Universality in random structures: Interfaces, Matrices, Sandpiles

Titles and Abstracts for Week 3, 28 January -01 February, 2019
Focus Area: In and around the KPZ Universality Class

Mini-Course by **Daniel Remenik**, University of Chile

Title : The KPZ fixed point

Abstract : The Kardar-Parisi-Zhang (KPZ) universality class is a broad class of models coming from mathematical physics which includes random interface growth, directed random polymers, interacting particle systems, and random stirred fluids. These models share a very special and rich asymptotic fluctuation behavior, which is loosely characterized by fluctuations which grow like t^{1/3} as time t evolves, decorrelate at a spatial scale of t^{2/3}, and have certain very special limiting distibutions; this fluctuation behavior is model independent but depends on the initial data, and in some important cases it is connected with distributions coming from random matrix theory.

A somewhat vague conjecture in the field was that there should be a universal, scaling invariant limit for all models in the KPZ class, containing all the fluctuation behavior seen in the class. In these lectures I will describe joint work with K. Matetski and J. Quastel [5] where we were able to construct and give a complete description of this limiting process, known as the KPZ fixed point. This limiting universal process is a Markov process, taking values in real valued functions which look locally like Brownian motion.

The construction follows from a novel exact solution for one of the most basic models in the KPZ class, the totally asymmetric exclusion process (TASEP), for arbitrary initial condition. This formula is given as the Fredholm determinant of a kernel involving the transition probabilities of a random walk forced to hit a curve defined by the initial data, and in the KPZ 1:2:3 scaling limit the formula leads in a transparent way to a Fredholm determinant formula given in terms of analogous kernels based on Brownian motion.

Two sets of lecture notes [6,7] serve as a good complement to the mini-course.

References:

1. A. Borodin, P. L. Ferrari, M. Prähofer, and T. Sasamoto. Fluctuation properties of the TASEP with periodic initial configuration. J. Stat. Phys. 129.5-6 (2007), pp. 10551080.
2. A. Borodin, I. Corwin, and D. Remenik. Multiplicative functionals on ensembles of non-intersecting paths. Ann. Inst. H. Poincaré Probab. Statist. 51.1 (2015), pp. 28–58.
3. I. Corwin, J. Quastel, and D. Remenik. Continuum statistics of the Airy 2 process. Comm. Math. Phys. 317.2 (2013), pp. 347–362.
4. J. Quastel, D. Remenik. How flat is flat in random interface growth? To appear in Trans. AMS. arXiv:1606.09228.
5. K. Matetski, J. Quastel and D. Remenik. The KPZ fixed point. arXiv:1701.00018.
6. K. Matetski, J. Quastel. From TASEP to the KPZ fixed point. arXiv:1710.02635.
7. D. Remenik. Course notes on the KPZ fixed point.
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Titles and Abstracts for Other Speakers

1. **Marc Brachet, Durham University.**
   Title : Crossover of equilibrium correlation time for the 1D Galerkin-truncated Burgers equation in the limit of vanishing noise and dissipation
   Abstract : Classical Galerkin-truncated systems have been studied since the early 50's in fluid mechanics [T.D. Lee, Quart Appl Math 10, 69 (1952)]. These spatially-periodic classical ideal fluids are known to admit, when spectrally truncated at wavenumber kmax, absolute equilibrium solutions with Gaussian statistics and equipartition of kinetic energy among all Fourier modes.
   The scaling of the correlation time around thermal equilibrium for the 1D Galerkin-truncated Burgers equation with equilibrium forcing is shown to display, as expected, a crossover from Edwards-Wilkinson to KPZ scaling when forcing and dissipation are jointly decreased. A new crossover to a third regime is characterized in the inviscid limit of vanishing forcing and dissipation.

2. **Abhishek Dhar, ICTS**.
   Title: Hydrodynamics and chaos in spin chains: connections to KPZ.
   Abstract: The first part of the talk will discuss the predictions of nonlinear fluctuating hydrodynamics for a one-dimensional chain of spins, described by the XXZ Hamiltonian and evolving with Hamiltonian dynamics. One of the interesting features, at low temperatures, is the emergence of "almost" conserved quantities leading to sound modes with KPZ scaling. Numerical results on equilibrium correlation functions, to check the predictions of the theory, will be presented. In the second part we discuss a different quantity which quantifies the chaotic spread and growth of localized perturbations. It is shown that this can be effectively described as a growing one-dimensional interface with KPZ scaling.

3. **Patrik Ferrari, University of Bonn.**
   Title : Time-time covariance for last passage percolation with generic initial profile
   Abstract : We consider time correlation for KPZ growth in 1+1 dimensions in a neighborhood of a characteristics. We prove convergence of the covariance with droplet, flat and stationary initial profile. In particular, this provides a rigorous proof of the exact formula of the covariance for the stationary case obtained in [SIGMA 12 (2016), 074]. Furthermore, we prove the universality of the first order correction when the two observation times are close and provide a rigorous bound of the error term. This result holds also for random initial profiles which are not necessarily stationary.

4. **Christopher Hoffman, University of Washington.**
   Title : Bigeodesics in fist and last passage percolation
   Abstract : First passage percolation is a model in statistical physics of random growth. A longstanding question, due to Furstenberg, is whether there are bi-infinite geodesics. This question is of interest to physicists due to its connections with ground states of the Ising model. We will discuss recent progress on this question. We will also show that in the related model of last passage percolation that there are no non-trivial bi-infinite geodesics a.s. This is based on joint works with Daniel Ahlberg, Riddhipartum Basu and Allan Sly.

5. **Manas Kulkarni, ICTS**.
Title : Connections between Classical Calogero-Moser, Log Gas and Random Matrix Theory
Abstract : We present a deep connection between the classical Calogero-Moser (CM) model, Log-gas (LG) model and Random Matrix Theory (RMT). We show that CM model has some remarkable connections with the 1D LG model. Both models have the same minimum energy configuration with the particle positions given by the zeros of the Hermite potential. Moreover the Hessian describing small oscilla tions around equilibrium are also related for the two models. We explore this connection further by studying finite temperature equilibrium properties of the CM model through Monte-Carlo simulations and comparing them with known LG results. In particular, our findings indicate that the single particle distribution and the marginal distribution of the boundary particle of CM model are also given by Wigner semi-circle and the Tracy-Widom distribution respectively (similar to LG model). Comparisons are made with analytical predictions from the small oscillation theory and we find very good agreement. Parallels are also drawn with rigorous mathematical results from RMT and implications of finite-size as well as finite-temperature effects are observed. We also present some preliminary results on large deviations in CM model by using field theory.

6. **Rahul Pandit, IISc.**
Title : Universal properties of the spatiotemporally chaotic state of the one-dimensional Kuramoto-Sivashinsky equation
Abstract : The spatiotemporally chaotic state of the one-dimensional Kuramoto-Sivashinsky (KS) equation is often characterised by height correlations. There is compelling numerical evidence that the long-distance and long-time behaviours of these height correlation functions is in the Kardar-Parisi-Zhang (KPZ) universality class. We use extensive direct numerical simulations to show that this spatiotemporally chaotic state of the KS equation also displays Tracy-Widom and related distributions that are now well known for the one-dimensional KPZ equation.
This work has been done with Dipankar Roy at the Indian Institute of Science

7. **Leandro Pimentel, UFRJ.**
Title : Ergodicity of the KPZ Fixed Point
Abstract : The Kardar-Parisi-Zhang (KPZ) fixed point is a Markov process in the space of upper semi-continuous functions, introduced recently by Matetski, Quastel and Remenik (2017). It describes the limit fluctuations of the height function associated to the totally asymmetric simple exclusion process (TASEP), and it is conjectured to be the limit fluctuations of a wide class of 1+1 interface growth process (KPZ universality class). Our main result is that the KPZ fixed point centred at the origin converges in distribution, as time goes to infinity, to a two-sided Brownian motion with zero drift and diffusion coefficient 2. The heart of the proof is the coupling method, that allows us to compare the TASEP height function started from a perturbation of density 1/2 with its invariant counterpart.

8. **Sunder Sethuraman, University of Arizona**
Title : On Hydrodynamic Limits of Young Diagrams
Abstract : We consider a family of stochastic models of evolving two-dimensional Young diagrams, given in terms of certain energies, with Gibbs invariant measures. `Static' scaling limits of the shape functions, under these Gibbs measures, have been shown by several over the years. In this talk, we discuss corresponding `dynamical' limits which are less understood. We show that the hydrodynamic scaling limits of the diagram shape functions may be described by different types parabolic PDEs, depending on the energy structure.

9. **Vladas Sidoravicius, NYU Shanghai.**
   Title : Random walks in growing domains - recurrence vs transience
   Abstract : We will discuss set of models where the random walk (or Brownian motion) moves in a restricted domain D(t), which itself evolves in time. This evolution could be independent of random walk evolution, but still affecting its motion, or evolution of D itself could be affected by random walk.
   Then I will focus on the phase transition (recurrence vs transience) for once reinforced random walks and some other self interacting processes.

10. **Allan Sly, Princeton University.**
    Title : The Slow Bond Model with Small Perturbations
    Abstract : The slow bond model is the totally asymmetric simple exclusion process (TASEP) in which particles cross the edge at the origin at rate 1-\epsilon rather than at rate 1. Janowsky and Lebowitz asked if there was a global slowdown in the current for all epsilon > 0. Using a range of of theory and simulations two groups of physicists came to opposing conclusions on this question. With Basu and Sidoravicius, this was settled establishing that there is a slowdown for any positive epsilon. In the current work we illuminate reason that this problem was difficult to resolve using simulations by analysing the effect of the perturbation at epsilon tends to 0 and showing it decays faster than any polynomial. Joint work with Lingfu Zhang and Sourav Sarkar.

11. **Herbert Spohn, TU Munich**.
    Title : The H_{XXZ} line ensemble and KPZ universality
    Abstract : The 2D statistical mechanics of the XXZ chain is a system of nonintersecting random walks, where Δ regulates the interaction between the lines (Δ = 0 is free fermion, Δ > 0 attractive, Δ < 0 repulsive). KPZ fluctuations are expected to show at the stochastic line and at facet edges. We discuss earlier results and explain more recent progress to establish such behavior. This is joint work with Michael Praehofer.