle = \frac{1}{2}\gamma_{nm}, \qquad \left\langle \mathcal{Q}^{\pm}_{\alpha}, \mathcal{Q}^{\mp}_{\beta}\right\rangle = C_{\alpha\beta}, \qquad \langle R, R\rangle = -\frac{1}{2}$$

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The barred sector satisfies exactly same algebra as the unbarred sector while its supertrace elements are equal but opposite in sign.

The asymptotic behaviours of the gauge fields can be taken as:

$$A_+ = \frac{dr}{2l} L_{-1} +$$
$$\left( L_1 + \frac{r}{l} L_0 + \frac{r^2}{4l^2} L_{-1} - \frac{1}{2} \mathfrak{L}_+ L_{-1} - \frac{1}{2} \psi_+ \mathcal{Q}_-^+ + \frac{1}{2} \psi_- \mathcal{Q}_-^- - i\phi_R^A R \right) dx^+$$
$$\bar{A}_- = \frac{dr}{2l} \bar{L}_1 +$$
$$\left( \bar{L}_{-1} - \frac{r}{l} \bar{L}_0 + \frac{r^2}{4l^2} \bar{L}_1 - \frac{1}{2} \mathfrak{L}_- \bar{L}_{-1} - \frac{1}{2} \bar{\psi}_+ \bar{\mathcal{Q}}_+^+ + \frac{1}{2} \bar{\psi}_- \bar{\mathcal{Q}}_+^- - i\bar{\phi}_R^A \bar{R} \right) dx^-$$

and the transformation parameters as:

$$\Lambda_+ = \chi^n L_n + \epsilon_+^\alpha \mathcal{Q}_\alpha^+ + \epsilon_-^\alpha \mathcal{Q}_\alpha^- + \lambda_R^A R$$
$$\Lambda_- = \bar{\chi}^n \bar{L}_n + \bar{\epsilon}_+^\alpha \bar{\mathcal{Q}}_\alpha^+ + \bar{\epsilon}_-^\alpha \bar{\mathcal{Q}}_\alpha^- + \bar{\lambda}_R^A \bar{R}$$

Following a similar procedure as in the BMS case, we get the Poisson brackets:

$$i\{\mathfrak{L}^+_n, \mathfrak{L}^+_m\}_{PB} = (n-m)\mathfrak{L}^+_{n+m} + \frac{c}{12} n^3 \delta_{n+m,0}$$

$$i\{\mathfrak{L}^+_n, \psi^\pm_\alpha\}_{PB} = \left(\frac{n}{2} - \alpha\right) \psi^\pm_{\alpha+n} \mp \frac{1}{2} \left[\psi^\pm R\right]_{n+\alpha}$$

$$i\{R_n, \psi^\pm_\alpha\}_{PB} = \pm \frac{1}{2} \psi^\pm_{\alpha+n} \qquad i\{R_n, R_m\}_{PB} = \frac{c}{12} n\, \delta_{n+m,0}$$

$$\{\psi^+_\alpha, \psi^-_\beta\}_{PB} = \mathfrak{L}^+_{\alpha+\beta} + (\alpha-\beta) R_{\alpha+\beta} + \frac{1}{2}[RR]_{\alpha+\beta} + \frac{c}{6}\alpha^2 \delta_{\alpha+\beta,0}$$

where $c = 6k_I$ and $k_I$ is the Chern-Simons level.

As before, the modes are defined, for example, as

$$[RR]_n = \frac{k_I}{4\pi} \int d\varphi e^{in\varphi} \phi^A_R \phi^A_R(\varphi)$$

Again for similar reasons as in BMS, we need to redefine

$$\mathfrak{L}_n^+ \to \hat{\mathfrak{L}}_n^+ = \mathfrak{L}_n + \frac{1}{2}(RR)_n$$

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$$\mathfrak{L}_n^+ \to \hat{\mathfrak{L}}_n^+ = \mathfrak{L}_n + \frac{1}{2}(RR)_n$$

After quantizing, we finally get the algebra:

$$[\hat{\mathfrak{L}}_n^+, \hat{\mathfrak{L}}_m^+] = (n-m)\hat{\mathfrak{L}}_{n+m}^+ + \frac{c}{12} n^3 \delta_{n+m,0}$$

$$[\hat{\mathfrak{L}}_n^+, \psi_\alpha^\pm] = \left(\frac{n}{2} - \alpha\right) \psi_{\alpha+n}^\pm \qquad [\hat{\mathfrak{L}}_n^+, R_m] = -m\, R_{n+m}$$

$$[R_n, \psi_\alpha^\pm] = \pm\frac{1}{2}\psi_{\alpha+n}^\pm \qquad [R_n, R_m] = \frac{c}{12} n\, \delta_{n+m,0}$$

$$\{\psi_\alpha^+, \psi_\beta^-\} = \hat{\mathfrak{L}}_{\alpha+\beta}^+ + (\alpha-\beta) R_{\alpha+\beta} + \frac{c}{6}\alpha^2 \delta_{\alpha+\beta,0}$$

Performing a similar analysis for the barred sector, we get an exact copy of the above algebra (with $\bar{c} = 6k_l$) where the Sugawara shift is now

$$\mathfrak{L}_n^- \to \hat{\mathfrak{L}}_n^- = \mathfrak{L}_n^- + \frac{1}{2}(\bar{R}\bar{R})_n$$

The only difference is that for the barred sector the Fourier modes are defined according to the convention

$$[\bar{R}\bar{R}]_n = \frac{k_l}{4\pi}\int d\varphi e^{-in\varphi}\phi_R^A\phi_R^A(\varphi)$$

This difference in sign in the Fourier transform is essential in deriving the BMS algebra by contracting the two copies of AdS algebras.

# $N = 4$ Super-BMS$_3$ from $N = (2,2)$ Super-AdS$_3$

Contraction of two copies of super-conformal algebra gives us the super-Poincare algebra.

The contraction is defined in the large AdS radius limit $l \to \infty$.

The level of the corresponding Chern-Simons actions are related as $k_l = k.l$.

The generators are related as

$$\mathcal{L}_n = L_n - \bar{L}_{-n}, \qquad M_n = \frac{L_n + \bar{L}_{-n}}{l}, \qquad \mathcal{R} = R - \bar{R}$$

$$q_\alpha^{1\pm} = \sqrt{\frac{2}{l}}\mathcal{Q}_\alpha^\pm, \qquad q_\alpha^{2\pm} = \sqrt{\frac{2}{l}}\bar{\mathcal{Q}}_{-\alpha}^\pm, \qquad \mathcal{S} = \frac{R + \bar{R}}{l}$$

The asymptotic gauge field and the gauge transformation parameters of the flat theory is obtained from the AdS ones in the limit $l \to \infty$

$$\mathcal{A} = A_+ + A_-, \qquad \Lambda = \Lambda_+ + \Lambda_-$$

We further need to contract the various parameters and charges.

Finally we can recover the $N = 4$ $\mathrm{BMS}_3$ algebra from the two identical copies of the asymptotic $(2, 0)$ and $(0, 2)$ $\mathrm{AdS}_3$ algebras with the identification:

$$\mathcal{J}_m = \lim_{\epsilon \to 0} (\mathfrak{L}^+_m - \mathfrak{L}^-_{-m}) \qquad \mathcal{M}_m = \lim_{\epsilon \to 0} \epsilon (\mathfrak{L}^+_m + \mathfrak{L}^-_{-m})$$

$$\mathcal{R}_m = \lim_{\epsilon \to 0} \epsilon (R_m - \bar{R}_{-m}) \qquad \mathcal{S}_m = \lim_{\epsilon \to 0} (R_m + \bar{R}_{-m})$$

$$\Psi^{1,\pm}_r = \lim_{\epsilon \to 0} \sqrt{\epsilon} \psi^\pm_r \qquad \Psi^{2,\pm}_r = \lim_{\epsilon \to 0} \sqrt{\epsilon} \bar{\psi}^\pm_{-r}$$

$$c_J = \lim_{\epsilon \to 0} (c - \bar{c}) \qquad c_M = \lim_{\epsilon \to 0} \epsilon (c + \bar{c})$$

where $\epsilon = \frac{1}{l}$.

This then gives the final $\mathrm{SBMS}_3$ algebra that we had obtained before, complete with the Sugawara shifts.

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Note that the Sugawara shifts in the two AdS sectors, $\hat{\mathfrak{L}}_+ = \mathfrak{L}_+ + \frac{1}{2}\phi_A^2$ and $\hat{\mathfrak{L}}_- = \mathfrak{L}_- + \frac{1}{2}\bar{\phi}_A^2$, when written in terms of the BMS fields, gives the correct Sugawara shifts of both $\mathcal{J}$ and $\mathcal{M}$ in the limit $l \to \infty$. For example,

$$\mathcal{M} = (\mathfrak{L}_+ + \mathfrak{L}_-) = (\hat{\mathfrak{L}}_+ + \hat{\mathfrak{L}}_-) - \frac{1}{2}(\phi_A^2 + \bar{\phi}_A^2)$$
$$= \hat{\mathcal{M}} - \frac{1}{4}\left(\rho^2 + (\phi/l)^2\right)$$

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# Conclusions:

- We constructed extended supersymmetric BMS algebra using Inonu-Wigner contraction of extended superconformal algebra.
- We showed that only asymmetric scaling of bosonic and symmetric scaling of fermionic generators give correct extended super BMS algebra.
- We were able to reproduce the correct BMS algebra by an asymptotic symmetry analysis using a suitable set of boundary conditions.
- We repeated the same analysis for AdS space and again obtained the BMS algebra as a correct contraction of two copies of AdS algebras.

# Current work and Future Directions:

Note that in the final AdS algebra we did not get the non-linear terms that appears explicitly in the paper mentioned before!

It appears that the case we considered is a very special representation in which the non-linear terms get cancelled/reabsorbed after the Sugawara shifts.

We have thereafter done the analysis for more generic representations and found the non-linear terms survive even after the Sugawara shifts (for both AdS and BMS).

Non-linear terms will also change the energy bounds and will have other effects.

We also want to apply our results on cosmological solutions.

Thank You!

# Appendix:

$$\Gamma_0 = i\sigma_2, \qquad \Gamma_1 = \sigma_1, \qquad \Gamma_2 = \sigma_3$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\gamma_{nm} = \begin{pmatrix} 0 & 0 & -2 \\ 0 & 1 & 0 \\ -2 & 0 & 0 \end{pmatrix}$$

$$C_{\alpha\beta} = C^{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$U_n^a = \begin{pmatrix} -1 & 0 & -1 \\ -1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$